NOTA 1077 II augustus 1978

Instituut voor Cultuurtechniek en Waterhuishouding
Wageningen

A NUMERICAL MODEL FOR NON-STATIONARY SATURATED GROUNDWATER FLOW IN A MULTI-LAYERED SYSTEM

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C O N T E N T S

blz.

I. INTRODUCTION

The groundwater system is part of the hydrological cycle and changes in this system affects phenomena such as plant production, river flow etc. Generally one tries to use groundwater resources in such a way that the utility for the community as a whole is maximal.

Numerical flow models can be valuable tools in the management of water resources. In many situations these models are the only possible way to indicate effects of certain interferences in the groundwater system.

In this paper a numerical model is presented which simulates non-stationary saturated groundwater flow in a certain region.

Purpose of the model is to predict effects of certain operations upon the various terms of the water balance of the groundwater basin and on the hydraulic head.

The basic idea of the model is that groundwater is flowing horizontally in waterbearing layers and vertically in less-permeable layers. Each layer is discretized into a number of elements. To each element both Darcy's law and the law of continuity are applied (chapter II).

In this way a set of equations is obtained, which are solved using a finite element method and the Gauss-Seidel iteration method (chapter III).

In chapter IV the input and output of the model are treated, while in chapter V some numerical experiments are presented.

Finally in chapter VI an evaluation of the model is given.

II. GENERAL DESCRIPTION OF THE PROBLEM

2.1. Physical background

Throughout this report water potential is expressed on a unit weight basis and is being called hydraulic head, with symbol h.

$$h = \frac{p}{\rho \cdot g} + z \qquad [L]$$

where

$\frac{p}{\rho g}$ = piezometer head [L]

z = gravitational head [L]

The hydraulic head is an expression for the capacity of a unit weight of water to do work as compared to the work capacity of the same weight of water with the same chemical composition at a certain reference level and under atmosferic pressure.

The problem is restricted to saturated flow of groundwater with one constant density.

2.1.1. The law of linear resistance (Darcy's law)

According to Darcy's law the rate of flow through a porous medium is proportional to the gradient in hydraulic head. It may be written as

$$q_x = -k_y \frac{\delta h}{\delta x}, \quad q_y = -k_y \frac{\delta h}{\delta y}, \quad q_z = -k_z \frac{\delta h}{\delta z} \qquad (1)$$

where

$q_{x,y,z}$ = volume flux of water passing through a unit area per per unit time, perpendicular to x, y, z direction, respectively $[L.T^{-1}]$

$k_{x,y,z}$ = hydraulic conductivity in x, y and z direction, respectively $[L.T^{-1}]$

The coefficient k in the Darcy flow equation is taken to be a constant, depending on both the properties of the porous medium and the fluid.

2.1.2. The law of continuity

The other important physical law states that no matter can be lost or created.

The conservation equation can be found by applying the principle of continuity to an infinite small volume

$$- \frac{\delta \Theta}{\delta t} = \frac{\delta v_x}{\delta x} + \frac{\delta v_y}{\delta x} + \frac{\delta v_z}{\delta z} + q \qquad (2)$$

where

Θ = volume of water per unit bulk volume of soil [-]
q = sink or source term $[T^{-1}]$

2.1.3. Combination of Darcy's law and the law of continuity

Darcy's law combined with the law of continuity gives:

$$- \frac{\delta \Theta}{\delta t} = \frac{\delta}{\delta x} (k_x \frac{\delta h}{\delta x}) + \frac{\delta}{\delta y} (k_y \frac{\delta h}{\delta y}) + \frac{\delta}{\delta z} (k_z \frac{\delta h}{\delta z}) + q \qquad (3)$$

or

$$- S_c \frac{\delta h}{\delta t} = \frac{\delta}{\delta x} (k_x \frac{\delta h}{\delta x}) + \frac{\delta}{\delta y} (k_y \frac{\delta h}{\delta y}) + \frac{\delta}{\delta z} (k_z \frac{\delta h}{\delta z}) + q \qquad (4)$$

where

S_c = volume of water released or stored per unit bulk volume of soil per unit change in hydraulic head $[L^{-1}]$

2.2. Hydraulic properties and types of aquifers

The flow in a natural groundwater basin takes place in layers with different hydraulic properties. Usually the layers are divided into good permeable layers or waterbearing layers (aquifers) and less permeable layers (aquitards).

The most important hydraulic properties are:

a. Transmissiviy - the product of the average hydraulic conductivity k and the saturated vertical thickness d of the waterbearing layer.
   It is a measure for the transport capacity of aquifers.
   Notation: $T = kd$ $[L^2.T^{-1}]$
b. Hydraulic resistance - the ratio of vertical thickness of aquitards d^1 and hydraulic conductivity k^1. It is a measure for the resistance capacity of aquitards, denoted as $c = d^1/k^1$ $[T]$
c. Specific yield - the volume of water released or stored in the phreatic zone per unit surface area of an unconfined aquifer per unit change in hydraulic head.
   It is also called effective porosity.
   The symbol used for specific storage will be S_y $[-]$.
d. Specific storage - the volume of water released or stored per unit volume of the saturated parts of an aquifer or aquitard per unit change in hydraulic head. It is a measure for the elasticity of the soil material and the fluid. Excluded are irreversible processes. Notation: S_s $[L^{-1}]$.
e. Storage coefficient - the volume of water released or stored per unit surface of a layer per unit change in hydraulic head. It is the sum of specific yield and the product of specific storage and thickness of the aquifer or aquitard:

$$S_c = S_y + S_s d \quad [-]$$

Note: in confined layers $S_y = 0$.

In fig. 1 a 3-layered configuration is given of two aquifers separated by an aquitard. Aquifer I has a free water surface and is an unconfined aquifer. Aquifer II is a completely saturated aquifer. Dependent on the hydraulic resistance of the aquitard, KRUSEMAN and DE RIDDER (1976) distinguish between confined, semi-confined and semi-unconfined aquifers. For our purpose it is not useful to make this distinction. We restrict the aquifer types to unconfined and confined aquifers.

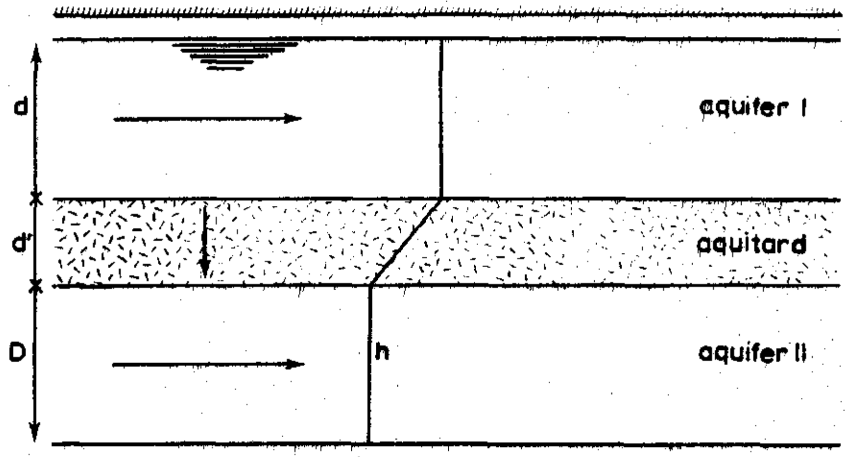


Fig. 1. Schematic representation of flow direction and variation of hydraulic head h in a 3-layered system

In an unconfined aquifers (I), the vertical thickness of the waterbearing layer, d, is a variable, while in a confined aquifer II, this is a constant (denoted as D). Other important properties of unconfined aquifers are the hydraulic head being the same as the phreatic surface and the specific yield being different from zero.

2.3. Schematization of flow

The groundwater flow in a groundwater basin is schematized into a horizontal flow in aquifers and into a vertical flow in aquitards. Consequently the vertical variation of the hydraulic head is as drawn in fig.1.

The validity of this assumption has been investigated by NEUMAN and WITHERSPOON (1969). The found that if the contrast in hydraulic conductivity between two adjacent layers is bigger than a factor two, the relative error caused by this assumption is usually smaller than 5%.

This schematization gives a simplification of the general flow equation (eq. 4). In an aquifer the vertical is zero and eq. (4) then reduces to

$$-S_c \frac{\delta h}{\delta t} = \frac{\delta}{\delta x} (k_x \, d \, \frac{\delta h}{\delta x}) + \frac{\delta}{\delta y} (k_y \, d \, \frac{\delta h}{\delta y}) + qd$$

or

$$-(S_y + S_s d) \frac{\delta h}{\delta t} = \frac{\delta}{\delta x} (k_x d \frac{\delta h}{\delta x}) + \frac{\delta}{\delta y} (k_y d \frac{\delta h}{\delta y}) + Q \quad (5)$$

where

Q = volume of water substracted from or added to unit surface of aquifer per unit time $[L.T^{-1}]$

In an aquitard, eq. (4) reduces to

$$-S_s \frac{\delta h}{\delta t} = \frac{\delta}{\delta z} (k_z \frac{\delta h}{\delta z}) \quad (6)$$

As will be discussed in the next paragraph, the term q in an aquitard is set equal to zero.

2.4. Water balance terms

In eq. (4) the term q is mentioned. With this sink or source term different terms of the water balance are taken into account. Fig. 2 depicts schematically the situation for an unconfined aquifer (for other layers some terms don't exist).

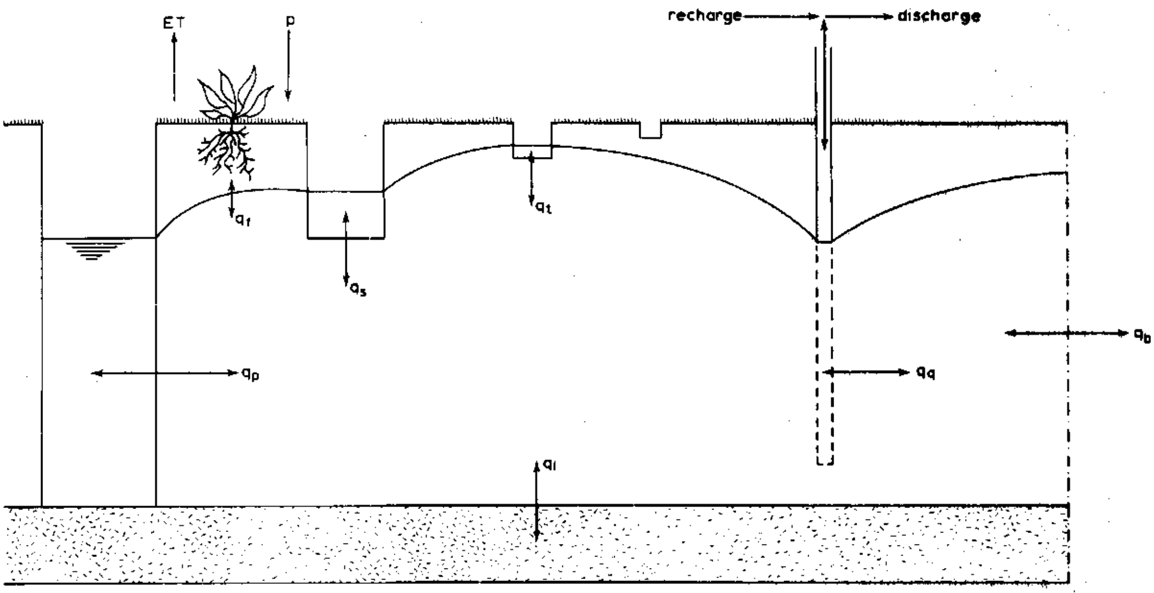


Fig. 2. Schematic representation of the various terms involved in the water balance

Description of the different terms:

a. The flow through the phreatic surface, q_f.
If this term is negative (water goes out of the saturated system), it is called capillary rise. If positive it is called percolation (or drainage).
This flow can be calculated as a rest term of the water balance of the unsaturated zone. Very often this term is expressed as a flow per unit surface (flux) and is called effective precipitation. For the time being this flux is a given input.

b. Flow to adjacent aquitards, q_l.
The flux to adjacent aquitards can be calculated by applying eq. (6) to the aquitard.

c. Flow to the primary surface water system, q_p.
The primary surface water system penetrates fully the groundwater basin. So it acts as a boundary with given values for the hydraulic head (is equal to the phreatic level of the primary system).

d. Flow to the secondary surface water system, q_s.
The secondary surface water system does not penetrate fully the groundwater basin, which causes a resistance against flow between groundwater and surface water. The flow q_s can be calculated as follows

$$q_s = \frac{h - h_{f,s}}{R} \qquad (7)$$

where

h = hydraulic head of the groundwater at the place of the secondary system [L]
$h_{f,s}$ = phreatic level of secondary system [L]
R = radial and entrance resistance of the secondary system, per unit surface [T]

According to ERNST (1962) the radial resistance Ω per unit surface, can be calculated with:

$$\Omega = \frac{1}{\pi k} \ln \frac{d}{B} \tag{8}$$

where

l	= area drained by 1 m conduit	[L]
d	= thickness of the layer near the conduit	[L]
B	= wet perimeter of the conduit	[L]

The entrance resistance is caused by a layer which shows a lower conductivity around the conduit than its surroundings. In practise, radial and entrance resistancies are difficult to deduct from hydraulic properties. They can be found indirectly by measuring simultaneously q, h and $h_{f,s}$.

e. Flow to the tertiary surface water system, q_t.
In smaller conduits usually the phreatic level is not known. So eq. (7) can't be applied. The flow to these conduits, however, is sometimes very important (e.g. in areas with shallow groundwater tables). One may assume (ERNST, 1978) that a relationship exists between the hydraulic head of an aquifer and the flow, q_t. According to Ernst the tertiary system can be divided into a number of categories with respect to their mutual distance and bottom height. For each category the flow is

$$q(\bar{h},b) = \frac{\bar{h} - h_f(b)}{\alpha \, \underline{\gamma}} \tag{9}$$

where:

$\bar{h}$	= average hydraulic head	[L]
$h_f(b)$	= phreatic level in the category under consideration; a function of bottom height	[L]
α	= geometry factor to convert h_m into $\bar{h}$	[-]
$\underline{\gamma}$	= drainage resistance	[T]
h_m	= hydraulic head midway between two conduits	[L]

Summation of the $q(\bar{h},b)$-relations of the different categories yields the overall relation between hydraulic head and flow to the tertiary system we are looking for.

f. Artificial recharge or discharge, q_a, and prescribed boundary flux, q_b.
   The magnitudes of these flows are not influenced by the conditions inside the groundwater basin but remain on a prescribed level. Thus q_a is an internal boundary condition, q_b an external boundary condition.

For confined aquifers some of the above mentioned terms don't exist. These terms are the flow through the phreatic surface, q_f and the flow to the tertiary surface water system, q_t.

Aquitards are considered only as transmitters of water from one aquifer to another, with possibilities of storage (eq. 6). This means that for aquitards the water balance terms a. through f. are set equal to zero.

2.5. Solution of groundwater flow problems

The basic equations for saturated groundwater flow are eq. (5) for aquifers and eq. (6) for aquitards. The solution of these equations require knowledge of the physical system and the auxiliary conditions describing the system constraints.

These auxiliary conditions are (REMSON et.al., 1971)

a. Geometry of the system.
   E.g. the diviation of the system into a restricted number of layers and the horizontal extension of the system under consideration.

b. The matrix of hydraulic properties, including in-homogeneity and an-isotropy.

c. Initial conditions. Because we are dealing with non-stationary flow, the values of pertinent system variables (such as hydraulic head at the initial time) must be known.

d. Boundary conditions. They describe the conditions at the boundaries of the system as a function of time.
   One can distinguish between:
   - hydraulic head boundary

- flow (including zero flow) boundary
- both head and flow boundary.

After specifying the auxiliary conditions an unique solution of the variation of the hydraulic head in space and time can be obtained.

For special shapes of flow and assumptions exact analytical solutions exist. In nature, however, one is usually confronted with problems like:

- spacial variation in the hydraulic properties of the soil
- irregular shape of the groundwater basin
- empirical relations between q and h
- d not being a constant.

For such situations by numerical and analogy models only an approximation of the actual situation can be obtained. In the next chapter a numerical solution, called the finite element method, is treated.

III. NUMERICAL SOLUTIONS

In this chapter the numerical solution of the 'aquifer equation' (eq. 5) and the 'aquitard equation' (eq. 6) will be treated. The method of solving is the finite element method. In section 3.1 only a short mathematical description of this method will be given. For more mathematical background, the reader is referred to publications on this subject (e.g. ZIENKIEWICZ, 1971, NEUMAN et.al., 1974 and WESSELING, 1976).

In section 3.2 the application of the finite element method on the flow in groundwater basins is treated while in section 3.3 the translation into a computer program is given.

3.1. Finite element method

For reasons of simplicity the finite element method will be explained for the stationary version of eq. 5, i.e.:

$$\frac{\delta}{\delta x}\left(k_x\, d\, \frac{\delta h}{\delta x}\right) + \frac{\delta}{\delta y}\left(k_y\, d\, \frac{\delta h}{\delta y}\right) + Q = 0 \qquad (10)$$

The basic idea of each numerical solution of eq. (10) is, to apply this equation on a small but finite volume. The flow region of interest, R, is divided into a finite number of volumes, called elements. The elements can have different shapes but for reasons of conveniency we only use triangles. Quadrilaterals can also be used because they can be divided into two triangles. The sizes of the triangles may vary. They can be adapted according to the geometry of the region and the accuracy desired (fig. 11 gives an example of the diviation of a region into triangles and quadrilaterals).

The corner points of the elements represent the nodal points. If there are N nodal points, then the continuous variation of h in region R is approximated by the expression

$$h_a\,(x,y) \approx \sum_{n=1}^{N} h_n\, f_n(x,y) \qquad (11)$$

where

$h_a(x,y)$ = approximation of $h\,(x,y)$
h_n = value of h in nodal point n
$f_n(x,y)$ = global coordinate function

In nodal point n h_a must be equal to h_n, so $f_n(x_n,y_n) = 1$ and $f_n(x_j,y_j) = 0$, $j \neq n$. Between $f_n(x_n,y_n)$ and $f_n(x_j,y_j)$ of the nodal points surrounding nodal point n, $f_n(x,y)$ varies linearly from 1 to 0.

In fig. 3 a picture of $f_n(x,y)$ is given.

If one knows the values h_n in the nodal points n, we know the approximation h_a of h. We can find these values of h_n by using for example the Galerkin method.

Galerkins principle states that an approximate solution of h in eq. (10) can be obtained from the following system of equations:

$$\iint_R \left\{\left[\frac{\delta}{\delta x}\left(k_x\, d\, \frac{\delta}{\delta x}\right) + \frac{\delta}{\delta y}\left(k_y\, d\, \frac{\delta}{\delta y}\right)\right] \sum_{n=1}^{N} h_n f_n(x,y)\right\} f_i(x,y)dxdy\; +$$

$$+ \iint_R Q\, f_i(x,y)\; dx\, dy = 0 \qquad i = 1, \ldots, N \qquad (12)$$

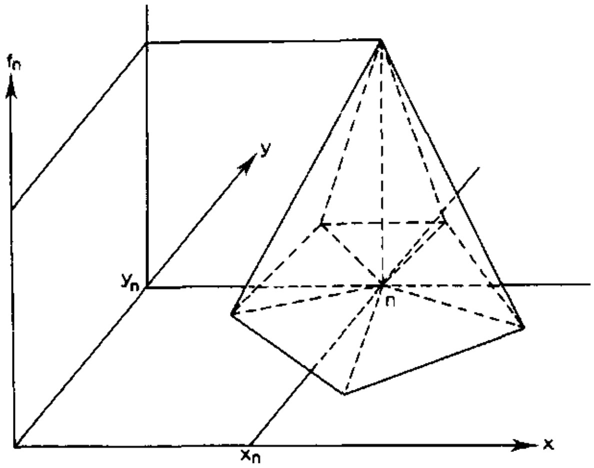


Fig. 3. The global coordinate function $f_n(x,y)$

The integration can be performed element by element with the aid of the coordinate functions f (NEUMAN et.al., 1974).

For non-stationary problems it is not possible to apply the Galerkin principle on the time derivative. But it is possible to discretize this derivative and to set up the system of equations (eq. 12) for each finite time step.

One obtains in this way a series of sets of equations that is an approximation of the non-stationary flow problem.

Treatment of boundary conditions

a. Initial conditions

   In non-stationary flow problems one must start with given values of h_n at time $t = t_o$

b. Boundary conditions

   - Prescribed head boundary

     Nodal points which lie on a prescribed head boundary have a prescribed value for h_n, so the number of unknown h_n's is reduced and accordingly so the number of equations.

   - Prescribed flux through the boundary

     If one knows the flux (inflow or outflow per unit area) and the area of the boundary per element, the inflow or outflow through the boundary per element can be calculated.

     One may assume this flow to be part of the sink term Q in eq. (12).

3.2. A p p l i c a t i o n   o f   t h e   f i n i t e   e l e m e n t   m e t h o d   o n   t h e   f l o w   i n   g r o u n d w a t e r   b a s i n s

From hydrogeological investigations the geometry and the matrix of the hydraulic properties are known. Knowing the geometry the flow can be split up into a finite number of layers. This holds also for the nature of the boundaries.

Now a nodal grid is superimposed on each layer in such a way that the nodal points of the different layers are situated right above each other (see fig. 4) and in each layer the nodal point lies in the middle of this layer.

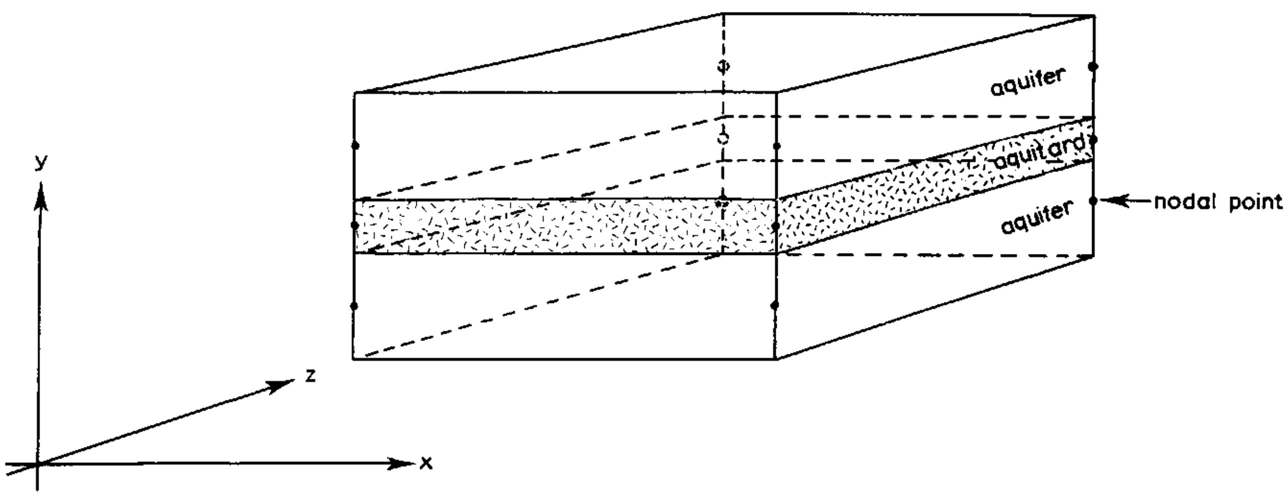


Fig. 4. Schematic representation of the nodal points in the x,y,z-space

Each layer has N nodal points of which P nodal points have a prescribed head. In each layer remain N-P = N* nodal points with unknown head. Eq. (12) can be applied to each point with unknown head in each separate layer.

As was stated in the last paragraph for each time step one can write

$$A_{in}h_n + B_{in}\frac{\Delta h_n}{\Delta t} + Q_i = 0 \qquad i,n = 1,2,\ldots N^* \tag{13}$$

where

$$A_{in} = \iint_R (k_x d\, f_n \frac{\delta f_i}{\delta x}\frac{\delta f_n}{\delta x} + k_y\, df_n \frac{\delta f_i}{\delta y}\frac{\delta f_n}{\delta y})\, dxdy$$

$$B_{in} = \iint_R (S_s\, d + S_y)\, f_i f_n\, dx\, dy \;;\quad 0 \text{ if } i \neq n$$

$$Q_i = \iint_R Q\, f_i\, dx\, dy$$

The matrices A_{in}, B_{in} and the vector Q_i have been defined elsewhere (NEUMAN et.al., 1974). A_{in} is the conductivity matrix, B_{in} the storage matrix and Q_i the source vector.

In order to clarify these rather complicate expressions apply equation (13) to point 4 of fig. 5.

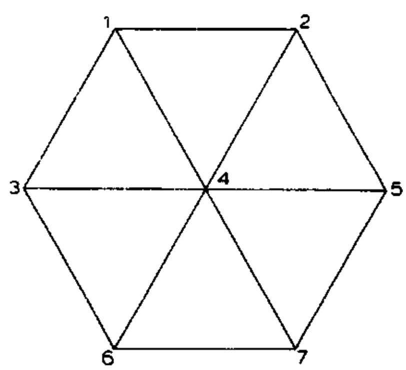


Fig. 5. Top view of part of the nodal grid

$$A_{41}h_1 + A_{42}h_2 + A_{43}h_3 + A_{44}h_4 + A_{45}h_5 + A_{46}h_6 + A_{47}h_7 + $$
$$+ B_{44}\frac{h_{4,n} - h_{4,n-1}}{t_n - t_{n-1}} + Q_4 = 0 \tag{14}$$

where $h_{4,n}$ and $h_{4,n-1}$ are values of h_4 at time t_n and t_{n-1}, respectively.

If the h-values belonging to the A-values are taken at time $t = t_{n-1}$, we get an equation with one unknown $h_{4,n}$ for time $t = t_n$. The equation can then be solved directly.

However this is an explicit formulation, which puts strong limitations on the magnitude of the time step, $t_n - t_{n-1}$ (REMSON et.al, 1971). Values larger than a certain value cause instability.

With an implicit method, the values of h are taken at time $t = t_n$. This method is unconditionally stable. Now one gets an equation with 7 unknowns: $h_{1,n}$ through $h_{7,n}$. If one has 7 equations for every point 1 through 7, we can solve the values $h_{i,n}$; $i = 1,\ldots,7$.

Of all the methods possible, the iterative method of Gauss-Seidel has been chosen. The Gauss-Seidel method starts by choosing a first estimate of $h_{i,n}$ at time $t = t_n$ (usually equal to $h_{i,n-1}$). At each iteration step a new value of $h_{i,n}$ is calculated using the last calculated values of $h_{i,n}$. If one passes the nodal points from 1 to 7, eq. (14) becomes

$$h^{j+1}_{4,n} = \{A_{41}h^{j+1}_{1,n} + A_{42}h^{j+1}_{2,n} + A_{43}h^{j+1}_{3,n} + A_{45}h^{j}_{5,n} + A_{46}h^{j}_{6,n} + A_{47}h^{j}_{7,n} + \frac{B_{44}h_{4,n-1}}{\Delta t} + Q^{j}_{4,n}\} \;/\; (A_{44} + \frac{B_{44}}{\Delta t}) \qquad (15)$$

where

$h^{j}_{i,n}$ =

j = iteration step or number of iteration

i = number of nodal point

n = time = t_n

The iteration process converges if the successive differences between $h^{k+1}_{i,n}$ and $h^{k}_{i,n}$ become continuously smaller.

The iteration process is stopped if at each nodal point the difference between $h^{k+1}_{i,n}$ and $h^{k}_{i,n}$ is smaller than a certain prescribed value.

To accelerate the convergence an overrelaxation factor W is used (for more information about W see FORSYTHE and WASOW, 1960).

$$h_{i,n}^{k+1} = h_{i,n}^{k} + W\,(h_{i,n}^{k+1} - h_{i,n}^{k}) \qquad (16)$$

If W = 1, there is no overrelaxation. Usually W varies between 1 and 2 and the optimum value very often is found by trial and error.

Eq. (15) is valid for points in aquifers. The connection with adjacent layers is made by the term Q, because in this term the flow from adjacent aquitards, q is included (two aquifers are always separated by an aquitard).

Aquitards.

We don't apply the finite element method on points in aquitards. As stated before there is no horizontal flow in aquitards. So it has no sense to replace the continuous variation of h in the aquitard by equation (11).

The nodal points in aquitards represent a certain volume of an aquitard which transmit water from one aquifer to another and which has storage capacity. The nodal points are included in the Gauss-Seidel scheme as follows (see fig. 6).

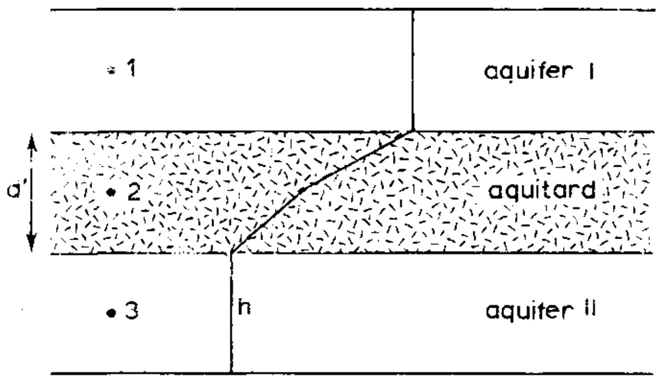


Fig. 6. Schematic situation for points in aquitards

Point 2 is situated in the middle of the aquitard with thickness d^1, storage coefficient S_c^1 and vertical hydraulic conductivity k_v^1. Suppose the Gauss-Seidel iteration starts in the upper layer and ends in the bottom-layer. Then the flux from point 1 in aquifer 1 to point 2 in the aquitard at time $t = t_n$ and in iteration step j + 1 is

$$q_1 = (h^{j+1}_{1,n} - h^{j+1}_{2,n}) \frac{k^1_v}{0,5xd^1} \qquad (17)$$

and the flux from point 3 in aquifer 2 to point 2 in the aquitard is

$$q_2 = (h^{j}_{3,n} - h^{j+1}_{2,n}) \frac{k^1_v}{0.5xd^1} \qquad (18)$$

The net flux q_1-q_2 must be equal to the change in storage per unit area per unit time. This change in storage can numerically be approximated by

$$q_1 - q_2 = S^1_c \frac{h^{j+1}_{2,n} - h_{2,n-1}}{t_n - t_{n-1}} \qquad (19)$$

Combination of (17), (18) and (19) gives

$$(h^{j+1}_{1,n} + h^{j}_{3,n} - 2\, h^{j+1}_{2,n}) \frac{k^1_v}{0.5xd^1} - S^1_c \frac{h^{j+1}_{2,n} - h_{2,n-1}}{t_n - t_{n-1}} = 0 \qquad (20)$$

From this equation $h^{j+1}_{2,n}$ can be solved.

3.3. Computer program

The problem outlined so far has been translated into a computer program, written in FORTRAN IV.

The flow chart of this program is given in fig. 7, whereas in appendix A the complete listing of the program can be found.

In the flow chart the 5 main parts of the program are indicated:

A. Reading and printing of input data

B. Determination of the matrices A and B and the vector Q per layer

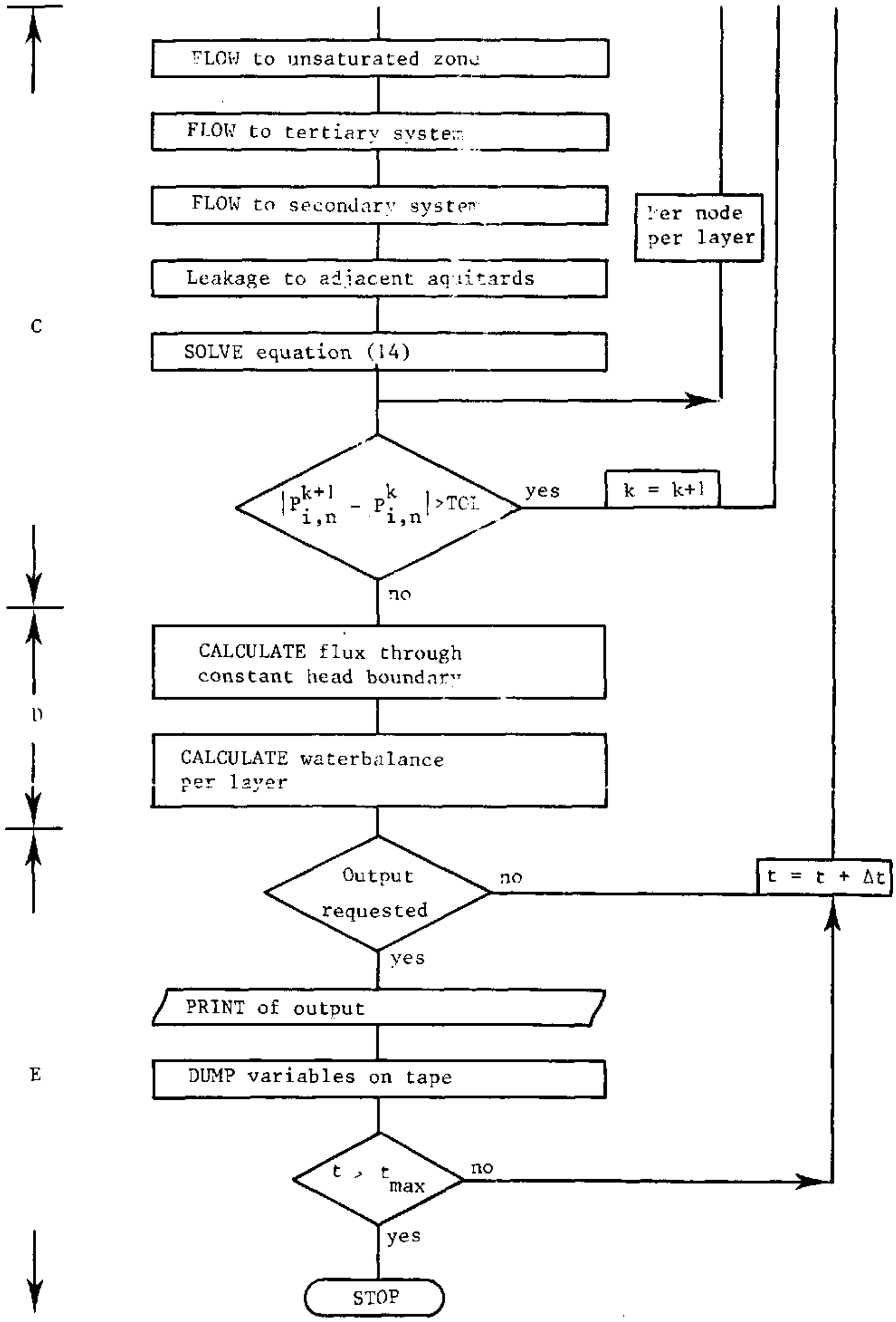
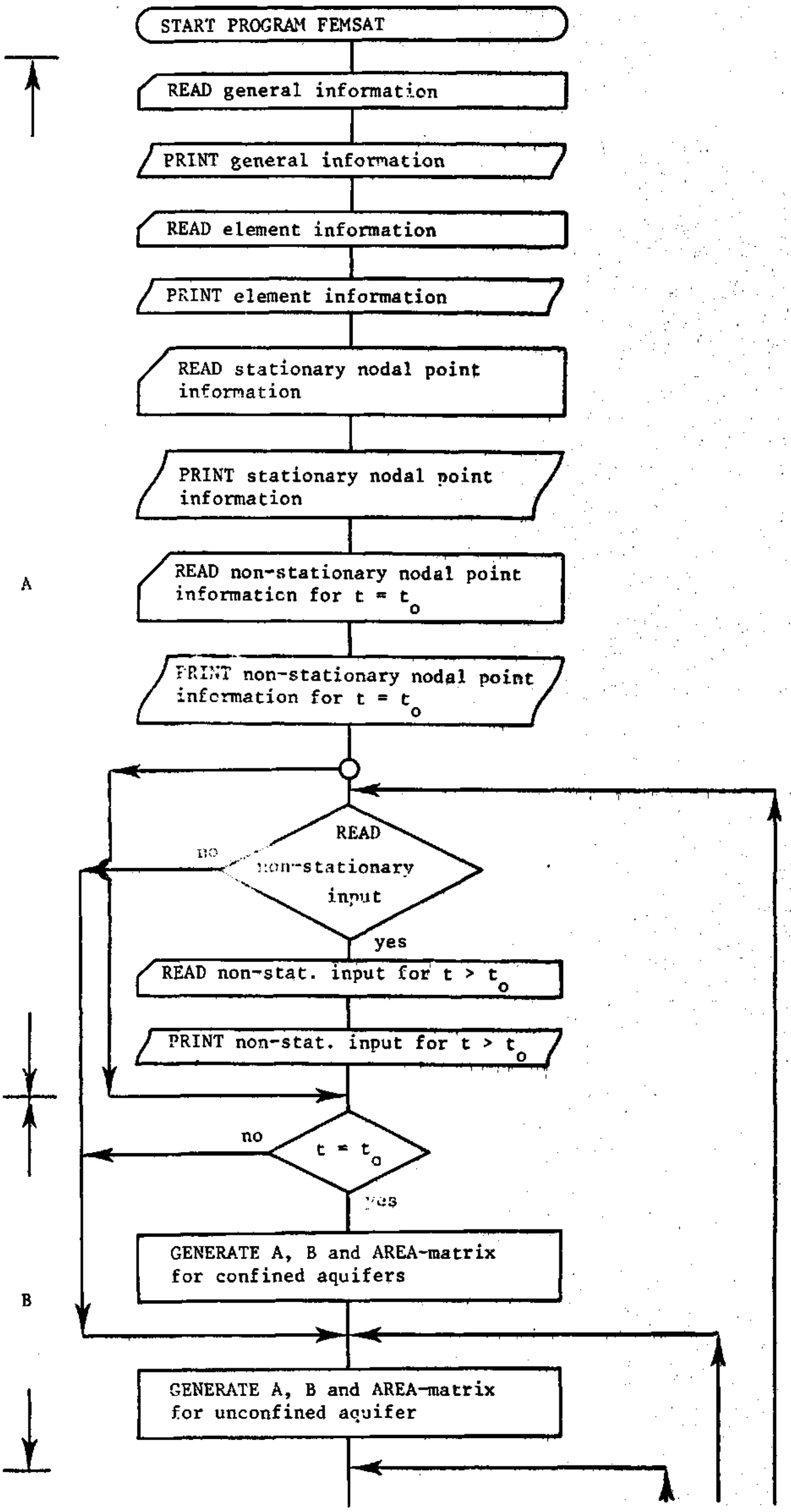


Fig. 7. Flow chart of the program FEMSAT with diviation in 5 main parts, A, B, C, D and E


START PROGRAM FEMSAT
READ general information
PRINT general information
READ element information
PRINT element information
READ stationary nodal point information
PRINT stationary nodal point information
A
READ non-stationary nodal point information for t = t0
PRINT non-stationary nodal point information for t = t0
READ non-stationary input
no
yes
READ non-stat. input for t > t0
PRINT non-stat. input for t > t0
no
t = t0
yes
GENERATE A, B and AREA-matrix for confined aquifers
B
GENERATE A, B and AREA-matrix for unconfined aquifer

C. Solving h with Gauss-Seidel

D. Determination of the water balance terms per layer

E. Printing of results

The program exists of a main program with several subroutines. The main program calls the subroutines. In the subroutines most calculations are performed.

In this way an easy adaptation to different problems is obtained (see also section 4.4). Comments in the program will help the reader to understand the program.

In the next chapter the preparation of the input data, executing of the program and the output will be treated.

Acknowledgement
Some parts of the computer program were taken from the program UNSAT2 of NEUMAN et.al. (1974).

IV. INPUT, EXECUTION AND OUTPUT

4.1. I n p u t

The input is divided into endogenous variables (chosen by the user or calculated by the program during execution) and exogenous variables (based on physical data).

4.1.1. Endogenous variables

Translation of groundwater basin into a nodal point network.

Probably the most important but also the most difficult stage in the process of simulating groundwater flow is the translation of the natural groundwater basin into a nodal point network. Questions which arise are

a. In how many layer the geo-hydrological system must/can be schematized.
b. The locations of the boundaries and the nature of these boundaries (impervious, prescribed head, prescribed flux).
c. The configuration of the nodal points.

The answers of these questions depend on the problem under consideration.

- Schematization into layers
  Usually our knowledge about the geo-hydrological situation is such that no more than 5 different layers can be distinguished (remember contrast rule)
- Boundary conditions
  The top boundary condition of a groundwater basin usually is a prescribed flux (effective precipitation).
  The bottom boundary condition usually is a prescribed flux boundary with flux zero (impervious hydrological base).
  For the boundaries at the other sides of the system one sees either prescribed head boundaries (e.g. a fully penetrating river) or prescribed flux boundaries (including zero flux).
- Nodal point configuration
  Nodal points are situated:
  . on the boundaries
  . along internal boundaries which indicate changes in hydraulic properties within the layer
  . along the tertiary surface water system
  . on places where artificial recharge or discharge will take place
  . elsewhere

Construction of the finite element mesh.

Assume we have schemetized the groundwater basin into a finite number of layers with different direction of flow (horizontal and vertical). Within one layer the hydrological properties may vary from place to place.

Now a nodal grid is superimposed on each layer in a way already described in section 3.2. Thus each layer has the same number of nodal points situated on the same places in the horizontal plane, but with different z-coordinates.

We are free to construct quadrilaterals or triangles, because the program automatically divides a quadrilateral into two triangles having identical material properties. The dimensions of the elements should be small at places where large changes in hydrological properties occur or where large gradients in hydraulic head are expected to occur.

Coding of nodal points.

Each node in the finite element network is assigned an integer code which indicates the type of calculation for this node. The coding is as follows:

- Code = 1 : nodal points with described head, as function of time
- Code = 2 : internal nodal points
- Code = 3 : internal nodal points which represent secondary surface water system
- Code = 5 : nodal points with prescribed artificial recharge or discharge and points with prescribed boundary flow (both as functions of time).

Other important endogenous variables are the magnitude of the time step, the value of the variable which stops the iteration process and the maximum number of iterations allowed per time step (see also group A and B of section 4.1.3.)

4.1.2. Exogenous variables

After constructing the nodal grid each node represents part of the groundwater basin. Geo-hydrological properties can be assigned to each mode, based on physical data (group C, D and E of section 4.1.3.). In group F of section 4.1.3. time-dependent variables can be assigned to each node.

4.1.3. Data input to Program FEMSAT

The data input must be punched on cards according to the following instructions.

GROUP	COLUMNS	FORMAT	SYMBOL	DESCRIPTION
A	1-80	20A4	HEAD	Desired heading to be printed at the beginning of the execution
	1- 5	I5	NUMNP	number of nodal points

GROUP	COLUMNS	FORMAT	SYMBOL	DESCRIPTION
	6-10	I5	NUMEL	number of elements
	11-15	I5	NUMLAY	number of layers
	16-20	I5	MBAND	maximum difference in node number between two adjacent nodes
	21-25	I5	NTSPI	number of time steps between a change in non-stationary input data
	26-30	I5	NTSPO	number of time steps between two successive outputs
	31-35	I5	MAXIT	maximum number of iterations per time step
	1-10	F10.3	ST	time to start with the execution
	11-20	F10.3	DELT	initial time interval
	21-30	F10.3	TMAX	maximum time to be reached before execution is stopped
	31-40	F10.5	TOL	maximum allowed absolute change in the values of h between any two successive iterations in a given time step. Iterations continue until all changes are less than or equal to TOL, or until MAXIT is exceeded
B	1- 5	I5	KX(N,1)	sequential number of i-th corner of element N
	6-10	I5	KX(N,2)	sequential number of j-th corner
	11-15	I5	KX(N,3)	sequential number of k-th corner
	16-20	I5	KX(N,4)	sequential number of l-th corner (see fig. 8)

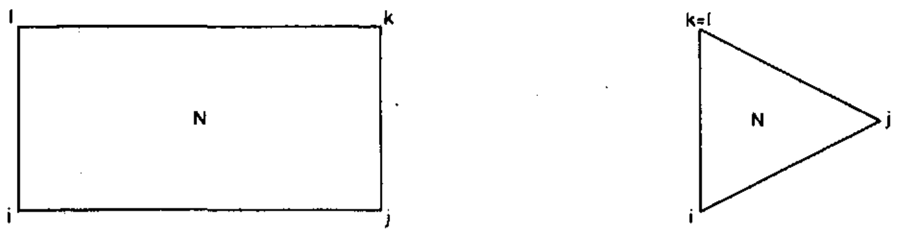


Fig. 8. Numbering of element corner nodes

GROUP	COLUMNS	FORMAT	SYMBOL	DESCRIPTION

One card must be provided for each element in sequential order, starting with N = 1 and ending with N = NUMEL

GROUP	COLUMNS	FORMAT	SYMBOL	DESCRIPTION
C	1-10	F10.3	SANG(N,LY)	angle in degrees between C1 and the x-coordinate, to be used in nodal print N in layer LY (see fig. 9)
	11-20	F10.3	C1(N,LY)	First principal conductivity of material in node N and layer LY
	21-30	F10.3	C2(N,LY)	idem second principal conductivity
	31-40	F10.4	THI(N,LY)	thickness of layer LY at nodal point N

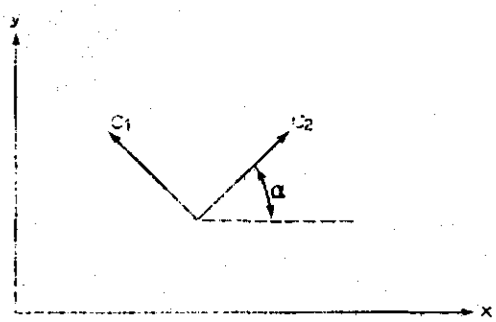


Fig. 9. Definition of SANG,C1 and C2

One card must be provided for each model point, starting with N = 1 and ending with N = NUMNP. If there is more than 1 layer, for each layer a set must be provided, starting with the upper layer, LY = 1, and ending with the bottom layer LY = NUMLAY. The variables SANG, C1 and C2 are not relevant for aquitards, but for reasons of conveniency this procedure is maintained (making SANG, C1 and C2 dummy variables)

GROUP	COLUMNS	FORMAT	SYMBOL	DESCRIPTION
D	1-10	I10	KODE(N)	coding of nodal points (see paragraph 4.2)
	11-20	I10	NTERR(N)	number of tertiary relation in node N. The q-h relationship of the flow to

GROUP	COLUMNS	FORMAT	SYMBOL	DESCRIPTION
				the tertiary system, as discussed in par. 2.4, are schematized in a small number of different relations
	21-30	F10.1	X(N)	x-coordinate of node N
	31-40	F10.1	Y(N)	y-coordinate of node N
	41-50	F10.1	HGL(N)	height ground level in node N
	51-60	F10.1	HBSC(N)	height bottom secondary conduit in node N. If node N is not a secondary conduit, this variable is a dummy variable
	61-70	F10.1	LESC(N)	length of secondary conduit in area represented by node N

One card must be provided for each node, starting with N = 1 and ending with N = NUMNP

GROUP	COLUMNS	FORMAT	SYMBOL	DESCRIPTION
E	1-10	F10.1	P(N,LY,1)	initial value of h at node N in layer LY
	11-20	F10.1	VERRES(N,LY)	vertical resistance at node N and layer LY. If the layer is an aquifer, this value is a dummy variable
	21-30	F10.1	RADRES(N,LY)	radial resistance for flow to secondary system at node N in layer LY
	31-40	F10.1	ENRES(N,LY)	entrance resistance for flow to secondary system at node N in layer LY
	41-50	F10.1	WP(N,LY)	wet perimeter of secondary system at node N in layer LY
	51-60	F10.1	POR(N,LY)	specific yield of the material at node N in layer LY
	61-70	F10.1	SS(N,LY)	specific storage of the material at node N in layer LY

GROUP	COLUMNS	FORMAT	SYMBOL	DESCRIPTION

One card must be provided for each node, starting with N = 1 and ending with N = NUMNP. If there is more than 1 layer, for each layer a set of cards must be provided, starting with LY = 1 and ending with LY = NUMLAY

F	1-10	F10.1	P(N,LY,1)	value of h at different times at node N in layer LY. Node N lies on a prescribed head boundary
	11-20	F10.1	WTSC(N)	water table in secondary system at node N at different times
	21-30	F10.1	ARRED(N,LY)	artificial recharge to or discharge from node N in layer LY or prescribed boundary flow at different times
	31-40	F10.1	QPHR(N)	flux through phreatic surface at node N at different times

The data in group F are non-stationary input data. One card must be provided for each node, starting with N = 1 and ending with N = NUMNP. In case of more than 1 layer, for each layer a set must be provided. Not relevant variables are taken as dummy variables.
After a certain number of time steps (NTSPI), the data of group E are read. So the set cards E must contain N×LY×C cards, where C = (TMAX/NTSPI×DELT) + 1, if DELT is a constant.

4.2. Running the program on the Cyber 7200

The program has been executed on a Cyber 7200 computer of IWIS-TNO in the Hague.

- The demanded input is punched on cards according to the instructions of the last paragraph and is put on a permanent file.
- For each specific problem in the program only the dimension statements in the main program have to be adjusted according to the instructions in the lines of comment.
- After compilation the program together with the input file is executed.

- Intermediate data necessary for continuation of the calculations, are saved on another permanent file. This gives the opportunity to restart the program at a time the calculation was stopped. The only change in the input file is to adjust the starting time.

Remark.
With the aid of a terminal small changes in the program and in the input file can be made easily and the output immediately appears on the terminal. In this way it is possible to stop erroneous calculations. This is especially useful when being in the testing phase of the program.

4.3. O u t p u t

Immediately after reading the input data they are printed so that possible errors can be detected. During execution after each certain number of time steps (NTSPO), calculated values of h and water balance terms are printed. In appendix B an example is given. The output data can be devided into two groups.

A. Data per node per layer
   - Coding of the nodal points, KODE
   - Hydraulic head, HEAD
   - Flow through phreatic surface at the time indicated in head of table, FLOWUN
   - Flow to tertiary surface water system, FLOWTS
   - Flow to secondary surface water system, FLOWSS
   - Flow to adjacent layers, LEAKAGE
   - Artificial recharge or discharge and prescribed boundary flow, ARRED
   - Change in storage during last time step, STORAGE
   - Flow through prescribed head boundary, LATFLOW

B. Data per layer
   Summation of the terms already mentioned under A gives the magnitude of the water balance terms per layer at the time indicated (instanteneous water balance terms). Summation of these instanteneous terms multiplied by their respective time intervals, since time is zero, gives the cumulative water balance terms per layer.

After each print out we are able to check the calculations, because the sum of the water balance terms per layer must be approximately zero.

4.4. Computing time

The computing time depends on the number of nodal points and the number of layers. But it depends also on the magnitude of the changes in hydraulic head and on the value which terminates the Gauss-Seidel iteration (TOL). To give an impression: TOL = .001 m.
Costs per day simulation (in hfl) on the Cyber 7200

$$\frac{\text{number of nodes * number of layer}}{\Delta t} \times a \times b \qquad (21)$$

where

a = costs in Dutch guilders per system second (ƒ 0.27)

b = empirical factor, with magnitude about 0.01.

V. NUMERICAL EXPERIMENTS

In general there are two possibilities to test the validity of numerical models. The first and most commonly used method is to compare numerical results with analytical solutions. The second possibility is to compare numerical results with measured field data. Since the last alternative requires many data that are difficult to obtain, we restrict ourselves for the time being to comparison with the analytical solutions.

5.1. Flow to a well in an unconfined circular-shaped aquifer

Flow to a well can be divided into steady-state flow and unsteady-state flow (KRUSEMAN and DE RIDDER, 1976).

5.1.1. Steady state flow in an unconfined aquifer

For this case the formula of Thiem-Dupuit is valid (see fig. 10)

$$Q = \frac{\pi k \ (h_2^2 - h_1^2)}{\ln(r_2/r_1)} \tag{22}$$

where

$h_1 h_2$ = hydraulic head in 1 and 2, respectively [L]

Q = discharge from well [$L^3.T^{-1}$]

r_1, r_2 = distance from well [L]

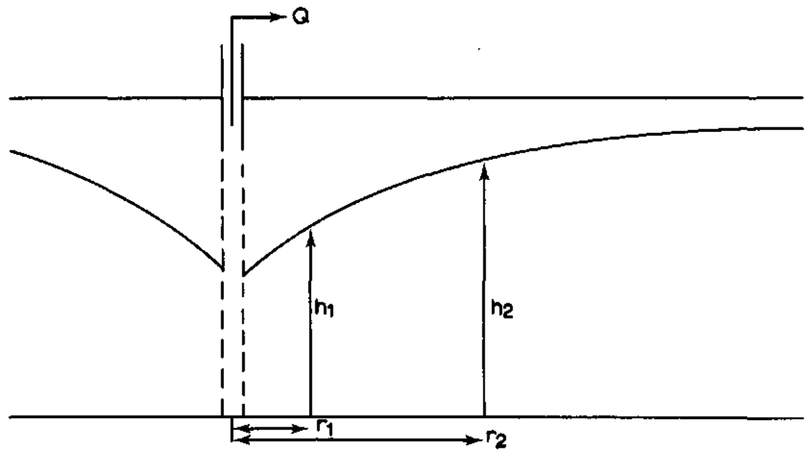


Fig. 10. Schematic cross-section of a pumped unconfined aquifer

Fig. 11 gives the nodal grid superimposed on a well in a circular shaped aquifer. Near the well the nodal point distance is made smaller. Quadrilaterals as well as triangles are used.

Numerical calculations with this finite element network and an extraction rate of 200 $m^3.day^{-1}$ were performed for 10 days. At that moment a steady-state situation was almost achieved.

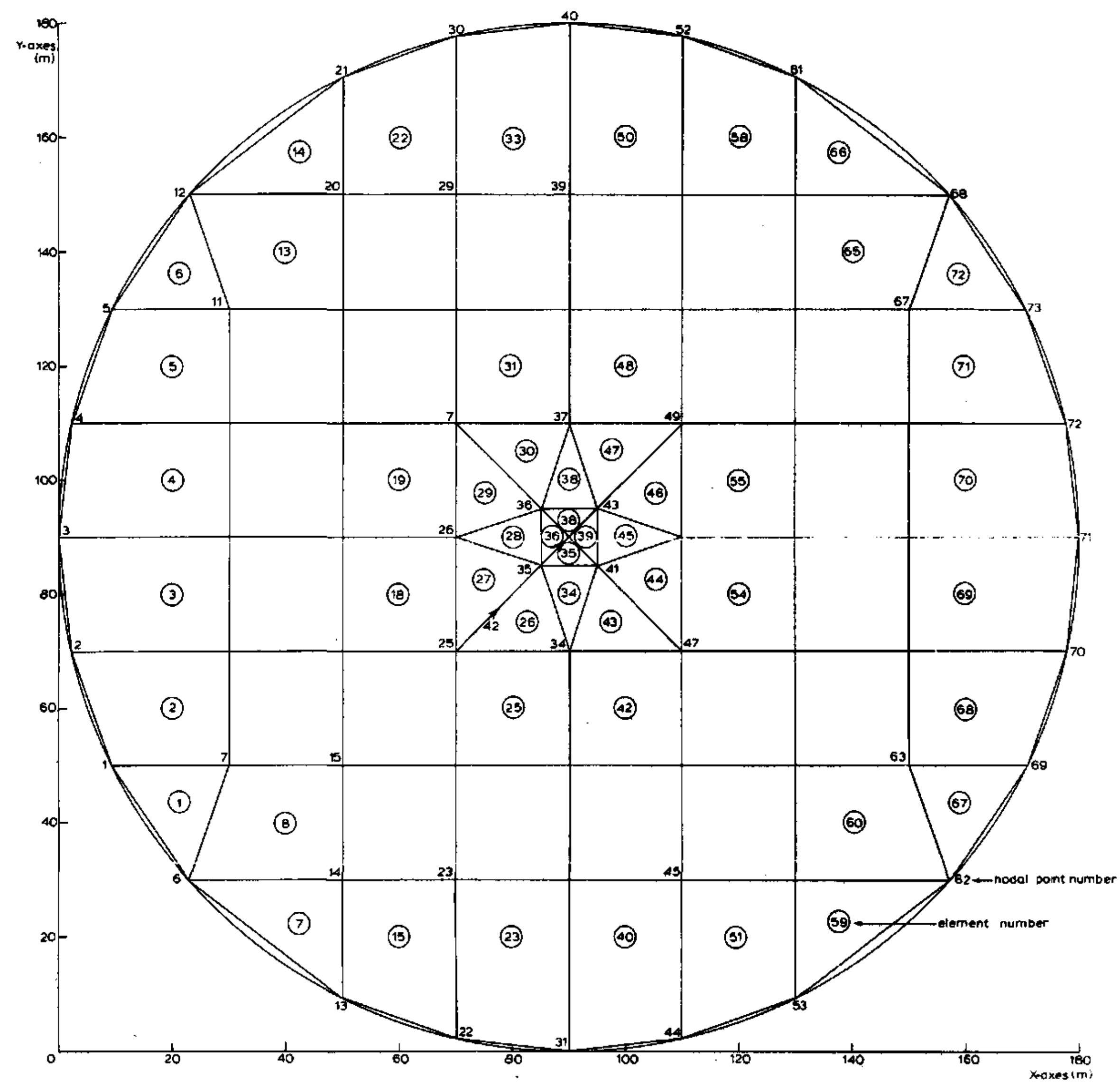


Fig. 11. Nodal point configuration for flow to a well in a circular-shaped aquifer. The outer boundary is taken to be constant. The well is situated in nodal point 42

In fig. 12 the results of the simulation are compared with the analytical solution. The agreement between numerical results and the analytical solution is excellent.

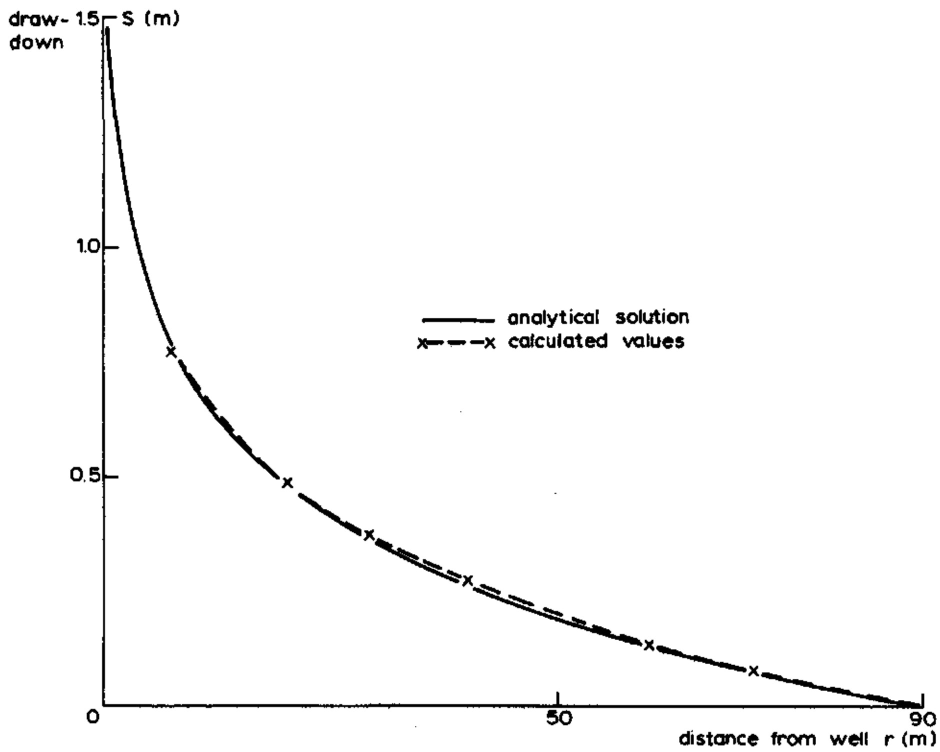


Fig. 12. Comparison between analytical and calculated steady-state drawdown for the problem of fig. 11, with:

$Q = 200\ m^3.day^{-1}$

$k = 1.0\ m.day^{-1}$

$d = 10.0\ m$

$S_y = .2$

$S_s = .0$

5.1.2. Unsteady-state flow in an unconfined aquifer

Analytical solutions for unsteady-state flow in an unconfined aquifer only exist if the aquifer extends infinitely. For the aquifer given in fig. 11 this condition is not fulfilled. However, during the first one or two days after starting the extraction, the influence of a constant hydraulic head at the outer boundary on the drawdown near the well can be neglected (see fig. 13). After one day the lateral inflow is only 2.5% of the extraction, the remainder being the change in storage due to drawdown.

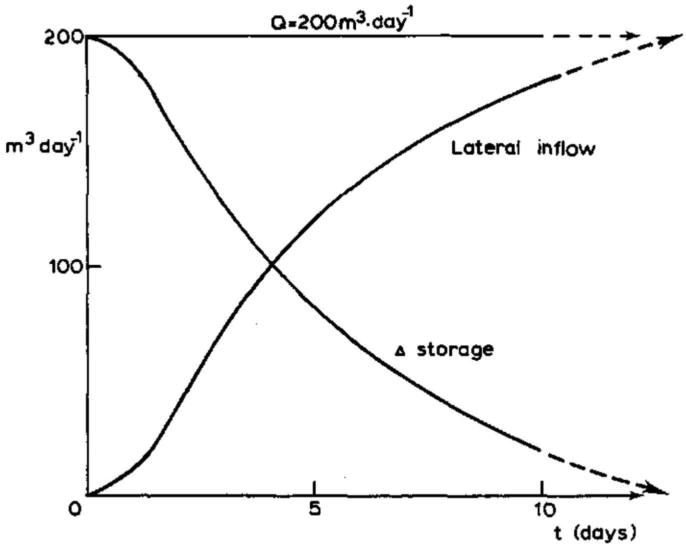


Fig. 13. Values of Q, lateral inflow and change in storage as functions of time

The nonsteady-state or Theis equation, can be written as

$$s^1 = \frac{Q}{4\pi kD} \int_u^\infty \frac{e^{-y}}{y} \, dy = \frac{Q}{4\pi kD} \, W(u) \qquad (23)$$

where

$u = \dfrac{r^2 S_c}{4\pi kDt}$ [-]

r = distance from well [L]

S_c = storage coefficient [-]

Q = constant discharge from well [$L^3.T^{-1}$]

D = thickness of the waterbearing layer at the beginning of pumping [L]

t = time after starting the extraction [T]

s^1 = corrected drawdown

= $s - s^2/2D$ [L]

s = calculated drawdown [L]

W(u) = Theis's well function

In fig. 14 the analytical unsteady-state solution, for the case described in section 5.1.1, is given together with the numerical results at times 0.5 and 1.0 days. The agreement between numerical results and the analytical solution is rather good.

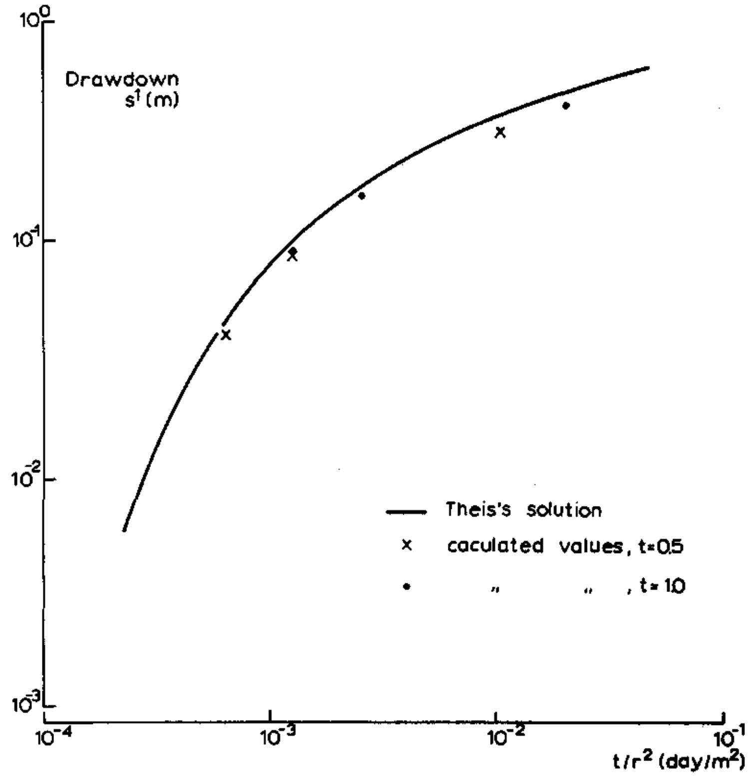


Fig. 14. Comparison between analytical and calculated steady-state drawdown for the problem of fig. 11, with:

$Q = 200\ m^3.day^{-1}$

$k = 1.0\ m.day^{-1}$

$d = 10.0\ m$

$S_y = .2$

$S_s = .0$

These results indicate that the assumption of a linear variation of h between two nodal points (see fig. 3), even with the rather rough element network of fig. 11, is acceptable. So there is no need to improve for example the finite element network by introducing higher order coordinate functions.

5.2. Flow to a well in a 3-layered system

In fig. 15 a schematic cross-section of a 3-layered system of two aquifers separated by an aquitard, is given. A well of infinitisemal radius is completed in the lower aquifer and discharges at a constant rate, Q_{II}. Each layer is homogeneous, isotropic and extending radially infinite.

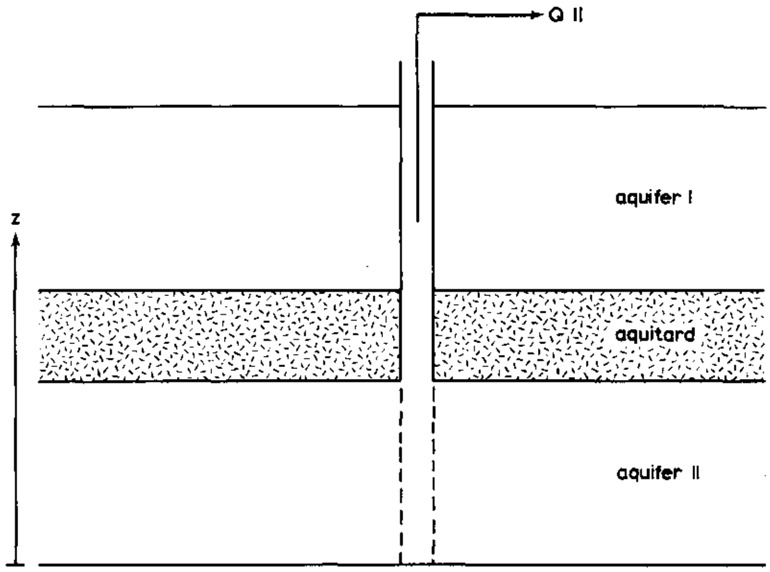


Fig. 15. Schematic cross-section of a 3-layered system

For this situation NEUMAN and WITHERSPOON (1969a) give rather complicated analytical solutions. In the special case where the hydraulic properties of the two aquifers are identical, they give graphical illustrations of their solutions for three different cases.

These graphical solutions give the opportunity to verify roughly numerical results of a 3-layered system. Therefore a nodal grid is superimposed on a circular shaped groundwater flow system with the extraction in the centre of the lower aquifer and with a constant head on the outer boundary. Fig. 16 gives the nodal point situation in the x - y plane of 1 layer. The nodes in the other two layers are situated in the middle of the layers at the same places in the x - y plane, only their z-coordinate is different.

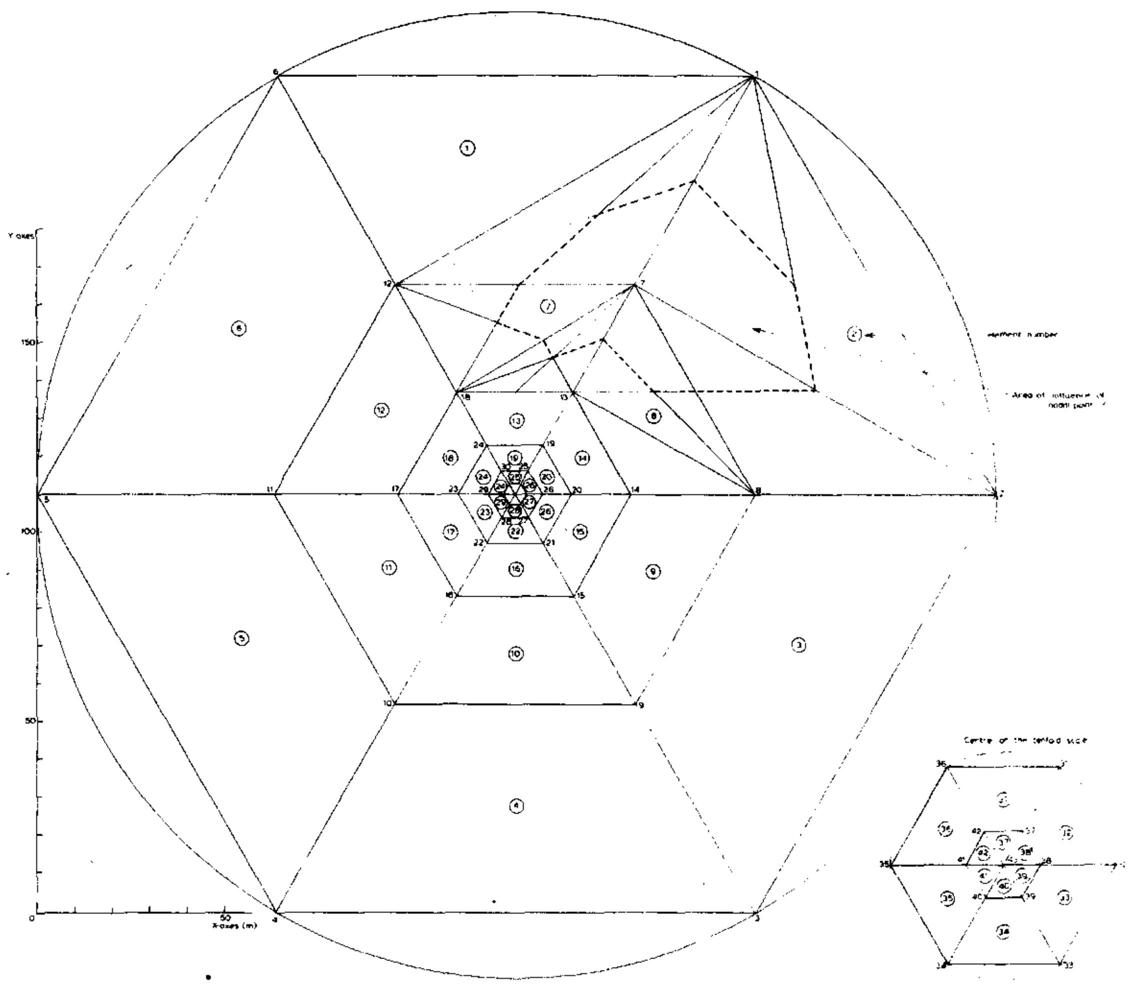


Fig. 16. Nodal point configuration for the 3-layered problem (top view)

In fig. 17 numerical results are compared with the analytical solution. The agreement is rather bad, especially for the two aquifers. But in terms of the water balance the difference between aquifers and aquitard is not so big, because the aquitard has about 16 times bigger storage capacity than the aquifers. The small value of the specific storage of the aquifers is the reason that small errors in the components of the water balance have a big influence on the calculated values of the hydraulic head.

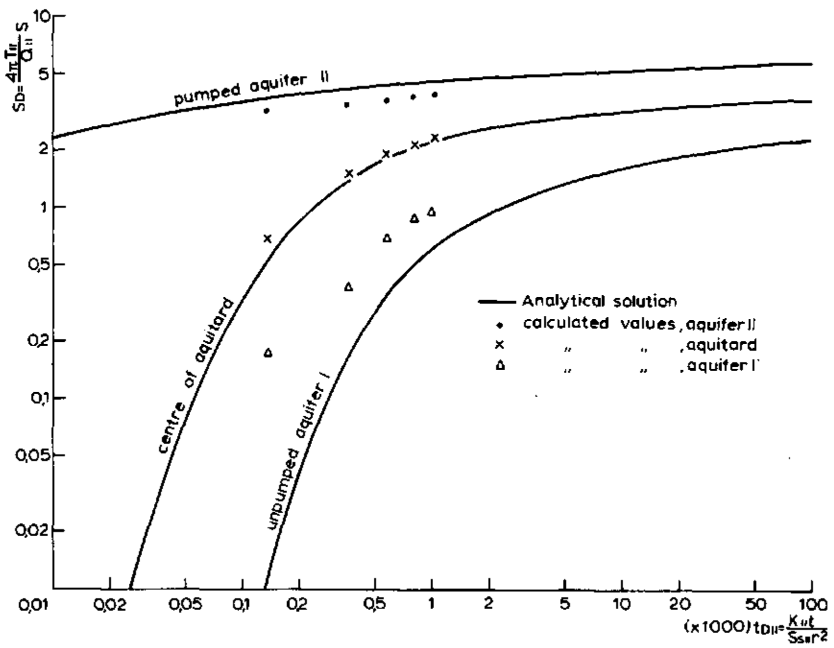


Fig. 17. Dimensionless drawdown vs. dimensionless time in a 3-layered system, with:

Q_{II} = 50 $m^3.day^{-1}$

r = 3 m (distance from well)

	Thi (m)	S_s	k($m.day^{-1}$)
aquifer I	10.0	.001	1.
aquitard	2.5	.0642	.0277
aquifer II	10.0	.001	1.

The discrepancies are partly caused by errors in discretization. In the z-direction these errors can be eliminated by taking the vertical resistance of the aquitard to be infinite. Then the calculated drawdown in the pumped aquifer can be compared with the analytical Theis's solution.

This is done in fig. 18 for different distances from the well. One sees that the agreement is better when the distance is bigger. Apparently the linearization of the logarithmic drawdown curve near the well is responsible for the differences. The differences for r = 3 m are of the same magnitude as for the pumped aquifer in the 3-layered case.

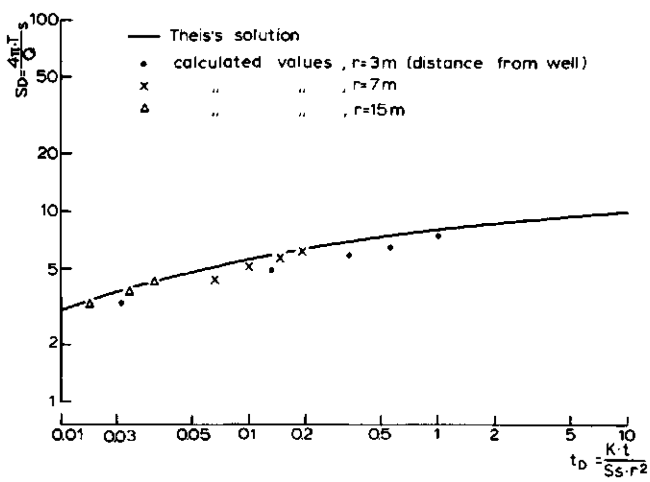


Fig. 18. Dimensionless drawdown vs. dimensionless time in a confined aquifer with:

$Q = 50.0\ m^3.day^{-1}$

$k = 1.0\ m.day^{-1}$

$D = 10.0\ m$

$S_s = .001$

The differences between numerical results and analytical solutions are not caused by taking a finite area, because changing the outer boundary into an impermeable boundary has only minor effects.

VI. SUMMARY AND CONCLUSIONS

In this report a numerical model for non-stationary saturated groundwater flow in a multi-layered system has been treated.

Groundwater flow can be described in different ways.

a. Physical-mathematical: by combining Darcy's law and the law of continuity (section 2.1)

b. Geo-hydrological: the flow in a natural groundwater basin takes place in layers with different hydraulic properties. Definitions of the most important hydraulic properties are given in section 2.2

c. Schematical: groundwater flows horizontally in aquifers and vertically in aquitards (section 2.3)

d. Approximating: by dividing the region of flow into a finite number of elements and applying Darcy's law and the law of continuity on each of these elements. In these ways one gets a set of equations, which can be solved given (initial and) boundary conditions (section 2.5)

e. Numerical: the set of equations has been solved using a Galerkin type finite element method and the implicit Gauss-Seidel method with overrelaxation factor (section 3.1 and 3.2)

f. Computer-solvable: the numerical solution has been translated in a FORTRAN IV program, called FEMSAT (section 3.3)

In chapter IV the input, execution and output of the program FEMSAT are treated.

Finally in chapter V some numerical applications are given: simulation of flow to a well in unconfined aquifer and simulation of flow to a well in a 3-layered configuration.

Comparison of hydraulic heads obtained by numerical calculations and analytical true solutions showed an excellent agreement in case of stationary flow and a good to moderate agreement in case of non-stationary flow. However, the diviations caused by the discretization process, must be seen in relation to the errors made in the establishment of the hydraulic properties.

As was stated in the introduction, numerical models can be valuable tools in the management of water resources. Therefore numerical models must be suitable to simulate effects of certain operations upon the spreading of water over saturated groundwater, unsaturated groundwater and surface water. The model described in this report is a model for saturated groundwater flow. The relation with the surface water can be taken into account via radial and entrance resistances or via a relation between flow to the surface water system and the hydraulic head. The relation with the unsaturated zone has been restricted to the input factor 'effective precipitation'. In practise however, the unsaturated and saturated system are interconnected. Therefore the model should be improved by taking into account the unsaturated zone. This can be done in at least two ways:

a. Replacing the unsaturated zone by a parametrical model
b. Coupling the unsaturated zone in an iterative way with the saturated system.

In order to find out the usefulness of the numerical model for water resources management, it will be applied to actual regional groundwater systems. Also sensitivity analyses will be carried out. The computer model itself can be improved by applying standard programs, e.g. automatical generation of a nodal grid and graphical plotting of the results.

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APPENDIXA A: LISTING OF PROGRAM FEMSAT

```
*DECK FEMSAT
      PROGRAM FEMSAT(INPUT,OUTPUT,TAPE1)
C
C THE SIZE OF THE NEXT ARRAYS MUST BE EQUAL TO THE NUMBER OF NODES
C
      DIMENSION X(43),Y(43),HGL(43),NTERR(43),HBSC(43),LESC(43),AREA(43)
     1,KODE(43),UN(43),TSYS(43)
C
C THE SIZE OF THE NEXT ARRAYS MUST BE EQUAL TO THE NUMBER OF NODAL
C POINTS AND THE NUMBER OF DISTINGUISHED LAYERS(NUMLAY)
C
      DIMENSION RADRES(43,3),ENRES(43,3),VERRES(43,3),WP(43,3),SSYS(43,3
     3),
     3LEAK(43,3),ACC(43,3),Q(43,3),ARRED(43 ,3),WTSC(43,3),C3(43,3),C2(7
     33,3),THI(43,3),SS(43,3),SANG(43,3)
      REAL LEAK
C
C THE SIZE OF THE NEXT ARRAYS MUST BE EQUAL TO THE NUMBER OF ELEMENTS
C AND THE NUMBER OF LAYERS
C
      DIMENSION EC1(42,3),EC2(42,3),ETHI(42,3),ESANG(42,3),ECANG(42,3)
C
C THE SIZE OF THE NEXT ARRAYS MUST BE EQUAL TO THE NUMBER OF LAYERS
C
      DIMENSION CFLSS(3),CFLLEA(3),CARRED(3),CSTOR(3),CINOUT(3)
      DIMENSION TFLSS(3),TFLLEA(3),TARRED(3),TSTOR(3),TINOUT(3)
C
C THE SIZE OF P:NUMNP*NUMLAY*2
C THE SIZE OF KX:NUMEL*4
C THE SIZE OF A:MBAND*NUMNP*NUMLAY
C THE SIZE OF B:NUMNP*NUMLAY
C THE SIZE OF AT:MBAND
C
      DIMENSION P(43,3,2),KX(42,4),A(8,43,3),B(43,3),AT(8)
C
      DIMENSION HEAD(20)
C
C READ GENERAL INFORMATION
C
      READ 10,HEAD
10    FORMAT(20A4)
      READ 20,NUMNP,NUMEL,NUMLAY,MBAND,NTSPI,NTSPO,MAXIT
20    FORMAT(7I5)
      READ 30,ST,DELT,TMAX,OMEGA,TOL
30    FORMAT(4F10.3,F10.5)
      PRINT 40,HEAD,NUMNP,NUMEL,NUMLAY,MAXIT,OMEGA,TOL
40    FORMAT(/20A4,///36H NUMBER OF NODAL POINTS              ,I5/,
     136H NUMBER OF ELEMENTS                  ,I5/,
     136H NUMBER OF LAYERS                    ,I5/,
     136H MAX.NUMBER OF ITERATIONS            ,I5/,
     136H OVERRELAXATIONFACTOR                ,F10.2/,
     136H MAX.ALLOWABLE CHANGE PER ITERATION  ,F10.5///)
      NERR=0
C
C READ AND GENERATE ELEMENT INFORMATION
C
      CALL ELIN(NUMEL,NUMLAY,SANG,C1,C2,THI,KX,NERR,ESANG,ECANG,EC1,EC2,
     1ETHI,NUMNP)
      IF(NERR.EQ.1)STOP
C
C READ AND GENERATE STATIONARY NODAL POINT INFORMATION AND INITIAL P
C
      CALL NPIN(NUMNP,NUMLAY,KODE,NTERR, X,Y,P,HGL,HBSC,LESC,RADRES,ENRE
     1S,VERRES,WP,NERR,POR,SS)
      IF(NERR.EQ.1)STOP
C
C IF ST>0,INITIAL VALUES OF P AND WATERBALANCE TERMS MUST BE READ
C  FROM TAPE1
C
      IF(ST.EQ..0)GO TO 80
      REWIND 1
      DO 60 LY=1,NUMLAY
      DO 50 N=1,NUMNP
      READ (1) P(N,LY,1)
50    CONTINUE
60    CONTINUE
      DO 70 LY=1,NUMLAY
      READ (1) CFLSS(LY),CFLLEA(LY),CARRED(LY),CSTOR(LY),CINOUT(LY)
70    CONTINUE
      READ (1) CFLTS,CFLUN,ST,DELT
      GO TO 100
80    DO 90 LY=1,NUMLAY
      CFLSS(LY)=.0
      CFLLEA(LY)=.0
      CARRED(LY)=.0
      CSTOR(LY)=.0
      CINOUT(LY)=.0
90    CONTINUE
      CFLUN=.0
      CFLTS=.0
C
C READ AND GENERATE  NON-STATIONARY INPUT
C
100   PROD=FLOAT(NTSPI)*DELT
      DIV=ST/PROD
      IDIV=1+IFIX(DIV)
      ITS=0
      CALL DYNAMIC(NUMNP,NUMLAY,KODE,P,ARRED,WTSC,IDIV,ST,PREV)
      ST=ST+DELT
      GO TO 120
      IDIV=1
110   ST=ST+DELT
      DO 116 LY=1,NUMLAY
      DO 114 N=1,NUMNP
      P(N,LY,1)=P(N,LY,2)
114   CONTINUE
116   CONTINUE
      REST=ST-IFIX(ST/PROD)*IFIX(PROD)
      IF(REST.EQ..0)CALL DYNAMIC(NUMNP,NUMLAY,KODE,P,ARRED,WTSC,IDIV,
     1ST,PREV)
120   IF(ITS.NE.0)GO TO 155
C
C GENERATE A-MATRIX
C
      CALL ELEM(KODE,P,EC1,EC2,X,Y,KX,ESANG,ECANG,NUMNP,NUMEL,MBAND,A,ET
     1HI,ITS,VERRES,NUMLAY,HGL,B,POR,SS,THI,AREA)
130   DO 150 LY=1,NUMLAY
      DO 140 N=1,NUMNP
      P(N,LY,2)=P(N,LY,1)
140   CONTINUE
150   CONTINUE
155   ITS=ITS+1
      ITER=0
      DIFF2=10000.
160   ITER=ITER+1
C
C A AND B-MATRIX IN FIRST LAYER ARE NOT CONSTANT
C
```

```
      CALL ELEM(KODE,P,EC1,EC2,X,Y,KX,ESANG,ECANG,NUMNP,NUMEL,MBAND,A,ET
     1HI,ITS,VERRES,NUMLAY,HGL,B,POR,SS,THI,AREA)
      DO 170 LY=1,NUMLAY
      TFLSS(LY)=.0
      TFLLEA(LY)=.0
      TARRED(LY)=.0
      TSTOR(LY)=.0
      TINOUT(LY)=.0
170   CONTINUE
      TFLTS=.0
      TFLUN=.0
C
C IN SOILWAT CALCULATION OF FLOW TO UNSATURATED ZONE FOR SOME NODAL
C POINTS
C
      DO 180 N=1,NUMNP
      UN(N)=.0
      IF(KODE(N).EQ.4)CALL SOILWAT(P,NUMNP,NUMLAY,AREA,FLOWUN,POR,UN)
      IF(KODE(N).NE.1)UN(N)=PREV*AREA(N)
      FLOWUN=UN(N)
      TFLUN=TFLUN+UN(N)
180   CONTINUE
C
C LOOPS ON NODES AND LAYERS
C
      DIFMAX=.0
      DO 200 LY=1,NUMLAY
      DO 190 N=1,NUMNP
      IF(VERRES(N,LY).GT.0.)GO TO 185
C
C IN TERSYS CALCULATION OF FLOW TO TERTIARY SYSTEM
C
      FLOWTS=.0
      IF(LY.EQ.1)CALL TERSYS(KODE,P,NUMNP,NUMLAY,FLOWTS,NTERR,AREA,N,HGL
     1)
      IF(LY.EQ.1)TSYS(N)=FLOWTS*AREA(N)
      TFLTS=TFLTS+FLOWTS
C
C IN SECSYS CALCULATION OF FLOW TO SECONDARY SYSTEM
C
      FLOWSS=.0
      IF(KODE(N).EQ.3)CALL SECSYS(P,NUMNP,NUMLAY,LESC,HBSC,RADRES,ENRES,
     1FLOWSS,N,LY,WP,WTSC,VERRES)
      IF(KODE(N).EQ.3)SSYS(N,LY)=FLOWSS
      TFLSS(LY)=TFLSS(LY)+FLOWSS
C
C IN LEAKAGE CALCULATION OF FLOW TO ADJACENT LAYERS
C
      FLOWLE=.0
      CALL LEAKAGE(P,NUMNP,NUMLAY,VERRES,THI,AREA,FLOWLE,N,LY,HGL)
      IF(N.EQ.43)FLOWLE=.0
      LEAK(N,LY)=FLOWLE
      TFLLEA(LY)=TFLLEA(LY)+FLOWLE
C
C IN ATBT DETERMINATION OF AT AND BT MATIX
C
      CALL ATBT(P,NUMNP,NUMLAY,A,B,DELT,ARRED,FLOWUN,FLOWTS,FLOWSS,FLOWL
     1E,N,LY,AT,BT,MBAND,VERRES,UN)
C
C IN SOLVE GAUSS-SEIDEL ITERATION
C
      CALL SOLVE(P,NUMNP,NUMLAY,PLAST,AT,BT,MBAND    ,KODE,N,LY,OMEGA,HG
     1L,A,VERRES)
185   RESIS=.5*VERRES(N,LY)/ABS(AREA(N))
      IF(VERRES(N,LY).NE.0.)P(N,LY,2)=(-P(N,LY-1,2)/RESIS-P(N,LY+1,2)
     1/RESIS+P(N,LY,1)*B(N,LY)/DELT)/(B(N,LY)/DELT-2./RESIS)
      DIFF1=ABS(P(N,LY,2)-PLAST)
      IF(DIFF1.GT.DIFMAX)NMAX=N
      IF(DIFF1.GT.DIFMAX)LMAX=LY
      IF(DIFF1.GT.DIFMAX)DIFMAX=DIFF1
      TARRED(LY)=TARRED(LY)+ARRED(N,LY)
      ACC(N,LY)=B(N,LY)*(P(N,LY,2)-P(N,LY,1))/DELT
      TSTOR(LY)=TSTOR(LY)+ACC(N,LY)
190   CONTINUE
200   CONTINUE
      IF(ITER.EQ.1)DMA=DIFMAX
      IF(ITER.EQ.1)NDMA=NMAX
      IF(ITER.EQ.1)LDMA=LMAX
C     PRINT 205,ITER,DIFMAX,LMAX,NMAX
205   FORMAT(/33H MAX.CHANGE IN P DURING ITERATION,I5,3H IS,F
     112.6,9H IN LAYER,I5,9H AND NODE,I5)
C
C TEST FOR CONVERGENCE
C
      IF(DIFMAX.GT.DIFF2)PRINT 210
210   FORMAT(/15H NO CONVERGENCE//)
      IF(DIFMAX.GT.DIFF2)OMEGA=1.0
      DIFF2=DIFMAX
C
C TEST FOR TERMINATION OF ITERATIONS
C
      IF(ITER.GT.MAXIT)GO TO 220
      IF(DIFMAX.GT.TOL)GO TO 160
C
C IN FIXQ CALCULATION OF INFLOW OR OUTFLOW
C
220   CALL FIXQ(KODE,P,NUMNP,NUMLAY,TINOUT,A,MBAND,Q,VERRES)
C
C DETERMINATION OF CUMULATIVE WATERBALANCE TERMS
C
      CFLTS=CFLTS+TFLTS*DELT
      CFLUN=CFLUN+TFLUN*DELT
      DO 230 LY=1,NUMLAY
      CFLSS(LY)=CFLSS(LY)+TFLSS(LY)*DELT
      CFLLEA(LY)=CFLLEA(LY)+TFLLEA(LY)*DELT
      CARRED(LY)=CARRED(LY)+TARRED(LY)*DELT
      CSTOR(LY)=CSTOR(LY)+TSTOR(LY)*DELT
      CINOUT(LY)=CINOUT(LY)+TINOUT(LY)*DELT
230   CONTINUE
C
C  OPTIONAL PRINTING OF RESULTS
C
      REST=ITS-ITS/NTSPO*NTSPO
      IF(REST.EQ.0.)CALL PRINTO(P,NUMNP,NUMLAY,ST,DELT,CFLTS,TFLTS,CFLSS
     1,TFLSS,CFLLEA,TFLLEA,
     1CARRED,TARRED,CSTOR,TSTOR,CINOUT,TINOUT,CFLUN,TFLUN,KODE,UN,TSYS
     1,SSYS,LEAK,ARRED,ACC,Q)
C
C  DETERMINATION OF MAX CHANGE IN P PER TIMESTEP
C
      DPMAX=.0
      DO 250 LY=1,NUMLAY
      DO 240 N=1,NUMNP
      ABSDIF=ABS(P(N,LY,1)-P(N,LY,2))
      IF(ABSDIF.GT.DPMAX)LM=LY
      IF(ABSDIF.GT.DPMAX)NM=N
      IF(ABSDIF.GT.DPMAX)DPMAX=ABSDIF
240   CONTINUE
250   CONTINUE
      ABSD=ABS(P(NDMA,LDMA,1)-P(NDMA,LDMA,2))
      OMEGA=OMEGA*ABSD/DMA
      IF(OMEGA.GT.1.6)OMEGA=1.6
```

```
      IF(OMEGA.LT..8)OMEGA=.8
      PRINT 260,LM,NN,ST,DPMAX,DELT
  260 FORMAT(/25H MAX.CHANGE IN P IN LAYER,I5,9H AND NODE,I5,8H AT TIME,
     1F8.2/
     1,8H  WAS   ,F10.3,4H PER,F10.3,5H DAYS//)
C
C WRITING OF CALCULATED DATA ON TAPE1
C
      REWIND 1
      DO 280 LY=1,NUMLAY
      DO 270 N=1,NUMNP
      WRITE (1) P(N,LY,2)
  270 CONTINUE
  280 CONTINUE
      DO 290 LY=1,NUMLAY
      WRITE (1) CFLSS(LY),CFLLEA(LY),CARRED(LY),CSTOR(LY),CINOUT(LY)
  290 CONTINUE
      WRITE (1) CFLTS,CFLUN,ST,DELT
      DELT=DELT*1.2
      IF(DELT.GE..2)DELT=.2
      IF(ST.GT..1)NTSPO=10
      IF(ST.LT.TMAX)GO TO 110
      STOP
      END
*DECK ELIN
C
C*******************************************************************
C
      SUBROUTINE ELIN(NUMEL,NUMLAY,SANG,C1,C2,THI,KX,NERR,ESANG,ECANG,E
     1C1,EC2,ETHI,NUMNP)
      DIMENSION ESANG(NUMEL,NUMLAY),ECANG(NUMEL,NUMLAY),EC1(NUMEL,NUMLAY
     1),EC2(NUMEL,NU
     1MLAY),ETHI(NUMEL,NUMLAY),KX(NUMEL,4),SANG(NUMNP,NUMLAY),C1(NUMNP,N
     1UMLAY),C2(NUMNP,NUMLAY),THI(NUMNP,NUMLAY)
      DO 10 N=1,NUMEL
      READ 20 ,(KX(N,I),I=1,4)
   10 CONTINUE
   20 FORMAT(4I5)
      DO 40 LY=1,NUMLAY
      DO 30 N=1,NUMNP
      READ 50,SANG(N,LY),C1(N,LY),C2(N,LY),THI(N,LY)
   30 CONTINUE
   40 CONTINUE
   50 FORMAT(4F10.3)
      DO 80 LY=1,NUMLAY
      DO 70 N=1,NUMEL
      KI=KX(N,1)
      KJ=KX(N,2)
      KL=KX(N,3)
      KM=KX(N,4)
      IF(KL.EQ.KM)GO TO 60
      ESANG(N,LY)=(SANG(KI,LY)+SANG(KJ,LY)+SANG(KL,LY)+SANG(KM,LY))/4.
      EC1(N,LY)=(C1(KI,LY)+C1(KJ,LY)+C1(KL,LY)+C1(KM,LY))/4.
      EC2(N,LY)=(C2(KI,LY)+C2(KJ,LY)+C2(KL,LY)+C2(KM,LY))/4.
      ETHI(N,LY)=(THI(KI,LY)+THI(KJ,LY)+THI(KL,LY)+THI(KM,LY))/4.
      GO TO 70
   60 ESANG(N,LY)=(SANG(KI,LY)+SANG(KJ,LY)+SANG(KL,LY))/3.
      EC1(N,LY)=(C1(KI,LY)+C1(KJ,LY)+C1(KL,LY))/3.
      EC2(N,LY)=(C2(KI,LY)+C2(KJ,LY)+C2(KL,LY))/3.
      ETHI(N,LY)=(THI(KI,LY)+THI(KJ,LY)+THI(KL,LY))/3.
   70 CONTINUE
   80 CONTINUE
C
C PRINTING OF ELEMENT INFORMATION
C
      PRINT 90
   90 FORMAT(/20H ELEMENT INFORMATION//)
      PRINT 100
  100 FORMAT(62H LAYER  ELEM   CORNERS       CONDUCTIVITIES  THICKN. ANG
     1LE    ,//)
      DO 130 LY=1,NUMLAY
      DO 110 N=1,NUMEL
      PRINT 120,LY,N,KX(N,1),KX(N,2),KX(N,3),KX(N,4),EC1(N,LY),EC2(N,LY)
     1,ETHI(N,LY),ESANG(N,LY)
  110 CONTINUE
  120 FORMAT(I5,I6,4I4,4F8.2)
  130 CONTINUE
      AA=3.14159264/180.
      DO 150 LY=1,NUMLAY
      DO 140 N=1,NUMEL
      BB=AA*SANG(N,LY)
      ESANG(N,LY)=SIN(BB)
      ECANG(N,LY)=COS(BB)
  140 CONTINUE
  150 CONTINUE
      RETURN
      END
*DECK NPIN
C
C*******************************************************************
C
      SUBROUTINE NPIN(NUMNP,NUMLAY,KODE,NTERR,X,Y,P,HGL,HBSC,LESC,RADRES
     1,ENRES,VERR
     1ES,WP,NERR,POR,SS)
      DIMENSION KODE(NUMNP),NTERR(NUMNP),X(NUMNP),Y(NUMNP),P(NUMNP,NUMLA
     1Y,2),HGL(NUMNP),
     1HBSC(NUMNP),LESC(NUMNP),RADRES(NUMNP,NUMLAY),ENRES(NUMNP,NUMLAY),V
     1ERRES
     1(NUMNP,NUMLAY),WP(NUMNP,NUMLAY),POR(NUMNP,NUMLAY),SS(NUMNP,NUMLAY)
      DO 10 N=1,NUMNP
      READ 20 ,KODE(N),NTERR(N),X(N),Y(N),HGL(N),HBSC(N),LESC(N)
   10 CONTINUE
   20 FORMAT(2I10,5F10.1)
      DO 40 LY=1,NUMLAY
      DO 30 N=1,NUMNP
      READ 50,P(N,LY,1),VERRES(N,LY),RADRES(N,LY)
     1,ENRES(N,LY),WP(N,LY),POR(N,LY),SS(N,LY)
   30 CONTINUE
   40 CONTINUE
   50 FORMAT(7F10.1)
C
C PRINTING OF NODAL POINT INFORMATION
C
      PRINT 60
   60 FORMAT(//35H STATIONARY NODAL PIONT INFORMATION/)
      PRINT 70
   70 FORMAT(52H  NODE  KODE    X        Y      HGL      HBSC    LESC  /)
      DO 80 N=1,NUMNP
      GO TO 80
      PRINT 90 ,N,KODE(N),X(N),Y(N),HGL(N),HBSC(N),LESC(N)
   80 CONTINUE
   90 FORMAT(I5,I6,5F8.2)
      DO 110 LY=1,NUMLAY
      PRINT 120,LY
      DO 100 N=1,NUMNP
      GO TO 100
      PRINT 130,N,P(N,LY,1),VERRES(N,LY),RADRES(N,LY),ENRES(N,LY),WP(N,L
     1Y),POR(N,LY),SS(N,LY)
  100 CONTINUE
  110 CONTINUE
  120 FORMAT(//24H INFORMATION OF LAYER NO,I5//,5H NODE,
     171H   INIT P      VERRES     RADRES     ENRES     WET PER  POROSITY
```

```
      1 SPEC ST/)
130   FORMAT(I5,7F10.3)
      RETURN
      END
*DECK DYNAMIC
C
C*********************************************************************
C
      SUBROUTINE DYNAMIC(NUMNP,NUMLAY,KODE,P,ARRED,WTSC,IDIV,ST,PREV)
      DIMENSION P(NUMNP,NUMLAY,2),ARRED(NUMNP,NUMLAY),WTSC(NUMNP),KODE(N
     1UMNP)
      DO 30 I=1,IDIV
      DO 20 LY =3,NUMLAY
      DO 10 N=1,NUMNP
      IF(KODE(N).EQ.2)GO TO 10
      READ 40 ,DUM1,DUM2,DUM3
      IF(KODE(N).EQ.1)P(N,LY,1)=DUM1
      IF(KODE(N).EQ.3.AND.LY.EQ.1)WTSC(N)=DUM2
      IF(KODE(N).EQ.5)ARRED(N,LY)=DUM3
10    CONTINUE
20    CONTINUE
30    CONTINUE
40    FORMAT(3F10.1)
      PRINT 50,ST
50    FORMAT(//47H NON-STATIONARY NODAL POINT INFORMATION AT TIME,
     1F8.2,5H DAYS//,39H LAYER NODE PRESC.HEAD   WTSC      ARRED//)
      DO 70 LY=1,NUMLAY
      DO 60 N=1,NUMNP
      IF(KODE(N).EQ.1)PRINT 80,LY,N,P(N,LY,1)
      IF(KODE(N).EQ.3)PRINT 90,LY,N,WTSC(N)
      IF(KODE(N).EQ.5)PRINT 100,LY,N,ARRED(N,LY)
60    CONTINUE
70    CONTINUE
80    FORMAT(2I5,F10.3)
90    FORMAT(2I5,10X,F10.3)
100   FORMAT(2I5,20X,F10.3)
105   RETURN
      END
*DECK ELEM
C
C*********************************************************************
C
      SUBROUTINE ELEM(KODE,P,EC1,EC2,X,Y,KX,ESANG,ECANG,NUMNP,NUMEL,MBAN
     1D,A,ETHI,ITS,VERRES,NUMLAY,HGL,B,POR,SS,THI,AREA)
      DIMENSION KODE(NUMNP),P(NUMNP,NUMLAY,2),EC1(NUMEL,NUMLAY),EC2(NUME
     1L,NUMLAY
     1),X(NUMNP),Y(NUMNP),KX(NUMEL,4),ESANG(NUMEL,NUMLAY),ECANG(NUMEL,NU
     1MLAY),
     2ETHI(NUMEL,NUMLAY),A(MBAND,NUMNP,NUMLAY),VERRES(NUMNP,NUMLAY),LM(4
     1),E(3,3)
     1,IX(3),S(4,4),HGL(NUMNP),B(NUMNP,NUMLAY),POR(NUMNP,NUMLAY),SS(NUMN
     1P,NUMLAY),THI(NUMNP,NUMLAY),AREA(NUMNP)
      I=1
      IF(ITS.EQ.0.AND.NUMLAY.GT.1)I=2
      IF(ITS.EQ.0.AND.NUMLAY.EQ.1)RETURN
      NLAYER=NUMLAY
      IF(ITS.GT.0)NLAYER=1
      DO 6 LY=I,NLAYER
      DO 4 N=1,NUMNP
      B(N,LY)=.0
      AREA(N)=.0
      DO 2 M=1,MBAND
      A(M,N,LY)=.0
2     CONTINUE
4     CONTINUE
6     CONTINUE
C
C DETERMINE CONDUCTIVITY TENSOR
C
      IAR=0
      DO 100 LY=I,NLAYER
      IAR=IAR+1
      DO 90 N=1,NUMEL
      SIN2=ESANG(N,LY)*ESANG(N,LY)
      COS2=ECANG(N,LY)*ECANG(N,LY)
      CONDI=(EC1(N,LY)*COS2+EC2(N,LY)*SIN2)*ETHI(N,LY)
      CONDJ=(EC1(N,LY)*SIN2+EC2(N,LY)*COS2)*ETHI(N,LY)
      CONDK=((EC1(N,LY)-EC2(N,LY))*ESANG(N,LY)*ECANG(N,LY))*ETHI(N
     1,LY)
      IF(LY.NE.1)GO TO 10
      KI=KX(N,1)
      KJ=KX(N,2)
      KL=KX(N,3)
      KM=KX(N,4)
      PGEM=(P(KI,1,2)+P(KJ,1,2)+P(KL,1,2)+P(KM,1,2))/4.
      IF(KL.EQ.KM)PGEM=(P(KI,1,2)+P(KJ,1,2)+P(KL,1,2))/3.
      HGLGEM=(HGL(KI)+HGL(KJ)+HGL(KL)+HGL(KM))/4.
      IF(KL.EQ.KM)HGLGEM=(HGL(KI)+HGL(KJ)+HGL(KL))/3.
      KHGEM=PGEM-(HGLGEM-ETHI(N,1))
      CONDI=CONDI*KHGEM/ETHI(N,LY)
      CONDJ=CONDJ*KHGEM/ETHI(N,LY)
      CONDK=CONDK*KHGEM/ETHI(N,LY)
      IF(CONDI.LT.0.)CONDI=.0
      IF(CONDJ.LT.0.)CONDJ=.0
      IF(CONDK.LT.0.)CONDK=.0
10    DO 40 I=1,4
      LM(I)=KX(N,I)
      DO 30 J=1,4
      S(I,J)=.0
30    CONTINUE
40    CONTINUE
      NUS=4
      IF(LM(3).EQ.LM(4))NUS=3
C
C LOOP ON SUBELEMENTS
C
      KK=2
      IF(NUS.EQ.3)KK=1
      DO 60 K=1,KK
      I=LM(1)
      J=LM(K+1)
      L=LM(K+2)
      CI=X(L)-X(J)
      CJ=X(I)-X(L)
      CK=X(J)-X(I)
      BI=Y(J)-Y(L)
      BJ=Y(L)-Y(I)
      BK=Y(I)-Y(J)
      DEL2=(CK*BJ-CJ*BK)/2.
C
C DEL2 IS AREA TRIANGLE
C
      E(1,1)=CONDI*BI*BI+2.*CONDK*BI*CI+CONDJ*CI*CI
      E(1,2)=CONDI*BI*BJ+CONDK*(BI*CJ+CJ*BJ)+CONDJ*CI*CJ
      E(1,3)=CONDI*BI*BK+CONDK*(BI*CK+CI*BK)+CONDJ*CI*CK
      E(2,1)=E(1,2)
      E(2,2)=CONDI*BJ*BJ+2.*CONDK*BJ*CJ+CONDJ*CJ*CJ
      E(2,3)=CONDI*BJ*BK+CONDK*(BJ*CK+BK*CJ)+CONDJ*CJ*CK
      E(3,1)=E(1,3)
      E(3,2)=E(2,3)
      E(3,3)=CONDI*BK*BK+2.*CONDK*BK*CK+CONDJ*CK*CK
      IX(1)=1
```

```
      IX(2)=K+1
      IX(3)=K+2
      DO 50 I=1,3
      II=IX(I)
      IJ=LM(II)
      BB=.0
      DO 45 J=1,3
      JJ=IX(J)
      JI=LM(JJ)
      BB=BB+SS(JI,LY)*THI(JI,LY)+POR(JI,LY)
      S(II,JJ)=S(II,JJ)+E(I,J)/(4.*DEL2)
45    CONTINUE
      B(IJ,LY)=B(IJ,LY)+(POR(IJ,LY)+SS(IJ,LY)*THI(IJ,LY)+BB)*DEL2/4.
      IF(IAR.EQ.1)AREA(IJ)=AREA(IJ)+DEL2/3.
50    CONTINUE
60    CONTINUE
C
C ADD ELEMENT CONTRIBUTION TO A-MATRIX
C
      DO 80 L=1,4
      I=LM(L)
      DO 70 K=1,4
      J=LM(K)-I+1
      IF(J.LT.1)GO TO 70
      A(J,I,LY)=A(J,I,LY)+S(L,K)
70    CONTINUE
80    CONTINUE
90    CONTINUE
100   CONTINUE
      RETURN
      END
*DECK SOILWAT
C
      SUBROUTINE SOILWAT(P,NUMNP,NUMLAY,AREA,FLOWUN,POR,UN)
      DIMENSION P(NUMNP,NUMLAY,2),AREA(NUMNP),POR(NUMNP,NUMLAY)
     1,UN(NUMNP)
      RETURN
      END
*DECK TERSYS
C
C********************************************************************
C
      SUBROUTINE TERSYS(KODE,P,NUMNP,NUMLAY,FLOWTS,NTERR,AREA,N,HGL)
      DIMENSION KODE(NUMNP),P(NUMNP,NUMLAY,2),AREA(NUMNP),NTERR(NUMNP),H
     1GL(NUMNP)
      DEPTH=HGL(N)-P(N,1,2)
      IF(DEPTH.LT.0.)DEPTH=.0
      IF(NTERR(N).EQ.1)FLOWTS=.0
      IF(NTERR(N).EQ.2)FLOWTS=.0
      IF(NTERR(N).EQ.3)FLOWTS=.0
      RETURN
      END
*DECK SECSYS
C
C********************************************************************
C
      SUBROUTINE SECSYS(P,NUMNP,NUMLAY,LESC,HBSC,RADRES,ENRES,FLOWSS,N,L
     1Y,WP,WTSC,VERRES)
      DIMENSION P(NUMNP,NUMLAY,2),LESC(NUMNP),HBSC(NUMNP),RADRES(NUMNP,N
     1UMLAY),
     1ENRES(NUMNP,NUMLAY),WP(NUMNP,NUMLAY),WTSC(NUMNP),VERRES(NUMNP,NUML
     1AY)
      IF(VERRES(N,LY).GT.0.)RETURN
      IF(WTSC(N).LE.HBSC(N))GO TO 10
      IF(P(N,LY,2).LE.WTSC(N))ENRES(N,LY)=2.0*ENRES(N,LY)
      FLOWSS=((P(N,LY,2)-WTSC(N))/RADRES(N,LY)+((P(N,LY,2)-WTSC(N))/ENRE
     1S(N,LY))*WP(N,LY))*LESC(N)
      IF(P(N,LY,2).LE.WTSC(N))ENRES(N,LY)=.5*ENRES(N,LY)
10    RETURN
      END
*DECK LEAKAGE
C
C********************************************************************
C
      SUBROUTINE LEAKAGE(P,NUMNP,NUMLAY,VERRES,THI,AREA,FLOWLE,N,LY,HGL)
      DIMENSION P(NUMNP,NUMLAY,2),VERRES(NUMNP,NUMLAY),THI(NUMNP,NUMLAY)
     1,AREA(NUMNP),HGL(NUMNP)
      IF(NUMLAY.EQ.1)RETURN
      IF(LY.EQ.1.AND.P(N,1,2).LT.(HGL(N)-THI(N,1)))RETURN
      IF(LY.EQ.1.AND.VERRES(N,1).GT.0.)FLOWLE=AREA(N)*(P(N,1,2)-P(N,2,2)
     1)/(
     1VERRES(N,LY)*(P(N,1,2)-(HGL(N)-THI(N,1)))/THI(N,1))
      IF(LY.EQ.1.AND.VERRES(N,LY+1).GT.0.)FLOWLE=AREA(N)*(P(N,1,2)-P(N,2
     1,2))/(.5*VERRES(N,LY+1))
      IF(LY.EQ.1)RETURN
      IF(LY.EQ.NUMLAY)FLOWLE=AREA(N)*(P(N,LY,2)-P(N,LY-1,2))/(.5*VERRES(
     1N,LY-1))
      IF(LY.EQ.NUMLAY)RETURN
      IF(VERRES(N,LY).EQ.0.)FLOWLE=AREA(N)*((P(N,LY,2)-P(N,LY-1,2))/(.5*
     1VERRES
     1(N,LY-1))+(P(N,LY,2)-P(N,LY+1,2))/(.5*VERRES(N,LY+1)))
      IF(VERRES(N,LY).NE.0.)FLOWLE=AREA(N)*((2.*P(N,LY,2)-P(N,LY-1,2)-P(
     1N,LY+1,2))/(.5*VERRES(N,LY)))
      RETURN
      END
*DECK ATBT
C
C********************************************************************
C
      SUBROUTINE ATBT(P,NUMNP,NUMLAY,A,B,DELT,ARRED,FLOWUN,FLOWTS,FLOWSS
     1,FLOWLE,N,LY,AT,BT,MBAND,VERRES,UN)
      DIMENSION P(NUMNP,NUMLAY,2),A(MBAND,NUMNP,NUMLAY),B(NUMNP,NUMLAY),
     1ARRED(NUMNP,NUMLAY),AT(MBAND),UN(NUMNP),VERRES(NUMNP,NUMLAY)
C     PRINT 5,N,LY,ARRED(N,LY),FLOWLE,B(N,LY)
C     IF(VERRES(N,LY).NE.0.)GO TO 20
      DO 10 M=1,MBAND
      AT(M)=A(M,N,LY)
10    CONTINUE
      AT(1)=AT(1)+B(N,LY)/DELT
      BT=ARRED(N,LY)-UN(N)-FLOWTS-FLOWSS-FLOWLE+(B(N,LY)/DEL
     1T)*P(N,LY,1)
      GO TO 40
20    DO 30 M=2,MBAND
      AT(M)=.0
30    CONTINUE
      AT(1)=+B(N,LY)/DELT
      BT=-FLOWLE+(B(N,LY)/DELT)*P(N,LY,1)
40    RETURN
      END
*DECK SOLVE
C
C********************************************************************
C
      SUBROUTINE SOLVE(P,NUMNP,NUMLAY,PLAST,AT,BT,MBAND,KODE,N,LY,OMEGA,
     1HGL,A,VERRES)
      DIMENSION P(NUMNP,NUMLAY,2),AT(MBAND),KODE(NUMNP),HGL(NUMNP),A(MBA
     1ND,NUMNP,NUMLAY),VERRES(NUMNP,NUMLAY)
      PLAST=P(N,LY,2)
      IF(KODE(N).EQ.1)RETURN
      SIGMA=.0
      IF(VERRES(N,LY).NE.0.)OMEGA=OMEGA*.5
C     IF(VERRES(N,LY).NE.0.)GO TO 32
```

```
      DO 20 M=2,MBAND
      K=N+M-1
      IF(K.GT.NUMNP)GO TO 20
      SIGMA=SIGMA+AT(M)*P(K,LY,2)
20    CONTINUE
      DO 30 M=2,MBAND
      L=N-M+1
      IF(L.LT.1)GO TO 30
      SIGMA=SIGMA+A(M,L,LY)*P(L,LY,2)
30    CONTINUE
C     PRINT 31,N,LY,AT(1),SIGMA,BT
31    FORMAT(2I5,3F10.3)
32    P(N,LY,2)=PLAST+OMEGA*(1./AT(1)*(-SIGMA+BT)-PLAST)
C     IF(VERRES(N,LY).NE.0.)PRINT 45,N,LY,AT(1),BT
45    FORMAT(2I5,2F12.5)
      IF(P(N,1,2).LT.0.)P(N,1,2)=0.
      IF(P(N,1,2).LT.0.)PRINT 35,N
35    FORMAT(/10H P IN NODE,I5,21H IS SMALLER THAN ZERO//)
      IF(P(N,1,2).GT.HGL(N))P(N,1,2)=HGL(N)
      IF(VERRES(N,LY).NE.0.)OMEGA=OMEGA*2.
      RETURN
      END
*DECK FIXQ
C
C*******************************************************************
C
      SUBROUTINE FIXQ(KODE,P,NUMNP,NUMLAY,TINOUT,A,MBAND,Q,VERRES)
      DIMENSION A(MBAND,NUMNP,NUMLAY),TINOUT(NUMLAY),KODE(NUMNP)
     1,P(NUMNP,NUMLAY,2),Q(NUMNP,NUMLAY),VERRES(NUMNP,NUMLAY)
      DO 40 LY=1,NUMLAY
      IF(VERRES(1,LY).GT.0.)GO TO 40
      DO 30 N=1,NUMNP
      IF(KODE(N).NE.1)GO TO 30
      Q(N,LY)=-A(1,N,LY)*P(N,LY,2)
      DO 20 J=2,MBAND
      K=N-J+1
      IF(K.LT.1)GO TO 10
      Q(N,LY)=Q(N,LY)-A(J,K,LY)*P(K,LY,2)
10    K=N+J-1
      IF(K.GT.NUMNP)GO TO 20
      Q(N,LY)=Q(N,LY)-A(J,N,LY)*P(K,LY,2)
20    CONTINUE
      TINOUT(LY)=TINOUT(LY)+Q(N,LY)
30    CONTINUE
40    CONTINUE
      RETURN
      END
*DECK PRINTO
C
C*******************************************************************
C
      SUBROUTINE PRINTO(P,NUMNP,NUMLAY,ST,DELT,CFLTS,TFLTS,CFLSS,TFLSS,C
     1FLLEA,
     1TFLLEA,CARRED,TARRED,CSTOR,TSTOR,CINOUT,TINOUT,CFLUN,TFLUN,KODE,
     1UN,TSYS,SSYS,LEAK,ARRED,ACC,Q)
      DIMENSION P(NUMNP,NUMLAY,2),KODE(NUMNP),UN(NUMNP),TSYS(NUMNP),SSYS
     1(NUMNP,
     1NUMLAY),LEAK(NUMNP,NUMLAY),ARRED(NUMNP,NUMLAY),ACC(NUMNP,NUMLAY),Q
     1(NUMNP,NUMLAY),
     1         CFLSS(NUMLAY),TFLSS(NUMLAY),CFLLEA(NUMLAY)
     1,TFLLEA(NUMLAY),CARRED(NUM
     1LAY),TARRED(NUMLAY),CSTOR(NUMLAY),TSTOR(NUMLAY),CINOUT(NUMLAY),TIN
     1OUT(NUMLAY)
      PRINT 10,ST
10    FORMAT(///50H HEAD AND TERMS OF WATERBALANCE PER NODE PER LAYER,//
     1,
     1,8H AT TIME,F15.3,19H IN M AND M**3/DAYS,//)
      PRINT 20
20    FORMAT(55H LAYER NODE KODE      HEAD      FLOWUN      FLOWTS      FLOWSS
     1,39H  LEAKAGE      ARRED      STORAGE  LAT FLOW)
      DO 30 LY=1,NUMLAY
      PRINT 22
22    FORMAT(//)
      DO 25 N=1,NUMNP,6
      IF(LY.EQ.1)PRINT 40,LY,N,KODE(N),P(N,LY,2),UN(N),TSYS(N),SSYS(N,1)
     1,LEAK(N,
     11),ARRED(N,1),ACC(N,1),Q(N,1)
      IF(LY.NE.1)PRINT 50,LY,N,    KODE(N),P(N,LY,2),SSYS(N,LY),LEAK(N,LY
     1),ARRED(N,LY),ACC(N,LY),Q(N,LY)
25    CONTINUE
30    CONTINUE
40    FORMAT(3I5,8F10.3)
50    FORMAT(3I5,F10.3,20X,5F10.3)
      PRINT 60
60    FORMAT(//57H TERMS OF WATERBALANCE PER LAYER DURING LAST TIMESTEP
     1AND,//,
     171H AND SINCE BEGINNING IN                                    M**3/DAY
     1 M**3
     1//)
      PRINT 70,TFLUN,CFLUN
70    FORMAT(45H FLOW TO UNSATURATED ZONE                         ,2F12.3/)
      PRINT 80,TFLTS,CFLTS
80    FORMAT(45H FLOW TO TERTIARY SYSTEM                          ,2F12.3/)
      DO 150 LY=1,NUMLAY
      PRINT 90,LY
90    FORMAT(/9H LAYER NO,I10/)
      PRINT 100,TFLSS(LY),CFLSS(LY)
100   FORMAT(45H FLOW TO SECONDARY SYSTEM                         ,2F12.3/)
      PRINT 110,TFLLEA(LY),CFLLEA(LY)
110   FORMAT(45H FLOW TO ADJACENT LAYER(S)                        ,2F12.3/)
      PRINT 120,TARRED(LY),CARRED(LY)
120   FORMAT(45H ARTIFICIAL FLOW AND PRESCRIBED BOUNDARY FLOW,2F12.3/)
      PRINT 130,TSTOR(LY),CSTOR(LY)
130   FORMAT(45H ACCUMULATION TERMS                               ,2F12.3/)
      PRINT 140,TINOUT(LY),CINOUT(LY)
140   FORMAT(45H LATERAL FLOW THROUGH PRESCR. HEAD BOUNDARY  ,2F12.3/)
150   CONTINUE
      RETURN
      END
```

APPENDIX B: EXAMPLE OF OUTPUT

HEAD AND TERMS OF WATERBALANCE PER NODE PER LAYER

AT TIME .260 IN M AND M**3/DAYS

LAYER	NODE	KODE	HEAD	FLOWUN	FLOWTS	FLOWSS	LEAKAGE	ARRED	STORAGE	LAT FLOW
1	1	1	125.000	0.000	0.000	0.000	-.000	0.000	0.000	.000
1	7	2	125.000	0.000	0.000	0.000	-.000	0.000	.000	0.000
1	13	2	125.000	0.000	0.000	0.000	-.000	0.000	.000	0.000
1	19	2	125.000	0.000	0.000	0.000	-.000	0.000	.000	0.000
1	25	2	125.000	0.000	0.000	0.000	-.000	0.000	.000	0.000
1	31	2	125.000	0.000	0.000	0.000	-.000	0.000	.000	0.000
1	37	2	125.000	0.000	0.000	0.000	-.000	0.000	.000	0.000
1	43	5	125.000	0.000	0.000	0.000	0.000	0.000	.000	0.000
2	1	1	125.000			0.000	.000	0.000	.000	0.000
2	7	2	125.000			0.000	-.001	0.000	.001	0.000
2	13	2	125.000			0.000	-.007	0.000	.007	0.000
2	19	2	125.000			0.000	-.023	0.000	.023	0.000
2	25	2	125.000			0.000	-.020	0.000	.020	0.000
2	31	2	125.000			0.000	-.009	0.000	.009	0.000
2	37	2	125.000			0.000	-.003	0.000	.003	0.000
2	43	5	125.000			0.000	0.000	0.000	.003	0.000
3	1	1	125.000			0.000	-.000	0.000	0.000	0.000
3	7	2	125.000			0.000	.001	0.000	.182	0.000
3	13	2	124.998			0.000	.007	0.000	1.982	0.000
3	19	2	124.972			0.000	.023	0.000	3.862	0.000
3	25	2	124.895			0.000	.020	0.000	1.674	0.000
3	31	2	124.787			0.000	.009	0.000	.449	0.000
3	37	2	124.647			0.000	.003	0.000	.078	0.000
3	43	5	124.358			0.000	0.000	50.000	.041	0.000

TERMS OF WATERBALANCE PER LAYER DURING LAST TIMESTEP AND

AND SINCE BEGINNING IN	M**3/DAY	M**3
FLOW TO UNSATURATED ZONE	0.000	0.000
FLOW TO TERTIARY SYSTEM	0.000	0.000
LAYER NO 1		
FLOW TO SECONDARY SYSTEM	0.000	0.000
FLOW TO ADJACENT LAYER(S)	-.000	-.000
ARTIFICIAL FLOW AND PRESCRIBED BOUNDARY FLOW	0.000	0.000
ACCUMULATION TERMS	.000	.000
LATERAL FLOW THROUGH PRESCR. HEAD BOUNDARY	.000	.000
LAYER NO 2		
FLOW TO SECONDARY SYSTEM	0.000	0.000
FLOW TO ADJACENT LAYER(S)	-.378	-.055
ARTIFICIAL FLOW AND PRESCRIBED BOUNDARY FLOW	0.000	0.000
ACCUMULATION TERMS	.381	.056
LATERAL FLOW THROUGH PRESCR. HEAD BOUNDARY	0.000	0.000
LAYER NO 3		
FLOW TO SECONDARY SYSTEM	0.000	0.000
FLOW TO ADJACENT LAYER(S)	.378	.055
ARTIFICIAL FLOW AND PRESCRIBED BOUNDARY FLOW	50.000	12.979
ACCUMULATION TERMS	49.376	12.909
LATERAL FLOW THROUGH PRESCR. HEAD BOUNDARY	0.000	0.000