## 4.2 Simulation of the soil water balance

L. Stroosnijder

### *4.2.1 Introduction*

As stated in the previous contribution (Section 4.1) the soil-water status influences crop growth in several ways, i.e. '... that prolonged water stress influences some of the basic plant properties', 'the functional balance between shoot and root implies that moisture stress in the plant leads to sub-optimal growth rates for the above ground plant parts, which results in increased growth of the roots and hence in a shift in the shoot-root ratio' and, finally, 'a second cause of death, especially important in the present context, is that of insufficient moisture in the soil'. Thus knowledge about the stock of moisture in the root zone is needed to be able to calculate crop growth. This includes knowledge about the spatial (in the vertical direction) distribution and availability of moisture to simulate root behaviour correctly.

The main source for the soil-water stock is rainfall (Figure 49). However part of the rain is lost due to interception by the plant cover and run-off. The remaining rain penetrates the soil by a process called infiltration and is distributed over different soil layers. The soil-water stock can also be replenished by water that flows upward from a water-table, i.e. by capillary action. Main sources of

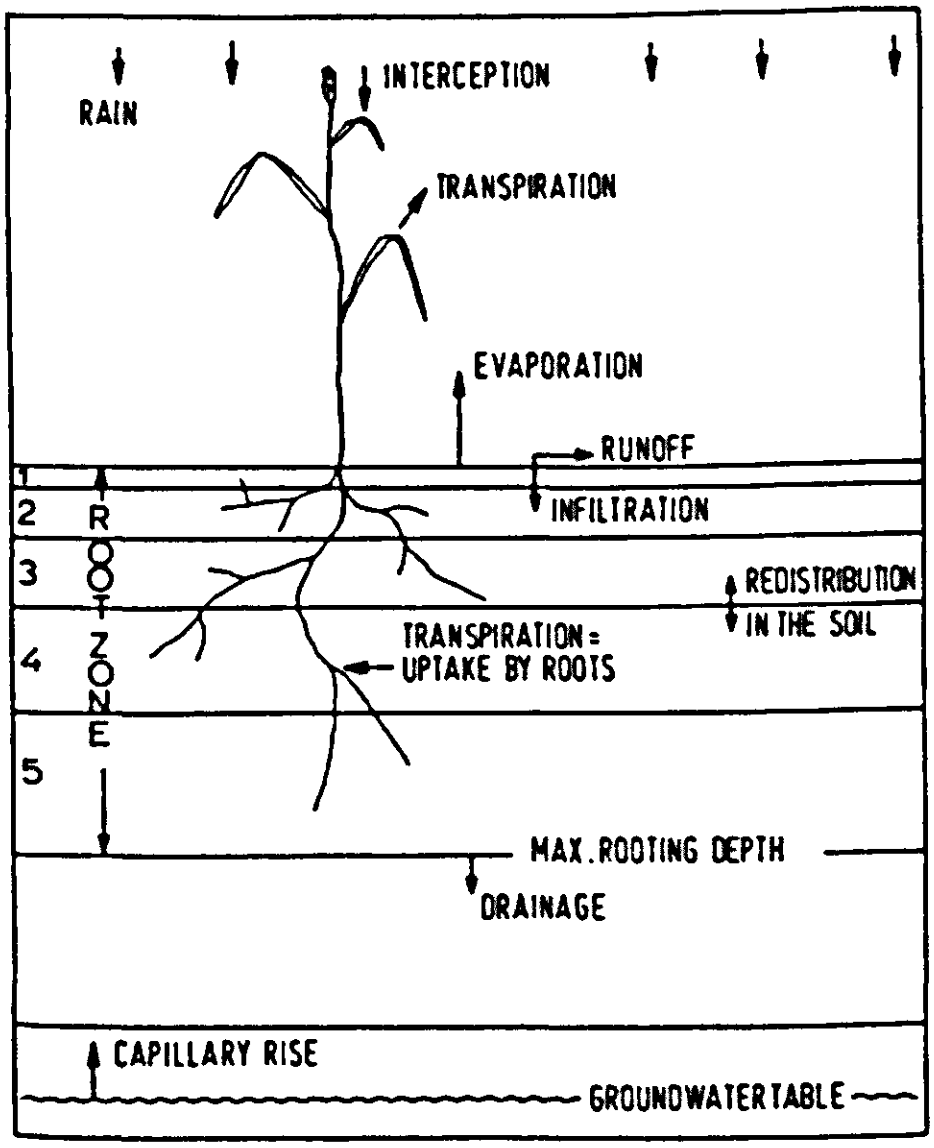


Figure 49. Schematic view of different elements and processes of the soil water balance.

depletion of the soil-water stock are uptake by plant roots (= transpiration), evaporation of soil water to the atmosphere, and drainage below the maximum rooting depth. To know how much and where water is available for plant growth, all sources of soil water must be considered, so that the soil-water balance can be quantitatively understood and simulated. As shown in Figure 49, the soil may be thought to be divided up into horizontal layers. Water is not static in the soil, but can be redistributed by flowing from one layer to another. A simple approach to the soil-water balance is to consider the water content of each layer as a separate state variable, and to describe the flow into and out of each compartment separately. This rate of flow depends on the driving force on the water, which is the sum of the gradient of the potential with which water is held by the soil and gravitational force, and on the hydraulic conductivity of the soil. If this force was proportional to the soil water content and if the conductivity was constant, the simulation model would be really simple. However, the relation of the potential of the soil water to the water content of the soil is quite non-linear, and the conductivity for water also depends very much on the water content, as illustrated in Figure 50. Because in reality there is a continuous gradient of the water content in the soil profile, the concept of a soil divided into layers makes it necessary to average water content, potentials and conductivities. Without going into detail, it will be clear that this aspect makes simulation of the soil water balance not a trivial problem. There are two approaches to its simulation. The first, described in this section, is to redefine from classical soil physics, concepts and parameters needed in the approximation of the water balance with a simple model as indicated. This may be called parametric modelling. The second is to follow the classical approach more closely and to develop the simulation program accordingly. This may be called deterministic modelling. This approach is emphasized in Section 4.3.

Subsection 4.2.3 describes a program to simulate the soil water balance as incorporated in the model ARID CROP and SAHEL GRASS NPK by the para-

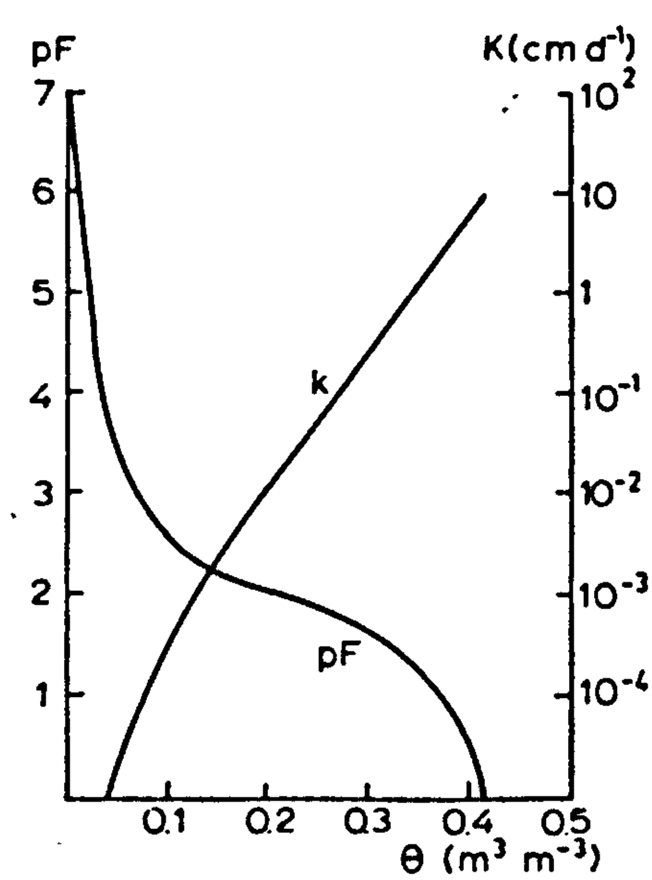


Figure 50. The relation of the soil water potential, expressed as pF (i.e. − lg (soil water potential in mbar)), and the conductivity, $K$, to the soil water content, $\theta$, of loamy soil from the Sahel.

metric approach. Examples are given that are oriented towards semi-arid zones like the West African Sahel zone (Subsection 4.2.3). But first the link between parametric and deterministic modelling is discussed (Subsection 4.2.2).

### *4.2.2 Deterministic modelling of the flow of water in soils*

The flow of water in a soil can be described mathematically with a partial non-linear differential equation – partial in time and space. This general flow equation is based on two basic (soil) physical principles (laws). For one-dimensional flow these are the (empirical) law of Darcy (Equations 54 and 55) and the mass continuity Equation 56:

$$q = -K(h)\,\frac{\delta H}{\delta z} \tag{54}$$

in which $q$ is the soil water flux ($m^3\ m^{-2}\ s^{-1}$); $K(h)$ the hydraulic conductivity ($m\ s^{-1}$), as a function of the soil water pressure head, $h$; $z$ the vertical coordinate (m), with origin at the soil surface and for which upwards is taken as positive; and $H$ the hydraulic head (m), which is the sum of the soil water pressure head, $h$, and the gravitational head, $z$. Thus Equation 54 can also be written as:

$$q = -K(h)\left(\frac{\delta h}{\delta z} + 1\right) \tag{55}$$

$$\frac{\delta\theta}{\delta t} = -\frac{\delta q}{\delta z} - S \tag{56}$$

where $\theta$ is the volumetric moisture content ($m^3\ m^{-3}$), $t$ the time (s) and $S$ the volume of water taken up by the roots per unit bulk volume of soil in unit time ($m^3\ m^{-3}\ s^{-1}$). Equation 56 states simply that in one soil element, the rate of change of the water content with time equals the flow out of the element through its boundaries plus its flow out through the roots that it contains. Darcy's law as well as the continuity principle are in Figure 51.

To simulate the soil water balance, the soil is considered to exist of horizontal layers, usually 3-10. Equation 54 is used to calculate the rate of flow of water between the centres of two adjacent homogeneous soil compartments in dependence on the value of the state variables. After having calculated all rates of flow, the state variables in each compartment are updated by an integration with respect to time (equivalent to Equation 56), after which new flow rates can be calculated for the following time step. Within a time step, water flow is by definition stationary (Subsection 1.1.3). Clearly, the rates used in a certain time step are calculated parallel and not mutually dependent, so that the order in which they are calculated does not matter. In essence this solution method is a matter of accurate bookkeeping.

Combination of Equations 54 and 56 leads to a non-linear partial differential equation of first order in $t$ and second order in $z$ (Equation 57) with two inde-

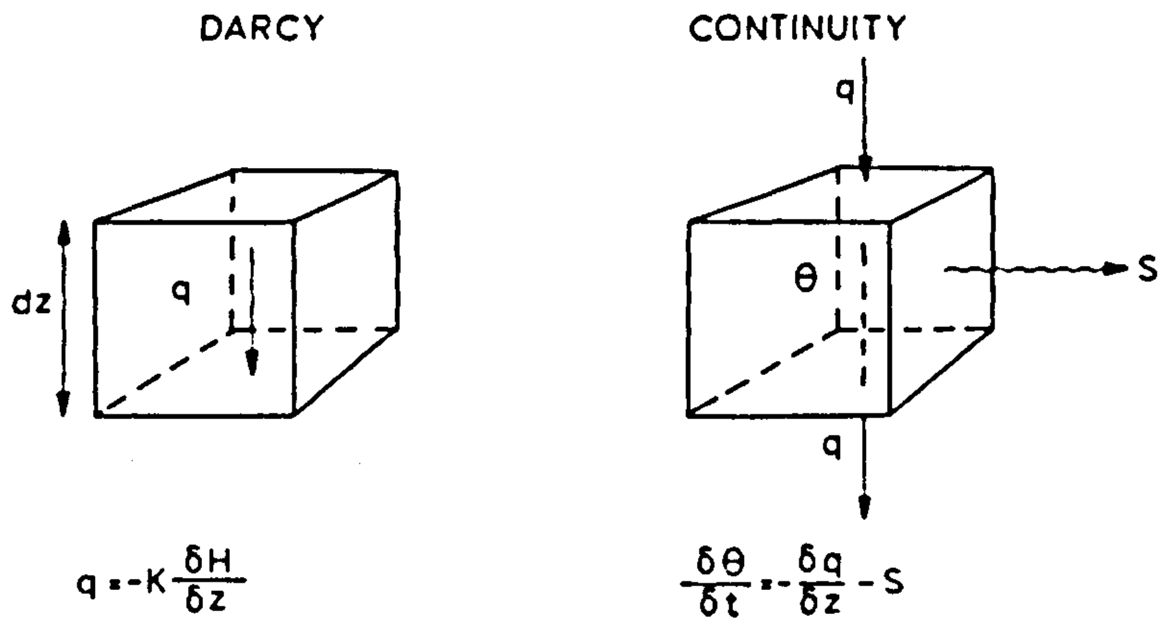


Figure 51. Schematic representation of the principle of flow and mass continuity including a sink term for water uptake by plant roots (Feddes et al., 1978).

pendent variables, $z$ and $t$ and two dependent variables, $\theta$ and $h$. The dependence between these last two variables is known as the soil-moisture characteristic or retention curve of the soil (Figure 50). Using this $h(\theta)$ relation, one may generate a general flow equation with one dependent variable only, either in $\theta$ or in $h$:

$$C(h) \frac{\delta h}{\delta t} = \frac{\delta}{\delta z} [K(h) (\frac{\delta h}{\delta z} + 1)] - S \tag{57}$$

with $C(h) = d\theta/dh$, the differential moisture capacity of the soil, which is a function of $\theta$ or of $h$. This form is more widely applicable, for positive (saturated soil) as well as for negative (unsaturated soil) values of $h$ and in heterogeneous soils, than the $\theta$ form (Stroosnijder, 1976).

With a computer available, this Equation 57 can be solved by the finite difference approach (Subsection 2.1.4); the variables place and time, which in reality are continuous, are divided into small intervals so that the situation approaches continuity. Space in the soil is divided in a number of gridpoints, while time is divided in time steps. Equation 57 can be expressed in finite difference form in many ways. There are elaborate implicit approximations (or schemes) where a whole matrix of equations must be solved for each time step (e.g. Section 4.3). There are also the more simple explicit approximations where an unknown value of the state variable is calculated from a number of known values of the same state variable (see Figure 52). An excellent review of the most commonly used approximations and related computer solution schemes has been published by Vauclin et al. (1979).

An example of a finite difference form of Equation 57 is:

$$C_i^j \frac{h_i^{j+1} - h_i^j}{\Delta t} = \frac{1}{\Delta z} [K_{i+½}^j ( \frac{h_i^j - h_{i+1}^j}{\Delta z} + 1) - K_{i-½}^j ( \frac{h_{i-1}^j - h_i^j}{\Delta z} + 1)] - S_i^j \tag{58}$$

which can be visualized with help of a gridpoint scheme as shown in Figure 52.

Emphasis is given here only to the above explicit approximation of Equation 57, since such an approximation is automatically used if the state variable approach is chosen. The latter is most commonly used in dynamic simulation models of the explanatory type (Section 1.1). Such models clearly distinguish state, rate and driving forces. As a consequence, the combined Equation 57 is not used in those models, but the more basic Equations 54 and 56.

Obviously, this simulation approach must be considered as an explicit solution for which stability and convergence determine rather stringently time step and compartment size. It was shown by van Keulen & van Beek (1971) that the time step taken must be small enough to avoid oscillation. The smallest time step is caused by the infiltration process, when water flows from a very wet compartment into a very dry one. The condition for the time step is then

$$\Delta t < (\Delta z)^2/D(\theta), \text{ with } D(\theta) \doteq K(\theta)/C(\theta) \qquad (59)$$

$D(\theta)$ is the soil water diffusivity ($m^2\ s^{-1}$). According to Stroosnijder (1976) values for $D(\theta)$ in wet soil vary between $10^{-2}\ m^2\ s^{-1}$ for sand to $10^{-4}\ m^2\ s^{-1}$ for clay. Thus for a layer near the soil surface of 2 cm thickness, the time step to be used in the simulation of infiltration of water in dry sand equals:

$$\Delta t \leq 0.04\ \text{s} \qquad (60)$$

Equation 59 implies that the execution time of a simulation run may be reduced by increasing layer thickness and adjusting the time step as a function of the soil water diffusivity, $D(\theta)$. Obviously, the choice of layer thickness is related to the problem and to the accuracy desired. If one's problem deals with very steep moisture gradients, as in evaporation, one is forced to use rather small layers (e.g. 2 cm) to solve the problem not only in a deterministic way but also in a physically realistic way. On the other hand, the above example of a time step indicates a kind of minimum; for other soil water flow processes, like evaporation, much smaller values of $D(\theta)$ are involved and hence much larger time steps can be used. For a discussion of time step size and integration method, see Subsections 2.3.5, 2.3.6 and 2.3.7.

As can be seen from Figure 52 and Equation 58, one has to choose some

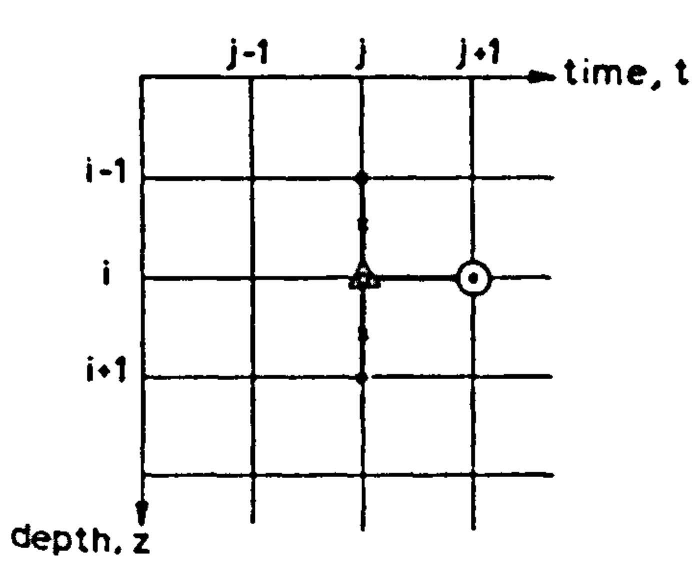


Figure 52. Illustration of Equation 58: the three known values of the state variable $h$ at known time level $j$ and gridpoints $i-1$, $i$ and $i+1$ (•) are used to calculate one new value of $h$ at time $j+1$ and place $i$ (o). Soil parameters are used at gridpoint $i$ (Δ) and in between the gridpoints $i-1$ and $i$ and $i+1$ (x).

method for obtaining the value of the soil parameter $K(\theta)$ between two adjacent compartments. Several averaging methods seem possible:
- averaging of the conductivities, as is done in electricity for series resistance;
- average first the moisture contents of the two adjacent compartments and determine the corresponding conductivity;
- average the conductivities according to different weighting procedures.

The averaging can either be done taking account of the compartment size or not. The latter procedure, combined with simple arithmetic averaging of conductivities often give the best results when compared to analytical solutions (Rietveld, 1978). It can also be seen from the Figure 52 that (since an explicit approximation is used) only one known value of the soil water capacity, $C(\theta)$, is used and no linearization (Vauclin et al., 1979) with respect to time.

In spite of the above difficulties, a variety of useful deterministic models were developed (van Keulen & van Beek, 1971; de Wit & van Keulen, 1972; Stroosnijder, 1976; Hillel, 1977; Shaykewich & Stroosnijder, 1977; Rietveld, 1978; van Loon & Wösten, 1979). Using the terminology of the Subsections 1.1.2 and 1.3.1, these are explanatory, comprehensive models.

The discussion above, in particular on the integration time step, makes it clear that only relatively short simulation runs can be made at relatively high computer costs. It appears that the time step necessary for a deterministic simulation of soil-water flow is several orders of magnitude smaller than necessary for other elements in simulation models for crop growth. Such models, like ARID CROP (Section 4.1, van Keulen, 1975; van Keulen et al., 1981) and SAHEL GRASS NPK (a model used for grass growth under tropical semi-arid conditions with optimal supply of nutrients and natural rain) use a time step of one day. Thus in these models soil-water flow cannot be simulated in a deterministic way but must be done in a parametric way, i.e. simplified submodels must be developed that simulate the various aspects of the soil-water balance as well as possible with a time step of one day. The term parametric model is used to indicate that if deterministic models cannot be used, alternative models have to be developed, but also those models need not be black-box models. Parametric models, discussed in the next subsection, describe processes in a distinctly physical way, but that way is a simplification of the fully physical understanding. Often use is made of overall parameters to describe physical processes on a large time scale that in reality take place on a small time scale. This explains the name parametric model. The necessity of development of parametric models from deterministic models when soil physical and crop physiological processes are combined in one model is a good illustration of the problems of coordination between models of different hierarchical levels (Subsection 1.4.3). The validation ('the best possible') of these simplified models is done with help of detailed deterministic submodels (i.e. following the hierarchical approach) and with experimental data. In some cases, simplification of different deterministic elements of the soil-water balance can be combined into one parametric element. So, infiltration of the water in the soil and the subsequent redistribution of this water over different soil layers were combined. This is due to the fact that redistribution is most important in the range of water contents between saturation and field capacity and is, by definition, very slow at lower moisture contents. This enables one to combine infiltration and redistribution in such a way that the water that enters the soil is directly distributed (in a parametric way) over different soil layers so that no layer becomes wetter than field capacity. This almost completely eliminates the need for a further computation of moisture redistribution. This has been proven for the prevailing choice of layer thickness and time step of integration as used for the description of crop growth processes, with refined deterministic models.

### *4.2.3 Parametric modelling of the soil water balance*

The following elements of the soil-water balance, as used in the whole crop models ARID CROP and SAHEL GRASS NPK will be briefly discussed under the headings *Rain, Interception, Runoff, Infiltration, Evaporation* and *Transpiration*. In this discussion the original CSMP statements, as used in the SAHEL GRASS NPK model (August 1980 version) will be used. Since both models were developed for use in semi-arid regions, where often a perma-dry subsoil exists, the following elements (which are necessary in the simulation of the water balance under more humid conditions; see Section 4.3) are omitted from discussion: drainage, water-table and capillary rise.

*Rain*

Rain (RAIN (in mm $d^{-1}$)) is an input to the model in the form of a table (RAINTB) of daily total rainfall; each day with its own number (DAYY).

RAIN = AFGEN (RAINTB, DAYY)

*Interception*

Not all rain reaches the soil surface due to interception (INTC) by the plant canopy. The amount of interception (in mm $d^{-1}$) is calculated according to Makkink & van Heemst (1975) as

INTC = AMIN1(RAIN, INTCAP/DELT)

where INTC is interception (mm $d^{-1}$)
INTCAP is interception capacity (mm)

INTCAP = (1. − FRLT) * FAC * FREWT

where FRLT is fraction of radiation reaching the soil (−)
FAC is the mass fraction interception capacity of fresh weight (kg water $kg^{-1}$ biomass)
FREWT is fresh weight of crop (kg $m^{-2}$)

PARAMETER FAC = 0.2

EFRAIN = RAIN − INTC

where EFRAIN is the effective amount of rain reaching the soil surface (mm $d^{-1}$)

*Runoff*

The procedure for calculating runoff is based on experimental data (Stroosnijder & Koné, 1982). The main input parameter is an average yearly runoff fraction of the rainfall. An additional table, which relates total daily rainfall to degree of runoff, enables the calculation of the amount of runoff on each individual day. Use of this table in combination with an estimated long-term average annual runoff factor enables automatic adaptation to individual years with more or less big rainstorms. This procedure is simulated with the following CSMP statements:

```
RRNOFF = EFRAIN*R*AFGEN(ROFINT, EFRAIN)
```

where RRNOFF is runoff (mm $d^{-1}$)
R is long-term average fraction of rain that runs off (−)
ROFINT is factor to adjust R (in dependence of the total amount of a rainstorm) in order to calculate runoff from an individual storm (−)
To cite an example for a fine sandy soil in Mali we used

```
FUNCTION ROFINT = 0., 0., 5., 0.2, 10., 0.5, 20., 1.2, 30., 1.55, 70., 1.7
PARAMETER R = 0.24
```

---

**Exercise 58**

a. Calculate the cumulative amount of runoff (in mm) of the following rainshowers if the long-term average yearly runoff is 30% (assume interception = 0); precipitation = 12,21,8,53 and 18 mm respectively and the above-mentioned table for ROFINT.
b. What is the average runoff percentage of these five showers?
c. How can this average differ from the R value?

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*Infiltration*

The infiltration rate (INFR) in mm $d^{-1}$ is written as

```
INFR = EFRAIN - RRNOFF
```

In the above-mentioned crop models, one does not calculate the flow of water between soil layers. But the soil is divided into a number of layers of unequal thickness and moisture content, and one must specify which layers are wetted by the infiltration and also up to which moisture content they are allowed to be wetted. We use a procedure of van Keulen (1975), which fills up the compartments successively from the soil surface further downwards and replenish the moisture content up to field capacity only. Simple but satisfactory, the model starts this procedure by taking the rate of water flow into the first (top) compartment to be equal to INFR and calculates what can be retained in this layer. The excess is the influx into the second compartment, and so on. To repeat these computations for all layers the statements are written in the following MACRO (for explanation of MACRO see Subsection 2.3.3.):

```
MACRO WATER, MWATER, RWFB = COMP(RWFT, THCKN, ...
                            TRR, ER, DRF)
WATER = 1000.*DRF*WLTPT*THCKN +
        INTGRL(0., RWFT-RWFB-TRR-ER)
```

where WATER is the actual amount of soil moisture in a compartment (mm)
DRF is initial dryness factor as a fraction of moisture content at wilting point (–)
WLTPT is wilting point of soil ($m^3$ $m^{-3}$)
THCKN is thickness of compartment (m)
RWFT is rate of water flow at the top of the compartment (mm $d^{-1}$)
RWFB is rate of water flow at the bottom of the compartment (mm $d^{-1}$)
TRR is rate of water uptake by plant roots (transpiration) from the compartment (mm $d^{-1}$)
ER is rate of evaporation from the compartment (mm $d^{-1}$)

```
MWATER = FLDCP*THCKN*1000.
```

where MWATER is maximum tolerated amount of soil moisture in a compartment (mm)
FLDCP is field capacity of soil ($m^3$ $m^{-3}$)

```
RWFB = AMAX1(0., RWFT-(MWATER-WATER)/DELT)
ENDMAC
```

Note that in the above and following MACROs often the same variable is used either with a subscript T (at the top of a soil layer) or a subscript B (at the bottom of a soil layer). Furthermore the variable at the bottom of a layer has the same value as the one at the top of the layer below, e.g. RWFT2 = RWFB1.

Example 1: *Infiltration*
Thickness layer 1 = 0.02 m, $\theta 1 = 0.10$
Thickness layer 2 = 0.03 m, $\theta 2 = 0.18$
Thickness layer 3 = 0.04 m, $\theta 3 = 0.12$
Thickness layer 4 = 0.05 m, $\theta 4 = 0.24$
WLTPT = 0.04, FLDCP = 0.25
INFR = 5.0 mm $d^{-1}$

The calculation of the amount of water necessary to wet the first layer to field capacity is $(0.25 - 0.10) \cdot 20 = 3$ mm. The calculation for the second layer is $(0.25 - 0.18) \cdot 30 = 2.1$. With 5 mm of infiltration the soil will not be wetted for more than 2 layers (= 5 cm)(see also Figure 53).

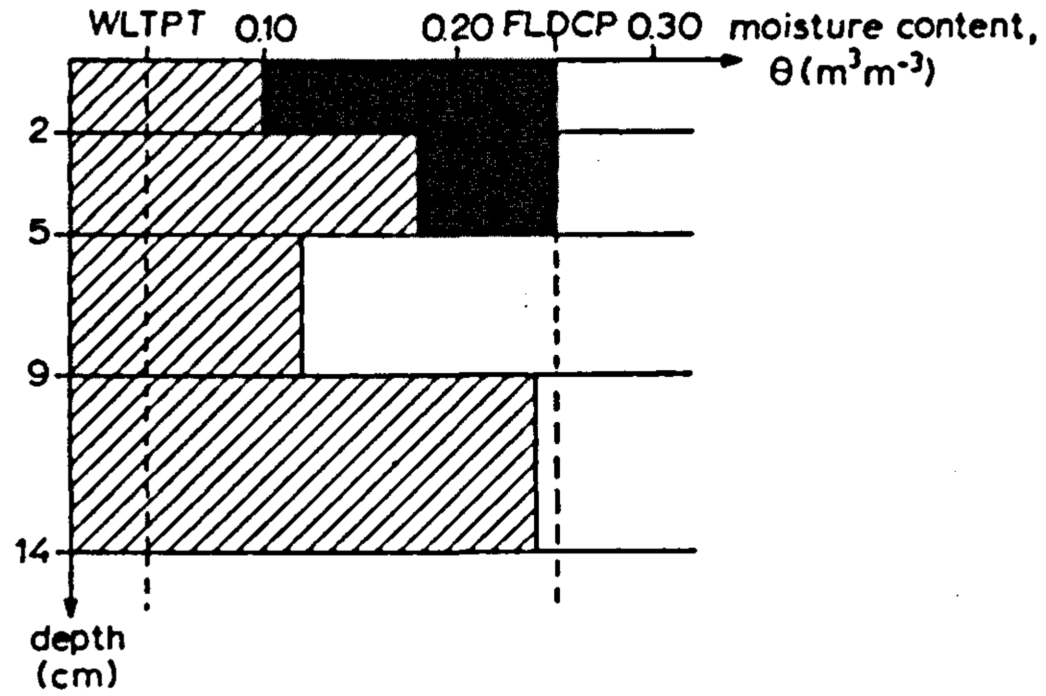


Figure 53. Example of the simulation of infiltration according to Example 1. WLTPT = permanent wilting point and FLDCP = field capacity (all variables in $m^3\ m^{-3}$).

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**Exercise 59**
Calculate the amount of infiltration necessary to wet all four layers of Example 1.

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*Evaporation*

Potential soil evaporation depends on how much energy for evaporation (one may distinguish a 'irradiation' and a 'drying power' term) passes through the leaf canopy and reaches the soil surface. Actual evaporation is a fraction of the potential evaporation and this fraction depends on the dryness of the soil surface and the soil's capability to transport water from deeper layers towards its evaporation surface. In ARID CROP this fraction was determined (deterministic) by the soil-water potential of the first soil layer, but wetting of this layer from below was not taken into account. This incomplete deterministic approach was replaced in SAHEL GRASS by a completely parametric approach based on experimental data from the Sahel (Stroosnijder, 1978) and a fully deterministic submodel for evaporation (van Loon & Wösten, 1979). One now assumes that actual evaporation equals potential evaporation during the day of the rainfall and that during the next days the cumulative actual evaporation is proportional to the square root of time. This goes on until the next rainfall. The following CSMP statements (including the calculation of potential evapotranspiration according to Penman, cf. Subsection 3.2.5) achieve such a computation:

```
AEVAP  = INSW(INFR-0.01, AEVAP2, AEVAP1)
```

where AEVAP is actual rate of evaporation ($mm\ d^{-1}$)
INFR is rate of infiltration ($mm\ d^{-1}$)
AEVAP2 is actual evaporation rate on rain-free days ($mm\ d^{-1}$)
AEVAP1 is actual evaporation rate on the day of the rainfall ($mm\ d^{-1}$)

```
AEVAP1 = AMIN1(PEVAP, INFR)
```

where PEVAP is potential evaporation rate as a function of soil cover and of radiation reaching the soil surface (mm $d^{-1}$)

```
AEVAP2 = AMIN1(PEVAP, EVAPC*(SQRT(DSLR)-...
         SQRT(DSLR-1.)))
PARAMETER EVAPC = 3.3
```

where EVAPC is evaporation constant, experimentally determined for Sahel conditions (mm $d^{-1}$)
DSLR is number of days plus 1. since the last rainfall

```
DSLR   = INTGRL(1.001, 1.-INSW(AFGEN(RAINTB,...
         DAYY+1.)-0.01, 0.,DSLR-0.001))
```

DAYY is number of days (Julian calender); the small value 0.001 was added to avoid division by zero. This statement requires the use of METHOD RECT.

```
PEVAP  = EVAPR*FRLT + EVAPD*FRDP
```

where EVAPR is potential evapotranspiration due to radiation only (mm $d^{-1}$)
EVAPD is potential evapotranspiration due to drying power air only (mm $d^{-1}$)
FRLT is fraction of radiation reaching the soil (–)
FRDP is fraction of drying power reaching the soil (–)

```
EVAPR  = ((DTR*(1.-REFCF)-LWR)*DELTA/GAMMA)/...
         (1.+DELTA/GAMMA)*1./LHVAP
PARAMETER GAMMA = 0.49
PARAMETER LHVAP = 262.E4
```

where DTR is daily total irradiation (J $m^{-2}$ $d^{-1}$)
REFCF is reflection coefficient for short-wave radiation (–)
LWR is outgoing long-wave radiation (J $m^{-2}$ $d^{-1}$)
DELTA is slope of saturated vapour pressure curve at air temperature (mm Hg $°C^{-1}$)
GAMMA is psychrometer constant (mm Hg $°C^{-1}$)
LHVAP is heat of vaporization of water (J $kg^{-1}$)

```
DTR    =AFGEN(DTRT, DAYY)
PARAMETER REFCF = 0.05
LWR    = 4.2E4*1.17E-7*(TMPA+273.)**4*(0.38-0.035*...
         SQRT(VPA))*(1.-0.9*FOV)
```

where TMPA is average daily air temperature (°C)
VPA is average vapour pressure in the air (mm Hg)
FOV is fraction of the day that is overcast (–)

```
DELTA  = 17.4*SVPA*(1.-TMPA/(TMPA+239.))/(TMPA+239.)
```

where SVPA is average saturated vapour pressure in the air (mm Hg)

```
EVAPD  = EA/(1.+DELTA/GAMMA)*1./LHVAP
```

where EA is contribution of drying power of the atmosphere to evaporative demand ($J\ m^{-2}\ d^{-1}$)

EA = 0.35*(SVPA − VPA)*(0.5+(WSR/1.6)/100.)*LHVAP

where WSR is measured windspeed ($km\ d^{-1}$)

FRLT = EXP(−0.5*LAI)

where LAI is leaf area index (−)

FRDP = EXP(−0.5*SQRT (0.2*CROPHT*LAI/(2.*0.5*...
SQRT(4.*WDL*CROPHT/PI/LAI))))

where CROPHT is crop height (m)
WDL is width of the leaves (m)
FRDP according to Goudriaan (1977) p. 109-110.

Example 2: *Evaporation*
Calculate the cumulative evaporation and the average daily evaporation from DAYY = 180 to DAYY = 188 with the following information:
– There is rain at DAYY = 180 (29 June) that results in 12 mm of infiltration into the soil. The Penman potential evaporation (EVAP = EVAPD + EVAPR) equals 6.0 mm $d^{-1}$. From DAYY = 180 to DAYY = 184 the plants are so small that one may take LAI = 0.0.
– There is more rainfall 4 days later, at DAYY = 184, that results in 8 mm of infiltration. The Penman potential evaporation has decreased to 5.0 mm $d^{-1}$ and from DAYY = 184 on plants are such that LAI = 0.5.
– Take (for simplicity) FRDP = FRLT.

DAYY=180: INFR>0, thus AEVAP=AEVAP1
LAI=0, thus FRLT=1.0
PEVAP=FRLT · (EVAPD+EVAPR)=1 · 6 = 6.00 mm $d^{-1}$
AEVAP1=PEVAP
DAYY=181: INFR=0, thus AEVAP=AEVAP2
DSLR=2, thus AEVAP2=3.3 · ($\sqrt{2}$ − $\sqrt{1}$)=1.37 mm $d^{-1}$
DAYY=182: DSLR=3, thus AEVAP2=3.3 · ($\sqrt{3}$ − $\sqrt{2}$)=1.05 mm $d^{-1}$
DAYY=183: DSLR=4, thus AEVAP2=3.3 · ($\sqrt{4}$ − $\sqrt{3}$)=0.88 mm $d^{-1}$
DAYY=184: INFR>0, thus AEVAP=AEVAP1
LAI=0.5, thus FRLT=0.78
PEVAP=FRLT·(EVAPD+EVAPR)=0.78·5=3.89 mm $d^{-1}$
AEVAP1=PEVAP
DAYY=185: DSLR=2, AEVAP2=1.37 mm $d^{-1}$
DAYY=186: DSLR=3, AEVAP2=1.05 mm $d^{-1}$
DAYY=187: DSLR=4, AEVAP2=0.88 mm $d^{-1}$

Total evaporation in 8 days equals 16.49 mm, equivalent to 2.06 mm $d^{-1}$.

All evaporation takes place at or near the soil surface, and this water is (partially) replaced by water flowing upward from deeper layers. However, since no flow between compartments is incorporated in the crop growth models considered, a method had to be found to extract the amount of evaporation from the successive soil compartments. Van Keulen (1975) developed a 'mimick procedure' with a moisture weighted exponential extinction with depth withdrawal function. Since this calculation must be repeated for all soil compartments, as for the computation of the infiltration, we must again use a MACRO:

```
MACRO TDB, EB, SUMB, ER = SOIL(TDT, ET, SUMT, THCKN,...
                              WATER)
ER = F*AEVAP
```

where ER is rate of moisture withdrawal from compartment (mm $d^{-1}$) and F is fraction of total actual evaporation withdrawn from compartment (–)

```
F = THCKN*VAR/(SUM10+NOT(SUM10))
```

where VAR is moisture weighted extinction (with depth) factor (–)
SUM10 is layer thickness weighted sum of VAR factors (m). The term NOT (SUM10) is introduced to avoid a possible division by zero.

```
VAR = AMAX1(0.001*WATER/THCKN-WCLIM, 0.)*
      EXP(-PROP*(TDT + 0.5*THCKN))
```

where WCLIM is volumetric moisture content at air dryness ($m^3$ $m^{-3}$)
PROP is extinction factor for moisture withdrawal (to be determined by validation with a deterministic simulation model)
TDT is depth of the top of the compartment below the soil surface (m)

```
SUMB    = SUMT+VAR*THCKN
EB      = ET+ER
TDB     = TDT + THCKN
ENDMAC
PARAMETER PROP = 50.0
```

Example 3: *Mimic extraction*
The situation in Example 1 will be used with WCLIM = 0.02, PROP = 15.0 and AEVAP = 6.0. The mimic extraction procedure proceeds as follows:

$VAR1 = (0.10-0.02)\cdot\exp(-15.0\cdot 0.010) = 0.069$
$VAR2 = (0.18-0.02)\cdot\exp(-15.0\cdot 0.035) = 0.095$
$VAR3 = (0.12-0.02)\cdot\exp(-15.0\cdot 0.070) = 0.035$
$VAR4 = (0.24-0.02)\cdot\exp(-15.0\cdot 0.115) = 0.039$

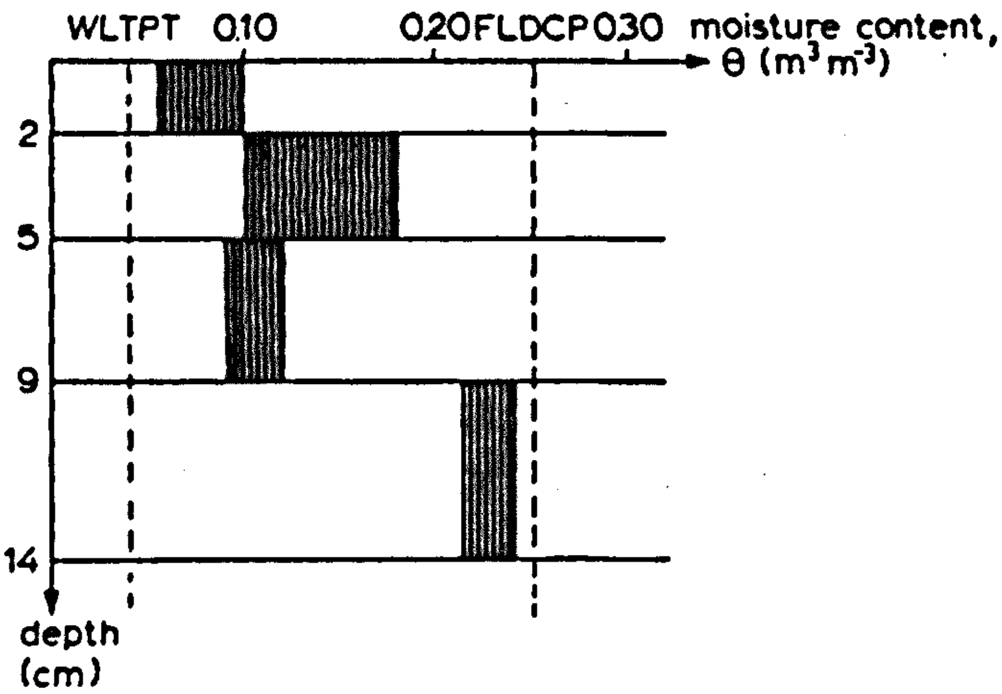


Figure 54. Example of the simulation of evaporation extraction from different soil layers (mimic extraction).

SUM1 = 0.069 • 0.02 = 0.00138
SUM2 = 0.00138 + 0.095 • 0.03 = 0.00423
SUM3 = 0.00423 + 0.035 • 0.04 = 0.00563
SUM4 = 0.00563 + 0.039 • 0.05 = 0.00758

F1 = 0.02 • 0.069/0.00758 = 0.182
F2 = 0.03 • 0.095/0.00758 = 0.376
F3 = 0.04 • 0.035/0.00758 = 0.185
F4 = 0.05 • 0.039/0.00758 = 0.257

ER1 = 0.182 • 6.0 = 1.1 mm $d^{-1}$ ($\Delta\theta = -0.05$)
ER2 = 0.376 • 6.0 = 2.3 mm $d^{-1}$ ($\Delta\theta = -0.08$)
ER3 = 0.185 • 6.0 = 1.1 mm $d^{-1}$ ($\Delta\theta = -0.03$)
ER4 = 0.257 • 6.0 = 1.5 mm $d^{-1}$ ($\Delta\theta = -0.03$)

6.0 mm

$\Delta\theta = 0.001 \cdot ER/THCKN$

This extraction is shown in Figure 54 as the shaded areas in the different soil layers.

---

**Exercise 60**
Calculate the mimic extraction from the situation in Example 3 with four different layers for a case where AEVAP = 4 mm $d^{-1}$. Take for the initial moisture contents the final situation of Example 3.

---

*Transpiration*

Only the soil's influence on transpiration and its reverse will be discussed. In the plant part of the program SAHEL GRASS NPK are calculated:

– the total rooting depth (and no rooting density)
– the potential transpiration

The soil's section of the program checks first how active the roots in the various layers within the rooting depth are and calculates a total effective rooting depth (for example, if there are 10 layers, this is ERLB10). The main program divides the potential transpiration, PTRANS, by this value to obtain the potential transpiration per metre of active root depth (TRPMM). Then the soil's section calculates per layer the actual transpiration as a function of potential transpiration in that layer (TRPMM * RTL), the effectiveness of the roots as a function of moisture content (EDPTF), the temperature of the soil (TEC) and the reduction effect of dryness of the soil on water uptake by the roots (WRED). All computations are again programmed in a MACRO:

```
MACRO TRR, ERLB, TDB, TRB = LAYER(ERLT, TDT, TRT,...
                            THCKN, WATER, TS, MWATER)
TRR = TRPMM*RTL*EDPTF*TEC*WRED
```

where TRR is transpiration rate of the soil layer (mm $d^{-1}$)
TRPMM is potential transpiration rate per metre rooting depth in wet soil (mm $m^{-1}$ $d^{-1}$)
RTL is rooting depth in a compartment (m)
EDPTF is reduction factor for root effectiveness as a function of soil moisture content (–)
TEC is reduction factor for root conductivity as a function of soil temperature (–)
WRED is reduction factor for water uptake as a function of soil moisture content (–)

```
RTL = LIMIT(0., THCKN, RTD-TDT)
```

where RTD is total rooting depth (m)
TDT is depth of the top of the compartment below the soil surface (m)

```
EDPTF = AFGEN(EDPTFT, AWATER/(MWATER-1000.*THCKN*
        WLTPT))
```

where EDPTFT is table of EDPTF versus reduced soil moisture content, $\bar{\theta}$ (see main program)

```
AWATER = AMAX1(0., WATER-1000.*THCKN*WLTPT)
```

where AWATER is available amount of soil moisture in a compartment (mm)

```
TEC = AFGEN(TECT, TS)
```

where TECT is table of TEC versus soil temperature (TS) (see main program)

```
WRED = AFGEN(WREDT, AWATER/(MWATER-1000.*THCKN*
       WLTPT))
```

where WREDT is table of WRED versus reduced soil moisture content, $\bar{\theta}$ (see main program)

ERLB = ERLT+RTL*EDPTF

where ERLB is effective rooting depth at bottom of the compartment (m)

TRB = TRT+TRR
TDB = TDT+THCKN

where TRB is cumulative sum of TRR

ENDMAC

Since the program SAHEL GRASS NPK simulates 10 soil layers (August 1980 version) the total actual rate of transpiration (TRAN in mm $d^{-1}$) is

TRAN = TRB10

In the above example not all the relations between soil wetness and growth are discussed. In the original MACRO of SAHEL GRASS NPK we used another two sets of statements which also refer to relations between growth and soil moisture. The statements SWP and SWPB check whether there are still soil layers wet enough for the roots to grow deeper. The statements RAWR and RAWRB check whether there is still enough available water for growth or whether plants will suffer from drought. The final statement in this MACRO refers to drainage below the maximum rooting depth. This element of the water balance is not discussed here.

Example 4: *Actual transpiration*
Calculation of the actual transpiration for the example with four layers given earlier and with the following data:
- rooting depth 0.09 m. Hence only the first three layers have roots.
- T1, T2, T3 and T4 all 20 °C
- FUNCTION EDPTFT = 0., .15, .15, .6, .3, .8, .5, 1., 1.1, 1.
- FUNCTION TECT = 0., 0.06, 3., 0.29, 10., 0.85, 16., 0.94, 20., ... 1., 31., 0.87, 40., 0.6, 50., 0.3
- FUNCTION WREDT = 0., 0., .1, .30, .15, .45, .3, .7, .5, .975, .75, ... 1., 1.1, 1.
- PTRANS = 2 mm $d^{-1}$

$\bar{\theta}$ is the reduced soil moisture content:
$\bar{\theta}_1 = (0.10-0.04)/(0.25-0.04) = 0.29$
$\bar{\theta}_2 = (0.18-0.04)/(0.25-0.04) = 0.67$
$\bar{\theta}_3 = (0.12-0.04)/(0.25-0.04) = 0.38$

| | | |
|---|---|---|
| EDPTF1 = 0.80 | EDPTF2 = 1.00 | EDPTF3 = 0.90 |
| TEC1 = 1.00 | TEC2 = 1.00 | TEC3 = 1.00 |
| WRED1 = 0.70 | WRED2 = 1.00 | WRED3 = 0.85 |

ERLB1 = 0.02 • 0.80 = 0.016
ERLB2 = 0.016 + (0.03 • 1.00) = 0.046
ERLB3 = 0.046 + (0.04 • 0.90) = 0.082

TRPMM = PTRANS/ERLB3 = 2.0/0.082 = 24.4 mm $m^{-1}$ $d^{-1}$

TRR1 = 24.4 • 0.02 • 0.80 • 1.0 • 0.70 = 0.27 mm $d^{-1}$ ($\Delta\theta_1 = 0.01$)
TRR2 = 24.4 • 0.03 • 1.00 • 1.0 • 1.00 = 0.73 mm $d^{-1}$ ($\Delta\theta_2 = 0.02$)
TRR3 = 24.4 • 0.04 • 0.90 • 1.0 • 0.85 = 0.75 mm $d^{-1}$ ($\Delta\theta_3 = 0.02$)
1.75 mm $d^{-1}$

TRAN = TRB3 = 1.75 mm $d^{-1}$; the extraction over different layers is shown in Figure 55 as the black areas.

---

**Exercise 61**
Calculate the actual transpiration for Example 4 and with the following data:
- rooting depth = 14 cm
- T1 = 50 °C, T2 = 35 °C, T3 = 25 °C, T4 = 20 °C
- potential transpiration = 2.5 mm $d^{-1}$

---

As already mentioned, the above discussed parametric elements were used to calculate the water balance of the semi-arid Sahel. One of the results is shown in Figure 56. As can be seen the above-ground dry matter can be adequately simulated till the period of flowering; measured and calculated cumulative evapotranspiration also correlate reasonably well over this period. During and after flowering, part of the biomass dies. During this reproductive phase not all the processes are understood well enough to permit their proper simulation.

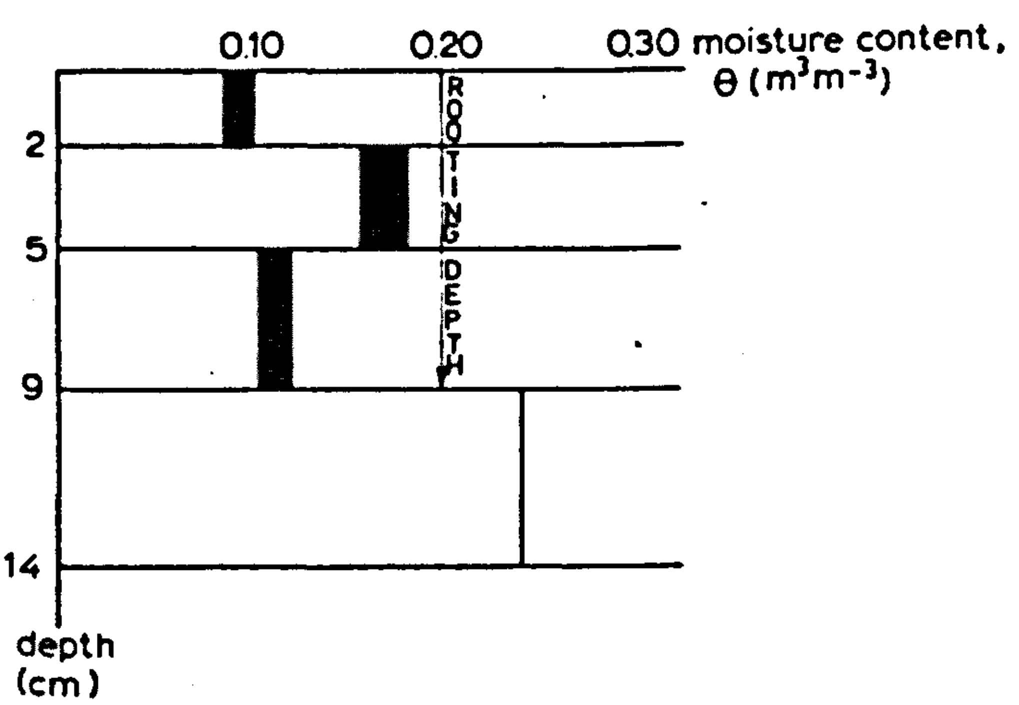


Figure 55. Example of the simulation of actual transpiration from different layers within the rooting depth.

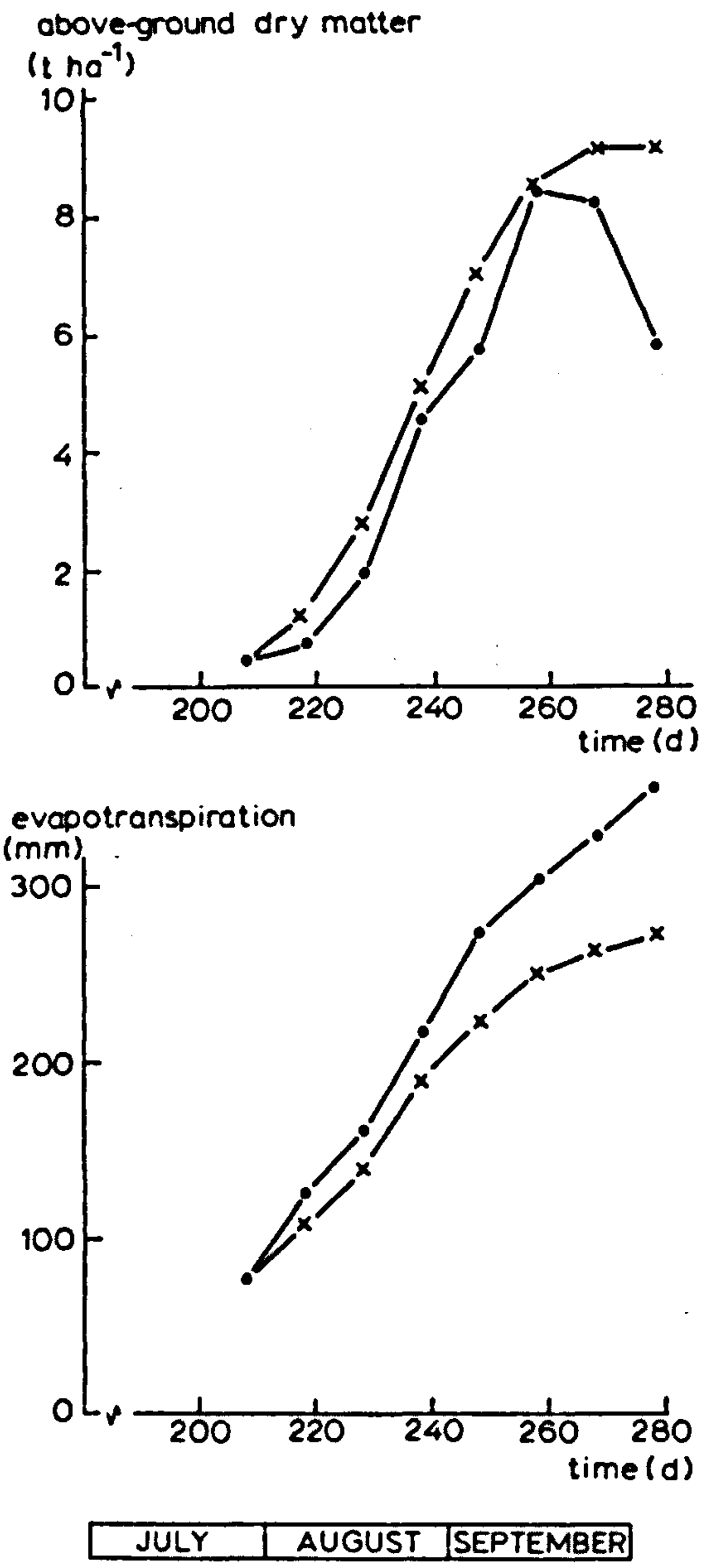


Figure 56. Calculated (x) and measured (•) above-ground dry matter and evapotranspiration of a natural Sahel vegetation with NPK fertilizer, on a clay soil, Niono, Mali, 1978; the program was initialized at Day 208.

### 4.2.4 *Conclusions*

Perhaps it is disappointing to hear that a fully deterministic approach to modelling of the important soil-water section of current whole-crop models is not possible because of the great difference in time coefficients of the various parts of such models. One reason for this is certainly the wish to use the state variable approach in combination with specially developed simulation languages like CSMP, which automatically leads to an explicit approximation of all differential equations. Good reasons for having a preference for the state variable approach and CSMP are its easy programming and the advantage that CSMP contains many preprogrammed routines, including those for data entry and output. Another reason is that one wishes to keep the soil divided into a number of layers of relatively small thickness. If one should change this attitude and consider only two layers (e.g. a rootzone and a subzone) this would omit a number (but not all) of problems with the soil-water section. However, reasons for keeping this number of layers are: the wish to include the phenology of roots and to be able to simulate nutrient uptake by roots from various depths (see Subsection 5.3.2).

Our experience with the various parametric soil-water submodels (of which some results are also presented in Section 4.1) is that they can be fruitfully used as long as enough attention is given to a really hierarchical approach to derive and to validate them. The latter shows the need for a stock of deterministic (soil physical) submodels that are already validated and a stock of useful experimental data, both of which have to be updated regularly. This makes clear the role of soil (physical) science in the multidisciplinary effort to understand and simulate crop growth.