Application of stochastic dynamic programming models in optimization of reservoir operations: A study of algorithmic aspects

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FOREWORD

Shortage of fresh water of adequate quality has been predicted to become one of the most pressing problems mankind must face in the foreseeable future. In order to counteract this situation storage of surplus flows in reservoirs and the careful management of this resource is thus going to gain in importance. In addition, the improved operation of existing reservoirs might contribute to postpone the construction of new storage elements, thus implicitly helping to mitigate the impact on the natural environment. Consequently the management aspects of reservoirs both at the planning and at the operational stage form an essential research area.

The present report summarizes the results of sensitivity analyses related to the application of stochastic dynamic programming (SDP) in optimization of reservoir operation in an uncertain environment. SDP has a strong potential to be used in deriving robust, detailed operational rules for reservoirs. The SDP-based policy is oriented towards the expected hydrological situation, thus its adequacy is largely depending on availability and accuracy of data and their numerical representation to capture the natural (hydrological) uncertainties inherent in the water resources system.

Artificial inaccuracies and simplifications have been introduced into the mathematical processing of inflow data in order to assess the potential impact of possibly biassed data both on the optimal reservoir policy itself and on the operational performance of reservoirs.

Next to the natural reservoir inflow uncertainties, objectives to be pursued may change as social preferences and aspirations undergo gradual changes. In order to model these possible scenarios optimal reservoir operational policies have been derived according to different anticipated objective functions and constraint sets. Furthermore alternative performance indicators such as reliability-type of criteria have been tested.

Model uncertainty and its impact are taken into account by testing different versions of SDP in the operational studies. All along these analyses both single and multiunit systems have been considered. Practical relevance of the results is ensured by using inflow data and salient features of existing "real world" reservoir systems.

Since the computations serve the purpose of clarification of algorithmic details and uncertainty aspects the results presented in this study do not refer to the actual operation and performance of the respective reservoirs.

This report is a publication in a series of dissertations, theses and reports concerning different research issues in reservoir operation. This broad-based research activities of a dedicated international team had started at the Asian Institute of Technology, Bangkok, Thailand in the mid eighties and continued at the Wageningen Agricultural University, Department of Water Resources from 1989 onwards.

The present report documents the joint efforts of the research team to provide a detailed analysis and practical recommendations towards the applicability of SDP in deriving reservoir operational rules.

Computational work and a draft of the report was done by Mrs. He. Next to the authors Prof. Dr. Paul van Beek, Department of Mathematics contributed substantially to this report through his advice, corrections and recommendations. His involvement as well as the critical review by Drs. P.J.J.F. Torfs, Department of Water Resources, Wageningen Agricultural University are most appreciated.

Prof. Dr.-Ing. J.J. Bogardi, Wageningen, February 1995
Chairman Department of Water Resources

Abstract

Stochastic Dynamic Programming (SDP) technique has been used in the operational policy analysis of water resources systems over the past several decades. However, those studies indicate that certain algorithmic aspects of SDP have to be studied further to facilitate the application of it to real world reservoir operational problems.

This study focuses on four major aspects of the SDP model: (a) Markov inflow transition probability matrix and its role in SDP models; (b) the influence of different decision variables and inflow state variables on the performance of the SDP model; (c) the suitability of the different inflow serial correlation assumptions; and (d) the appropriateness of the objective function in the SDP model and the performance evaluation criteria.

The characteristics of a Markov chain and the convergence behaviour of the SDP model are analyzed through a real world application. Large number of zero elements in transition probability matrices seems to be the cause for failing to satisfy the convergence criterion, stabilization of expected annual increment of the objective function value, in the SDP model. The study shows that the substitution of these zeros with reasonably small values is a suitable method to overcome the above problem.

Several versions of the SDP model with different decision variables and inflow state variables are employed to study their influence on the performance of the SDP model. The variable, which is directly related to the objective of optimization, seems to be preferred as the decision variable. The choice of the inflow state variable considerably affects the operation of the system if the selected decision variable is not directly related to the objective of optimization.

The suitability of different inflow serial correlation assumptions in the SDP model is examined through models formulated based on Markov-I, Markov-II, independence and deterministic inflow assumptions. The analysis indicates that the SDP model becomes insensitive to the above inflow assumptions if the selected decision variable is directly related to the objective of optimization. A comparison among the above assumptions is made based on the complexity involved in the computations, the length of inflow time series available, time step length considered in optimizations and errors possible in inflow forecast.

Several different objective functions are introduced into the SDP model to study their influence on the resulting reservoir operation performance. The selection of the most appropriate objective in the formulation of the SDP optimization seems to be important for its success. The simulated objective function value is an inadequate indicator to measure the performance of the optimization. The study shows that some risk-related performance indices such as reliability, vulnerability and resilience are more suitable in the evaluation of the reservoir performance.

Table of Contents

ABSTRACT i

TABLE OF CONTENTS ii

LIST OF TABLES v

LIST OF FIGURES vii

LIST OF ABBREVIATIONS viii

1 INTRODUCTION 1

1.1 Reservoir Operation Optimization 1
1.2 Reservoir Operation Optimization Under Uncertainty 2
1.3 Review of the Explicit Stochastic Models 3
1.4 Identification of the Task 5
1.5 Objectives and Scope of the Study 5

2 LITERATURE REVIEW 8

3 MODEL DESCRIPTION 16

3.1 Stochastic Dynamic Programming (SDP) Models for Single Reservoir Operation 16
3.1.1 Calculation Procedure of the Model 16
3.1.2 Stage 16
3.1.3 State and Decision Variables 16
3.1.4 Discretization of Storage, Inflow and Release Variables 18
3.1.5 Inflow Serial Correlation 19
3.1.6 Markov (or Markov-I) Inflow Process 19
3.1.7 Estimation of Inflow Probabilities 20
3.1.8 State Transformation Equation 22
3.1.9 Physical Constraints of Reservoir Systems 22
3.1.10 Objective Functions 23
3.1.11 Recursive Equation 23
3.1.12 Convergence Criteria 24
3.2 SDP Models for Two Reservoirs in Series 25
3.3 Performance Assessment of SDP based Operation Policy 26

4 DESCRIPTION OF THE CASE STUDY SYSTEMS 28

4.1 Mahaweli Reservoir System 28
4.2 Kariba Reservoir System 32
4.3 Joumine Reservoir System 35

5 MARKOV INFLOW TRANSITION PROBABILITIES 39

5.1 Role of Markov Inflow Transition Probabilities in the SDP Model 39
5.1.1 Problems in SDP Convergence Behaviour 39
5.1.2 Reasons for the Violation of SDP Convergence Criteria 40
5.2 Sensitivity Analysis of Markov Inflow Transition Probabilities 44
5.2.1 Computer-Experiments 44
5.2.2 Results and Discussion 46
5.3 A Methodology to Eliminate the Zero Elements in Transition Probability Matrices 48

6 STATE AND DECISION VARIABLES 52

6.1 Review of the Existing Models 52
6.2 Comparison of the Models with Different Decision and Inflow State Variables 57
6.2.1 Computer-Experiments 58
6.2.2 Analysis of the Results and Discussion 59
6.3 Comparison of the Models with Different Decision Variables 69
6.3.1 The Structure of the SDP Models and Resulting Policies 70
6.3.2 Simulation and Results 71

7 INFLOW SERIAL CORRELATION ASSUMPTIONS 78

7.1 The Four Serial Correlation Assumptions and Their Modelling Complexities 79
7.2 The Best Serial Correlation Assumption 83
7.3 Computer-Experiments 85
7.3.1 Kariba System Operation Based on the Models with Release as Decision Variable (Experiment 7.1) 85
7.3.2 Kariba System Operation Based on the Models with Storage as Decision Variable (Experiment 7.2) 87
7.3.3 Kariba System Operation Based on the Models with Markov-I and Independence Assumption (Experiment 7.3) 89
7.3.4 Kariba System Operation Based on Imperfect Forecast (Experiment 7.4) 90
7.3.5 Mahaweli System Operation (Experiment 7.5) 92
7.3.6 Joumine System Operation (Experiment 7.6) 93

8 OBJECTIVE FUNCTIONS AND PERFORMANCE EVALUATION 96

8.1 Theoretical Discussion 96
8.2 Computer-Experiments 98
8.3 Analysis and Results 105

9 CONCLUSIONS AND RECOMMENDATIONS 113

9.1 Conclusions 113
9.2 Recommendations 116

10 REFERENCES 117

APPENDIX A: Data of the Case Study Systems 124

APPENDIX B: Definition of Ergodicity 131

APPENDIX C: Example for Problems in Convergence Behaviour 134

List of Tables

4.1	Principal Features of the Mahaweli Reservoirs	31
4.2	Monthly Evaporation Values from the Victoria and Randenigala Reservoirs	31
4.3	Elevation-Storage-Area Relationships of the Victoria and Randenigala Reservoirs	31
4.4	Principal Features of the Kariba Reservoir	34
4.5	Monthly Evaporation Values from the Kariba Reservoir	34
4.6	Elevation-Storage-Area Relationship of the Kariba Reservoir	34
4.7	Multiple Regression Analysis of the Kariba Reservoir Inflow	35
4.8	Principal Features of the Joumine Reservoir	37
4.9	Monthly Evaporation Values from the Joumine Reservoir	37
4.10	Elevation-Storage-Area Relationship of the Joumine Reservoir	37
4.11	Water Demands of the Joumine Reservoir	38
5.1	Model Setups for the Mahaweli and Kariba Reservoir Systems	44
5.2	Example of Modifications of the Markov Inflow Transition Probabilities of the Kariba Reservoir	45
5.3	Operational Performance of Kariba the Reservoir	46
5.4	Operational Performance of the Mahaweli System	47
5.5	Example of the Smoothing Method	49
5.6	Simulated Performance after Smoothing	51
6.1	Operational Policy Identifying the Current Release	53
6.2	Operational Policy Identifying the Final Storage	54
6.3	Summary of the Three Computer-Experiments	59
6.4	Derived SDP-based Policy Tables for the Kariba Reservoir (May)	60
6.5	Simulated Average Annual Performances (Experiment 6.1)	61
6.6	Simulated Average Annual Performances (Experiment 6.2)	64
6.7	Simulated Average Annual Performances (Experiment 6.3)	64
6.8	SDP Model Setup for the Joumine Reservoir	70
6.9	Derived SDP-based Policy Tables for the Joumine Reservoir (March)	72
6.10	Simulated Performance (Experiment 6.4 and 6.5)	76
7.1	Serial Correlation Coefficients of the Three Case Study Systems	84
7.2	The Key Points of the Design of Experiments 7.1 to 7.6	85
7.3	Simulated Average Annual Performance (Experiment 7.1)	86
7.4	Simulated Average Annual Performance (Experiment 7.2)	88
7.5	Simulated Average Annual Performance (Experiment 7.3)	90
7.6	Simulated Average Annual Performance (Experiment 7.4)	91
7.7	Simulated Performance (Experiment 7.5)	92
7.8	Simulated Performance (Experiment 7.6)	95
8.1	Combinations of Objective Functions and Constraints of the Tested Models	100
8.2	Simulated Performance Indices Referring to Energy Output	106
8.3	Simulated Performance Indices Referring to Irrigation Supply	107

8.4	Derived SDP Policy Tables for the Mahaweli System	110
A.1	Estimated Monthly Spillages over the Polgolla Barrage (MCM)	124
A.2	Monthly Incremental Inflows into the Victoria Reservoir (MCM)	125
A.3	Monthly Incremental Inflows into the Randenigala Reservoir (MCM)	126
A.4	Monthly Incremental Inflows into the Rantembe Reservoir (MCM)	127
A.5	Estimated Monthly Irrigation Demands at Minipe (MCM)	128
A.6	Monthly Inflows into the Kariba Reservoir (MCM)	129
A.7	Monthly Inflows into the Joumine Reservoir (MCM)	130
C.1	Inflow Transition Probabilities	134
C.2	Calculation Progress for the Example	134

List of Figures

3.1	Calculation Procedure of the SDP Model	17
3.2	Relationship Between Stage Notations	18
3.3	Relationship Between Stage Notation t and Other Variables	18
3.4	Calculation Flowchart of Recursive Equation	25
3.5	Simulation Procedure	27
4.1	Layout of the Mahaweli Development Scheme and the Climatological Regions in Sri Lanka	29
4.2	Mahaweli Development Scheme	30
4.3	Zambezi River Basin	33
4.4	Tunisia Reservoir System Scheme	36
5.1	Deriving Inflow Transition Probabilities by Distribution Fitting	42
5.2	Simulated Monthly Performance After Smoothing	50
6.1	Simulated Monthly Performance of Experiment 6.1	62
6.2	Simulated Monthly Performance of Experiment 6.2	63
6.3	Simulated Monthly Performance of Experiment 6.3	65
6.4	Quantity-based Water shortage Results from Experiment 6.4	74
6.5	Quantity-based Water shortage Results from Experiment 6.5	75
7.1	Graphical Illustration of the Three-dimensional (Markov-II) Transition Probabilities	80
7.2	Graphical Illustration of the Two-dimensional (Markov-I) Transition Probabilities	81
7.3	Graphical Illustration of the One-dimensional (independence) Probabilities	82
7.4	Quantity-based Water shortage Results from Experiment 7.6	94
8.1	Utility Functions	102
8.2	Relation of Deficits and α	111

List of Abbreviations

AVG	Average Transition Probabilities
AVM	Modified Average Transition Probabilities
DE	Deficiency of Energy (from firm power requirement)
DP	Dynamic Programming
DR	Deficiency of Release (from water demand)
EG	Energy Generation
GWh	Giga Watt hours
ha	hectares (10000 square meters)
IDP	Incremental Dynamic Programming
LP	Linear Programming
MCM	Million Cubic Meters
mm	millimetres
MDF	Modified Transition Probabilities
MW	Mega Watts
O.F.	Objective Function
OMS	Operational Mode Switch
ORG	Original Transition Probabilities
pdf	probability density function
SDP	Stochastic Dynamic Programming
SSDP	Sampling Stochastic Dynamic Programming

1 Introduction

1.1 Reservoir Operation Optimization

The history of man-made reservoirs can be traced back hundreds of years. Perhaps at the beginning, the "water reservoir" was not more than a huge tank to store water during the wet season for the use during the dry season. Today, with the development of the civilization, reservoirs can be found all over the world. The reservoirs can serve single or multiple purposes including hydropower generation, water supply for irrigation, industrial and domestic use, flood control, improvement of water quality, recreation, wildlife conservation and navigation. The effective use of reservoir systems has become increasingly important.

For many years the rule curves, which define ideal reservoir storage levels at each season or month, have been the essential operational rule. Reservoir operators are expected to maintain these levels as closely as possible while generally trying to satisfy various water needs downstream. If the levels of reservoir storage are above the target or desired levels, the release rates are increased. Conversely, if the levels are below the targets, the release rates are decreased. Sometimes operation rules are defined to include not only storage target levels, but also various storage allocation zones, such as conservation, flood control, spill or surcharge, buffer, and inactive or dead zones. Those zones also may vary throughout the year and advised release range for each zone is provided by the rules. The desired storage levels and allocation zones mentioned above are usually defined based on historical operating practice and experience. Having only these target levels for each reservoir, the reservoir operator has considerable responsibility in day-to-day operation with respect to the appropriate trade-off among storage levels and discharge deviations from ideal conditions. Hence, such an operation requires experienced operators with sound judgement.

To counteract the inefficiency in operating a reservoir system only by the "rule curves", now additional policies for operation have been incorporated into most reservoir operation rules. Those operation policies define precisely when conditions are not ideal (e.g., when the maintenance of the ideal storage levels becomes impractical), the decisions to be made for various combinations of hydrological and reservoir storage conditions. For some reservoir systems, this type of operation policy has already taken over the rule curves and is acting as the principal rule for reservoir operation.

Over the past two to three decades, increasing attention has been given to system analysis techniques for deriving operation rules for reservoir systems. As a result, a variety of methods are now available for analyzing the operation of reservoir systems. In general, these techniques lead to models, which can be classified into two categories: optimization models and simulation models. These categories are complementary. Simulation models can effectively analyze the consequences of various proposed operation rules and indicate where marginal improvements in operation policy might be made. The technique is not very appropriate in selecting the best alternative rule from the set of possible alternatives. Usually there are too many alternatives to be simulated and compared. Therefore, optimization models are often used to indicate which alternatives are most likely to be better than the others.

Linear Programming (LP) and Dynamic Programming (DP) have been the most popular among the optimization models in deriving optimum operation rules for reservoir systems. Linear Programming is concerned with solving problems in which all relations among the variables are linear, both in the constraints and in the objective function to be optimized. The fact that most of the functions encountered in problems with reservoir operation are nonlinear has been the main obstacle to the successful use of LP in this area. Although linearization techniques can be employed, this might not be satisfactory. The degree of the approximation required in the linearization process can seriously affect the reliability associated with this technique.

Dynamic Programming, a method that breaks down a multi decision problem into a sequence of subproblems with few decisions, is ideally suited for time sequential decision problems like deriving operation policies for reservoirs. Hall and Dracup (1970) stated that DP possesses substantial advantages for analysis of such a system. Because it can treat the nonconvex, nonlinear discrete variables and is generally more amenable to stochastic inputs. However, in the DP model, the separability condition of the objective function limits some applications. Thus a careful choice of the setup of the model (e.g., stage, state, decision, objective and constraint) is essential.

1.2 Reservoir Operation Optimization Under Uncertainty

Uncertainty has always been a serious problem in reservoir operation optimization. Uncertainty is present in many factors that affect the performance of systems, such as future hydrological, economic, human, technological conditions, etc.. Among them the uncertainty of future hydrology (i.e., reservoir inflow) is of extreme importance to water reservoir systems and is evidently regarded as a major issue in reservoir operation optimization.

There are many ways to deal with the uncertainty of reservoir inflows, depending on its severity and on its influence on the operation of the system. The simplest approach is to replace the uncertain inflows, either by their expected mean values or by some critical values, and then proceed with a deterministic approach. However, substituting the random inflows by an expected mean value is unacceptable when there is a large variation in the inflow time series. In this situation, any single value would not be a safe approximation of the inflow variable.

Besides the deterministic approach of using the expected mean inflow value, there exists several stochastic models that can be applied to solve the reservoir operation problem considering the inflow uncertainty. These models can be classified into two major schemes: "implicit" and "explicit".

In the so called "implicit" stochastic model approach, a number of synthetic inflow sequences are generated using a time series model. The system is then optimized for each inflow sequence, and operation rules are found by using multiple regression on the optimized operation sequences. During the optimization phase the synthetic data series are considered deterministic. Monte Carlo Dynamic Programming (Young, 1967) is a well known example of the implicit stochastic model approach. A serious drawback inherent in the implicit approach is that it may never be possible to derive the theoretical optimum. Besides, the form of the equation for regression analysis (the independent variables to be included and the way they should be treated) and the error estimation are continuously open to discussion (Loucks and Sigvaldason, 1979).

The "explicit" stochastic approach uses the probability distribution of the inflows. It is introduced into the optimization formulation by either a substitution of the expected value for the system objective or a failure chance permitted for the system. Most of the so called explicit models are extensions of well known deterministic models, either LP or DP, to the stochastic situation. Well developed explicit stochastic models include the so called Policy Iteration Method (Howard, 1960), Stochastic Dynamic Programming (Butcher, 1968), Stochastic Linear Programming (Loucks, 1968), Chance-constrained Linear Programming (ReVelle *et al.*, 1969), Reliability-constrained Dynamic Programming (Askew, 1974a), and Reliability Programming (Colorni and Fronza, 1976).

1.3 Review of the Explicit Stochastic Models

Three of the explicit stochastic models, among those mentioned above, can take the serial correlation of the random inflow process into consideration. These are Stochastic DP, Policy Iteration and Stochastic LP.

Stochastic DP simply combines the stochastic nature of inflows into the deterministic DP by optimizing the expectation of the original objective. In stochastic DP, the release decision is found at each stage upon each storage and inflow state, which maximizes the expectation of the objective value in the remaining stages (calculated with an appropriate recursive equation). The procedure is applied successively at each stage going backward until the policy becomes steady.

The Policy Iteration is, as its name implies, an iteration method. It generally aims to improve the expectation of the objective value through a trial-and-error strategy. This method has two phases. One is called Value-Determination Operation, and the other Policy-Improvement Routine. During the Value-Determination Operation, a set of linear simultaneous equations has to be solved to find objective related values for any given release policy. During the Policy-Improvement Routine, for each (storage and inflow) state in each stage, a new set of policies is determined (through an appropriate recursive equation) based on the previous knowledge of the objective related values. The iteration cycle is supposed to terminate on the achievement of a steady state policy.

The Stochastic LP, however, has certain significant differences as compared with the other two models. In Stochastic LP, the decision is defined as the steady state joint probability instead of steady state policy. Like the other two models, Stochastic LP also maximizes the expectation of the objective. The objective function is the sum of all states, stages, inputs and decisions (the joint probabilities that the steady state policies have). Therefore, a large set of linear equations has to be solved simultaneously to find the set of probabilities that maximize the expectation of the objective. Knowing the optimal values of joint probabilities of the system states, stages and inputs, the conditional probabilities of the final storage volumes for the given initial storage volumes and inflows in the corresponding stages are calculated for the derivation of the final operation policy.

Many comparison studies among these three explicit stochastic models have been carried out (Gablinger and Loucks, 1970; Loucks and Falkson, 1970). All the results are in favour of Stochastic DP model. Although the information derived from each model yields an identical policy, the computational efficiencies of each model differ considerably. Stochastic DP takes the least amount of computer time. For a simple problem presented by Gablinger and Loucks (1970), Stochastic DP obtained a steady state policy in about one twentieth time required by the Stochastic LP.

Perhaps, the accuracy is more important than the speed. There is no doubt that the number of simultaneous linear equations that can be accurately solved on present computers is much less than the number that may be required for any real world reservoir operation problem. While solving large numbers of simultaneous linear equations, computer round-off and truncation errors may result in an initially feasible solution rendering it infeasible. This limits the size of the problem that can be examined using techniques such as Stochastic LP and Policy Iteration (Chaturvedi, 1987).

Furthermore, the stochastic DP model is very flexible. It has the ability to adjust easily to various problem environments by varying the state variables, decision variables, objective functions and constraints, etc., of the model. It can deal not only with Markov inflow process (lag-one serial correlation) as it was originally introduced for, but also with more (or less) complicated stochastic inflow processes. For example, the model Reliability-constrained Dynamic programming is in fact an extension of the version of Stochastic DP that assumes inflow as an independent process. The only difference in the Reliability-constrained DP is that an additional constraint or a penalty is introduced to limit the failures that could happen during the operation horizon.

The three remaining explicit stochastic models, which were mentioned in Section 1.2, i.e., Chance-constrained LP, Linear Decision Rule and Reliability programming can be considered as one group. Among this group, Chance-constrained LP model is the basic model. Linear Decision Rule is introduced to allow easy formulation of chance constraints. Reliability Programming is developed to overcome difficulties in identifying a specific reliability as a chance constraint.

The optimum operation policies designed to maximize expectation of the objective value, if followed strictly, may sometimes allow the system to fail on many occasions. The probability of such failures may be greater than can be permitted. In the Chance-constrained LP type of models, the inflow probability condition is reflected in the constraints. They aim to constrain the optimization to those decisions that represent a failure probability smaller than an acceptable level. A major advantage of this type of models is that they can be converted into deterministic equivalent after the accepted level is specified.

In general, the usefulness of this group of Chance-constrained LP models is seriously limited due to the following facts: (a) they can only deal with linearly structured problems, whereas the problems in reservoir operation are mainly nonlinear; (b) they derive "rule curve" type of operating rules than a detailed operation policy, which is more needed in the modern reservoir operation; (c) they are based on the too rigid assumption that each inflow in each period is critical.

1.4 Identification of the Task

The previous review reveals that Stochastic Dynamic Programming (SDP) is a model with great potential. Having the nature of Dynamic Programming, SDP can handle non convex, nonlinear discrete variables. Furthermore, this approach generates an operation policy comprising storage targets or release decisions for all the possible reservoir storages and inflow states in each month (i.e., precise operation policy), than a mere single schedule of reservoir releases (rule curve operation rules). After all, it is a flexible model that could be adjusted easily to various problem environments.

Since it has inherent merits, SDP has been well received as a long term (monthly or annually) reservoir operation optimization model. Over the past twenty years it has attracted considerable attention and has resulted in a long list of related studies (see Chapter 2).

Many of those studies, however, indicate that certain algorithmic aspects have to be studied further to facilitate the application of SDP model to real world reservoir operation problems. For example, although many SDP formulations (different choices of state variables, decision variables, inflow serial correlation assumptions, objectives and constraints, etc.) are feasible, the suitability of each formulation for a particular problem at hand is still to be analyzed. Yet, another problem experienced is the large number of zero elements in the estimated inflow transition probability matrices, due to the absence of long time series of hydrological data.

It is now an appropriate time to step back and try to view the structure of the model in its proper perspective and to develop some guidelines for the appropriate application of the SDP model in real world reservoir operation problems.

1.5 Objectives and Scope of the Study

The general goal of this research study is to obtain some insight or perception of SDP model construction and its application. This research aims at achieving this general goal by focusing on the following four specific objectives.

I. In most applications of SDP based reservoir operation optimization, a Markov inflow process has been assumed. The stochasticity of inflow is expressed by inflow transition probability matrices for each successive time step, based on observed inflow records. The transition probability matrices coupled with the Bellman recursive relation lead to expectation oriented optimal strategies. Due to the limited length of the historical inflow time series, the estimated numerical values of the elements in the probability matrices are unreliable. Also, many elements remain void. Under certain circumstances, it causes the problem that the convergence criterion of the SDP model can only be partially fulfilled.

The first objective of the research is to find effective means to circumvent the problem caused by the poorly structured estimation of the Markov inflow transition probability matrices.

The study attempts to reach the first objective though the following three steps. (a) The interconnection between the characteristics of a Markov chain (the discretized presentation of the Markov process) and the convergence behaviour of the SDP model is analyzed. (b) The influence of the Markov inflow transition probability matrices has on the resulting SDP policy is studied through a real world case study. (c) Based on the studies of (a) and (b) a simple and effective method that would alleviate the problem is proposed.

II. For any DP type of model, the careful choice of state and decision variables is crucial for the success of the model. There are two versions of stationary SDP models, which have been widely applied in reservoir operation optimization. One is the model having release as the decision variable, with previous inflow and initial storage as state variables. The other is the model with final storage as the decision variable, with present inflow and initial storage as state variables. These models have been developed and used by different groups of researchers in different problem environments. However, little has been known about their relative performance during reservoir operation optimization. Until now, the importance of the choice of the decision variable has been neglected. There also exist controversial remarks in literature regarding the choice of different inflow state variables.

The second objective of this research is to study the characteristics of different SDP models that are defined with different decision variables and inflow state variables.

The study attempts to reach the second objective through the following three steps. (a) The relevant studies in literature regarding the choice of decision and inflow state variables in SDP model are reviewed. (b) Besides the two existing versions, two more alternatives of the model are developed and a comparative study between these model versions is carried out using the same decision base (thus comparing the different choices of inflow state variables) or the same inflow state base (thus comparing the different choices of decision variables). (c) The suitability of the choices of decision variables and inflow state variables in the SDP model are tested and evaluated through this comparative study.

III. An important issue in the literature on reservoir operations concerns the appropriate serial correlation assumptions for stochastic inflow sequences. When the SDP model was originally introduced into reservoir operation optimization, the inflow sequence had been assumed to be a Markov (i.e., Markov-I) process. Later, independence assumption has also been introduced into SDP based reservoir operation optimization. However, a clear picture on the pros and cons of the two assumptions is not available up to date. In theory, the model should reflect the nature of the inflow serial correlation, if this correlation has been identified as important. Yet, the growing error in parameter estimation with the growing complexity of the model sets practical limits to the validation of such a requirement.

The third objective of the research is to obtain insight into the characteristics of different SDP models that are defined by different inflow serial correlation assumptions.

The study attempts to reach the third objective through the following three steps. (a) Besides the models with Markov-I and independence assumptions, two other models are developed. They are used to obtain an overall picture of the relation between the serial correlation assumptions and performance of the SDP model. One of the models developed considers the serial correlation one step further than the Markov-I assumption: i.e., SDP model with Markov-II assumptions. The other model interprets the inflow process even simpler than the independence assumption does: the model with the assumption that the inflow is deterministic. (b) The inherent connection among each of the transition probability matrices, which correspond to different inflow serial correlation assumptions, are discussed from a theoretical point of view. (c) The applicability and suitability of each inflow serial correlation assumption is tested and evaluated by means of six experiments.

IV. Simulation studies of reservoir system operation utilizing SDP based rules revealed that the simulated objective function value as an inadequate indicator to characterize the performance of a reservoir system. Bogardi *et al.* (1991) have noticed, for example, that for the Mahaweli reservoir system in Sri Lanka the value of the simulated average annual energy generation varies very little when different objective functions are used in SDP models. Besides the simulated objective function value, a number of (reliability-related) performance indices can be used to describe the operational behaviour of the reservoir system upon the application of a certain release policy (Bogardi and Verhoef, 1991).

The fourth objective of the research is to obtain more insight and systematic knowledge on the subject of objective functions and performance evaluation criterion. This part of the work is in fact a continuation of the initial studies by Bogardi *et al.* (1991).

The study attempts to reach the fourth objective through the following four steps. (a) The difficulties in the selection of the objective functions are discussed and some possible improvements are considered. (b) Several reasonable choices of the objective function are introduced into the SDP model to study their influence on the resulting reservoir operation performance. (c) Besides the often used performance indices of simulated objective values, some risk-related performance indices are also adopted as performance evaluation criteria to obtain a more complete picture of the reservoir performance. (d) Experiments are carried out with a real case study to examine the interaction between objective functions and the performance evaluation criterion.

This report is organized into 9 chapters. Chapter 1 to 4 contain general information and background knowledge of the present research. Chapter 1 is a general introduction to the research. Chapter 2 gives a "bibliography" of the application of the SDP model in the reservoir operation optimization. In this chapter many important related publications are briefly reviewed chronologically. Chapter 3 gives a general description of the formulation and calculation procedure in the SDP approach. This chapter introduces the terminology and concepts for the study reported in the following parts of the report. Chapter 4 describes the reservoir systems selected for the case study. In the present research three different reservoir systems have been selected as reference systems. The reason for the selection of these systems is briefly explained at the beginning of Chapter 4. Chapter 5 to 8 are the four major chapters of the report that contain the contribution of the present research with respect to the earlier mentioned four research objectives. The Markov inflow transition probability matrix is the subject matter of Chapter 5. Decision and inflow state variables are studied in Chapter 6. Chapter 7 focuses on inflow serial correlation assumption. Objective and performance evaluation are analyzed in Chapter 8. Conclusions and recommendations are presented in Chapter 9. References and appendices are provided at the end of the report.

2 Literature Review

There are many reviews of mathematical programming models in reservoir operation. Yakowitz (1982) has provided a thorough insight on the application of dynamic programming models to various water resources problems. Yeh (1985) reviewed the state-of-the-art of the reservoir management models. Reznicek and Cheng (1991) presented a review of the implementation of uncertainties in reservoir management models.

This chapter is focused on the literature regarding the application of stochastic dynamic programming models in the field of reservoir operation optimization. It aims at giving a general view upon the development of optimization models. In this chapter, all the relevant studies are listed and reviewed chronologically. The literature regarding the four subject matters, i.e., Markov transition probability matrices, decision and inflow state variables, inflow serial correlation assumptions, objective function and performance evaluation are discussed further in Chapters 5, 6, 7 and 8 respectively.

The origins of dynamic programming, inventory theory and reservoir management are intimately interconnected. Masse (1946) is considered to be the first (Arrow *et al.*, 1958; Hadley and Whitin, 1963; and Sobel, 1975) to achieve a satisfactory solution to an inventory problem with non negative variables. Masse's study concerned reservoir operations and he employed the functional equation approach, which lies at the base of dynamic programming. The earliest stochastic reservoir operation optimization study published in english language appears (c.f., Yakowitz, 1982) to be the work of Little (1955), who has considered the operation of a simplified reservoir system.

Little's model departs from the model for deterministic reservoir operation by assuming inflows to be observations of a stochastic sequence. Little chose the Markov assumption that the conditional probabilities for the present inflows can be defined completely by the previous inflow. The Markov assumption was not supported by statistical analysis. But Little mentioned that the much more convenient independence assumption was discarded because it is "... untenable for river flow". Little applied his model to data from the Grand Cooley generation plant on the Columbia River, USA. The time horizon was taken to be one year, and it was divided into 26 decision periods with a time interval of 2 weeks. The optimization was carried out backward through a recursive equation, and the transition matrix was inferred from 39 years of historical flows. The highly nonlinear single-stage loss function for the numerical study reflected the amount of water at a given head required to generate a given amount of electricity, and the cost of failing to meet a specified demand. His model derived the optimal release strategy as a function of the storage volume at the start of each time interval and the inflow at the previous time interval. The computed optimal strategy was compared with that of the "rule curves" then in use, over the 39 year historical record. A relative improvement using the optimal strategy was detected.

The early work of Little in this direction was modified by many researchers, based on the theory of Dynamic Programming (Bellman, 1957).

Butcher (1968, 1971) adapted the model of Little (including the Markov inflow assumption) to a realistic case study. Butcher (1971) applied the so called (discrete) stochastic dynamic programming to find the optimal stationary strategy for operating the Wataheamu Dam along the California-Nevada border, USA. In his model the release decision was defined by initial storage and previous inflow states. The optimization calculation process was carried out based on Bellman's Principle of Optimality. Starting at sometime in future and using the connection between the flow in one time period and the adjacent time period, the value of release at each time period was calculated backward through a recursive equation. Butcher specially noted that under certain circumstances, this policy is said to converge when the values of release, which are used to evaluate the objective value, repeat for all of time periods t, as t becomes large enough. These steady state releases then form an optimal policy for the operation of that reservoir.

Schweig and Cole (1968) adapted the Little (1955) model to a two-reservoir problem. Through discrete stochastic dynamic programming, they computed an optimal strategy for a problem based on data from the Lake Vyrnwy, Wales. Despite very coarse discretization (e.g., the state coordinates for past inflows were discretized into only two levels), the authors reported severe computational difficulties; the so called "curse of dimensionality".

Gablinger and Loucks (1970) examined discrete stochastic reservoir operating models based on serially correlated Markov inflows with both linear and dynamic programming techniques. In the original version of the Stochastic LP model, the decision was defined based on the initial storage and present inflow instead of the previous inflow. Therefore, to make the two models (linear and dynamic) comparable, they introduced a new version of the SDP model. It used the present inflow instead of the previous inflow as the inflow state variable. The version of SDP assumed that the present inflow was known at the beginning of the period (or a forecast is possible with 100% certainty); thus the present return from the recursive relation of the SDP model was deterministic. The detailed formulation of the model was well described by Loucks *et al.* (1981). Their comparative study revealed that both DP and LP models result in the same optimal policy, but the requirements in computing time are different. The stochastic dynamic programming approach seemed faster. The authors suggested that the reason for the relatively poor performance of the linear programming method was due to its requirement of more solution variables in the transition compared with the dynamic programming method. Specially, the number of control values to be determined by the linear programming solution equals the product of the number of the discretized points in state and policy spaces, multiplied by the number of decision times. Whereas the number of control variables to be solved in the discrete dynamic programming formulation is only the product of the number of state values multiplied by the number of decision times. If there are 10 decision times, and if everything in a bivariate space (as in the Little model) is discretized into 10 levels, the number of solution variables to be dealt with by dynamic programming is 10^3, whereas the number of variables to be dealt with by linear programming is 10^4.

Loucks and Falkson (1970) examined three types of discrete stochastic reservoir operating models based on serially correlated Markov inflows: linear, dynamic and policy iteration (Howard, 1960). They all lead to the same optimal policy but the requirements in computing time were different. The stochastic dynamic programming approach was observed to be the fastest.

Arunkumar and Yeh (1973) used SDP to maximize the firm power output accompanied by a penalty function for not meeting the specified firm power level. They also proposed a heuristic decomposition approach for a multireservoir system. The approach consists of fixing a stationary policy for (m-1) reservoirs (i.e., 2, ..., m) and optimizing with respect to reservoir 1. The optimized policy of reservoir 1 replaces the initial policy of reservoir 1, and then reservoir 2 is chosen for optimization while release rules for reservoir 1, 3, ..., m are fixed, and so on. This procedure is continued until either the policies do not change or the successive improvement in the infinite time horizon return functions are uniformly bounded by some desired level. The existence of stationary optimal policies has been shown by Ross (1970). Arunkumar and Yeh applied the decomposition approach to a two parallel reservoir system, the Shata and Folsom reservoirs of the California Central Valley Project, USA. The algorithm started with the determination of the optimal release rules for Shata, independent of Folsom. The "flip-flop" decomposition algorithm was applied repeatedly interchanging the two reservoirs until the improvement between successive approximations of the reward function was uniformly bounded by a small number.

Su and Deininger (1974) applied the model and methodology of Little (1955). They examined both independent and serially correlated Markov flows and derived the probabilities of occurrence of the flow intervals from the relative frequency with which the historical data fell into these intervals. As the flow intervals and storage state intervals were not of the same size, the state transition for a given inflow interval might not fall into a single output storage state interval in Su and Deininger's scheme. They proposed a second order interpolation scheme to compute the probabilities of occurrence of the output storage states in the optimization of the objective function. Data from the Lake Superior served as a basis for their computations. They noted that the "unreal" assumption of independent inflows (the inflows are serially correlated) did not influence very much the optimal strategy for the studied problem.

Askew (1974a, 1974b) used stochastic dynamic programming (with independent inflow assumption) and simulation technique to derive the optimal policy that maximizes the expected net benefits. By introducing a penalty function in the recursive equation, to reduce the net benefits every time the demand is not met, an amended policy can be derived that has lower target releases and hence a smaller associated probability of failure. A simulation technique was used to estimate the value of the average number of failure associated with the optimum policy.

Klemes (1977) studied the discrete representation of storage for stochastic reservoir optimization. He pointed out that the number of storage states is subjected to some absolute constraints. Also, it must increase linearly with the reservoir storage capacity so that comparability of results is assured. He demonstrated, both theoretically and with the aid of a numerical example, that a too coarse discrete storage representation can not only impede accuracy but may completely distort reality in most unexpected ways.

Alarcon and Marks (1979) presented a SDP model to study guidelines for the operation of the High Aswan Dam in Egypt. The study considered the conflicting nature of the purposes for which the dam was to be operated. The policies obtained by the SDP model were tested using a simulation model. The results were compared with the ones obtained by operating the system using a simple heuristic approach.

Gal (1979) presented a pilot study of a method for finding an approximation for the optimal policy of a system, which contains one surface water reservoir and two underground aquifers. In the model, the state of the reservoir was represented by the water storage volume in the reservoir at the beginning of each time period and the inflows into the reservoir for the two previous time periods. Since this system was too large to be solved by the usual SDP approach, a method was devised to obtain an approximate solution that did not consume too much time or space. This method was referred to as the parameter iteration method. However, it was noted that contrary to the usual SDP approach, the parameter iteration method is not fully automatic. Further, the user was expected to have a good understanding and intuition about the behaviour of the considered system. Because good results depended on successful choice of some parameters.

Turgeon (1980) proposed two methods to alleviate the problem of dimensionality. The first, called one-at-a-time method, consists of breaking up the original problem into a series of one state variable sub-problems, which are solvable by DP. The second method, called aggregation/decomposition method, consists of breaking up the original *n*-state variables stochastic optimization problem into a *n* stochastic optimization sub-problems of two state variables each, which are also solvable by DP. However, the final result was an optimal local (or a sub optimal global) feedback operating policy for each reservoir of the system.

Loucks *et al.* (1981) presented a SDP model, which is different compared with the Butcher's (1971) version of the model. In Loucks' model, the generated sequential operating policies define the final storage volume as a function of the initial storage volume, which is known, and the inflows in the current period, which are not known until the end of the period. Since the policy is to be implemented starting at the beginning of each period prior to a knowledge of the inflow at that period, the above policy cannot be implemented right away. One way to implement this type of operating policy in real time operation is to reformulate the sequential operating policy in a way that does not depend on unknown future inflows. It can be done by identifying either a final storage volume target, subject to limitations on the releases, or by identifying reservoir release targets subject to limitations on the final storage volumes, in each period. Another way to implement this type of policy is to employ inflow forecast. In spite of additional errors involved in forecasting inflows, the model does open the way for reservoir operators to operate the system based on the most up-to-date knowledge of inflow.

Stedinger *et al.* (1984) presented a stochastic dynamic programming model, which is based on Louck's formulation. It uses the best forecast for the current period's inflow to implement the reservoir release policy. They claimed that the use of the best inflow forecast as an inflow state variable, instead of the preceding periods inflow, results in substantial improvements in simulated reservoir operations with derived stationary reservoir operation policies. However, the optimal policy derived in this way was conditioned on inflow forecast, which had been integrated in the model. This unnecessary additional restraint limited the applicability of the model.

Goulter and Tai (1985) applied SDP to model a small hydroelectric dam system. They addressed the aspect of discretization of the storage space in the modelling process. They found that using too small numbers of storage states can result in unrealistic high skewness in the storage probability distribution functions and therefore, affect the optimal operation policy. On the other hand, the computational burden may limit the number of applicable storage states that can be considered in the model.

Nandalal and Bogardi (Nandalal, 1986; Bogardi and Nandalal, 1988) used a SDP model (which is similar to Louck's version with respect to decision and inflow state variables) to derive operation policies for two serially linked multipurpose (irrigation and energy generation) reservoirs on the Mahaweli River in Sri Lanka. The joint transitional probabilities of inflows were defined based on a Markov chain. The discrete time series and their probabilities were used to approximate the continuous distribution of the inflows. The model has the objective of maximizing expected annual energy generation subjected to the constraint of satisfying average annual irrigation requirement. Besides the SDP model, a deterministic DP model based on the incremental DP (IDP) was also formulated. The developed operational policies based on the SDP model were verified through simulation and were compared with the optimum operation of the system obtained by the deterministic method. They pointed out again the problem of the "curse of dimensionality".

Budhakooncharoen (1986) studied the operation of a hydro-power plant using the IDP and SDP (Louck's version) formulations. The models have the objective to maximize the expected annual energy generation. The derived operational policies were compared with historical operational records of the Kariba Reservoir on the Zambezi River, Central Afrika. A sensitivity analysis was carried out by varying the installed capacity of the power plant, the size of the reservoir, and the minimum drawdown level to identify potential increase of the installed power generation and/or the reservoir capacity.

Karamouz and Houcks (1987) formulated two dynamic programming models, one deterministic and the other stochastic (Butcher's version), to determine reservoir operating rules. These formulations were then tested with 12 cases of monthly operation of single reservoirs. The deterministic model (named DPR) constituted an algorithm that cycled through three components: a dynamic programme, a regression analysis and a simulation. The stochastic dynamic programme (SDP) considered the inflow with a discrete lag-one Markov process. To test the usefulness of both models in generating reservoir operating rules, real-time reservoir operation simulation models were constructed for three hydrologically different sites. The rules generated by DPR and SDP were then applied in the operation simulation model and their performances were evaluated. It was concluded that the DPR generated rules are more effective in the operation of medium to very large reservoirs and the SDP generated rules are more effective for the operation of small reservoirs. They showed that the DPR model is more sensitive to the number of characteristic storages and requires usually a large number of storage state variables to function properly. Especially when the reservoir is fairly large (1.0 - 1.7 times the mean annual flow).

Tai and Goulter (1987) developed a heuristic stochastic dynamic programming model to derive a monthly operation policy for a "Y" shaped hydroelectric system consisting of three reservoirs. The unique feature of this system was that it had two upstream reservoirs without any hydroelectric generating capacity and with only the storage regulation structures. The author concluded that two upstream reservoirs must respond to the requirements of the downstream station. Under this principle, they described a heuristic approach for this particular system. The method started with finding an optimum operation of the downstream reservoir using the historical data. Then the resulting optimal operation policies were used to determine relative weights or targets for finding the optimal operation policies for the upstream reservoirs. New input inflows to the downstream reservoir were then computed by running the historical inflow records through the optimal policies for the upstream reservoirs. Subsequently, optimal policies for downstream units were computed. This resulted in new sets of targets for upstream units. This iterative process terminated when the same overall system benefits for two successive iterations were achieved. It was shown that the best results with respect to accuracy and the requirement of computational efforts, could be obtained with nine storage state variables. The authors also mentioned that the number of storage states causes the problem of the "trapping states".

He and Bogardi (He, 1987; He and Bogardi, 1990a; He and Bogardi, 1990b) studied a strategy based on a single reservoir stochastic dynamic programming (Louck's version) concept to obtain operation policies for a system of three tandem reservoirs in Northern China. In the study, the concepts such as hypothetical composite reservoir and iterative SDP/simulation were introduced to break up the large and complicated original problem of multireservoir optimization into some simpler sub-problems that can be solved separately. Besides, the impact of different objective functions such as minimizing reservoir spillage, minimizing quadratic deviation from downstream water demand and maximizing reservoir releases were investigated.

Shrestha (1987) applied SDP to derive optimal operation policies for a hydropower system, which was in the planning stage. Simulation of the system operation was carried out based on the SDP based optimum policy to evaluate the system performance. Finally the optimum system configuration was selected by comparing the performance values obtained for the different configurations.

Bogardi *et al.* (1988) investigated the impact of varying the number of storage and inflow classes upon the operational performance of SDP for both single and multiunit reservoir systems. Their results indicated that by simply increasing the number of storage classes beyond certain limits, the system performance would not improve dramatically. They stated that emphasis should be placed on the "synchronization" of the number and size of storage and inflow classes, to check whether any improvement can be obtained this way.

Laabs and Harboe (1988) presented three models based on dynamic programming technique including a deterministic model, a probabilistic model and a stochastic model (Butcher's version) for finding Pareto-optimal operating rules for a multipurpose reservoir. The complex stochastic model included several objective functions and weighting factors for each objective as needed in a compromise analysis of multiobjective decision making. As a result many pareto-optimal operating rules for the reservoir were obtained. The final selection of an optimal policy can be done only after real-time simulations with these operating rules (with historical and synthetic flow records) have been performed and a multiobjective selection criterion is applied to the results.

Kularathna and Bogardi (Kularathna, 1988; Kularathna and Bogardi, 1990) extended the two serially linked reservoir system of the Mahaweli Development Scheme in Sri Lanka studied by Nandalal (1986) to a system comprising three reservoirs. Four different SDP based techniques were tested for their applicability in deriving optimal operation policies for the system. The first technique was a sequent optimization approach, which optimized the cascade reservoir system starting from the uppermost reservoir, and proceeding downwards. The second approach considered the two downstream reservoirs as a single hypothetical composite reservoir. An iterative SDP/simulation based approach was introduced as the third technique. A conventional SDP algorithm had been used as the fourth approach, which considered the three-reservoir-in-series configuration in the optimization process simultaneously. Although not applicable in general, they concluded that the sequent optimization as the best approach to derive operation policies for that reservoir system. The operation policy derived by the three reservoir model can hardly be used as an adequate guideline for the actual operation because of its rough discretizations of river inflow and reservoir storage.

Shrestha (1988) studied the optimum operation of a multipurpose water resources system based on a SDP model, considering both inflows and downstream irrigation demands as stochastic variables. The analysis was carried out with two types of objective functions: (a) maximization of expected annual energy generation, and (b) minimization of deviation of the release from the irrigation demand. Two serially linked reservoirs on the Mahaweli River in Sri Lanka were used for the case study. The simulation of the reservoir operations was carried out based on the SDP optimum policy to evaluate the system performance. The problem of the "curse of dimensionality" again limits the applicability of the model.

Huang *et al.* (Huang, 1989; Huang *et al.*, 1991) developed an Operational Mode Switch (OMS) system for the on-line operation of the Feitsui multipurpose reservoir situated in a typhoon-prone area, in Northern Taiwan. The decision about whether to and when to shift the operation back and forth between the long term "normal mode" and short term "emergency mode" was determined by the OMS model. The reservoir operation in "normal mode" followed SDP based release policies. Four types of SDP models were considered. Those are the SDP models based on current or past inflows, and conditional and unconditional inflow transition probabilities. Among the proposed four types of SDP, they concluded that the one with observed inflows performs better than the others with forecasted inflows. However, they pointed out that the conclusion may not hold under different hydrological regimes. Locating in a typhoon area, the case study system has the prominent feature that there will be inevitable substantial errors in the forecasted inflows.

Kelman *et al.* (1990) developed an implicit type of stochastic model called Sampling Stochastic Dynamic Programming (SSDP) based on SDP. The model captures the complex temporal and spatial structure of the streamflow process by using a large number of sample streamflow sequences. The best inflow forecast can be included as a hydrological state variable to improve the reservoir operating policy. The authors illustrated the SSDP approach and its performance through its application to a case study of the hydrological system on the North Fork of the Feather River in California.

Bogardi *et al.* (Bogardi and He, 1991; Bogardi *et al.*, 1991) revealed considerable insensitivity of the SDP based reservoir operation performance with respect to inaccuracies of the inflow data and their model representation. Furthermore, the simulated value of the objective function appeared to be an inadequate indicator to measure the impact of the selection of the objective function and constraint set, to be relied upon in the SDP computation. In the study, the potential reasons of the above described phenomena were analyzed. The results of the study clearly showed the need to identify the key indices for various combinations of reservoir systems, objective functions and constraint sets.

Bogardi and Verhoef (1991) demonstrated an example of evaluating the operation policies of a reservoir system derived according to SDP, by using performance evaluation indices, such as reliability, reparability and vulnerability. The reference reservoir system selected was the Mahaweli reservoir system in Sri Lanka. Their research confirmed that the traditional evaluation indices, such as the simulated value of the objective function and the output figures related to the constraints, are not sufficient for an adequate characterization of the SDP based operation performance.

Kularathna (1992) applied aggregation/disaggregation techniques based on SDP (Louck's model) and simulation to analyze a complex water resources system in Sri Lanka. The identification of subsystems by their functional and physical characteristics was an important first step in the analysis. Subsequently each subsystem was represented by a hypothetical composite reservoir to arrive at an operation policy for the interface point of the subsystem. A more detailed analysis that considers the real configurations of the subsystems was performed by following this operation policy of the interface point. Two approaches: sequential optimization and iterative optimization were presented. In those approaches, each subsystem was individually analyzed using a two reservoir SDP model.

3 Model Description

3.1 Stochastic Dynamic Programming (SDP) Models for Single Reservoir Operation

3.1.1 Calculation Procedure of the Model

Stochastic Dynamic Programming is an extension of DP that considers the stochastic feature of inflow. Therefore, the structure of SDP model, similar to that of the conventional DP model, is defined by stage, state and decision variables, objective function, constraints and recursive equation. To incorporate the stochastic feature of inflow, it is defined as an additional state variable. The probability of inflow can be estimated from the observed inflow time series, and thus the optimization is based on the expectation of objective values. Optimum operation policies are obtained by iterating the recursive equation for each stage in successive years until they become stable (converged). The procedure for calculation of the SDP model is summarized in Figure 3.1. Each step used in the calculation procedure is explained in the following subchapters.

3.1.2 Stage

In a SDP formulation of reservoir operation, time periods (e.g., month or week) are often considered as stages. Thus, one period represents one stage and the total number of periods in a year represent a cycle. Since a SDP model has to be solved backward, the last period of the cycle becomes the initial stage of the next cycle. Two indices have been used to describe the stage of the model clearly. One is the absolute index n, which denotes the total number of periods passed in the backward moving optimization. The other is the within-cycle index t, which denotes the number of the time period within the year cycle. The relationship between these two notations is shown in Figure 3.2. T in Figure 3.2 is the total number of time periods in one year cycle (which is 12 if the length of time period is month).

3.1.3 State and Decision Variables

The storage volume of water in the reservoir at the beginning of the time periods represents the state of the system. To incorporate the Markovian nature of the streamflow, it is also defined as a state variable in SDP formulations. Therefore, a SDP formulation of a reservoir operational problem will have a two-dimensional state variable consisting of the storage volume and the inflow to the reservoir.

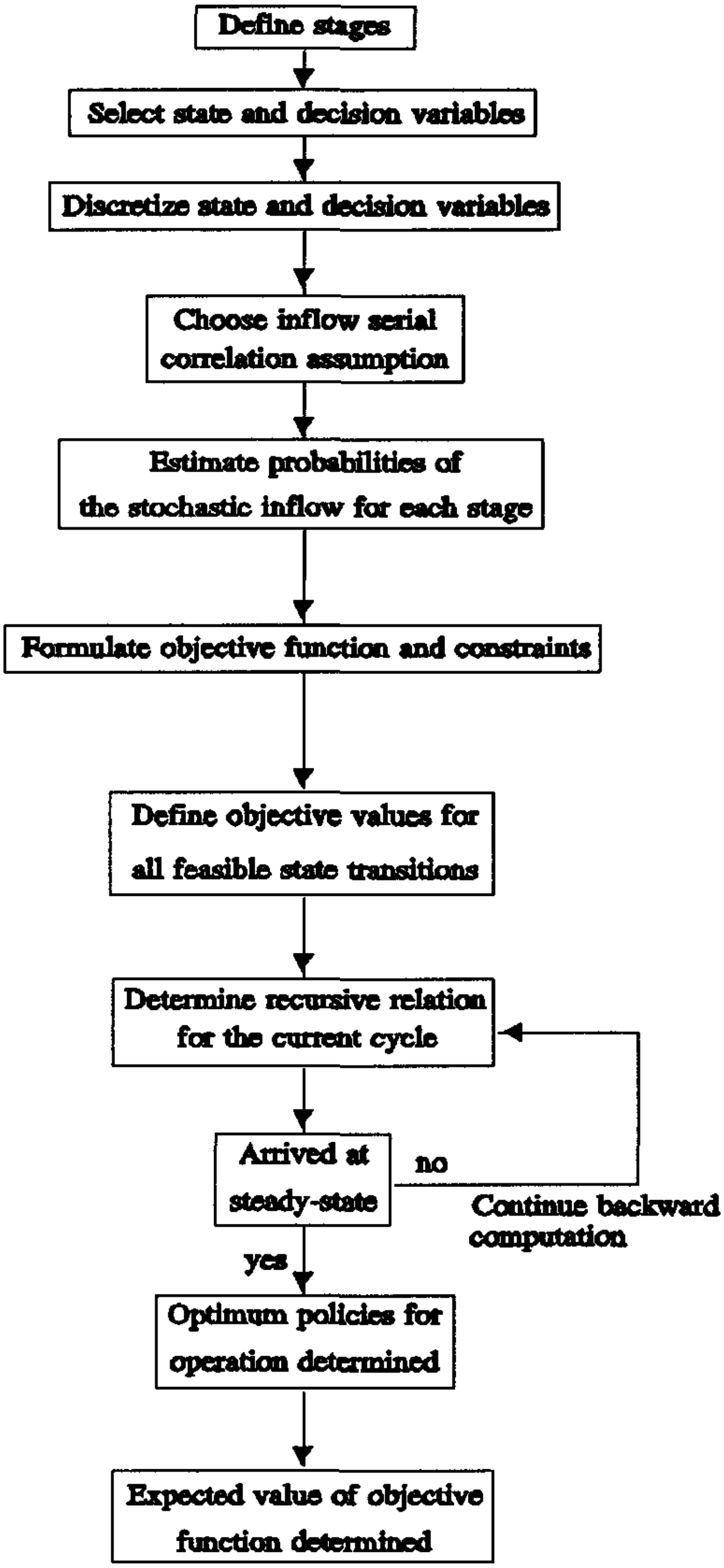


Figure 3.1 Calculation Procedure of the SDP Model

The decisions to be made at every stage are concerned with the reservoir releases contributed to the objective formulated in the objective function. Therefore, release during the period t, R_t, is frequently the choice of the decision variable. Sometimes the idea can be also implicitly identified by specifying the storage at the end of stage t (i.e., at the beginning of stage $t+1$), S_{t+1}, as the decision variable. The relationship between the stage notation t and the other variables are as shown in Figure 3.3. Q_t is inflow to the reservoir during period t. The suitability of various choices of decision and inflow state variables in SDP model will be investigated in Chapter 6.

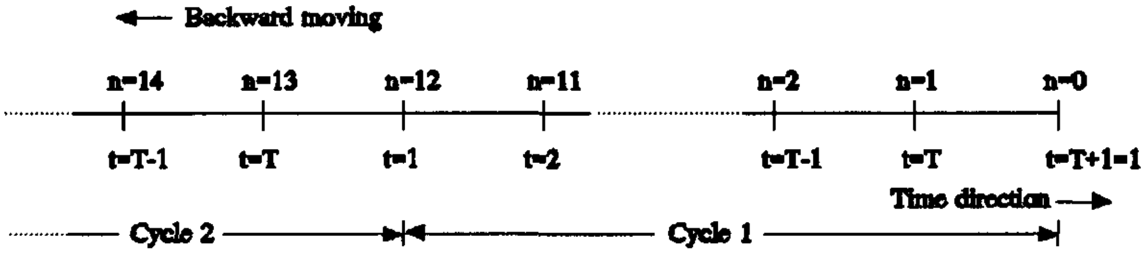


Figure 3.2 Relationship Between Stage Notations

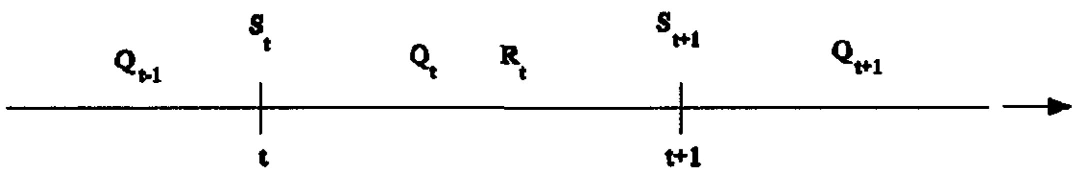


Figure 3.3 Relationship Between Stage Notation t and Other Variables

3.1.4 Discretization of Storage, Inflow and Release Variables

The active storage volume of the reservoir is divided into equally spaced storage intervals. The boundary values of these storage intervals are taken as the characteristic storage volumes falling within halves of the storage intervals above and below them. The smallest and the largest storage states correspond to the storage volumes at minimum operating level and at the full supply level, respectively. They represent only the feasible halves of the smallest and the largest storage classes.

The historically observed minimum and maximum inflow values for each time period (stage) form the range of possible inflow for that time period. Two discretization schemes of inflow states are considered. In the first scheme, the domain of inflows is divided into equally spaced (uniform) inflow intervals. In the second scheme, the domain of inflows is divided into intervals in such a way so that an equal number of inflows fall into each interval. This may result in non-uniform interval sizes. In both schemes, the averages of the inflows fall into the intervals are chosen as the discrete values to represent inflow classes.

The range of profitable release volume is divided into equally spaced intervals. This range is from 0 upto the downstream water demand for the water supply system or upto a value that is comparable with the release capacity of turbine pipes for the hydro-power system. The boundaries of these release intervals are taken as the characteristic release states, representing release volumes falling within halves of the intervals above and below these states. Thus, the smallest and the largest release states, correspond to the release volumes of 0 and the largest profitable release volume level, respectively. They represent the feasible halves of the smallest and largest intervals.

3.1.5 Inflow Serial Correlation

It is not uncommon that an observation at one time period is correlated with the observation in the preceding time period in an inflow time series. Such a correlation is termed "serial correlation" or "autocorrelation".

Serial correlation also can exist between an observation at one time period and an observation k time periods earlier for $k=1, 2, \ldots$. In this discussion it is assumed that observations are equally spaced in time and that the statistical properties of the process do not change with time (stationary process). The population serial correlation coefficient is denoted by $\rho(k)$ where k is the lag or number of time intervals between the observations being considered. The sample serial correlation coefficient will be given by $r(k)$. The sample serial correlation coefficient for a sample of size n is given by;

$$r(k) = \frac{\sum_{t=1}^{n-k} Q_i * Q_{i+k} - \frac{\sum_{i=1}^{n-k} Q_i * \sum_{i=1}^{n-k} Q_{i+k}}{n-k}}{\left[\sum_{i=1}^{n-k} Q_i^2 - \frac{(\sum_{i=1}^{n-k} Q_i)^2}{n-k}\right]^{1/2} * \left[\sum_{i=1}^{n-k} Q_{i+k}^2 - \frac{(\sum_{i=1}^{n-k} Q_{i+k})^2}{n-k}\right]^{1/2}} \tag{3.1}$$

From Equation 3.1 it is seen that $r(0)$ is unity. That is, the correlation of an observation with itself is one. If $\rho(k)=0$ for all $k=0$, the process is said to be an uncorrelated stochastic process. The trade-off among various inflow correlation assumptions will be investigated in Chapter 7.

3.1.6 Markov (or Markov-I) Inflow Process

"Markov process" (or "Markov chain" - the discretized presentation of the Markov process) is the most widely applied inflow serial correlation assumption for the SDP models. It is the key to the understanding of other inflow serial correlation assumptions.

In general, a Markov process describes only one-step dependence, called a first-order process, or exhibiting lag-one serial correlation (Markov-I). The process has the property that the dependence of future values of the process on past values is summarized by the current value.

$$P(Q_{t+1} | Q_t, Q_{t-1}, Q_{t-2}, \ldots) = P(Q_{t+1} | Q_t) \tag{3.2}$$

In a Markov chain, the move from a state at period t to a subsequent state at period $t+1$ is called "transition". assume that state Q at any time step can assume values from the set q,

$q=(q_1, q_2, \ldots)$. The transition probability P_{ij}^t is the conditional probability of the transition from state q_i at period t to state q_j at period $t+1$. When it is stationary it can be expressed as:

$$P_{ij}^t = P(Q_{t+1}=q_j/Q_t=q_i) \tag{3.3}$$

which satisfies:

$$0 \leq P_{ij}^t \leq 1 \qquad \forall\ i,j$$

$$\sum_j P_{ij}^t = 1 \qquad \forall\ i$$

A Markov chain is completely described by the initial state and the complete matrix of transition probabilities. The initial state is the value of the variable at the beginning, $p_i=P(Q_0=q_i)$. The vector $p=[p_1,p_2,\ldots]$ is called the "initial probability distribution" for the Markov chain. The complete matrix of transition probabilities is a matrix with elements P_{ij} for all possible states i and j in the Markov chain.

$$P = \begin{bmatrix} P_{11} & P_{12} & \cdot \\ P_{21} & P_{22} & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix} \tag{3.4}$$

Let $P_{ij}(n)$ denote the n-step transition probability, which is the probability that n periods from now the state will be j, given that the current state is i. It holds (Feller, 1968) that,

$$P_{ij}(n) = ij \text{ th element of } P^n.$$

After a large number of periods have elapsed, the n-step transition probabilities P^n approach a matrix with identical rows, for a certain class of Markov chains. These are called the steady-state probabilities. These conditions will be discussed in Chapter 5.

$$\lim_{n\to\infty} P^n = \begin{bmatrix} P_1 & P_2 & \cdot & \cdot & P_n \\ P_1 & P_2 & \cdot & \cdot & P_n \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ P_1 & P_2 & \cdot & \cdot & P_n \end{bmatrix} \tag{3.5}$$

The steady state probabilities are independent of the initial states. They describe the long-run behaviour of Markov chains.

3.1.7 Estimation of Inflow Probabilities

The inflow discrete probabilities (either independent or dependent) are estimated from the historical inflow time series. In the present research, three inflow correlation assumptions will be adopted into a SDP model. They are,

(i) the independent assumption:

$$P(Q_{t+1}|Q_t,Q_{t-1},Q_{t-2},....) = P(Q_{t+1}) \tag{3.6}$$

(ii) the Markov-I assumption:

$$P(Q_{t+1}|Q_t,Q_{t-1},Q_{t-2},....) = P(Q_{t+1}|Q_t) \tag{3.7}$$

(iii) the Markov-II assumption:

$$P(Q_{t+1}|Q_t,Q_{t-1},Q_{t-2},....) = P(Q_{t+1}|Q_t,Q_{t-1}) \tag{3.8}$$

In general, the independent probabilities $P_t(Q_t=q_j)$ are estimated from the observed inflow records by counting the number of times the recorded inflow values fall into interval j in period t. i.e.,

$$P_t(Q_t=q_j) = \frac{N(Q_t=q_j)}{N(Q_t)} \quad \forall\ j,t \tag{3.9}$$

Where, $N(Q_t=q_j)$ is the number of inflows that fall into interval j in period t; $N(Q_t)$ is the number of inflow samples in period t.

The Markov-I transition probabilities $P_t(Q_t=q_j|Q_{t-1}=q_i)$ are estimated from the observed inflow records by tabulating the number of times the observed data went from state i to state j from period t-1 to period t. That is,

$$P_t(Q_t=q_j|Q_{t-1}=q_i) = \frac{N(Q_t=q_j,Q_{t-1}=q_i)}{N(Q_{t-1}=q_i)} \quad \forall\ j,i,t \tag{3.10}$$

Where, $N_t(Q_t=q_j,Q_{t-1}=q_i)$ is the number of inflows that fall into interval j and i in period t and t-1; $N_t(Q_{t-1}=q_i)$ is the number of inflows that fall into interval i in period t-1.

The Markov-II transitional probabilities $P_t(Q_t=q_j,Q_{t-1}=q_i|Q_{t-1}=q_i,Q_{t-2}=q_h)$ are estimated from the observed inflow records by tabulating the number of times the observed data went from state h,i to state i,j from period t-2, t-1 to period t-1, t.

$$P_t(Q_t=q_j,Q_{t-1}=q_i|Q_{t-1}=q_i,Q_{t-2}=q_h) = \frac{N(Q_t=q_j,Q_{t-1}=q_i,Q_{t-2}=q_h)}{N(Q_{t-1}=q_i,Q_{t-2}=q_h)} \quad \forall\ j,i,h,t \tag{3.11}$$

Where, $N(Q_t=q_j,Q_{t-1}=q_i,Q_{t-2}=q_h)$ is the number of inflows that fall into interval j, i and h in period t, t-1 and t-2; $N(Q_{t-1}=q_i,Q_{t-2}=q_h)$ is the number of inflows that fall into interval i and h in period t-1 and t-2.

However, due to the limited length of the historical inflow time series, the estimated numerical values of the elements in the probability matrices are rather unreliable (errors in estimations) and many elements remain void (ill-structured probability matrices). These problems will be discussed in depth in Chapter 7 and Chapter 5, respectively.

3.1.8 State Transformation Equation

The state transformation equation in the SDP model is expressed by the principle of continuity for the water quantity in the reservoir.

$$S_{t+1} = S_t + Q_t - R_t - SP_t - E_t \qquad \forall t \qquad (3.12)$$

Where, t is time period within year; S_t is storage in reservoir at the beginning of period t; Q_t is inflow to the reservoir during period t; R_t is release from the reservoir during period t; SP_t is spillage from the reservoir during period t; E_t is evaporation from the reservoir during period t; the relation between time notations and variables are as shown in Figure 3.3.

3.1.9 Physical Constraints of Reservoir Systems

Typical physical constraints of the reservoir systems include maximum and minimum storages, maximum and minimum releases, penstock and equipment limitations.

The storage of the reservoir during any time period must be within the limits of maximum and minimum live storage capacity.

$$S_{t,min} \leq S_t \leq S_{t,max} \qquad \forall\, t \qquad (3.13)$$

Where, t and S_t are as defined in Equation 3.12; $S_{t,min}$ is minimum live storage of the reservoir at the beginning of period t; $S_{t,max}$ is the maximum live storage of the reservoir at the beginning of period t.

The maximum releases are usually defined by the conveyance capacity of downstream channels. For those systems with hydro-power plants, the capacity of the power generators set a maximum limit to the reservoir releases. The minimum release is the compulsory release from the system, if any. The releases during any period should be within the feasible range.

$$R_{t,min} \leq R_t \leq R_{t,max} \qquad \forall\, t \qquad (3.14)$$

Where, t and R_t are as defined in Equation 3.12; $R_{t,min}$ is minimum release from the reservoir during period t; $R_{t,max}$ is the maximum release from the reservoir during period t.

For those reservoir systems with hydro-power plants, energy generation during each period should satisfy the firm power of the system. Also, it cannot be more than the generation capacity.

$$EG_{t,min} \leq EG_t \leq EG_{t,max} \qquad \forall\, t \qquad (3.15)$$

Where, t is as defined in Equation 3.12; $EG_t = 9.81 * \eta * R_t * H_t * T_t / 10^6$ (MWh); R_t is the release from reservoir during period t in m^3/s; $H_t = EL_t - TW_t$ (m); EL_t is elevation of water level in reservoir during period t (m); TW_t is tail water level of power station during period t (m); T_t is time in hours in period t; $\eta = 0.75$, the overall efficiency (turbines + generator transmission); $EG_{t,min}$ is minimum energy generation during period t; $EG_{t,max}$ is maximum energy generation during period t.

3.1.10 Objective Functions

A reservoir can serve single or multiple purposes, which range from hydro-power generation, water supply for irrigation, industrial and domestic use, flood control, water quality improvement, recreation to navigation. A properly formulated objective function of the optimization model should reflect the purposes and key concerns of the reservoir system.

Three reservoir systems have been selected as case studies for this research. They are operated for the purposes of water supply (Tunisia), or hydro-power generation (Kariba), or multi-purposes of both irrigation water supply and hydro-power generation (Mahaweli).

For reservoir systems with hydro-power generation as their operation purpose, the objective can be, for example, maximizing annual energy generation of the system or minimizing energy deficit from a target.

For maximizing annual energy generation, the formulation is as follows:

$$\max \mathscr{E} [\sum_{t=1}^{N} (EG_t)] \qquad (3.16)$$

Where, EG_t is defined as in Equation 3.15; $\mathscr{E}$ is the expectation operator; N is number of periods in a year.

For minimizing energy deficit from a target, the formulation is as follows:

$$\max \mathscr{E} [\sum_{t=1}^{N} (DE_t)] \qquad (3.17)$$

Where, $DE_t = max\{0, (EG_{t,tar} - EG_t)\}$, is deficit of energy generation from energy target during period t (MWh), $EG_{t,tar}$ is the energy target for period t (MWh); EG_t is defined as in Equation 3.15.

Minimizing the water deficit from the requirement can be the objective function for systems with water supply as their operation purpose. This can be formulated as:

$$\max \mathscr{E} [\sum_{t=1}^{N} (DR_t)] \qquad (3.18)$$

Where, $DR_t = max\{0, (R_{t,tar} - R_t)\}$, is deficit of release from release target during period t (MCM); $R_{t,tar}$ is release target for period t (MCM); t and R_t are defined as in Equation 3.12.

In some cases, contractual, legal, and institutional obligations arising from the various purposes of the reservoir system can be considered as constraints of the model. The applicability of various objective functions to various situations and the evaluation of their performance are investigated in Chapter 8.

3.1.11 Recursive Equation

There are different forms of recursive equation associated with different correlation characteristics of inflow time series (time dependent or independent), decision variables (release or final storage), inflow state variables (previous inflow or present inflow) and objective patterns (maximization or minimization). They can be generalized in the following form;

$$f_t^n(S_t,SQ_t)=\underset{D_t\ \ Q_t}{opt.}[\Sigma PI_t*[B_t(S_t,Q_t,D_t)+f_{t+1}^{n-1}(S_{t+1},SQ_{t+1})]] \qquad \forall S_t,Q_t,D_t\ feasible \qquad (3.19)$$

subject to,

$$R_t = S_t + Q_t - S_{t+1} - SP_t - E_t$$

$$f_T^1(S_t,SQ_t)=\underset{D_t\ \ Q_t}{opt.}[\Sigma PI_t*B_t(S_t,Q_t,D_t)]$$

Where, t, S_t, Q_t, R_t, E_t and SP_t are defined as in Equation 3.12; n is total number of time periods passed, $n=1,2,...$; D_t is decision variable, can either be R_t or S_{t+1}; SQ_t is state of inflow at period t, which can either be Q_{t-1} or Q_t; PI_t is probability of inflow at either period t or $t+1$, which can be dependent or independent of previous inflow; $B_t(S_t,Q_t,D_t)$ is increment of objective value for the transition state when the decision reaching D_t at the end of period t starting from S_t and having Q_t inflow during the period; $f_t^n(S_t,SQ_t)$ is (sub) optimal value of the recursive equation at stage n (period t) as function of S_t and SQ_t; the relation between time notations and variables are as shown in Figure 3.3.

To solve the optimization problems defined by the recursive equation, a backward DP algorithm is used. Note that in the case of SDP a forward algorithm has no sense as the expectation over the future states has to be considered. The value of the objective function at the last state is initialized. Computations proceed backward by stages until a steady state is reached. The procedure of carrying out the recursive calculation is shown in Figure 3.4.

3.1.12 Convergence Criteria

There are two convergence criteria that mark the so called "steady state" condition.

(a) The first criterion is the stabilization of the operating policy. At each stage of the SDP algorithm, an operation policy for that stage is determined. After continuing backward computation for a couple of years, a stable operation policy can be obtained. This implies that the operation policy for each period will not change from year to year. When this condition is reached, the stabilization of the operation policy is achieved.

(b) The second criterion is the stabilization of the expected annual increment of the objective value. During the continued backward computation of the SDP algorithm, the optimum expected return for all possible initial states $(f_t^{n+T}(S_t,SQ_t)-f_t^n(S_t,SQ_t))$ will be determined for each stage (time period). After continuing with backward computation for a couple of years, the expected annual increments of objective values for all initial states in each stage tend to be constant. This phenomenon is called the stabilization of annual return of objective values.

Studying the relationship between convergence criteria of the SDP model and the structure of Markov transition probability matrices is one objective of this research study. It is further discussed in Chapter 5.

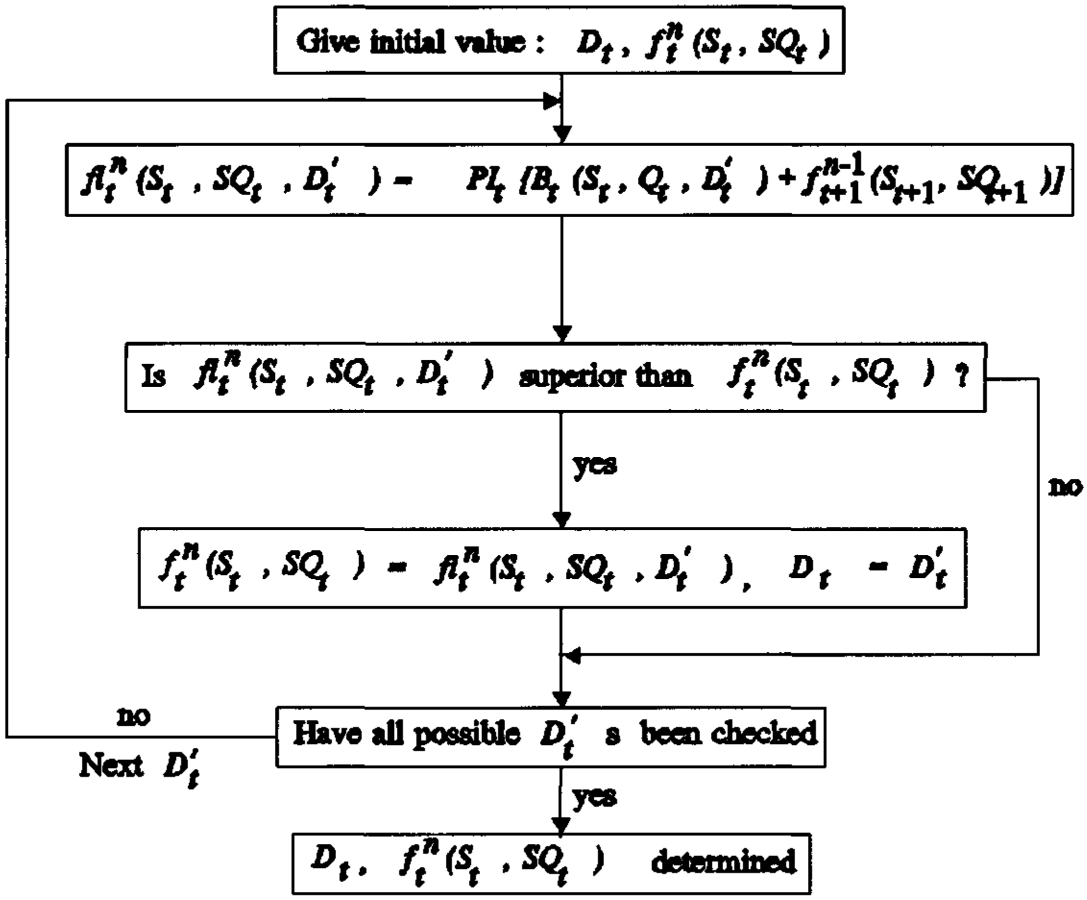


Figure 3.4 Calculation Flowchart of Recursive Equation

3.2 SDP Models for Two Reservoirs in Series

In this research, one case study concerns a system with two reservoirs in series. The formulation of a SDP model for such a system is almost similar to that for a single reservoir system. The difference is due to the increase of the number of state and decision variables.

Its stage, state and decision variables, objective function and constraints are defined similar to those of a single reservoir SDP model. However, it is worth noting the following points;

(a) The continuity equation should reflect the relation between the two reservoirs in the system,

$$S_{t+1,u} = S_{t,u} + Q_{t,u} - R_{t,u} - SP_{t,u} - E_{t,u} \qquad \forall t \tag{3.20}$$

$$S_{t+1,d} = S_{t,d} + Q_{t,d} + SP_{t,u} + R_{t,u} - R_{t,d} - SP_{t,d} - E_{t,d} \qquad \forall t \tag{3.21}$$

Where, t, S_t, Q_t, R_t, E_t and SP_t are as defined in Equation 3.12; u denotes upstream reservoir; d denotes downstream reservoir.

(b) In the recursive equation, the inflow probabilities for single reservoir SDP model will be replaced by joint inflow probabilities for the two reservoir system.

For example, the joint independent probability $JP_t(Q_t=q_j)$ is the probability that the inflow to the reservoirs upstream and downstream at period t will fall in states $q_{j,u}$ and $q_{j,d}$ respectively.

The joint dependent (transition) probability $JP_t(Q_t=q_j|Q_{t-1}=q_i)$ is the probability that the inflow to the reservoirs upstream and downstream at period t will fall in states $q_{j,u}$ and $q_{j,d}$ given that at time period t-1 the inflow to the reservoirs upstream and downstream were in states $q_{i,u}$ and $q_{i,d}$ respectively.

3.3 Performance Assessment of SDP based Operation Policy

The SDP based optimum operation policy has been evaluated by operation simulation. The simulation has been carried out by using either (a) the same piece of historical inflow series, or (b) forecasted inflow series. Case (a) opens the way to make a comparison between the system performance values obtained from steady state solution of SDP (which relies on discrete representation of input states) and system performance values obtained from a simulation that uses actual inflow series. Case (b) shows how the policy performs in a real world situation.

The simulation procedure begins with the assumption of an initial storage state and an inflow state representing chosen historical inflow. For this combination, storage and inflow state, the decision (either final storage state or current release level) to be reached is defined by the SDP model. These storage state(s), release state and inflow state are then used for computing the objective values, their standard deviations and variance and associated performance indices. Those are used in the evaluation of the system performance. The suitability of various performance evaluation criteria is evaluated in this research study in Chapter 8.

Since the optimum operation policies are determined using expected system performance based on discrete storage and inflow states, there is possibility that in some periods the actual releases or final storage will be out of their feasible range. That is, the release may be less than 0 or larger than the down stream channel capacity or final storage may be less than dead storage or larger than reservoir capacity. In such instances, corrections (e.g., over-ruling the SDP optimum operation policy) are made in the simulation model. The schematic diagram of the simulation procedure is shown in Figure 3.5.

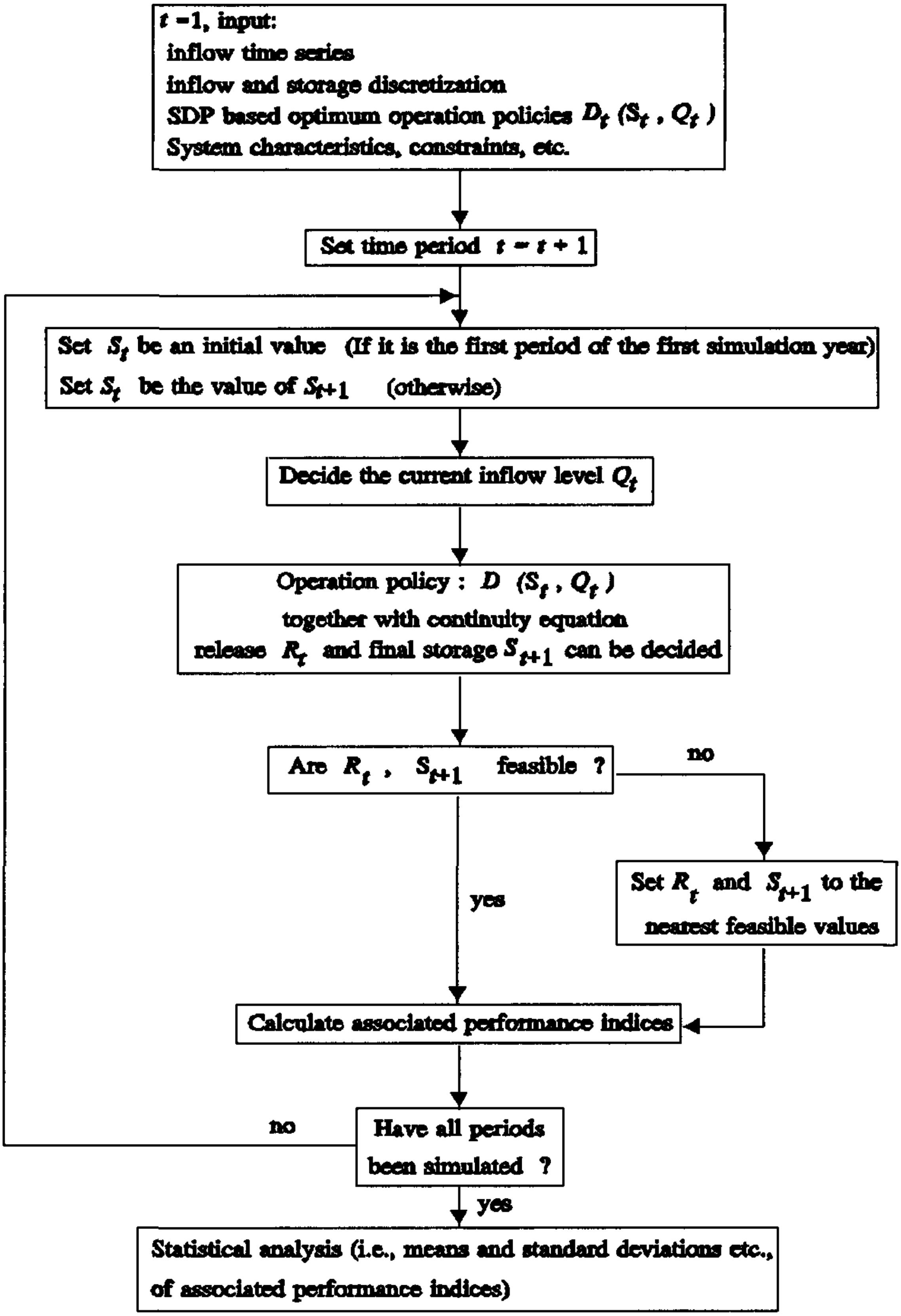


Figure 3.5 Simulation Procedure

4 Description of the Case Study Systems

To investigate the performance of SDP three reservoir systems were selected as test systems in this study. They are the Mahaweli reservoir system in Sri Lanka, the Kariba Lake in Zambia and Zimbabwe, and the Joumine reservoir in Tunisia.

With the common hydrological characteristic of strong seasonal fluctuation that makes them interesting to stochastic optimization, these three cases represent three different configurations of a reservoir system. Both Kariba and Joumine are single-unit/single-purpose systems. However, the Kariba is a robust system with huge reservoir capacity and huge reservoir catchment. Its turbine capacity is relatively small and it serves for energy generation. The Joumine is a more stressful system whose demand almost equals the mean annual inflow. It serves for water supply. The Mahaweli system is a multi-unit/multi-purpose system. The details of these three reservoir systems, Mahaweli, Kariba and Joumine are presented in the following sections.

4.1 Mahaweli Reservoir System

The Mahaweli reservoir system is a comprehensive multipurpose water resources development scheme planned to harness hydroelectric and irrigation potential of Mahaweli Ganga (river), Sri Lanka's largest and the most important river.

The climate conditions in Sri Lanka are dominated by two monsoons, the Southwest monsoon (April to September) and the Northeast monsoon (October to March). The central hills impose a strong orographic influence. This, with other factors lead to the subdivision of the country into three climatic zones; wet zone, intermediate zone and dry zone as shown in Figure 4.1. The Mahaweli reservoir system has been based on using the naturally diverse flow pattern of the Mahaweli Ganga, regulated where possible with storage reservoirs, to satisfy irrigation demands in the dry zone. Hydroelectric energy is generated at storage dams and along some diversion routes, and fed into the National Electricity Grid.

Figure 4.2 presents the schematic diagram of the Mahaweli system. For this study, the part of the system from the Polgolla diversion to the Minipe anicut (Victoria, Randenigala and Rantembe cascade three-reservoir system) was selected. The Rantembe was considered as a run-off reservoir. The acceptability of this simplification has been confirmed by Nandalal (1986). The principal features of the relevant reservoirs are summarized in Table 4.1.

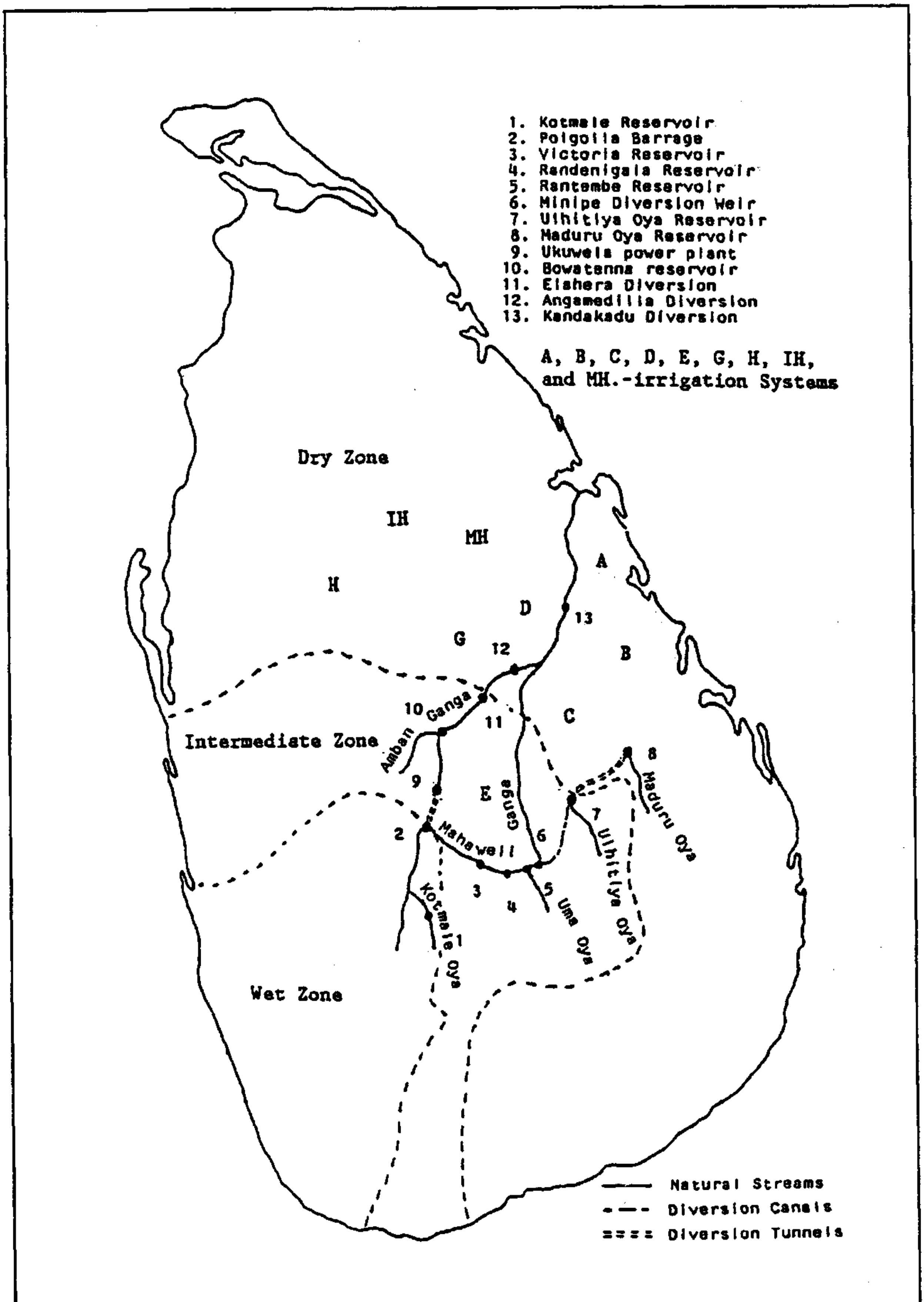


Figure 4.1 Layout of the Mahaweli Development Scheme and the Climatological Regions of Sri Lanka

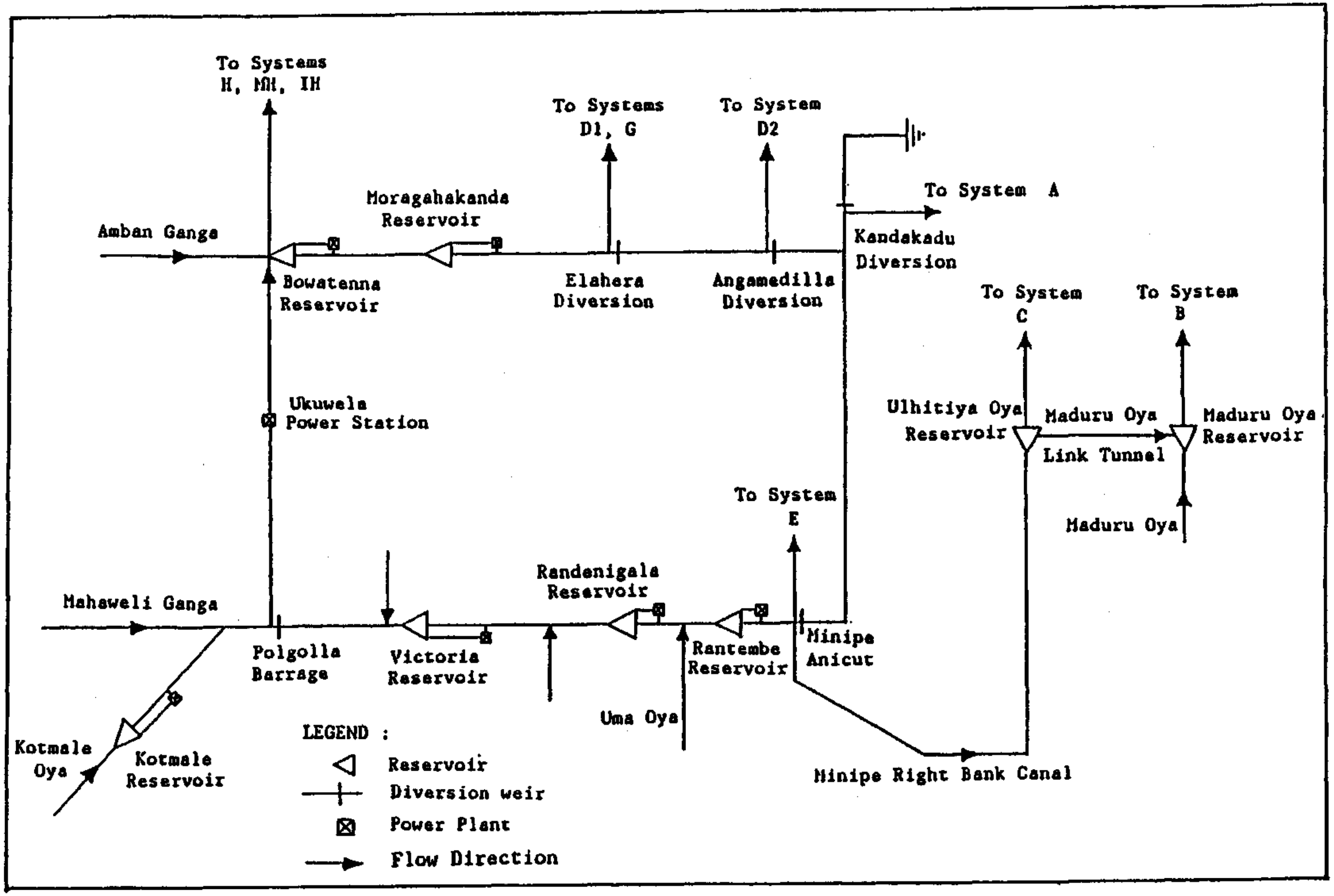


Figure 4.2 Mahaweli Development Scheme

Table 4.1 Principal Features of the Mahaweli Reservoirs

Characteristics		Victoria	Randenigala	Rantembe
Catchment Characteristics				
Catchment area	km^2	1891	2365	3111
Average annual inflow	MCM	1984	2528	3126
Completion of construction		1984	1986	1990
Reservoir Characteristics				
Retention water level	m	438.0	232.0	152.0
Design flood level	m	441.2	236.2	155.0
Maximum water level	m	370.0	203.0	140.0
Active storage	MCM	686.0	580.0	17.0
Reservoir volume factor	%	23.5	16.5	4.1
Power House Characteristics				
Average net head	m	190.0	78.0	32.7
Turbine discharge	m^3/s	140.0	180.0	180.0
Turbine		3 Francis	2 Francis	2 Francis
Installed capacity	MW	210	126	49
Firm energy	GWh/yr	446	304	176

The monthly evaporation values from the Victoria and Randenigala reservoirs are given in Table 4.2. Table 4.3 shows the elevation-storage-area relationships for the Victoria and Randenigala reservoirs.

Table 4.2 Monthly Evaporation Values from the Victoria and Randenigala Reservoirs

Reservoir	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep
Victoria (mm)	50	26	28	66	143	159	103	124	125	131	147	138
Randenigala(mm)	48	6	3	48	129	149	105	137	154	160	178	165

Table 4.3 Elevation-Storage-Area Relationships of the Victoria and Randenigala Reservoirs

Victoria			Randenigala		
Elevation (m)	Storage (MCM)	Area (km^2)	Elevation (m)	Storage (MCM)	Area (km^2)
355.0	9.0	1.1	201.5	270.0	13.8
365.0	24.0	1.9	203.0	295.0	14.4
375.0	47.0	2.7	206.5	355.0	15.8
385.0	80.0	4.0	210.0	415.0	17.2
395.0	129.0	5.9	215.0	503.0	18.9
405.0	200.0	8.7	220.0	590.0	20.6
415.0	306.0	12.6	225.0	708.0	21.9
425.0	455.0	17.2	230.0	825.0	23.2
435.0	651.0	22.2	233.1	903.0	23.8
440.0	768.0	24.8	236.2	980.0	24.3

The releases of the Kotmale reservoir and incremental inflows upto Polgolla are diverted by a barrage at Polgolla to meet the requirements of the irrigation area in the Amban Ganga basin. The excess, which is spilled over the Polgolla barrage has to be estimated and added to the incremental inflow between the Polgolla barrage and the Victoria reservoir, to get the total inflows to the Victoria reservoir. Nandalal (1986) simulated the operation of the system upstream of Polgolla barrage according to the operation rules recommended by the Acres International Limited (1985). The spillages over the Polgolla barrage obtained from that simulation were used in this study. These values are given in Table A.1 in Appendix A.

Observed records of monthly incremental inflows to the Victoria, Randenigala and Rantembe reservoirs are presented in Table A.2, Table A.3 and Table A.4, respectively in Appendix A. Further, the irrigation demands at Minipe were obtained from a system simulation carried out on Ulhitiya-Maduru Oya basins by Nandalal (1986). This simulation has been carried out using the water demands for irrigation areas A, B, C and E (see Figure 4.2). The rule curves developed for the Ulhitiya Oya and Maduru Oya reservoirs by Acres International Ltd. (1985) have been adopted in the reservoir operations in that simulation. The estimated irrigation demands at Minipe are presented in Table A.5 in Appendix A.

4.2 Kariba Reservoir System

The Zambezi River rises in northern Zambia. After flowing through Angola, it forms the boundary between Zambia and Zimbabwe. At the end, it passes through Mozambique to discharge into the Indian Ocean, north of Beira. The Zambezi River basin is shown in Figure 4.3.

The catchment area upstream of Kariba Gorge is approximately 664,000 km^2. Rainfall over the catchment is strongly seasonal. Normally the rainy season extends from November to March. The river's peak discharge reaches the Victoria Falls, approximately 400 km upstream of Kariba Gorge, at the beginning of May. The discharge at the Falls returns to base flow in October or November. In January the flows begin to rise. Additional inflow to Lake Kariba comes from the lower catchment located between Victoria Falls and Kariba. About 60% of the average annual flow from the lower catchment is concentrated between January and March (Santa Clara, 1988).

Since completed, the hydro-power plant of Kariba has been the main source of energy for the Zambian-Zimbabwean interconnected electricity supply system. The principal features of the reservoir system are summarized in Table 4.4.

The average monthly evaporations are shown in Table 4.5. These values are the depths of water evaporated from the reservoir (mm) derived by water balance calculations (Budhakooncharoen, 1986). The elevation-storage-area relationship of the Kariba reservoir is given in Table 4.6.

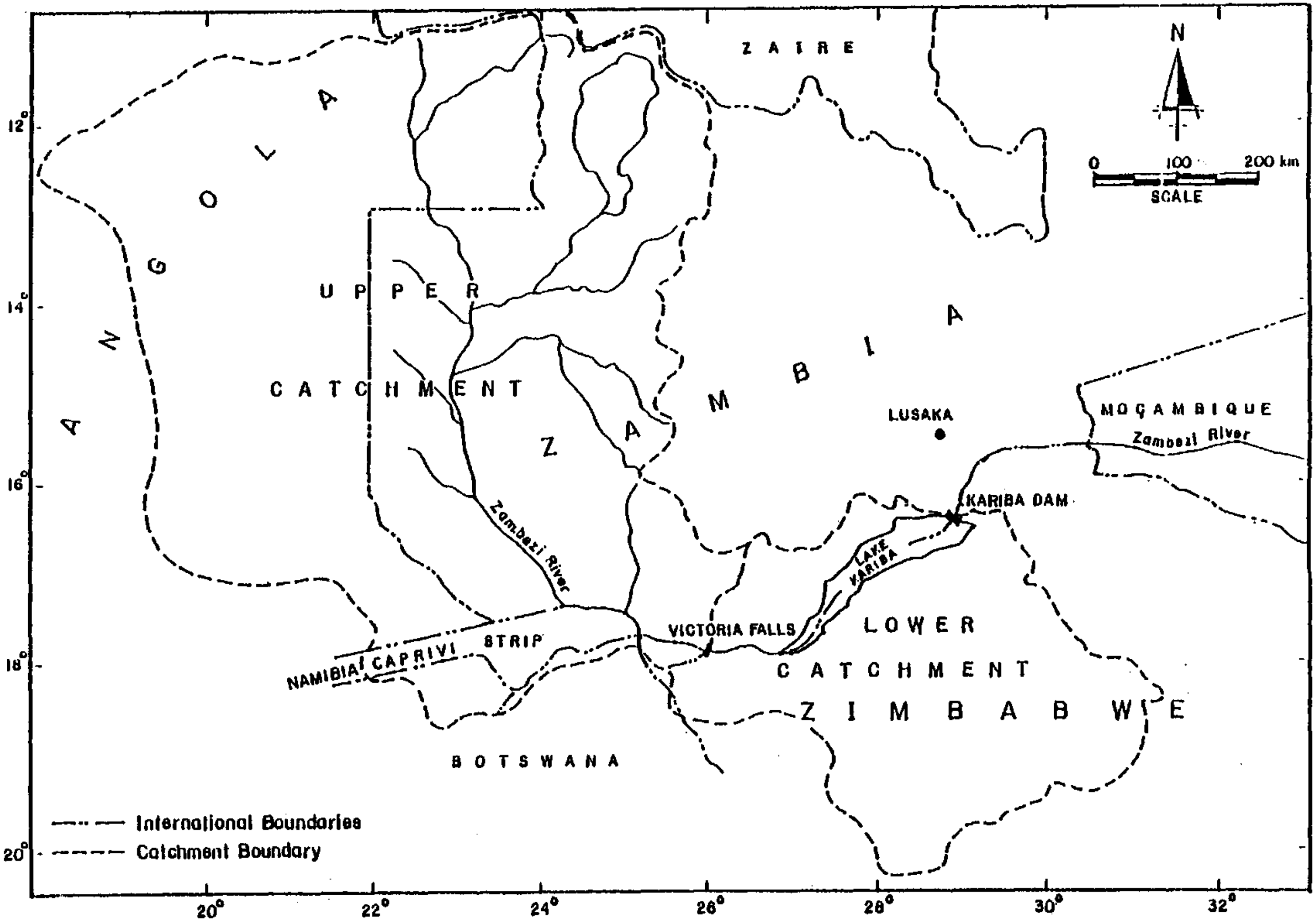


Figure 4.3 Zambezi River Basin

Table 4.4 Principal Features of the Kariba Reservoir

Characteristics		Kariba reservoir
Catchment characteristics		
Catchment area	km^2	664,000
Average annual inflow	MCM	54,689
Completion of Construction		1977
Reservoir characteristics		
Retention water level	m	488.5
Design flood level	m	489.6
Minimum water level	m	475.5
Active storage	MCM	64,750
Flood gate discharge	m^3/s	9,400
Reservoir volume factor	%	118.4
Power House characteristics		
Average net head	m	86
Turbine discharge	m^3/s	277.6
Turbine		12 Francis
Installed capacity	MW	1200

Table 4.5 Monthly Evaporation Values from the Kariba Reservoir

Reservoir	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep
Kariba (mm)	19.7	17.6	14.9	14.6	12.8	14.4	13.5	12.3	10.9	11.2	12.9	16.2

Table 4.6 Elevation-Storage-Area Relationship of the Kariba Reservoir

Elevation (m)	Storage (MCM)	Area (km^2)
475.5	50	4354
476.0	2270	4405
477.0	6710	4507
478.0	11280	4608
479.0	15910	4709
480.0	20610	4811
481.0	25480	4901
482.0	30410	4991
483.0	35430	5081
484.0	40570	5171
485.0	45780	5261
486.0	51090	5350
487.0	56510	5440
488.0	62000	5531
489.0	67600	5623
490.0	72086	5719
490.8	76304	5792

The Zambezi River hydrological data at the Kariba Reservoir, provided by the Central African Power Corporation (CAPCO, 1985), are available from 1961 to 1984. Historical monthly discharge into the Kariba Reservoir including rainfall directly on the lake surface are tabulated in Table A.6 in Appendix A.

For the assessment of the quality of derived reservoir operation policies, the inflow forecasting is sometimes required during operation simulation. Budhakooncharoen (1986) used regression analysis to forecast inflow to the Kariba reservoir. In that study, multiple regression has been used to evaluate the forecasted inflow (dependent variables) from the known inflows in previous stages (independent variables) by assuming that these variables are linearly correlated. That study investigated the appropriate number of independent variables in previous months, and selected three months lag time after trials with various number of independent variables. The results of that analysis are summarized in Table 4.7 and that forecast is used in this study.

Table 4.7 Multiple Regression Analysis of the Kariba Reservoir Inflow

Multiple Regression Equation : Y(j,X1,X2,X3) = A1(t)*X1 + A2(t)*X2 + A3(t)*X3 + A4(t)

	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep
A1	0.8773	0.3269	-0.8018	0.7772	0.8494	0.3063	1.0595	0.5854	0.6720	0.3446	0.4389	0.2915
A2	0.2070	0.4326	3.5237	-0.5743	0.2684	0.4652	-0.2041	-0.0272	-0.1394	0.1413	-0.1464	0.0385
A3	-0.2469	-0.1821	2.0885	-3.2355	0.6903	0.7587	-0.3044	0.0612	0.0088	0.0068	0.0727	0.0057
A4	310.48	780.55	-2288.14	6967.94	899.77	1734.72	2845.92	2081.14	604.85	-166.32	602.65	752.66
R^2	0.382	0.192	0.227	0.353	0.441	0.431	0.843	0.841	0.896	0.970	0.924	0.637

where, R^2 is the determination coefficient

4.3 Joumine Reservoir System

The major part of the Tunisia is in arid or semi-arid area. Different studies have identified a total of 39 major reservoirs and transfer constructions (partly terminated and partly planned), which play an important role in the national system of water management. Figure 4.4 shows the configuration of the complex system. Most of the reservoirs in the system are independent and the utilization of water depends only on their active storage volume, demands and their inflows. Among them, the Joumine reservoir is the one that plays a crucial role in supplying drinking water to Tunis.

The principal features of the Joumine reservoir are summarized in Table 4.8. Table 4.9 presents the monthly evaporation values from the reservoir. The elevation-storage-area relationship for the Joumine reservoir is given in Table 4.10.

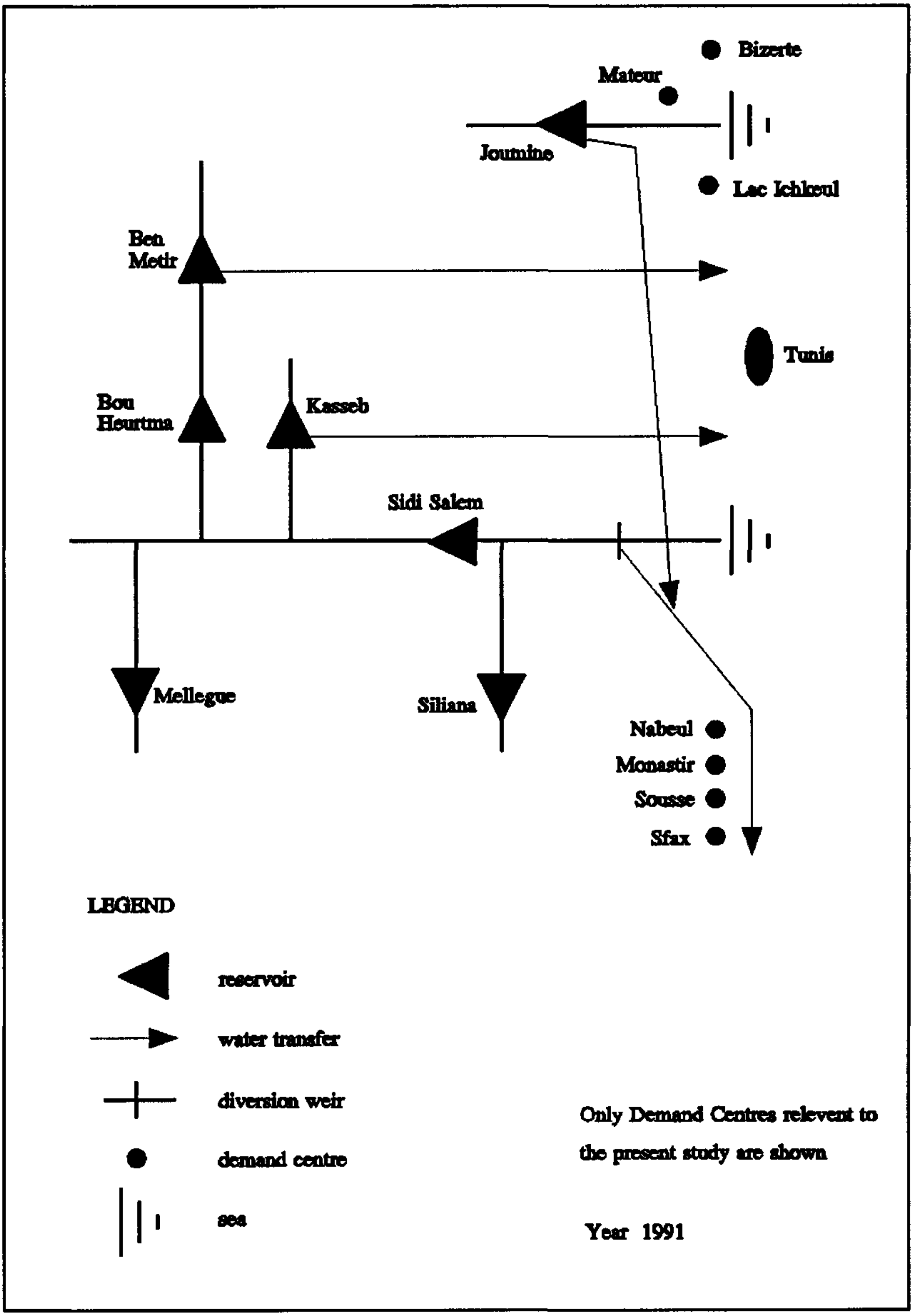


Figure 4.4 Tunisia Reservoir System Scheme

Table 4.8 Principal Features of the Joumine Reservoir

Characteristics		Joumine reservoir
Catchment characteristics		
Catchment area	km^2	418
Average annual inflow	MCM	133
Completion of Construction		1983
Reservoir characteristics		
Retention water level	m	90.0
Design flood level	m	95.0
Minimum water level	m	58.0
Active storage	MCM	121.3
Flood gate discharge	m^3/s	59.0
Reservoir volume factor	%	91.2

Table 4.9 Monthly Evaporation Values from the Joumine Reservoir

Reservoir	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep
Joumine (mm)	101	52	41	38	34	55	73	162	245	349	314	203

Table 4.10 Elevation-Storage-Area Relationship of the Joumine Reservoir

Elevation (m)	Storage (MCM)	Area (km^2)
0.0	0.00	0.0
4.0	0.14	13.3
6.0	0.66	38.6
8.0	1.55	50.5
18.0	9.30	144.0
20.0	14.00	174.0
22.0	17.90	214.0
24.0	22.50	254.0
32.0	47.60	375.0
38.0	73.20	476.0
42.0	93.70	547.0
48.0	129.90	651.0
54.0	172.60	763.0

The inflow time series at the Joumine reservoir provided by Agrar-und Hydrotechnik GMBH (1991) are available from 1946 to 1989. These inflows are tabulated in Table A.7 in Appendix A.

The Joumine reservoir alone supplies the following water demands:
(a) drinking water to Bizerte,
(b) irrigation water to Mateur region, and
(c) water demand of Lac Ichkeul (recharge of Lake Ichkeul).
These are called the "Joumine's local demands".

Besides these demands, the Joumine reservoir contributes towards satisfying the following demands together with some other reservoirs.

(a) drinking water to city of Tunis,

(b) drinking water demands of remote urban areas and coastal tourist centres (Nabeul, Monastir, Sousse, Sfax); water for these users is transferred via the Medjerdah-Cap Bon canal.

These demands are called "Joumine's system demands". These demands are given in Table 4.11.

Table 4.11 Water Demands of the Joumine Reservoir

System Demand (MCM)	Local Demand (MCM)
8.003	0.539
8.081	1.030
8.554	2.358
9.923	4.089
11.026	4.972
10.027	4.482
10.415	3.086
8.901	1.939
9.027	1.536
9.409	1.445
10.014	1.063
10.199	1.014

5 Markov Inflow Transition Probabilities

Application of the SDP technique for the optimization of reservoir system operations is based on the idea that the policies will converge to a "steady-state" policy after several iterations of the recursive relation. The steady-state policy achieved in this way will be a global optimum. In several applications of the SDP technique in reservoir operation optimizations, one convergence criteria given in Chapter 3.1.12, that is the stabilization of the expected annual increment of objective function value, could not be realized. In those studies only the stabilization of the operating policy after a few iteration cycles was reported to be achieved. Besides the effect of possible deterministic constraints (Nandalal, 1986), the cause is believed to be related to the structure of the Markov transition probability matrices, which have been derived from inflow records. The large number of zero elements resulting in the transition probability matrices due to the limited length of those inflow time series is considered the cause.

This chapter presents a study carried out to find an effective means to circumvent the problem caused by the poorly structured Markov inflow transition probability matrices in SDP models. It starts with the introduction of some basic terminologies used in the Markov process (and Markov chains). The relationship between the convergence of the SDP model (converging to the steady state) and the ergodicity of the Markov chains will be discussed next. This is followed by a sensitivity analysis of the SDP based operational performance of reservoir systems with respect to inaccuracies in the estimation of Markov inflow transition probability matrices. Then an example method to guarantee the ergodicity by modifying the transition probability matrices will be presented. The conclusions drawn from this chapter are presented in Chapter 9.

5.1 Role of Markov Inflow Transition Probabilities in the SDP Model

5.1.1 Problems in SDP Convergence Behaviour

In stochastic dynamic programming model, the inflow process Q_t is usually assumed as a "Markov Process" (or "Markov chain"). In general, a Markov process describes only one-step dependence, called a first-order process, or exhibiting lag-one serial correlation (Markov assumption). A SDP model is the application of the "Principle of Optimality" of dynamic programming (Bellmann, 1957) to the Markovian sequential decision process.

As an example, consider the version of the SDP model described by Loucks *et al.* (1981). The time period is defined as stage. The storage volume at the beginning of the time period and the inflow level at the present time period are the state variables. The storage volume at the end of the time period is defined as the decision variable. The Markov transition probabilities of the inflow (from present time period to the subsequent time period) can be incorporated into the recursive relation to derive optimal values.

$$f_t^n(S_t,Q_t) = \underset{S_{t+1}}{opt}[B_t(S_t,Q_t,S_{t+1}) + \sum_{Q_{t+1}} P_{t+1}(Q_{t+1}/Q_t) * f_{t+1}^{n-1}(S_{t+1},Q_{t+1})] \quad \forall S_t,Q_t,S_{t+1} feasible \quad (5.1)$$

subject to,

$$R_t = S_t + Q_t - S_{t+1} - SP_t - E_t$$

$$f_T^1(S_t,Q_t) = \underset{S_{t+1}}{opt}[B_t(S_t,Q_t,S_{t+1})]$$

Where, t, n, S_t, Q_t, R_t, SP_t and E_t are as defined as in Equation 3.12. $P_{t+1}(Q_{t+1}/Q_t)$ is Markov transition probability of inflow falls into level Q_{t+1} in period $t+1$, given the inflow Q_t in period t; $B(S_t,Q_t,S_{t+1})$ is objective increment for the transition state when the decision reaching S_{t+1} at the end of period t starting from S_t and having Q_t inflow during the period; $f_t^n(S_t,Q_t)$ is (sub) optimal value of the recursive equation at stage n (period t) as function of S_t and Q_t; the relation between time notations and variables are shown in Figure 3.3.

The optimization process starts with a set of initial values $f_T^1(S_t,Q_t)$. Due to the characteristics of Markov sequential decision process, after a large number of iterations of the recursive relation (Equation 5.1) the "steady state" for each period in successive years will finally be reached. It is independent of the initial state.

There are two criteria marking convergence of the steady state: (i) stabilization of the policy; (ii) stabilization of the expected annual increment of the objective values. The interpretation of the two criteria is presented in Chapter 3.1.12. However, experimental evidence shows that often the second criterion of convergence cannot be achieved. This is illustrated in Appendix C.

5.1.2 Reasons for the Violation of SDP Convergence Criteria

As has been reviewed in Chapter 2, the SDP model was originally developed based on the model of Little (1955) by introducing the dynamic programming recursive relation. It is similar in form to the method that has been developed to derive the solution of Markov sequential decision process (Howard, 1960). A step-by-step discussion regarding the links among Markov process, Markov process with reward (which is the gain associated with each transition), and the solution of Markov sequential decision process was given by Howard, (1960). The behaviour of the policy convergence after many iterations is the resulting performance of the Markov transition probability matrix incorporated in the recursive relation that converges to its steady state probabilities. Under the condition of the bounded reward, Howard has proved that it is generally true the policies would converge to the global optimum, if the associated Markov transition probability matrices were "stationary" and "ergodic".

Stationarity means that the probability distribution of the inflow process is not changing over time cycle (Loucks *et al.*, 1981). This is the condition to ensure that the policies will become stable after a certain number of iterations (the first convergence criterion).

This condition may not always hold for reservoir systems. Urban development, deforestation, agricultural development, climatic shifts, and changes in regional resource management can alter the distribution of inflow over time. If the stochastic inflow process is not essentially stationary over the time span, adjustments have to be made to account for the changes in the inflow data. Otherwise the SDP model that relies on the stationary assumption cannot be applied. Since this condition has always been much better taken care by the users of SDP model, it will not be discussed at this stage.

The precise definition of ergodicity is presented in Appendix B. In short, a Markov chain is said to be ergodic if all the members in the chain form a single recurrent chain. Immaterial of the starting point the process would end making jumps among all the members in the chain. In other words, ergodic implies that the final state of the system is independent of the initial state (Howard, 1960). This condition ensures that the stable policies would be the global optimum.

To illustrate it, consider a simple three-state process with two recurrent chains (non-ergodic Markov chain) as shown below.

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1/3 & 1/3 & 1/3 \end{bmatrix}$$

State 1 constitutes one recurrent chain. State 2 the other (both are trapping states). State 3 is a transient state that may lead the system to either of the recurrent chains.

The n-step transition probability matrix when $n \to \infty$ is,

$$\lim_{n \to \infty} P^n = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1/2 & 1/2 & 0 \end{bmatrix}$$

It says that if the system is started in State 1 or State 2 it will eventually remain in its starting state indefinitely. If it is started in State 3, after many steps it will be State 1 with probability ½ and in State 2 with probability ½.

The above simple example shows that for a non-ergodic Markov process the rows of the n-step transition probability matrix P^n when $n \to \infty$ are no longer identical. Because the limiting state probability distribution is now dependent on how the system is started. The i^{th} row of matrix P^n, when $n \to \infty$ represents the limiting state probability distribution that would exist if the system were started in the i^{th} state. It is easy to see that having non-ergodic

Markov inflow transition probability matrices, the set of derived SDP based policy would be separated into more than one group, which have no communication among each other. The reservoir operation following such policy will confine itself to the optimal strategies in one of the groups where it initially starts.

Fortunately, from theoretical point of view the ergodicity is a condition that is naturally hold for reservoir inflow process in reality. Many theoretical probability distributions have been used to describe reservoir inflow processes. Even though they only approximate the hydrological processes, it has been proved to be useful, at least for understanding the phenomenon. The most commonly used probability distributions for the description of the reservoir inflow processes are Normal, Log-normal and Pearson-III.

Assume, for example, both reservoir inflows of subsequent periods (Q_{t-1} and Q_t) follow the Log-normal probability density function. Each row of the transition probability matrices of the two periods is also Log-normal distribution. The discrete transition probability equals to the area of the stripe that has the class boundaries as sides and the shape of the Log-normal probability distribution as the top (see Figure 5.1). By estimating mean, standard deviation and other useful statistical parameters, the shape of the probability distribution can be determined. Therefore, the discrete transition probability can be calculated by computing the area of the stripe.

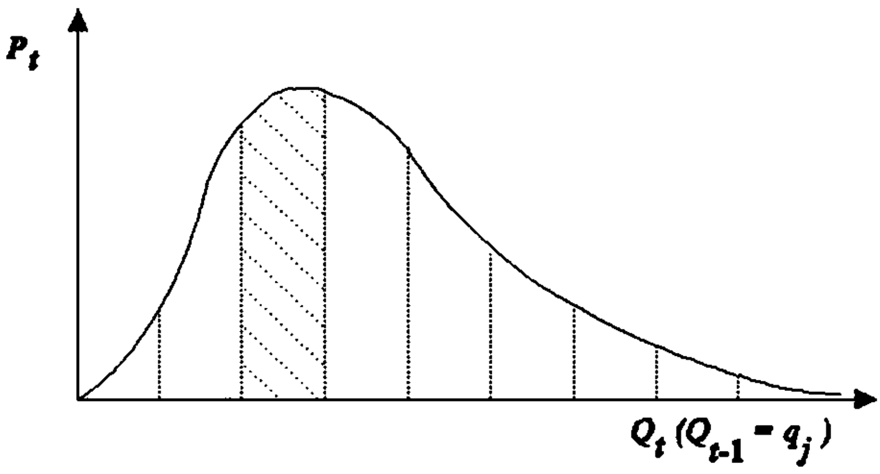


Figure 5.1 Deriving Inflow Transition Probabilities by Distribution Fitting

It is obvious that the inflow transition probability matrix derived through the above procedure will be ergodic since a matrix without any vanished element will surely form a single recurrent chain. However, this rigorous methodology is hampered by its relatively large computational requirement.

In practice, the transition probabilities are usually estimated from observed inflow records. This is done by counting the number of times the observed data transit from state Q_{t-1} in period t-1 to Q_t in period t (see section 3.1.7). This simple method is suitable when the number of inflow classes is small. However, it has the drawback of limiting the accuracy of the SDP model.

When number of inflow classes is larger than 3 or 4, the difficulty arises. For example, if the inflow of subsequent periods are discretized into 10 classes, the number of elements to be estimated in a matrix during a period is 10*10 = 100. In practice, the historical reservoir inflow time series are seldom longer than 50 years. For developing countries, a 30 year record is considered as a long record. A large number of elements will remain void if 30 years of monthly inflow data are used to estimate 12 matrices (each with 100 elements) in a year. This would cause the real danger of loosing the ergodicity of the matrices. Note that those zeros are "artificial" in the sense that those are the results from the error estimations of statistics due to the sample size.

In SDP model, the problem is more aggravated as there are T (e.g., 12 with a monthly time period) Markov inflow transition probability matrices in one year cycle. The optimal policies are produced after many cycles (years) of iterative calculation. Therefore, it is not easy to judge whether the ergodicity requirement is satisfied by looking at the combination of T matrices. For example, some vectors that do not communicate in the matrix of transitions from October to November may communicate in the matrix of November to December.

One definite sign of the combination matrices having ergodicity is that the second criterion for steady state policy, that is a constant value of annual increment of objective function can be achieved.

Up to now, the importance of the ergodicity property of the Markov chains has been disregarded in their application in the SDP model. The problem stated at the beginning of this chapter, that is the difficulty in achieving the second convergence criterion in many SDP applications can be explained from the above discussion. For those cases, the stable policies can be reached after a few iteration cycles while the annual increment of the objective function value converges to more than one constant (instead of one). The failure to converge to a single constant in the annual increment of objective value is due to the violation of ergodicity of the Markov inflow processes. At this time although the first convergence criterion (i.e., stable policies) is obtained, the set of stable policies is separated to more than one group, which have no communication among each other. Each group obtains its optimal with respect to the initial state where the reservoir operation starts.

The cause of violating the ergodicity can be traced back to the large number of zero-elements in the estimated transition probability matrices. Therefore, an important point obtained from this analysis is that in applying SDP model, the number of zero-elements in the reservoir inflow transition probability matrices should be kept within a limit to guarantee that the derived policies will be a global optimum.

Therefore, an approach to satisfy the ergodicity requirement is needed while keeping the computing effort requirements at a reasonable level. The method to derive inflow transition probabilities by distribution fitting involves a considerably large amount of computing effort. This suggests the necessity to find an alternative method to smooth out the zero-elements in the matrices derived from the simple tabulating method. Therefore, the elimination of zero-elements while maintaining the performance of the derived optimal operation policies is of interest.

Initially the sensitivity of the SDP based operational performance of reservoir systems with respect to inaccuracies in the estimation of Markov inflow transition probability matrices was investigated.

5.2 Sensitivity Analysis of Markov Inflow Transition Probabilities

5.2.1 Computer-Experiments

The Kariba lake and the Mahaweli reservoir system were selected to study the impact of the transition probability matrices have on the reservoir operational performance. In this study several hypothetical transition probability matrices reflecting different flow regimes were used.

The sensitivity is carried out for both systems in the following steps. (a) Set up the SDP models. (b) Create several sets of extremely different inflow transition probability matrices and incorporate them into the SDP model to derive several sets of operation policies. (c) Simulate with the historical inflow time series according to the derived sets of policies and compare the resulting performance.

The setups of the SDP models used to derive operation policies for the two reservoir systems are described in Table 5.1.

Table 5.1 SDP Model Setups for the Mahaweli and Kariba Reservoir Systems

	Mahaweli System Victoria and Randenigala reservoirs	Kariba reservoir
Objective	Maximize expected annual energy generation	
Constraint	Irrigation demand	---
Inflow Discretizations	equal size intervals 4*4=16 combinations	equal occupancy varying number of classes (from 2 to 8)
Storage Discretizations	7*7=49 combinations	33 classes
Time step length	one month	one month

The version of SDP model with the recursive relation described in Equation 5.1 has been adopted. For both cases the time period is month; the objective is to maximize the expected annual energy generation. For the Mahaweli system the constraint of satisfying irrigation requirement was added. For the Mahaweli system 4 inflow classes and 7 storage classes with equal size intervals have been considered for each reservoir in cascade. This yields 4*4=16 inflow class combinations and 7*7=49 storage class combinations. For the Kariba system varying number (2 classes for August, September, October and November; 3 classes for December and July; 4 classes for January and June; 6 classes for February and May; 8 classes for March and April) of inflow classes with equal occupancy frequencies and 33 storage classes with equal size have been used.

The observed Original Transition Probabilities (ORG) and some modified forms, namely, Modified Transition Probabilities (MDF), Average Transition Probabilities (AVG) and Modified Average Transition Probabilities (AVM) have been adopted in the SDP models to derive respective optimal operational policies.

Original Transition Probabilities are derived from the historical inflow series. Modified Transition Probabilities are obtained from the ORG version by overemphasizing the maximum probability (or -is) occurring in every line. Average Transition Probabilities are assumed to characterize a hypothetical uniform frequency distribution of inflow class transitions. Modified Average Transition Probabilities combine the principles of MDF and AVG. Zero elements are kept zero at the beginning and at the end of each row. Internal sequence of 3 or more zero elements remain as such, while uniform frequency distribution is assumed row-wise over the non-zero and imbedded single or double zero elements of the ORG matrices. Table 5.2 shows a few example lines of these inflow transition probabilities calculated for the Kariba reservoir.

Table 5.2 Example of Modifications of the Markov Inflow Transition Probabilities of the Kariba Reservoir

ORG	.500	.250	.250	.000	.000	.000	.000	.000
	.000	.000	.000	.333	.000	.333	.000	.333
	.000	.000	.000	.000	.000	.000	.330	.670
	....	....						
MDF	1.000	.000	.000	.000	.000	.000	.000	.000
	.000	.000	.000	.333	.000	.333	.000	.333
	.000	.000	.000	.000	.000	.000	.000	1.000
	....	....						
AVG	.125	.125	.125	.125	.125	.125	.125	.125
	.125	.125	.125	.125	.125	.125	.125	.125
	.125	.125	.125	.125	.125	.125	.125	.125
	....	....						
AVM	.333	.333	.333	.000	.000	.000	.000	.000
	.000	.000	.000	.200	.200	.200	.200	.200
	.000	.000	.000	.000	.000	.000	.500	.500
	....	....						

In the subsequent simulation, the historical inflow time series have been used "strictly" relying on the SDP based operational policies obtained according to the different sets of transitional probabilities. The optimum operation policies are determined using the expected system performance based on discrete storage and inflow states. Therefore, it is possible that in some periods the actual releases resulted from continuity equation would be out of their feasible range (e.g., release less than 0 or larger than the downstream channel capacity). In such instances, corrections (i.e., over-ruling the SDP optimum operation policy) are required in the simulation model. The releases are made equal to the nearest feasible value and the consequent final storage is defined by the continuity equation. The final storage (or the closest discrete value) obtained is used as the initial storage for next time step.

5.2.2 Results and Discussion

The operation performances of Kariba and Mahaweli systems are shown in Table 5.3 and Table 5.4 respectively. The expected annual energy outputs shown are obtained from the SDP optimization. They are the expected gains for the different sets of inflow transition probabilities considered in this study and do not represent the real gain of the reservoir operation.

The simulated mean annual energy outputs are obtained from the real time operational model for the given time period. Those results are based on the operation policies derived from different sets of inflow transition probabilities. The performance based on "ORG policy" is observed to be better than the others. That operation has the largest average energy output, least standard deviation and largest minimum energy output. The differences among mean annual energy outputs appear to be very limited (less than 2%). The standard deviations and the minimum energy output seem to be more sensitive for assumed changes in the inflow regime. However, the variation among the simulated performances of the reservoir system for different sets of assumed transition probability matrices is observed to be very much limited.

Table 5.3 Operational Performance of the Kariba Reservoir

	ORG	MDF	AVG	AVM
Indices referring to energy output:				
(1) Expected annual energy (GWh)	8679	9164	9291	8967
(2) Simulated mean annual energy (GWh)	8502	8355	8494	8478
(3) Standard deviation of (2) (GWh)	847	1226	914	996
(4) Minimum annual energy (GWh)	6157	4606	5383	5358
Indices referring to reservoir:				
(5) Mean utilized storage volume as % of available reservoir capacity	58.7%	57.5%	44.4%	50.6%
(6) Standard deviation of (5)	26.6%	26.9%	24.2%	26.0%
(7) Minimum storage drawdown as % of available reservoir capacity	0.0%	0.0%	0.0%	0.0%

The mean utilized reservoir storage volumes, their standard deviations and the minimum storage drawdowns are also presented in Table 5.3. The mean utilized reservoir storage volume related to "AVG policy" is much smaller than that value from "ORG policy". The "AVG policy" is derived based on the assumption that the reservoir inflow transition probabilities are uniform. This implies that anyhow there is a moderate (not low and not high) incoming inflow. If this assumption is valid, the reservoir storage capacity needed to regulate the over year inflow is low. Therefore, decisions (the storage volumes at the end of each time period) made from the SDP model lead to the smaller mean utilized reservoir storage volume.

Table 5.4 shows the performance of the Victoria and Randenigala reservoirs in the Mahaweli system. In the SDP model, the objective of maximizing energy generation is subjected to the constraint of irrigation requirement. When both the inflow to the reservoir and the initial reservoir storage are very small it may not possible to make a decision (a reservoir storage at the end of the period) that falls into the feasible region. This occurs as the irrigation demand is introduced to the system as a constraint that has to be satisfied always. Zeros represent these cases in the policy tables. Since the optimization does not hold for the whole set of decisions in the annual cycle, the expected annual energy output is not obtained.

Table 5.4 presents the simulated mean annual energy outputs corresponding to the operation policies derived for different sets of inflow transition probabilities. The mean utilized reservoir storage volumes, their standard deviations and the minimum drawdown of the reservoir storages are also shown in this table. Except the indices referring to reservoir storage volume, most of the other performance indices referring to the objective function (energy generation) display little variation among different sets of policies derived from different inflow series. These results are similar to those for Kariba reservoir.

The Mahaweli system serves multipurposes: energy generation and irrigation supply. The irrigation requirement is set as a constraint in the SDP model. In Table 5.4 the performance indices referring to the irrigation supply are also presented. They display little variation among different policies derived from different sets of inflow series.

Table 5.4 Operational Performance of the Mahaweli System

	ORG	MDF	AVG	AVM
Indices referring to energy output:				
(1) Mean annual energy (GWh)	1390	1364	1342	1384
(2) Standard deviation of (1)	308	302	311	321
(3) Minimum annual energy (GWh)	841	838	774	730
Indices referring to reservoirs:				
(4) Mean utilized storage volumes as % of available reservoir capacity				
Victoria	80.1%	89.1%	42.4%	65.6%
Randenigala	91.2%	89.1%	86.5%	91.5%
(5) Standard deviation of (4)				
Victoria	6.6%	7.8%	11.3%	8.0%
Randenigala	10.2%	10.0%	11.8%	9.9%
(6) Minimum storage drawdown as % of available reservoir capacity				
Victoria	20.6%	36.5%	4.7%	20.6%
Randenigala	44.8%	44.8%	33.7%	44.8%
Indices referring to irrigation shortage:				
(7) Time-based reliability[1]	86.2%	85.9%	84.6%	86.2%
(8) Quantity-based reliability[2]	95.9%	95.9%	95.5%	95.7%
(9) Repairability[3] (month)	1.57	1.64	1.64	1.47
(10)Vulnerability[4] (MCM)	60.5	61.6	62.0	59.6

[1] - % of time steps with fulfilled irrigation demand
[2] - % of the accumulated irrigation demand met
[3] - Average duration of an irrigation failure (shortage) event
[4] - Average accumulated irrigation shortages per failure

These results show that for the given inflow and storage discretizations, a limited impact of the different transition probabilities can be detected as long as the objective factors are concerned. This fact implies that the inherent inaccuracy in estimating the transition probabilities is unlikely to have a considerable impact on the SDP based operational performance of reservoir systems. This "insensitivity" phenomenon may be interpreted from the following two aspects.

(i) The transition probability matrices with large variations may derive similar steady state policies. As mentioned before, the SDP optimization process is an iteration of the Bellman recursive relation with the incorporated transition probability matrices (Equation 5.1). Each row of the transition probability matrix is associated with an expected objective value of a feasible region. The optimal decision for each state is selected among the whole set of feasible decisions for that state by comparing the expected value of those decisions. It is clear from the behaviour of Markov chains that the influence of the initial transition probabilities is decreasing along the iterations. After many cycles the decisions are mainly weighted by the steady probabilities of the transitions.

However, the foregoing discussion does not imply that all the transition probability matrices with the same steady probabilities will always derive the same steady state policy. Two transition probability matrices with large variation at the starting of the optimization process may be deformed to have less variation along the way of optimization iterations. Thus they may derive similar optimal policies at the end.

(ii) The derived operation policies, which vary to a certain extent may satisfy the purpose of the reservoir system to similar standards.

Water reservoirs are expensive long-live investment projects. Once they are built, they are often operated for decades. Therefore, when designing reservoirs the uncertainty on the future supplies, flows, qualities, costs, benefits and so on has to be considered. While the forecast for the future conditions is never perfect, well designed reservoirs are needed to be sufficiently flexible to permit their adaptation to a wide range of possible future conditions. Nowadays, many professionally designed reservoirs have to a certain extent built in robustness for dealing with the future uncertainties. Therefore, the policies, which vary to a certain extent from the optimal policies may not cause much worse performance of reservoir systems.

In the two case studies presented, the operational policies derived from transition probability matrices "AVG" vary from the policies derived from the matrices "ORG". This can be detected from the indices referring to the reservoir storage level (policy is a set decisions defining the reservoir storage at the end of each time period). However, the difference among the performance indices referring to the key concerns of the reservoir systems (energy output and irrigation supply) are limited.

5.3 A Methodology to Eliminate Zeros in Transition Probability Matrices

As shown in Chapter 5.1, the large number of zero elements in the transition probability matrices resulted due to the limited length of inflow record are the cause of the violation of the second convergence criterion of SDP model. Therefore, when using SDP model, it is safer to make sure that most of the elements in each row in the transition probability matrices are non-zeros.

It can be easily seen that a transition probability matrix is ergodic (irreducible) if more than half the elements in each row are non-zero. This can be proved by the reduction to absurdity. Assume a n*n non-ergodic matrix having more than n/2 non-zero elements in each row. This matrix can be divided at least into two groups of rows (each non-ergodic matrix is reducible and can be reduced to at least two non-communicating groups of rows). These two groups of rows do not have non-zero elements at the same column. Therefore, these groups of rows form a matrix with more than n columns. This is in contradiction with the given fact that the matrix is a n*n matrix.

The results of the sensitivity analysis in Chapter 5.2 reveal that the inherent inaccuracy in estimating the transition probability matrices is unlikely to have a considerable impact on the SDP based operational performance of reservoir systems. Therefore, it is proposed that some zeros in the transition probability matrices may be easily smoothed out by a reasonably small value. The following example applied to Kariba reservoir illustrates this.

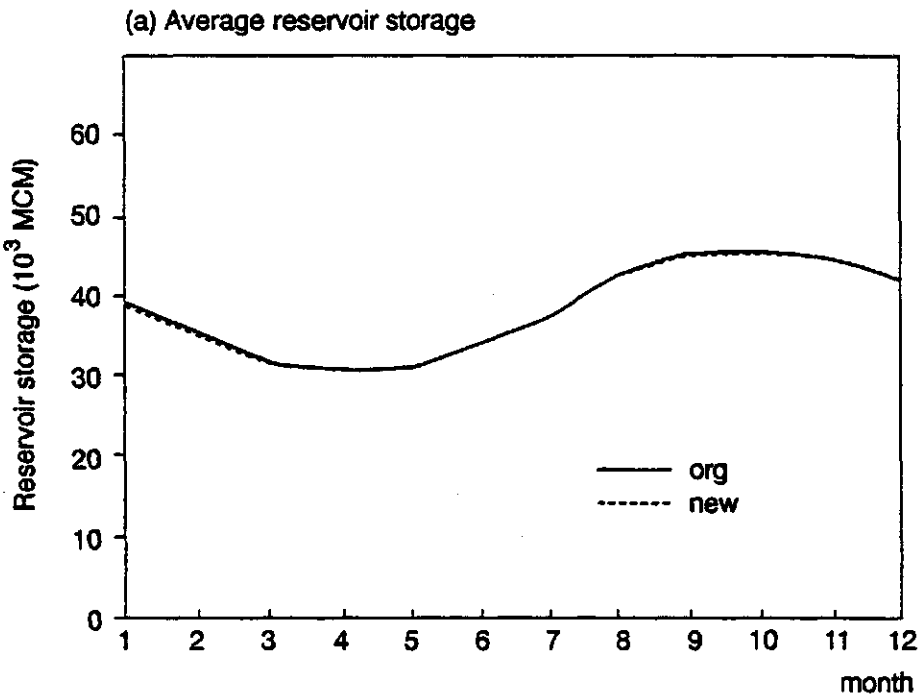
The method targets to make, most of the elements in the transition probability matrices, non-zeros. When a row is empty, a uniform frequency distribution is assumed row-wise. When more than half the elements of a row are non-zeroes, the row is kept unchanged. Otherwise, each zero in the row is replaced by 0.01 while the non-zeros in the row are accordingly reduced slightly to maintain the sum of the row to 1.0. Table 5.5 shows an example of how a given transition probability matrix is transformed into a new transition probability matrix by smoothing out the zeros according to the method.

Table 5.5 Example of the Smoothing Method

ORG	.500	.250	.250	.000	.000	.000	.000	.000
	.000	.000	.000	.333	.000	.333	.000	.333
	.000	.000	.000	.000	.000	.000	.330	.670
	....	....						
NEW	.480	.240	.230	.010	.010	.010	.010	.010
	.010	.010	.010	.313	.010	.323	.010	.313
	.010	.010	.010	.010	.010	.010	.310	.630
	....	....						

The simulated reservoir operation performance based on the policy derived by incorporating the new transition probability matrices (after partially smoothing out zeros) in the SDP model are compared with that from the original transition probability matrices. The results are shown in Table 5.6 and Figure 5.2.

Table 5.6 shows the average annual performance indices from the simulated reservoir operation concerning both reservoir storage and energy output. Figure 5.2a and Figure 5.2b show the average monthly reservoir storage and energy output respectively. The similarity in the performance indices for reservoir storage can be interpreted as the similarity of the operation policies derived from both "ORG" and "NEW" transition probability matrices. Their performance indices referring to the energy output are also similar.



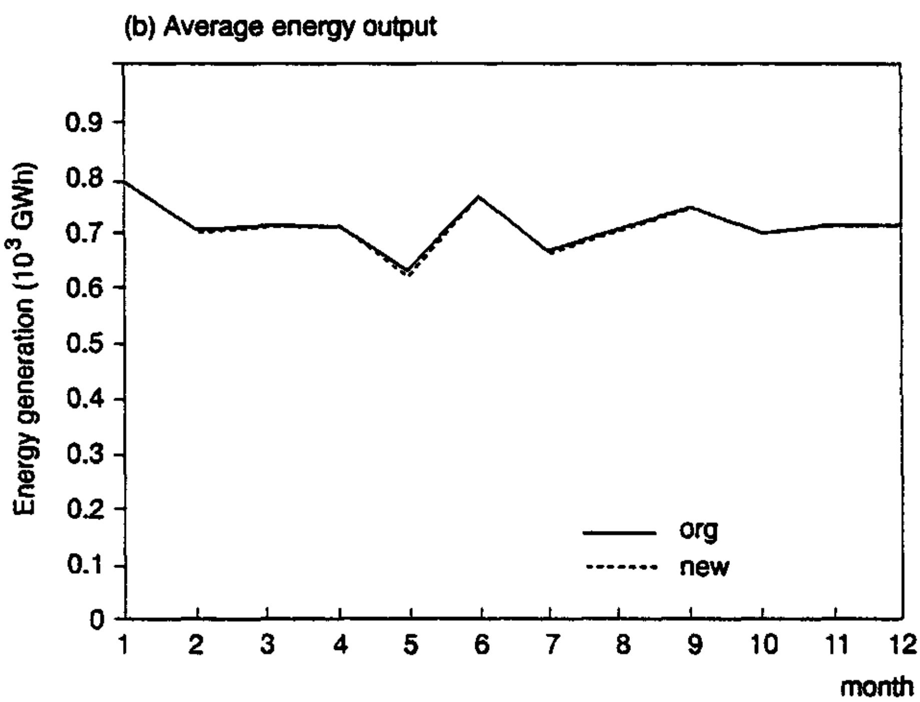


Figure 5.2 Simulated Monthly Performance After Smoothing

Table 5.6 Simulated Performance after Smoothing

	ORG	NEW
Indices referring to energy output:		
(1) Expected annual energy (GWh)	8679	8693
(2) Simulated mean annual energy (GWh)	8502	8493
(3) Standard deviation of (2) (GWh)	847	847
(4) Minimum annual energy (GWh)	6157	6157
Indices referring to reservoir:		
(5) Mean utilized storage volumes as % of available reservoir capacity	58.7%	58.8%
(6) Standard deviation of (5)	26.6%	26.7%
(7) Minimum storage drawdown as % of available reservoir capacity	0.0%	0.0%

The same method was applied for Mahaweli system and identical results were obtained. That is, the simulated reservoir operation performance based on the policy derived by incorporating the new transition probability matrices (after partially smoothing out zeros) in the SDP model is almost the same as that from the original transition probability matrices.

6 State and Decision Variables

For any DP type of model, the careful choice of state and decision variables is crucial to the success of the model. There are two versions of stationary SDP models, which have been widely applied in reservoir operation optimization. One is the model having release as a decision variable, with previous inflow and initial storage as state variables. The other is the model having final storage as decision variable, with present inflow and initial storage as decision variables. These models have been developed and used by different groups of researchers in different problem environments. However, little is known about their relative performance during reservoir operation optimization. The importance of the choice of the decision variable has been neglected; and there exist controversial remarks in literature regarding the choice of different inflow state variables. This chapter aims at obtaining an insight into the roles of different decision variables and state variables in the SDP model.

The chapter starts with a review of the existing models and relevant research. It is followed by a comparative study carried out for a real case, the Kariba reservoir system. To enable making comparisons on the decision base (with the same decision variable when comparing the different choices of inflow state variables) or the inflow state base (with the same inflow state variable when comparing the different choices of decision variables), two more alternatives of the SDP models have been developed. One model has the final storage as a decision variable and previous inflow and initial storage as state variables. The other model has release as decision and present inflow and initial storage as state variables. One phenomenon in the comparative study, which catches attention is that the overwhelming influence of the decision variable (which is directly related to the objective) has on the suitability of the SDP model for the system to be optimized. This was analyzed with case study system, the Joumine reservoir in Tunisia.

6.1 Review of the Existing Models

In reservoir operation, the decision has to be made at the beginning of the time period. But at that instant, the inflow to the reservoir during the time period is not known. However, by considering the probability relations between the inflows in succeeding periods (Markov process), the reservoir operation policy can be set up based on the knowledge of previous inflow and the reservoir storage level at the beginning of the period. The SDP model was originally introduced into reservoir operation in this way (Little 1955; Buras 1966; Butcher 1968, 1971) and has been widely applied in reservoir operation. Butcher (1971) described the model as follows;

$$f_t^n(S_t,Q_{t-1}) = \underset{R_t}{opt}\left[\sum_{Q_t} P_t(Q_t/Q_{t-1}) * [B_t(S_t,Q_t,R_t) + f_{t+1}{}^{n-1}(S_{t+1},Q_t)]\right] \quad \forall S_t,Q_{t-1},R_t \; feasible \tag{6.1}$$

subject to,

$$R_t = S_t + Q_t - S_{t+1} - SP_t - E_t$$

$$f_T^1(S_t,Q_{t-1}) = \underset{R_t}{opt}\left[\sum_{Q_t} P_t(Q_t/Q_{t-1}) * B_t(S_t,Q_t,R_t)\right]$$

Where, t, S_t, Q_t, R_t, SP_t and E_t are as defined as in Equation 3.12. n is total number of time periods passed, $n=1,2,...$; $P_t(Q_t/Q_{t-1})$ is Markov transition probability of inflow falls into level Q_t in period t, Given the inflow Q_{t-1} in period t-1; $B_t(S_t,Q_t,R_t)$ is objective increment for the transition state when the release decision is R_t in period t starting from the initial storage S_t and having Q_t inflow during the period; $f_t^n(S_t,Q_{t-1})$ is (sub) optimal value of the recursive equation at stage n (period t) as function of S_t and Q_{t-1}; the relation between time notations and variables are shown in Figure 3.3.

The general sequential operating policies define the release for each time period as a function of the initial storage volume S_t and the previous inflow Q_{t-1}. These current releases $R_t(S_t,Q_{t-1})$'s form the optimal policy of operation of a reservoir. Table 6.1 shows an example of a possible policy set for a problem with two periods in an operational cycle. It has two inflow and two storage states during each period.

Table 6.1 Operational Policy Identifying the Current Release

Period $t=1$		
S_t \ Q_{t-1}	30	40
20	20	20
30	20	20

Period $t=1$		
S_t \ Q_{t-1}	10	20
10	20	20
20	20	30

At the beginning of the time period, when the decision is to be made, both S_t and Q_{t-1} are known. Thus the forgoing type of policy has the advantage that it can be easily implemented. However, such a policy has been criticized as making no explicit attempt to redefine the release in response to the actual inflow levels observed during the current period t (Stedinger *et al.*, 1984).

At the same time when Butcher was working on the Stochastic Dynamic Programming model, Loucks (1968) developed another interesting approach. It is the Stochastic Linear Programming (SLP) for the optimization of reservoir operation with the Markov inflow process. In his original version of the SLP model, the present inflow is used instead of the previous inflow to determine the present decision. Later Loucks and his associates (Gablinger and Loucks, 1970; Loucks and Falkson, 1970) carried out studies on the relationship between the SDP and SLP models. To make the two models comparable, they introduced a new version of SDP model which uses the present inflow instead of the previous inflow as the hydrologic state variable. This version of the SDP assumes that the present inflow is known at the beginning of the period (or that a forecast is possible with 100% certainty); thus the present return from the recursive relation of the SDP model is deterministic (Stedinger *et al.*, 1984). The model has been described by Loucks *et al.* (1981) as follows:

$$f_t^n(S_t,Q_t) = \underset{S_{t+1}}{opt}\left[B_t(S_t,Q_t,S_{t+1}) + \sum_{Q_{t+1}} P_{t+1}(Q_{t+1}/Q_t) * f_{t+1}^{n-1}(S_{t+1},Q_{t+1})\right] \quad \forall S_T, Q_t, S_{t+1} \ feasible \tag{6.2}$$

subject to,

$$R_t = S_t + Q_t - S_{t+1} - SP_t - E_t$$

$$f_T^1(S_t,Q_t) = \underset{S_{t+1}}{opt}[B_t(S_t,Q_t,S_{t+1})]$$

Where, t, S_t, Q_t, R_t, SP_t and E_t are as defined in Equation 3.12; n, B_t, f_t^n *and* P_{t+1} are as defined in Equation 6.1. The relation between time notations and variables are as shown in Figure 3.3.

The generated sequential operating policies define the final storage volume as a function of the known initial storage volume S_t and the current period's inflow Q_t, which can only be observed during the period. These final storage volumes $S_{t+1}(S_t,Q_t)$'s then form an optimal policy for the operation of a reservoir. Table 6.2 shows an example of a possible policy set. By comparing Table 6.1 and Table 6.2, one can notice the difference between the derived policies from the two versions of the SDP model.

Table 6.2 Operational Policy Identifying the Final Storage

Period t=1		
S_t \ Q_t	10	20
20	10	20
30	20	20

Period t=1		
S_t \ Q_t	30	40
10	20	20
20	20	30

To implement this policy, the inflow during the period should be known at the beginning of each period. Therefore, the above policy cannot be implemented straight away.

One way of implementing this type of operation policy in real time operation is to reformulate the sequential operating policy in a manner that does not depend on unknown future inflows. This could be done by identifying either a final storage volume target subject to the releases, or reservoir release targets subject to limitations to the final storage volumes in each period t. The implementation of such an operating policy does not guarantee smooth reservoir releases throughout a period. However, such a policy can be used by the reservoir operators as a guideline at the beginning of a period without actual knowledge of the period's inflow (Loucks *et al.*, 1981).

Another way of implementing a $S_{t+1}(S_t,Q_t)$ policy, which demonstrates the advantage of this version of SDP is to employ the forecast of Q_t. As the goal of making the storage volume of the reservoir at the end of the period, $S_{t+1}(S_t,Q_t)$ is realized by a continuous release process throughout the period, and Q_t will be known by the end of that period, the operator could redefine the release during the period in response to the best available forecast of Q_t. In spite of the difficulties and additional errors involved in forecasting inflows, the model does open the way for reservoir operators to operate the system based on the most up-to-date knowledge of inflow.

Besides the inflow state variable the choice of the decision variable is important. The storage volume at the end of the period and release within the period could be the most reasonable choices. The suitability of the decision variable varies from case to case. The choice depends on the purposes and the characteristics of the reservoir system. For example, the policy that defines the storage volume would be a better choice for a reservoir system if it aims at flood control and if the level of water in the reservoir can be easily controlled by a sluice gate. On the other hand, the policy that defines the release volume would be a better choice for a reservoir system for energy generation and hence, the key task for the operators is to regulate the quantity of water entering the generator.

The optimal policy derived from the SDP model with previous inflow as state variable is used in real time operation, without inflow forecasting. In this case there is no even a rough estimation between S_{t+1} and R_t. Thus, clearly for this type of models the decision indicator, which is needed during the operation of the reservoir system has to be the decision variable.

The policy derived from a SDP model with present inflow as state variable, is usually implemented with the forecast of the present inflow. Huang *et al.* (1991) argued that since Q_t is assumed to be known, S_{t+1} and R_t are mutually determined by the continuity equation. Thus, the decision S_{t+1} is preferred over R_t in the optimization process because the discretization of the release variable is eliminated. Their first argument can hardly be accepted when the spillage is significant, which is usually the case for reservoir systems for hydro-power generation. In such instances the so-called release is only the amount of water passing through the turbines that is used for energy generation. The balance can be called spillage and it is usually not negligible.

Even if their first argument is tenable, it is still disputable whether the use of the storage instead of the release as the decision variable is a good choice for those cases where release is the direct target. For a yearly regulated reservoir system aiming at hydro-power generation, the feasible range of storage is usually much larger than the release. The feasible range of the storage volume is the reservoir capacity, which is almost comparable with the reservoir annual inflows. The feasible range of release in a month (if the time period is a month) is constrained by the capacities of the turbines, which is about 10 or 20 times smaller than the reservoir storage capacity. Therefore, for the same discretization accuracy, many more discretization points in the decision variable (storage) are required if the storage is used as the decision variable instead of release. This implies that much unnecessary search-work has to be performed during the optimizing process in checking the discretized points that are not feasible. Therefore, the decision variable, which mostly facilitates the operation of the reservoir system is probably always the better choice for both versions of the SDP model (version with the previous inflow state variable or version with the present inflow state variable). A further discussion regarding the selection of decision variables is presented in Chapter 6.3 with case studies.

Huang *et al.* (1991) compared the performance of the stationary SDP model with different choices of inflow state variables. They have structured four types of stationary SDP models to derive optimal operation policies for the Feitsui Reservoir System in Thaiwan. The four models are;

Type 1 - Model described by Equation 6.2
State : Initial storage + Present inflow
Decision : Final storage
Markov inflow processes

Type 2 - A version of Type 1, assuming that the inflow is an independent variable
(Model described by Equation 7.6)
State : Initial storage + Present inflow
Decision : Final storage
Independent inflow processes

Type 3 - Model described by Equation 6.1
State : Initial storage + Previous inflow
Decision : Current release
Markov inflow processes

Type 4 - A version of Type 3, assuming that the inflow is an independent variable
State : Initial storage + Previous inflow
Decision : Current release
Independent inflow processes

The comparison has been made based on the simulation of the reservoir system operation with the derived policies. They concluded that the derived policies, which can be implemented with observed inflows (Type 3 and Type 4) perform better than the policies that have to be implemented with forecasted inflows (Type 1 and Type 2). However, they have immediately added that this conclusion may not hold under different hydrological regimes. The studied case has the prominent feature that there will be inevitable substantial errors in the forecast inflows as it is situated in a typhoon area. This feature may have largely weaken the models whose derived policies have to be implemented with forecast inflows.

There is another factor that makes the generality of their conclusion questionable. It is clear from the structure of their models that the final storage has been adopted as the decision variable while the present inflow is the state variable. Current release has only been adopted as the decision variable for the models with the previous inflow as the state variable. As previously shown in this section, the selection of either the final storage or the current release as the decision variable may largely alter the optimization searching process. The computer-experiment in the next section will show that the robustness of the policies derived from these two different decision variables also can differ considerably. Therefore, from their comparison hardly any conclusion on the performance of different inflow state variables can be drawn.

6.2 Comparison of the Models with Different Decision and Inflow State Variables

This section compares the performance of the different SDP models. These models differ in the choice of inflow state variables and decision variables. For this purpose two more alternatives of the SDP model (Model 2 and Model 3) have been formulated based on the models described by Equation 6.1 (Model 4) and the model described by Equation 6.2 (Model 1), respectively. Model 2 has the same decision variable (final storage) as Model 1 while it has the same inflow state variable (previous inflow) as Model 4. Model 3 has the same decision variable as Model 4 and the same inflow state variable as Model 1. The formulations of the four SDP models are listed below.

Model 1 State : Initial storage + Present inflow
Decision : Final storage
The recursive equation is defined as in Equation 6.2

Model 2 State : Initial storage + Previous inflow
Decision : Final storage

$$f_t^n(S_t,Q_{t-1})=\underset{S_{t+1}}{opt}\left[\sum_{Q_t} P_t(Q_t/Q_{t-1})*[B_t(S_t,Q_t,S_{t+1})+f_{t+1}^{n-1}(S_{t+1},Q_t)]\right] \quad \forall S_t,Q_{t-1},S_{t+1} feasible \qquad (6.3)$$

subject to,

$$R_t = S_t + Q_t - S_{t+1} - SP_t - E_t$$

$$f_T^1(S_t,Q_{t-1}) = \underset{S_{t+1}}{opt}\left[\sum_{Q_t} P_t(Q_t/Q_{t-1})*B_t(S_t,Q_t,S_{t+1})\right]$$

Model 3 State : Initial storage + Present inflow
Decision : Current release

$$f_t^n(S_t,Q_t)=\underset{R_t}{opt}\left[(S_t,Q_t,R_t)+\sum_{Q_{t+1}} P_{t+1}(Q_{t+1}/Q_t)*f_{t+1}^{n-1}(S_{t+1},Q_{t+1})\right] \quad \forall S_t,Q_t,R_t feasible \qquad (6.4)$$

subject to,

$$R_t = S_t + Q_t - S_{t+1} - SP_t - E_t$$

$$f_T^1(S_t,Q_t) = \underset{R_t}{opt}[B_t(S_t,Q_t,R_t)]$$

Model 4 State : Initial storage + Previous inflow
Decision : Current release
The recursive equation is defined as in Equation 6.1

Where, t, S_t, Q_t, R_t, SP_t and E_t are as defined in Equation 3.12; n, B_t, f_t^n *and* P_t are as defined in Equation 6.1. The relation between time notations and variables are as shown in Figure 3.3.

The four SDP formulations are compared by applying to a real case, the Kariba reservoir system. The Kariba reservoir system is described in Chapter 4. Initially the optimum operation policies were derived and then the system operations were simulated according to the derived policies. The comparison is made based on the performance indices obtained from the simulation.

6.2.1 Computer-Experiments

The structure of the SDP model is similar to the model described in Table 5.3 except the different sets of decision and inflow state variables. The stage is the time period, which is one month. The objective is to maximize the expected annual energy generation. The optimization is subjected to the physical constraints of the reservoir system (e.g., storage constraints, release constraints, etc.). The whole reservoir storage is discretized into 42 classes of equal size, each having the interval of 1579 MCM. For the models in which policies are defined as optimal releases (Model 3 and Model 4), the release levels up to twice of the monthly release capacity of the turbines are to be optimized. They are discretized into 6 classes with equal size, each having an interval of about 1575 MCM. The monthly inflows are discretized into varying numbers of classes (2 classes for August, September, October and November; 3 classes for December and July; 4 classes for January and June; 6 classes for February and May; 8 classes for March and April) with equal occupancy frequencies. The median of each inflow class is defined as the representative value of that class. Uniform frequency distribution is assumed row-wise for the empty rows if they occur in the inflow transition probability matrices. Empty rows may occur due to the limited length of the observed historical inflow time series. For all the SDP based optimizations, both convergence criteria have been obtained.

In the subsequent simulations, the historical inflow time series were used "strictly" relying on the derived SDP based operation policies subjected to the physical constraints of the reservoir system. Based on the setup of these models, the following three computer-experiments have been carried out.

Experiment 6.1
Derive optimal operation policies for the reservoir system using the four SDP models based on 24 years (1961 - 1984) of historical inflow time series. Then simulate the performance of the reservoir system according to the derived operation policy sets. Use the last 12 years (1973 - 1984) of historical inflow time series at this step. Assume that the perfect forecast is available at the beginning of each time period.

Experiment 6.2
Derive optimal operation policies for the reservoir system using the four SDP models based on 12 years (1961 - 1972) of historical inflow time series. Then simulate the performance of the reservoir system according to the derived operation policy sets using the last 12 years (1973 - 1984) of historical inflow time series. Assume that the perfect forecast is available at the beginning of each time period.

Experiment 6.3

Derive optimal operation policies for the reservoir system using the four SDP models based on 12 years (1961 - 1972) of historical inflow time series. Then simulate the performance of the reservoir system according to the derived operation policy sets using the last 12 years (1973 - 1984) of historical inflow time series using the imperfect inflow forecast at the beginning of each time period. The inflows are forecast by using the regression equations derived by Budhakooncharoen (1986) for this case study system. The regression equations obtained from that study are shown in Table 4.7. The three computer-experiments are summarized in Table 6.3.

Table 6.3 Summary of the Three Computer-Experiments

	Experiment 6.1	Experiment 6.2	Experiment 6.3
The inflow time series used to derive release policy	1961 - 1984	1961 - 1972	1961 - 1972
The inflow time series used to simulate system operation	1973 - 1984	1973 - 1984	1973 - 1984
The type of inflow forecast available at the beginning of each period (during simulation)	Perfect forecast	Perfect forecast	Imperfect forecast

In Experiment 6.1, the part of historical inflow series (1973 - 1984) that has been employed for deriving the optimal policies is used to simulate the performance of the system. This enables to observe the difference between system performance values obtained from the steady state solution of SDP and that from simulation. SDP relies on discrete representation of input states while simulation uses actual inflow series.

In Experiment 6.2, the first 12 years of the historical inflow series are employed to derive optimal policies. The second 12 years of the historical inflow series are used to simulate the performance of the system according to the derived policies. This corresponds to the situation that exist in reality if the system is operated based on a perfect inflow forecast.

Derivation of optimum operation policies in Experiment 6.2 and Experiment 6.3 are similar. But in the implementation of the derived policies in the operation simulations, the forecasted inflow data are used in Experiment 6.3 while observed inflow data are used in Experiment 6.2. Nevertheless, when simulating the operation the inflows used in the continuity equation are still the actual inflows. In Experiment 6.3, the models whose derived policies can be implemented with known previous inflow (Model 2 and Model 4) will perform the same way as in Experiment 6.2. Variation from Experiment 6.2 occurs only in those models whose derived policies have to be implemented using inflow forecast data.

6.2.2 Analysis of the Results and Discussion

For each experiment each SDP model derives one optimal policy. Each policy contains 12 tables (12 months). Out of the large number of policies derived, the policy tables for the month of May from Experiment 6.2 are presented in Table 6.4 for all the four models, for example. Table 6.4a, Table 6.4b, Table 6.4c, and Table 6.4d refer to the policies from Model 1, Model 2, Model 3 and Model 4 respectively. Note that the policies from

Table 6.4 Derived SDP-based Policy Tables for the Kariba Reservoir (May)

(a) Model 1

Period: May						
Inflow:	1	2	3	4	5	6
Initial Storage	Final Storage					
1	2	1	1	1	1	1
2	3	2	1	1	1	1
3	4	3	2	1	1	1
4	5	4	3	2	1	1
5	6	5	4	3	2	1
6	7	6	5	4	3	1
7	7	6	6	5	4	2
8	8	7	7	6	5	3
9	9	8	8	7	6	4
10	10	9	9	8	7	5
11	11	10	10	9	8	6
12	12	11	11	10	9	7
13	13	12	12	11	10	8
14	14	13	13	12	11	9
15	15	14	14	13	12	10
16	16	15	15	14	13	11
17	17	16	16	15	14	12
18	18	17	17	16	15	13
19	19	18	17	17	16	14
20	20	19	18	18	17	15
21	21	20	19	19	18	16
22	22	21	20	20	19	17
23	23	22	21	21	20	18
24	24	23	22	22	21	19
25	25	24	23	23	22	20
26	26	25	24	24	23	21
27	27	26	25	25	24	22
28	28	27	26	26	24	23
29	29	28	27	27	25	24
30	30	29	28	28	26	25
31	31	30	29	29	27	26
32	32	31	30	30	28	27
33	33	32	31	31	29	28
34	34	33	32	32	30	29
35	35	34	33	32	31	30
36	36	35	34	33	32	31
37	37	36	35	34	33	32
38	37	37	36	35	34	33
39	38	37	37	36	35	34
40	39	38	37	37	36	35
41	40	39	38	38	37	36
42	41	40	39	39	38	37

(b) Model 2

Period: May								
Inflow:	1	2	3	4	5	6	7	8
Initial Storage	Final Storage							
1	1	1	1	1	1	1	1	1
2	2	2	1	1	1	1	1	1
3	3	3	2	2	1	1	1	1
4	4	4	3	3	2	2	1	1
5	5	5	4	4	3	3	2	1
6	6	6	4	4	4	4	2	1
7	7	7	5	5	5	5	3	2
8	8	7	6	6	6	6	4	3
9	9	8	7	7	7	7	5	4
10	10	9	8	8	8	8	6	5
11	11	10	9	9	8	8	7	6
12	12	11	10	10	9	9	8	7
13	13	12	11	11	10	10	9	8
14	14	13	12	12	11	11	10	9
15	15	14	13	13	12	12	11	10
16	16	15	14	14	13	13	12	11
17	17	16	15	15	14	14	13	12
18	18	17	16	16	15	15	14	13
19	19	18	17	17	16	16	15	14
20	20	19	18	18	17	17	16	15
21	21	20	19	19	18	18	17	16
22	22	21	20	20	19	19	17	17
23	23	22	21	21	20	20	18	17
24	24	23	22	22	21	21	19	18
25	25	24	23	23	22	22	20	19
26	26	25	24	24	23	23	21	20
27	27	26	25	25	24	24	22	21
28	28	27	26	26	25	25	23	22
29	29	28	27	27	26	26	24	23
30	30	29	28	28	27	27	25	24
31	31	30	29	29	28	28	26	25
32	32	31	30	30	29	29	27	26
33	33	32	31	31	30	30	28	27
34	34	33	32	32	31	31	29	28
35	34	34	33	33	32	32	30	29
36	35	35	34	34	33	33	31	30
37	36	36	35	35	34	34	32	31
38	37	37	36	36	35	35	33	32
39	38	38	36	36	36	36	34	33
40	39	39	37	37	37	37	35	34
41	40	40	38	38	38	38	36	35
42	40	40	39	39	39	39	37	36

(c) Model 3

Period: May						
Inflow:	1	2	3	4	5	6
Initial Storage	Final Storage					
1	1	1	1	1	1	1
2	1	1	1	1	1	1
3	1	1	1	1	1	1
4	1	1	1	1	1	1
5	1	1	1	1	1	1
6	1	1	1	1	1	1
7	1	1	1	1	1	1
8	1	1	1	1	1	1
9	1	1	1	1	1	1
10	1	1	1	1	1	1
11	1	1	1	1	1	1
12	1	1	1	1	1	1
13	1	1	1	1	1	1
14	1	1	1	1	1	1
15	1	1	1	1	1	1
16	1	1	1	1	1	1
17	1	1	1	1	1	1
18	1	1	1	1	1	1
19	1	1	1	1	1	1
20	1	1	1	1	1	1
21	1	1	1	1	1	1
22	1	1	1	1	1	1
23	1	1	1	1	1	1
24	1	1	1	1	1	1
25	1	1	1	1	1	1
26	1	1	1	1	1	1
27	1	1	1	1	1	1
28	2	1	1	1	1	1
29	2	1	1	1	1	1
30	2	1	1	1	1	1
31	2	2	1	1	1	1
32	2	2	2	1	1	1
33	2	2	2	1	1	1
34	2	2	2	1	1	1
35	2	2	2	1	1	1
36	2	2	2	2	1	1
37	2	2	2	2	2	1
38	3	3	2	2	2	1
39	3	2	3	2	2	1
40	3	3	2	2	2	1
41	4	4	3	2	2	2
42	3	4	4	3	2	2

(d) Model 4

Period: May								
Inflow	1	2	3	4	5	6	7	8
Initial Storage	Final Storage							
1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1
3	1	1	1	1	1	1	1	1
4	1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1
6	1	1	1	1	1	1	1	1
7	1	1	1	1	1	1	1	1
8	1	1	1	1	1	1	1	1
9	1	1	1	1	1	1	1	1
10	1	1	1	1	1	1	1	1
11	1	1	1	1	1	1	1	1
12	1	1	1	1	1	1	1	1
13	1	1	1	1	1	1	1	1
14	1	1	1	1	1	1	1	1
15	1	1	1	1	1	1	1	1
16	1	1	1	1	1	1	1	1
17	1	1	1	1	1	1	1	1
18	1	1	1	1	1	1	1	1
19	1	1	1	1	1	1	1	1
20	1	1	1	1	1	1	1	1
21	1	1	1	1	1	1	1	1
22	1	1	1	1	1	1	1	1
23	1	1	1	1	1	1	1	1
24	1	1	1	1	1	1	1	1
25	1	1	1	1	1	1	1	1
26	1	1	1	1	1	1	1	1
27	1	1	1	1	1	1	1	1
28	2	1	1	1	1	1	1	1
29	2	1	1	1	1	1	1	1
30	2	2	1	2	1	1	1	1
31	2	2	2	2	1	1	1	1
32	2	2	2	2	1	1	1	1
33	2	2	2	2	2	2	1	1
34	2	2	2	2	2	2	1	1
35	3	2	2	2	2	2	1	1
36	2	2	2	2	2	2	1	1
37	3	3	2	3	2	2	2	1
38	3	3	3	3	3	2	2	1
39	3	4	4	4	3	3	2	1
40	4	3	3	3	3	2	2	2
41	4	4	3	4	3	3	2	2
42	3	4	4	4	3	4	2	2

Experiment 6.3 are the same as those from Experiment 6.2. The values in Table 6.4a and Table 6.4b are the targeted storage classes in the reservoir at the end of the month. The values in Table 6.4c and Table 6.4d are the targeted release classes during the month. The policy tables of the other months and the tables from Experiment 6.1 have a similar structure as the policies shown in Table 6.4.

The simulated performance is presented based on the following three aspects: (a) the average reservoir storage; (b) the average release through turbine; and (c) the average energy generation. The energy generation, being the objective of the optimization is the most important performance index. The release through turbine is directly proportional to the energy generation. Reservoir storage shows the behaviour of the reservoir very clearly.

Table 6.5 and Figure 6.1 present the results from Experiment 6.1. Table 6.5 shows the average annual performance indices regarding the reservoir storage, the release through turbines and the energy generation. For each of the three performance indices, the simulated mean, the standard deviation and the minimum value are presented. For energy generation the expected annual gain obtained from the SDP based optimization is also included. To make the comparison easy, each value corresponding to reservoir storage, turbine release and energy generation are presented as the percentages of reservoir capacity, the average annual inflow and the generation capacity, respectively. Figure 6.1 shows the average monthly performances (average reservoir storage in (a), average turbine release in (b) and average energy generation in (c)).

Table 6.5 Simulated Average Annual Performances (Experiment 6.1)

	Model 1	Model 2	Model 3	Model 4
Indices in storage as % of reservoir capacity:				
(1) Mean utilized storage	62.3%	63.7%	68.0%	67.2%
(2) Standard deviation of (1)	29.8%	21.1%	28.1%	28.4%
(3) Minimum drawdown	7.0%	24.7%	10.6%	11.0%
Indices in releases as % of annual inflow:				
(4) Mean annual release	70.7%	66.5%	70.3%	70.1%
(5) Standard deviation of (4)	7.3%	8.1%	4.7%	5.2%
(6) Minimum annual release	51.3%	47.4%	57.6%	57.7%
Indices in energy as % of power capacity:				
(7) Expected mean annual energy output	93.5%	86.6%	91.7%	91.4%
(8) Simulated mean annual energy output	90.8%	85.5%	91.0%	90.7%
(9) Standard deviation of (8)	11.4%	11.3%	8.3%	9.0%
(10)Firm annual energy output	62.0%	59.4%	73.3%	73.8%

Table 6.6 and Figure 6.2 show the average annual performance indices and the average monthly performances, respectively, from Experiment 6.2.

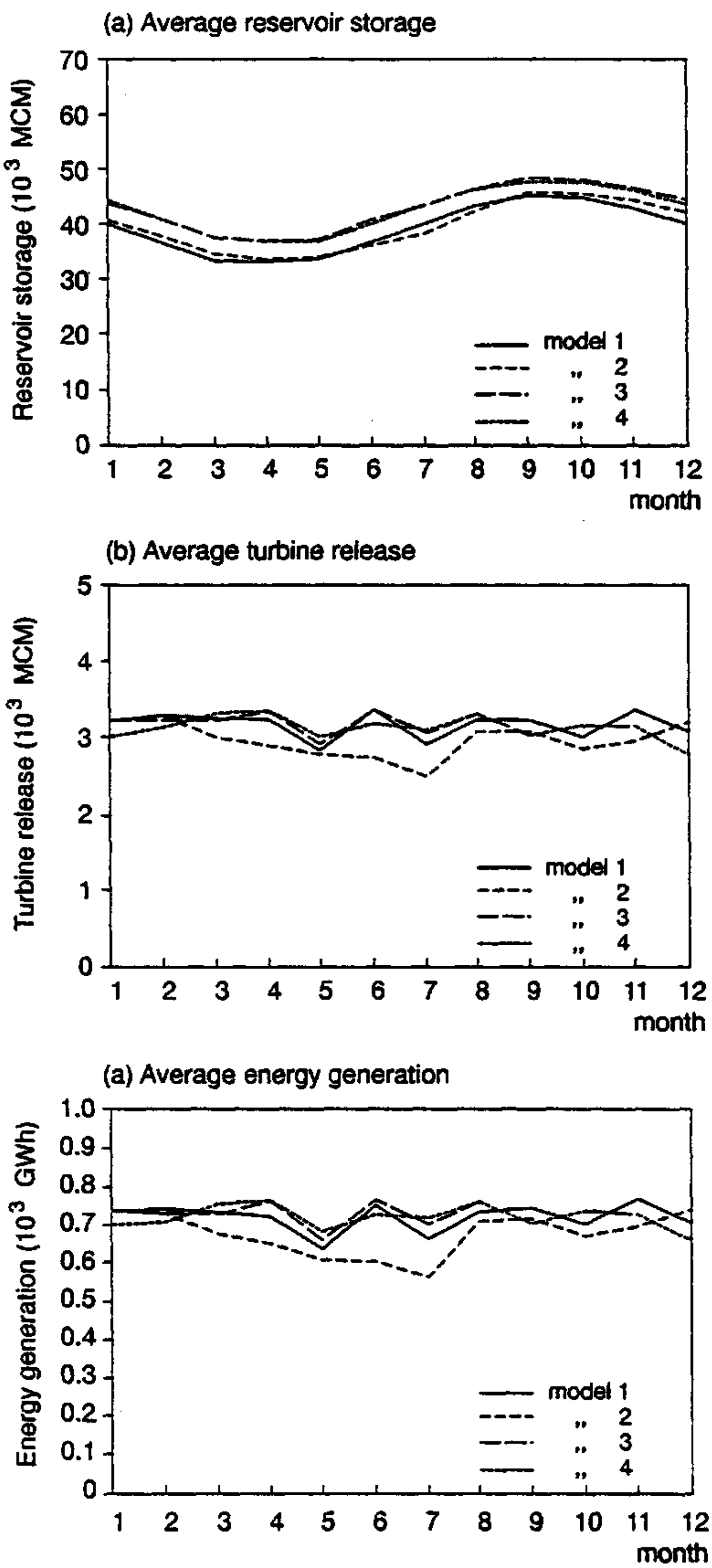


Figure 6.1 Simulated Monthly Performance of Experiment 6.1

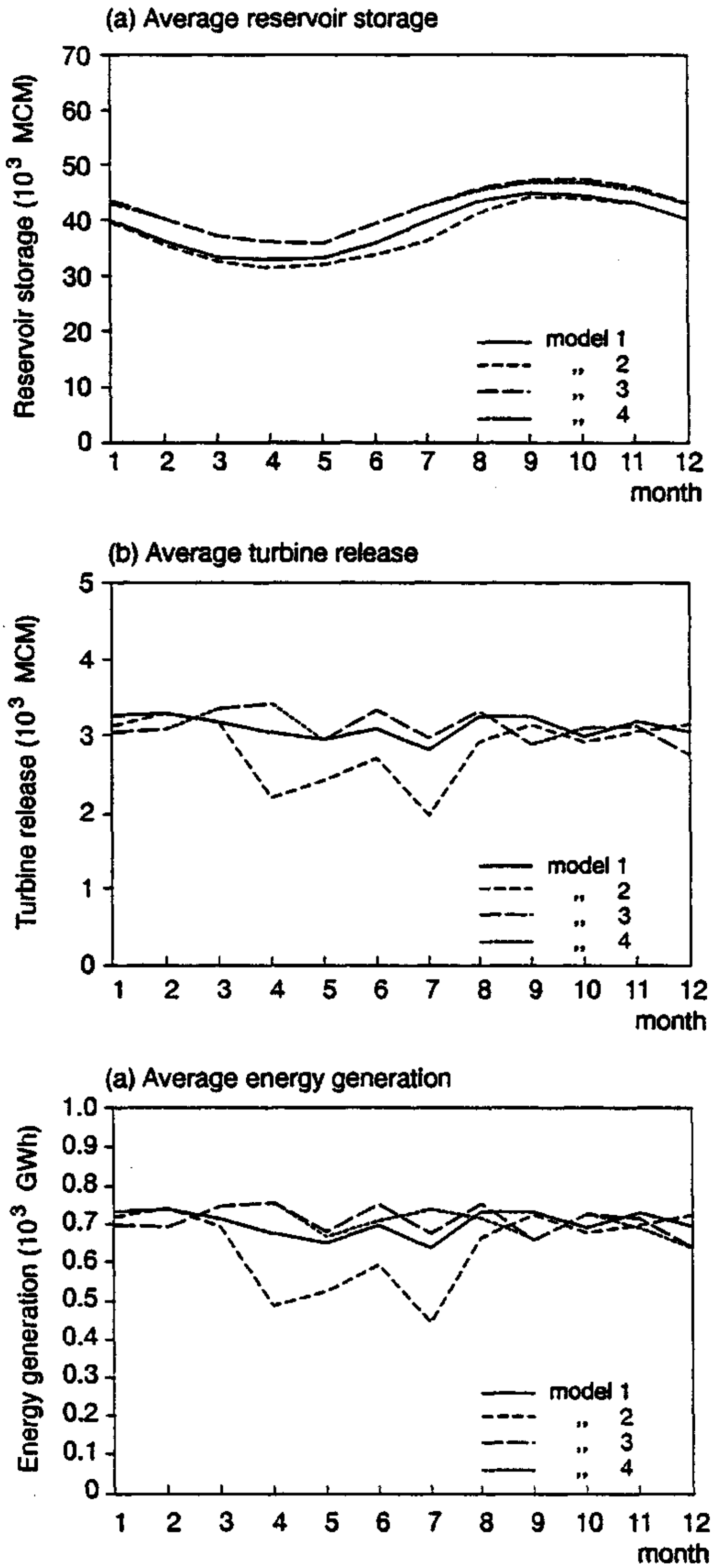


Figure 6.2 Simulated Monthly Performance of Experiment 6.2

Table 6.6 Simulated Average Annual Performances (Experiment 6.2)

	Model 1	Model 2	Model 3	Model 4
Indices in storage as % of reservoir capacity:				
(1) Mean utilized storage	61.8%	59.8%	66.5%	66.0%
(2) Standard deviation of (1)	30.2%	21.1%	29.6%	29.2%
(3) Minimum drawdown	6.2%	22.0%	8.2%	11.1%
Indices in releases as % of annual inflow:				
(4) Mean annual release	69.8%	64.5%	69.9%	69.7%
(5) Standard deviation of (4)	7.9%	7.2%	6.0%	6.2%
(6) Minimum annual release	48.4%	50.4%	55.6%	54.6%
Indices in energy as % of power capacity:				
(7) Expected mean annual energy output	92.7%	87.7%	91.0%	91.1%
(8) Simulated mean annual energy output	89.6%	82.4%	90.3%	90.0%
(9) Standard deviation of (8)	11.9%	10.0%	10.1%	10.2%
(10)Firm annual energy output	58.6%	62.7%	67.1%	66.5%

Table 6.7 and Figure 6.3 show the average annual performance indices and the average monthly performances, respectively, from Experiment 6.3.

Table 6.7 Simulated Average Annual Performances (Experiment 6.3)

	Model 1	Model 2	Model 3	Model 4
Indices in storage as % of reservoir capacity:				
(1) Mean utilized storage	66.3%	59.8%	66.3%	66.0%
(2) Standard deviation of (1)	22.5%	21.1%	29.3%	29.2%
(3) Minimum drawdown	19.6%	22.0%	10.4%	11.1%
Indices in releases as % of annual inflow:				
(4) Mean annual release	67.2%	64.5%	69.9%	69.7%
(5) Standard deviation of (4)	5.1%	7.2%	6.3%	6.2%
(6) Minimum annual release	58.8%	50.4%	54.7%	54.6%
Indices in energy as % of power capacity:				
(7) Expected mean annual energy output	92.7%	87.7%	91.0%	91.1%
(8) Simulated mean annual energy output	86.4%	82.4%	90.3%	90.0%
(9) Standard deviation of (8)	7.5%	10.0%	10.2%	10.2%
(10)Firm annual energy output	73.3%	62.7%	66.6%	66.5%

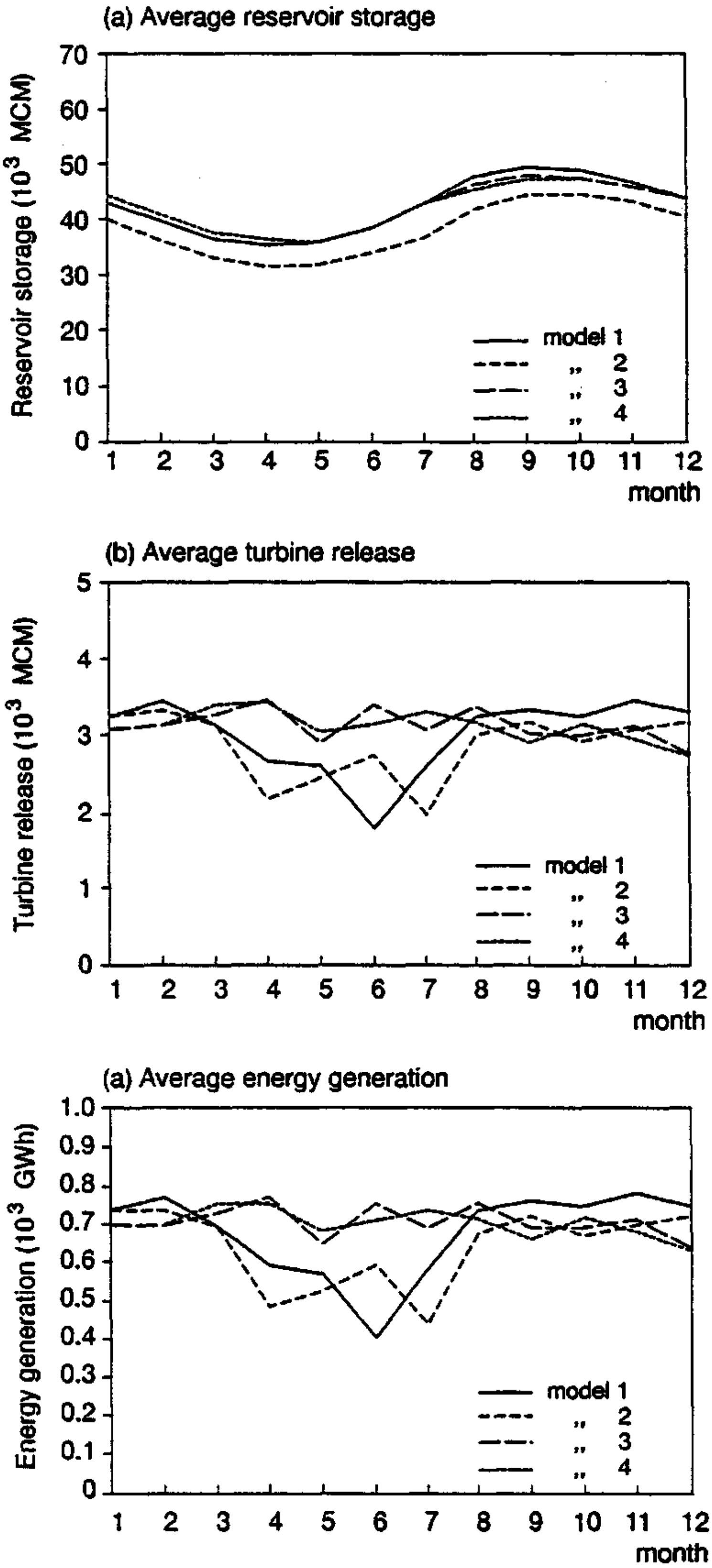


Figure 6.3 Simulated Monthly Performance of Experiment 6.3

In Table 6.4 the particular structure of the policy tables for models with release as the decision variable (Table 6.4c and Table 6.4d) attracts attention. It is noticeable that most of the decisions are 3212 MCM, which is close to the capacity of the turbines (3937 MCM). The decision changes to the value of one class larger (4818 MCM) only if both initial storage and the inflow are very large, or to the value of one class smaller (1606 MCM) only if both initial storage and the inflow are very small. This structure of the policy table can be explained as follows. The objective of the optimization is to generate as much energy as possible. The maximum energy that a hydro-power plant can generate is the capacity of its generators. The amount of energy generation is proportional to the release through the penstocks. Corresponding to the capacity of the generators there is an imaginary capacity release (R_c). Therefore, optimal decisions can be seen as attempts to approach R_c.

In contrast to Table 6.4c and Table 6.4d, models with storage as the decision variable (Table 6.4a and Table 6.4b) have policy tables that show large variations in the decisions. The above mentioned feature of "stability" is achieved for the policy with release as decisions, only when release is the direct target of the optimization (e.g., to satisfy downstream water requirement or to satisfy energy requirement, etc.). The feature of "stability" would occur to the policy with storage as the decision, if storage is the direct target of the optimization. For example, if the water level in a reservoir is important, the objective of the optimization could be minimizing the deviation from a target storage level at each time period. Thus the derived policy would have the feature that many decisions (storage) in each table were equal to the unique value of the target (or the discretized value closest to that target) for that time period. The more robust the reservoir system is, the more decisions will be the target values.

The "stability" of the policies defined with release as decision intuitively explains the following three aspects of the simulation results in this study.

(i) Figure 6.2 and Figure 6.3 show the simulated average monthly performance of Experiment 6.2 and Experiment 6.3, respectively. As previously described in Chapter 6.2.1, the Experiment 6.2 corresponds to the operation of the system with perfect inflow forecasting. The Experiment 6.3 corresponds to the operation of the system with perfect inflow forecasting. With respect to models with the previous inflow as a state variable (Model 2 and Model 4), the performance indices do not change. Model 1 performs considerably worse in Experiment 6.3 than in Experiment 6.2, as expected. However, the performance of Model 3 in Experiment 6.3 is unexpectedly close to the performance in Experiment 6.2.

This unexpected result can be explained with policy Table 6.4c. For a wide range of initial storage values (from 11105 MCM to 52166 MCM), the release decisions are independent of the present inflow values. The errors in the present inflow forecast only affect the release decision in a very few cases (the top-right and bottom-left triangle), and the maximum deviation in decision is only one class. Therefore, the (simulated) operation based on this type of policy is insensitive to the errors in the inflow forecast.

(ii) From Table 6.5 to Table 6.7 and from Figure 6.1 to Figure 6.3, it is observed that the models with release as a decision variable (Model 3 and Model 4) differ very little from each other with respect to all three simulated performance indices (reservoir storage, turbine release and energy generation). In contrast, models with storage as a decision variable

(Model 1 and Model 2) show much larger difference between them with respect to performance indices.

In Table 6.4c (Model 3) and Table 6.4d (Model 4), for a wide range of initial storage values (from 12684 MCM to 52166 MCM) the optimal decisions (release) are independent of both the present and the previous inflow values. The differences in the present or the previous inflows only affect the release decisions in a very few cases (the top-right and bottom-left triangle). The maximum deviation in decision is one class. In contrast, Table 6.4a (Model 1) and Table 6.4b (Model 2) show that the optimal decisions (final storage) for the operation are strongly determined by the initial storage values and the inflow values. A small variation in the inflow value can lead to a different decision. Therefore, whether the previous or the present inflow is used as a state variable strongly influences the decisions during operations (and hence, the performance indices). Comparing the structures of Table 6.4c and Table 6.4d with those of Table 6.4a and Table 6.4b, it can be concluded that the policies derived from models with release as a decision variable (Model 3 and Model 4) are much less sensitive to variations in the initial storage and inflow than policies derived from models with storage as a decision variable (Model 1 and Model 2).

(iii) From Table 6.5 to Table 6.7 and from Figure 6.1 to Figure 6.3 (particularly in Table 6.7 and Figure 6.3 when the imperfect inflow forecast is adopted as a guidance during operation simulation), indicate that the reservoir performances obtained from the models with release as a decision variable (Model 3 and Model 4) are better than that obtained from the models with storage as a decision variable (Model 1 and Model 2). Model 3 and Model 4 results in larger mean values and smaller fluctuations in energy generation, both in annual and monthly performance indices. These results can be related to the "stability" of policy Table 6.4c and Table 6.4d.

From this discussion, it can be concluded that the models with release as the decision variable considerably outperform the models with storage as the decision variable. If the "right" decision has been made regarding the decision variables, the different choices of inflow state variables would not much affect the performance of the system. However, in reality the selection of the "right" decision variable can not always be realized. For example, for multipurpose reservoir systems, sometimes more than one objective has to be optimized at the same time. Some objectives might be directly related to release and others might be directly related to storage. As has been shown in the case study that when storage is selected as a decision variable, models will become sensitive to the choice of inflow state variable.

Figure 6.1b and Figure 6.1c show that Model 2 considerably deviates from the optimum during the wet season (January, February, March and April) when the stochastic inflow has large distribution ranges (see Table A.6, the 24 years of historical inflow into the Kariba reservoir). The course for this behaviour is explained below.

Equation 6.3 (Model 2) can be written in the following form;

$$f_t^n(S_t,Q_{t-1})=\underset{S_{t+1}}{opt}\Big[\sum_{Q_t} P_t(Q_t/Q_{t-1})*B_t(S_t,Q_t,S_{t+1})+\sum_{Q_t} P_t(Q_t/Q_{t-1})*f_{t+1}{}^{n-1}(S_{t+1},Q_t)]\Big| \qquad (6.5)$$

When Equation 6.2 and Equation 6.5 are compared, the major difference between them is observed to be in the first item (present return). For the model with the present inflow as a state variable (Equation 6.2), the present return in the recursive relation is deterministic. For the model with previous inflow as a state variable (equation 6.5), the present return is stochastic. Thus, its expected value is instead adopted as the index for finding an optimal decision. When the possible range of the stochastic inflow is small, an expected value would be a good representative. The larger the possible range of the stochastic inflow, the poorer the expected value as a representative. Table A.6 shows that in April the minimum value is 3010 MCM and the maximum value is 21390 MCM, the mean value is 9373 MCM. The possible deviation from the mean could be 12017 MCM, which is more than three times the release capacity of the generator. Therefore, an expected value of the present return from such a series of inflow is likely to mislead the SDP optimization search slightly.

The same problem does not occur in Model 4. It has the previous inflow as a state and thus has a stochastic present return. Because the much smaller feasible range of the release decision base restricts the scale of possible error.

The above argument can be explained in a simple way by an example with an optimization horizon of one time period. Assume there is an empty reservoir with storage capacity of 64800 MCM. The incoming inflow is stochastic and it has a distribution range from 3010 MCM to 21390 MCM and the mean value is 9373 MCM. The feasible release is from 0 to 3937 MCM. Releasing 3937 MCM is assumed as the target of optimization. These assumed figures are picked up from the historical inflow time series for Kariba reservoir (April, Table A.6).

First, consider the case that the release is selected as a decision variable. If the estimated inflow is 3010 MCM, the release decision should be 3010 MCM; if the estimated inflow is 3937, the release decision should be 3937; if the estimated inflow is 9373, the release decision should still be 3937; if the estimated inflow is 21390, the release decision is still 3937. Therefore, the error in the estimation of incoming inflow for the range from 3937 to 21390 MCM does not affect the release decision. Only the error in the estimation of inflow within the range of 3010 to 3937 MCM alters the release. Now, let the final storage be selected as decision variable. If the estimated inflow is 3010 MCM, the storage decision should be 0 and thus release 3010 MCM; if the estimated inflow is 3937, the storage decision should still be 0; if the estimated inflow is 9373, the storage decision should be 5436; if the estimated inflow is 21390, the storage decision should be 17463. The error in the estimation of coming inflow for the range from 3937 to 21390 MCM does influence the storage decision. Although the fluctuation on inflow within the range of 3010 to 3937 does not alter the decision of storage, the resulting release does change with the actual inflow. Therefore, for the policy defined with final storage as a decision, the error in the estimation of inflow for the whole possible range (from 3010 to 21390 MCM) can strongly alter the actual release.

Now, assume that the policies are obtained from the mean value of inflow. The policy defined with release as a decision is 3937 MCM; the policy defined with storage as a decision is 5436 MCM. In real time operation if the actual inflow is 5436, for example, according to the policy defined with release as a decision, 3937 MCM should be released and 1499 MCM should be left in the reservoir; according to the policy defined with final storage as a decision, 5436 MCM should be kept in the reservoir and 0 should be released. Therefore, for this particular inflow situation, the policy defined with release as a decision satisfies the release target 3937 MCM more than the policy defined with final storage as a decision. This again demonstrates the importance of selecting the variable that is directly related to the target as a decision.

The same example also can be used to explain the simulation results that during the dry season (September, October and November) the models with final storage as a decision variable slightly outperform the models with release as a decision variable (see Figure 6.1b, Figure 6.1c, Figure 6.2b and Figure 6.2c).

Consider the inflow time series of October as an example (Table A.6). The inflow distribution range is from 843 to 1918 MCM and the mean is 1197 MCM. Now the obtained policies based on the mean value of inflow are releasing 1197 MCM with release as a decision and storing 0 MCM in the reservoir with final storage as a decision. In real time operation if the actual inflow is 1918 MCM, for example, according to the policy defined with release as a decision, 1197 MCM should be released and 721 MCM should be left in the reservoir; according to the policy defined with final storage as a decision, 0 MCM should be kept in the reservoir and 1918 MCM should be released. Therefore, for this inflow situation, the policy defined with final storage as a decision satisfies the release target with 721 MCM more than the policy defined with release as a decision. However, as compared with the wet season, the scale of possible errors is limited.

The comparison of the simulation results from Experiment 6.2 and Experiment 6.3 (Table 6.6, Figure 6.3, Table 6.7 and Figure 6.4) shows the influence of the imperfect inflow forecast on the Model 1 based performance. The influence is limited during wet season, as can be expected. During wet season the distribution range of inflow is large, and thus the possible errors in the inflow forecasting are large enough to alter the SDP based policy. It is obvious that the quality of the forecasting model has a great effect on the performance. The better the reliability of the forecast inflow, the closer the reservoir operation performance to its real optimum. In Experiment 6.3, the inflow is forecast by a simple linear regression model (Budhakooncharoen, 1986) with considerable errors (see Table 4.7). Even so, the Model 1 based simulation still performs better than the Model 2 based simulation that depends on the previous inflow. This makes the model with present inflow as a state, better than the model with previous inflow as a state. Although it cannot be concluded from the current results that this type of models with present inflow as a state will always outperform the type of models with previous inflow as a state, it does illustrate that the policy derived from Model 1 can sustain errors in the inflow forecast. Besides, in real time operation, having large amounts of up-to-date information regarding rainfall, river channel flow, ground water and catchment area characteristics etc., a good inflow forecast can be easily produced.

6.3 Comparison of the Models with Different Decision Variables

One of the phenomena from the comparative study of Chapter 6.2, which draws attention is the overwhelming influence of the release as the decision variable has on the suitability of the SDP model for the system optimized. As compared with final storage volume, release is more directly related to the objective of hydro-power generation, which is the objective of the system analyzed.

However, though the above example (Kariba reservoir) is good to illustrate the influence of decision variable and state variable, it may not be sufficient to draw conclusions. First, Kariba is not a water supply system and thus the objective is not purely decided by release. The Kariba system serves as a hydropower plant. Hydropower generation is proportional to the water head and the amount of water passing through the turbines. The water head depends on the water storage level in the reservoir. In Kariba reservoir the change in the water head is very limited compared with the penstock release. Nevertheless, regarding it as a problem based on release decision is an approximation. Second, Kariba is a robust system with huge reservoir storage (active capacity is 64750 MCM) and a huge annual inflow (average is 54689 MCM), but with relatively small capacity of penstocks (about 4000 MCM water can pass through monthly). This fact may slightly abate any inherent advantages or disadvantages of the optimization models.

To verify the hypothesis regarding the choice of decision variables, another case study is carried out on the Joumine reservoir. It has characteristics different from the Kariba reservoir.

The Joumine reservoir, situated in Tunisia, is a water supply system. Its live storage capacity is about 121.3 MCM and the mean annual inflow is 133 MCM. The Joumine reservoir is supposed to satisfy the immediate downstream water demand and supply water through an inter-basin transfer tunnel to several remote places (together with some other reservoirs). In this study the immediate water demand is called "local demand"; the demand from remote places is called "system demand". The "local demand" and the "system demand" together is called the "total demand" in this study. Detailed information about the Joumine reservoir is described in Chapter 4.3.

The present study confines to the comparison between the models with the present inflow as the inflow state variable, but with different decision variables; i.e., Model 1 and Model 3 are applied to derive SDP based optimal operation policies. The system operation is then simulated according to the derived policies and the discussion is made based on the simulated water shortage or the water requirement fulfilment.

6.3.1 The Structure of the SDP Models and Resulting Policies

The structure of the SDP model is summarized in Table 6.8.

Table 6.8 SDP Model Setup for the Joumine Reservoir

Stage	Time period in month
Objective	Minimize expected squared deficit from the "total demand"
Inflow discretizations	Equal size intervals and varying number of classes (from 4 to 12)
Storage discretizations	Equal size intervals. 18 classes
Release discretizations	Equal size intervals. 3 or 4 classes

The stage is the time period, which is one month. The objective is to minimize the squared deficit from the "total demand", subjected to the physical constraints of the reservoir system (e.g., storage constraints, release constraints, etc.). Total reservoir storage is discretized into 18 classes of equal size. For the model with the policy defined as release (Model 3) the release levels upto the monthly "total demands" are to be optimized. They are discretized into 3 classes of equal size when the demands are smaller than 9 MCM and into 4 classes otherwise. The monthly inflow is discretized into varying number of classes (from number 4 to 12 according to the distribution range of the 44 years (1946-1989) of historical monthly inflow) with equal intervals.

Table 6.9 shows the derived policy tables for the month of March. Table 6.9a and Table 6.9b are the policy tables from Model 1 and Model 3, respectively. The numbers in Table 6.9a are the targeted storage classes at the end of March; the numbers in Table 6.9b are the targeted release classes during March.

In the policy table defined in terms of release (Table 6.9b), most of the optimal decisions are the value of the "total demand" in March (class 1). The decision changes to the value of one class smaller (class 2) or two classes smaller (class 3) only if both the initial storage and the inflow are very small (see the small bottom-left triangle in Table 6.9b). This feature of the policy table clearly shows the tendency that decisions are equal to the water demand in that time period as long as the reservoir system is capable of doing so.

In the policy table defined in terms of storage (Table 6.9a), the decision for most of initial storage and inflow conditions (except the bottom-left triangle) are also aiming at releasing the amount at the discretized point closest to the water demand. However, since the optimal decisions are defined as final storage volumes, the resulting policy table has a different feature (as compared with Table 6.9b). The figures gradually change with the different combination of initial storage and inflow conditions from the smallest value at the left bottom corner to the storage capacity of the reservoir (up-right).

6.3.2 Simulation and Results

To compare the performances of Model 1 and Model 3, the reservoir operation has been simulated according to the above two sets of derived policies (from Model 1 and Model 3). The Model 3 based simulation "strictly" relies on the derived policies (as long as the physical constraints of the reservoir system are not violated), whereas the Model 1 based simulation includes certain modifications to the derived policy. These modifications are; (a) if the release according to the policy table is larger than the "total demand", then release an amount of water equal to the "total demand" and save the balance in the reservoir (if there is still room in the reservoir); (b) if the initial storage level is higher than half the full capacity and the resulting release according to the policy table were smaller than the "total demand", adjust the targeted final storage so that the release satisfies the "total demand".

Table 6.9 Derived SDP-based Policy Tables for the Joumine Reservoir (March)

(a) Model-1

Period: March											
Inflow	1	2	3	4	5	6	7	8	9	10	11
Initial Storage											
1	1	1	1	1	1	1	1	1	1	1	1
2	2	1	1	1	1	1	1	1	1	1	1
3	3	2	1	1	1	1	1	1	1	1	1
4	4	3	2	1	1	1	1	1	1	1	1
5	4	3	3	1	1	1	1	1	1	1	1
6	5	4	4	2	1	1	1	1	1	1	1
7	6	5	5	3	1	1	1	1	1	1	1
8	7	6	6	4	2	2	1	1	1	1	1
9	8	7	7	4	3	3	1	1	1	1	1
10	9	8	8	5	4	4	2	1	1	1	1
11	10	9	9	6	5	5	3	1	1	1	1
12	11	10	9	7	5	6	4	2	1	1	1
13	12	11	10	8	6	7	5	3	2	1	1
14	13	12	11	9	7	8	6	4	3	2	1
15	14	13	12	10	8	9	6	4	4	2	1
16	15	14	13	11	9	10	7	5	5	3	2
17	16	15	14	12	10	11	8	6	6	4	3
18	17	16	15	13	11	12	9	7	7	5	3

(b) Model-2

Period: March											
Inflow	1	2	3	4	5	6	7	8	9	10	11
Initial Storage											
1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1	1
3	1	1	1	1	1	1	1	1	1	1	1
4	1	1	1	1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1	1	1	1
6	1	1	1	1	1	1	1	1	1	1	1
7	2	1	1	1	1	1	1	1	1	1	1
8	2	1	1	1	1	1	1	1	1	1	1
9	2	2	1	1	1	1	1	1	1	1	1
10	2	2	1	1	1	1	1	1	1	1	1
11	2	2	1	1	1	1	1	1	1	1	1
12	2	2	1	1	1	1	1	1	1	1	1
13	3	2	1	1	1	1	1	1	1	1	1
14	3	2	1	1	1	1	1	1	1	1	1
15	3	2	1	1	1	1	1	1	1	1	1
16	3	2	1	1	1	1	1	1	1	1	1
17	3	2	1	1	1	1	1	1	1	1	1
18	3	2	2	1	1	1	1	1	1	1	1

The reason for having these modifications for the Model 1 based simulation is the following. As has been described when setting up the SDP model in Chapter 6.3.1, in Model 3 the release up to the amount of the "total demand" has been optimized. The exact demands are discretized and the reservoir release is never larger than the demand unless storage is full. Besides, since the decision base of Model 3 (monthly release varying from 0 up to the amount of "total demand") is much smaller than the decision base of Model 1 (the whole reservoir storage capacity), it can be easily discretized into much smaller classes. Therefore, there is a smaller discrepancy between the release and demand caused by rough dicretization in Model 3 compared with that for Model 1. These are the advantages of the models that use release as decision variable. Therefore, to illustrate the additional superiority inherent in Model 3, the Model 1 is reinforced with the foregoing modifications during the simulation process.

The following two simulation experiments were designed based on the above discussion.

Experiment 6.4
Simulate the performance of the reservoir system based on the two derived operation policy sets (from Model 1 and Model 3) using the 44 years (1946 - 1989) of historical inflow time series. Assume the perfect forecast is available at the beginning of each time period.

Experiment 6.5
Simulate the performance of the reservoir system based on the two derived operation policy sets (from Model 1 and Model 3) using the 44 years (1946 - 1989) of historical inflow time series. An imperfect inflow forecast is used at the beginning of each time period.

The inflow forecast at this time step is according to the so-called "lag-one multi-period Markov model" (Viessman *et al.*, 1989), which has the following formula;

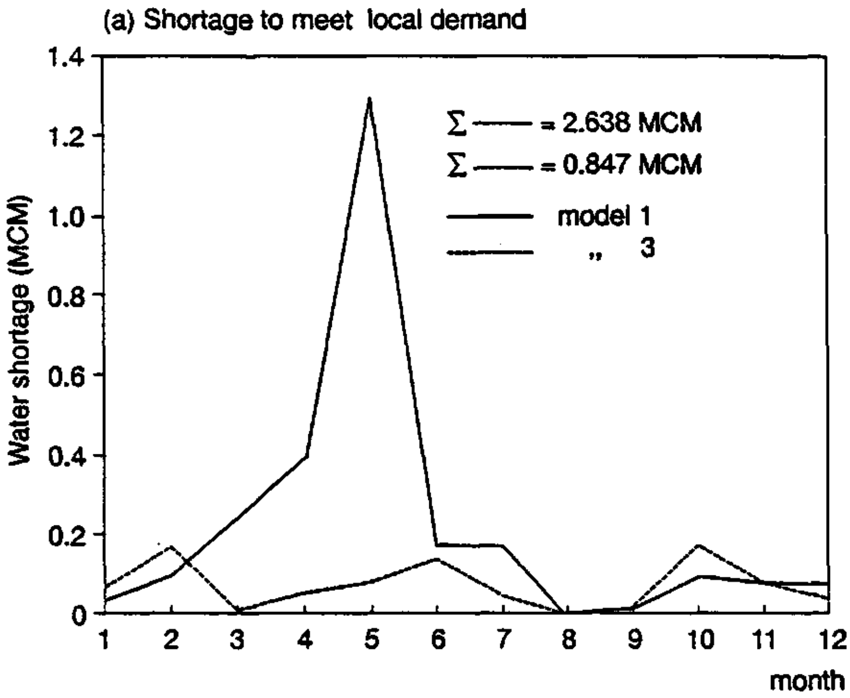
$$\hat{Q} = \bar{Q}_t + r_{t,t-1}*(\sigma_t/\sigma_{t-1})*(Q_{t-1}-\bar{Q}_{t-1}) + e_t*\sigma_t*(1-r^2_{t,t-1})^{1/2} \tag{6.6}$$

Where, $\hat{Q}_t$ is the forecast inflow in period t; Q_{t-1} is the observed inflow in period t-1; $\bar{Q}_t$ is the mean of observed inflows in period t; $r_{t,t-1}$ is the correlation coefficient for the relation of inflows from period t to period t-1; σ_t is the standard deviation of observed flows for the period t; e_t is a random number selected from a normal distribution having a zero mean and a unit variance.

The results from the two simulation experiments have been summarized in Figure 6.4, Figure 6.5 and Table 6.10. The quantity-based water shortage from simulation Experiment 6.4 and simulation Experiment 6.5 are shown in Figure 6.4 and Figure 6.5 respectively. Table 6.10 lists the monthly distribution of the level of time-based fulfilment of the "local demand" from the two experiments.

The following two major points can be deduced from the simulation results.

(i) When the perfect inflow forecast is available (Experiment 6.4), the system operation according to the release-based policy satisfies the water requirement slightly better than that according to the storage-based policy (both from the points of view of quantity-base and time-base). However, the difference is limited.



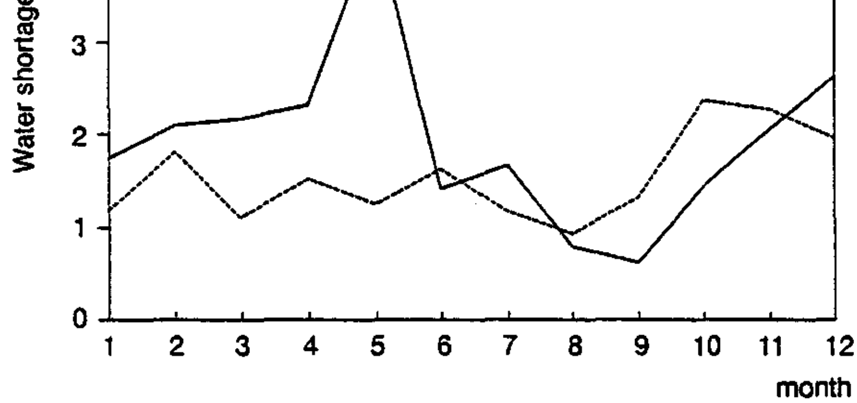
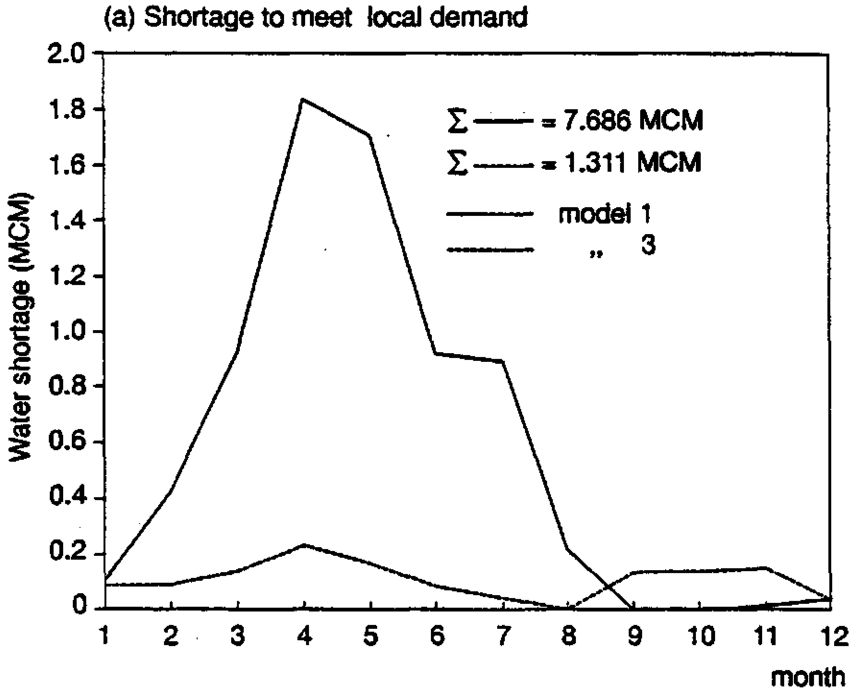


Figure 6.4 Quantity-based Water Shortage Result from Experiment 6.4



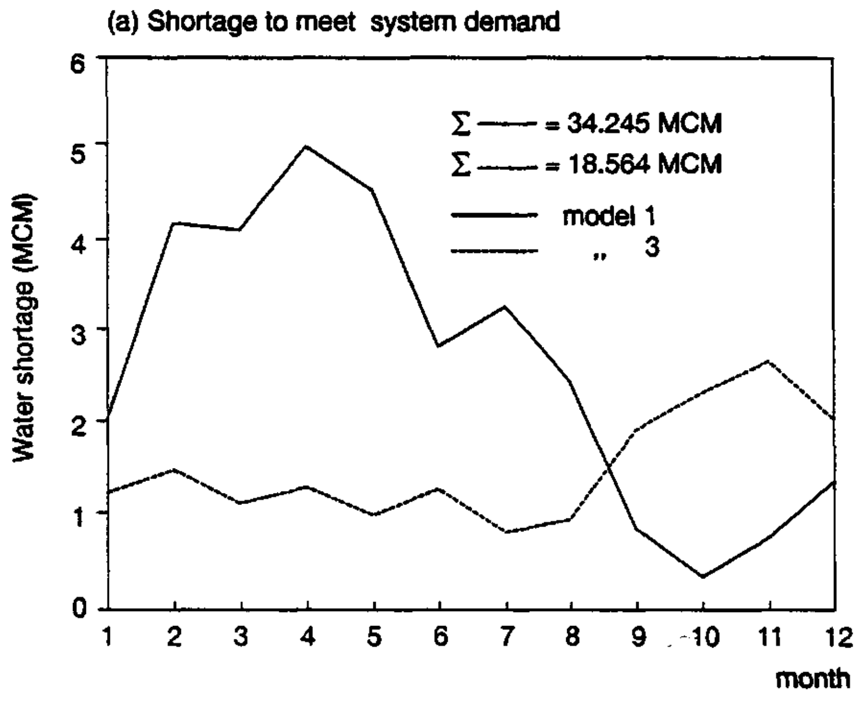


Figure 6.5 Quantity-based Water Shortage Results from Experiment 6.5

(ii) When the imperfect inflow forecast is used (as is the case in real time operation) as a guidance to implement the derived policy (Experiment 6.5), the system operation according to the release-based policy satisfies the water requirement much better than that according to the storage-based policy.

Table 6.10 Simulated Performance (Experiment 6.4 and 6.5)
(Percentages of the months corresponding to Different Levels of Local Demand Fulfilment)

Simulation Results from Experiment 6.4													
Model-1													
month	1	2	3	4	5	6	7	8	9	10	11	12	whole
100%	93.2	90.9	86.4	84.1	59.1	86.4	86.4	100.0	97.7	93.2	93.2	93.2	88.6
80% or more	93.2	90.9	86.4	86.4	61.4	93.2	90.9	100.0	97.7	93.2	93.2	93.2	90.0
60% or more	93.2	90.9	88.6	90.9	65.9	97.7	95.5	100.0	97.7	93.2	93.2	93.2	91.7
40% or more	95.5	90.9	90.9	93.2	72.7	97.7	100.0	100.0	100.0	93.2	93.2	93.2	93.4
more than zero	95.5	93.2	95.5	95.5	93.2	100.0	100.0	100.0	100.0	93.2	93.2	93.2	96.0
zero	4.5	6.8	4.5	4.5	6.8	0.0	0.0	0.0	0.0	6.8	6.8	6.8	4.0
Model-3													
month	1	2	3	4	5	6	7	8	9	10	11	12	whole
100%	86.4	81.8	97.7	95.5	95.5	86.4	97.7	100.0	97.7	86.4	90.9	95.5	92.6
80% or more	86.4	81.8	100.0	97.7	97.7	88.6	97.7	100.0	97.7	86.4	90.9	95.5	93.4
60% or more	88.6	81.8	100.0	100.0	97.7	100.0	97.7	100.0	100.0	86.4	90.9	95.5	94.9
40% or more	88.6	84.1	100.0	100.0	97.7	100.0	100.0	100.0	100.0	88.6	90.9	95.5	95.5
more than zero	88.6	88.6	100.0	100.0	100.0	100.0	100.0	100.0	100.0	90.9	93.2	95.5	96.4
zero	11.4	11.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	9.1	6.8	4.5	3.6
Simulation Results from Experiment 6.4													
Model-1													
month	1	2	3	4	5	6	7	8	9	10	11	12	whole
100%	81.8	61.4	56.8	52.3	61.4	75.0	70.5	81.8	100.0	97.7	97.7	95.5	77.7
80% or more	81.8	61.4	56.8	52.3	63.6	77.3	70.5	84.1	100.0	100.0	97.7	95.5	78.4
60% or more	81.8	61.4	56.8	54.5	65.9	79.5	75.0	86.4	100.0	100.0	97.7	95.5	79.5
40% or more	81.8	61.4	63.6	56.8	68.2	81.8	79.5	88.6	100.0	100.0	97.7	95.5	81.3
more than zero	81.8	63.6	63.6	56.8	68.2	81.8	81.8	93.2	100.0	100.0	97.7	95.5	82.0
zero	18.2	36.4	36.4	43.2	31.8	18.2	18.2	6.8	0.0	0.0	2.3	4.5	18.0
Model-3													
month	1	2	3	4	5	6	7	8	9	10	11	12	whole
100%	86.4	90.9	90.9	90.9	93.2	93.2	97.7	100.0	90.9	88.6	84.1	93.2	91.7
80% or more	86.4	90.9	93.2	90.9	93.2	93.2	97.7	100.0	90.9	88.6	84.1	93.2	91.9
60% or more	88.6	90.9	95.5	93.2	95.5	100.0	97.7	100.0	90.9	88.6	84.1	93.2	93.2
40% or more	88.6	90.9	95.5	97.7	97.7	100.0	100.0	100.0	90.9	90.9	86.4	93.2	94.3
more than zero	88.6	95.5	95.5	100.0	100.0	100.0	100.0	100.0	93.2	93.2	86.4	93.2	95.5
zero	11.4	4.5	4.5	0.0	0.0	0.0	0.0	0.0	6.8	6.8	13.6	6.8	4.5

The intuitive explanation for the results obtained is the feature of "stability" in the release based policy tables. In those tables, more than 80% of the release decisions are equal to the amount of water demand for the period. This feature of "stability" makes the release-based policy insensitive to possible errors in inflow estimation.

The underlying reason can be found by looking into the details of the system operation process (which goes from period t to period $t+1$). When the policy is defined as release, the operation aims directly at target release and the amount of water, which is the result of the error in the inflow estimation during period t will be left in reservoir storage. In the next time step $t+1$, the system will automatically adjust its operation according to the resulting storage level from the last time period t (since the decisions for period $t+1$ are based on the storage levels at end of period t). In this way the non-optimal operation in the current time step t will be amended with future operations in time $t+1$, $t+2$, . . etc., . Thus the error in inflow estimation does not greatly disturb the system operation in the current periods nor the system operation on the long run. Whereas if the policy is defined as targeted storage, the operation aims at the "ideal" storage and the amount of water, which is the result of the error in the inflow estimation during the operation period t will be released. If the forecast inflow is larger than the actual inflow, for example, the selected targeted final storage may result in water shortage; and if the forecasted inflow is smaller than the actual inflow, the selected targeted final storage may result in a wasted release. In the next time step $t+1$ the system operation continues according to the final storage level from time step t. Nothing can be done in future time periods to amend what was done wrong during time step t.

The case study in the present section verifies the phenomena observed in the Kariba reservoir case study, regarding the choice of decision variables. The different choice of decision variables in the SDP model has a considerable influence on the suitability of the model to the optimized system. Choosing the variable (either release or storage) that is directly related to the objective of the optimization will largely reinforce the power of the SDP model.

7 Inflow Serial Correlation Assumptions

Serial correlation or autocorrelation means that the value of the stochastic variable under consideration at one time period is correlated with the values of the stochastic variable at earlier time periods. The correlation between an observation at certain time period with an observation k time periods earlier is called the k^{th} order serial correlation. It is denoted by correlation coefficient $\rho(k)$ (see Section 3.1.5).

The serial assumption for the stochastic inflow sequences is an important issue in reservoir operation optimization. In SDP models, serial correlation assumption is used to describe the inflow sequence. The stochastic nature of inflow sequence is a generally observed fact. However, the choice of the serial correlation assumption is an unsolved controversy in literature of reservoir operation optimization.

When SDP model was first introduced into reservoir operation, the inflow sequence was assumed as a Markov-I process. Later, the independence assumption has also been used in SDP models. The Markov-I assumption is more popular than the independence assumption. But supporters of the use of each of these assumptions have presented arguments to favour one above the other. These arguments are often supported by experimental results on the reservoir operation optimization either for different real problems or with different SDP model setups (e.g., different discretization, different decision or state variables, and different objectives, etc.; See discussions in Chapter 5, 6 and 8). Thus the results are not often comparable. The discussion, on which inflow assumption is the best, remains undecided.

The purpose of this chapter is to study the influence of different inflow serial correlation assumptions on the performance of the SDP models. To obtain an overview of the problem, besides models with the Markov-I and the independence assumptions, another two models have also been developed. One model considers the serial correlation one step further than the Markov-I assumption: SDP model with Markov-II assumption. The other model interprets the inflow process even simpler than the independence assumption does: the model with the assumption that the inflow is deterministic.

This chapter is organized as follows. Initially, the four serial correlation assumptions are described and their modelling complexities are analyzed. Next, the issue about the best serial correlation assumption is discussed and the topics of the current study are identified. It is followed by several case studies (experiments) carried out to investigate these topics. The conclusions with respect to the use of serial correlation assumption in SDP models are given in Chapter 9.

7.1 The Four Serial Correlation Assumptions and Their Modelling Complexities

It has been known for long that many river flow time series exhibit serial correlation. That is, high flows follow high flows and low flows follow low flows. This phenomena is particularly evident for short time intervals. Annual and seasonal flows (the total flow amount of an entire period) are seldom highly correlated, while monthly, weekly, and especially daily, hourly flows generally exhibit high serial correlations. The "high" is in the sense of both the value of lag k and the value of correlation coefficient $\rho(k)$.

For the application of SDP model in the optimization of reservoir operation, the inflow serial correlation is interpreted by transition probabilities. These transition probabilities are coupled with recursive relation of Dynamic Programming to derive expectation-oriented optimal values. The formulations for the SDP models with Markov-II, Markov-I, independence and deterministic inflow serial correlation assumptions are given in this section. The modelling and computational complexities of these models are analyzed. In all four models, the present inflow is selected as the inflow state variable.

The Markov-II inflow process assumption

The Markov-II assumption takes lag-two and lag-one serial correlations of the inflow process into consideration. The transition probability of the Markov-II process can be characterized as:

$$P_{t+1}(Q_{t+1}|Q_t,Q_{t-1},Q_{t-2},\ldots) = P_{t+1}(Q_{t+1}|Q_t,Q_{t-1}) \tag{7.1}$$

The SDP model with Markov-II assumption can be formulated with the following recursive relation:

$$f_t^n(S_t,Q_t,Q_{t-1}) = \underset{D_t}{opt}[B_t(S_t,Q_t,D_t) + \sum_{Q_{t+1}} P_{t+1}(Q_{t+1}/Q_t,Q_{t-1}) * f_{t+1}^{n-1}(S_{t+1},Q_{t+1},Q_t)] \quad \forall S_t,Q_t,D_t \; feas. \tag{7.2}$$

subject to,

$$R_t = S_t + Q_t - S_{t+1} - SP_t - E_t$$

$$f_T^1(S_t,Q_t,Q_{t-1}) = \underset{D_t}{opt}[B_t(S_t,Q_t,D_t)]$$

Where, t, S_t, Q_t, R_t, E_t and SP_t are defined as in Equation 3.12; n is total number of time periods passed, $n=1,2,\ldots$; D_t is decision variable, can either be R_t or S_{t+1}; $B_t(S_t,Q_t,D_t)$ is increment of objective value for the transition state when the decision is D_t in the end of period t starting from initial storage S_t and having Q_t inflow during the period; $f_t^n(S_t,Q_t,Q_{t-1})$ is (sub) optimal value of the recursive equation at stage n (period t) as function of S_t, Q_t and Q_{t-1}; $P_{t+1}(Q_{t+1}/Q_t,Q_{t-1})$ is transition probability of inflow in class Q_{t+1} in period $t+1$, given the inflow classes are Q_t and Q_{t-1} in period t and t-1 respectively; the relation between time notations and variables are as shown in Figure 3.3.

The transition probabilities, $P_{t+1}(Q_{t+1}/Q_t,Q_{t-1})$ of a Markov-II process can be represented as a three-dimensional array. Figure 7.1 shows a graphical illustration of such a three-dimensional array.

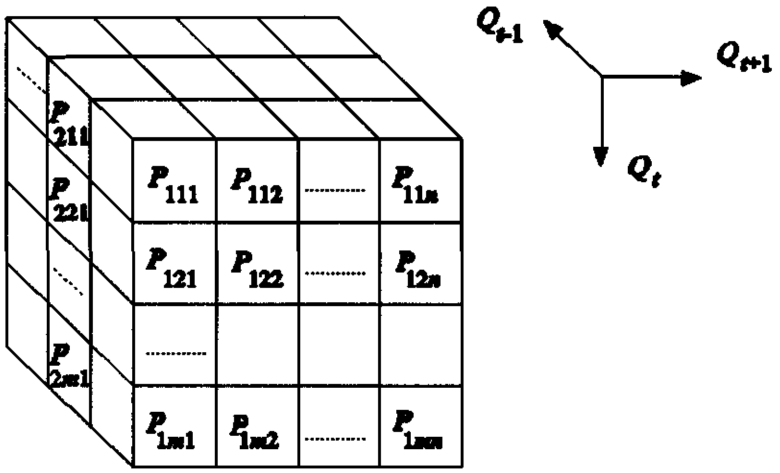


Figure 7.1 Graphical Illustration of the Three-dimensional (Markov-II) Transition Probabilities

The Markov-I inflow process assumption

The Markov-I assumption takes the lag-one (first order) serial correlation of inflow process into consideration. The transition probability of the Markov-I process can be expressed as:

$$P_{t+1}(Q_{t+1}|Q_t,Q_{t-1},Q_{t-2},....) = P_{t+1}(Q_{t+1}|Q_t) \tag{7.3}$$

The SDP model with Markov-I assumption can be formulated with the following recursive relation:

$$f_t^n(S_t,Q_t)=\underset{D_t}{opt}[B_t(S_t,Q_t,D_t)+\sum_{Q_{t+1}} P_{t+1}(Q_{t+1}/Q_t)*f_{t+1}^{n-1}(S_{t+1},Q_{t+1})] \quad \forall S_t,Q_t,D_t\ feas. \tag{7.4}$$

subject to,

$$R_t = S_t + Q_t - S_{t+1} - SP_t - E_t$$

$$f_T^1(S_t,Q_t) = \underset{D_t}{opt}[B_t(S_t,Q_t,D_t)]$$

Where, t, S_t, Q_t, R_t, E_t and SP_t are defined as in Equation 3.12; D_t, $B_t(S_t,Q_t,D_t)$ and n are as defined in Equation 7.2; $f_t^n(S_t,Q_t)$ is (sub) optimal value of the recursive equation at stage n (period t) as function of S_t and Q_t; $P_{t+1}(Q_{t+1}/Q_t)$ is Markov transition probability of inflow in period $t+1$ is in class Q_{t+1}, Given the inflow class is Q_t in period t; the relation between time notations and variables are as shown in Figure 3.3.

The transition probabilities, $P_{t+1}(Q_{t+1}/Q_t)$ of a Markov-I assumption can be represented as a two-dimensional array as shown in Figure 7.2. It can be considered as a special case of the transition probabilities of a Markov-II process, that each layer along the axis of Q_{t-2} has the unique probability distributions.

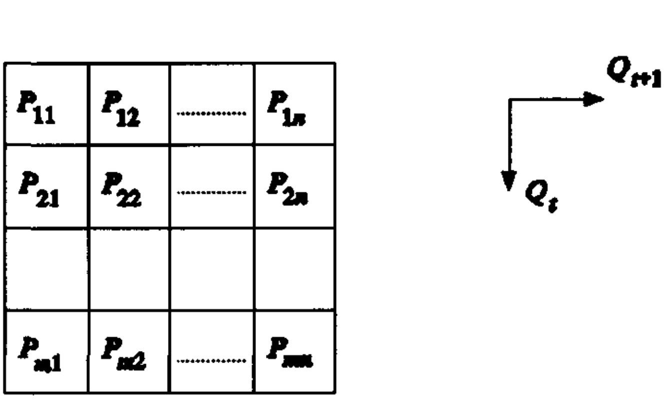


Figure 7.2 Graphical Illustration of the Two-dimensional (Markov-I) Transition Probabilities

The independence or random inflow process assumption

The inflow process is considered as exhibiting no serial correlation with inflows of the previous time periods with the independence or random inflow process assumption. That is the probability $P_{t+1}(Q_{t+1})$ is independent of the previous inflow station:

$$P_{t+1}(Q_{t+1}|Q_t,Q_{t-1},Q_{t-2},....) = P_{t+1}(Q_{t+1}) \tag{7.5}$$

The SDP model with independence inflow assumption can be formulated with the following relation:

$$f_t^n(S_t,Q_t)=\underset{D_t}{opt}[B_t(S_t,Q_t,D_t)+\sum_{Q_{t+1}} P_{t+1}(Q_{t+1})*f_{t+1}^{n-1}(S_{t+1},Q_{t+1})] \quad \forall S_t,Q_t,D_t\ feas. \tag{7.6}$$

subject to,

$$R_t = S_t + Q_t - S_{t+1} - SP_t - E_t$$

$$f_T^1(S_t,Q_t) = \underset{D_t}{opt}[B_t(S_t,Q_t,D_t)]$$

Where, t, S_t, Q_t, R_t, E_t and SP_t are defined as in Equation 3.12; D_t, $B_t(S_t,Q_t,D_t)$ and n are as defined in Equation 7.2; $f_t^n(S_t,Q_t)$ is the same as in Equation 7.4; $P_{t+1}(Q_{t+1})$ is the probability of the inflow in period $t+1$ is in class Q_{t+1}; the relation between time notations and variables are as shown in Figure 3.3.

The transition probabilities resulted from independence inflow assumption can be represented as a one-dimensional array as shown in Figure 7.3. It can be considered as a special case of the transition probabilities of the Markov-I assumption, that each line along the axis of Q_{t-1} has the unique probability distribution.

Figure 7.3 Graphical Illustration of the One-dimensional (independence) Probabilities

The deterministic inflow process assumption

The deterministic assumption assumes that there is a predetermined inflow Q_t for each time period t, thus:

$$P_{t+1}(Q_{t+1}|Q_t,Q_{t-1},Q_{t-2},\ldots) = P_t(\bar{Q}_t) = 1.0 \tag{7.7}$$

The mean inflow value of a time period t (e.g., the average value of inflows of the month over a number of years) is used as Q_t.

The (deterministic) model deterministic inflow assumption can be expressed in the following recursive relation:

$$f_t^n(S_t)=\underset{D_t}{opt}[B_t(S_t,Q_t,D_t)+*f_{t+1}^{n-1}(S_{t+1})] \quad \forall S_t,Q_t,D_t\ feas. \tag{7.8}$$

subject to,

$$R_t = S_t + Q_t - S_{t+1} - SP_t - E_t$$

$$f_T^1(S_t) = \underset{D_t}{opt}[B_t(S_t,Q_t,D_t)]$$

Where, t, S_t, Q_t, R_t, E_t and SP_t are defined as in Equation 3.12; D_t, $B_t(S_t,Q_t,D_t)$ and n are as defined in Equation 7.2; $f_t^n(S_t)$ is (sub) optimal value of the recursive equation at stage n (period t) as function of S_t; the relation between time notations and variables are as shown in Figure 3.3.

The deterministic assumption may be considered as a special case of the independence inflow process assumption. In that case the inflow of each time period is discretized into only one class and thus the occurrence probability is 1.0.

The complexities involved in the modelling of these four types of models will be discussed and compared with each other.

First, consider the Markov-II assumption. From Figure 7.1 it can be seen that if the inflow is divided into m classes for each time period, then the total number of transition probabilities is equal to m^3. These m^3 transition probabilities of the inflow process have to be estimated from historical inflow data. To estimate such a large amount of transition probabilities (parameters), not only many calculations have to be performed, but more importantly a large estimation error may occur on the transition probabilities. To see this, let us consider the way the transition probability $P_t(Q_t/Q_{t-1},Q_{t-2})$ is calculated:

$P_t(Q_t/Q_{t-1},Q_{t-2})$ = occurrence frequency of Q_t at time period t given the inflows are Q_{t-1} and Q_{t-2} at time period t-1 and t-2 respectively.

If the total number of available historical observation data is not very large, say less than $1/2*m^3$, then most of the transition probabilities will be zero. Because there are simply not enough occurrences. Thus it requires a large number of historical observation data to obtain a reasonably accurate estimation of the transition probabilities. This is however a big difficulty, because historical inflow data covering more than 30 or 40 years are hardly available. Also, consider the computational complexity of a SDP model with the Markov-II assumption. Equation 7.1 indicates that the number of evaluations is proportional to m^3.

Second, consider the Markov-I assumption. From Figure 7.2 it is clear that if inflow is divided into m classes, then the total number of transition probabilities is equal to m^2. Thus the number of transition probabilities to be estimated is a factor of m less than that with the Markov-II assumption. Consequently, with the same amount of historical inflow data, the estimation of the transition probabilities will be more accurate than the estimation of the Markov-II transition probabilities. Furthermore, from Equation 7.3, it can be seen that the number of evaluations is proportional to m^2. Thus, the computational complexity of a SDP model with the Markov-I assumption is a factor m less than that of a SDP model with the Markov-II assumption.

Similar analysis can be made for the SDP model with the independence inflow assumption and (deterministic) model with the deterministic inflow assumption respectively. The number of "transition" probabilities to be estimated is m for independence assumption (Equation 7.5) and is 1 for deterministic inflow assumption (Equation 7.7).

7.2 The Best Serial Correlation Assumption

The Markov-I assumption has been adopted to model the inflow by most of the researchers as mentioned at the beginning of this chapter. The independence inflow process assumption has also been used by some researchers (Su and Deininger, 1974; Laabs and Harboe, 1988; Huang *et al.*, 1991). However, the simpler model with the independence inflow process assumption has never enjoyed the same popularity that the Markov-I assumption has in the application of SDP in reservoir operation. The independence assumption was criticized as too simple to describe the stochasticity of inflow time series accurately. However, this argument as not sufficient to decide which inflow assumption should be used in the SDP models. In fact, it is important to have an inflow assumption that reflects the real nature of the inflows. It is also very important not to make the model unnecessarily complicated. The best model should be the simplest model, which still "sufficiently" reflects the reality. The term "sufficient" depends on the application. For SDP models applied to reservoir operation optimization, a model is sufficiently good if it produces approximately the same performance as the best (complicated) models.

The modelling and computational complexity of SDP models decrease with a factor m from Markov-II, Markov-I, independence inflow to deterministic inflow assumption as shown in Section 7.1.

With respect to the errors in the SDP model caused by the inflow assumption, two types of errors can be distinguished. One is when the model is too simple to describe those properties of the natural phenomena that are important for the decision. The other is when the set of historical samples is too small. In Section 7.1, it has been shown that the error caused by a small sample set increases as the complexity of the model increases (the number of parameters to be estimated increases). The difficulty in making a good choice among the serial correlation assumptions is mainly due to the difficulty in determining the appropriate interchange between these two types of errors.

The question about which serial correlation assumption gives the best SDP model cannot be easily answered. It depends on many factors, and there may even not be a single best SDP model. It depends on the situation for which the SDP models are applied. Therefore, this problem is further studied through several case studies.

The SDP models have been mainly applied in reservoir operations for their long-term management. The time period usually is a month or half a month or so. For this level of time it is not uncommon to have systems with low serial correlation for many time periods within a year. To illustrate, the monthly serial correlation coefficients of the three available inflow time series from the systems Kariba, Victoria/Randenigala and Joumine are listed in Table 7.1.

Table 7.1 Serial Correlation Coefficients of the Three Case Study Systems

	Kariba		Victoria	Randenigala	Joumine
Month	lag-one	lag-two	lag-one	lag-one	lag-one
1	0.36	0.07	0.22	0.22	0.08
2	0.39	0.22	0.29	0.07	0.03
3	0.10	0.42	0.50	0.36	0.29
4	0.52	0.11	0.39	0.21	0.20
5	0.65	0.45	0.51	0.53	0.31
6	0.53	0.55	0.68	0.55	0.11
7	0.89	0.31	0.31	0.44	0.04
8	0.92	0.82	0.55	0.49	0.55
9	0.93	0.79	0.19	0.36	0.08
10	0.97	0.97	0.07	0.11	0.28
11	0.94	0.88	0.01	0.05	0.32
12	0.79	0.77	0.54	0.33	0.04

From the serial correlation coefficients in Table 7.1, the following facts can be observed. For many months, the inflow series of the Kariba system is highly serially correlated (both lag-one and lag-two). Inflow series of the Mahaweli (Victoria/Randenigala) system can be considered as moderately serially correlated. The serial correlation of inflow time series of the Joumine system is low for almost all the months.

In Section 7.3, case studies will be carried out to investigate the pros and cons of the four serial correlation assumptions described in Section 7.1. The study will be carried out with the three reservoir systems, Kariba, Victoria/Randenigala and Joumine.

7.3 Computer-Experiments

In this section six experiments have been carried out. The detailed design and result of each of the experiments will be presented from Section 7.3.1 to 7.3.6. Table 7.2 gives a summary of the six experiments.

Table 7.2 The Key Points of the Design of Experiments 7.1 to 7.6

	Experiment 7.1	Experiment 7.2	Experiment 7.3	Experiment 7.4	Experiment 7.5	Experiment 7.6
Study case	Kariba	Kariba	Kariba	Kariba	Victoria/Randenigala	Joumine
Inflow assumption of the models	1)Markov-II 2)Markov-I	1)Markov-II 2)Markov-I 3)Independent 4)Deterministic	Markov-I if 1)0.0, 2)0.5 3)0.75, 4)0.9 5)1.0 otherwise independent	1)Markov-I 2)Independent	1)Markov-I 2)Independent	1)Markov-I 2)Independent
Decision variable	release	final storage	final storage	final storage	final storage	final storage
State variable	present inflow, initial storage	present inflow, initial storage	present inflow, initial storage	present inflow, initial storage	present inflow, initial storage	present inflow, initial storage
Objective	max. expected annual energy	max. expected annual energy	max. expected annual energy	max. expected annual energy	max. expected annual energy	min. expected annual energy
Constraint	-	-	-	-	irrigation demand	-
Simulation	according to policy based on perfect forecast	according to policy based on perfect forecast	according to policy based on perfect forecast	according to policy based on imperfect forecast	according to policy based on perfect forecast	according to policy based on perfect forecast

7.3.1 Kariba System Operation Based on the Models with Release as Decision Variable (Experiment 7.1)

In this experiment the Kariba reservoir, which has relatively high inflow serial correlation coefficients (see Table 7.1) is selected as the study case.

This experiment aims to screen the performance of the SDP models with the four different inflow assumptions: Markov-II (Equation 7.2), Markov-I (Equation 7.4), independence (Equation 7.6) and deterministic (Equation 7.8). Based on the conclusion of Chapter 6, the release (which is more directly related to the objective of energy generation) is selected as the decision variable. The decision variable D_t in Equation 7.2, 7.4, 7.6 and 7.8 can now be specified as R_t.

In this experiment the structure of the SDP model is similar to the model described in Section 6.2 (Equation 6.4), except the different inflow assumptions. The stage is the time step, which is one month. The objective is to maximize the expected annual energy generation. The optimization is subjected to the physical constraints of the reservoir system (e.g., storage constraints, release constraints, etc.). The reservoir storage is discretized into 42 classes with equal size. The release levels upto twice the monthly release capacities of the penstocks are to be optimized. They are discretized into 6 classes with equal size. For the three stochastic inflow assumptions the monthly inflows are discretized into varying numbers of classes (from 2 to 8, according to the discretization range of the 24 years historical monthly inflow; 1961-1984) with equal occupancy frequencies. For the deterministic inflow assumption the monthly inflows are "discretized" into one class. The median of each inflow class is the representative value of that class. With the derived SDP based optimal operation policies, the performances of the reservoir system were simulated with the 12 years (1973-1984) historical inflow time series. Perfect forecasting is assumed to be available at the beginning of each time period. The simulations "strictly" rely on the derived optimal operation policies, as long as the physical constraints of the reservoir system are not violated. Table 7.3 presents the simulated average annual performance indices from Experiment 7.1.

Table 7.3 Simulated Average Annual Performance (Experiment 7.1)

	Markov-II	Markov-I	Independent	Deterministic
Indices referring to energy as % of power capacity:				
(1) expected mean annual energy output	91.3%	91.7%	96.6%	95.6%
(2) simulated average annual energy output	91.0%	91.0%	90.6%	90.0%
(3) standard deviation of (2)	8.3%	8.3%	10.0%	9.4%
(4) 95% confidence interval for mean energy	(86.5%, 95.5%)	(86.5%, 95.5%)	(85.2%, 96.0%)	(84.9%, 95.1%)
(5) minimum annual energy output	73.7%	73.3%	69.6%	70.0%
Indices referring to storage as % of reservoir capacity:				
(6) average utilized storage	68.1%	68.0%	65.2%	67.1%
(7) standard deviation of (6)	27.7%	28.1%	29.4%	30.0%
(8) minimum drawdown	11.9%	10.6%	8.6%	10.8%
Indices referring to release as % of annual inflow:				
(9) average annual release	70.3%	70.3%	70.3%	69.6%
(10)standard deviation of (9)	4.7%	4.7%	6.0%	5.5%
(11)minimum annual release	60.7%	57.6%	57.6%	57.7%

Same as in Section 6.2, the simulated performance is presented according to the following three aspects: (a) the energy generations; (b) the reservoir storages; and (c) the releases through turbine. For each of the three performance indices, the simulated mean, the standard deviation and the minimum value are presented. For the energy generation, the expected annual gain obtained from the SDP based optimization is also presented. The so called "expected annual energy output" itself does not tell much about the real performance of the system. However, the difference between the expected value (item 1 in Table 7.3) and the simulated value (item 2) exhibits how far is the optimization model from the real nature of the problem being optimized.

According to the results, although the ways of inflow been considered differ very much for the four optimization models, the simulated performance based on the four derived "optimal" policies differ very little. For example, for the objective value of annual energy output (item 2), the smallest simulated energy output from deterministic inflow assumption is only 1% less than largest output from Markov-II and Markov-I assumptions.

The present result can be understood by recalling the conclusion of Chapter 6 regarding the influence of the decision variable of SDP. When the variable directly related to the objective of the optimization is selected as decision variable, the model becomes insensitive to the way how the inflow is considered. In this situation, the simpler models (either SDP model with independence inflow or even deterministic model based on mean value of inflow) would almost perform as well as the more complicated models that consider inflow serial correlations.

7.3.2 Kariba System Operation Based on the Models with Storage as Decision Variable (Experiment 7.2)

This experiment also aims to screen the performance of the SDP models with the four different inflow assumptions: Markov-II (Equation 7.2), Markov-I (Equation 7.4), independence (Equation 7.6) and deterministic (Equation 7.8). Same as in Experiment 7.1, the Kariba reservoir is selected as case study in this experiment. The final storage will be chosen to be the decision variable in the present experiment. Therefore, the decision variable D_t in Equation 7.2, 7.4, 7.6 and 7.8 can now be specified as S_{t+1}. The balance setup of the SDP models and the way of subsequent simulations remain the same as in Experiment 7.1.

Table 7.4 presents the simulated average annual performance indices from Experiment 7.2. Similar to Table 7.3, they are presented according to the following three aspects: (a) the energy generations; (b) the reservoir storages; and (c) the releases through turbine.

From the result of the present experiment the influence of SDP model with different inflow assumptions can be much better detected than from Experiment 7.1.

First, the policy derived from the model with deterministic inflow assumption leads to a considerably worse performance of the reservoir system as compared with that from the models with stochastic inflow assumption. For example consider the indices for energy. The simulated mean annual energy output (item 2) resulted from the model with deterministic inflow assumption about 12% less than that of the models with stochastic inflow assumptions. The fluctuation or standard deviation of annual energy generation (item 3) is almost 50% more than that of the other three models. The firm annual energy output (item 4) is about one quarter less than that of the other three models.

This result clearly illustrates the drawback of the model with deterministic inflow assumption in deriving reservoir operation policies. The deterministic model based on mean value of inflows seems too simple to represent the nature of reservoir inflows sufficiently. In general, a deterministic model is probably a good tool to screen the best performance a system could have if the historical inflow observations (which are already known) repeat in the future. If the issue is to derive reservoir operation policies, the deterministic model probably may function better within the framework of the so called "implicit" type stochastic approach (see Section 1.2 regarding the "implicit" stochastic approach).

Table 7.4 Simulated Average Annual Performance (Experiment 7.2)

	Markov-II	Markov-I	Independent	Deterministic
Indices referring to energy as % of power capacity:				
(1) expected mean annual energy output	93.0%	93.7%	99.9%	99.2%
(2) simulated average annual energy output	90.9%	90.8%	90.9%	80.1%
(3) standard deviation of (2)	11.3%	11.4%	12.9%	17.8%
(4) 95% confidence interval for mean energy	(84.8%, 97.0%)	(84.6%, 97.0%)	(83.9%, 97.9%)	(70.5%, 89.7%)
(5) minimum annual energy output	62.2%	62.0%	58.4%	48.7%
Indices referring to storage as % of reservoir capacity:				
(6) average utilized storage	62.4%	63.3%	62.0%	52.9%
(7) standard deviation of (6)	29.5%	29.8%	30.9%	6.0%
(8) minimum drawdown	8.2%	7.0%	3.1%	43.7%
Indices referring to release as % of annual inflow				
(9) average annual release	70.7%	70.7%	70.7%	63.4%
(10)standard deviation of (9)	7.3%	7.3%	8.5%	14.5%
(11)minimum annual release	51.2%	51.3%	48.6%	38.2%

Second, the policy derived from the SDP model with the Markov-II assumption does not show much improvement in the reservoir system performance as compared with that of the model with the Markov-I assumption. Compare the two columns of performance indices corresponding to the Markov-II and Markov-I assumptions. It seems that except the unimportant indices referring the reservoir minimum drawdown (item 7), the difference of all the indices are less than 1%.

This result, as we have expected, indicates that the improvement resulted from the model with Markov-II assumption does not justify the additional complication of the model. Generally, it can be concluded that the SDP models with the second or higher order inflow serial correlation assumption are not practical. The difficulty lies in the estimation of the three-dimensional inflow transitional probabilities. The availability of observed data of inflow time series for over 40 years is scarce. Such a limited length of historical inflow time series is bound to result in considerable errors in the estimation of three-dimensional inflow transition probabilities. Those errors may significantly diminish the merit of the Markov-II based model even when it better reflects the characteristics of the inflow.

When comparing the performance between Markov-I and independence inflow assumption, a firm conclusion is somehow difficult to draw based on the present results. The objective value of mean annual energy outputs (item 2) resulted from both assumptions are almost the same. The model with the Markov-I assumption leads the system to slightly better performance in the sense of standard deviation of annual energy output (item 3) and firm annual energy output (item 4). However, as compared with the model with deterministic inflow assumption, the model with independence assumption does not vary considerably from the model with Markov-I assumption.

7.3.3 Kariba System Operation Based on the Models with Markov-I and Independence Assumption (Experiment 7.3)

The present experiment aims to obtain additional insight into the performance of the SDP models with the Markov-I and independence assumptions. The reservoir system in consideration is still the Kariba system.

Referring to Table 7.1, it can be noticed that the correlation coefficients of Kariba are high for some months and low for the rest. The idea of using a model with Markov-I assumption for the months with high correlation coefficient and independence assumption for the months with low correlation coefficient may be a reasonable choice for such a system. This type of model can both reflect the serial correlation of the inflow time series for the months when the serial correlation is high. It also can avoid the unnecessary additional parameter estimation errors for the months when the serial correlation is low.

To, investigate this idea, five models are set up in the present experiment. For each model a critical point (cp) is defined. They are (a) 0.0, (b) 0.5, (c) 0.75, (d) 0.9 and (e) 1.0, respectively. For the months with lag-one serial correlation coefficients are larger than or equal to the critical point, the Markov-I transition probabilities will be coupled into the recursive relation of SDP model (Equation 7.4). For the months with lag-one serial correlation coefficients are smaller than the critical point, the independent probabilities will be coupled into the recursive relation of SDP model (Equation 7.6). Consider model-c as an example. There are 6 months (from month 7 to month 12) whose lag-one correlation coefficients are larger than 0.75 (see Table 7.1). Therefore, for those 6 months the Markov-I transition probabilities and for the remaining 6 months the independent probabilities will be coupled into the recursive relation. Similarly, for model-b there will be 9 months (from month 4 to 12) with Markov-I transition probabilities and 3 months with independent probabilities. For model-d there will be 4 months (from month 8 to 11) with Markov-I transition probabilities and 8 months with independent probabilities. The model-a with 0.0 as critical point is the model with Markov-I assumption for all the 12 months in a year. The model-e with 1.0 as critical point is the model with independence assumption for all the 12 months in a year.

The balance set up of the SDP models and the procedure adopted in the subsequent simulations are the same as in Experiment 7.2. Table 7.5 presents the simulated annual performance indices from Experiment 7.3.

Table 7.5 shows that the variation among the simulated performances resulted from the five models are very limited. For the indices to storage, gradual minor changes can be observed from the model with Markov-I inflow assumption (cp=0.0) to the model with independence assumption (cp=1.0), with the increase of the value of critical point value. For the indices to energy and release, a very small jump at the critical point 0.9 can be detected. The standard deviations (item 3 and 10) are smaller and the minimum values (item 4 and 11) are bigger at the critical point 0.9 when compared with the two neighbouring points, 0.75 and 1.0. This can be interpreted as a positive sign for the idea of using a model with Markov-I assumption for the months with high correlation coefficients and independence assumption for the months with low correlation coefficients. However, the improvement is so small to justify the additional complications involved with the model.

Table 7.5 Simulated Average Annual Performance (Experiment 7.3)

	cp=0.0 Markov-I	cp=0.50	cp=0.75	cp=0.90	cp=1.0 Indep.
Indices referring to energy as % of power capacity:					
(1) expected mean annual energy output	93.7%	93.7%	95.3%	97.3%	99.9%
(2) simulated average annual energy output	90.8%	90.8%	91.0%	91.0%	90.9%
(3) standard deviation of (2)	11.4%	11.4%	12.3%	12.1%	12.9%
(4) 95% confidence interval for mean energy	(84.6%, 97.0%)	(84.6%, 97.0%)	(84.3%, 97.7%)	(84.4%, 97.6%)	(83.9%, 97.9%)
(5) minimum annual energy output	62.0%	62.0%	58.6%	61.9%	58.4%
Indices referring to storage as % of reservoir capacity:					
(6) average utilized storage	63.3%	63.3%	62.3%	62.2%	62.0%
(7) standard deviation of (6)	29.8%	29.8%	30.2%	30.5%	30.9%
(8) minimum drawdown	7.0%	7.0%	5.8%	4.5%	3.1%
Indices referring to release as % of annual inflow:					
(9) average annual release	70.7%	70.7%	70.8%	70.8%	70.7%
(10)standard deviation of (9)	7.3%	7.3%	8.1%	7.9%	8.5%
(11)minimum annual release	51.3%	51.3%	48.4%	51.4%	48.6%

7.3.4 Kariba System Operation Based on Imperfect Forecast (Experiment 7.4)

From the results of both Experiment 7.2 and 7.3, it seems that the model with the Markov-I assumption leads the Kariba system to a slightly better performance when the perfect forecasting is available (as they have been used in the experiments). However, the policy derived from the model with Markov-I inflow assumption is likely to be more sensitive to the accuracy of inflow forecasting. In real-time operation when the inflow forecasting is not perfect, the trade-off between the two SDP models with Markov-I and independence inflow assumption may be different.

The present experiment aims to obtain insight into the performance of the SDP models with the Markov-I and independence inflow assumptions, when the inflow forecasting is not perfect during operation simulation. The reservoir system in consideration is still the Kariba system.

The setting up of the SDP models are the same as the two models (Markov-I and Independence) in Experiment 7.3. The way of subsequent simulations differs from Experiment 7.3. The derived optimal policies are implemented at the beginning of each time period with forecasted inflow instead of the actual inflow. The inflows are forecasted according to the readily available regression analysis of Budhakooncharoen (1986). This has also been applied in Section 6.2 of this report. Table 7.6 presents the simulated annual performance indices from Experiment 7.4.

Table 7.6 Simulated Average Annual Performance (Experiment 7.4)

	Markov-I	Independent
Indices referring to energy as % of power capacity:		
(1) expected mean annual energy output	93.7%	99.9%
(2) simulated average annual energy output	86.6%	87.1%
(3) standard deviation of (2)	7.4%	7.4%
(4) 95% confidence interval for mean energy	(82.6%, 90.6%)	(83.1%, 91.1%)
(5) minimum annual energy output	76.5%	73.6%
Indices referring to storage as % of reservoir capacity:		
(6) average utilized storage	67.1%	66.2%
(7) standard deviation of (6)	24.2%	24.7%
(8) minimum drawdown	14.8%	14.3%
Indices referring to release as % of annual inflow:		
(9) average annual release	67.2%	67.7%
(10)standard deviation of (9)	4.6%	4.2%
(11)minimum annual release	60.4%	61.0%

Table 7.6 shows that the difference between all the simulated performance indices resulted from the two models (with Markov-I and independence assumptions) becomes even less, as compared with that when the perfect inflow forecasting is available (see Table 7.5, the models with Markov-I and independence assumptions). As for the simulated mean annual energy output (item 2), the model with independence assumption slightly outperforms the model with Markov-I assumption. As for the firm annual energy output (item 4), the model with Markov-I assumption slightly outperforms the model with independence assumption. The standard deviations of mean annual energy output (item 3) are the same for the two models. The impression obtained from Table 7.6 is that when the inflow forecasting is not perfect, it is difficult to identify the most suitable model (with Markov-I or independence assumptions) for Kariba system in the sense of energy production.

From the results of Experiments 7.2, 7.3 and 7.4, it can be observed that the model with the Markov-I assumption leads Kariba system to a slightly better performance when the perfect forecasting is available (as they have been defined in the Experiments 7.2 and 7.3). However, by considering the additional complexity of the SDP model with Markov-I assumption, the improvement does not seem to be substantial. Besides, in real-time operation when the inflow forecasting is not perfect, the small improvement of the model with Markov-I assumption will diminish. From these points of view, the SDP model with independence inflow assumption can be entitled a better suitable one than the SDP model with Markov-I assumption for a system like Kariba with high inflow serial correlation coefficients for many months but with short observed inflow data.

7.3.5 Mahaweli System Operation (Experiment 7.5)

This experiment aims to compare the model with the Markov-I assumption with the model with independence inflow assumption. In the experiment the two-unit reservoir system of Victoria/Randenigala, whose inflow time series has a low serial correlation compared with Kariba, is selected as case study.

The setup of the SDP models are similar to that described in Section 5.2, except for the different inflow assumptions. The recursive relations of the two models are defined as in Equation 7.4 and 7.6, respectively, while the D_t in the equations is final storage S_{t+1}. The stage is time step, which is one month. The objective is to maximize the expected annual energy generation subject to the constraints of satisfying downstream irrigation requirement. The 32 years of (1949-1980) available observed inflow data is used to obtain statistical parameters of the stochastic inflow. 4 inflow classes and 7 storage classes with equal size intervals have been considered for both reservoirs in cascade, thus yielding 4*4=16 inflow class combinations and 7*7=49 storage class combinations. The median of each inflow class is the representative value of that inflow class. In the subsequent simulations the historical inflow time series have been used "strictly" relying on the derived optimal operation policies, as long as the physical constraints of the reservoir system are not violated. During the simulation, perfect forecasting is assumed to be available at the beginning of each time period. Table 7.7 presents the simulated performance indices from Experiment 7.5.

Table 7.7 Simulated Performance (Experiment 7.5)

	Markov-I	Independence
Indices referring to energy as % of power capacity:		
(1) simulated average annual energy output	52.5%	52.2%
(2) standard deviation of (1)	11.6%	11.6%
(3) 95% confidence interval of mean energy	(49.0%, 56.0%)	(48.7%, 55.7%)
(4) minimum annual energy output	31.7%	28.2%
Indices referring to irrigation supply:		
(5) Time-based Reliability[1]	86.2%	86.2%
(6) Quantity-based Reliability[2]	95.9%	96.0%
(7) Repairability (month)[3]	1.57	1.47
(8) Vulnerability (MCM)[4]	60.5	56.1

1 % of time-based steps with fulfilled irrigation demand
2 % of the accumulated irrigation demand met
3 Average duration of an irrigation failure (shortage) event
4 Average accumulated irrigation shortage per failure

The simulated performance is presented in two aspects: (a) energy generation; and (b) the irrigation supply. For the energy generation, the simulated mean, the standard deviation and the minimum value are presented. As has been explained in Section 5.2, for this system the optimization does not hold for the whole set of decisions in the annual cycle due to the constraint of irrigation demand. Therefore, the expected annual energy output is not obtained. For the irrigation supply, the performance indices of reliability (both for time-based and quantity-based), repairability and vulnerability are presented.

The performance indices of energy output in Table 7.7 indicate that the model with Markov-I inflow assumption leads to slightly better system performance (e.g., larger minimum annual energy in item 4) compared with independence assumption. However, the indices of irrigation supply imply that the model with independence inflow assumption leads to slightly better system performance (e.g., smaller reparability in item 7 and vulnerability in item 8) compared with the other. When the most important indices (mean annual energy and reliability of irrigation supply) are concerned, there is hardly any difference between the two models.

The results from the experiment show that the two SDP models (with Markov-I and independence assumptions) lead Victoria/Randenigala system to almost equal utilization of the water in the reservoirs, when the perfect inflow forecasting is available. Therefore, the SDP model with independence inflow assumption can be considered better than the SDP model with Markov-I assumption due to its simplicity involved in modelling.

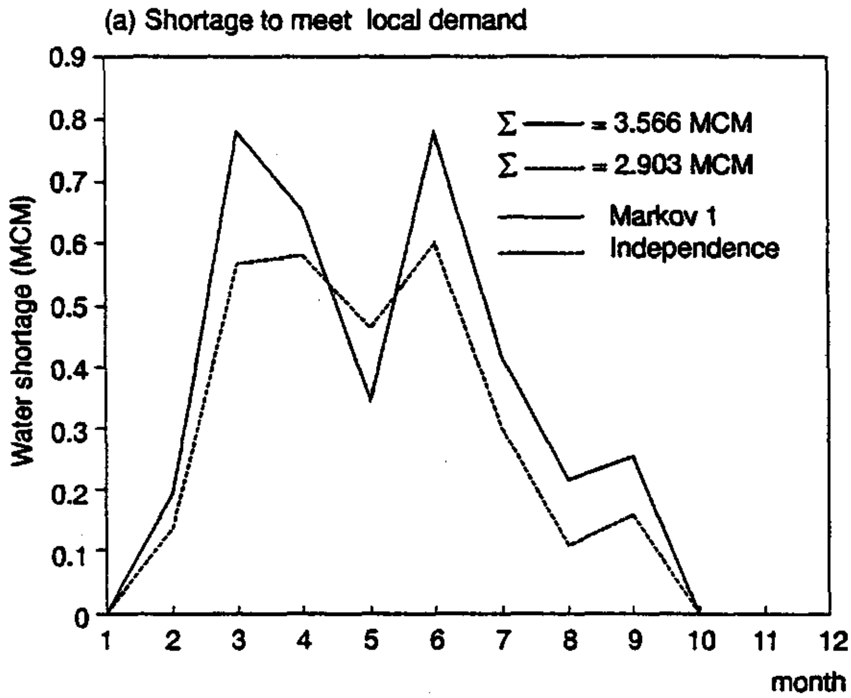
7.3.6 Joumine System Operation (Experiment 7.6)

In this experiment the comparison between the two models with the Markov-I and independence inflow assumptions will be carried out with Joumine reservoir. Its inflow time series shows very low serial correlation (see Table 7.1).

The setting up of the models are similar to the model described in Section 6.3, except for the different inflow assumptions. The recursive relation of the two models are defined as in Equations 7.4 and 7.6, while the D_t in the equations is final storage S_{t+1}. The stage is the time step, which is one month. The objective is to minimize the squared deficit from the "total demand", subject to the physical constraints of the reservoir system (e.g., storage constraints, release constraints, etc.,). The whole reservoir storage is discretized into 18 classes with equal size. The monthly inflows are discretized into varying numbers of classes (from 4 to 12, according to the distribution range of the 44 years historical monthly inflow :1946-1989) with equal intervals. The median of each inflow class is the representative value of that inflow class. In the subsequent simulation the historical inflow time series have been used "strictly" relying on the derived optimal operation policies, as long as the physical constraints of the reservoir system are not violated. During the simulation, perfect forecasting is assumed to be available at the beginning of each time period.

Figure 7.4 and Table 7.8 present the simulated performance indices from Experiment 7.6. Figure 7.4 shows the quantity-based water shortage from the "total demand" and from the "local demand". Table 7.8 presents the monthly distribution of the level of time-based fulfilment of the "local demand".

In the experiment, the SDP model with independence inflow assumption leads to a remarkably better system performance than the SDP model with Markov-I assumption. Figure 7.4 shows the amount of water shortages both from the "local demand" and from the "total demand". For both shortages, the simulated values resulted from the SDP model with Markov-I inflow assumption are higher than that from the SDP model with independence inflow assumption. As for the time-based fulfilment of the "local demand" (Table 7.8), the simulated values resulted from the SDP model with Markov-I inflow assumption are smaller than that from the model with independence assumption.



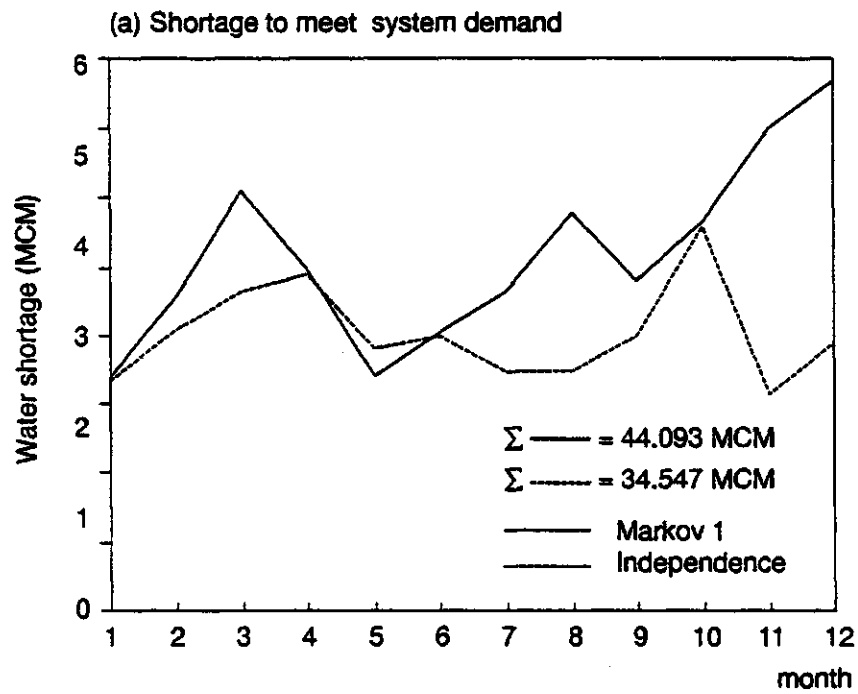


Figure 7.4 Quantity-based Water Shortage Results from Experiment 7.6

Table 7.8 Simulated Performance (Experiment 7.6)

(Percentages of the Months Corresponding to Different Levels of Demand Fulfilment)

Model with Markov-I Assumption

month	1	2	3	4	5	6	7	8	9	10	11	12	whole
100%	100.0	79.5	59.1	68.2	84.1	81.8	77.3	72.7	79.5	100.0	100.0	100.0	83.5
80% or more	100.0	81.8	61.4	75.0	88.6	81.8	81.8	81.8	81.8	100.0	100.0	100.0	86.2
60% or more	100.0	81.8	65.9	79.5	93.2	84.1	86.4	84.1	81.8	100.0	100.0	100.0	88.1
40% or more	100.0	81.8	65.9	86.4	93.2	86.4	95.5	93.2	84.1	100.0	100.0	100.0	90.5
more than zero	100.0	81.8	79.5	100.0	100.0	88.6	97.7	97.7	88.6	100.0	100.0	100.0	94.5
zero	0.0	18.2	20.5	0.0	0.0	11.4	2.3	2.3	11.4	0.0	0.0	0.0	5.5

Model with Independence Inflow Process Assumption

month	1	2	3	4	5	6	7	8	9	10	11	12	whole
100%	100.0	84.1	70.5	70.5	81.8	84.1	84.1	86.4	88.6	100.0	100.0	100.0	87.5
80% or more	100.0	86.4	72.7	77.3	86.4	84.1	84.1	90.9	88.6	100.0	100.0	100.0	89.2
60% or more	100.0	86.4	75.0	81.8	90.9	86.4	88.6	93.2	88.6	100.0	100.0	100.0	90.9
40% or more	100.0	86.4	77.3	90.9	90.9	88.6	97.7	97.7	88.6	100.0	100.0	100.0	93.2
more than zero	100.0	86.4	86.4	100.0	97.7	90.9	100.0	97.7	93.2	100.0	100.0	100.0	96.0
zero	0.0	13.6	13.6	0.0	2.3	9.1	0.0	2.3	6.8	0.0	0.0	0.0	4.0

The result from the present experiment clearly illustrates the drawback of unnecessary complication in the modelling. It can be concluded that when the inflow serial correlation coefficients are low for most of the time periods, the SDP model with independence inflow assumption should be applied.

8 Objective Functions and Performance Evaluation

Simulation studies of reservoir system operation utilizing Stochastic Dynamic Programming (SDP) based rules revealed that the simulated objective function value as an inadequate indicator to measure the impact of the selection of the objective function and constraint set. Bogardi *et al.* (1991) reported that for Mahaweli reservoir system the value of the simulated average annual energy generation varies very little when different objective functions are used in SDP models. Bogardi and Verhoef (1991) revealed that some (reliability-related) performance indices fit better the task of measuring the operational behaviour of reservoir systems upon the application of a certain release policy.

The present chapter is a continuation of the initial studies by Bogardi *et al.*, (1991). It aims to obtain more insight and systematic knowledge on the subject of objective functions and performance evaluations.

An introduction to the problems of selecting an appropriate objective function will be given in Section 8.1. The difficulties in selecting the objective function and some possible improvements will be discussed. In Section 8.2, several experiments will be set up. These experiments are designed to study the relation between the performance and various setups of the objectives. To obtain a more complete picture of the reservoir performance, besides the often used performance index of simulated objective values, some risk-related performance indices are also adopted as performance evaluation criteria. The results of these experiments will be analyzed in Section 8.3. The conclusions obtained from the results of these experiments are given in Chapter 9.

8.1 Theoretical Discussion

A major difficulty in the construction of a reservoir operation model is to define a suitable objective function. This is primarily due to (a) the stochastic nature of inflow to the reservoir, (b) the difficulty in identifying a unique (economic) measure of performance, (c) the inadequacy of expected performance criteria to reflect the typical decision maker's aversion to the poor outcome of an adopted set of policies, and (d) the multi-objective nature of reservoir operation.

Since the objective function is a function of random variables, in the SDP model it is averaged by an expectation factor to make it mathematically tractable. This leads to the computation of policies that maximize the expected average performance of the reservoir but ignores serious negative effects that are unusual, yet still probable (Loaiciga and Marino, 1986).

Generally, maximizing the expected economic performance is the commonly aimed goal in reservoir operation optimization. However, it is not easy to define the values of benefit and cost quantitatively. Therefore, some directly measurable quantities are usually substituted as objectives in optimizing reservoir operations. For example, if the purpose of a reservoir is water supply, then the objective can be to satisfy a certain demand, or to have the greatest possible average annual release, etc.

The appropriateness of the objective depends to a large extent on the anticipated system performance and system characteristics. Comparing the above two example objectives, the objective of satisfying a demand may usually be more consistent with the goal of maximizing the expected economic performance. The benefit from a function of this type has high value when there is demand and lacks value for the part exceeding the demand. Having the greatest possible annual release type of objective can find its application only in arid areas when the system is seriously short of water (i.e., demand is always larger than supply, and the critical anticipation of the system is to save every drop of incoming water). However, even in this situation, having a well-distributed release pattern, which satisfies the most essential part of the demand is still to be preferred. The appropriateness of different objective functions to a specific purpose is one issue studied in this chapter. Further study and discussion will be carried out by means of a few experiments.

If the goal of reservoir operation is to meet a specific (release) demand, the objective can be set to minimize the expected value of deficits of actual releases from target levels, for example. With such an objective function the SDP model will result in release policies that averagely satisfy the targeted releases. However, since the noncommensurate consequences of risk have not been explicitly considered during the optimization process, the derived release policies can be better trusted under normal than under extreme conditions (such as prolonged droughts).

According to the shortcomings of the expected performance criterion, there have been attempts in the direction of embedding a utility function. A utility function is defined as the numerical representation of the relationship between the set of possible decisions and the decision maker's preference (Bogardi, 1987) into the objective function to reflect the risk-attitude of a decision maker (e.g., Keeny and Wood, 1977; Loaiciga and Marino, 1986). Unfortunately, there are certain drawbacks to this methodology. In short, such a method requires a utility function that incorporates a decision maker's attitude towards risk. Identifying such a function for a single decision maker will not only be difficult, but also will not reflect the priorities of all the groups in a multiple-decision-maker system (Hashimoto *et al.*, 1982).

Another attempt to overcome the shortcomings of the expected performance criterion is to develop additional performance criteria that capture particular performance aspects that are specially important in extreme situations (Hashimoto *et al.*, 1982; Moy *et al.*, 1986; Duckestain and Plate, 1987; Bogardi and Verhoef, 1991). The addition of the risk-related performance evaluation criteria to the already used objective (with or without the risk-attitude of a decision maker) illustrating the expected performance would help the decision makers to better understand the performance of a system in the uncertain future. In the remaining sections of the present chapter, number of utility functions and risk-related performance evaluation criteria are introduced into the case study. This is carried out to investigate how they can work together to overcome the shortcomings of the expected performance criterion.

So far the discussion has been confined to the single purpose reservoir operation delivering a targeted release. In practice, most of the reservoirs serve more than one purpose and most of those purposes are competing with each other to some extent. For example, a reservoir may have the function to satisfy both energy and irrigation demands from its surrounding area. A serious difficulty in the optimization of operations of a multi-purpose reservoir is how to combine the different units and the often competing demands into the objective function.

In the present research, the background philosophy of one linear optimization technique "goal programming" is applied. The basic approach of goal programming is to establish a specific numerical goal for each of the objectives, formulate an objective function for each objective, and then seek a solution that minimizes the (weighted) sum of deviations of these objective functions from their respective goals (Hillier and Lieberman, 1990). In some situations one of the objectives of the system may not be specified in terms of a numeric goal, but as maximizing or minimizing a quantity. In these cases, one can first determine the best expected optimal value (maximum or minimum) of that objective through the corresponding single-objective optimization. This best expected optimal value can be then considered as a pseudo-system goal with respect to this objective in the multi-objective optimization.

The following three cases can be distinguished. (a) There is a hierarchy of priority levels for the goals. The goals of primary importance receive first-priority attention, those of secondary importance receive second-priority attention, and so forth (if there are more than two priority levels). (b) All the goals are of roughly comparable importance. (c) The mixture of (a) and (b), which means there are different priority levels for the goals, and for some priority levels there is more than one goal with roughly comparable importance.

One way of dealing with type (a) problem is to select the goals with lower level importance to be targeted in the objective function by minimizing the deficit from it, and to set the goals with higher level importance as constraints during the optimization. A method to deal with the type (b) problem is to measure the degree of deficiency of each goal as the percentage of the goal not being satisfied, and define the objective as minimizing the (weighted) sum of the degrees of deficiency of the system goals. This type of approach, in fact, has also the potential to deal with type (a) and (c) problems. Assigning much higher weights to the degree of deficiency of those particular goals can lead to giving higher preference for some goals over others. The mixture of the above two approaches also can deal with the type (c) problem. The interchange among the different methods of handling multiobjective problems will be another major issue of the present study. Investigations into these issues will be further carried out by means of experiments in the remaining sections of the present chapter.

8.2 Computer-Experiments

For the computer experiments in the present chapter, the Mahaweli multi-reservoir/ multi-purpose system has been selected as the case study. The detailed description of the reservoir system is presented in Section 4.1.

This reservoir system serves two major purposes: hydro-power generation and irrigation supply. There is a firm power requirement for energy generation. A 32-year record (1949-1980) of monthly irrigation requirements at Minipe has been used as the irrigation demand of the system. This record is obtained from the study of Nandalal (1986).

In accordance with the two purposes of the system, the following four types of objectives and constraints are proposed: (a) the objective function is maximizing annual energy generation under the constraints of satisfying the average irrigation demand and firm power of the system; (b) the objective function is minimizing the deficit from the firm power requirement under the constraint of satisfying the average irrigation demand of the system; (c) the objective function is minimizing the deficit from the average irrigation demand under the constraint of satisfying firm power of the system; and (d) both the requirements of the firm power and average irrigation demand are targeted simultaneously in the objective function.

In the present study, several additional variations in objective functions and constraints are developed from the above four types, to obtain a wider view on the studied subject. All together 25 models with different sets of objective functions and constraints have been tested in the study. They are listed in the Table 8.1.

Table 8.1 Combinations of Objective Functions and Constraints of the Tested Models

1	Objective :	max. E (Σ EG_t)	(8.1)
	Constraint :	$R_t \geq ID_{t,f}$	
		$EG_t \geq EG_{t,f}$	
2	Objective :	same as (8.1)	
	Constraint :	$R_t \geq ID_{t,avg}$	
3	Objective :	same as (8.1)	
	Constraint :	$EG_t \geq EG_{t,f}$	
4	Objective :	same as (8.1)	
5	Objective :	min. E (Σ DE_t)	(8.2)
	Constraint :	$R_t \geq ID_{t,avg}$	
6	Objective :	same as (8.2)	
7	Objective :	min. E (Σ DR_t)	(8.3)
	Constraint :	$EG_t \geq EG_{t,f}$	
8	Objective :	same as (8.3)	
9 - 25	Objective :	$\min.E\{\Sigma[\alpha*(DE_t/EG_{t,f})^{\gamma1}+(1-\alpha)*(DR_t/ID_{t,avg})^{\gamma2}]\}$	(8.4)

For 9	α = 1.00	$\gamma1$ = 1.0	$\gamma2$ = 1.0
For 10	α = 0.75	$\gamma1$ = 1.0	$\gamma2$ = 1.0
For 11	α = 0.50	$\gamma1$ = 1.0	$\gamma2$ = 1.0
For 12	α = 0.25	$\gamma1$ = 1.0	$\gamma2$ = 1.0
For 13	α = 0.10	$\gamma1$ = 1.0	$\gamma2$ = 1.0
For 14	α = 0.01	$\gamma1$ = 1.0	$\gamma2$ = 1.0
For 15	α = 0.00	$\gamma1$ = 1.0	$\gamma2$ = 1.0
For 16	α = 0.01	$\gamma1$ =10.0	$\gamma2$ = 1.0
For 17	α = 0.01	$\gamma1$ = 5.0	$\gamma2$ = 1.0
For 18	α = 0.01	$\gamma1$ = 2.0	$\gamma2$ = 1.0
For 19	α = 0.01	$\gamma1$ = 0.5	$\gamma2$ = 1.0
For 20	α = 0.01	$\gamma1$ = 0.1	$\gamma2$ = 1.0
For 21	α = 0.01	$\gamma1$ = 1.0	$\gamma2$ =10.0
For 22	α = 0.01	$\gamma1$ = 1.0	$\gamma2$ = 5.0
For 23	α = 0.01	$\gamma1$ = 1.0	$\gamma2$ = 2.0
For 24	α = 0.01	$\gamma1$ = 1.0	$\gamma2$ = 0.5
For 25	α = 0.01	$\gamma1$ = 1.0	$\gamma2$ = 0.1

Where, E is expectation; Σ is summation over the annual cycle of 12 months, t=1,2,.....,12; max. is maximization; min. is minimization; DE_t is energy deficit from the system firm power, which is defined as $DE_t=\max.\{0,EG_{t,f}-EG_t\}$; EG_t is total energy generation of the system during the month t (which is the sum of the energy generation of each hydro-power plant ($EG_{t,i}$) in the system: Victoria, Randenigala and Rantembe); $EG_{t,i}=\Sigma 9.81*\eta*R_{t,i}*H_{t,i}*T_t/10^9$ (GWh); $R_{t,i}$ is release from reservoir i during period t (m^3/s); $H_{t,i}=EL_{t,i}-TW_{t,i}$ (m); $EL_{t,i}$ is elevation of water level in reservoir i during period t (m); $TW_{t,i}$ is tail water level of power station of reservoir i during period t (m); T_t is length of period t in hours; η=0.75, which is overall efficiency (turbines+generators transmission); $EG_{t,f}$ is firm power required for the system during the month t (GWh); DR_t is release deficit from the system irrigation demand, which is defined as $DR_t=\max.\{0,ID_{t,avg}-R_t\}$; R_t is total release at Rantembe during the month t (MCM); $ID_{t,avg}$ is average irrigation water demand at Minipe during month t (MCM); α is a parameter which determines the weight of each objective in the objective function, $0.0 \leq \alpha \leq 1.0$; $\gamma1$ is a parameter, which defines the shape of the deficiency function from firm power demand; $\gamma 2$ is a parameter, which defines the shape of the deficiency function from irrigation demand.

Model-1 to Model-4 have the type-I objective functions. Model-1 has the objective function of maximizing annual energy generation under the constraints of satisfying the average irrigation demand and firm power of the system. Model-2 to Model-4 have the same objective function as Model-1. The differences are in their constraints. Model-2 is under the constraint of satisfying the average irrigation demand. Model-3 is under the constraint of satisfying the firm power. Model-4 has no other constraint but the physical constraints of the reservoirs.

Model-5 and Model-6 have the type-II objective functions. Model-5 has the objective function of minimizing the deficit from power requirement under the constraint of satisfying average irrigation demand of the system. Model-6 has the same objective function as Model-5, however has no other constraint except the physical constraints of the reservoirs.

Model-7 and Model-8 have the type-III objective functions. Model-7 has the objective function of minimizing the deficit from the average irrigation demand under the constraint of satisfying the firm power of the system. Model-8 has the same objective function as Model-7, however, it has no other constraint but the physical constraints of the reservoirs.

Model-9 to Model-25 have the type-IV objective functions and no constraints but the physical constraints of the reservoirs. In those models, both the requirements of firm power and average irrigation demand are targeted in the objective function. To put the two objectives with different dimensions into one formula, the degree of deficiency, which is the deficit normalized by the demand is introduced. Therefore, the objective function is minimizing the sum of degrees of deficiency from the firm power and from the average irrigation demand of the system, respectively. The differences among Model-9 to Model-15 in this group are their weights on each of the two objectives. From Model-9 to Model-15 the weight of the degree of deficit from the firm energy decreases from 1.0 to 0.0 and the weight of the degree of deficit from the irrigation water supply increases from 0.0 to 1.0.

In this group, Model-16 to Model-20 differ in the exponent $\gamma 1$ in their objective functions, and Model-21 to Model-25 differ in the exponent $\gamma 2$ in their objective functions. The terms $(DE_t/EG_{t,f})$ and $(DR_t/ID_{t,avg})$ have a value between 0 and 1. It holds $(DE_t/EG_{t,f})^{\gamma 1} > (DE_t/EG_{t,f})$ if $\gamma 1 < 1$, and $(DE_t/EG_{t,f})^{\gamma 1} < (DE_t/EG_{t,f})$ if $\gamma 1 > 1$. This is illustrated in Figure 8.1.

This means that for $\gamma 1 < 1$ the effect of a small deficit $(DE_t/EG_{t,f})^{\gamma 1}$ is considered more serious. Whereas, for $\gamma > 1$ the effect of a small deficit $(DE_t/EG_{t,f})^{\gamma 1}$ is considered less serious than it really is. Therefore, by choosing the values of $\gamma 1$ and $\gamma 2$, it is possible to incorporate a certain preference and risk-attitude of a decision maker into the objective function. From Model-16 to Model-20, the exponent $\gamma 1$ decreases from 10.0 to 0.01, while $\gamma 2$ remains a constant 1.0. From Model-21 to Model-25, the exponent $\gamma 2$ decreases from 10.0 to 0.01, while $\gamma 1$ remains a constant 1.0.

Except the objective functions and the constraints, the setups of the SDP models are the same for Model-1 to Model-25. The stage is time period, which is one month. The state variables are reservoir storage at the beginning of each time period and the inflow during the current time period. Reservoir storage at the end of each time period is selected as the decision variable. The available 32 years (1949-1980) of observed inflow data are used to obtain statistical parameters of the stochastic inflow. Since the serial correlation coefficients of the inflow time series of the system are not high, the independent inflow process is assumed. 4 inflow classes and 7 storage classes with equal size intervals were considered for both reservoirs in cascade, thus yielding 4*4=16 inflow class combinations and 7*7=49 storage class combinations. The median of each inflow class is the representative value of that inflow class.

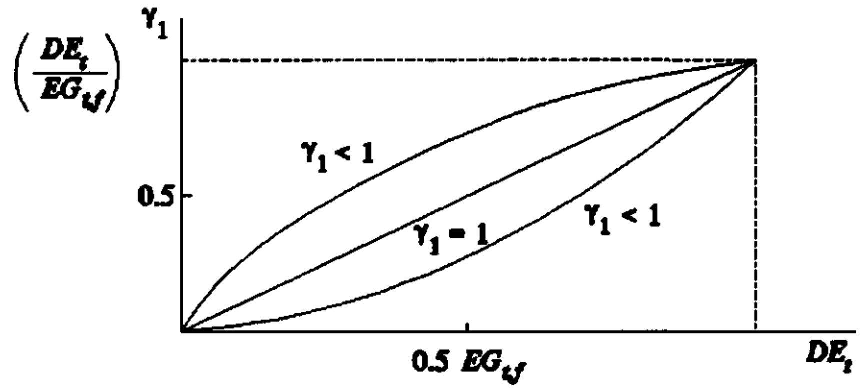


Figure 8.1 Utility Functions

The reason for selecting final storage as the decision variable is as the following. Most of the objective functions of the 25 models concern energy generation. For the Mahaweli system, reservoir elevation considerably changes with the reservoir storage. Therefore, for this system both reservoir storage and release are closely related to the objectives. On this basis, final storage is preferred as a decision variable since it saves the additional work of discretizing the reservoir release (refer to Section 6.1). Furthermore, as has been discussed, the SDP models with release as a decision variable would outperform the model with final storage as a decision variable. This occurs particularly when there are errors in the assumption regarding the inflow process (refer Chapter 7) or forecast (refer Chapter 6) of the inflow process. Otherwise the reservoir operation performance resulting from the two types of models would not differ more than that caused by the accuracy of the discretization of the variables. To verify this argument, one additional model (Model-8') has been developed based on Model-8. Model-8' has the same setup as Model-8, except that the decision variable is the current release of each time period instead of the final storage volume. Model-8' has been designed to illustrate the closeness of the reservoir operation performance resulting from the SDP model with release as a decision variable (Model-8') with that from the model with final storage as the decision variable (Model-8).

The recursive relation of the models can be summarized in the following formulation:

$$f_t^n(S_t,Q_t) = \underset{D_t}{opt}[B_t(S_t,Q_t,D_t) + \sum_{Q_{t+1}} P_{t+1}(Q_{t+1}) * f_{t+1}^{n-1}(S_{t+1},Q_{t+1})] \quad \forall S_t,Q_t,D_t\ feas. \qquad (8.5)$$

subject to,

$$R_t = S_t + Q_t - S_{t+1} - SP_t - E_t$$

$$f_T^1(S_t,Q_t) = \underset{D_t}{opt}[B_t(S_t,Q_t,D_t)]$$

Where, t, S_t, Q_t, R_t, SP_t and E_t are as defined as in Equation 3.12; n is total number of time periods passed, $n=1,2,...$; D_t is decision for the time period t, which is R_t for Model-8 and S^{t+1} for all other models. $B_t(S_t,Q_t,D_t)$ is objective increment for the transition state when the decision is D_t in period t starting from the initial storage S_t and having Q_t inflow during the period; This is the annual energy generation for Model-1 to Model-4, deficit from firm power for Model-6 and Model-6, deficit from irrigation demand for Model-7 and Model-8 (and Model-8'), and (weighted) sum of degree of deficiency from firm power and irrigation demand for Model-9 to Model-25. $f_t^n(S_t,Q_t)$ is (sub) optimal value of the recursive equation at stage n (period t) as function of S_t and Q_t; $P_t(Q_t)$ is probability of inflow in period t is in class Q_t; opt. is optimization, which is maximization for Model-1 to Model-4, and minimization for all the other models; the relation between time notations and variables are shown in Figure 3.3.

After the SDP based optimal operation policies have been derived, the performances of the reservoir system are simulated with the same part of historical inflow time series. The simulations "strictly" follow the derived optimal operation policies, as long as the physical constraints of the reservoir system are not violated. During simulations it has been assumed that the perfect forecast is available at the beginning of each time period.

The simulated performances of the system are described not only in terms of simulated values that are directly related to the objectives (average objective values, their standard deviations and 95% confidence interval for their means), but also in terms of the risk-related performance indices. Generally, reliability is a risk-related performance criterion, which receives the most attention in reservoir design and operation. There have been quite a number of attempts to maximize system reliability in practice. Still, few systems are so perfect that failures are impossible. Even when it is possible, it is often not economical to do so. After a certain point, efforts are better made to make the consequences of failure less severe and more acceptable than to try to eliminate the possibility of failure alone. In the present study, the following three risk-related performance indices: (a) reliability, (b) resilience, and (c) vulnerability are adopted. To avoid confusion in terminology, the definitions of the selected risk-related performance indices are given below.

Reliability

Reliability (*PI1*) indicates how likely a system is to fail. Two measures of reliability are adopted in the present study. They are called time-based reliability and quantity-based reliability.

Time-based reliability ($PI1_{tb}$) is the percentage of the total time period that the system can satisfy a specific demand (or in other words, never flips into failure mode during the simulated operation). Let the indicator function $\delta(k)$ be 1, if the system is in failure mode in month k, or be 0 otherwise. The time-based reliability is then defined as:

$$PI1_{tb} = 1.0 - \frac{\sum_{k=1}^{JK} \delta(k)}{JK} \tag{8.6}$$

Where, JK is the total number of time periods (months) in the simulated operation (JK=384 in this study).

Quantity-based reliability ($PI1_{qb}$) is the percentage of the total sum of a specific demand that has been satisfied during the simulated operation. Let the deficiency function $DRO(k)$=max.$\{0,RO(k)-O(k)\}$. Where, $O(k)$ is simulated system output for month k; $RO(k)$ is the corresponding target output. The quantity-based reliability is then defined as:

$$PI1_{qb} = 1.0 - \frac{\sum_{k=1}^{JK} DRO(k)}{\sum_{k=1}^{JK} RO(k)} \tag{8.7}$$

The reliability indices vary from 0 to 1. The reservoir system is more reliable for larger values of this index.

Resilience

Resilience ($PI2$) indicates how quickly the system recovers from failure. Two measures of resilience are adopted in the present study. One is the average number of consecutive periods of failures that occurred prior to recovery during the simulated operation. It is denoted by $PI2_{avg}$. The other is the maximum number of consecutive periods of failures that occurred prior to recovery during the simulated operation, and it is denoted by $PI2_{max}$. They can be formulated as follows:

$$PI2_{avg} = \frac{1}{N}\sum_{n=1}^{N} d(n) \tag{8.8}$$

$$PI2_{max} = \max_{n} d(n) \qquad \forall\, n \tag{8.9}$$

Where, $d(n)$ is the duration of the n-th failure incident (n=1,2,....,N); N is the total number of periods.

In the present study these resilience indices are presented as the length of failure in months. The reservoir system is less resilient for large values of this number.

Vulnerability

Vulnerability ($PI3$) indicates how severe the consequences of failure may be. Two measures of vulnerability are adopted in the present study. One is the average of the accumulated deficit per failure event during the simulated operation, which is denoted by $PI3_{avg}$. The other is the largest deficit occurred in one failure event during the whole simulated operation and it is denoted by $PI3_{max}$. They can be formulated as follows:

$$PI3_{avg} = \frac{1}{N}\sum_{n=1}^{N}\sum_{k=1}^{d(n)} DRO(k) \tag{8.10}$$

$$PI3_{max} = \max_{n}\sum_{k=1}^{d(n)} DRO(k) \quad \forall\ n \tag{8.11}$$

Vulnerability is a measure of the magnitude of a failure. Large vulnerability indices imply that the failure is a significant one.

8.3 Analysis and Results

Table 8.2 and Table 8.3 present the simulated performance indices of the experiments. Table 8.2 presents the indices referring to energy output and Table 8.3 presents the indices referring to irrigation supply.

The reservoir operation simulation results for Model-8 and Model-8' (Table 8.2 and 8.3) are almost the same. This fact confirms the argument that for the study case the choice of the decision variable will not greatly affect the comparison among the various objective functions. Therefore, the comparison among the various objective functions will be carried out based only on the SDP models in which the decision variable is the average volume at the end of time period *t*.

First consider the two simulated operations in which energy generation have been targeted in the objective functions: annual energy output and annual deficit from firm power (Table 8.2). The average annual deficits do vary considerably with the different setups of objectives. While the average annual energy output remains consistently uniform, despite whether it has been targeted as an objective during optimization (for Model-1 to Model-4) or not (for the other models). The difference between the largest value 1393.9 GWh (Model-4) and the smallest value 1248.0 GWh (Model-1) is about 10%. Except the models with firm power as a constraint (Model-1, Model-3 and Model-4), the average annual energy output varies less than 3.5% for all the models.

To facilitate the interpretation of this phenomenon, the term "challenging" is introduced. First, consider a simplified situation in which a reservoir system serves to save every drop of inflow for downstream water supply. If the reservoir is large enough, it will not be difficult to operate the system without any spillage. For this case the long-run average annual release will always be equal to the average annual inflow, no matter how dramatically the actual monthly (or weekly) releases vary from each other. For a system of this nature the optimization of maximizing the expected annual release would not be a "challenging" task. If the reservoir becomes smaller or the fluctuation of monthly inflow becomes larger, the spillage becomes more difficult to avoid. For this situation the optimization of maximizing the expected annual release may become a "challenging" task. Consider another example where the downstream demand affects the operation of a reservoir system. If the demand is so small that there is no difficulty at all for the system to satisfy it, then the optimization of minimizing the expected deficit from the demand would not be a "challenging" task. If the demand becomes larger or the fluctuation of inflow becomes larger, then it becomes more difficult for the system to satisfy the demand. For this situation the optimization of minimizing the expected deficit from the demand may become a "challenging" task.

When the purpose is energy generation, the problem becomes more complicated due to the involvement of the water head. However, one thing remains the same as in the release problem. That is, there is no general conclusion about which types of objectives are more "challenging". It varies from case to case, depending on the characteristics of the reservoir, fluctuation of inflows and how exigent the demand is, etc. For the case study, the invariability of the simulated average annual energy generation implies that this objective of maximizing expected annual energy generation is not "challenging" to this system. On the other hand, satisfaction of the firm power is obviously a more "challenging" goal. Thus, the optimization had better taken satisfying the monthly firm power as an objective, the average annual energy generation would not vary much anyway.

Table 8.2 Simulated Performance Indices Referring to Energy Output

	annual energy output			annual deficit from firm power			reliability		resilience		vulnerability	
	avg. (GWh)	std. (GWh)	95% confidence interval (GWh)	avg. (GWh)	std. (GWh)	95% confidence interval (GWh)	time-based	quantity-based	avg. (month)	max. (month)	avg. (GWh)	max. (GWh)
1	1248.0	304.4	(1155;1341)	123.8	121.7	(87;161)	0.727	0.865	2.5	9.0	94.3	341.4
2	1384.2	308.3	(1290;1478)	162.0	80.8	(137;187)	0.659	0.823	2.0	7.0	77.4	310.1
3	1300.0	282.5	(1214;1386)	92.1	105.6	(60;124)	0.776	0.900	2.2	9.0	75.6	341.4
4	1393.9	335.9	(1291;1495)	172.8	96.2	(144;202)	0.667	0.812	1.9	6.0	83.8	274.5
5	1358.9	290.2	(1271;1447)	88.9	97.4	(59;119)	0.734	0.903	1.9	7.0	52.7	313.4
6	1348.5	273.8	(1265;1432)	56.1	63.7	(37; 75)	0.789	0.964	2.0	6.0	43.8	184.9
7	1303.3	286.9	(1216;1391)	90.0	110.4	(56;124)	0.794	0.902	1.9	7.0	70.2	312.3
8	1358.6	295.4	(1269;1448)	156.3	83.8	(130;181)	0.627	0.830	2.3	5.0	80.7	274.6
8'	1358.6	290.2	(1270;1447)	155.9	81.3	(131;181)	0.625	0.831	2.2	5.0	79.8	264.6
9	1342.3	278.8	(1257;1427)	56.0	66.8	(36; 76)	0.786	0.939	1.8	9.0	39.8	227.0
10	1341.4	272.6	(1258;1424)	55.0	67.2	(35; 75)	0.784	0.940	1.8	9.0	37.5	227.0
11	1343.7	269.6	(1262;1426)	54.7	64.7	(35; 74)	0.789	0.940	1.8	6.0	38.9	189.1
12	1341.8	267.6	(1260;1423)	59.5	65.5	(40; 79)	0.781	0.935	1.8	6.0	40.5	189.1
13	1338.8	273.6	(1256;1422)	62.1	73.6	(40; 85)	0.768	0.932	1.9	8.0	43.2	256.4
14	1346.2	278.6	(1261;1431)	66.2	84.5	(40; 92)	0.797	0.928	1.7	8.0	45.1	329.8
15	1358.8	295.7	(1269;1449)	155.4	82.7	(130;181)	0.628	0.831	2.2	6.0	76.5	274.6
16	1358.1	275.3	(1274;1442)	72.9	78.4	(49; 97)	0.760	0.921	1.8	7.0	45.7	294.5
17	1356.8	274.2	(1273;1440)	72.0	77.4	(48; 96)	0.758	0.922	1.8	8.0	44.3	297.6
18	1349.1	283.6	(1263;1435)	67.8	83.8	(42; 93)	0.784	0.926	2.2	8.0	57.0	310.1
19	1338.8	281.2	(1253;1424)	67.7	85.3	(42; 94)	0.784	0.926	2.0	7.0	52.8	310.0
20	1335.0	281.9	(1249;1421)	80.1	92.3	(52;108)	0.771	0.913	1.9	7.0	55.7	320.6
21	1343.0	272.8	(1260;1426)	54.6	66.0	(35; 75)	0.784	0.940	1.9	6.0	39.7	189.1
22	1343.2	269.4	(1261;1425)	55.0	64.3	(35; 75)	0.786	0.940	1.8	6.0	39.1	189.1
23	1343.9	278.0	(1259;1429)	63.4	69.9	(42; 85)	0.766	0.931	2.0	9.0	44.1	261.5
24	1344.0	283.7	(1258;1430)	69.1	94.7	(40; 98)	0.792	0.925	1.9	7.0	51.4	293.2
25	1344.8	285.4	(1258;1432)	68.8	94.9	(40; 98)	0.792	0.925	1.8	7.0	50.0	293.2

Table 8.3 Simulated Performance Indices Referring to Irrigation Supply

	annual deficit from irrigation demand			reliability		resilience		vulnerability	
	avg. (MCM)	std. (MCM)	95% confidence interval (MCM)	time-reliabiliy	quantity-reliability	avg. (month)	std. (month)	avg. (month)	max. (month)
1	176.7	160.6	(128;226)	0.810	0.886	1.8	6.0	137.9	681.8
2	63.1	47.2	(49; 77)	0.859	0.959	1.5	6.0	56.1	246.2
3	161.7	143.4	(118;205)	0.810	0.896	1.6	6.0	112.5	492.2
4	189.4	180.6	(134;244)	0.750	0.878	2.0	9.0	126.3	731.3
5	55.3	33.5	(45; 65)	0.862	0.964	1.6	6.0	52.1	251.9
6	140.3	126.8	(102;179)	0.807	0.910	1.5	4.0	91.6	292.5
7	174.7	158.1	(127;223)	0.799	0.888	1.6	6.0	118.9	529.7
8	63.9	46.3	(50; 78)	0.849	0.959	1.7	6.0	60.	232.0
8'	63.9	44.3	(50; 77)	0.849	0.959	1.7	6.0	60.1	232.0
9	137.4	123.6	(100;175)	0.807	0.912	1.5	4.0	89.7	291.9
10	122.6	115.1	(88;150)	0.818	0.921	1.5	4.0	85.3	291.1
11	115.5	111.5	(82;149)	0.815	0.926	1.5	4.0	78.6	291.8
12	106.5	101.9	(75;133)	0.833	0.931	1.5	4.0	79.3	291.9
13	68.0	65.0	(48; 88)	0.867	0.956	1.4	4.0	58.8	273.0
14	63.5	17.5	(58; 69)	0.859	0.959	1.5	7.0	54.9	182.6
15	64.4	47.1	(50; 79)	0.846	0.959	1.7	6.0	60.6	231.0
16	51.8	47.1	(39; 66)	0.875	0.967	1.4	6.0	48.8	242.1
17	50.9	46.1	(37; 65)	0.877	0.967	1.4	6.0	47.9	242.1
18	58.6	32.4	(49; 68)	0.867	0.962	1.4	6.0	52.0	242.1
19	70.3	35.3	(60; 81)	0.867	0.955	1.4	4.0	60.8	202.8
20	67.2	58.0	(50; 85)	0.867	0.957	1.3	4.0	56.6	201.8
21	133.6	124.7	(96;172)	0.810	0.914	1.5	4.0	87.2	273.0
22	122.8	119.3	(86;159)	0.812	0.921	1.5	4.0	81.9	273.0
23	95.8	90.3	(68;123)	0.836	0.938	1.4	4.0	68.1	217.8
24	56.9	27.1	(49; 65)	0.867	0.963	1.5	7.0	52.0	261.4
25	57.7	31.9	(48; 67)	0.865	0.963	1.4	7.0	51.3	261.4

A second phenomenon, which draws attention is that the reservoir performance turns considerably worse for those models with firm energy requirements as constraint (Model-1, Model-3 and Model-7). This can particularly be noticed from the simulated performance indices of the average annual energy output (Table 8.2), vulnerability of firm power (max.) (Table 8.2), average annual irrigation deficit (Table 8.3), reliability of irrigation water supply and vulnerability of irrigation (max.) (Table 8.3) etc.

The poor performance of those models with firm energy requirement as constraint can be explained as follows. In Section 8.1 it has been mentioned that there are two ways of targeting a demand in SDP optimization: by objective function, or by constraint. If a demand is taken in the objective function as minimizing the deficit, the system will try to satisfy the demand as much as possible. While if demand is taken as a constraint, it becomes an "absolute" requirement in the optimization process, which must be met. A demand subjected to the constraint is being treated with priority compared to the demand targeted in the objective function during the optimization process. However, putting any demand into constraint of the optimization is a potential danger. When the demand is too exigent for a system to satisfy, there will be no feasible solution in certain circumstances (for certain initial storage and inflow states). This leads to distortion of the whole optimization process (see the next two paragraphs). The more severe the demand is, the more severe the optimization process will be distorted.

When comparing Table 8.2 to Table 8.3, it can be observed that the reliability of power, particularly the time-based reliability, is considerably lower than that of the irrigation demand. This implies that the firm power is more difficult for the system to satisfy than the irrigation demand. The policy tables are analyzed below, to study the extent to which the firm power requirement affects the system and to study the frequency of occurring distortions due to the inability to satisfy the firm power requirement.

Table 8.4 shows the derived SDP-based policy tables for the hydrological month 6 for the type-I models. Table 8.4a, Table 8.4b, Table 8.4c and Table 8.4d are the policy tables from Model-1, Model-2, Model-3 and Model-4, respectively.

The values in Table 8.4 are the targeted numbers of the final storage class. 1 represents class 1, which means both reservoirs should be in class 1. 49 represents class 49, which means both reservoirs should be class 7. 0 means there is no feasible solution available for that state. During reservoir operation simulation, the 0 decisions encountered will be replaced by the closest feasible decisions (minimum storage volumes for both reservoirs, for the case under study).

The large amount of zeros in Table 8.4a (Model-1) and Table 8.4c (Model-3) clearly illustrate the difficulty for the system to satisfy the firm power constraint. For Model-2 (with satisfaction of average irrigation demand as a constraint), there are no zeros in the policy table of month 6. Although there are some zeros in some other months, the severity is much less when compared to Model-1 and Model-3. The occurrence of non-feasible solutions (zeros) causes distortions during the optimization process. This means the optimization in Model-1 and Model-3 has been frequently distorted.

Both from the theoretical discussion and from the experimental results, it can be concluded that the tough requirement, such as the firm power in this system, had better be considered in the objective function than be set as a constraint. This conclusion is also supported by the result of Model-5. That model has the objective function of minimizing the deficit from firm power subjected to the constraint of satisfying the irrigation demand. Model-5 gives a rather good performance both in terms of satisfying firm power (Table 8.2) and satisfying irrigation demand (Table 8.3).

Next, consider the type-IV models (models with both the requirements of firm power and average irrigation demand are targeted in the objective function). One obvious advantage of this type of model is that it has the flexibility to change the preference of one objective to another by changing the weight corresponding to that objective in the objective function. This type of approach can deal with the multiobjective problems of both type (a) and type (b) cases, which have been discussed in Section 8.1. In the present study, the influence of the weight α has been studied for 7 different values (see Table 8.1). From Model-9 to Model-15, the weight of the energy generation decreases from 1.0 to 0.0 and the weight of the irrigation water supply increases from 0.0 to 1.0. Figure 8.2 shows the changing curves of the average annual deficit from firm power (Table 8.2) and from irrigation demand (Table 8.3) respectively, along the axis of α.

Table 8.4 Derived SDP-based SDP Policy Tables for the Mahaweli system (May)

(a) From Model-1

Period: May

Inflow	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Initial Storage	Final Storage															
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
9	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2
11	1	1	1	2	1	1	2	2	2	2	2	2	2	2	2	3
12	2	2	2	3	2	2	3	3	3	3	3	3	3	3	3	4
13	3	3	3	4	3	3	4	4	4	4	4	4	4	4	4	5
14	4	4	4	5	4	4	5	5	5	5	5	5	5	5	5	6
15	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
16	1	1	1	1	1	1	1	9	1	1	9	9	9	9	9	9
17	9	9	9	9	9	9	9	10	9	10	10	10	10	10	10	10
18	10	10	10	11	10	10	10	11	11	11	11	11	11	11	11	12
19	11	11	11	12	11	12	12	12	12	12	12	12	12	12	12	13
20	12	12	12	13	12	13	13	13	13	13	13	13	13	13	13	14
21	13	13	13	14	13	14	14	14	14	14	14	14	14	14	14	14
22	1	1	1	1	8	8	8	9	8	8	9	15	9	9	15	15
23	15	15	15	10	15	15	10	16	15	10	16	16	16	16	22	11
24	16	16	16	11	16	11	11	17	11	17	17	17	17	23	23	23
25	17	17	17	18	17	17	18	18	18	18	18	18	24	24	24	19
26	18	18	18	19	18	19	19	19	19	19	19	19	25	25	25	25
27	19	19	19	20	19	20	20	20	20	20	20	20	26	26	26	26
28	20	20	20	21	20	20	21	21	21	21	21	21	27	27	27	27
29	15	15	15	10	15	15	15	16	15	15	16	22	16	16	22	22
30	22	22	11	17	22	22	17	23	22	17	17	23	23	23	23	18
31	23	23	23	18	23	18	18	24	18	24	24	24	24	30	30	19
32	24	24	19	25	24	19	25	25	25	25	25	25	31	31	31	26
33	25	25	25	26	25	25	26	26	26	26	26	26	32	32	32	27
34	26	26	26	27	26	26	27	27	27	27	27	27	33	33	33	33
35	27	27	27	28	27	27	28	28	28	28	28	28	34	34	34	34
36	22	22	22	17	22	22	22	23	22	22	23	23	23	23	23	23
37	23	23	18	24	23	23	23	24	23	24	24	30	30	30	30	25
38	30	19	19	25	30	25	25	31	25	31	31	31	31	37	37	26
39	31	31	26	32	31	26	32	32	32	32	32	32	38	38	38	33
40	32	32	27	33	27	27	33	33	33	33	33	33	39	39	39	34
41	33	33	33	34	33	33	34	34	34	34	34	34	40	40	40	35
42	34	34	34	35	34	34	35	35	35	35	35	35	41	41	41	41
43	23	23	23	24	29	29	29	30	29	29	30	30	30	30	30	30
44	30	30	25	31	30	30	30	31	30	31	31	37	37	37	37	32
45	37	26	26	32	37	32	32	38	32	38	38	38	38	38	38	33
46	38	27	33	39	38	33	39	39	39	39	39	39	45	45	45	40
47	39	39	34	40	39	34	40	40	40	40	40	40	46	46	46	41
48	40	40	35	41	40	35	41	41	41	41	41	41	47	47	47	42
49	41	41	41	42	41	41	42	42	42	42	42	42	48	48	48	48

(b) From Model-2

Period: May

Inflow	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Initial Storage	Final Storage															
1	1	1	1	1	1	1	1	2	1	1	2	2	2	2	2	2
2	2	2	2	2	2	2	2	3	2	2	3	3	3	3	3	3
3	3	3	3	3	3	3	3	4	3	3	4	4	4	4	4	4
4	4	4	4	4	4	4	4	5	4	4	5	5	5	5	5	5
5	5	5	5	5	5	5	5	6	5	5	6	6	6	6	6	6
6	6	6	6	6	6	6	6	7	6	6	7	7	7	7	7	7
7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
8	8	8	8	3	8	8	8	9	8	3	9	9	9	9	9	9
9	9	9	9	4	9	9	9	10	9	4	10	10	10	10	10	10
10	10	10	10	10	10	10	10	11	10	5	11	11	11	11	11	11
11	11	11	11	11	11	11	11	12	11	6	12	12	12	12	12	12
12	12	12	12	12	12	12	12	13	12	7	13	13	13	13	13	13
13	13	13	13	13	13	13	13	14	13	13	14	14	14	14	14	14
14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14
15	15	15	15	4	15	15	4	16	4	10	16	16	16	16	16	5
16	16	16	16	5	16	16	5	17	5	11	17	17	17	17	17	17
17	17	17	17	17	17	17	17	18	17	12	18	18	18	18	18	18
18	18	18	18	18	18	18	18	19	18	13	19	19	19	19	19	19
19	19	19	19	19	19	19	19	20	19	14	20	20	20	20	20	20
20	20	20	20	20	20	20	20	21	20	20	21	21	21	21	21	21
21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21
22	22	22	5	11	22	22	11	23	11	17	23	23	23	23	23	12
23	23	23	23	12	23	23	12	24	12	18	24	24	24	24	24	13
24	24	24	24	13	24	24	13	25	13	19	25	25	25	25	25	14
25	25	25	25	14	25	25	14	26	14	20	26	26	26	26	26	26
26	26	26	26	26	26	26	26	27	26	21	27	27	27	27	27	27
27	27	27	27	27	27	27	27	28	27	27	28	28	28	28	28	28
28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28
29	29	29	12	18	29	29	18	30	18	24	30	30	30	30	30	19
30	30	30	13	19	30	30	19	31	19	25	31	31	31	31	31	20
31	31	31	14	20	31	31	20	32	20	26	32	32	32	32	32	21
32	32	32	32	21	32	32	21	33	21	27	33	33	33	33	33	33
33	33	33	33	33	33	33	33	34	33	28	34	34	34	34	34	34
34	34	34	34	34	34	34	34	35	34	34	35	35	35	35	35	35
35	35	35	35	35	35	35	35	35	35	35	35	35	35	35	35	35
36	30	30	19	25	30	30	25	31	25	31	37	31	37	37	37	26
37	31	31	20	26	31	31	26	32	26	32	32	32	38	32	32	27
38	32	32	21	27	32	32	27	33	27	33	39	33	39	39	39	28
39	33	33	33	28	33	33	28	40	28	34	40	40	40	40	40	40
40	34	34	34	34	34	34	34	41	34	35	41	41	41	41	41	41
41	41	41	41	41	41	41	41	42	41	41	42	42	42	42	42	42
42	42	42	42	42	42	42	42	42	42	42	42	42	42	42	42	42
43	31	31	26	32	31	31	32	32	32	32	32	32	38	32	32	33
44	32	32	27	33	32	32	33	33	33	39	39	33	39	39	39	34
45	33	33	28	34	33	33	34	40	34	40	40	40	40	40	40	35
46	34	34	34	35	34	34	35	41	35	41	41	41	41	41	41	41
47	41	41	41	41	41	41	41	48	41	42	42	48	48	48	48	48
48	48	48	48	48	48	48	48	49	48	48	48	49	49	49	49	49
49	49	49	49	49	49	49	49	49	49	49	49	49	49	49	49	49

Table 8.4 Derived SDP-based SDP Policy Tables for the Mahaweli system (May) (cont.)

(c) From Model-3

Period: May

Inflow	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Initial Storage	Final Storage															
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
9	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2
11	1	1	1	2	1	1	2	2	2	2	2	2	2	2	2	3
12	2	2	3	3	2	3	3	3	3	3	3	3	3	3	3	4
13	3	3	3	4	3	3	4	4	4	4	4	4	4	4	4	5
14	4	4	4	5	4	4	5	5	5	5	5	5	5	5	5	6
15	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
16	1	1	1	1	1	1	1	9	1	1	9	9	9	9	9	9
17	9	9	9	9	9	9	9	10	9	10	10	10	10	10	10	10
18	10	10	10	11	10	10	10	11	11	11	11	11	11	11	11	12
19	11	11	11	12	11	12	12	12	12	12	12	12	12	12	12	13
20	12	12	12	13	12	13	13	13	13	13	13	13	13	13	13	14
21	13	13	13	14	13	14	14	14	14	14	14	14	14	14	14	14
22	1	1	1	1	8	8	8	9	8	8	9	15	9	9	15	15
23	15	15	15	10	15	15	10	16	15	10	16	16	16	16	22	11
24	16	16	16	11	16	11	11	17	11	17	17	17	17	23	23	23
25	17	17	17	18	17	17	18	18	18	18	18	18	24	24	24	19
26	18	18	18	19	18	19	19	19	19	19	19	19	25	25	25	25
27	19	19	19	20	19	20	20	20	20	20	20	20	26	26	26	26
28	20	20	20	21	20	20	21	21	21	21	21	21	27	27	27	27
29	15	15	15	10	15	15	15	16	15	15	16	22	16	16	22	22
30	22	22	11	17	22	22	17	23	22	17	17	23	23	23	23	18
31	23	23	23	18	23	18	18	24	18	24	24	24	24	30	30	19
32	24	24	19	25	24	19	25	25	25	25	25	25	31	31	31	26
33	25	25	25	26	25	25	26	26	26	26	26	26	32	32	32	27
34	26	26	26	27	26	26	27	27	27	27	27	27	33	33	33	33
35	27	27	27	28	27	27	28	28	28	28	28	28	34	34	34	34
36	22	22	22	17	22	22	22	23	22	22	23	23	23	23	23	23
37	23	23	18	24	23	23	23	24	23	24	24	30	30	30	30	25
38	30	19	19	25	30	25	25	31	25	31	31	31	31	37	37	26
39	31	31	26	32	31	26	32	32	32	32	32	32	38	38	38	33
40	32	32	27	33	27	27	33	33	33	33	33	33	39	39	39	34
41	33	33	33	34	33	33	34	34	34	34	34	34	40	40	40	35
42	34	34	34	35	34	34	35	35	35	35	35	35	41	41	41	41
43	23	23	23	24	29	29	29	30	29	29	30	30	30	30	30	30
44	30	30	25	31	30	30	30	31	30	31	31	37	37	37	37	32
45	37	26	26	32	37	32	32	38	32	38	38	38	38	38	38	33
46	38	27	33	39	38	33	39	39	39	39	39	39	45	45	45	40
47	39	39	34	40	39	34	40	40	40	40	40	40	46	46	46	41
48	40	40	35	41	40	35	41	41	41	41	41	41	47	47	47	42
49	41	41	41	42	41	41	42	42	42	42	42	42	48	48	48	48

(d) From Model-4

Period: May

Inflow	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Initial Storage	Final Storage															
1	1	1	1	1	1	1	1	2	1	1	2	2	2	2	2	2
2	2	2	2	2	2	2	2	3	2	2	3	3	3	3	3	3
3	3	3	3	3	3	3	3	4	3	3	4	4	4	4	4	4
4	4	4	4	4	4	4	4	5	4	4	5	5	5	5	5	5
5	5	5	5	5	5	5	5	6	5	5	6	6	6	6	6	6
6	6	6	6	6	6	6	6	7	6	6	7	7	7	7	7	7
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37	37	37	37	32	37	37	26	38	26	32	38	38	38	38	38	27
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42	42	42	42	42	42	42	42	42	42	42	42	42	42	42	42	42
43	43	43	26	38	43	43	32	44	32	38	44	44	44	44	44	33
44	44	44	27	39	44	44	33	45	33	39	45	45	45	45	45	34
45	45	45	28	34	45	45	34	46	34	40	46	46	46	46	46	35
46	46	46	46	35	46	46	35	47	35	41	47	47	47	47	47	47
47	47	35	35	35	47	35	35	48	47	42	48	48	48	48	48	48
48	42	42	42	42	42	42	42	49	42	42	49	49	49	49	49	49
49	49	49	49	49	49	49	49	49	49	49	49	49	49	49	49	49

Figure 8.2 shows that along the axis α, the two curves go in (two) opposite directions. As α decreases, the average annual deficit from firm power keeps quite stable at the level of 50 - 60 GWh except at α=0.0 where a jump to about 150 GWh occurs. The average annual deficit of irrigation demand decreases relatively gradually from about 140 to 60 MCM as α decreases from 1.0 to 0.0. The characteristics of these curves indicate that depending on preference of a decision maker an interchange point between α=0.0 and α=1.0 can be chosen. For example at α=0.01 (Model-14), the deficit from irrigation demand is already its lowest value (64 MCM, when α=0.0) while the deficit from firm power is still quite stable at a constant level (from α=1.0 till α=0.01). Model-14 is considerably better at satisfying firm power with hardly any sacrifice in satisfying irrigation demand compared with the other relatively good model, Model-5.

Model-16 to Model-25 are designed to investigate the influence of the utility function in the objective function on the risk-related performance indices. As has been described in Table 8.1, Model-16 to Model-20 differ in the exponent $\gamma 1$ in their objective functions. The exponents $\gamma 1$ and $\gamma 2$ define the shape of the function of deficiency from the system firm power and the irrigation demand, respectively. The values of $\gamma 1$ and $\gamma 2$ varying between 10 and 0.1 (see Table 8.1) have been considered to provide a set of policies, for comparison.

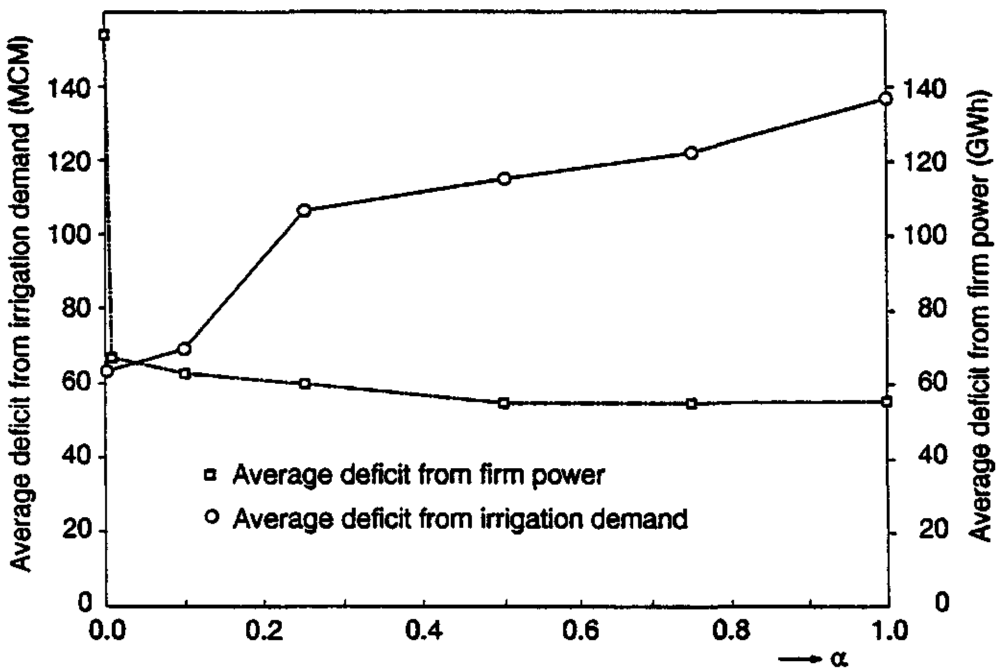

Figure 8.2 Relation of Deficits and α

Theoretically, if it is a single goal optimization, for $\gamma=1$ one obtains the "standard" operating policy. For $\gamma>1$, operating policies exhibit "hedging": they sometimes provide only a portion of the target value, where the total amount of the target value or at least more could be provided. This saves water to protect against future deficits, which could be even larger. A very different operating policy behaviour results for $\gamma<1$. In this case the marginal disutility of deficits is a decreasing function of the total deficit. As a result, optimal policies cover the entire target when it is possible, but sometimes fail to release any water when a modest failure should occur.

The problem of the present case study is a two-goal optimization. Therefore, there are mutual influences between the two goals in the objective function when either $\gamma 1$ or $\gamma 2$ changes. As Table 8.3 shows, the reliability of satisfying the irrigation demand for $\gamma 1 > 1$ is higher than that for $\gamma 1 = 1$. This is because when $\gamma 1 > 1$, the deficit from system firm energy is less emphasized, and therefore the deficit from system irrigation demand becomes relatively more emphasized in the optimization. Similarly, Table 8.2 reveals that the reliability of satisfying firm power energy for $\gamma 2 > 1$ is higher than that for $\gamma 2 = 1$. This is because when $\gamma 2 > 1$, the deficit from system irrigation demand is less emphasized. Therefore, the deficit from system firm energy becomes relatively more emphasized in the optimization.

From the investigation of type-IV models (from Model-9 to Model-25), a general trend can be observed. When α decreases from 1.0 to 0.0, the reliability (both time-based and quantity-based) of firm energy decreases and the reliability of irrigation demand increases. When $\gamma 1$ increases, the reliability of irrigation demand increases. When $\gamma 2$ increases, the reliability of firm energy increases.

The interchange among these tested models is rather a practical than a scientific issue. It largely depends on the key concern of the system, which is closely related to the political, economical, administrative, environmental, etc., factors of the local area. They are beyond the scope of the present study. The aim of the present study is to examine the influence of the different approaches of considering objectives and constraints on the SDP based reservoir operation performance.

9 Conclusions and Recommendations

9.1 Conclusions

The study was focused on four major aspects of the SDP model: (a) the Markov inflow transition probability matrix and its role in SDP models; (b) the influence of different decision variables and inflow state variables on the performance of the model; (c) the suitability of the different inflow serial correlation assumptions; and (d) the appropriateness of the objective function in the SDP model and the performance evaluation criteria.

A large number of zero elements may occur in transition probability matrices due to the limited length of inflow time series usually available in practice. This is regarded as the cause for failing to satisfy the second convergence criterion (stabilization of the expected annual increment of the objective function value) in the SDP model. Also, the violation of the second convergence criterion is the sign that the ergodicity condition of the Markov inflow transition probability matrices has not being fulfilled. The corresponding derived "stable policy" is only a locally optimum for the initial state of reservoir operation. Therefore, when using the SDP model, most of the elements in each row in the transition probability matrices should not be zero.

The sensitivity study carried out in Chapter 5 reveals that the inaccuracies in estimating the transition probability matrices do not have much impact on the SDP based operational performance of reservoir systems. The "insensitivity" may have partially resulted from the robustness of the structure design of the reservoir system. The inflow transition probability matrices, which vary to a certain extent may yield steady state policies with considerable similarity. The above observed results are possible due to this reason also. Due to the above fact and the possible errors in "historical" transition probability matrices (estimated from limited length of inflow records), a procedure to ensure ergodicity can be applied by replacing zeros in the transition probability matrices by reasonably small values.

The selection of the decision variable in the SDP model has an overwhelming influence on the suitability of the model for the system to be optimized. This was observed in Chapter 6. The variable (either release or storage) that is directly related to the objective of optimization was shown to be always preferred as the decision variable.

The SDP model with the decision variable directly related to the objective has the following advantages: (i) the policy derived leads to better performance of the reservoir system, particularly during real-time operation when the policy has to be implemented with the guidance of imperfect inflow forecasting; (ii) the selection of the inflow state variable may not greatly affect the performance of the system operation; (iii) reservoir operators can implement the policy table more easily (when the decision is not the variable that indicates the operation target, it has to be converted by the continuity equation together with an estimation of the inflow for that operation period), and the principle is consistent with use of a rule curve, which has been the common practice in the past.

The choice of different inflow state variables considerably affects the operation of the system if the selected decision variable is not directly related to the objective of optimization in the SDP model. In those cases, present inflow should be taken as inflow state variable. The main advantage of this type of model is that it opens the way for reservoir operators to regulate the system with the most up-to-date information of inflow. If the perfect inflow forecast is available, the system operation based on present inflow will perform better.

The conclusion from Chapter 6 regarding the influence of the decision variable of SDP has been verified by the experiments carried out in Chapter 7. If the variable directly related to the objective of optimization is selected as decision (Experiment 7.1), the SDP model becomes insensitive to the way inflow is considered. In this situation, the simplest SDP model with independence inflow assumption or even the deterministic model based on mean value of inflow was shown to be always preferred over the complicated SDP models that consider inflow serial correlation.

Drawbacks in the deterministic and Markov-II assumptions were observed when the decision variable is not directly related to the objective of optimization in the SDP model (Experiment 7.2). The simplest deterministic model based on mean value of the inflows is too simple to represent the characteristics of reservoir inflow. The policy derived from this model results in a system performance considerably worse than those of the other models. On the other hand, the insignificant improvement resulting from the model with Markov-II assumption does not justify the additional complication of the model. In fact, the experiments show hardly any improvement by the Markov-II model over the Markov-I model.

The main conclusion of Chapter 7 is concerning the trade-off between the independence and the Markov-I assumptions. The SDP model with independence inflow process assumption seems more practical than the one with the Markov-I assumption. First, the limited length of historical inflow time series is the main obstacle for making a good estimate of Markov-I transition probabilities. A good estimation of a one-dimensional occurrence probability distribution that refers to the independence inflow assumption is much easier to obtain (e.g., it requires fewer observations). Second, if the time interval is a month or half a month or so, systems do not generally have high serial correlation among all the time periods within a year. Third, the model that assumes independent inflow is less sensitive to the errors in inflow forecast, which is unavoidable during real-time operation.

There should always be an inflow statistical analysis for the case under consideration before making the inflow correlation assumptions. Naturally the independence inflow assumption is preferred if the inflow correlation coefficients are low for most of the time periods (Experiments 7.5 and 7.6). Even for the situation that the inflow correlation coefficients are high for many time periods but the length of the available historical inflow time series is not very long (Experiments 7.3 and 7.4), the assumption of independent inflow still looks a better choice for its simplicity in modelling and its insensitivity to the imperfect inflow forecasting.

Probably, the more complicated Markov-I assumption would only be justified if the inflow serial correlation coefficients are high for almost all the time periods in a year and the observed historical time series is also long enough.

Chapter 8 shows that the selection of the most appropriate objective in the formulation of the SDP optimization model is important for its success. For example, if the system serves for energy generation, the objective can be formulated to maximize the expected annual energy generation or to minimize the expected squared deficit from firm power. For the case studied, the second formulation is more consistent with the idea of maximizing the expected economic benefit of the system, and more "challenging" for the system to be optimized. Therefore, this formulation should be selected as objective.

One way of dealing with multiobjective optimization is to transform some objectives into constraints. However, additional care must be taken to determine which objectives should be transformed, and which should not. Those objectives that are difficult for the system to satisfy should not be set as constraints. Otherwise the optimization process will be seriously distorted.

For example, for the case studied, the firm power is not a good choice as a constraint. First, because it is too difficult for the system to satisfy. Second, because in practices, it is not an "absolute" requirement but a kind of target that the system aims to satisfy with certain reliability. Therefore, setting it as an "absolute" constraint is also not consistent with its role in reality.

When all the objectives can be formulated as approaching certain targets (e.g., demands), all the objectives should be in the objective function (e.g., defined as minimizing the sum of the degree of deficiency). This type of approach will overcome the drawbacks of treating an objective as a constraint in the optimization. Furthermore, it has the flexibility to allow gradual change in the preference for one objective over another by changing the weight corresponding to that objective in the objective function. This offers opportunities to obtain the best trade-off among different goals. For example, for the case study the relatively good models (Model-14 or Model-18) were identified from this group of models.

Regarding the performance indices, the simulated objective values (which are targeted either in objective functions or in constraints) are the most direct measure of the performance of the optimization. However, these indices give only an "averaged" picture of the system operations based on SDP optimization. To gain a more complete picture of the system operations, the risk-related performance indices should be examined.

The risk-related performance indices provide a means of monitoring the vulnerability spots of the SDP based operation under extreme circumstances. Among the three selected risk-related performance criteria, reliability has provided the essential information on system performance. It offers a view on the long-term satisfaction of a certain demand. Time-based reliability shows how frequently a deficit may (or may not) occur during system operation. Quantity-based reliability (although somewhat closely related to the objective value - expected deficit) gives an additional insight into how significant the deficit is, compared with system demand. Resilience and vulnerability together give an insight into the severity or likely consequences of an individual failure. Resilience gives an insight into how long a failure persists. Vulnerability gives an insight into how bad things may become.

9.2 Recommendations

The elimination of the zeros (if a large number occurs) in the Markov inflow transition probability matrices was observed to be important in SDP models. In the present study a simple method was suggested to eliminate zeros. It is worthwhile to compare this method with other potential techniques that can be used to minimize or eliminate zeros in transition probability matrices. Some potential techniques are: (a) generating inflow time series before deriving transition probabilities; (b) fitting the transition probability matrices row by row, by suitable one dimensional probability distribution function; and (c) fitting the transition probability matrices by suitable two dimensional probability distribution function.

Throughout this study, one month was considered as the length of the stage (time period). In SDP models the choice of the length of the time period (one stage) could have an influence, specially regarding the inflow serial correlation assumption used in the model. Future research towards studying the influence of the length of the time period on the performances of the SDP models is valuable.

An important aspect, which was not addressed in this research is studying the influence of the discretization of state and decision variables have on the performance of SDP models. Discretization of state and decision variables could influence the accuracy and also the complexity of the model. It would be interesting to study the relationship between the variable discretization and the performance of the SDP models.

In this study only three case studies were used. Each system considered in the study has certain representative characteristics as discussed in Chapter 4. However, they do not represent all possible systems. Therefore, studying some extreme systems (in hydrological characteristics or reservoir features, etc.) with respect to the key aspects examined in this research is important. That would assist to verify the conclusions made from this research.

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Appendix A: Data of the Case Study Systems

Table A.1 Estimated Monthly Spillages over the Polgolla Barrage (MCM)

YEAR	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP
1950	102.4	75.5	77.0	.1	.0	.0	.0	.0	101.4	218.9	221.8	339.8
1951	99.5	75.1	.0	42.2	.0	.0	11.7	.0	520.0	196.4	11.4	71.7
1952	174.9	211.1	30.9	46.5	.0	.0	.0	328.8	271.4	181.9	200.8	77.5
1953	340.9	59.7	.0	.0	.0	.0	.0	.0	.0	69.5	88.7	61.1
1954	118.1	86.8	54.4	21.8	.0	.0	37.4	100.3	82.8	102.6	239.0	93.8
1955	300.4	61.9	135.4	43.7	3.4	4.2	15.0	309.7	712.1	253.5	104.0	149.3
1956	184.3	141.8	13.4	.0	.0	.0	.0	.0	205.0	104.3	185.0	154.8
1957	255.6	332.5	74.0	.0	.0	.0	.0	14.1	301.6	357.7	107.0	53.5
1958	68.7	247.2	622.1	69.2	.0	11.1	42.0	110.0	117.1	219.8	178.7	38.9
1959	307.3	231.9	54.2	.0	.0	.0	.0	.0	355.8	418.6	113.9	119.9
1960	188.7	145.2	49.0	23.0	56.7	6.1	77.2	89.0	163.7	214.5	204.8	287.3
1961	265.5	314.6	62.8	.0	.0	.0	.0	90.0	125.8	123.3	307.5	101.7
1962	88.5	117.4	56.5	18.6	.0	.0	.0	140.9	84.2	203.8	128.3	242.5
1963	233.0	117.3	36.5	24.4	.0	.0	.0	.0	73.9	158.1	125.2	106.3
1964	245.5	148.8	185.2	47.7	5.1	1.8	.0	18.5	53.6	193.8	226.3	220.9
1965	110.6	293.1	31.6	.0	.0	.0	.0	261.1	224.3	30.0	162.3	99.7
1966	206.5	133.8	101.3	7.4	.0	.0	.0	.0	.0	.0	28.6	249.4
1967	262.2	185.9	47.8	.0	.0	.0	.0	.0	28.3	60.5	81.9	10.1
1968	310.1	171.4	209.4	.0	.0	.0	.0	25.7	59.4	436.3	299.0	236.0
1969	193.7	135.5	45.1	.0	.0	.0	.0	75.9	262.4	82.3	19.5	135.9
1970	143.2	69.5	71.1	54.5	12.3	.0	18.8	35.4	117.0	135.5	252.4	73.4
1971	185.8	169.8	184.4	57.7	.0	.0	17.4	106.7	197.0	268.6	272.0	538.2
1972	246.9	81.4	137.0	.0	.0	.0	.0	134.2	.0	227.2	171.0	79.3
1973	343.1	254.3	157.1	.0	.0	.0	.0	.0	.0	.0	72.7	12.7
1974	.0	71.5	85.0	12.9	.0	.0	26.0	146.2	305.8	486.4	455.0	353.4
1975	263.4	64.3	65.7	14.4	.0	.0	.0	.0	367.0	105.6	343.7	253.7
1976	330.9	483.5	125.3	25.8	.0	.0	.0	.0	.0	2.7	.0	.0
1977	121.0	149.5	63.8	.0	.0	.0	.0	71.4	145.4	181.2	83.8	24.1
1978	272.5	164.9	46.2	.0	.0	.0	.0	120.6	106.3	295.3	397.9	139.5
1979	216.0	487.7	132.0	1.8	.0	.0	.0	.0	56.2	208.0	86.1	205.2
1980	264.2	297.2	165.0	1.0	.0	.0	.0	.0	.0	55.1	144.9	46.9
1981	132.6	104.8	56.2	6.0	.0	.0	.0	.0	78.7	115.9	114.2	387.8

Table A.2 Monthly Incremental Inflows into the Victoria Reservoir (MCM)

YEAR	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP
1950	40.9	74.2	143.2	80.5	67.1	44.9	17.3	18.7	16.5	8.9	30.2	27.1
1951	28.4	33.7	23.0	282.5	87.4	19.6	34.5	23.0	17.8	27.4	7.1	28.0
1952	23.6	82.6	98.7	174.7	52.0	12.8	33.0	58.4	37.5	34.1	19.7	22.2
1953	43.0	68.5	58.9	72.5	24.0	21.1	31.9	4.3	13.9	22.2	18.9	3.0
1954	34.4	40.1	62.5	64.7	40.8	44.2	22.9	14.6	6.6	9.4	19.4	18.5
1955	65.1	66.0	175.1	240.2	121.5	37.4	52.6	17.3	11.2	13.1	18.7	28.7
1956	27.1	32.1	67.4	73.2	26.4	20.8	13.2	11.4	67.6	13.2	4.4	1.1
1957	28.8	83.2	107.3	97.4	78.6	41.0	14.5	14.0	28.7	43.3	9.7	15.4
1958	25.0	151.8	447.0	270.9	65.8	65.4	59.1	65.7	42.1	17.5	33.8	21.6
1959	46.2	62.2	82.3	57.3	22.3	9.6	27.7	20.4	18.0	25.6	10.2	16.3
1960	92.5	121.7	123.0	142.7	248.0	71.2	26.8	23.5	17.0	54.9	29.7	49.8
1961	50.0	88.6	54.9	67.7	42.4	27.9	19.9	33.3	15.5	19.1	19.0	17.6
1962	31.2	132.7	130.2	167.3	89.0	30.6	40.7	74.2	25.4	25.4	27.9	51.1
1963	73.9	92.3	123.0	140.3	106.9	27.3	40.0	33.3	18.8	28.5	21.3	32.8
1964	50.8	95.6	160.5	229.2	175.6	76.6	23.9	26.4	7.6	41.8	24.9	22.5
1965	25.8	63.8	82.3	68.3	141.6	20.6	55.1	96.3	10.7	6.0	32.5	9.5
1966	47.4	99.1	129.0	110.4	46.9	35.0	42.7	31.3	10.9	12.3	25.5	44.3
1967	50.4	87.2	57.6	87.6	105.4	42.4	26.1	11.9	18.2	17.5	14.8	20.5
1968	57.4	153.7	167.4	81.9	17.4	30.6	19.5	28.9	6.7	32.9	17.6	25.5
1969	73.3	104.8	144.7	79.0	35.3	19.1	64.8	33.9	26.6	20.0	19.7	35.2
1970	84.5	70.4	140.2	166.9	155.3	25.8	36.4	26.6	15.1	15.4	37.3	17.4
1971	58.3	78.1	206.8	209.1	26.1	23.5	29.6	35.8	22.0	21.2	58.8	63.1
1972	44.3	14.5	230.7	81.8	46.1	7.1	16.7	39.2	4.5	28.0	11.8	12.5
1973	72.6	145.9	295.2	49.5	16.4	3.3	11.0	2.3	7.2	17.6	21.4	15.7
1974	24.8	68.2	116.0	40.5	10.6	8.8	12.4	12.6	7.7	18.4	57.5	28.6
1975	18.2	28.7	85.3	67.9	12.4	16.5	13.6	31.9	34.2	7.3	27.2	29.7
1976	15.7	65.8	98.3	109.7	31.2	8.2	13.5	6.8	1.6	5.3	9.9	8.8
1977	36.5	63.5	50.6	18.9	10.9	10.3	13.2	38.2	13.8	18.8	8.3	7.1
1978	92.0	59.9	95.1	65.8	27.1	29.2	11.8	44.6	4.4	27.1	27.0	4.1
1979	56.8	108.5	113.3	46.7	5.8	10.6	8.8	7.4	5.3	5.5	13.7	11.2
1980	31.0	109.4	81.9	53.2	7.5	1.1	10.9	14.7	4.3	9.3	15.5	8.2
1981	40.3	79.5	50.5	48.4	34.8	10.9	16.5	17.0	14.1	17.2	7.6	23.0
MEAN	71.0	103.8	117.5	85.4	50.5	29.9	40.5	52.4	52.1	53.5	52.8	47.5
STD.	32.0	42.4	89.4	61.9	51.3	21.5	22.7	36.6	35.4	28.7	33.6	37.9
COR.COE.	.220	.290	.500	.390	.510	.680	.310	.550	.190	.070	.010	.540

Table A.3 Monthly Incremental Inflows into the Randenigala Reservoir (MCM)

YEAR	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP
1950	69.3	88.0	144.8	57.8	72.3	44.8	29.8	34.4	63.6	27.6	89.8	82.9
1951	55.4	64.6	22.7	282.5	85.5	24.7	47.8	24.9	44.0	72.7	10.1	59.3
1952	45.5	115.0	79.6	138.2	57.6	16.2	51.6	118.0	126.9	74.0	103.4	26.3
1953	98.0	73.7	45.9	46.3	19.8	24.6	45.4	15.3	33.1	49.7	38.3	6.0
1954	59.0	45.4	42.4	45.8	38.8	58.3	35.0	52.9	36.6	29.5	40.7	47.8
1955	95.4	71.9	181.8	181.2	103.0	47.9	81.4	52.7	41.7	44.7	24.2	57.3
1956	50.3	52.0	45.0	46.2	15.1	29.1	30.9	40.5	168.8	47.4	15.0	4.0
1957	51.8	104.6	82.2	97.8	81.4	37.6	25.5	26.9	102.2	119.3	33.9	22.8
1958	39.8	191.3	492.0	170.1	40.4	88.2	92.0	117.2	108.0	52.2	105.0	22.3
1959	80.1	90.9	68.9	43.6	17.2	11.5	47.3	35.3	43.5	84.1	26.2	30.0
1960	134.6	140.4	105.6	134.8	230.7	61.8	54.3	57.5	69.6	100.9	53.4	150.0
1961	71.9	144.2	45.8	55.2	37.1	26.6	28.9	93.1	47.4	58.4	57.1	38.9
1962	45.5	127.5	116.8	106.4	62.7	24.5	61.3	157.1	78.9	68.0	49.5	140.6
1963	135.6	91.7	111.0	141.3	83.1	37.5	54.8	34.0	55.9	67.4	76.1	49.5
1964	83.1	111.1	169.8	196.6	139.6	73.0	34.6	49.2	26.3	82.3	52.5	58.1
1965	46.3	113.2	78.4	49.8	127.3	31.1	83.3	130.2	39.5	10.8	51.1	24.9
1966	66.4	116.9	124.4	80.1	35.7	41.3	50.4	20.3	25.0	39.3	34.1	98.8
1967	67.5	106.8	58.9	79.3	104.1	52.5	37.5	21.5	50.3	71.0	52.5	34.2
1968	89.4	165.5	217.2	62.5	25.1	36.3	40.7	52.5	21.6	123.7	70.0	68.3
1969	108.4	154.5	146.0	61.9	22.8	19.1	87.3	80.2	94.7	39.7	38.1	69.3
1970	129.7	87.9	140.0	102.6	163.0	31.0	60.4	39.4	57.9	53.8	79.8	39.1
1971	123.2	108.7	196.8	141.3	29.4	29.5	49.0	56.4	68.3	51.8	140.9	149.2
1972	66.8	13.7	244.0	63.2	20.4	8.7	28.3	73.3	13.6	80.4	16.9	15.4
1973	86.3	157.6	302.9	24.9	11.8	3.7	17.9	4.0	17.3	48.4	74.9	21.0
1974	27.6	75.8	122.2	26.3	11.2	10.5	22.3	26.6	25.1	49.7	143.0	75.6
1975	31.9	38.0	70.9	44.9	13.0	24.0	20.8	55.1	92.9	16.0	84.4	64.5
1976	43.9	141.0	88.3	83.5	21.6	8.5	20.5	3.7	4.7	15.3	24.6	24.6
1977	72.2	92.0	59.8	12.2	9.3	12.9	24.6	82.7	55.2	35.9	16.7	7.3
1978	141.2	91.7	76.0	34.4	27.2	36.8	15.9	96.4	21.9	69.5	89.7	14.6
1979	83.6	205.3	98.5	35.0	5.3	16.6	14.4	17.3	13.5	12.4	21.9	23.0
1980	39.7	166.8	83.1	24.1	1.5	0.8	15.2	23.0	16.7	35.6	47.3	19.0
1981	54.6	96.5	50.6	46.5	24.9	13.0	21.3	20.4	53.9	35.0	20.9	52.3
MEAN	44.2	78.7	120.3	108.6	57.4	26.9	26.0	28.0	16.6	20.1	20.9	21.4
STD.	21.0	33.7	80.0	68.9	55.2	22.9	15.4	20.0	12.9	11.5	12.2	14.2
CO.CO.	.220	.070	.360	.210	.530	.550	.440	.490	.360	.110	.050	.330

Table A.4 Monthly Incremental Inflows into the Rantembe Reservoir (MCM)

YEAR	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP
1950	45.6	67.5	54.1	59.8	33.8	26.5	46.4	45.5	29.3	30.3	22.8	28.7
1951	54.1	100.7	194.4	122.7	73.2	58.0	64.1	69.7	43.3	27.6	26.0	19.3
1952	67.5	127.0	151.2	122.1	166.5	80.2	92.9	66.2	35.9	47.3	31.0	26.2
1953	50.4	59.6	26.1	81.0	54.9	35.1	66.9	72.2	41.5	35.6	25.1	28.6
1954	43.8	73.2	77.4	82.8	60.6	47.9	45.6	52.6	23.4	26.8	24.7	29.4
1955	81.6	138.1	524.7	275.5	149.3	125.2	84.6	69.8	42.7	37.1	31.6	22.4
1956	39.5	40.4	20.9	20.4	15.4	6.8	10.5	20.0	21.9	40.5	18.0	39.5
1957	40.9	65.1	52.3	52.6	47.9	35.0	39.0	48.4	31.0	27.0	18.8	23.5
1958	67.2	149.7	338.6	166.4	70.6	66.4	76.8	70.8	33.6	31.2	31.7	19.9
1959	47.3	64.5	58.8	47.7	28.4	15.7	60.8	51.1	42.3	35.0	19.5	22.8
1960	57.1	80.2	60.1	86.7	146.5	72.8	80.0	55.3	28.9	47.3	26.3	35.7
1961	53.4	83.5	34.5	75.4	60.0	49.4	59.9	77.1	37.1	29.5	28.8	28.9
1962	31.0	65.6	82.7	80.8	47.6	37.4	43.2	58.7	27.0	25.5	24.3	28.7
1963	49.8	42.6	54.0	83.7	59.8	40.5	53.1	45.8	29.9	24.9	19.4	22.4
1964	56.1	84.8	107.1	110.3	61.2	68.2	37.6	35.4	26.4	26.8	23.9	24.6
1965	45.3	69.7	55.1	79.1	53.6	35.5	58.7	50.7	31.1	28.7	22.6	25.5
1966	49.3	83.1	92.0	64.1	53.4	35.6	46.1	35.3	21.5	30.4	22.7	29.5
1967	41.8	52.9	37.8	64.6	53.6	38.4	38.0	40.3	24.5	32.0	21.4	29.3
1968	47.5	72.7	73.7	48.0	31.5	18.8	32.7	40.5	27.6	34.7	23.3	25.4
1969	50.1	72.2	104.7	85.3	68.0	44.7	45.5	43.7	26.3	30.8	19.6	28.8
1970	52.9	89.1	135.8	110.4	68.4	49.3	64.9	67.7	31.8	29.4	26.4	24.6
1971	73.2	94.7	151.8	108.6	106.7	62.2	126.7	72.5	45.0	47.0	29.8	35.2
1972	41.5	53.9	52.8	52.5	45.7	26.5	42.1	58.2	25.2	27.0	21.3	23.0
1973	51.9	73.2	61.7	94.0	166.6	63.1	74.7	79.6	48.1	36.3	29.3	24.4
1974	39.8	65.9	131.6	100.4	60.4	30.6	104.2	74.8	47.8	45.6	31.3	27.1
1975	49.5	54.9	44.7	95.7	66.0	51.9	67.0	53.2	39.3	34.3	26.6	28.8
1976	40.0	40.2	33.4	62.9	27.2	23.7	43.1	45.1	22.1	17.1	18.0	19.3
1977	46.0	62.6	45.8	64.7	39.7	34.5	40.8	43.5	25.1	28.3	19.4	24.7
1978	58.9	79.7	32.0	80.5	70.8	50.0	65.9	74.8	36.6	36.6	30.1	28.9
1979	48.0	70.4	84.4	85.5	48.6	38.8	30.7	32.4	23.4	28.1	21.3	26.9
1980	39.1	50.1	39.1	73.1	76.4	45.7	61.9	46.4	34.1	28.1	20.3	23.4
1981	43.4	68.3	53.1	39.9	24.4	14.6	19.9	27.9	18.6	35.2	19.6	38.5
MEAN	45.1	61.3	83.9	86.0	63.8	45.7	54.5	50.8	32.6	32.6	30.2	39.0
STD.	12.1	21.1	95.5	53.6	33.5	20.2	22.2	15.5	6.7	5.8	4.8	6.0

Table A.5 Estimated Monthly Irrigation Demands at Minipe (MCM)

YEAR	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP
1950	347.2	25.8	.0	.0	32.2	.0	240.2	233.8	226.2	172.0	93.9	75.2
1951	337.7	196.3	116.7	.0	18.7	5.0	140.6	236.0	213.2	165.3	84.3	68.7
1952	368.3	64.9	8.2	.0	14.8	40.7	176.3	218.9	227.3	150.2	100.3	81.4
1953	373.4	128.4	113.6	31.8	68.3	20.5	150.4	297.9	213.0	93.7	92.3	70.1
1954	232.4	142.7	8.1	8.9	.0	.0	149.3	266.9	225.6	167.7	89.5	88.4
1955	288.4	173.7	.0	.0	.0	.0	129.7	263.2	224.4	171.0	73.1	49.6
1956	378.9	241.9	109.0	89.3	113.0	42.9	250.0	300.7	166.8	164.9	96.6	86.3
1957	289.7	81.6	37.7	66.8	.0	15.8	254.9	217.8	225.2	161.3	100.3	82.5
1958	302.9	.0	.0	.0	9.2	.0	105.9	204.5	235.2	176.6	96.0	83.9
1959	368.2	170.1	.0	76.7	113.6	64.0	208.1	198.7	221.9	171.2	94.5	80.7
1960	242.4	110.0	.0	.0	.0	4.2	66.5	202.4	239.8	106.5	102.5	84.0
1961	343.2	116.4	123.3	.0	.0	15.9	201.8	223.7	221.9	152.8	104.7	77.3
1962	335.9	.0	.0	.0	40.0	17.9	209.0	188.6	216.8	163.1	64.5	77.0
1963	261.1	109.6	37.7	.0	.0	1.7	171.1	242.8	220.8	164.0	98.1	68.1
1964	285.9	.6	.0	.0	.0	.0	236.5	206.6	216.0	101.0	84.8	82.7
1965	320.1	240.4	73.1	146.4	.0	28.4	155.7	211.6	222.4	169.4	55.2	84.8
1966	310.6	50.1	41.1	41.3	76.8	.0	210.5	277.9	221.9	171.2	70.0	68.1
1967	183.8	57.0	71.7	92.8	7.6	10.7	228.0	291.6	215.9	170.9	99.3	79.7
1968	257.6	.0	.0	.0	117.4	.0	250.1	292.1	222.1	181.4	109.1	77.4
1969	355.7	125.9	61.9	68.5	26.1	52.2	71.7	292.3	230.7	180.1	67.4	77.2
1970	210.6	229.1	.0	.0	.0	4.8	133.8	196.7	227.5	185.8	97.8	67.7
1971	368.8	132.6	30.0	5.0	71.2	26.6	183.1	264.7	212.3	136.0	67.8	60.0
1972	311.1	161.0	.0	165.1	134.1	58.2	254.2	187.6	214.2	163.8	82.4	50.8
1973	142.0	68.3	.0	133.9	48.9	40.4	242.2	250.7	175.8	105.7	97.2	53.8
1974	292.8	37.4	.0	100.7	67.8	51.0	172.1	195.2	222.6	166.8	96.4	60.1
1975	427.4	243.6	4.7	77.3	55.4	.0	118.3	225.1	223.7	77.9	72.5	81.2
1976	353.3	147.9	8.8	66.0	104.5	55.4	226.4	292.9	216.2	161.5	92.3	81.8
1977	380.3	121.6	50.9	188.5	121.7	26.9	195.3	254.4	229.3	136.9	95.3	69.7
1978	252.2	114.2	.0	153.8	99.8	29.0	268.8	274.0	235.0	182.9	116.2	87.3
1979	252.8	126.8	.0	3.2	58.4	35.2	248.0	286.2	224.7	167.9	103.8	66.2
1980	261.8	3.6	79.5	245.0	150.9	57.2	130.3	273.6	241.4	193.1	112.1	83.7
1981	280.1	107.1	130.7	148.0	48.4	43.2	237.2	232.1	227.4	127.1	96.1	78.3
MEAN	303.6	110.3	34.6	59.7	50.0	23.4	188.0	243.8	220.5	155.0	90.8	74.8

Table A.6 Monthly Inflows into the Kariba Reservoir (MCM)

	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP
1961	1300	1460	1710	5830	6550	9900	13230	10680	6290	3420	1860	1410
1962	1220	1430	8130	8080	14080	18740	16420	10420	5260	3250	1900	1560
1963	1510	1730	4900	5080	6010	5630	6020	5500	3150	1710	1330	1300
1964	1160	1130	3450	5230	3570	4950	6790	5850	3200	1860	1331	990
1965	980	1720	1780	2620	6450	6730	11750	7820	4650	2680	1560	1260
1966	1103	1218	2374	2744	3816	4267	5307	8069	5078	3088	1617	1159
1967	1078	1434	2178	3679	11567	12339	12717	8996	4872	3112	1955	1506
1968	1281	1249	3218	4713	4502	19365	21390	13847	6145	4354	2785	1735
1969	1918	1722	7410	5295	6511	12537	11036	7382	3803	2239	1624	1475
1970	1257	1996	2197	7468	6253	7040	7932	5297	3804	2063	1504	1370
1971	999	1822	1995	5950	7395	4538	4807	5000	4011	2106	1315	1288
1972	1098	1106	1230	2128	3410	2701	3010	3648	2201	979	964	1254
1973	843	1155	6957	11166	13277	13856	7769	6923	4489	2341	1451	1084
1974	1073	1524	6542	6321	11254	11487	12819	8950	5020	2723	1476	1495
1975	974	1496	2533	3195	4128	12594	16734	12079	7659	4024	2339	1482
1976	1185	1510	1661	2514	5049	9800	7134	7502	5558	3124	1568	1450
1977	1144	1096	4436	11798	11473	17486	16978	15042	8773	5138	2682	1856
1978	1470	1830	6054	3588	4501	8115	10827	10504	5499	2945	1862	1547
1979	1514	1659	5866	4229	7326	10149	8094	7018	4599	2379	1645	1595
1980	1139	1584	2484	5211	15407	8794	8090	8860	4868	2777	1657	1348
1981	1051	1599	1553	2489	2794	2869	3224	3928	2670	1409	1408	1032
1982	1290	1274	1976	4620	3727	3609	3429	3219	2091	1106	914	1156
1983	1007	1112	2559	2188	3252	3986	5292	4356	2483	1088	1001	1017
1984	1125	1194	2747	4839	9304	4188	4157	5390	4059	1947	1269	1077
MEAN	1197	1460	3580	5041	7150	8986	9373	7762	4593	2578	1626	1352
STD.	229	267	2127	2558	3778	5013	5023	3158	1628	1036	472	231
CO.CO.	.36	.39	.10	.52	.65	.53	.89	.92	.93	.97	.94	.79

Table A.7 Monthly Inflows into the Joumine Reservoir (MCM)

	SEP	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG
1946	.15	1.59	7.05	192.00	38.60	8.81	3.59	7.59	.38	.02	.00	.00
1947	.00	4.82	2.55	18.10	30.30	7.89	5.76	6.32	3.86	.71	.02	.00
1948	.00	.00	31.10	27.90	69.40	25.90	40.70	5.57	2.44	.71	.10	.00
1949	.00	.00	4.02	2.06	23.30	21.40	61.30	44.10	2.81	.60	.00	.00
1950	.00	.73	5.50	5.49	19.00	32.70	9.11	2.36	.83	.14	.00	.00
1951	.00	9.29	18.50	30.00	49.60	58.10	19.60	22.10	6.80	1.53	.29	.00
1952	.00	.00	11.80	85.40	76.60	18.00	22.20	4.56	11.40	1.26	.24	1.02
1953	.00	.00	27.70	13.60	84.40	68.90	32.10	15.20	3.21	.00	.00	.00
1954	.00	.16	1.79	9.08	15.00	8.54	29.50	10.40	4.90	1.26	.00	.00
1955	3.84	28.90	23.90	34.60	43.90	87.70	11.60	2.02	3.54	.00	.00	2.71
1956	30.10	.11	6.56	84.10	66.40	7.77	.42	.91	1.41	.00	.00	.00
1957	.00	10.90	35.00	46.60	76.30	8.10	36.20	10.90	.30	.00	.00	.00
1958	.63	6.78	34.70	23.90	53.30	14.80	22.40	20.10	6.43	2.27	.12	.00
1959	.19	9.13	16.40	9.96	24.80	7.14	3.99	1.42	22.00	.00	.19	.00
1960	.00	.00	.00	7.47	18.70	4.98	.94	.00	.00	.00	.00	.00
1961	.00	.00	12.50	4.12	4.71	72.80	5.97	1.01	.00	.00	.00	.00
1962	.00	57.90	42.00	34.30	22.90	103.00	10.40	4.61	.52	.11	.06	.00
1963	.43	.73	.36	1.68	33.20	12.90	8.46	2.75	1.13	.52	.07	.06
1964	.00	13.90	13.20	8.36	73.10	73.30	15.60	7.83	2.67	.52	.06	.04
1965	.56	.55	.98	3.80	6.64	5.47	18.00	11.50	2.42	.69	.10	.01
1966	.16	.00	4.61	35.40	44.70	15.10	4.63	2.26	1.07	.29	.01	.00
1967	.00	.00	.14	1.82	32.70	5.54	3.29	1.58	.43	.33	.00	.00
1968	.00	.00	.21	5.54	12.70	9.39	6.24	4.48	.76	.17	.00	.00
1969	.01	34.30	2.95	112.00	15.50	35.30	18.90	3.42	1.39	.50	.11	.00
1970	.00	.00	.03	2.71	21.00	96.30	27.60	28.00	2.81	.73	.21	.02
1971	.50	1.61	.67	2.79	22.50	12.00	9.80	6.40	3.13	.85	.15	.00
1972	.07	.39	.39	.45	45.50	31.70	192.00	26.70	2.45	.94	.13	.00
1973	.04	12.10	1.52	1.72	1.28	18.10	11.00	4.67	1.71	.45	.04	.01
1974	.00	1.38	47.70	14.50	3.37	60.00	12.50	4.17	2.20	.72	.11	.01
1975	1.53	2.38	37.60	14.90	8.12	18.70	17.80	3.89	2.54	.71	.29	.04
1976	.07	2.95	22.30	11.20	14.00	5.60	2.13	2.02	.76	.21	.02	.02
1977	.00	.01	2.27	2.14	21.40	70.30	6.81	21.30	3.42	.93	.14	.00
1978	.00	.00	3.90	11.10	3.55	36.40	11.70	9.60	2.70	.80	.05	.00
1979	1.52	.55	43.70	5.63	48.20	11.20	41.10	6.66	4.18	.77	.05	.00
1980	.00	.65	4.06	47.60	53.10	20.60	6.56	2.89	1.68	.13	.02	.00
1981	.00	.00	.23	7.95	25.60	21.50	46.50	12.20	3.70	.92	.07	.00
1982	.00	1.63	63.60	90.70	25.40	5.30	39.70	4.44	1.02	.34	.08	.00
1983	.00	.00	.79	5.22	7.95	17.64	15.08	6.08	.00	.17	.00	.01
1984	.06	.11	.11	17.05	46.04	16.12	14.99	3.58	2.36	1.04	1.00	.02
1985	.36	.13	.34	2.74	15.33	18.05	9.51	2.89	3.07	.85	.05	.00
1986	1.55	5.19	6.72	12.37	34.40	46.12	16.53	14.66	2.06	.34	.24	.00
1987	.53	.25	.10	.62	3.33	3.53	17.53	1.44	1.19	.49	.01	.00
1988	.07	.27	.28	2.09	2.05	5.11	3.87	2.31	2.71	.53	.81	.00
1989	.16	1.77	1.14	6.68	13.59	6.49	2.08	1.47	1.11	.33	.01	.11
Mean	.96	4.80	12.29	24.40	30.71	28.06	20.36	8.14	2.85	.54	.11	.10
STD.	4.54	10.89	16.23	37.08	23.01	27.51	29.87	8.95	3.61	.47	.19	.43
COR.CO.	.08	.03	.29	.20	.31	.11	.04	.55	.08	.28	.32	.04

Appendix B: Definition of Ergodicity

The definitions given here on transition probability matrices are obtained from the book "Dynamic Probability Systems" by Howard (1970).

Definition 1 Given two states i and j, a **path** from i to j is a sequence of transitions that begins in i and ends in j, such that each transition in the sequence has a positive probability of occurrance.

Definition 2 A state j is **reachable** from a state i if there is a path leading from i to j.

Definition 3 Two states i and j are said to **communicate** if j is reachable from i, and i is reachable from j.

The following transition probability matrix (Winston, 1987) is used to illustrate the above definitions (* represents a non-zero element).

$$P = \begin{bmatrix} * & * & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 \\ 0 & 0 & * & * & 0 \\ 0 & 0 & * & * & * \\ 0 & 0 & 0 & * & * \end{bmatrix} \qquad \text{(B.1)}$$

For the matrix B.1, state 5 is reachable from state 3 (via the path 3-4-5) and state 3 is reachable from state 5. Therefore, state 3 and 5 communicate with each other. State 5 is not reachable from state 1 (there is no path from 1 to 5). Therefore, state 5 and state 1 do not communicate.

Definition 4 A state i is a **trapping (or absorbing) state** if $p_{ii} = 1$ and $p_{ij} = 0$, for $i = j$.

Definition 5 A set of states S in a Markov chain is a **recurrent (or closed) chain** if no state outside of S is reachable from any state in S.

In other words, a set of states S in a Markov chain is a recurrent chain if the states inside S do not communicate with any state outside S. Each recurrent chain can be regarded as a generalized trapping state: once it is entered, it can never be left.

As Howard (1970) pointed out, Markov chains have at least one recurrent chain. Most Markov chains in practice have exactly one, but some have more than one. The largest number of recurrent chains a Markov chain can have is its number of states.

Definition 6 A state i is a **transient state** if there exists a state j that is reachable from i, but the state i is not reachable from state j.

That is, a transient state is a state that the system occupies before it becomes committed to one of the recurrent chains.

For the matrix B.1, $S_1=\{1,2\}$ and $S_2=\{3,4,5\}$ are both recurrent chains. Once S_1 or S_2 has been entered, all state changes will remain in that recurrent chain.

The following transition probability matrix (Feller, 1968) illustrates the above definitions.

$$P = \begin{bmatrix} 0 & 0 & 0 & * & 0 & 0 & 0 & 0 & * \\ 0 & * & * & 0 & * & 0 & 0 & * & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & * & 0 \\ * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & * & 0 & 0 & 0 & 0 \\ 0 & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & * & 0 & 0 & 0 & * & * & 0 & 0 \\ 0 & 0 & * & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & * & 0 & 0 & 0 & 0 & * \end{bmatrix} \qquad \text{(B.2)}$$

For the matrix B.2, in the fifth row a * appears only at the fifth place. Therefore, $p_{55}=1$ and the state 5 is a trapping state. The third and the eighth row contain only one positive element each, and it is clear that state 3 and state 8 form a recurrent chain. From state 2 direct transitions are possible to itself and to state 3, 5 and 8. The pair {3,8} forms a recurrent chain while state 5 is trapping state. Accordingly, the set of smallest chain containing state 2 is the set {2,3,5,8} and state 2 is a transient state.

From state 6 the only possible transition is to state 2, which will end up in either the recurrent chain {3,8} or the trapping state {5}. Therefore, the smallest chain containing state 6 is the set of {2,3,5,6,8} and state 6 is also a transient state. Similarly, the smallest chain containing state 7 is {2,3,5,6,7,8} and state 7 is a transient state. From state 1 transition into state 4 and 9 are possible, and from there only to state 1, 4 and 9. Accordingly, the three states, state 1, state 4 and state 9 form another recurrent chain {1,4,9}.

Definition 7 A Markov Process is said to be **ERGODIC** if it forms a single recurrent chain.

When a process has a single recurrent chain, no matter where the process started, it would end making jumps among the members of the recurrent chain. However, when a process has two or more recurrent chains, the ergodic property no longer holds. In such a situation, if the system is started in a state of one chain, then it will continue to make transitions within that chain and will never make transitions to another chain.

Both matrices B.1 and B.2 are not ergodic. Matrix B.1 contains two recurrent chains, {1,2} and {3,4,5}. The matrix B.2 has three recurrent chains, {1,4,9}, {3,8} and {5}.

The following two matrices are examples for ergodic chains.

$$\boldsymbol{P} = \begin{bmatrix} * & * & 0 \\ * & 0 & * \\ 0 & * & * \end{bmatrix} \tag{B.3}$$

$$\boldsymbol{p} = \begin{bmatrix} * & * & * \\ * & * & 0 \\ 0 & * & * \end{bmatrix} \tag{B.4}$$

Appendix C: Example for Problems in Convergence Behaviour

In some cases one of the convergence criteria, that is the stabilization of the expected annual increment of the objective value can not be achieved. The following example similar to that given by Loucks *et al.*, (1981) (pp.327-332) illustrates it. Instead of the original inflow transition probabilities in that example a new set of probabililities as given in Table C.1 are assumed in this study.

Table C.1 Inflow Transition Probabilities

	i	Q_t \ j Q_{t+1}	Period T=2: 1 (30)	2 (40)
Period t=1	1	10	1	0
	2	20	0	1

	i	Q_t \ j Q_{t+1}	Period T=1: 1 (10)	2 (20)
Period t=2	1	30	1	0
	2	40	0	1

Table C.2 shows the computation process. As the table reveals, a steady-state policy $S_{t+1}(S_t,Q_t)$ is reached after 1 annual cycle while the expected annual increments of objective value tend to be two different constants, 100 and 300.

Table C.2 Calculation Progress for the Example

Stage 1: $t=2$, $n=1$

State		$B_t(S_{t-1},Q_t)$			
S_{t-1}	Q_t	S_t : 1	2	$f_2^1(S_{t-1},Q_t)$	S_t^*
1	1	200	500	200	1
1	2	100	200	100	1
2	1	0	121	0	1
2	2	0	1	0	1

Stage 2: $t=1$, $n=2$

State		$f_1^2(S_{t-1},Q_t,S_t)$			
S_{t-1}	Q_t	S_t : 1	2	$f_1^2(S_{t-1},Q_t)$	S_t^*
1	1	300	441	300	1
1	2	100	121	100	1
2	1	301	244	244	2
2	2	200	104	104	2

Stage 3: $t=2$, $n=3$

State		$f_2^3(S_{t-1},Q_t,S_t)$			
S_{t-1}	Q_t	S_t : 1	2	$f_2^3(S_{t-1},Q_t)$	S_t^*
1	1	500	744	500	1
1	2	200	304	200	1
2	1	300	365	300	1
2	2	100	105	100	1

Stage 4: $t=1$, $n=4$

State		$f_1^4(S_{t-1},Q_t,S_t)$			
S_{t-1}	Q_t	S_t : 1	2	$f_1^4(S_{t-1},Q_t)$	S_t^*
1	1	600	741	600	1
1	2	200	221	200	1
2	1	601	544	544	2
2	2	300	204	204	2

Stage 5: $t=2$, $n=5$

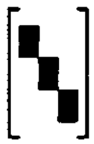
State		$f_2^5(S_{t-1},Q_t,S_t)$			
S_{t-1}	Q_t	S_t : 1	2	$f_2^5(S_{t-1},Q_t)$	S_t^*
1	1	800	1044	800	1
1	2	300	404	300	1
2	1	600	665	600	1
2	2	200	205	200	1

Stage 6: $t=1$, $n=6$

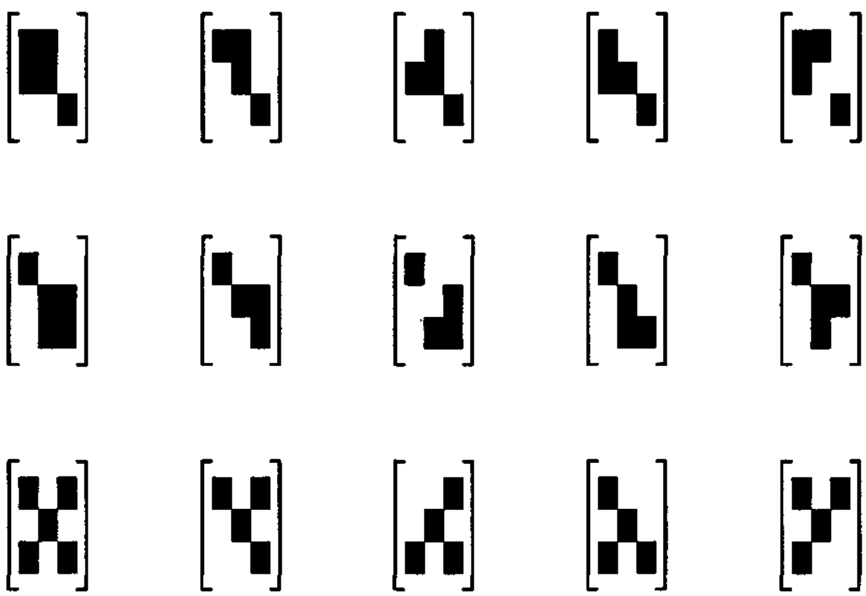
State		$f_1^6(S_{t-1},Q_t,S_t)$			
S_{t-1}	Q_t	S_t : 1	2	$f_1^6(S_{t-1},Q_t)$	S_t^*
1	1	900	1041	900	1
1	2	300	321	300	1
2	1	901	844	844	2
2	2	400	304	304	2

S_{t-1}	Q_t	$f_2^5(S_{t-1},Q_t)-f_2^3(S_{t-1},Q_t)$	$f_2^6(S_{t-1},Q_t)-f_2^4(S_{t-1},Q_t)$
1	1	800 - 500 = 300	900 - 600 = 300
1	2	300 - 200 = 100	300 - 200 = 100
2	1	600 - 300 = 300	844 - 544 = 300
2	2	200 - 100 = 100	304 - 204 = 100

He and Bogardi (1989) reported that the transition probability matrices with 'zero rows' or 'zero columns', and with 'a large number of zero elements having symmetric shape of the non-zero elements' cannot achieve the second convergence criterion. They demonstrated this behaviour using a 3*3 transition probability matrix, as an example. In the following figures (from He and Bogardi, 1989), each small square in the matrices presents an element or a collection of elements (blocks). The empty square means a zero element (or elements). A shaded square means a non-zero element (or elements).



The above transition probability matrices will converge to 3 constants (objective function values) instead of one.



The above transition probability matrices will converge to 2 constants (objective function values) instead of one. Based on the above results they suggested that the number of zero elements in the transition probability matrices should be kept as few as possible in order to facilitate the achievement of the second convergence criterion.