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Thermal runaway and bistability in microwave heated isothermal slabs

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The dissipation of electromagnetic energy within a microwave heated layer has been analyzed. It has been shown that the dissipation oscillates as a function of temperature, regardless of the specimen material. This oscillation, combined with the heat loss, is found to be responsible for thermal runaway phenomenon in isothermal slabs. Based on such an observation, a general rule to prevent thermal runaway was developed. Slab temperature analysis for time dependent microwave power indicates that the concept of bistability is not the appropriate term to describe the observed jumps in temperature. © 1996 American Institute of Physics. [S0021-8979(96)08302-5]

I. INTRODUCTION

The application of microwave heating in, for example, the food industry is seriously hampered by two problems which have their roots in the basic physics of the heating process. The first difficulty is the uneven spatial dissipation of energy within foodstuffs. The second difficulty is the catastrophic phenomenon of thermal runaway in which a slight change of microwave power causes the temperature to increase rapidly. In this article we will study the second problem. The motive for this research was the introduction of the concept of bistability in microwave physics. According to Stuerger,¹ the temperature of a microwave heated slab, when irradiated by microwave power directly proportional to time, shows bistable behavior. As usual the bistable phenomenon is accompanied by a hysteresis loop. Stuerger also claims to have found experimental evidence for this idea. However, up to now the predicted hysteresis loop has never been found.

A number of bistable phenomena exist in physics. Probably the one which is best known is the phase transition of vapor into liquid. A vapor may be compressed to a pressure well above the vapor pressure of the liquid without condensation taking place and, on the other hand, a liquid may be heated well above its boiling point without boiling. Both processes are limited to certain values of pressure above (below) at which condensation (boiling) starts. This is a typical metastable or bistable phenomenon. This bistable behavior can be described according to the Van der Waals equation, which is very remarkable because it is an equation of state related to a gas and contains nothing about phase transitions. Van der Waals law suggests bistability, but for a complete understanding the theory of phase transitions has to be added.

It seems that the same kind of situation is present in microwave physics. Fourier's differential equation suggests bistability (for time dependent microwave power), but this is insufficient for a complete understanding. The purpose of the study described in this article was to develop a kind of phase transition theory, aiming to explain and support the idea by Stuerger.

The phenomena of thermal runaway and bistability are closely related. In fact the idea of bistability was inspired by thermal runaway. This is why the first part of this article contains a study of dissipation which, combined with heat

losses, might result in thermal runaway. The aim was to find the origin of the runaway process and hope that it would lead to the real explanation of bistability. A side product of this part of the investigation resulted in a rule to eventually prevent thermal runaway. In the final part of the article the concept of bistability in microwave heating is discussed.

II. THE ORIGIN OF THERMAL RUNAWAY IN ISOTHERMAL SLABS

Consider a layer of specimen material, irradiated from one side by microwave radiation with a frequency of 2450 MHz. The wave is a plane, harmonic one and impinges normally upon the material. In order to explain the principles of thermal runaway, the simplest possible system was conceived. The absence of reflection from the cavity in which the actual experiment was performed is assumed. This means that the initial wave is reflected, absorbed, and transmitted (Fig. 1). It is also assumed that the temperature throughout the layer is the same at every moment. This can be achieved by taking a liquid as the medium and mixing it in such a way that the spatial equalization of temperature is much faster than the process causing temperature increase. This process is due to the dissipation of electromagnetic energy within the slab. Under these conditions simple relationships evolve, which indicate the reasons for thermal runaway in isothermal slabs.

Initially a system without convective and radiative thermal losses is considered. The differential equation, describing the relationship between temperature T and time t , (Fourier's law) reads:

$$\rho C_p L \frac{dT}{dt} = D, \quad (1)$$

where ρ is the density, C_p is the thermal capacity, L is the layer thickness, and D the production of heat (total amount of power per square meter generated along the z axis, extending from $z=0$ to $z=L$). This power follows from Maxwell's equations, together with the appropriate boundary conditions at the surfaces of the slab. According to Stratton² or Ayappa³ for a pure dielectric one has:

$$D = P \frac{\omega}{c} K_2'' |T_{12}|^2 \frac{\text{numerator}}{\text{denominator}} \quad (2)$$

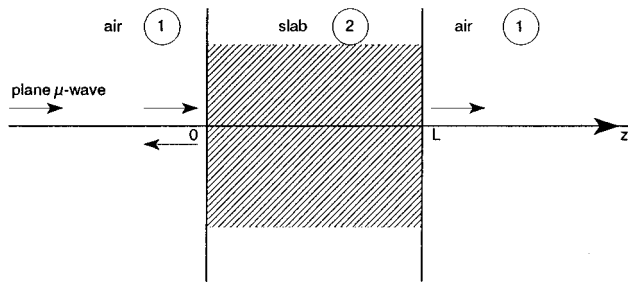


FIG. 1. A layer, being irradiated from one side, in an echofree cavity.

with

$$\frac{1}{2\beta_2} (1 - e^{-2\beta_2 L})(1 + |R_{12}|^2 e^{-2\beta_2 L}) - \frac{2}{\alpha_2} |R_{12}| e^{-2\beta_2 L} \sin \alpha_2 L \cos(\alpha_2 L + \delta_{12}) \quad (3)$$

as a numerator and

$$1 - 2|R_{12}|^2 e^{-2\beta_2 L} \cos(2\delta_{12} + 2\alpha_2 L) + |R_{12}|^4 e^{-4\beta_2 L} \quad (4)$$

as a denominator, where P is the intensity of the initial microwave, better known as the microwave power. The frequency has the symbol ω and c refers to the speed of light in vacuum.

The dielectric constant K is written in the usual way as the difference between a real part K' and an imaginary part K'' :

$$K = K' - iK'' \quad (5)$$

K' and K'' are material constants, independent of the geometry of the system. They only depend on frequency and temperature. They are not very appropriate to describe the propagation of waves, however, and this is why the wavenumber (phase constant) α and the attenuation constant β are introduced. For this free layer model α and β are related to the dielectric properties of a medium and frequency of radiation by

$$\alpha = \frac{\omega}{c} \sqrt{\frac{K'[\sqrt{1 + \tan^2 \delta} + 1]}{2}} \quad (6)$$

$$\beta = \frac{\omega}{c} \sqrt{\frac{K'[\sqrt{1 + \tan^2 \delta} - 1]}{2}} \quad (7)$$

with

$$\tan \delta = K''/K' \quad (8)$$

The reflection coefficient R_{12} and the transmission coefficient T_{12} are related to α and β by

$$|R_{12}|^2 = \frac{(\alpha_1 - \alpha_2)^2 + (\beta_1 - \beta_2)^2}{(\alpha_1 + \alpha_2)^2 + (\beta_1 + \beta_2)^2} \quad (9)$$

$$|T_{12}|^2 = \frac{4(\alpha_1^2 + \beta_1^2)}{(\alpha_1 + \alpha_2)^2 + (\beta_1 + \beta_2)^2} \quad (10)$$

$$\delta_{12} = \tan^{-1} \left[\frac{-2(\alpha_1 \beta_2 - \alpha_2 \beta_1)}{(\alpha_1 - \alpha_2)^2 + (\beta_1 - \beta_2)^2} \right] \quad (11)$$

The objective now is to find the solution to Eq. (1), i.e., the temperature as a function of time.

The electro-magnetic dissipation is always directly proportional to K'' , regardless of product's geometry. From this it follows that in crude approximation the temperature-time graph is governed by the temperature behavior of K'' . For instance, if K'' decreases exponentially in temperature, the temperature will exponentially increase to a constant value in time. According to this approximation no sudden jumps in temperature or other non-linear behavior will ever occur. So thermal runaway, if present, is caused by the interference of waves within the irradiated material. In order to investigate this process in detail the above relationships have to be simplified. Only then will it be possible to see which phenomenon is responsible for thermal runaway.

In experimental set-ups medium 1 is usually air. Air hardly absorbs electro-magnetic energy, so $\beta_1 = 0$. Medium 2 is a liquid or solid, so α_1 will be smaller than α_2 . From the definition of α Eq. (6) and β Eq. (7) it is obvious that β is always smaller than α . With these considerations the $2/\alpha_2$ term in the numerator can be neglected and thus the numerator is proportional to $1/2\beta_2$. For the absolute value of the transmission coefficient one roughly obtains:

$$|T_{12}| = \frac{2\alpha_1}{\alpha_2} \quad (12)$$

Without the denominator the dissipation D is roughly proportional to $K_2'' |T_{12}|^2 / 2\beta_2$. From Eqs. (6) and (7) it follows that $\alpha_2 \beta_2 = \omega^2 K_2'' / 2c^2$. With this expression and Eq. (12) one can conclude that for a slab the dissipation, as far as its temperature behavior is concerned, is proportional to $1/\alpha_2$. In the 2450 MHz region α_2 hardly varies with temperature, so our final conclusion is that in the first approximation the dissipation in a slab is independent of temperature! The temperature dependency of K' and K'' is more or less canceled out and the temperature as a function of heating time will be represented as a straight line (Fig. 5).

In the denominator the small temperature dependency of α_2 becomes very important, because α_2 is part of the argument of a cosine. With increasing temperature the denominator oscillates. In the temperature-time diagram this oscillation is superimposed on the straight line. Here we see the reason for "sudden" temperature jumps in isothermal slabs.

To illustrate these ideas a numerical example for demineralized water is given. Water has been taken, because its dielectric constant is well-known. Kaatz⁴ describes the behavior of K_2' and K_2'' as a function of temperature between 0 and 50 °C. In this example his formulae are extrapolated to 100 °C. Figure 2 shows the temperature behavior of K_2' and K_2'' . More important is the behavior of α_2 and β_2 (Fig. 3). Based on these data the relative dissipation D/P for several layer thicknesses L , is calculated and plotted in Fig. 4. The oscillating character of the dissipation is clearly seen. This oscillation is caused by the interference of waves. In a slab without damping, the wave reflected on the rear side of the layer is in phase with the initial wave if $L = n\lambda_2/2$ for

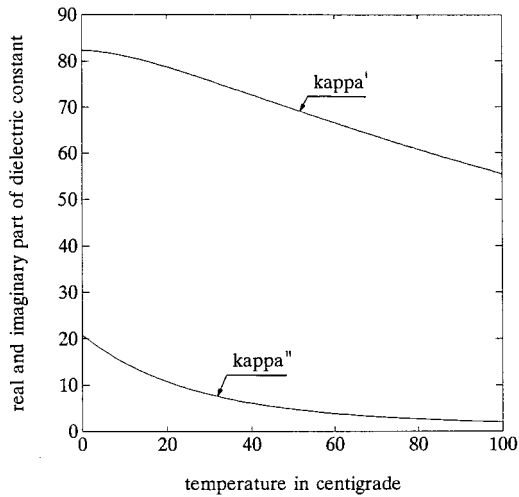


FIG. 2. The real and imaginary part (K' and K'') of the dielectric constant of water plotted versus temperature, at 2450 MHz, according to Kaatze.

$n = 1, 2, 3, \dots$. This situation corresponds to maximal dissipation. The dissipation is small when reflected wave and initial wave more or less cancel each other. This is the situation for $L = (2n + 1)\lambda_2/4$ and $n = 0, 1, 2, \dots$. The wavelength λ_2 of water varies from 1.34 cm at 0 °C to 1.65 cm at 100 °C. For example, if $L = 4$ cm, the dissipation starts at 0 °C at a maximum ($L = 3\lambda_2$). With increasing temperature it will run to a minimum ($L = 11\lambda_2/4$), followed by a maximum ($L = 5\lambda_2/2$), and finally it falls off to a neutral situation at 100 °C (Fig. 4). For a complete period of oscillation the layer thickness L must be equal to $\pi/\Delta\alpha_2$, where $\Delta\alpha_2$ is the difference between the maximum and the minimum value of α_2 . For water this results in a layer thickness of 3.5 cm. Important for this oscillation is the behavior of its amplitude as a function of temperature. The amplitude is proportional to $\exp(-2\beta_2 L)$. If β_2 decreases strongly with temperature (as is the case for water), the amplitude increases strongly and the deviations

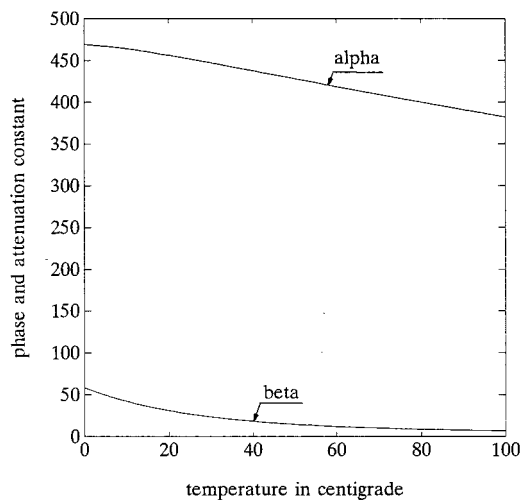


FIG. 3. The phase and attenuation constant (α and β) of water plotted versus temperature at 2450 MHz.

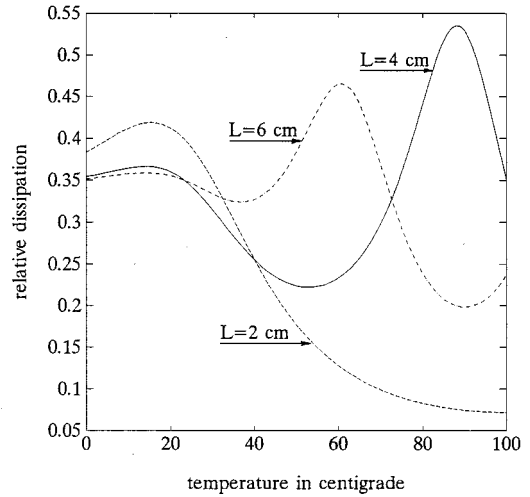


FIG. 4. The relative dissipation D/P oscillates as a function of temperature.

from the straight line in the temperature-time diagram are larger. In such a case the oscillation becomes significant at higher temperatures. For large L the amplitude is small. Many periods of oscillation exist, but they are hardly noticeable. A small $L (< \pi/\Delta\alpha_2)$ results in a large deviation from the straight line, but no complete period exists. Applied to water, the combination of all of these effects, results in a significant temperature jump for a thickness of about 4 cm. Figure 5 shows temperature-time diagrams for several values of L . In these plots the quotient P/L is constant.

III. THERMAL RUNAWAY AND HEAT LOSS

In this section the heat loss at the surfaces of the slab is taken into account. The convective heat loss is proportional to the temperature difference ($T - T_0$) to the ambient. For small temperature differences this is also the case for radia-

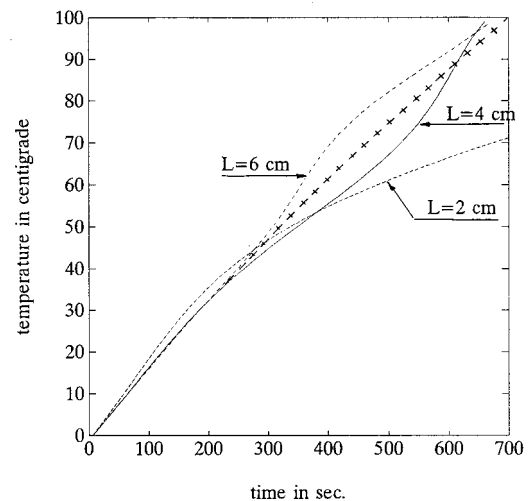


FIG. 5. In the first approximation the temperature-time diagram is a straight line (++++ curve for $L = 4$ cm). Superimposed on this line is an oscillation of which the amplitude increases with temperature, but decreases with layer thickness ($P = 0.2 * L$ kW/cm²).

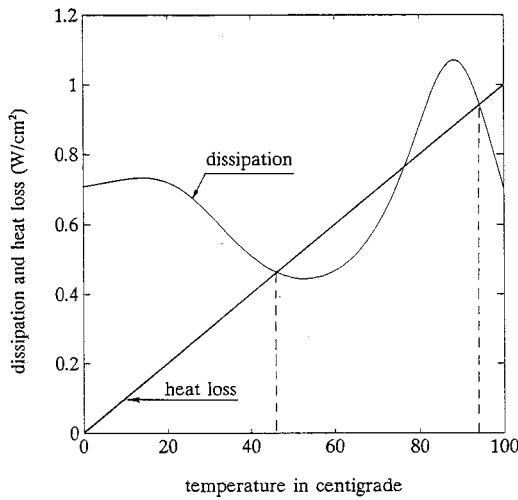


FIG. 6. The marked intersection points of dissipation and heat loss correspond to stable final slab temperatures ($L=4$ cm, $P=2$ W/cm², $h=0.01$ W/K cm², $T_0=0$ °C).

tive loss. Thus the total heat loss can be described by some effective heat transfer coefficient h , multiplied by the temperature difference ($T-T_0$). Equation (1) is then transformed into

$$\rho c_p L \frac{dT}{dt} = D - h(T - T_0). \quad (13)$$

Because the right-hand side of Eq. (13) is only dependent on temperature, the final temperature T_f of the slab follows from

$$D - h(T_f - T_0) = 0. \quad (14)$$

Depending on the microwave power P and the layer thickness L one has one or more intersecting points of the functions D and $h(T-T_0)$, as shown in Fig. 6. For certain layer thicknesses L this results in a S -shaped or multi S -shaped curve in a plot showing the final slab temperature versus the microwave power P (Fig. 7). Only the upper and lower branch of the S -shape represent stable final temperatures. Between these two branches an unstable temperature region exists. Depending on the initial temperature of the slab the final temperature will either be "high" or "low" as a result of the microwave heating. It is seen that a small increase of the microwave power might lead to the well-known thermal runaway phenomenon, i.e., a jump from the lower to the upper branch. For more details on these phenomena see Kriegsmann⁵ (for a ceramic slab) and Stuerger⁶ (for water). The S -shape or multi S -shape is caused by oscillations of the dissipation. Without oscillations there is no S -shape and hence no thermal runaway. For half a period of oscillation there is only a single point of intersection of the dissipation D and the heat loss $h(T-T_0)$, which suggests the following rule: If the layer thickness is smaller than $\pi/2\Delta\alpha_2$, where $\Delta\alpha_2$ is the difference between the maximum and minimum value of the phase constant α_2 , no thermal runaway will ever occur. No materials will be damaged by thermal runaway if the layer thickness is smaller as $\pi/2\Delta\alpha_2$. For water this re-

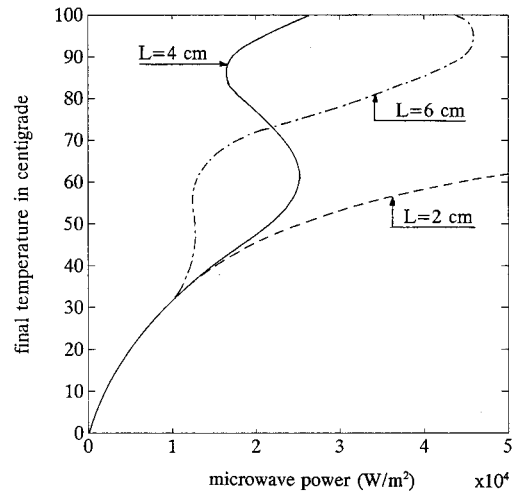


FIG. 7. Only the upper and lower branch of S -shaped curves represent stable final slab temperatures. The final temperature will be "high" or "low," depending on the initial slab temperature.

sults in a thickness of 1.75 cm. A detailed analysis gives 2.2 cm as a safety thickness. For the drying of food products this observation might be of great importance.

IV. BISTABILITY

Besides instability and thermal runaway, the oscillation of the dissipation, combined with heat loss, causes a third problem. If the dissipation takes a minimum value with increasing temperature and this minimum value is of the same order of magnitude as the heat loss at that very same temperature, it takes a lot of time to reach a desired final temperature. This is a very inefficient situation. For a fixed L , only one degree of freedom exists to accelerate this process

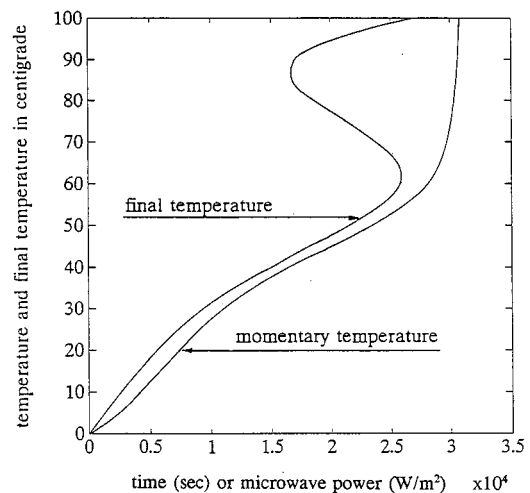


FIG. 8. Plot of final temperature T_f as a function of time-independent microwave power, compared to a plot of temperature versus time for microwave power directly proportional to time, resulting in artificial bistability ($L=4$ cm, $P=t$ W/m², $h=100$ W/K m², $T_0=0$ °C).

and that is the variation of the microwave power in time. For this reason the heat balance is written down as:

$$\rho C_p L \frac{dT}{dt} = P(t)D_1 - h(T - T_0). \quad (15)$$

D_1 is the temperature dependent factor calculated from $D = P(t)D_1$. Kriegsmann⁵ and Tian⁷ suggest several functions $P(t)$ to accelerate the sintering process of a ceramic slab.

Physically very interesting is a control of heat in such a way that the temperature, as a function of time, describes the S -shape as shown in Fig. 7. This will be the case when the microwave power P of Eq. (14) is replaced by γt , where γ is a constant. This leads to the requirement:

$$\gamma t D_1 - h(T - T_0) = 0 \quad (16)$$

from which

$$\frac{dT}{dt} = \frac{-\gamma D_1}{\gamma t \frac{dD_1}{dT} - h}. \quad (17)$$

Substitution of Eq. (16) and Eq. (17) into Eq. (15) yields a desired function $P(t)$:

$$P(t) = \gamma t - \frac{\rho c_p L}{t \left[\frac{dD_1}{dT} - \frac{D_1}{T - T_0} \right]}. \quad (18)$$

With this $P(t)$ the temperature-time curve is S -shaped. Because the temperature cannot go back in time, all kinds of remarkable effects such as bistability and hysteresis, occur.

As explained in the introduction, bistability is a rare physical phenomenon that needs a more complete explanation. In the case considered here the explanation is very simple. The function $P(t)$ becomes negative in the unstable temperature region. One has negative irradiation. The best way to avoid this is to omit the second term in equation $P(t)$. If the second term is small in comparison with γt , the temperature-time curve will follow the S -shape, except in

the unstable regions. Figure 8 shows a plot for $P(t) = t$ and $L = 4$ cm. The temperature increases rapidly from the lower to the upper branch of the S -shape in the unstable region. This temperature jump has been found experimentally,⁸ but it is incorrect to interpret this phenomenon as evidence of bistability. As a consequence of the absence of bistability, hysteresis will never be found.

V. CONCLUSIONS

In the first approximation the dissipation in a slab is independent of the temperature within the slab, so that the temperature-time diagram is a straight line. An oscillation is superimposed on this straight line. This is caused by the temperature dependency of the wavenumber. For isothermal slabs the convective and radiative heat losses amplify the effects of oscillation, resulting in instability and thermal runaway. Thermal runaway never occurs for a layer thickness smaller than $\pi/2\Delta\alpha_2$, where $\Delta\alpha_2$ is the difference between the maximum and minimum value of α_2 .

Finally, it has been shown that the concept of bistability is not the appropriate term to describe the temperature jumps for a system for which the microwave power is directly proportional to time.

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