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Linear programming to meet management targets and restrictions

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Objectives

From this chapter the reader should gain knowledge of:

- the characteristics of linear programming
- the formulation of a linear programming model
- the solving procedure
- the assumptions used

The method is introduced by a simplified example and further illustrated with an application to optimizing a dairy herd calving pattern.

6.1 Introduction

Being managers, farmers need to allocate limited resources to competing activities in the best possible way. The allocation problem can be solved by analysing a large number of alternatives, using techniques such as partial and whole-farm budgeting. With the budgeting procedure, however, each single plan has to be identified and evaluated by detailed calculations. For complex problems, such calculations are time-consuming and become tedious and burdensome. Moreover, it is only by chance that the plans chosen for budgeting analysis include the optimal one.

Linear programming uses the same type of information as is done in the budgeting procedure, but the mathematical technique involved guarantees that the optimal plan is determined. The essential **characteristics** of a linear programming model are that (1) there is a function to be maximized or minimized, (2) there are limited resources that can be used to satisfy the objective, and (3) there are several ways of using the resources (Heady & Chandler, 1958). In agriculture, linear programming is most extensively used in determining least-cost rations and in planning the farm business organization (Boehlje & Eidman, 1984). In determining the least-cost ration, the objective can be to minimize the cost of meeting the nutritional requirements of a certain type of livestock while using particular feed ingredients. In planning the farm business, land, labour and machinery and equipment resources are allocated to competing activities such as production of different crops and different types of livestock production in order to maximize income.

The method of linear programming is first introduced and illustrated with an example.

Assume that at the start of the grazing season, all beef cattle on a farm are treated with an anthelmintic. For years the farmer has used anthelmintic A. Recently, anthelmintic B has come on the market. Anthelmintic A costs US\$2 per animal and B US\$6. The firms that produce the anthelmintics have carried out experiments to estimate the effect of applying the anthelmintics. According to these experiments, A leads to an increase in live weight of 20 kg, while B leads to an increase of 40 kg, if compared with animals that have not been treated. Although the veterinarian is enthusiastic about anthelmintic B, the farmer is a bit reluctant to use it because of the higher costs. The farmer decides that (s)he wants to maximize the total effect of the therapy, but does not want to spend more than US\$600 on anthelmintics. Furthermore, the number of animals to be treated should not exceed 150.

This problem can be formulated as a linear programming problem. It refers to a situation of allocating limited resources (money, number of treatments) to competing activities (treatment with anthelmintics A and B) in order to maximize a certain objective (the total effect of the treatment). To formulate the mathematical model of the problem, let x_1 and x_2 represent the number of treatments with A and B respectively. Let Z be the resulting effect on extra live weight gain for the herd as a whole; x_1 and x_2 are the decision variables (activities) for the model. The relationship between Z and x_1 and x_2 is as follows: $Z = 20x_1 + 40x_2$. The objective is to choose the values of x_1 and x_2 so as to maximize Z , subject to the restrictions on their values imposed by the farmer. Thus the mathematical formulation of the problem is as follows:

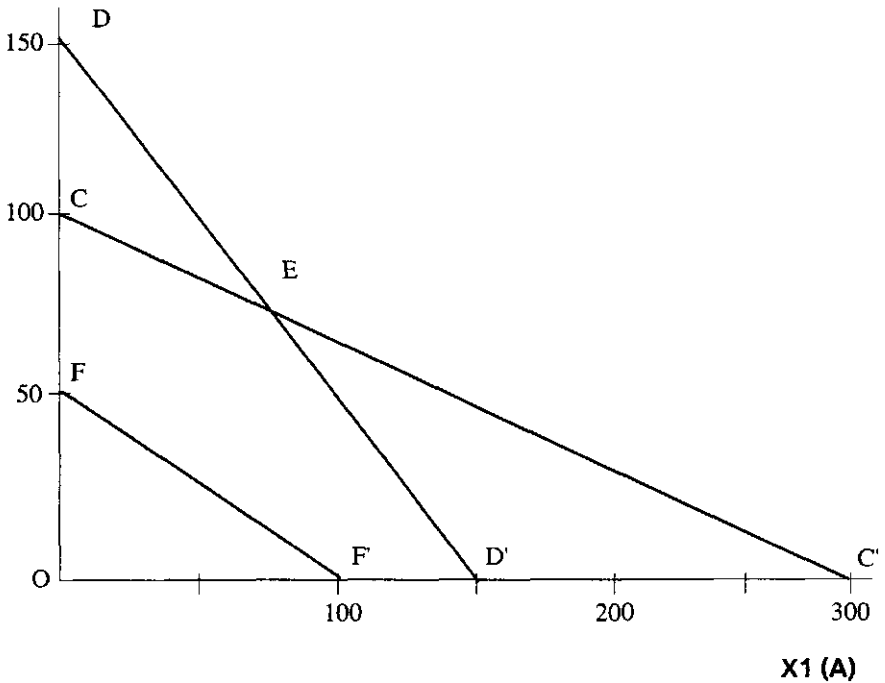
$$\text{Maximize } Z = 20x_1 + 40x_2$$

subject to the restrictions

$$\begin{aligned} 2x_1 + 6x_2 &\leq 600 \\ x_1 + x_2 &\leq 150 \\ x_1 \geq 0, x_2 &\geq 0 \end{aligned}$$

It is necessary to define the decision variables to be nonnegative. This small problem has only two decision variables, and therefore only two dimensions, and hence a graphical procedure can be used to solve it. This procedure involves constructing a two-dimensional graph with x_1 and x_2 as the axes. The first step is to identify the values of x_1 and x_2 that are permitted given the restrictions. This is done by drawing the lines that border the range of permissible values. The graphical representation of the linear programming problem is given in Figure 6.1. First, each restriction is treated separately. Resulting from the restriction on the amount of money to be spent on the treatments (ie, US\$600), 300 animals can be treated with A, 100 animals with B, or any linear combination of A and B. This is depicted in Figure 6.1 as line CC'. The constraint on the total number of animals to be treated results in 150 animals treated with A, 150 animals treated with B or any linear combination of A and B (line DD'). If all constraints are considered simultaneously, any points inside the area or at the boundary of OCED' are feasible combinations of treatments with A and B. This area is known as the 'feasible set'.

X2 (B)



The final step is to identify the point in this region that maximizes the value of Z . The optimal combination of the number of treatments A and B depends on their relative efficiency. Two animals treated with A will have the same total weight gain as one animal treated with B. The line FF' in Figure 6.1 denotes those combinations of x_1 and x_2 that generate a total effect of 2000 kg. With line FF' moving to the right, higher levels of total weight gain are obtained. In case of minimization the line FF' has to be moved into the direction of lower levels of Z . The line is moved to the right until it touches the farthest edge of the feasible set point E). Combinations of numbers of treatments are not possible beyond this point because adequate quantities of the money and/or animal resources are not available. If we draw a line from point E to each axis of the graph, they indicate that 75 treatments with A and 75 treatments with B will maximize the effect of the total treatment ($Z = 75 \times 20 + 75 \times 40 = 4500$ kg). When more constraints and decision variables are added, the example becomes a multidimensional problem that will be impossible to represent graphically. The underlying principle of the optimization procedure of linear programming, as illustrated in this simplified example, remains the same, however.

6.2 Linear programming models in general

6.2.1 General formulation

With the above-mentioned simplified example in mind, we will now give the general formulation of the linear programming model. A linear programming model has the objective to select the values for x_1, x_2, \dots, x_n (the decision variables or activities) so as to

$$\text{Maximize or Minimize } Z = c_1x_1 + c_2x_2 + \dots + c_nx_n$$

subject to the restrictions

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n &\leq b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n &\leq b_2 \\ &\vdots \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n &\leq b_m \\ \text{and } x_1 \geq 0, x_2 \geq 0, \dots, x_n &\geq 0 \end{aligned}$$

The function being maximized or minimized, $c_1x_1 + c_2x_2 + \dots + c_nx_n$, is called the **objective function**, with x_j being the decision variables. The restrictions are normally referred to as **constraints**. The first m constraints, representing the total usage of resource b_i , are called functional constraints. Similarly, the $x_j \geq 0$ restrictions are called nonnegative constraints. The model may also include 'greater than or equal to' constraints, as well as equality constraints. The input constants, a_{ij} , b_i and c_j , are referred to as parameters of the model.

The basic structure of any linear programming model is a matrix, with the columns in that matrix being the processes or activities and the rows the resource constraints. Three general types of constraints are usually included:

- real constraints which limit physical resource availability;
- institutional and subjective constraints which reflect limits imposed by the outside institutions or personal preferences of the operator; and
- accounting constraints which are used to keep track of resources or will provide structure to the model.

A linear programming problem has different types of solutions. The **feasible solution** is a solution for which all constraints are satisfied. The feasible region is the collection of all feasible solutions. It is possible that there is no feasible solution to a problem. This would be the case if in the linear programming example from the previous section at least 110 animals were treated with anthelmintic B. If there are feasible solutions, linear programming should find which one is best, measured against the value of the objective function of the model. An **optimal solution** is a feasible solution that results in the most favourable value of the objective function. So, the optimal solution maximizes or minimizes the objective

function over the entire feasible region. Most problems will have just one optimal solution. It is possible, however, to have more than one. This would be so in the example above if the effectiveness of treatment B was modified such that all points on line segment EC' in Figure 6.1 would be optimal.

6.2.2 Solving procedure

For the sake of convenience, a precise set of mechanical rules has been developed to solve a linear programming model. These rules specify each step that is to be taken during the solution process, and are actually a trial-and-error procedure for problem solving. However, they have been constructed in such a way that each trial results in an improved answer. The rules also guarantee that, if an optimal value exists, it will be found within a finite number of steps (Heady & Chandler, 1958).

The mechanical rules for solving linear programming problems are collectively known as the **simplex method**. The previously discussed characteristics of a linear programming model and the example presented will be used to provide a basic understanding of the simplex procedure¹.

It was shown earlier that the optimal combination of treatments with A and B occurred at the boundary of the feasible set at the point of intersection between the constraints with respect to the maximum amount of money to be spent and the maximum number of animals to be treated. This intersection is referred to as a 'corner point'. It can be proven mathematically that the optimal solution will always be at a corner point. Thus, to determine the optimum, the only points that need to be investigated during the trial-and-error process of the simplex method are the corner points at the boundary of the feasible set. This is exactly how the mechanical rules of the simplex procedure operate: they search the corner points of the boundary of the feasible set in a sequential fashion. For example, the procedure starts at the original corner point of Figure 6.1 and moves along the axis of treatment with A to the corner point denoted by D'. Profit is evaluated at that point and then the next corner point of the feasible space, corner point E, is investigated. Once corner point E has been evaluated, the simplex method investigates the possibility of moving to corner point C. Since corner point C has a lower objective function value than corner point E, the procedure will stop and declare corner point E the optimum. The mechanical rules of the simplex method are structured such that each following corner point should result in a higher value of the objective function. Once a corner point that has a lower value is reached, further investigation is unnecessary because no other corner point in the feasible set has a higher value of the objective function, so an optimal solution has been found.

Information on the economic contribution of the various resources to the measure of performance (Z) is extremely useful. The simplex method provides this information in the form of **shadow prices** for the respective resources.

¹No time will be devoted to discussing this method in detail. Those who are interested may consult numerous works which provide detailed discussions on this procedure along with examples that can be solved by hand (see, for example, Heady & Chandler, 1958; Boehlje & Eidman, 1984; Dent *et al.*, 1986; Hillier & Lieberman, 1990).

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The shadow price for resource i (denoted by y_i) measures the marginal value of this resource, that is, the rate at which Z would be increased by (slightly) increasing the amount of this resource (b_i).

The shadow prices for the anthelmintic problem are calculated as follows:

$$\begin{aligned}y_1 &= 5 \text{ is shadow price for resource 1} \\y_2 &= 10 \text{ is shadow price for resource 2}\end{aligned}$$

where these resources represent the maximum amount of money the farmer wants to spend on anthelmintics and the maximum number of animals to be treated. These numbers can be verified by checking in Figure 6.1 that individually increasing each b_i by 1 would indeed increase the optimal value of Z by 5 and 10 respectively. For example, the optimal solution, (75, 75) with $Z = 4500$, changes to $(76\frac{1}{2}, 74\frac{1}{2})$ with $Z = 4510$ when b_2 is increased by 1 (from 150 to 151), so that

$$y_2 = \Delta Z = 4510 - 4500 = 10$$

The kind of information provided by shadow prices is especially valuable to the management when it considers reallocations of resources within the organization. It is also helpful when an increase in b_i can be achieved only by purchasing more of the resource. For example, suppose that Z represents profit and the unit profits of the activities include the costs of all the resources consumed. Then a positive shadow price of y_i for resource i means that the total profit Z can be increased by y_i by purchasing one more unit of this resource at its regular price. Alternatively, if a premium price has to be paid for the resource, then y_i represents the maximum premium that will be worth paying. So, based on the shadow prices of a certain resource, it may be decided to increase its amount.

Sensitivity analysis can be used to identify the most sensitive parameters. One parameter at a time is changed and its influence on the optimal solution determined. Typically, more attention is given to performing sensitivity analysis on the b_i and c_j parameters than to that on the a_{ij} parameters. In many cases, a_{ij} values are determined by the technology being used, so there may be relatively little (or no) uncertainty about their final values. Parametric programming involves the systematic study of how the optimal solution changes if many of the parameters change simultaneously over some range. This technique can be used to study the effects of trade-offs in parameter values.

Because of their mechanical nature, the rules representing the simplex method have been computerized so that only the model of the problem needs to be developed and submitted to the computer for 'number-crunching' (optimal solution and sensitivity analysis). Various software packages that can carry out these tasks on mainframe and personal computers are available. Some examples are XA (Sunset Software Technology, San Marino, Ca, USA), LINDO (The Scientific Press, Palo Alto, Ca., USA), MICRO-LP (Scicon Ltd, Milton Keynes, UK) and OMP (Beyers & Partners nv, Brasschaat, Belgium).

6.3 Assumptions of linear programming

Several assumptions are used in linear programming. Four basic assumptions are essential to determine whether linear programming is applicable to a particular problem and whether it will provide a meaningful and precise answer (Heady & Chandler, 1958; Boehlje & Eidman, 1984; Hillier & Lieberman, 1990).

Additivity and linearity in input and output coefficients

The additivity assumption specifies that the total amount of resources used for two or more activities must be the sum of the amounts of resources used for each separate activity. The same assumption applies to products produced. The implication of this is that interaction between activities is not allowed. If necessary, this can be included by adding a new process that reflects the complementarity between two activities (eg, crop rotation).

The assumption of linearity follows directly from that of additivity. Linearity implies that multiplying all inputs used in an activity by a constant results in a constant change in the output of that process. Thus, the production function for an activity is linear. To reflect nonlinear production relationships, these relationships are approximated by linear segments, with each linear segment representing a separate activity or decision variable. The assumptions of linearity and additivity refer to relationships between activities.

Divisibility in resources and products

The divisibility assumption is that activity units can be divided into any fractional level, so that noninteger values for the decision variables are permissible. Frequently linear programming is applied, even if an integer solution is required. If the solution obtained is a noninteger one, then the noninteger variables are merely rounded to integer values. However, the optimal linear programming problem is not necessarily feasible after the variables have been rounded. Even if it is feasible, there is no guarantee that this rounded solution will be the optimal integer one. Because the mathematical procedure requires complete divisibility of inputs and outputs, a practical interpretation of the results requires the judgment of the user.

Finiteness

This assumption sets a limit to the number of alternative processes and resource restrictions that can be included in the analysis. In fact, this number will depend on the software package used to solve the linear programming problem. Most packages will allow thousands of activities and restrictions.

Single-valued expectations

The single-valued expectations assumption essentially eliminates the important dimension of risk from linear programming analysis. This assumption specifies that resource supplies, input-output coefficients, and commodity and input prices must be known with certainty. Although many will argue that this assumption is unrealistic and makes the results suspect, it should not lead to rejecting linear programming altogether. First, the same assumption is

required in many other analysis procedures used in animal health economics, including partial budgeting and gross margin analysis. Second, prices and production coefficients can easily be varied in the linear programming framework, and this 'sensitivity analysis' can illustrate the resource allocation and income impacts of alternative sets of prices and production efficiencies.

The assumptions in perspective

A mathematical model is intended to be only an idealized representation of reality. Approximations and simplifying assumptions generally are required in order to keep the model tractable. It is very common in real applications that almost none of the four assumptions hold completely. If the assumptions of linear programming are considered too restrictive, techniques are available to relax them. Nonlinear, separable and quadratic programming techniques can be used to handle nonlinear functions. Integer programming can be used in situations where fractional amounts of inputs or outputs are not feasible from a technical nor from a practical point of view. Stochastic and quadratic programming procedures can be used to incorporate risk dimensions in the analysis, replacing the single-valued expectations requirement. Applying these techniques results in increased complexity of model construction, reduced model size and higher costs of solving the model.

Exercise

In Chapter 19 you can find a computer exercise similar to the example given in the introduction of this chapter. You will practise the principles of linear programming by allocating the limited resources labour and grass to the competing activities sheep and cows in order to maximize the net returns. You have to define the restrictions to get a graphical representation of this problem. With the objective function you can find the optimal solution. Some sensitivity analyses can be carried out to see the effect of changes in net returns. The whole exercise takes approximately 40 minutes.

6.4 A more realistic application to herd calving pattern

6.4.1 Outline of the linear programming model

Jalvingh (1993) developed a dynamic probabilistic simulation model for dairy herds. The model simulates the technical and economic consequences of decisions concerning production, reproduction, replacement and calving pattern and can be tailored to individual farm conditions. The user has, for instance, the possibility of comparing different calving patterns and studying the effects on gross margin and labour. From this comparison the user has to choose the calving pattern that suits the objectives best. This is not necessarily the optimal calving pattern for that specific farm. The optimal calving pattern of a herd can be derived by combining the results of the simulation model with linear programming. The basic ingredients of the linear programming are the technical and economic results of the twelve so-called **single-month equilibrium herds** (SME-herds). An SME-herd represents a herd in which all heifers calve in the same month. The resulting herd dynamics

are based on biological variables on the one hand (eg, conception rate and oestrus detection rate) and management strategies on the other (eg, insemination and replacement policy). Input variables of performance and prices are combined with the information on herd dynamics to obtain the technical and economic results of each SME-herd. The technical and economic results of any herd can easily be obtained by weighing the results of the twelve SME-herds according to the proportion of heifer calvings per month. The technical and economic results of the twelve SME-herds and the weighing of the SME-herds form the basic ingredients of the linear programming problem. The major results of the SME-herds are given in Table 6.1. The major input variables used to determine the results of the SME-herds are in Appendix 6.1.

The linear programming problem uses the number of heifers calving per month as decision variables. The objective is to choose the number of heifer calvings per month so as to maximize gross margin of the resulting herd, with the restriction that the annual milk production of the herd should not exceed the available milk quota. The following linear programming problem can be formulated:

$$\text{Maximize } Z = \sum_{i=1}^{12} gm_i x_i$$

subject to

$$\sum_{i=1}^{12} mp_i x_i \leq \text{quota}$$

where

- x_i = number of heifers calving in month i ;
- gm_i = gross margin of the SME-herd expressed per heifer calving, in case the heifer calves in month i (see Table 6.1); and
- mp_i = milk production of the SME-herd expressed per heifer calving, in case the heifer calves in month i (see Table 6.1).

As can be seen, the coefficients of the objective function and the constraint are derived from the results of the SME-herds. For that reason, the results of the SME-herds are expressed per calving heifer (Table 6.1).

The optimal solution of the linear programming problem represents the optimal **heifer** calving pattern. The optimal **herd** calving pattern and the corresponding technical and economic results can be derived by weighing the results of the SME-herds according to the optimal heifer calving pattern.

The linear programming problem just presented is referred to as **set I**. This set can be extended by adding other constraints. The optimal herd calving pattern was also determined for two different sets of additional constraints.

Table 6.1 Results of SME-herds

	Single-month equilibrium herd											
	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Annual culling rate(%)	37.4	40.1	40.8	38.9	35.8	34.3	33.0	31.7	31.6	30.0	31.7	33.9
Calving interval (days)	370	370	371	372	372	372	372	373	372	371	370	370
<i>Per calving heifer</i>												
Number of cows	2.67	2.49	2.45	2.57	2.79	2.92	3.03	3.16	3.17	3.34	3.16	2.95
Number of calvings	3.12	2.95	2.94	3.07	3.30	3.44	3.55	3.67	3.67	3.78	3.59	3.38
Milk production (kg)	19266	17825	17339	17969	19375	20110	20803	21750	22113	23740	22747	21336
Gross margin (US\$)	6364	5844	5702	6021	6626	6964	7266	7605	7705	8246	7783	7178
<i>Per average cow</i>												
Milk production (kg)	7214	7150	7069	6997	6940	6893	6861	6891	6978	7116	7209	7242
Gross margin (US\$)	2383	2344	2325	2344	2374	2387	2396	2409	2431	2472	2467	2436
<i>Per 100 kg of milk</i>												
Average monthly deviation in base price (US\$)	0.38	-0.63	-0.66	-0.49	-0.22	0.12	0.47	0.73	0.87	0.80	0.47	0.02
Gross margin (US\$)	33.03	32.79	32.89	33.51	34.20	34.63	34.92	34.97	34.84	34.73	34.22	33.62

In **set II**, an additional restriction is used, which specifies that all replacement heifers entering the herd should come from heifer calves that were born in the herd 24 months before. This can be formulated in twelve constraints:

$$\sum_{i=1}^{12} f_j x_{ij} x_i \geq x_j, \text{ for all } j$$

where

- y_{ij} = number of herd calvings in month j in SME-herd corresponding to one heifer calving in month i ; and
 f_j = factor representing the number of 24-month-old replacement heifers per calving in month j that becomes available in month j 2 years later (f_j is set at 0.4 for all months).

All replacement heifers are assumed to calve at the age of 24 months, but this age at first calving can easily be changed to include variation.

A concentration of calvings within a few months results in a large variation in the monthly herd size. In **set III**, variation in monthly herd size is restricted by using a lower and upper limit between which monthly herd size is allowed to vary. The limits are formulated in terms of a proportion of the average annual herd size. In formula:

$$\sum_{i=1}^{12} h s_{ij} x_i \geq \sum_{i=1}^{12} \min a h s_i x_i, \text{ for all } j$$

$$\sum_{i=1}^{12} h s_{ij} x_i \leq \sum_{i=1}^{12} \max a h s_i x_i, \text{ for all } j$$

where

- $h s_{ij}$ = herd size of SME-herd in month j , in case one heifer calves in month i ;
 $a h s_i$ = average annual herd size in SME-herd, in case one heifer calves in month i (see Table 6.1);
 \min = lower limit of the variation in herd size per month, expressed as a proportion of the average annual herd size; and
 \max = upper limit of the variation in herd size per month.

The lower and upper limits for variation in monthly herd sizes are set at 95 and 105% of the annual average herd size respectively. The constraints used in set II also hold for set III.

6.4.2 Results

The optimal heifer and herd calving patterns for the different sets of constraints are presented in Table 6.2, together with the technical and economic results of the herds.

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Table 6.2 Results of the optimum herd calving pattern for different sets of constraints

	Set of constraints		
	I	II	III
Milk production herd (kg)	500000	500000	500000
Calving pattern heifers (%)			
January	0.0	0.0	0.0
February	0.0	0.0	0.0
March	0.0	0.0	0.0
April	0.0	0.0	0.0
May	0.0	0.0	0.0
June	0.0	0.0	13.7
July	0.0	23.9	21.8
August	100.0	32.4	23.7
September	0.0	37.2	10.2
October	0.0	6.5	19.6
November	0.0	0.0	9.6
December	0.0	0.0	0.0
Calving pattern herd (%)			
January	1.6	2.1	2.6
February	0.9	1.1	1.5
March	0.5	0.6	1.0
April	0.6	0.8	1.6
May	1.8	2.1	4.3
June	4.9	5.5	10.6
July	13.1	16.4	15.0
August	43.2	22.2	16.4
September	14.5	25.5	15.5
October	10.2	13.2	16.8
November	5.6	6.6	10.1
December	3.1	3.9	4.6
Average number of cows	72.6	72.1	71.7
Range herd size (% of average)	87-117	90-113	95-105
Number of calvings	84.3	83.7	83.0
Annual culling rate(%)	31.7	31.8	32.0
Calving interval (days)	373	372	372
Milk per average cow (kg)	6891	6932	6972
Average monthly deviation in base price milk (US\$/100 kg)	0.73	0.73	0.59
<i>Economic results (US\$/100 kg of milk)</i>			
Revenues			
- milk	46.91	46.87	46.66
- calves	3.41	3.33	3.31
- cullings	3.85	3.86	3.88
Costs			
- feed	12.45	12.43	12.36
- heifers	6.74	6.72	6.70
Gross margin	34.98	34.91	34.78
Gross margin herd (US\$)	174913	174551	173922

As expected, the highest gross margin per 100 kg of milk is realized when only the milk production of the herd is restricted (set I). In that case, all heifer calvings take place in August, which could be expected from the information presented in Table 6.1. The resulting herd calvings, including heifer calvings, take place mainly from July to October. The proportional monthly milk production varies from 4.3% in June to 11.3% in September. The variation in monthly milk production is much smaller than the variation in monthly herd calvings. The monthly herd size, expressed as a percentage of the average annual herd size, varies from 87% in July to 117% in August.

If the number of heifers calving per month is restricted by the number of heifer calves born in the herd in each month (set II), heifer calvings occur from August to October. The resulting herd calvings are still concentrated in the period from August to October. The gross margin is reduced by only US\$0.07 per 100 kg of milk, which is US\$362 at herd level. The reduction in gross margin is a result of the reduction in milk and calf revenues. The milk revenues are reduced because of the reduction in average-realized monthly deviation in the base price of milk, whereas the revenues from calves are reduced because of the reduction in the number of calvings in the herd. In set II, the monthly herd size varies from 90% in June to 113% in September of the average annual herd size (Table 6.2).

In set III, the monthly herd size is restricted to vary between 95 and 105% of the average annual herd size, resulting in an optimal heifer calving pattern that is spread over a longer period than in set II. The gross margin is reduced by US\$0.20 per 100 kg of milk compared with set I, which equals US\$991 at herd level.

The optimal herd calving pattern can also be determined for herds with a lower reproductive performance, or different prices, performance etc. Only a few constraints have been demonstrated in this chapter. However, it is possible to include other constraints, such as restrictions on roughage supply or available labour, as well. The objective function of the problem can also be modified. Gross margin of the herd can be maximized while herd size rather than the annual milk production is restricted (ie, for a situation without a milk quota system).

6.5 Concluding remarks

Linear programming is a very useful tool in finding the optimal solution for complex problems. The technique has not much been applied yet in animal health economics. The reason for that might be unfamiliarity with the technique or that it is not considered useful. Linear programming has several clear underlying assumptions (limitations). Many modifications have been made to the technique to deal with these limitations, such as mixed-integer, nonlinear and quadratic programming. As a result of the advances in computer technology (higher speed and larger memory capacities), computing facilities are now easily available for everybody. Furthermore, recently developed interactive menu-driven packages may greatly facilitate the application of these techniques.

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Appendix 6.1

See Jalvingh (1993) for more details on input variables and for a complete overview. The given input values are assumed to represent typical Dutch herds, but they can easily be modified to suit other farm and price conditions.

Herd dynamics model

Proportions of first inseminations for months 2 to 5 after calving are 44, 41, 11 and 4% respectively. After second calving and later these proportions are 49, 38, 10 and 3%. Conception rate after insemination depends on lactation number. Conception rate per lactation number weighed according to an average herd composition results in 62%. Oestrus detection rate is 70%. Probability of involuntary disposal is 12% in lactation 1 and increases to 23% in lactation 10.

Performance model

In Table A6.1 the base prices of milk, calves, replacement heifers and carcass weight are presented, together with the monthly deviation in prices. In Table A6.2 energy content and price of grass, silage and concentrates are presented. In summer (May-October) cows feed on grass and concentrates. In winter the ration consists of silage and concentrates.

Table A6.2 Energy content and prices of different kinds of feed

	Energy content (VEM) ^a	Price (US\$/1000 VEM)
Grass	951	0.122
Silage	850	0.167
Concentrates	1045	0.194

^a VEM = Dutch Feed Unit; 1000 VEM = 6.9 MJ NE_L

Table A6. 1 Base price and monthly deviations in prices of milk, calves, replacement heifer and carcass weight

	Monthly deviations in price (US\$)											
	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Milk, 100 kg ^a	-1.61	+1.00	-0.39	-0.83	-2.50	-2.50	-2.50	-2.06	-0.39	2.94	+4.06	+3.89
Calves, kg ^b	5.86	-0.83	-0.92	-0.09	0.98	1.09	+1.14	+0.44	-0.04	-0.27	-0.4	-0.54
Replacement heifer	1444	-21	-37	-25	7	+7	+15	+26	+24	+36	+11	+13
Carcass weight, kg ^c	3.56	-0.13	0.03	+0.04	+0.18	+0.18	+0.13	+0.08	+0.06	-0.02~	-0.12	-0.17

^a Prices of milk fat and protein are US\$4.44 and 8.06 per kg respectively. Price of 100 kg of milk is based on the negative base price, the monthly deviation in base price and the price of fat and protein contents in 100 kg of milk. Mature equivalent milk production is set at 7750 kg of milk, 4.35% of fat, and 3.39% of protein.

^b Refers to male calves. Base price for heifer calves is US\$3.67.

^c Refers to price of a kilogram of carcass weight of a first parity cow 210 days in lactation.