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The Relation between Crop Yield, Nutrient Uptake, Nutrient Surplus and Nutrient Application

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Abstract

This study reexamines the hypothesis that if the availabilities of several limiting nutrients are raised simultaneously and proportionally, the nutrients are most productive at high, rather than at low input rates. It was concluded that in most cases rather the opposite holds.

It was argued that crop production as a function of different nutrient inputs may best be described by the theory of Liebscher, and that this theory may be represented satisfactorily by a Michaelis-Menten mathematical model. A theoretical derivation of this model is presented. From the Michaelis-Menten model it follows that both the "total productivity" (kg product/kg available nutrients) and the "system productivity" (kg product/kg applied nutrients) decrease when the nutrient application rate is raised. The optimum productivity of crops in which the harvest index decreases concomitantly with decreasing production (e.g. cereals) the optimum productivity will not be found at zero nutrient availability but at a rather low value.

An analysis of data sets from the literature reveals that the Michaelis-Menten model is reasonably empirically valid, not only in terms of the relationship between availability of different nutrients and uptake, but also in terms of the relation between uptake and yield. Thus by mathematical inference, the model is also valid for the relation between nutrient availability and yield.

In empirical data the relation between production and (weighted) sum of proportional application of several nutrients is rarely S-shaped, but is usually an increasing curve with an declining slope. A secondary analysis of data from some experiments with production factors other than nutrients indicates that the Michaelis-Menten equation has even broader applicability (CO_2, water, radiation).

On the basis of the Michaelis-Menten model it is also expected that the nutrient surplus per kg product will increase with an increase in nutrient availability. It is demonstrated analytically that this relation applies not only to individual nutrients, but also to the sum of proportionally available nutrients. Only in the case of a linear model the result was a constant surplus per kg product.

In empirical data, the *observed nutrient residue* often appears to be lower than the *calculated surplus*, because of feedback mechanisms between crop and

soil and other processes within the soil. In some cases in the range of zero to medium application, the residue per kg product decreases when increasing external nutrients, but this seems only to occur in crops with an extensive, long and active root system.

If the prices of nutrients are relatively low and, at the same time, the prices of products, labour and land are high (as at present in Western countries), profit is highest at very high nutrient applications per ha. This implies that high external input agriculture on a limited area of the best soil available may be advisable for reasons of farm economics. This strategy, however, runs counter to ecological arguments in favour of lower nutrient application per kg product and less nutrient surplus per kg product. How to reconcile the divergent goals of resource productivity, pollution, economy, and sustainability, appears to be a political problem.

From the different models different indications follow whether extensification or intensification of farming practices, at regional level as well as at field level, is more appropriate if the aim is (for economic reasons) a certain target of production and (for environmental reasons) a maximum nutrient productivity. It was derived that if the Mitscherlich model is appropriate, the best way to reduce production is to maintain high application rates per ha and take the less endowed fields out of production. However, from the Michaelis-Menten model (Liebscher theory) follows that it may be better to reduce production by reducing the application of nutrients per ha and keeping all categories of soil quality in production. It was argued that low levels of available nutrients should not necessarily be associated with unharmonious ratios between them (the practice of best ecological means). Harmonizing the ratios of nutrients increases productivity; increasing the magnitudes of them does the opposite.

1 Introduction, problem definition, research questions

In the last decades serious continuity problems have arisen in West European agriculture. In several respects there is an imbalance. In the sector of controlled market products, in some crops, large production surpluses have occasionally been produced. Moreover, the production increase has been accompanied by considerable pollution of the environment by nutrients and biocides. Even if government plans are realized in the Netherlands, concentrations of nitrogen in surface water and emission of ammonium is unacceptable high (WRR, 1992a, p. 26 and p. 124). Hence a proper policy with nutrients is of utmost importance. Especially as a result of the study "Bases for choices" (Grond voor keuzen, WRR, 1992b), the discussion about the agronomically, economically and ecologically most efficient method to attain a temporary reduction of crop production, got a new impetus. Scientists and agro-politicians do not agree at all about the best approach.

On the one hand, in some literature (e.g. WRR, 1992b, p. 77) it was stated that intensive high external input agriculture has advantages over low external input agriculture. These advantages do not only concern the conceived contribution to a high world food production or farm economic advantages. When applying relatively high rates of well-balanced resources, resources are thought to be used more efficiently. The productivity (quotient of output and input) is higher and less surplus per physical unit of product is produced, than with a low external input agriculture (for instance DTO, 1995, p. 14/15). According to these authors, not only is the fraction of the available nutrients taken up as high as possible, but the production per kg of nutrient uptake is also higher and the nutrient surplus per kg product lower. If uptake of a particular nutrient increases then uptake of other nutrients (if available) will increase too. It is plausible e.g. that in case of increasing one specific limiting nutrient, the roots of the crop expand. This results in increasing availability of other nutrients, mineralization might increase and leaching diminishes - a chain of more favourable events is evoked, thus improving the recovery. At incremental doses of a fertilizer, in which all nutrients are present in harmonious proportions, each additional kg of this fertilizer will give a greater production increase than the preceding kg. The increased productivity continues up to a certain optimum. If other essential production factors are improved concomitantly, this optimum will be at a high availability of nutrients, rather than at a low one. De Wit (1994, p. 46) calls this "increasing returns to

scale". This application of "best technological means" also includes site specific and time specific fine tuning of management.
The policy advice that has been connected to this view, is that low external input agriculture should not be stimulated; not only because of economic arguments, but also because of arguments of inefficient resource utilization. If fertilization is profitable, then it follows from the reasoning above, that large amounts should be used rather than small amounts. So, if national or regional production should be reduced, this can be achieved best by setting aside the poor land areas and pursuing high production on the fertile land. Only in situations where a high nutrient residue per ha of land is harmful, low external input may yet be advised.
This we suppose as the reasoning behind "best technological means" agriculture.

On the other hand, in other literature (Parlevliet, 1993, p. 3; Middelkoop et al., 1993; Van der Meer, 1994; De Vries et al., 1997, p. 12; Van der Werff, 1993), another view is presented, on the basis of farm data. A high production, generally attained by means of a high application of nutrients, is accompanied by inefficient nutrient utilization and a considerable pollution of the environment. They conclude that a high application of nutrients cannot be defended with arguments of environmental friendliness. Supposedly, a high application rate is associated with a high concentration of soluble nutrients in the soil. This not only enhances the rate of crop growth, but may raise the relative rate of nutrient loss as well, especially if a considerable spatial and temporal heterogeneity in the production system occurs. In recent publications attention is drawn to the relevance of environmental variation with respect to resource productivity and resource emission (Almekinders et al., 1995; Goewie, 1995; Van Noordwijk & Wadman, 1992).
Moreover, in the situation of high availability of external nutrients, the root system may only develop superficial, as Donald (1951) noted at a very high density in Wimmery ryegrass *(Lolium rigidum)*. This high availability of external nutrients brings about inefficient utilization of the "internal nutrients", which remain in deeper layers of the soil. Besides, a high availability of nutrients often causes "luxury consumption" of nutrients by the crop. These redundant nutrients may even be harmful for consumption of the crop because of an excess of nutrients (e.g. nitrate), which may reduce the (fraction of) marketable product.
This we suppose as the reasoning behind "best ecological means" agriculture.

Given those divergent opinions, the matters of nutrient application, nutrient uptake, production and nutrient surplus require a new analysis. The central issue is which manner of total yield reduction would most reduce the application of nutrients needed and the emission to the environment:

a) reducing the productive land area and maintaining or even enhancing application rates and production per ha on the remaining crop land? or,
b) reducing the application rates and production per ha and maintaining the current area of cultivated land.

This is a research question both at the farm and the regional level of aggregation. The answer to this question is dependent on the production functions at the plant and plot level, and on the relation between these relations at the plant and plot level at the one hand and at the farm and regional level on the other. An important issue regarding the relation between nutrient input and production is, whether increasing or decreasing productivities are found if the availabilities of several nutrients are **proportionally raised** (if the availability of one nutrient is raised by a factor f, the other nutrients are also raised by a factor f, no less and no more). Because most theoretical and empirical research concerning these relationships is done at the aggregation level of the small experimental plot, they are first elaborated at that level and next the consequences will be drawn for the regional level.

We pose the following queries:

- Will, according to theory, nutrient productivity be enhanced or reduced when the availabilities of several nutrients are increased proportionally? (Chapter 3)
- Do empirical data corroborate the hypothesis that nutrient productivity increases when the availability of several nutrients is increased proportionally? (Chapter 4)
- To what extent should a distinction be made between production enhancement by innovations and/or by increase of nutrient application? How can the productivity discussion be clarified by differentiating nutrient input into internally available and externally applied nutrients? (Chapter 5)
- Is current high external application in farm practice a consequence of the entanglement of the concepts of *agronomic nutrient productivity* and *farm economic profit*? (Chapter 6)
- To what extent do the answers to the questions change if the analyses are scaled up from the plot level to the regional level? (Chapter 7)
- To what extent does maximum nutrient productivity correlate with minimum nutrient surplus, and how far are the conclusions for surplus identical to those for productivity? and for plot and regional level? (Chapters 8 and 9)
- Do empirical data support the Michaelis-Menten theory with regard to nutrient surplus? (Chapter 10)
- What are the implications of the results of this research for nutrient management and agricultural policy? (Chapter 11)

In order to elaborate these research questions it is crucial to carefully elaborate the definitions of the concepts, and the terms and symbols used. In this study, eco-physiological, agronomical, economic and environmental aspects of nutrient use are to be integrated. Terms such as productivity and efficiency are defined differently in these different disciplines. An important term in this respect is "Resource use efficiency" which is replaced here by "Resource productivity". This gives the opportunity to reserve the term "efficiency" for the concept: "quotient between an observed actual performance and the theoretical maximal performance", or "quotient between an observed actual performance and the observed maximum performance" (apparent best performance), as used in economics.
In the next chapter the research material on the nutrient productivity problem mentioned above is introduced, together with the concepts used throughout this study. In the subsequent chapters the questions are theoretically elaborated and empirically tested.

2 Research material and methods of analysis

The research material of the study consists of:

- revisiting different theoretical concepts in the literature, particularly Von Liebig, Mitscherlich, Liebscher, Michaelis-Menten, De Wit.
- reanalysis of data from various investigations, among others: Greenwood (1971), Penning de Vries et al. (1982) and Nielsen (1963).

The following methods have been used:

- derivations of features of the models by means of mathematical analysis, such as common algebra and differential calculus,
- numerical simulation (and graphical representations) of the model relations by means of numerical simulations with a complete model of the relations between application, availability, uptake and surplus of three nutrients, dry matter yield, harvest index and profit,
- statistical methods of empirical testing.

The relations are simulated with three theories: the Von Liebig, the Mitscherlich and the Liebscher theories, for three macronutrients: nitrogen, phosphorus and potassium and for four or five fixed levels of maximum production capacity. The results were obtained with a nitrogen availability range of 0 - 500 (kg N/ha)) and keeping the levels of the other nutrients constant (three P levels and three K levels), but also under the assumption of keeping the levels of the other two nutrients proportional to the nitrogen level. The ratios between the different nutrients remain constant then, e.g. doubling N from 50 to 100 kg per ha is accompanied by doubling of P from 10 to 20 kg per ha and doubling K from 25 to 50 kg per ha [1]). The levels of internally available nutrients, as well as the prices of products, the response coefficients of uptake with respect to nutrient availability, of yield with respect to nutrient uptake and the ratios between the three types of nutrient may be varied.
The results of the calculations are applied throughout this report, especially for the graphical representations of the model relations.

In Appendix 12.1 the definitions of all concepts and variables in this study are given. In Appendix 12.2 some definitions of **nutrient productivity** and **nutrient surplus productivity** are elaborated. The model relations and coefficients are given in Appendix 12.3 and 12.4 respectively. In Appendices 12.6 - 12.12 mathematical inferences have been made for check and better understanding of the results found by simulation. The symbols are as much as possible in agreement with Van Noordwijk & Wadman (1992) and Vos et al. (1997). At the end of the report notes on specific concepts in the text may be found.

3 Nutrient productivity and theoretical models

3.1 Introduction

For the relation between application of nutrients and production several mathematical functions are found in the literature. Mostly these functions do not only give a partial description of the observed empirical reality, but are also more or less incomplete representations of intended mental or verbal theories. The formal models may function as a bridge between theory and observed reality.

Most production functions have the character of saturation curves, which belong to the same family of mathematical functions (Goudriaan, 1979, p. 783). These functions may be formalizations of different plant-physiological or ecological concepts. However, they may also be regarded as variants of the same underlying dynamic theories at lower levels of aggregation (De Wit, 1993, unpublished). Both errors in observation and errors in formalization make it very difficult to decide about the validity of theories.

Here only the theories (and models) of Von Liebig (1855), Mitscherlich (1924), Liebscher (1895) and Michaelis-Menten (formalization of Liebscher theory) are dealt with. ***In this chapter the question is analyzed whether, departing from these different theoretical models, increased or decreased productivity is to be expected when availabilities of several nutrients are increased proportionally. In Chapter 4 the empirical validity is treated.***

3.2 Von Liebig

In the Von Liebig "model of the minimum" [2]), production is determined solely by the production factor that is "at its minimum". The production increases proportionally to an increase in that minimum factor, up to the point at which another factor becomes limiting. From that point on, the production may be increased by raising that other new limiting factor, but not any further by raising the first factor. So any factor may become limiting under changing conditions of the other factors.
For three factors N, P, and K the Von Liebig model can be mathematically represented by the following equation (if error terms are neglected):

$$\mathbf{Y = MIN\{MY\ ,\ \alpha_V{*}N\ ,\ \beta_V{*}P\ ,\ \gamma_V{*}K\}}$$

in which:

Y	=	Yield (synonym = production): primary biological production at prevailing values of N, P and K, expressed in (kg dm/ha).
MY	=	Maximum production: attainable production if nitrogen, phosphorus and potassium do not limit the production (kg dm/ha).
MIN(.)	=	A logical function selecting the minimum outcome of any expression, separated by commas, in the brackets.
N,P,K	=	Amounts of available (internally delivered + externally applied) nitrogen, phosphorus and potassium in (kg/ha).
$\alpha_V, \beta_V, \tau_V$	=	Coefficients of response of **production** to the **availability** of nitrogen, phosphorus and potassium, expressed as increase of yield per kg increase of available nutrient. (kg dm/kg nutrient).

The model relations are represented schematically in Figure 3.2.1.

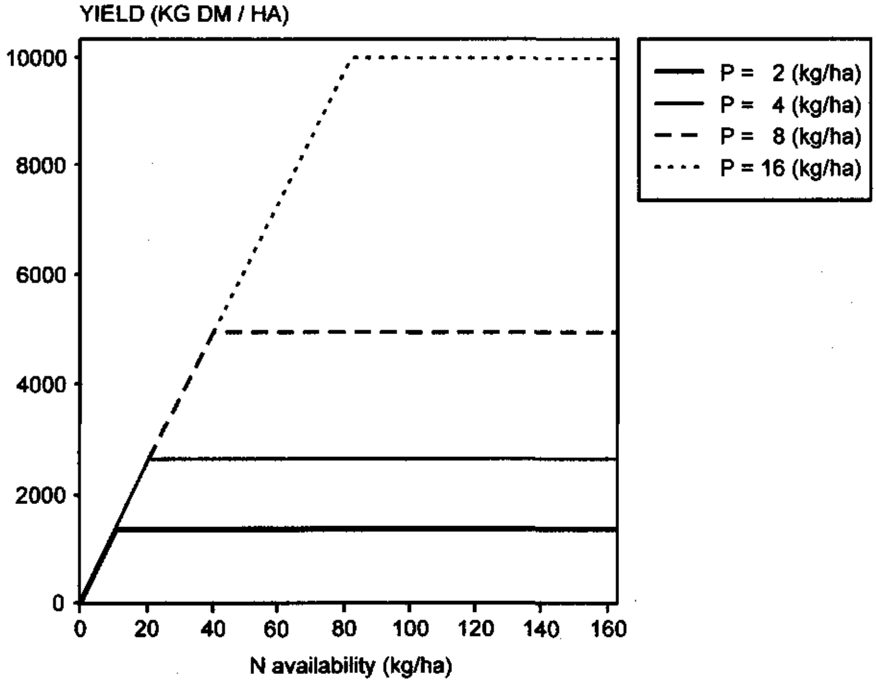


Figure 3.2.1 Relation between crop production and the separate availability of two resources (N and P) in the Von Liebig model. The values of the coefficients are: α_V = 125 kg dm/kg N, β_V = 625 kg dm/kg P, MY = 10000 kg dm/ha.

The Von Liebig model seems to be appropriate in homogeneous situations at the plant-physiological level, especially for growth factors which are material constituents of biomass, such as nitrogen, phosphorus, potassium. At the level of the individual plant, and at low availability levels, doubling the availability of a certain input then produces a double yield. According to Wallace (1989, p. 469): "... severe deficiencies are generally of the Liebig type, and slight deficiencies of the Mitscherlich type ..." and, according to the same author "... little or no response can be expected to inputs to correct Mitscherlich type limiting factors until those of the Liebig type are removed...".

In the Von Liebig model production factors cannot be substituted by other factors. The model has a kind of "discontinuous interaction" among production factors. One unit of a factor has an influence on production, which is a maximum influence or no influence at all, dependent on the level of other factors. In the theory of Von Liebig productivity will not increase under the conditions of an increase of proportionally available production factors.

In this formalization of the Von Liebig theory, production and factor availability are linearly related. That linear relation, however, is actually not an implication of this theory. Theories, at least the older ones, have mostly been formulated in verbal terms which leave open a number of possibilities for formalizations.

In the well known "barrel with staves" representation of the Von Liebig theory, production is indeed a linear function of the availability. But linearity is not essential in the physical model; it depends on the form of the barrel. There are also non-linear representations of the Von Liebig theory (Paris, 1992, p. 1021; De Wit 1994, p. 42). In Figure 3.2.2 the Paris formalization of the Von Liebig theory is represented. What is essential for the Von Liebig theory is not the linearity of the relations, but the non-substitutability of the factors.

It should be added, however, that the temporal and spatial variations at **higher levels of aggregation** (field, farm, region) mean that at every point in time and at every location, different production factors may be limiting. If so, this causes the aggregate production to respond to changes in a broad set of growth factors, thus creating substitution and interaction effects. This would also be applicable for factors which do not exhibit these effects at the plant-physiological level.These effects may be **statistically summarized** in terms of soil heterogeneity (Appendix 12.6), or they may also be **theoretically explained** in terms of plant physiology. In the literature (De Wit & Van Keulen, 1987, p. 253; Rabbinge & Van Ittersum, 1994, p. 34) there is a serious warning against generalization of models from the micro-level to the macro-level. The consequence is that the Von Liebig model is expected to become less valid as it is scaled up from the plot to the field level or from the field to the farm level and from the farm to the regional level.But it may be possible to start from this Von Liebig model, valid for the micro-scale (at one spot and at one moment), then taking into account the scale effects of variability over time and space, and thus arriving at a model for the macro-level, without being forced to elaborate all system dynamics processes into a very detailed theoretical model. In § 3.4 (Figure 3.4.1) it will be demonstrated that the Von Liebig relation transfers to a Michaelis-Menten relation when accounting for heterogeneity.

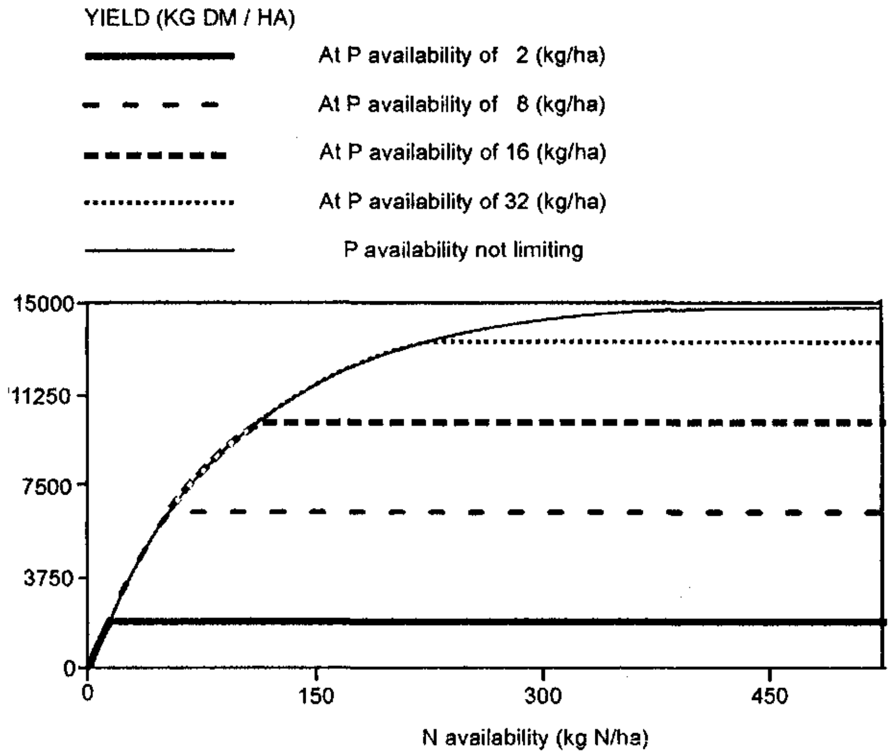


Figure 3.2.2 Relation between crop production and the separate availability of two resources (N and P) in the Paris formalization of the Von Liebig model. The values of the coefficients are: α_P = 0.010 proportion of maximum dm/kg N, β_P = 0.070 proportion of maximum dm/kg P, MY = 15000 kg dm/ha.

3.3 Mitscherlich

In Appendix 12.5, [eq. 6] the general mathematical form of the Mitscherlich model (Mitscherlich, 1924) [3]) and the features which may be inferred from it have been represented. For three nutrients, N, P and K, this theory is formalized mathematically (see also e.g. Meyer (1926/1927, p. 150-151)) as follows [eq. 1]:

$$\mathbf{Y = MY \cdot \{1\text{-}EXP(-\alpha_M \cdot N)\} \cdot \{1\text{-}EXP(-\beta_M \cdot P)\} \cdot \{1\text{-}EXP(-\tau_M \cdot K)\}}$$

in which the new symbols are:

EXP(.) = $e^{(.)}$ (in which e = 2.7... = base of natural logarithms).

α_M = Coefficients of response of **production** to nitrogen **availability** expressed as the proportion of the maximally attainable production that has not yet been realized by means of nitrogen, per kg of available nitrogen. (the subscript M refers to Mitscherlich).

β_M, τ_M = Same definitions for phosphorus and potassium.

The production increase (δY) per unit increase (δN) of nitrogen, being the first derivative ($\delta Y/\delta N$) of yield Y to one of the nutrients, nitrogen, is an important derivative of the model because the optimal amount of input is often chosen as the amount where the marginal productivity equals the price of the last applied unit of nutrient. The marginal productivity is expressed as [eq. 2]:

$$\delta Y/\delta N = \alpha_M \cdot EXP(-\alpha_M \cdot N) \cdot MY \cdot \{1-EXP(-\beta_M \cdot P)\} \cdot \{1-EXP(-\tau_M \cdot K)\}$$

Considering only one variable nutrient, the maximum attainable production ($MY_{P,K}$) at not limiting N and at current levels of P, K and all other factors except N is represented by [eq. 3]:

$$MY_{P,K} = MY \cdot \{1-EXP(-\beta_M \cdot P)\} \cdot \{1-EXP(-\tau_M \cdot K)\}$$

Then for only one variable factor N, eq. 2 is represented by [eq. 4]:

$$\delta Y/\delta N = \alpha_M \cdot (MY_{P,K} - Y)$$

This "short equation" is further elaborated in Appendix 12.5, part C.

From the fundamental assumptions, represented in the mathematical form of this theory, the following features of this model may be noticed:

a A positive interaction between production factors: the effect of a factor on the production is greater when other factors are at a higher level (see eq. 2 and Figure 3.3.1).

b Partial substitution among production factors (the same production may be realized by different combinations of factors).

c Increasing marginal (and average) production (up to a certain availability level) when the availability of a number of inputs are increased proportionally, and beyond that level decreasing marginal production. It means that, as De Wit (1993, p. 6) showed that the relation between production and the increase of several, proportionally available, nutrients produces a **sigmoid production curve** (see Figure 3.3.1). This feature of the Mitscherlich model is mathematically derived in Appendix 12.5, part A.

d For any level of one specific variable production factor (x) (given a fixed combination of other production factors) the marginal production is the same proportion of the difference between maximum and actual production (see eq. 4). For the derivation of this feature see Appendix 12.5, part C.

e Feature of constant ratios between productions. This feature may be derived from the form of the Mitscherlich model, as represented in eq. 1. This is a variant with three nutrients of the general model in Appendix 12.5, eq. 6. The actual production is the same proportion of the maximal production at a **certain level of that specific production factor**, at any combination of other production factors [4]). This implies that all production functions of one variable production factor coincide if the productions are expressed as fractions of the maximum productions at any level of the other production factors. This also means that the responses ($\delta y/\delta x$) are proportional to the maximum productions. See for the derivation of this feature Appendix 12.5 part B.

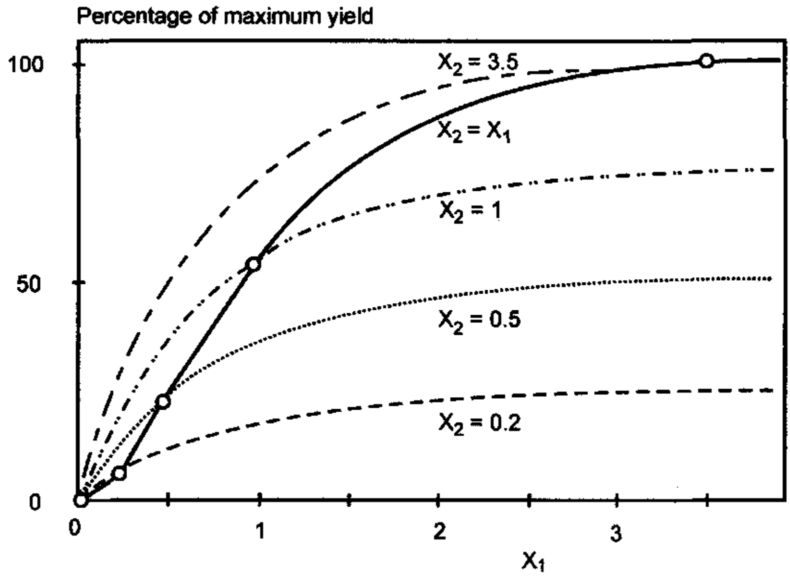


Figure 3.3.1 Schematic representation of the relation in the Mitscherlich model between crop yield and the availability of a resource x_1, both at different constant levels of another resource x_2 (dashed lines) and the sigmoid curve resulting from a proportional availability of these two resources (solid line). (Adapted from De Wit, 1993, p. 6.)

The (in Appendix 12.5 part A mentioned) **S-form** implies that every next kg of proportionally available nutrients raises the **(average) productivity** of the nutrient up to the availability at which the sigmoid curve makes only one point of contact with a straight line through the origin. (N.B. This is not the break point in the sigmoid curve, up to which point the **marginal productivity** [5]) increases (see Figure 12.5.1 in Appendix 12.5). When input is applied beyond the point of maximum productivity, the causes for decreasing productivity become, on the average, dominant over the effects of positive interaction between the nutrients. The effect of increasing productivity at low inputs does not show up very clearly when only few inputs are included. It becomes more apparent as the number of factors involved increases. Moreover, the more factors that are involved, the higher the required nutrient availability that gives maximum productivity (De Wit, 1992a, p. 43) [6]).

The Mitscherlich model, however, does not seem theoretically valid. It is not very plausible that the actual production is the same proportion of the maximal production at each level of the factor concerned, independent of other factors (Van der Paauw, 1938, p. 800; Von Boguslawski, 1958, p. 964). Empirical support for the rejection of this model follows in § 4.3 and in Appendix 12.7.

3.4 Liebscher and Michaelis-Menten

The theory of Liebscher (1895) states that the interaction between inputs increases when the availability levels increase, up to a certain amount. Beyond that, the interaction decreases again. So the activity of a limiting nutrient is more pronounced as the other nutrients are closer to the optimum. This theory is called the "principle of variable endurance" by Van der Paauw (1938). A crop turns out to be relatively more resistant against low levels of a growth factor if other conditions are more favourable.

In the availability range from zero to the optimum, the Liebscher theory may be represented as a Michaelis-Menten equation (also called hyperbolic function). As far as we know, Liebscher himself, primarily following an inductive empirical approach, did not formalize his theory into a mathematical model. Our hypothesis is that the Michaelis-Menten model is an adequate formalisation of Liebscher's theory.

The Michaelis-Menten model (applied for only one substrate factor N) is in the literature mostly represented as:

$$\mathbf{Y = (\alpha.N \cdot MY) / (\alpha.N + MY)}$$

For the meaning of the symbols see Appendix 12.1 and endnote [7]).

Taking the inverses of the left and right parts of the equation gives:

$$\mathbf{1/Y = 1/MY + 1/(\alpha.N)}$$

As far as we know, Michaelis-Menten himself did not generalize his model for more than one substrate factor. For the extension of this model from one substrate to more substrates see Fell (1997, p. 58) and Thornley & Johnson (1990, p. 463, cited by Langeveld (1997, personal communication)). Our model, based on ecological subspaces occupied by the crop, resembles the multi-dimensional "standruimte-model" developed by De Wit (1960). In Appendix 12.6 our agronomic derivation of the Michaelis-Menten formalization of the Liebscher theory is given, which also considers the spatial and temporal variation and co-occurrence of several nutrients in the soil. The model may be written as follows [eq. 4]:

$$\mathbf{1/Y = 1/MY + 1/(\alpha_* N) + 1/(\beta_* P) + 1/(\tau_* K)}$$

This form is equivalent to the Michaelis-Menten function without explicit interaction terms, a form which may be found in Fell (1997, p. 58).
Eq. 4 is not the most customary graphical presentation for production functions, but the explicit function of Y against the nutrient availability is less transparent than the inverse representation:

$$\mathbf{Y = (MY \,_*\, \alpha_* N \,_*\, \beta_* P \,_*\, \tau_* K) \,/\, \{(\alpha_* N \,_*\, \beta_* P \,_*\, \tau_* K)\ \hat{}\ + (MY \,_*\, \beta_* P \,_*\, \tau_* K) + (MY \,_*\, \alpha_* N \,_*\, \tau_* K) + (MY \,_*\, \alpha_* N \,_*\, \beta_* P)\}}$$

We demonstrated that also the reciprocals of production (Y) and of the availability of N, with proportional co-availability of P and K ($N\phi$) are **linearly** related (Appendix 12.8). This feature implies that in the interval - zero to infinity - productivity cannot increase, only decrease. So the maximum productivity (kg product/ kg available nutrient) occurs at an application rate of zero (also if the availability of internal nutrients is not zero).

A qualitative derivation of the Liebscher model from the Von Liebig model may be obtained by taking into account the temporal and/or spatial variations of the substrate on which the crop grows or of other environmental growth factors or of the variability of the biological material itself from which the yield results. For temporal variation this derivation is illustrated in Figure 3.4.1 in which the relation between nutrient availability and yield according to the Von Liebig model has been schematically portrayed with a crop at different points in time. Simple averaging of the different Von Liebig curves produces a curvilinear relationship over the total time span. The more variation over time the more the average production curve departs from linearity and the more it resembles a Michaelis-Menten relation.

An analogous figure can be given for crops with variable genetic production potentials, or for places within the field with different production capacities (Nijland, 1994; Whitmore & Van Noordwijk, 1995, p. 275). Resemblance also exists with the non-linear representation of the Von Liebig model by Paris (see Figure 3.2.2).

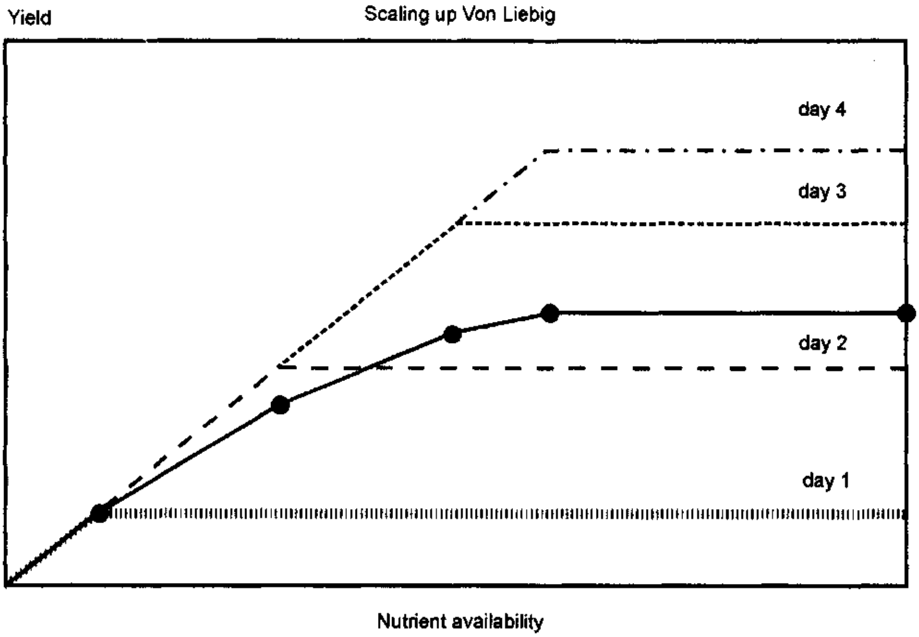
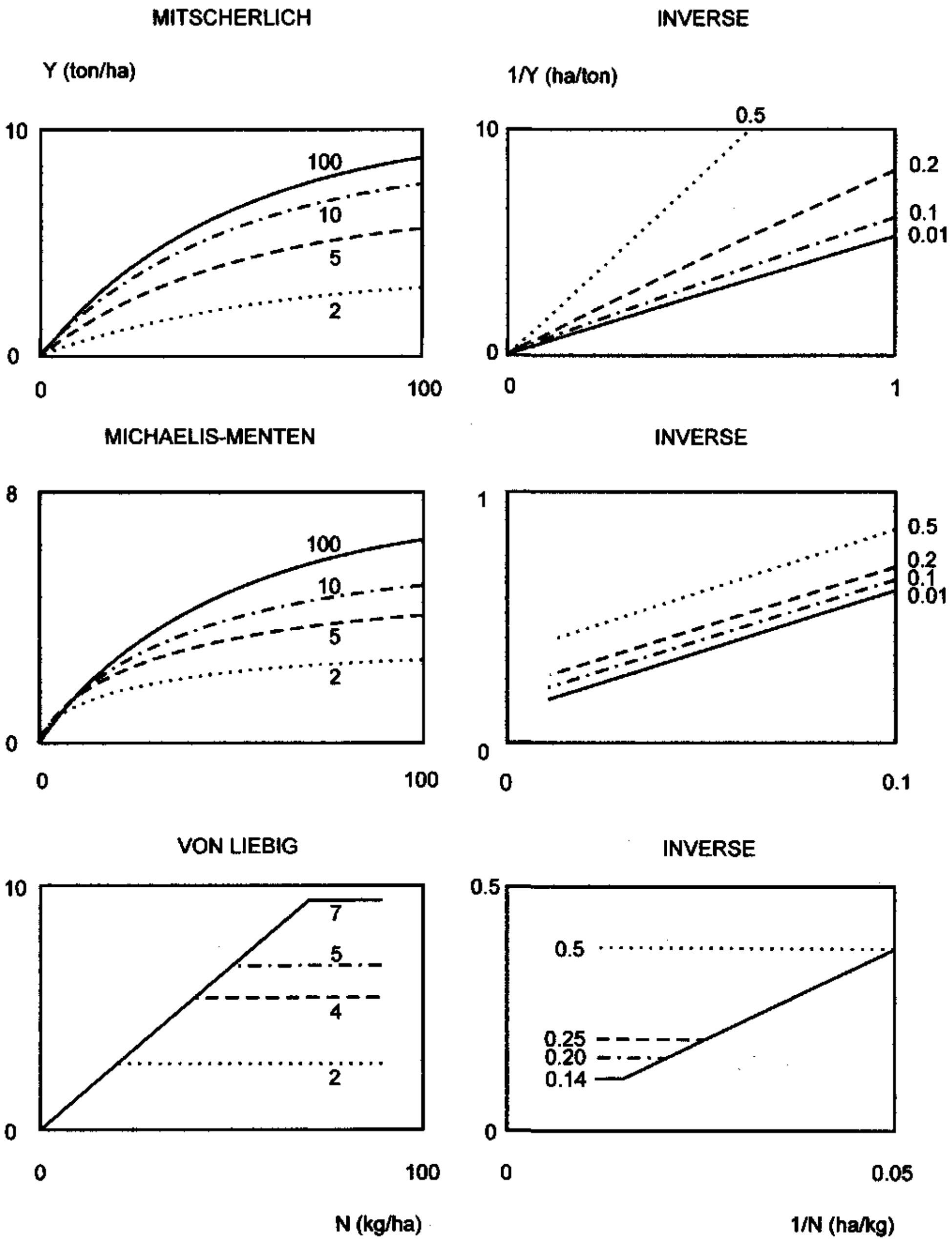


Figure 3.4.1 Schematic relation between yield and nutrient availability on days with different potential production capacities according to the Von Liebig production function. The drawn curve represents the average relation between yield and nutrient availability over 4 days.

In Figure 3.4.2 the Mitscherlich, the Liebscher (Michaelis-Menten formalization), and the Von Liebig models are compared graphically. Each curve of the Mitscherlich function (left plots of Figure 3.4.2), may be derived from another by multiplying with a constant. The curves are nearly coincident in Liebscher at the lower values of available N, almost approaching the lines of a Von Liebig function.

Figure 3.4.2 Yield (Y) as a function of available nitrogen (N) at 4 levels of phosphorus (P) in the Von Liebig, the Mitscherlich and the Michaelis-Menten model. The right side plots give a reciprocal representation of the variables.
The coefficient values are: maximum yield 10 ton dm/ha (all models); P availabilities = 2, 5, 10 and 100 kg/ha (Mitscherlich and Michaelis-Menten) and 2, 4, 5, and 7 kg/ha (Von Liebig); response coefficients $\alpha_M = 0.02$ proportion dm/kg N and $\beta_M = 0.2$ proportion dm/kg P (Mitscherlich), $\alpha = 0.2$ ton dm/kg N and $\beta = 2.0$ ton dm/kg P (Michaelis-Menten), $\alpha_L = 0.125$ ton dm/kg N and $\beta_L = 1.25$ ton dm/kg P (Von Liebig).


MITSCHERLICH
Y (ton/ha)
10
0
100
10
5
2
0
100
INVERSE
1/Y (ha/ton)
10
0
0.5
0.2
0.1
0.01
0
1
MICHAELIS-MENTEN
8
0
100
10
5
2
0
100
INVERSE
1
0
0.5
0.2
0.1
0.01
0
0.1
VON LIEBIG
10
0
7
5
4
2
0
100
N (kg/ha)
INVERSE
0.5
0
0.5
0.25
0.20
0.14
0
0.05
1/N (ha/kg)

On the other hand, at higher nutrient rates, the Michaelis-Menten curves show more increase than the Mitscherlich curves, which all become saturated at lower availability values than the Michaelis-Menten curves.

The different patterns of the three models become more obvious when the reciprocal values of the availabilities and yields are plotted against each other. In the Mitscherlich model, the curves of the reciprocals are **divergent.** In the Michaelis-Menten formalization of the Liebscher theory they are **parallel**, and in the Von Liebig model they partly **coincide** and are partly **horizontally parallel.** At first glance, the Michaelis-Menten model seems intermediate between the Mitscherlich and the Von Liebig model (at low input levels it shows less positive substitutability between factors than the Mitscherlich model, but more than the Von Liebig model). This does not imply that the optimum availability of an input lies somewhere between the optimum rates of the Mitscherlich model (at an availability far greater than zero) and the Von Liebig model (at availability giving maximum production). The influence of increasing productivity through positive physiological "interaction" is in the Michaelis-Menten model not strong enough to compensate for the decreasing productivity due to:

a) effects of the separate factors approaching a "ceiling of saturation",
b) effects of temporal and spatial soil heterogeneity (imperfectly correlated occurrence of growth factors).

An alternative mathematical formalization of the Liebscher theory, in which increasing productivity is present, just as in the Mitscherlich model, is given by De Wit (1993, unpublished). This concerns a system dynamics model of the underlying physiological nutrient conversion processes. The model is an analogue of a chemical model of two reactants in a vessel. This chemical analogue integrates the Von Liebig, Liebscher and Mitscherlich theories in an extremely elegant, generic system dynamics model. However elegant the model, we think that it has the disadvantage of omitting the concept of substrate heterogeneity within the system. We think that the analogon is theoretically valid for rather homogenous situations in small vessels. This latter homogeneity assumption of the substrate is possibly the cause that the formalization of the Liebscher theory by De Wit shows an increasing productivity with proportionally increasing availability of inputs, while the formalization of Michaelis-Menten does not.

The relation between availability and uptake and the relation between uptake and production.

Claassen et al. (1986, p. 218) argue that the relation between the concentration of nutrients in the soil and the rate of uptake, can often be quantitatively described by Michaelis-Menten kinetics. This is supposedly because the transport of nutrients from soil to plant roots mainly proceeds by mass flow and diffusion processes. In soils this biologically passive diffusion processes seem dominant over the biologically active process of collecting of nutrients by plants through root activity. Also Wadman (1983, p. 14 and 34) found empirical support for the validity of the Michaelis-Menten model.

If a linear relation exists between the reciprocals of availability and uptake (quadrant IV) on the one hand, and a linear relation between the reciprocals of uptake and yield (quadrant I) on the other hand, it can be mathematically inferred that the relation between the reciprocals of availability and yield (quadrant II) is also a linear relation. (See Appendix 12.9, and Figure 4.3.1 for an example of the three quadrant representation). Analogue features of the Michaelis-Menten kinetics of chain reactions has been reported by Fell (1997, p. 50) in the domain of biochemistry.

3.5 Inferences

- The Von Liebig theory appears to be a plausible description of input-output relations for growth factors of the category of constituents of biomass, at low levels of aggregation (homogenous experimental plots).
- An increase of proportionally available nutrients neither improves nor lowers nutrient productivity in the Von Liebig theory. Productivities are constant until the maximal production has been reached.
- For higher levels of aggregation (field, farm, region), models with positive interaction and substitution [8]) between nutrients give a better description of the process of application, uptake and production.
- The Mitscherlich theory implies positive interaction and substitution among nutrients. The positive interaction gives rise to a sigmoid relation between yield and proportional available nutrients. And the implication is that at low availability levels of proportional nutrients there is hardly any production. This and the feature of constant activity, are ecologically not very probable.
- The Liebscher theory is theoretically more appropriate in those situations than the Mitscherlich, and Von Liebig theories.
- The Michaelis-Menten model may be considered as a plausible mathematical formalization of the Liebscher theory. An alternative derivation of the Michaelis-Menten model can be given in terms of multidimensional ecological space, and its heterogeneity. This model may also be regarded as an approximation of a scaled up Von Liebig model.
- If the Liebscher theory (Michaelis-Menten formalization) is valid, then (as opposed to the Mitscherlich model) no increasing but only decreasing productivity occurs at an increasing proportional availability of nutrients.

4 Nutrient productivity and empirical data

4.1 Introduction

In this chapter the question is posed how far empirical data corroborate the hypotheses of increasing or decreasing nutrient productivity when increasing the availability of several nutrient inputs proportionally. Data from different publications (Greenwood, Van Keulen, Nielsen, Mitscherlich) were reanalysed to test the empirical validity of different theoretical models.

4.2 Von Liebig

Most empirical data at the field level of aggregation on production at different combinations of production factors do not correspond to the Von Liebig law. According to the conventional graphical image of the Von Liebig function (Figure 3.2.1) the response curves of a production factor should be linear up to a certain maximum, beyond which the production no longer increases, thus giving rise to a sharp break in the curve. In most empirical data those breaks are not clearly observed. However, as De Wit pointed out (1992a, p. 42) the break is not the essential feature in the Von Liebig model. The linear relation is not inevitable in the theory of Von Liebig, as was explained in § 3.2.

Paris noticed that most empirical data (e.g. the data set of Heady (1961)) are statistically better described by a formalization of the Von Liebig model with non-linear relations, which are coinciding in the lower range of a limiting input (see Figure 3.2.2 for the graphical representation).

However, in many data sets the production curves for different values of other factors only coincide for very low values of an input. At higher availabilities of an input a gradually increasing divergence of production curves may be observed. Apparently, not just a single factor is limiting at a specific availability. Neither the Von Liebig model nor the model of Paris (Figure 3.2.2) accounts for this pattern of diverging curves. The adaptation from Paris seems to be a kind of mixture of the Von Liebig and the Mitscherlich model, which cannot be interpreted theoretically very easily. In terms of empirical validity, however, this model remains a competitor for the Michaelis-Menten model.

The divergence between the Von Liebig model and experimental data may be largely ascribed a) to substrate heterogeneity within the production system (Berck & Helfand, 1990), b) to genetic variability of the crop (Kuhlmann,

1992) and c) to positive interactions between production factors. So, the Von Liebig model has a rather restricted validity.

4.3 Mitscherlich

In Chapter 3 it was argued that both the Von Liebig model and the Mitscherlich model do not properly theoretically explain the crop production proce ss **at field and region level.** The weak theoretical validity of the Mitscherlic h model appears to be revealed as a discrepancy between theoretical expectation and empirical observations in the field (Von Boguslawski, 1958, p. 962, Van der Paauw, 1938). Apart from these citations in the literature, additional tests were performed (Appendix 12.7).

As was theoretically elaborated in 3.3 the Mitscherlich model implies that, if yields are standardized as a fraction of the maximum level, all production curves for one factor coincide. In Appendix 12.7 it is demonstrated for data from Penning de Vries & Van Keulen (1982, p. 196-226) and Mitscherlich (1923, p. 201) that the expected constancy of the quotient could not be shown. In data from Penning de Vries c.s. (De Wit, 1992b, p. 138) it appeared also that a sigmoid curve of increased productivity at proportional N, and P application (as expected from Mitscherlich) could not be demonstrated. This may be observed by inspection of Figure 4.3.1, where the relation between grass production, uptake and application of nitrogen and phosphorus in various combinations, has been presented.

In Figure 4.3.1 (quadrant II of the figures) it may be seen that a curve drawn through proportionally applied amounts of nitrogen and phosphorus does not reveal an application interval of increasing productivity (the curves of production against proportional availability of N and P are not sigmoid curves like in Figure 3.3.1). This also applies to the intermediate relations (**between production and uptake**) in quadrant I and (**between uptake and application**) in quadrant IV). There is one exception (in quadrant IV of the lower plot in Figure 4.3.1): The curve of uptake against application of phosphorus shows increased uptake per kg application. The effect, however, is not transposed to the relation between application and production in quadrant II (upper left). It is possible that this increasing uptake-application ratio is compensated through a simultaneously decreasing production/uptake ratio [9]). This is in accordance with observations in the literature that the relation between application and uptake is sometimes more or less Mitscherlich-like (quadrant IV of the lower plot in Figure 4.3.1), and the relation between uptake and production sometimes more or less Von Liebig-like (e.g. in Van Heemst et al. 1978, Figures 3, 4 and 5).

In the Figures 4.3.2 and 4.3.3 diagrams from some other publications are reproduced.

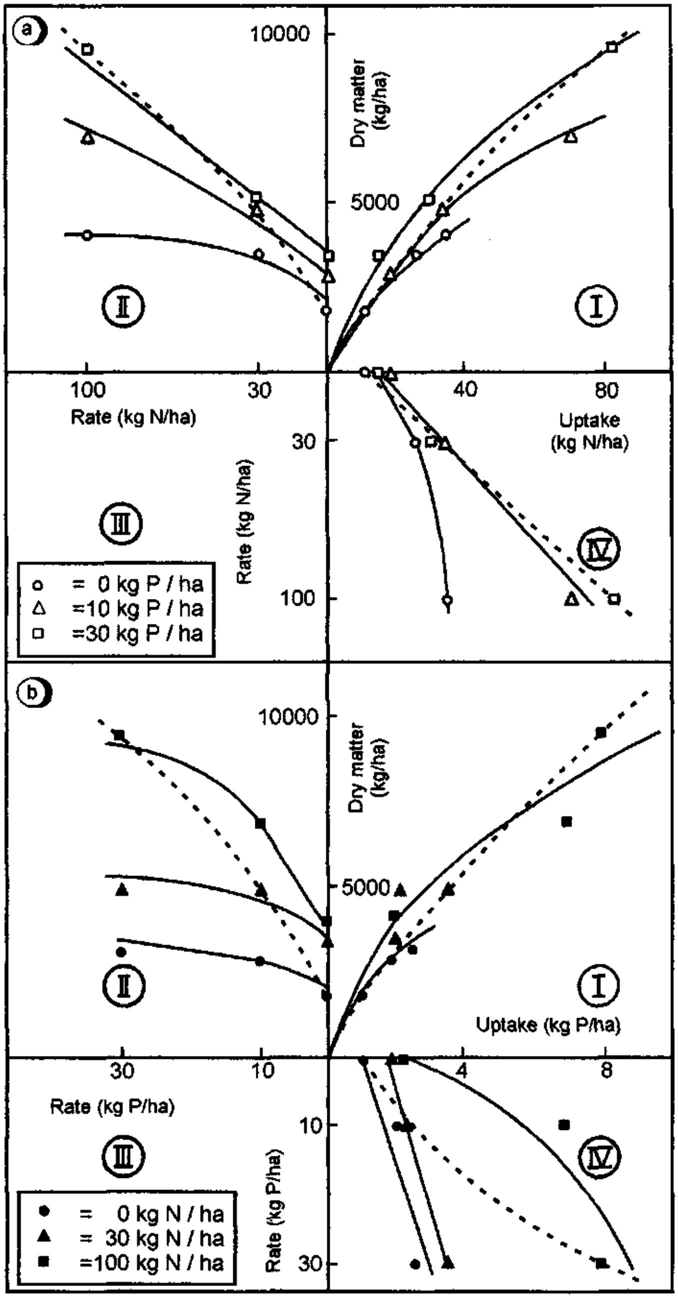


Figure 4.3.1 "Four-quadrant" diagrams for a field trial on natural grasslands; with (a) the three levels of N and (b) the three levels of P, in all nine combinations (Penning de Vries & Van Keulen, 1982, referred to by De Wit, 1992b, p. 138).

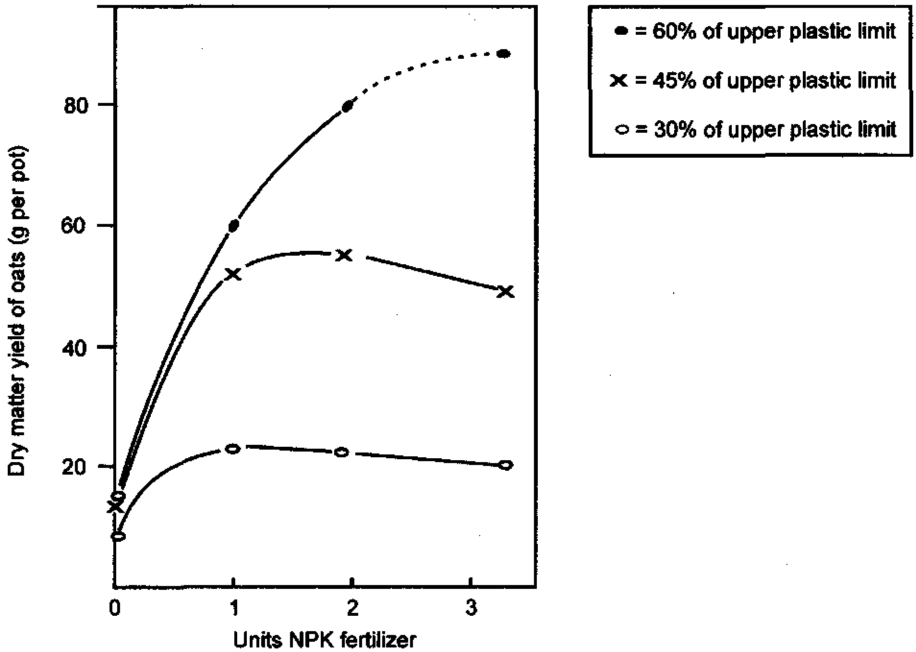


Figure 4.3.2 Relation between production and proportional availability of nitrogen, phosphorus and potassium, for three different situations (after Van Diest (1971, p. 25)).

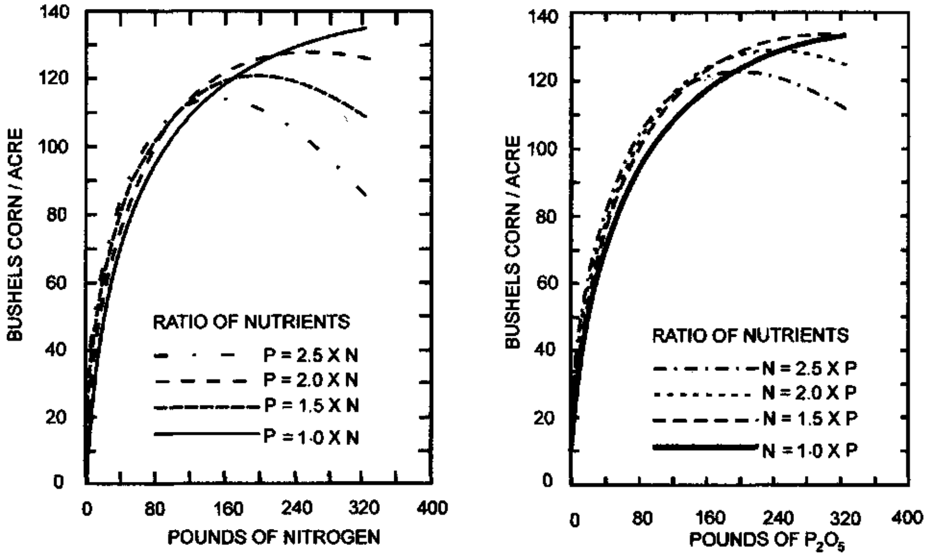


Figure 4.3.3 Relation between yield of corn and nutrients (nitrogen and phosphorus) increased in constant proportions (after Heady (1961, p. 495).

Figures 4.3.2 and 4.3.3 are also in accordance with the Liebscher hypothesis of decreasing productivity at an increase of proportional availability of different nutrients. The curves in these figures represent the relation between production and - proportionally applied - nutrients. In Figure 4.3.2 it is not known in which ratios N, P, and K are available. Most likely they are applied in proportional ratios. In Figure 4.3.3 of Heady et al. (1961) the relation between yield and proportional application of N and P is presented for different ratios of N and P. As all these curves have almost the same slopes near the origin, these empirical data give little support to the model of Mitscherlich with increasing productivity at increase of proportionally available nutrients. For increasing productivity the curves should be divergent as in Figure 3.3.1.

4.4 Liebscher and Michaelis-Menten

As stated in § 3.4 the Michaelis-Menten model (Appendix 12.6 [eq. 5] can be considered as a mathematical representation of the theory of Liebscher. The empirical validity of this model was tested by examining if, and to what extent, the reciprocal values of nutrient availability and production correlate linearly. This is the so called Lineweaver-Burk transformation of the data. First the relation between applied (or available) nutrients and production was treated, because for this relation more data are available. Next the intermediate relation between nutrient application (or nutrient availability) and nutrient uptake, and finally the intermediate relation between nutrient uptake and production is represented.

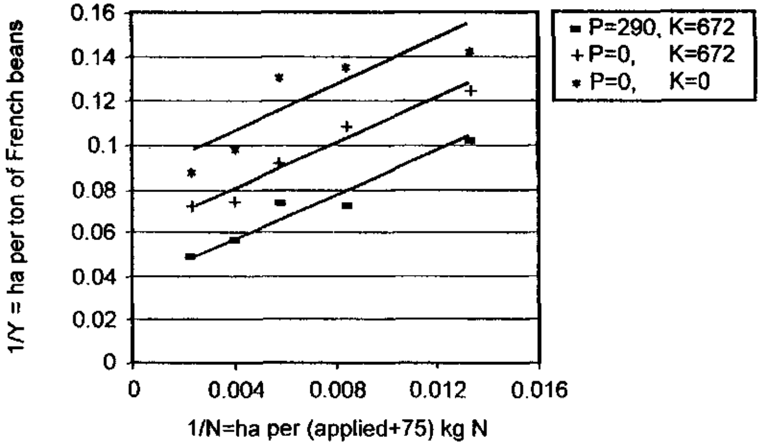
The relation between nutrient availability and production.

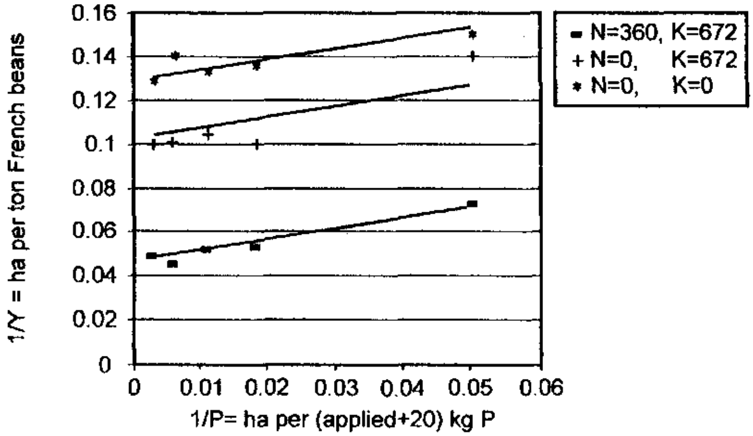
These relations have been tested by a number of data sets from literature (Figure 4.4.1 - Figure 4.4.3).

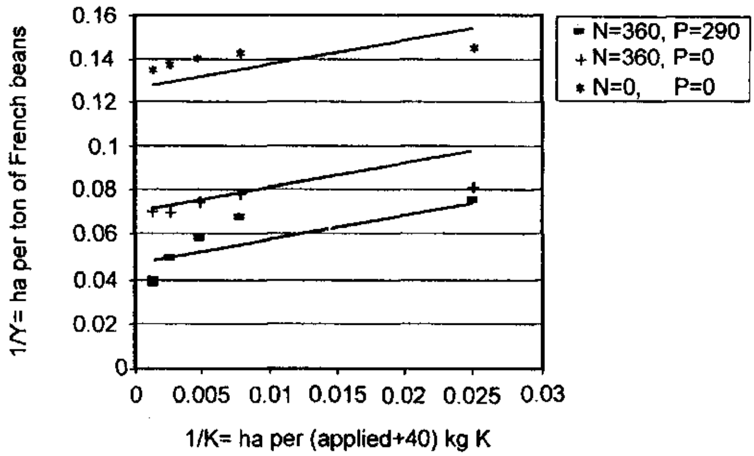
Figure 4.4.1 concerns French beans (Greenwood et al., 1971, p. 515). The linear relation between the reciprocals of available nutrients and production appears to be fairly suitable, if constant quantities for internally available nutrients are included ((75 kg N + 20 kg P + 40 kg K)/ha). These quantities were iteratively determined. The best fitting Michaelis-Menten model without interaction terms was:

$$\mathbf{1/Y = 1/MY + (1/\alpha_M) \cdot (1/N) + (1/\beta_M) \cdot (1/P) + (1/\tau_M) \cdot (1/K)}$$

Figure 4.4.1 Relation between the reciprocal values of production (1/Y) and of nutrient availability (1/N, 1/P and 1/K) at different combinations of nitrogen, phosphorus and potassium fertilization (after Greenwood et al., 1971, p. 515). The solid lines correspond to the for all data points best fitting Michaelis-Menten model. For further explanation see text.


P=290, K=672
P=0, K=672
P=0, K=0
1/Y = ha per ton of French beans
1/N=ha per (applied+75) kg N


N=360, K=672
N=0, K=672
N=0, K=0
1/Y = ha per ton French beans
1/P= ha per (applied+20) kg P


N=360, P=290
N=360, P=0
N=0, P=0
1/Y= ha per ton of French beans
1/K= ha per (applied+40) kg K

The coefficients in the equation: $1/MY = 0.032913.10^{-3}$; $1/\alpha_M = 5.269.10^{-3}$, standard error = $0.229.10^{-3}$; $1/\beta_M = 0.500.10^{-3}$, standard error = $0.051.10^{-3}$; $1/\tau_M = 1.111.10^{-3}$, standard error = $0.109.10^{-3}$; Total explained variance 95.7%. Standard error of estimated $1/Y = 0.007.10^{-3}$.

So the maximum production is 30383 kg dm/ha , and the following response coefficients were found: $\alpha_M = 0.19.10^3$, $\beta_M = 2.0.10^3$ and $\tau_M = 0.9.10^3$ kg/kg N, P and K, respectively (values attained by linear regression).

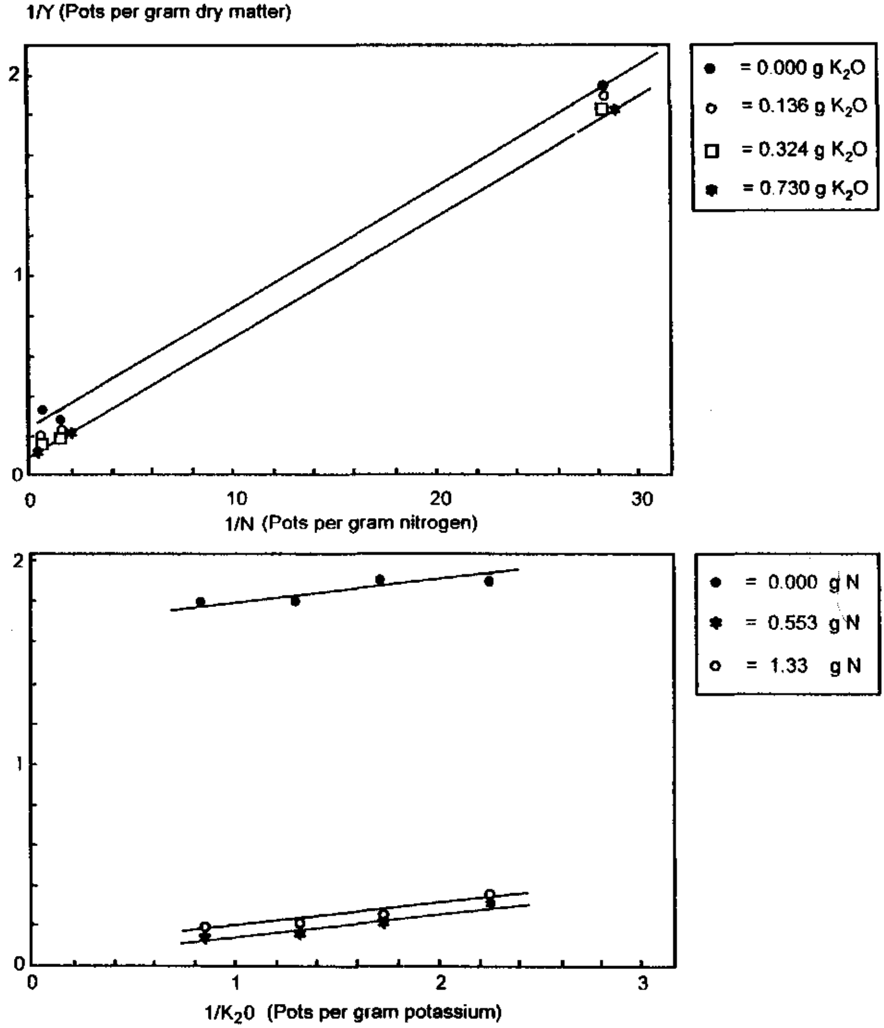


Figure 4.4.2 Relation between the reciprocals of production (1/Y) and of nutrient availability (1/N, 1/K_2O) (after Mitscherlich, 1923, p. 201). In the upper plot only the lines for the lowest and the highest K_2O applications are given.

Besides the interaction effects, covered by the **basic structure** of the model, no explicit **extra interaction terms** appear to be needed. For very high availabilities, especially of nitrogen, the model is not suitable because it does not account for a decrease of production under that condition [10]). Our study is, however, not concerned with very high applications, but rather with the interval between very low and medium applications. Clearly the lines are not diverging, but very close to parallel, which endorses the Michaelis-Menten model rather than the Mitscherlich model.

Figure 4.4.2 shows data from a pot trial by Mitscherlich with oats (1923 p. 201). The reciprocal of production (pots per gram dry matter) has been plotted against the reciprocal of nutrient availability (pots per gram N, respectively K_2O). The Michaelis-Menten model appears to represent these data satisfactorily, if for a delivery from the soil 0.035 gram N and 0.442 gram K_2O per Mitscherlich pot are assumed. The other coefficient values are approximately as follows: α = 176 gram dm/gram availability of N, and τ = 90 gram dm/gram availability of K_2O (equivalent to 108 gram dm per gram availability of K), maximum production = 90 gram dm per Mitscherlich pot (all values were attained iteratively, after reconstruction of data sets from graphs). For a good fit no interaction terms are needed; the structural interaction within the model suffices. Again, the lines are very close to parallel, endorsing the model of Michaelis-Menten.

Figure 4.4.3 presents data from Nielsen (1963) for oats in years with bad, mediocre and good weather conditions. The lines appear to be diverging. This gives support for a Michaelis-Menten model with small extra interaction effects or for a model with some Mitscherlich characteristics.

The Michaelis-Menten model was also tested with data from Klapp (1958, p. 11), Penning de Vries & Van Keulen (De Wit, 1992, p. 138), Baan Hofman & Van der Meer (1986, p. 19). In most of those cases the Michaelis-Menten model is satisfactory, with no extra interaction terms needed. However some theoretical assumptions about the unknown amounts of internal nutrients were required. So - as De Wit (1992b, p. 146) stated - most of the data support the theory of Liebscher.

The relation between availability and uptake and the relation between uptake and production.

Van Keulen (1986, different graphs) presents the relation between application and uptake mostly as a linear relation. Regarding the scatter of the points in his figures, however, it appears that most of those curves may be described as well or even better by non-linear Michaelis-Menten relations.

Figure 4.4.4 applies to data for grass (Nielsen, 1963, referred to by De Wit, 1992b, p. 136). The reciprocals of availability, uptake and production are plotted against each other. It appears that the relation between availability and production (a) as well as the relation between uptake and production (c) may be fairly well described through the Michaelis-Menten model.

The relation between availability and uptake (b) however shows tendencies to Mitscherlich characteristics, as the lines of the reciprocal relations are slightly diverging. Just as in the pattern of one of the curves in Figure 4.3.1, this pattern of diverging lines is not merged into the same diverging pattern for the production/uptake relations. A Michaelis-Menten relation with a small tendency to a Mitscherlich model for the availability/uptake relation (b), compensated by a Michaelis-Menten relation with a small tendency to the Von Liebig model for the uptake/production relation (c) appears to produce a nearly perfect Michaelis-Menten model for the availability/production relation (a).

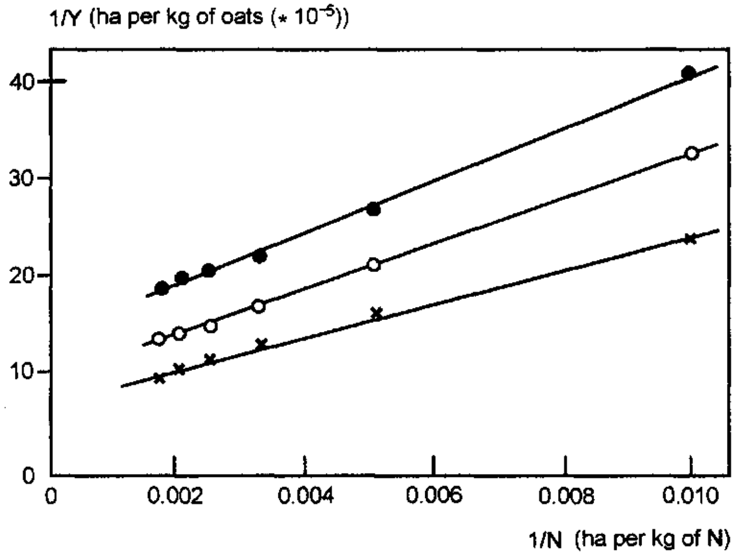


Figure 4.4.3 Relation between the reciprocals of production (1/Y) and of nutrient availability (1/N) in oats (referred to by De Wit, 1992b, p. 135, after Nielsen, 1963). The upper, middle and lower lines indicate years with bad, mediocre and good weather conditions.

When the relation between application and uptake is nearly linear, like in the data from Spiertz (1980, referred to by De Wit, 1992b, p. 139), it may, all the same, be formally represented by a Michaelis-Menten equation. But in such cases, the estimated constant "maximum uptake" has an unrealistic, very high value.

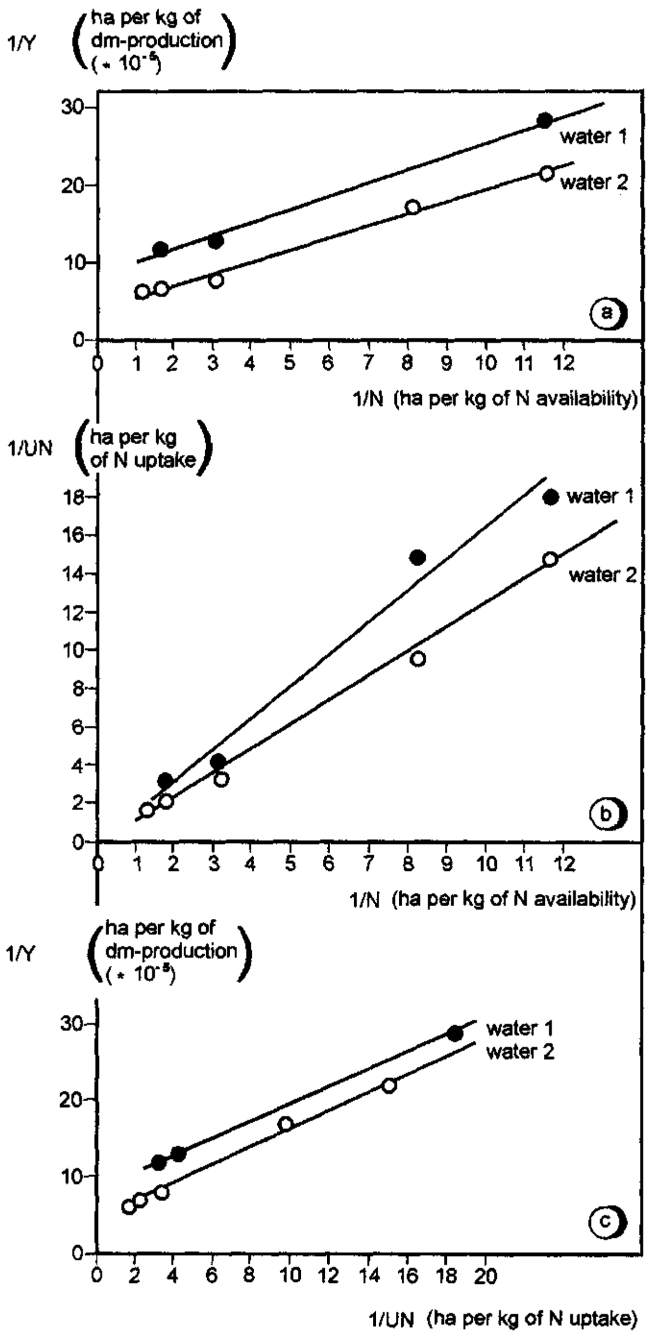


Figure 4.4.4 Relation between reciprocals of dry matter production (1/Y) and of N uptake (1/UN) and of N availability (1/N), for grass at two levels of water supply (De Wit, 1992b, p. 136, data from Nielsen, 1963).

4.5 Validity of the Michaelis-Menten model for other factors

The Michaelis-Menten model seems to have a rather broad applicability for the relation between production and production factors. We tested the model on data from Gaastra (1959, p. 38-39). The model is:

1/UC = 1/MUC + 1/(α.R) + 1/(ß.C)

where in this case:

UC	=	C uptake as CO_2 ($mm^3.cm^{-2}.h^{-1}$)
MUC	=	maximum CO_2 uptake ($mm^3.cm^{-2}.h^{-1}$)
R	=	available radiation ($J.cm^{-2}.h^{-1}$)
C	=	available CO_2 (volume %)
α,ß	=	coefficients of response of C uptake on C availability and radiation respectively.

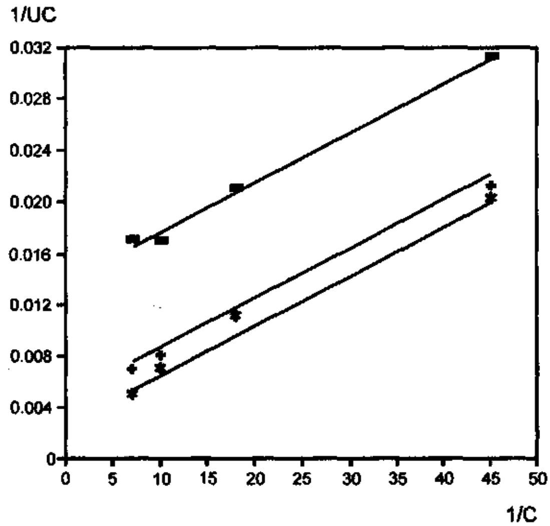


Figure 4.5.1 Relation between the reciprocal (1/UC) of CO_2 uptake (cm^2. h^1.mm^{-3}), and the reciprocal (1/C) of CO_2 concentration, in cucumber, for three intensities (R) of radiation (upper line 10, middle line 28 and lower line 55 $J.cm^{-2}.h^{-1}$. The lines represent the Michaelis-Menten equation with CO_2 uptake response coefficients of about α = 7 $mm^3.J^{-1}$ for radiation, of about ß = 26.10^2 $mm^3.cm^{-2}$.volume%$^{-1}$.h^{-1} for CO_2 concentration and a maximum CO_2 uptake of 4.10^2 $mm^3.cm^{-2}.h^{-1}$. The points represent reconstructed data from a graph of Gaastra, 1959, p. 38-39.

From Figure 4.5.1 it may be concluded that the model also appears to fit fairly well for radiation and CO_2. We suggest that the model is also applicable for production relations which are usually referred to as "yes-no relations", such as the relation between production and the application of biocides. At the plant level (or pot level) these relations will probably be step functions. Up to a certain dose, the production does not respond at all to an increase of the dose. If the dose is raised beyond that point, there is a sudden change in response, and if it is raised still further, production no longer responds. However, at the level of the **field, farm or region**, the crop will not respond so abruptly to increase of doses, because of spatial and temporal variability in other growth factors. At that aggregation level the response curve may resemble Michaelis-Menten response curves of fertilizers, water, radiation and CO_2.

It has been explained and demonstrated (De Wit, 1960 p. 37-39, Schouls, 1968, p. 18-43), that the Michaelis-Menten equation also holds for the factor space (as term for **total resources**). The authors show that for the relevant intervals the reciprocals of production and plant density are linearly related. Only for very high densities (because of mortality of individuals) and very low densities (because of no interaction between individuals) the model does not hold.

4.6 Inferences

- Plotting of the reciprocals of nutrient availability, nutrient uptake and yield appears to be a good tool for analyzing the empirical validity of the Michaelis-Menten characteristics.
- Both the Von Liebig and the Mitscherlich models were not supported by the reanalysed data from literature in this study.
- The Michaelis-Menten model appeared to give a good description of most empirical input-output relations from reanalysed data in the literature. This applies to the relations between application and uptake (provided that plausible values for internally available nutrients are estimated), but also for the relation between uptake and yield, and as a consequence for the relation between application and yield.
- In a part of the application-uptake relations there is a small tendency towards the Mitscherlich model, and in a part of the uptake-yield relations a small tendency towards the Von Liebig model.
- The Michaelis-Menten model appears also to have fair validity for input-output relations other than those of the macronutrients, such as radiation, water and CO_2.

5 Increase of nutrient application and innovations

5.1 Introduction

In this chapter the question is dealt with, to what extent a distinction should be made between production enhancement by innovations versus production increase as a result of increase of nutrient application. Occasionally time series in the literature give the impression that the output of products in agriculture is proportional, or sometimes even more than proportional, to the application of nutrients. Thus constant or increased marginal and average productivities may be supposed.

The phenomena of increased nutrient application and technological change are correlated in time, however. For analytical reasons these two should be conceptually disentangled as much as possible. Moreover it is very important in the interpretation of productivity data to differentiate between alternative definitions of the term productivity. Inclusion or exclusion of yield from nutrients internally generated within the production system, makes a decisive difference with regard to the evaluation of agro-ecosystems.

5.2 Examples of analysis of historical data series

As an example see Figure 5.2.1 after De Wit (1992b, p. 127). In this figure the relation between application and yield is not ceteris paribus. Two factors (nitrogen application and genotypic/technological levels) improve together in time.

In Figure 5.2.2 (adapted after Antle & McGuckin, 1993) at every separate point in time the yield per kg application is smaller at a high application than at a low application. The conclusion is that the productivity may be raised by lowering the nutrient application (moving to the left on one of the curves in the graph) or by amelioration of the efficiency of the production process (leap to the next higher curve). So a sharp distinction between changes in nutrient application and technical innovations in the production process may keep the discussion clear.

To avoid this entanglement it might be better to present them in another way by separating "effects through innovations" from "effects from increased inputs". Figure 5.2.2 is such a schematic representation which may be more clarifying.

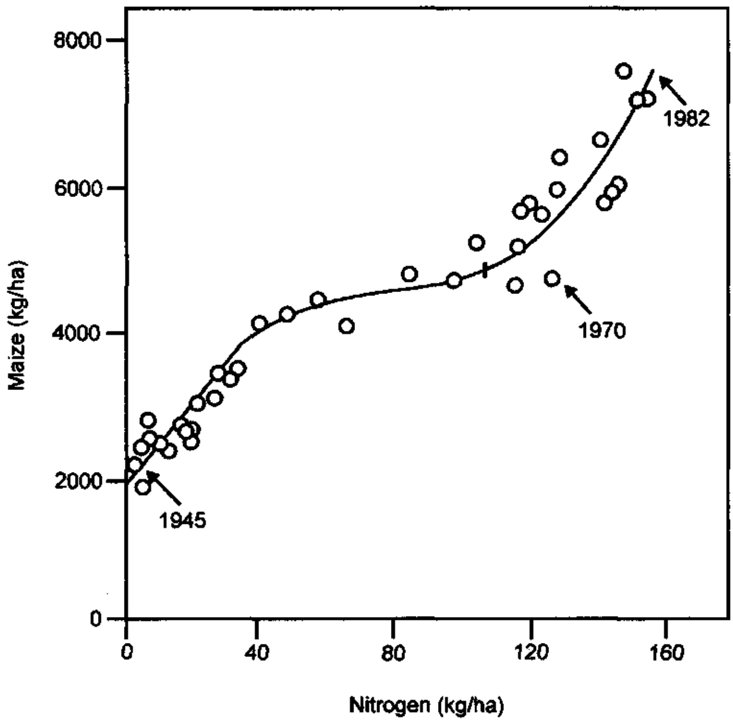


Figure 5.2.1 Maize production and fertilizer application in the USA during the period 1945-1982 (De Wit, 1992b, p. 127, data from Sinclair).

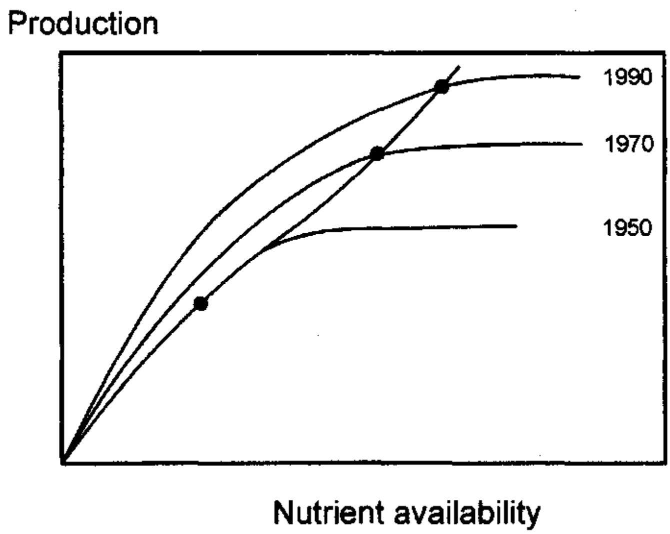


Figure 5.2.2 Change in the relation between production and application of nutrients over time (schematically).

5.3 Influence of internal nutrients on the productivity measure

Regarding Figure 5.2.1 it is striking that even if technological change and nutrient inputs are not disentangled in the analysis, no real increase (or even maintenance) in nutrient productivity may be observed.

Around 1945 a maize production of about 2000 kg/ha was obtained with 0 kg N/ha. Around 1982, these figures were about 7200 kg/ha with an application of 160 kg N/ha. It seems that about 2000 kg dm was obtained from internally available nitrogen (at a nitrogen application = 0).
The question is how to calculate productivity. In some publications apparently the ratio between marginal application and marginal yield is taken (e.g. De Wit, 1992, p. 127). In that conception the production at an external application of zero is obviously totally attributed to mining. But even if calculated as marginal productivity of external nutrients (MZP), (see Appendix 12.2 for definition of the concepts) there is no increase, of productivity at an increase of N application between 1945 and 1982, as is shown by the calculation below.

1945	(4000-2000)/40	= 50 kg maize per kg N
1982	(7200-5200)/40	= 50 kg maize per kg N

Decreased productivity on account of increased application of N just compensated by the technological improvements of that period. When ecological agriculturists calculate productivity, they prefer to calculate the ratio of total production to external application (e.g. Besson et al. 1995, p. 73). This productivity measure may be called system productivity (see Appendix 12.2). Their arguments are that the internal nutrients, if sustainably delivered, should not be regarded as input from outside the boundary of the production system. The total yield is regarded as the result of the achievement of the system with mainly internal nutrients (for mixed farming also internal farm manure) and few external nutrients. A measure for the performance regarding the shift from external to internal nutrients is the quotient of the total yield and the external nutrients, to indicate the productivity increase of ecological measures, which are aimed at a shift from external application to internal generation of resources. They calculate the "system productivity = SZP" (see Appendix 12.2) and the calculation runs as follows:

1945	3000/30	= 100 kg maize per kg N
1982	7200/160	= 45 kg maize per kg N

In this calculation the N productivity at low applications (in 1945) is about twice as high than at high applications (in 1982). The question is whether both these calculations adequately indicate the relevant N productivity. A calculation that does more justice to the difference between mining and internal generation of nutrients will be obtained if the internal availability of nitrogen is divided into a component that originates from depletion of soil stocks (mining) and a component which originates from the deposition and fixation of N. This procedure yields the "Sustainable System Productivity" (SSZP, see Appendix 12.2). Let us assume here that from the 2000 kg dm at application zero about 1000 kg dm (corresponding to about 30 kg N uptake) originates from N fixation and N deposition and the other 1000 kg from mining out of stocks. The production originating from mining should not be taken into account when calculating the nutrient productivity in the current year, but the production originating from deposition and N fixation should be. To prevent extraordinary high values of the quotient productivity at values of external inputs close to zero this measure may be more appropriately expressed as the reciprocal of system productivity (so "consumptivity"). The measures are compared in Table 1.

Table 1 Marginal productivity and kg external N use per kg dm at external application levels of zero in 1945 until 160 in 1982. Data reconstructed from Figure 5.2.1.

N	Marginal N per ton extra dm	Kg external N per ton dm	Kg external N (per ton dm, excl. 1 ton for mining)
0		0/2 = 0	0/1 = 0
	40/(4.0-2.0) = 20		
40		40/4 = 10	40/3 = 13
	40/(4.5-4.0) = 80		
80		80/4.5 = 18	80/3.5 = 23
	40/(5.2-4.5) = 57		
120		120/5.2 = 23	120/4.2 = 29
	40/(7.2-5.2) = 20		
160		160/7.2 = 22	160/6.2 = 26

So if only a relatively small amount of the production out of internal N supply is taken into account, then the system productivity at 40 kg N (in 1945) is still greater than at 160 kg N (in 1982), despite the technological progress during that period. Calculated without controlling for technological improvement, the productivity reaches at 120 and 160 kg N a stable level.

5.4 Inferences

- In historical time series of the relation between application and yield, decreasing productivities at increasing application may be hidden by a trend of increased technological innovation.
- If controlled for this trend in the statistical data decreasing marginal and average productivities was clearly observed in a specific case where increasing productivities could be interpreted.
- Even if not corrected for a technological trend, no increase of productivity with increased application was found in the analyzed data, when the part of the internal nutrients, which may be renewed each year, was taken into account.
- For ecosystems the most appropriate measure of productivity is stated to be the "(sustainable) system productivity" [11]).

6 Productivity and profit

6.1 Introduction

Important sources of confusion in the optimization discussion may be the unclear definitions and use of terms as productivity, total productivity, system productivity and financial productivity, and an insufficient distinction between the concepts productivity and profit. This is also partly related to differences in objectives of research between agronomists and economists. At first glance it may be argued that farmers aim for maximum financial productivity instead of a maximum agronomic productivity, the former being dependent on prices and the latter not. However, farmers will not even aim for maximum financial productivity (*f* yield per *f* nutrient) at all, but for maximum profit (*f* yield minus *f* nutrient). The optimal applications then also depend on the prices of inputs and products. Since in Western countries, in the past decades, prices of nutrients have been relatively low, application rates of nutrients have been and have remained very high.

But there may be still some other determinants. It may be questioned if, and to what extent, current high external application of nutrients in farm practice may not only be a consequence of low nutrient prices, but also of the entanglement of the concepts of productivity and profit, and/or of the predominance of specific production functions in education and extension, and/or of the use of specific operational definitions of the concepts productivity and profit.

In this chapter we try to elucidate the relation between some economic and agronomic output criteria and to evaluate some consequences when optimizing.

6.2 Profit and productivity measures

A starting point for our discussion will be the formal definitions of the concepts "financial yield" (Y_F) and "variable nutrient costs" (C_F).

$$\mathbf{Y_F = Y * PRI_Y}$$

$$\mathbf{C_F = N_E * PRI_N + P_E * PRI_P + K_E * PRI_K}$$

in which Y may be calculated from different production functions, and in which as new variables:

PRI_Y	=	Price of product (*f*/kg dm)
PRI_N	=	Price of nitrogen (*f*/kg N)
PRI_P	=	Price of phosphorus (*f*/kg P)
PRI_K	=	Price of potassium (*f*/kg K)
C_F	=	Variable nutrient costs (*f*/ha)
Y_F	=	Financial production (synonym = economic return): production in monetary units (*f*/ha). (F refers to financial.)
N_E	=	Externally applied nitrogen (kg N/ha)
P_E	=	Externally applied phosphorus (kg P/ha)
K_E	=	Externally applied potassium (kg K/ha)

The difference between financial yield (Y_F) and nutrient costs (C_F) give a partial measure of financial revenue (R_F):

$$\mathbf{R_F = Y_F - C_F}$$

in which: R_F = Profit (financial revenue, gross margin). In general defined as output minus input (*f*/ha) and here specified for nutrient input: yield minus costs of nutrients (*f*/ha).

The quotient between financial yield and costs gives a measure of financial productivity restricted to only nutrients.

$$\mathbf{PR_F = Y_F / C_F}$$

Symbols defined before, or see Appendix 12.1.

Provisionally we will assume that the internal nutrients have the same price as the applied nutrients. This is in accordance with environmental economists, who propose to include environmental depletion of the system in cost calculation (see e.g. Van Ierland, 1993, p. 29). (In § 5.3 it was argued that such is only justified for real depletion but not for internal nutrients generated within the boundaries of the system [12]). So yield, costs, profit and productivity are all functions of nutrient availability. In this chapter we compare financial productivity with profit. We refer to Appendices 12.1 and 12.2 for the definitions of the profit and productivity concepts.

In § 6.4 some simulation experiments are presented in which the sensitivity of the productivity indicator for small changes in the amount of internal nutrients is shown.

6.3 Comparison of different models

In Figure 6.3.1 the variables "financial yield" and "nutrient costs" are plotted against the proportionally available (N+P+K). This is done for the Mitscherlich model (above) as well as for the Michaelis-Menten model (below).

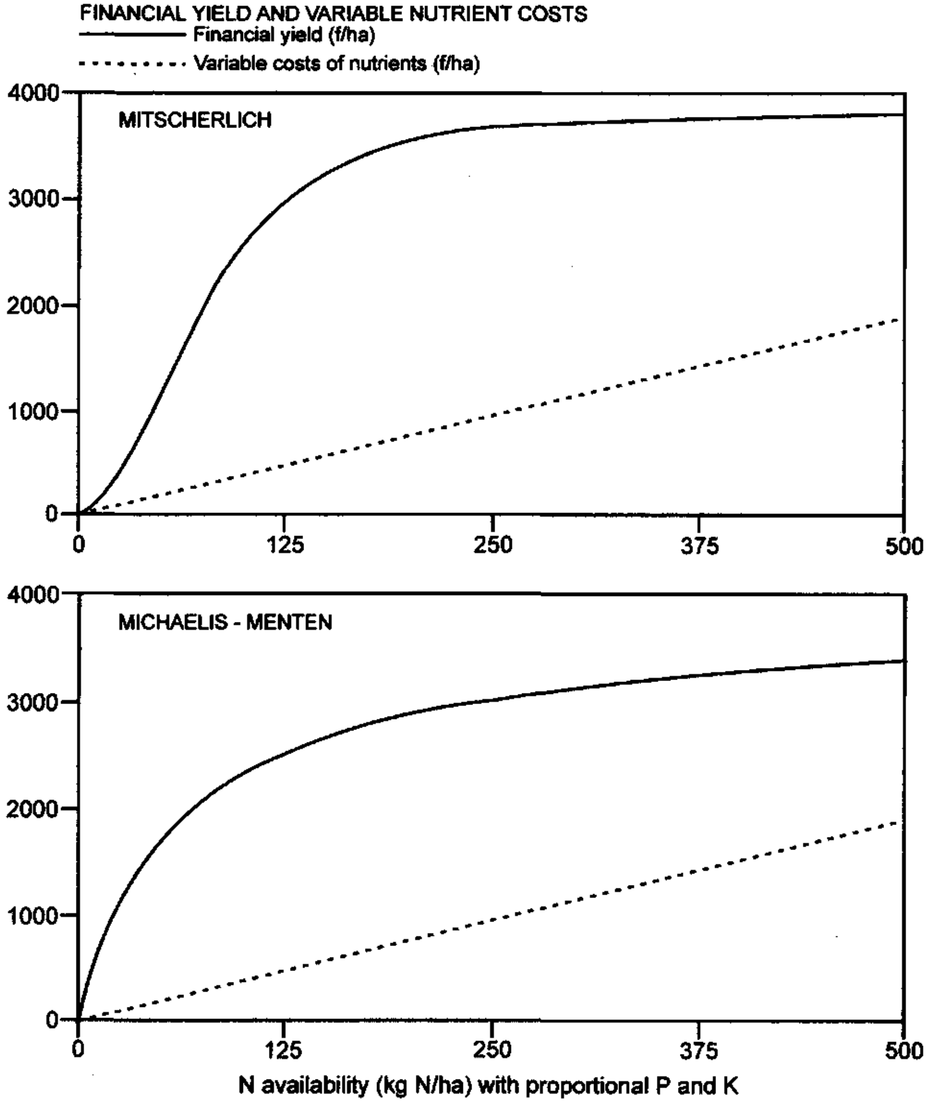


Figure 6.3.1 Financial yield (*f*/ha) and variable nutrient costs (*f*/ha) against N availability (with proportional N, P and K availability) of the Mitscherlich and Michaelis-Menten production functions. Coefficient values deviating from the values of the model in Appendix 12.4 are: MY = 12000, PRI_Y = 0.55, PRI_N = 2.3, PRI_P = 3.5, PRI_K = 2.5, f_H = 0.60.

The curves are derived from the simulation models (Appendix 12.3). The maximum profit will be found at the application rate at which the vertical distance between the production curve and the cost curve in Figure 6.3.1 is maximal.

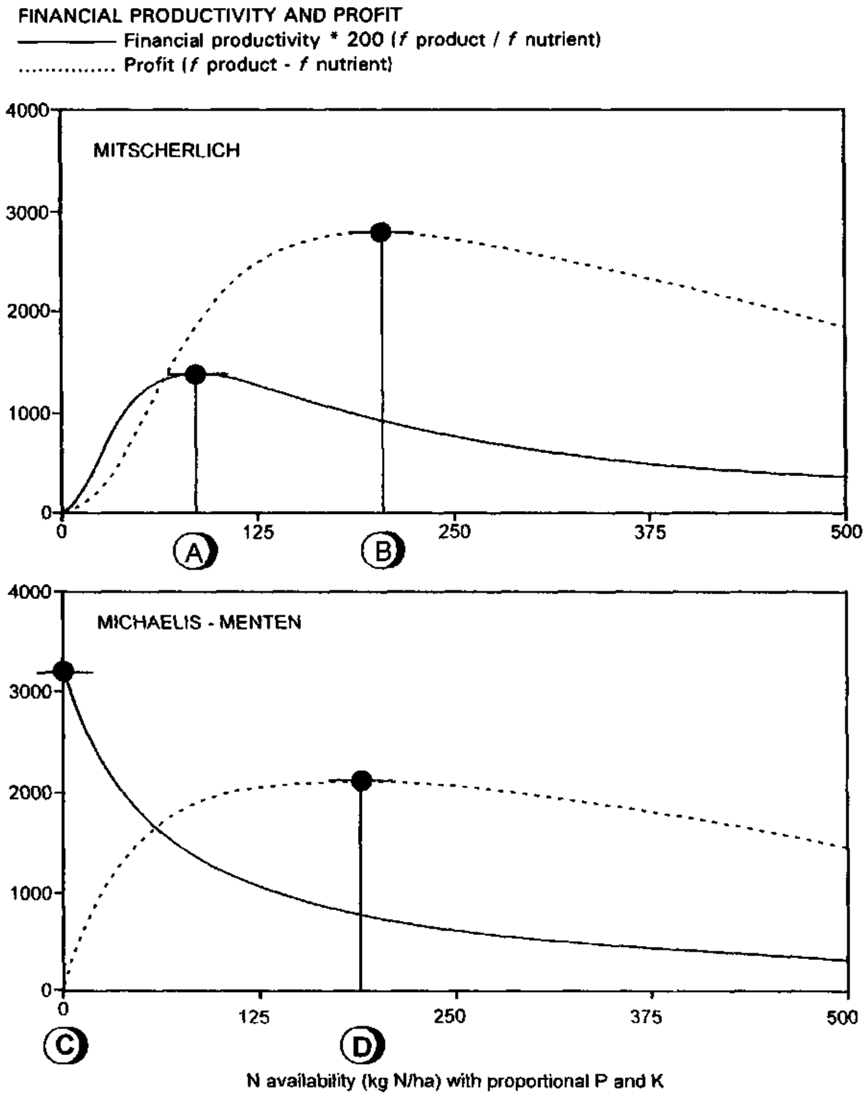


Figure 6.3.2 Financial productivity (*f* yield / *f* nutrient) and profit (*f* yield - *f* nutrient) against N availability (with proportional P, and K availability) of the Mitscherlich, and Michaelis-Menten production functions. Coefficient values deviating from the values of the model in Appendix 12.4 are: MY = 12000, PRI_Y = 0.55, PRI_N = 2.3, PRI_P = 3.5, PRI_K = 2.5, f_B = 0.60.

That is the point where the first derivative of the production curve is equal to a constant (price of nutrients per unit), which is formally the first derivative of the cost curve (or the point where marginal production equals marginal costs). In Figure 6.3.2 financial productivity (not marginal but average productivity) and profit are plotted jointly for the Mitscherlich model (above) and the Michaelis-Menten model (below). Comparing both models in Figure 6.3.2 it may be observed that both quantities (financial productivity and profit) have their optima at different levels of nutrient availability. In both the Michaelis-Menten model and the Mitscherlich model, the maximum profit is found at a higher availability level than the level which gives maximum (financial) productivity. This discrepancy will decrease as nutrient prices rise and product prices fall (not shown in our figures). For the Michaelis-Menten model the maximum (financial) productivity is situated at the lowest possible availability level. Another observation is that there is much more discrepancy in optimal input between economic and ecological goals in the Michaelis-Menten model than in the Mitscherlich model. The availability levels that are optimal for financial productivity and profit differ much more in the Michaelis-Menten model than in the Mitscherlich model (distance A-B is less than distance C-D in Figure 6.3.2.

According to the Michaelis-Menten model, the greatest (financial) productivity is situated at application zero (C), but the highest profit is obtained at a higher nutrient availability (D), though at a lower level than in the Mitscherlich model.

The conflicts between economically optimal input and agronomically optimal input appear to contradict statements in other research (Janssen, Braakhekke & Catalan, 1994). These authors depart from other production functions that resemble the Mitscherlich function more, and they conclude that the economic optimum input is close to the physiological and environmental optimum input. And, indeed, in the Mitscherlich model, the greatest productivity is not found at the lowest application but at a value between zero and the application giving maximum production. Optimum agronomic productivity then approaches optimum financial productivity, especially if nutrient prices are not too high.

The optimum **agronomic** ratio between nutrients (N.B. not rate of nutrients) is not dependent on the prices of the nutrients, but the optimum **economic** ratio seems to be dependent on prices in models with substitutability of nutrients. It seems profitable to use relatively more of cheap and less of expensive nutrients because they are substitutable in the Michaelis-Menten model. This was inferred in Appendix 12.10. With a more careful analysis, however, as Janssen et al. (1994) state, it is only at the short term that it is profitable, because this practice is at the expense of the harmonious ratios between nutrients in the soil over the long run [13])!

The Mitscherlich production function not only underestimates the productivity of "low external input agriculture", especially at low proportional availability levels, but also indicates too high economic optimum application rates. The model underestimates profit too. This will be further worked out in Figure 6.3.3, where also different production capacities and an extra model (Von Liebig) are introduced in the analysis.

In Figure 6.3.3 the profit curves of the Von-Liebig, the Mitscherlich and the Michaelis-Menten model were compared at production capacities of 2000, 6000, 12000 kg dm/ha (20000 and 30000 kg dm/ha also simulated but not shown in the figure). The response coefficient values were chosen, such that at the production capacity of 12000 kg dm/ha equal yields (4820 kg harvestable dm/ha) were attained, at equal N availabilities (of 145 kg N, with proportional co-availability of P and K). Because of those conditions the coefficients are not the same in the different models.

From Figure 6.3.3 conclusions may be made with respect to a) the level of profit and b) the nutrient availability level which gives maximum profit.

a) <u>level of profit</u>:

The Mitscherlich model underestimates the level of profit in comparison with the Michaelis-Menten model at the lower levels of nutrient availability. For low production capacities the calculated profit in the Mitscherlich model is even negative, whereas the Michaelis-Menten model still achieves a positive profit. The discrepancy between these two models decreases as the production capacity increases. In the Von Liebig model the levels are close to those of the Michaelis-Menten model.

b) <u>optimum availability</u>:

For very poor soils (production capacity of 2000) the optimum application of nutrients is about 67% lower in case of the Michaelis-Menten model compared to the Mitscherlich model. (Actually, in this analysis, the optimum input for Von Liebig and Michaelis-Menten even coincide almost with the local minimum profit in Mitscherlich!). For higher production capacities the discrepancy between the models decreases. For a production capacity of 6000 the optimum application level lies about 48% lower for the Michaelis-Menten model than for the Mitscherlich model. For a capacity of 12000 the optimum of Michaelis-Menten is about 15% lower than that of Mitscherlich, and for a capacity of 20000 kg dm the economic optimum application of Michaelis-Menten is about the same for both models (result not shown in Figure 6.3.3). The optimum application in the Von Liebig model is not much different from that of the Michaelis-Menten model. Only for very productive soils is the optimum application in Von Liebig considerably lower than in the other models.

Models departing from Mitscherlich functions seriously underestimate both the productivity and the profit of low external input agriculture.

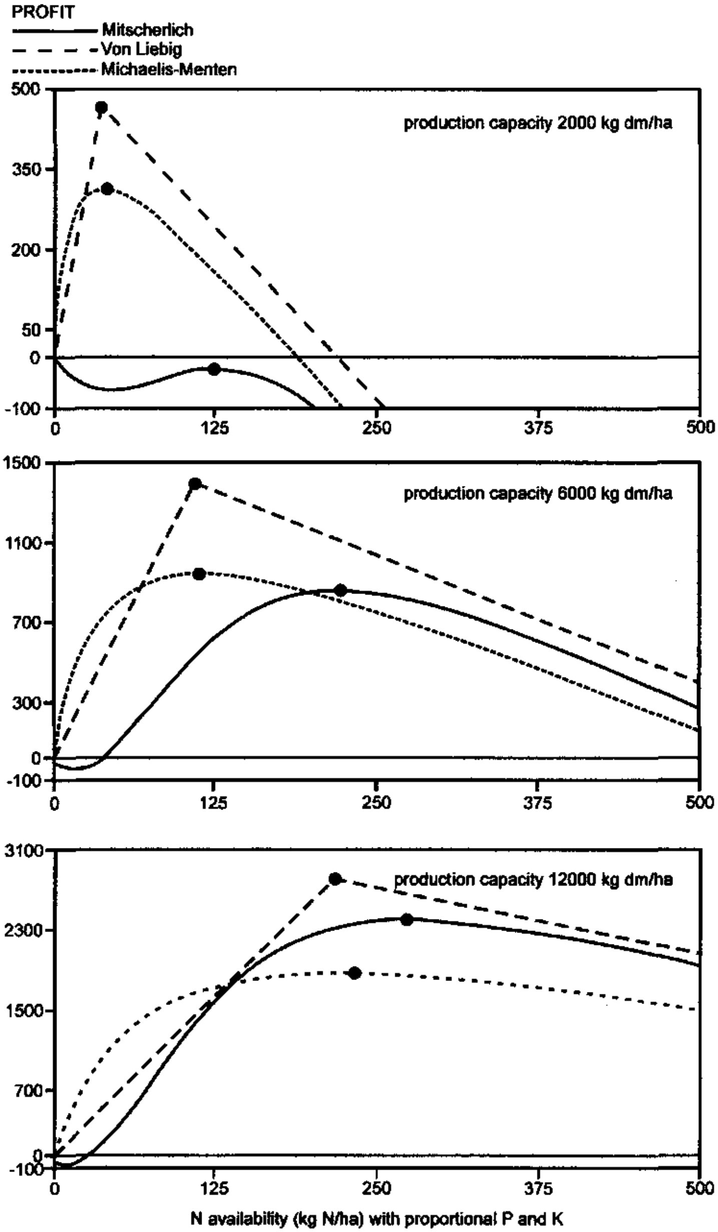


Figure 6.3.3 Relation between profit (yield minus variable nutrient costs) and N availability (with proportional N,P and K), for the Mitscherlich, Michaelis-Menten and Von Liebig models, at three levels of production capacity (2000, 6000 and 12000 kg dm/ha). For coefficient values see Appendix 12.4. The maxima of the curves are indicated with ●

This is especially so for soils with low to medium production capacities and in case of low external inputs. With the current input, and output prices, however, high external input agriculture, though less productive, continues to be more profitable than low external input agriculture, and tend to indicate too high optimum application rates.

The Mitscherlich model seems to underestimate the performances of "low external input agriculture" not only with respect to productivity but also with respect to profit.

6.4 Behaviour of the models under other conditions

Productivity measures appear to be very sensitive to the part of internal nutrients included in the calculation, especially at low values of application. We demonstrate that for the Mitscherlich model. For example if it is assumed that, besides the external application, relatively small amounts of internal nutrients (25 kg N, 5 kg P and 10 kg K) are available, not only the Michaelis-Menten model, but also the Mitscherlich model has its maximum nutrient productivity at an external application rate of zero. This sensitive dependence of the productivity measure on internal nutrients is illustrated in Figure 6.4.1 for the Mitscherlich model.

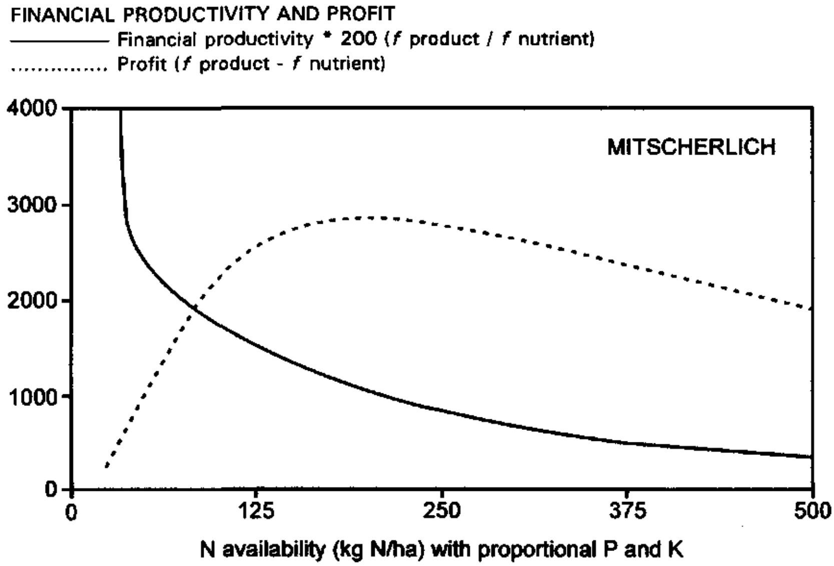


Figure 6.4.1 Financial productivity and profit, against N availability (with proportional availability of P and K with the Mitscherlich production function, taking into account internally available N, P and K values of 25, 5 and 10 kg/ha respectively. Coefficient values deviating from the values of the model in Appendix 12.4 are: MY = 12000, PRI_Y = 0.55, PRI_N = 2.3, PRI_P = 3.5, PRI_K = 2.5, f_H = 0.60.

As in ecological agriculture systems a large fraction of yield is produced by internally generate nutrients, this is a relevant sensitivity analysis.

The conclusions of § 6.3 may also change if the harvestable or marketable production, rather than biological production, is taken as a criterion and at the same time the harvestable yield is not linearly related with the biological yield.

The net harvestable production (Y_H) is a proportion, called harvest index (f_H), multiplied by the total dry matter production (Y):

$$\mathbf{Y_H = f_H * Y}$$

For some crops the harvest index is not dependent on production level (sugar beet) and for some crops (grass) the harvest index may even decrease with production level. For cereals the harvest index increases mostly with production level. In that case, in the Michaelis-Menten model too, the maximum productivity may not be situated at zero nutrient availability, but at some higher value. We demonstrate this with a simulation experiment, in which the relation between harvest index and production is taken as in Table 2.

Table 2 Relation between harvest index and production in cereals.

Biotic production as proportion of maximum	0.00	0.20	0.40	0.60	0.80	1.00
Harvest index	0.00	0.40	0.65	0.73	0.76	0.78

Source: The shape of the relationship between harvest index and production is derived from Meyer (1928, p. 339), but it is arbitrarily assumed that the level of the harvest index for cereals at present is about twice that of 1928.

Integrating this table function in the model (Appendix 12.3), the relation between harvestable productivity (Y_H/NPK) and proportional available nitrogen, phosphorus and potassium (NPK) is illustrated in Figure 6.4.2 for the Michaelis-Menten model.

The maximum (harvestable) productivity is now found at a low level of nutrient availability, but no longer at the level of zero, as in the case of Figure 6.3.2 with a constant harvest index.

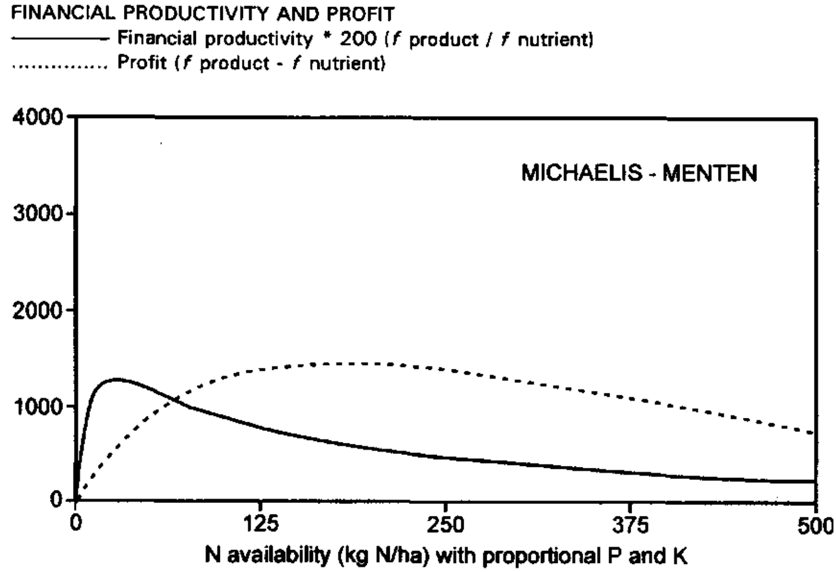


Figure 6.4.2 Financial productivity and profit, against N availability (with proportional P, and K availability) of the Michaelis-Menten production function, taking into account a harvest index which depends on the yield level. Coefficient values deviating from the values of the model in Appendix 12.4 are: MY = 12000, PRI_Y = 0.55, PRI_N = 2.3, PRI_P = 3.5, PRI_K = 2.5, f_H = see *Table* 2.

6.5 Inferences

- Comparing the Mitscherlich and the Michaelis-Menten model, a bigger difference between the agronomic optimum and the economic optimum is found in the case of the Michaelis-Menten production function than in the Mitscherlich model. For that model the maximum financial (as well as agronomic) productivity (but not the maximum profit) lies at the lowest nutrient availability (Figure 6.3.2).
- At relatively low nutrient prices and relatively high product prices, as presently in Western countries, maximum profit is situated at a much higher application than the application resulting in maximum productivity. This difference is smaller in the Mitscherlich model than in the Michaelis-Menten model (Figure 6.3.2).
- The Mitscherlich model underestimates the performance (in terms of productivity as well as in terms of profit) of low external input production systems and indicates too high optimum applications, especially in the situation of a low production capacity (Figure 6.3.2 and 6.3.3).
- The position of the optima is sensitive to different model assumptions. In the Mitscherlich model the maxima of system productivity measures shift to application zero (not the total nutrient productivity (TZP)) if it is

assumed that a reasonable amount of nutrients remains internally available. This will be the situation in most practical cases. In that case also with Mitscherlich decreasing system productivities will be found with increasing availability (Figure 6.4.1).

- In the Michaelis-Menten model, the availability at which maximum productivity is reached moves from zero to a small positive value if instead of the physiological, the agronomic productivity is chosen and at the same time the harvestable part of the yield increases with increasing yield (Figure 6.4.2).
- Distinguishing between the different ecological, agronomic and economic concepts it becomes clear why farmers choose much higher rates of application than would be advisable from a standpoint of maximizing nutrient productivity.

7 Nutrient productivity and set-aside policy

7.1 Introduction

The relation between nutrient availability and production at **plot and field levels** was discussed in the preceding sections. Certain other aspects also have to be considered for an analysis at **farm and region level**. Especially where the consequences of set-aside policy will be analyzed. Do the answers to the productivity questions change if the analyses are scaled up from the plot level to the regional level?

7.2 Distribution of fields over soil qualities

Considerable differences in "soil quality" exist between fields within a farm or a region.

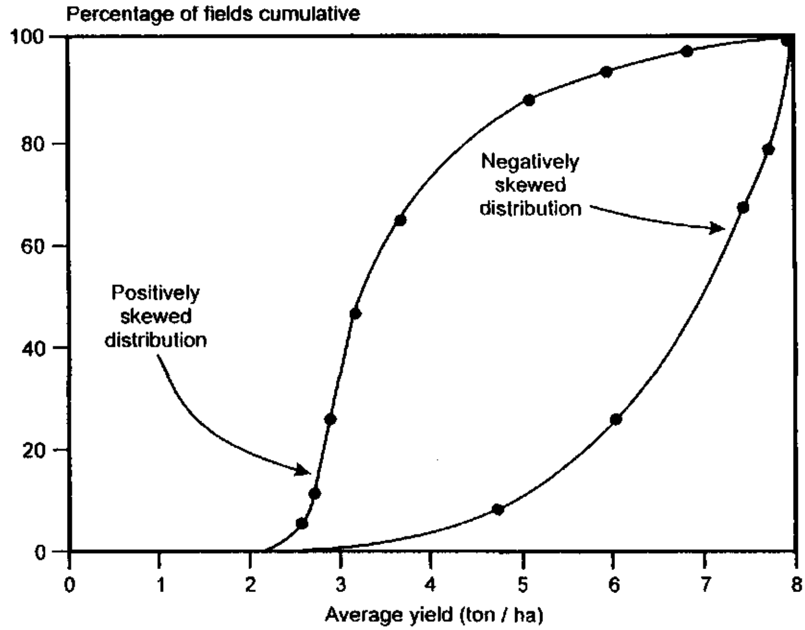


Figure 7.2.1 Cumulative distribution of the fields (as a fraction of the total number of fields) of a region over average production per ha. The figure gives a graphical presentation of data from Loomis (1992, p. 62-63).

The distribution of fields over various soil qualities is not the same for every region as may be demonstrated in Figure 7.2.1 (after data from Loomis, 1992, p. 62-63). The distribution may be symmetrical, negatively skewed and positively skewed.

Apart from the availability of any internal sources of nutrients, one may analyze whether - aiming at a predetermined (reduced) regional production level - it will be more efficient to exclude the least fertile land from cultivation, or to reduce the external applications per ha or that a combination is most appropriate.

7.3 Comparison of the three models

We will demonstrate that, independent of the type of distribution of fields over fertility categories, it appears to be relatively more efficient to fallow land, when the Mitscherlich model would be applicable and in case the Liebscher theory (= Michaelis-Menten model) is valid, it appears to be relatively more efficient to reduce the level of available nutrient per ha, at least, if nutrient productivity **would be the only decision criterion.**

The mathematical relation between dry matter production (Y-axis) and several proportionally available nutrients (X-axis) for the production functions, is as follows:

Mitscherlich (See § 3.3 and Appendix 12.5, [eq. 6]):

$$\mathbf{Y = MY \cdot \{1\text{-}EXP(-\alpha_M \cdot N\phi)\} \cdot \{1\text{-}EXP(-\beta_M \cdot qq \cdot N\phi)\} \cdot \{1\text{-}EXP(-\tau_M \cdot rr \cdot N\phi)\}}$$

Michaelis-Menten (See § 3.4 and Appendix 12.6, [eq. 5]):

$$\mathbf{1/Y = 1/MY + 1/(\alpha \cdot N\phi) + 1/(\beta \cdot qq \cdot N\phi) + 1/(\tau \cdot rr \cdot N\phi)}$$

in which the new symbols are:

qq	=	Ratio of P to N (kg P/kg N).
rr	=	Ratio of K to N (kg K/kg N).
$N\phi$	=	Available N, if the availabilities of P and K are proportional to that of N (kg N/ha).

For both models the total nutrient availability is given by:

$$\mathbf{NPK = N + P + K = (1+qq+rr) \cdot N\phi}$$

in which:

NPK = Available nutrients for the crop: available internal plus external nutrients (kg NPK/ha); not synonymous with nutrient uptake.

In order to simulate the implications of different categories of soil quality in a region, the models were run with five different maximum production capacities and associated five maximum nutrient uptake capacities. The values of maximum yield (MY) were: MY = 2000, 6000, 12000, 20000 and 30000 (kg dm/ha)) for both models.

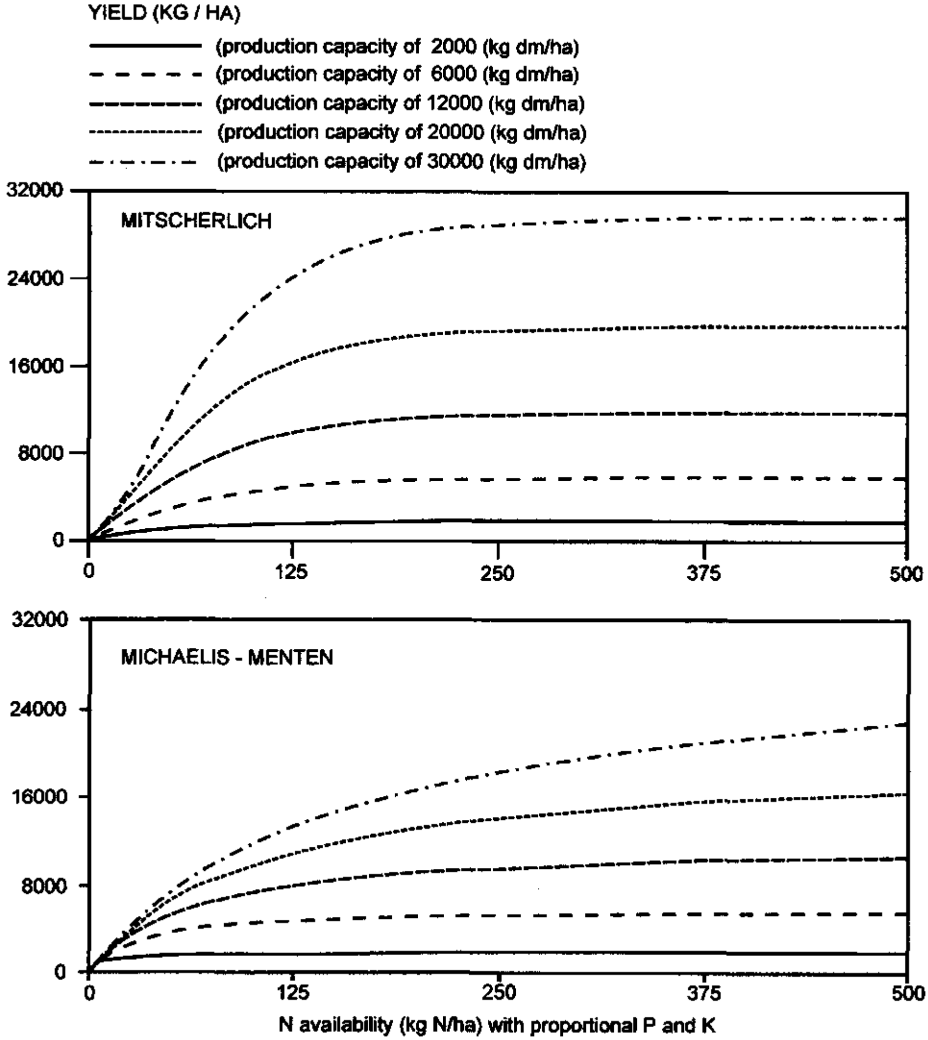


Figure 7.3.1 Relation between production and available N (with P, and K availability proportional to that of N), for five levels of maximum production capacity (representing five soil qualities of fields) for the Mitscherlich model (above), and the Michaelis-Menten model (below). For coefficient values see Appendix 12.4.

The relations between yield and nutrient availability for these five production categories according to the Mitscherlich and the Michaelis-Menten models are plotted in figure 7.3.1. (N.B. In this figure the Mitscherlich model has not the form usually found in the literature, but is represented in its sigmoid shape of in the situation when relating yield and proportional nutrient availabilities.

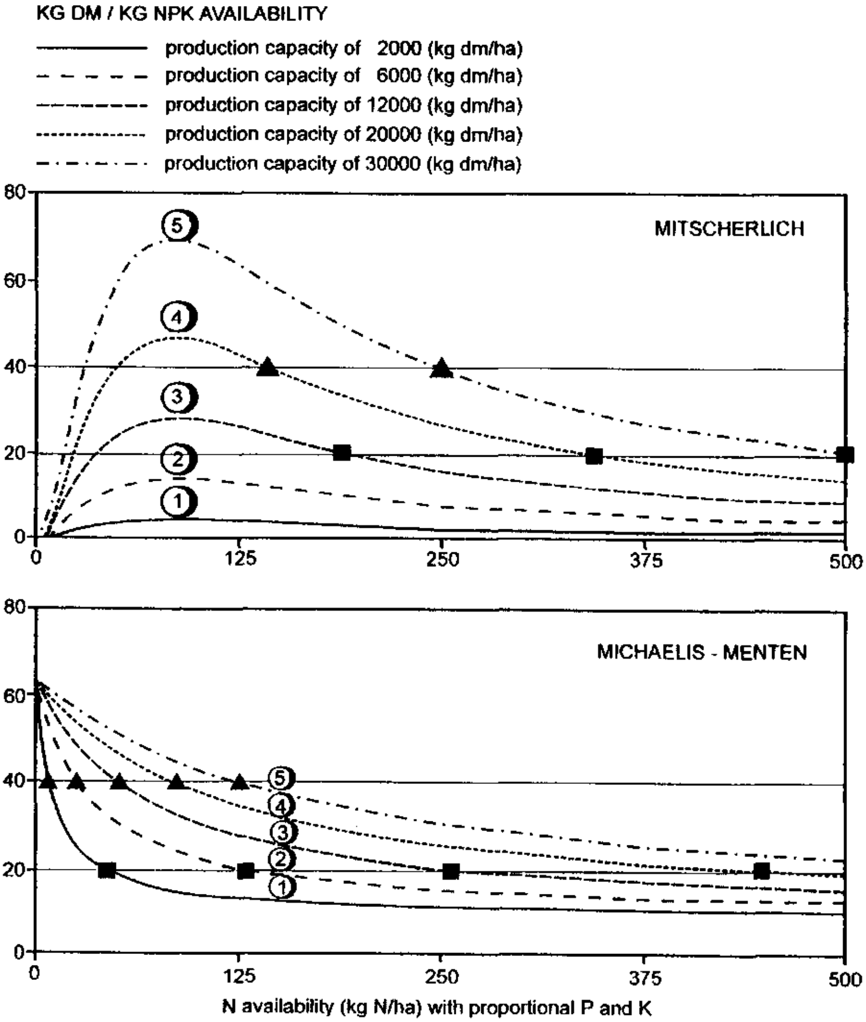


Figure 7.3.2 Relation between the nutrient productivity and available N (with P, and K availability proportional to that of N), for five levels of maximum production (the encircled numbers represent five categories of soil qualities for fields). The horizontal lines connect points of equal average productivities (Y/NPK) of the production curves (Mitscherlich model above, Michaelis-Menten model below). For coefficient values see Appendix 12.4. For the meaning of ■ and ▲ see text.

The average nutrient productivities may be derived by taking the quotient between yield and nutrient availability. These quotients are plotted against nutrient availability in Figure 7.3.2. In the graphs, points of equal productivities have been connected by horizontal lines at 20 and 40 kg dm/kg N. The encircled numbers indicate the order in which the use of land of different qualities and/or the proportional availability of several nutrients should be reduced, if the regional production aim will be reduced, and a minimum regional nutrient productivity is aimed at. If e.g. all situations with a nutrient productivity of less than that corresponding to a production of 20 kg dry matter per kg NPK should be avoided, then in case of the Mitscherlich model, all the fields with a soil fertility of the lowest two fertility categories should be taken out of production, and the availabilities on the others should not be higher than (crossings between curves and lines indicated by squares in Figure 7.3.2 upper part) circa 190, 340, and 500 kg NPK per ha. In case of the Michaelis-Menten model no category of soil fertility should disappear; only the nutrient availabilities should not exceed (crossings between curves and lines indicated by squares in Figure 7.3.2, lower part) about 45, 125, 250, 440 and >500 kg NPK per ha. The reasoning may become perhaps more clear if still more strict norms are chosen, avoiding all production with productivity of 40 or less kg dry matter per kg NPK. In case of the Mitscherlich model, only the highest two fertility categories remain in production. The nutrient availabilities on these plots should be reduced to (triangles in Figure 7.3.2, upper part) 140 and 250 kg NPK per ha. In case of the Michaelis-Menten model however, all the fertility categories remain in production, but at different reduced levels of available nutrients of about 10, 25, 50, 90 and 125 kg NPK per ha (triangles in Figure 7.3.2, lower part).

In the case of the Von Liebig model, these optimum conditions are met by taking as many of the fields out of production as necessary, in order to produce at the maximum possible rate on the remaining fields. This maximum production per ha is reached at a relatively high nutrient availability on the better fields, and at a relatively low availability on the less endowed fields. But on none of the fields this rate is reached at a nutrient availability that exceeds the one needed to just realize the target production. The Von Liebig theory implies that nutrients are used optimally at the minimum (harmonized) nutrient availabilities, which just realize the target yield per ha, and which should also be equal to the maximum attainable yield per ha. Such will be reached by putting aside as much of the soil with the worse quality as needed to reach this equity.

As noted in the introduction, the abovementioned differences in policy resulting from the two production functions are qualitatively independent of how fields are distributed over land quality categories. Quantitatively, however, in the case of the Michaelis-Menten model, a greater difference in land qualities within the region would only result in greater differences in the external input levels indicated, and not in different areas of land being taken out of production. In the case of the Mitscherlich model, however, the more the land qualities differ, the greater also the indicated contribution of removing fields from production, and the smaller the indicated contribution of reducing inputs in the remaining fields. The difference in external input on the remaining fields will not be as large as in the Michaelis-Menten model, because the greater differences in land quality will be cancelled out by taking the poorest land out of production.

7.4 Inferences

- In terms of nutrient productivity, the reduction of nutrient applications, given a certain politically determined (lower) regional production goal, was considered optimal (maximum yield per kg available nutrient) when the following situation has been obtained:
  a) the regional production goal has just been reached,
  b) the nutrient productivities on all fields within the region are equal.
- In the case of the Von Liebig model, these optimum conditions are met by taking as many of the fields out of production as necessary, in order to produce at the maximum possible rate on the remaining fields.
- In the case of the Mitscherlich model, the abovementioned conditions (a and b) are met if the nutrient availabilities per ha are as low as possible, but sufficiently high to just realize the regional production goal with equal productivities on all the fields. This implies that the indicated nutrient availability per ha should be lower on less fertile soils than on more fertile soils, because on less fertile soils the same productivity is reached at lower availability. On none of the fields, however, should it be lower than what corresponds to equal productivities for all the individual fields. This implies that fields with a lower maximum productivity than corresponding with the "target productivity" should be taken out of production. This also means - and that is an important difference compared to the Michaelis-Menten model - that when moving from too high a regional production target (accompanied by too low a productivity and too high a nutrient availability level) in the direction of a lower regional production target (the horizontal line in Figure 7.3.2 moves upward then), it will initially be more efficient to reduce the application per ha on all the fields. But after a certain reduction of application on all the fields (different for different plot qualities) it will be more rational to take the least fertile fields totally out of production.
- In the case of the Michaelis-Menten model (representing the Liebscher theory), conditions a and b will be met if all fields are kept in production and the external applications are minimized such that equal nutrient

productivities are realized, under condition of realization of the target production too. On the better fields this minimization of nutrients is reached at a high level, and on the less endowed fields at a low level of available nutrients.

8 Nutrient surplus at plot and field level (theory)

8.1 Introduction

In this chapter we pose the question how nutrient surplus [14]) changes if nutrient availability increases. Are the surplus relations different from the productivity relations? An intuitive idea is, that when uptake and productivity are high, surplus will be low. Is this different if measured per unit input, per unit output or per unit area? As with productivity the question will be elaborated for the studied theoretical models (Von Liebig, Mitscherlich and Michaelis-Menten model), both for one variable nutrient and the others constant and for proportional availability of several nutrients. After the elaboration concerning the field level of aggregation in this chapter, we shift to the regional level in Chapter 9.

8.2 General considerations about the issue

Producing a CERTAIN output:

To decrease costs of nutrient application,

increase the agronomic productivity (harvestable yield per kg available nutrients [15]). So, to attain lower costs, nutrient availability may be decreased as much as possible, by means of reducing the application, following from:

yield = availability . (yield/availability)

where availability is defined as: applied + internal nutrients

To decrease nutrient surplus,

decrease nutrient surplus per kg product, following from:

surplus = yield . (surplus/yield)

where surplus is defined as the calculated amount of available nutrient which has not been taken up.

Two criteria may be used when considering the acceptability of production systems, an agronomic criterion with the variable "productivity", the other a ecological criterion, with the variable "total surplus per kg of yield". Though these relations, as such, are analytically independent of acreage, the inferences may change when scaled up to the regional level, because of the possibly different type of distribution of the fields in the region over the quotients (yield/availability) and (surplus/yield).

There are different reasons which make it useful to analyze the relation between nutrient application and nutrient surplus separately from the relation between nutrient application and production.

Firstly: Maximizing the different indicators for productivity does not for **all models** simply linearly correlate with minimizing the different indicators for "nutrient surplus per kg dm". For production is not always a **linear function** of uptake, as the nutrient concentration is not a constant. Luxury consumption decreases the productivity and decreases the nutrient surplus too.

Secondly: In the different productivity, surplus and recovery quotients **internal nutrients** are dealt with differently (see Appendix 12.2). For example the available nutrients internally generated by the system should not be included as input in calculating the productivity quotient of the whole system, but nutrient surplus from all internal nutrients should be included in calculating the environmental load of the system. This means that the optima for productivity and for surplus per kg dm do not necessarily coincide, especially because different indicators (see Appendix 12.2) for these concepts were used, dependent on the policy questions which should be answered. The indicator surplus of available N (SN) seems ecologically more relevant than the indicator surplus of applied N (SN_E), because also nutrient surplus of internal nutrients is potentially environmentally harmful. In situations where drinking water is produced the indicator surplus of available N per ha of land (SN) is more relevant than (SN/Y). The indicator BAN gives information about possible depletion of the stocks of the system, which process is not indicated by any of the other indicators.

As in the case of productivity, there might also be a difference in the relation between surplus and availability of a nutrient under conditions of constant amounts of other nutrients and this relation under conditions of proportionally increased rates of other nutrients. Experiments (Middelkoop, Ketelaars, & Van der Meer (1990, p. 32) have shown that when the N application was increased and the other nutrients remain constant, the nutrient surplus increases considerably, both per ha of land and per kg product. But this is no evidence that the same conclusion will be valid if **the availabilities of all nutrients will be kept proportional**. Actually proportionality of available nutrients at low levels is hardly to realize in the Netherlands, so these hypotheses are hard to test. However, in spite of lack of empirical observations, it seems worthwhile to pay theoretical attention to the relations between nutrient availability and nutrient surplus, according to the elaborated theoretical models. First for only one variable nutrient (N), and the others nutrients constant, assuming a simple linear relation (§ 8.3), then (in an excursion out of the main text), for several proportional nutrients, assuming a linear model (Appendix 12.11). Next we treat the more complicated Michaelis-Menten model for three nutrients available in proportional ratios (§ 8.4 and Appendix 12.12). For the Mitscherlich model no mathematical/analytical derivations are presented, but instead comparisons of output with that of other models by means of numerical simulation were performed.

8.3 One variable nutrient, other factors constant, linear model

In the following it is demonstrated that, keeping other nutrients constant, and assuming a linear uptake function, surplus of available nutrient **per ha of land** increases, but **per kg product** is constant with increase of the nutrient application rate, while both the **external** input-output balance as well as the real surplus of applied nutrients per kg yield increases. As variable nutrient to consider we take here an important one: nitrogen (N).

Assume the production function is:

$\mathbf{Y = Y_{N0} + \alpha . N_E}$, or in another formulation: $\mathbf{Y = \alpha . N}$

in which as new variables:

N_E	=	Externally applied nitrogen (kg N/ha).
Y_{N0}	=	Production at nitrogen application of zero (kg dm/ha).

The total N uptake (UN) is:

$$\mathbf{UN = N_C . (Y_{N0} + \alpha . N_E)}$$

Note that $\alpha . N_C$ represents the **fraction** uptake of nitrogen

where:

N_C	=	Amount of N per unit dry matter of yield (kg N/kg dm).
UN	=	Uptake of N (kg/ha).
α	=	Coefficient of response of yield on N.

At a N application of zero the N uptake (UN_{N0}) equals:

$UN_{N0} = N_C \cdot Y_{N0}$

Available N is equal to applied N + internal N. The best estimate for internal N is $N_I = Y_{N0}/\alpha$, assuming that the internal N has the same effect on dry matter as the applied N.

So the total available nitrogen (N) is:

$$N = N_E + Y_{N0}/\alpha$$

Recapitulating (in terms of N and N_E respectively):

Internal N	:	$N_I = Y_{N0}/\alpha$
Uptake internal N	:	$UN_I = N_C \cdot Y_{N0}$ [16])
Applied N	:	$N_E = N - Y_{N0}/\alpha$
	:	N_E
Total available N	:	N
	:	$N = Y_{N0}/\alpha + N_E$
Uptake external N	:	$UN_E = \alpha \cdot N_C \cdot N - N_C \cdot Y_{N0}$
	:	$UN_E = \alpha \cdot N_C \cdot N_E$
Total N uptake	:	$UN = \alpha \cdot N_C \cdot N$
	:	$UN = \alpha \cdot N_C \cdot N_E + N_C \cdot Y_{N0}$
Total yield	:	$Y = \alpha \cdot N$
	:	$Y = Y_{N0} + \alpha \cdot N_E$

(I refers to internal, E to external).

A. Surplus measures per ha of land

The surplus and balance indicators per ha of land may be calculated in various tautological ways from these quantities. Below they are expressed in terms of N or N_E respectively:

Surplus of internal N:
- internal N minus uptake from internal N.

Surplus of external N:
- external N minus uptake external N,
- total availability minus internal availability minus external uptake.

Surplus of available N:
- total availability minus total uptake,
- external surplus plus internal surplus.

Gross balance of N:
- total availability minus total uptake minus internal availability,
- external availability minus total uptake.

In symbols:

$$SN_I = Y_{N0}/\alpha - N_{C*}Y_{N0}$$
$$SN = N - \alpha_* N_{C*} N$$
$$SN = N_E - \alpha_* N_{C*} N_E + Y_{N0}/\alpha - N_{C*} Y_{N0}$$
$$SN_E = N_E - UN_E = N_E - \alpha_* N_{C*} N_E$$
$$SN_E = N - N_I - UN_E = N - Y_{N0}/\alpha - \alpha_* N_{C*} N + N_{C*} Y_{N0}$$
$$BAN = N - Y_{N0}/\alpha - \alpha_* N_{C*} N$$
$$BAN = N_E - \alpha_* N_{C*} N_E - N_{C*} Y_{N0}$$

All these equations have the same basic structure with respect Z. (Z being N available or N applied, accordingly as the equation concerned):

Indicator = $(1- \alpha_* N_C)_* Z$ + constant

As long as $\alpha_* N_C < 1$, the indicator increases with increasing Z. We note that the fraction uptake from available is always smaller than 1. So the nitrogen surplus of available N per ha of land (SN) increases linearly with the nitrogen availability per ha (N) and because of linearity between N and N_E, also with the nitrogen application per ha (N_E). The same holds for the surplus of applied N per ha (SN_E) and for the input-output balance (external application minus total uptake) of applied nitrogen per ha land (BAN).

B. <u>Surplus measures per kg production</u>

We aim for a use of land such that total surplus for the production needed is lowest. So a term for N surplus per kg product (or for the total desired production) is needed. What about the surplus of available N per kg product in our linear model?

B.1. The surplus of available N per kg product (SN/Y) is:

$$SN/Y = N - \alpha_* N_{C*} N) \,/\, (\alpha_* N) = 1/\alpha - N_C = \text{constant}$$

So the N surplus of available N per kg of product is constant at increasing N availability.

B.2. The surplus of applied N per kg product (SN_E/Y) is:

$$SN_E/Y = (N - Y_{N0}/\alpha + N_{C*} Y_{N0} - \alpha_* N_{C*} N) \,/\, (\alpha_* N) =$$

$$= 1/\alpha - Y_{N0}/(N_* \alpha^2) + (N_{C*} Y_{N0}) \,/\, (\alpha_* N) - N_C =$$

$$= 1/\alpha - N_C + Y_{N0*}\{N_C/\alpha - 1/\alpha^2\}/N$$

SN_E/Y increases as N increases, for $\alpha.N_C$ is always < 1, and then $(N_C.\alpha-1)/\alpha^2$ in the equation above is negative.

B.3. The gross balance of N per kg product:

$$\mathbf{BAN/Y = (N - Y_{N0}/\alpha - \alpha.N_C.N) \;/\; (\alpha.N) =}$$

$$\mathbf{= 1/\alpha - Y_{N0}/(\alpha^2.N) - N_C =}$$

$$\mathbf{= 1/\alpha - N_C - (Y_{N0}/\alpha^2)/N}$$

This is of the form: **BAN/Y = constant - constant/N**

So BAN/Y increases as N increases, as α and Y_{N0} are always positive.

In Appendix 12.11 it is demonstrated that - just as with the Michaelis-Menten model - these relations do not essentially change if the model is extended to proportional availability of several nutrients. This is different in the Mitscherlich model, which model gives a quite different type of curve when extended to different proportionally available nutrients. Also changing the linear additive model to a Von Liebig model does not change the linear relationship.

C. <u>Surplus measures per kg of nutrient uptake</u>

C.1. Surplus of available nitrogen per kg uptake of available N gives:

$$\mathbf{SN/UN = \{N - \alpha.N_C.N\} \;/\; \alpha.N_C.N = 1/(\alpha.N_C) - 1 = constant}$$

The nutrient surplus of available N per kg of N uptake is constant with increasing N availability.

C.2. Nutrient surplus of applied N per kg uptake of applied N:

$$\mathbf{SN_E/UN_E = (N_E - \alpha \cdot N_C \cdot N_E)/\; (\alpha \cdot N_E \cdot N_C) \quad = 1/(\alpha \cdot N_C) - 1 = constant}$$

The nutrient surplus of applied N per kg of external N uptake is constant with increasing availability.

D. <u>Surplus measures per kg of nutrient applied</u>

D.1. Surplus of applied N per kg applied N

$$\mathbf{SN_E/N_E = \{N_E - \alpha.N_C.N_E\} \;/\; N_E = 1 - \alpha.N_C} = \text{constant}$$

The surplus of applied N per kg applied N is constant at increasing N application.

8.4 Several proportional nutrients, non-linear models

In Appendix 12.12 it has been derived that for the Michaelis-Menten model for one single nutrient as well as for several proportionally available nutrients the nutrient surplus per ha of land, as well as the nutrient surplus per kg of yield, increases at increasing nutrient availability.

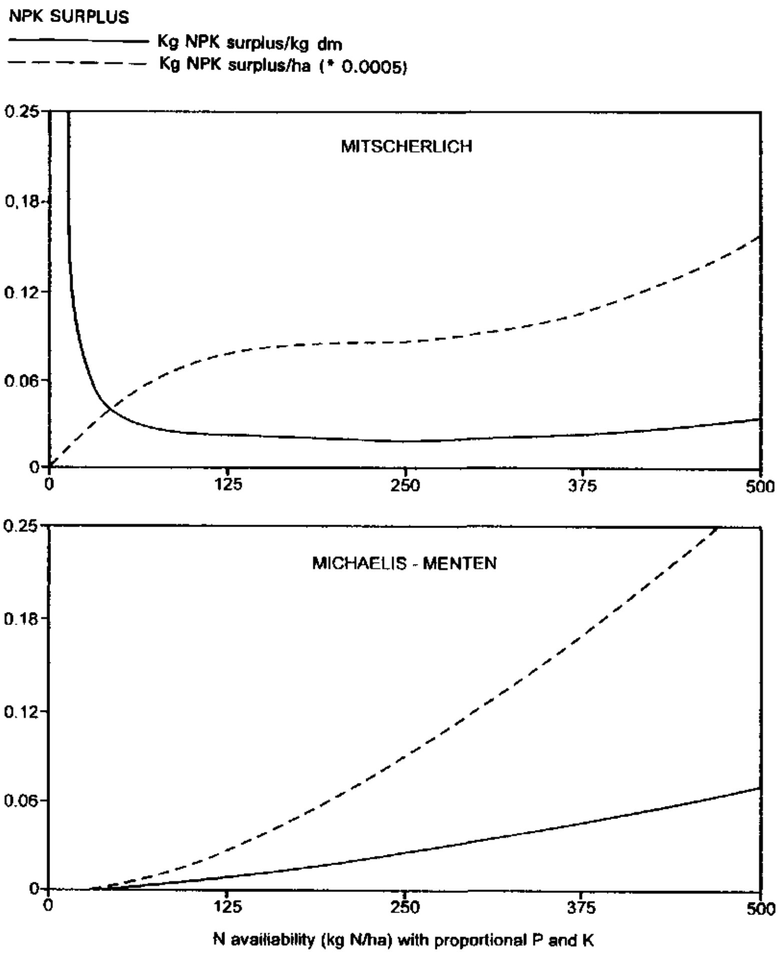


Figure 8.4.1 Nutrient surplus/ha and nutrient surplus per kg dm against N availability, with proportional N, P, and K availability. Mitscherlich model above. Michaelis-Menten model below. For coefficient values see Appendix 12.4.

These relations are visualized in Figure 8.4.1 by numerical simulation with the model of Appendix 12.3. For Mitscherlich the results are only simulated, because of difficulties with the mathematical analysis of that model.

In the Mitscherlich model the optimum nutrient availability for minimizing nutrient surplus per kg dm appears somewhere along the x-axis at about 250 kg NPK.

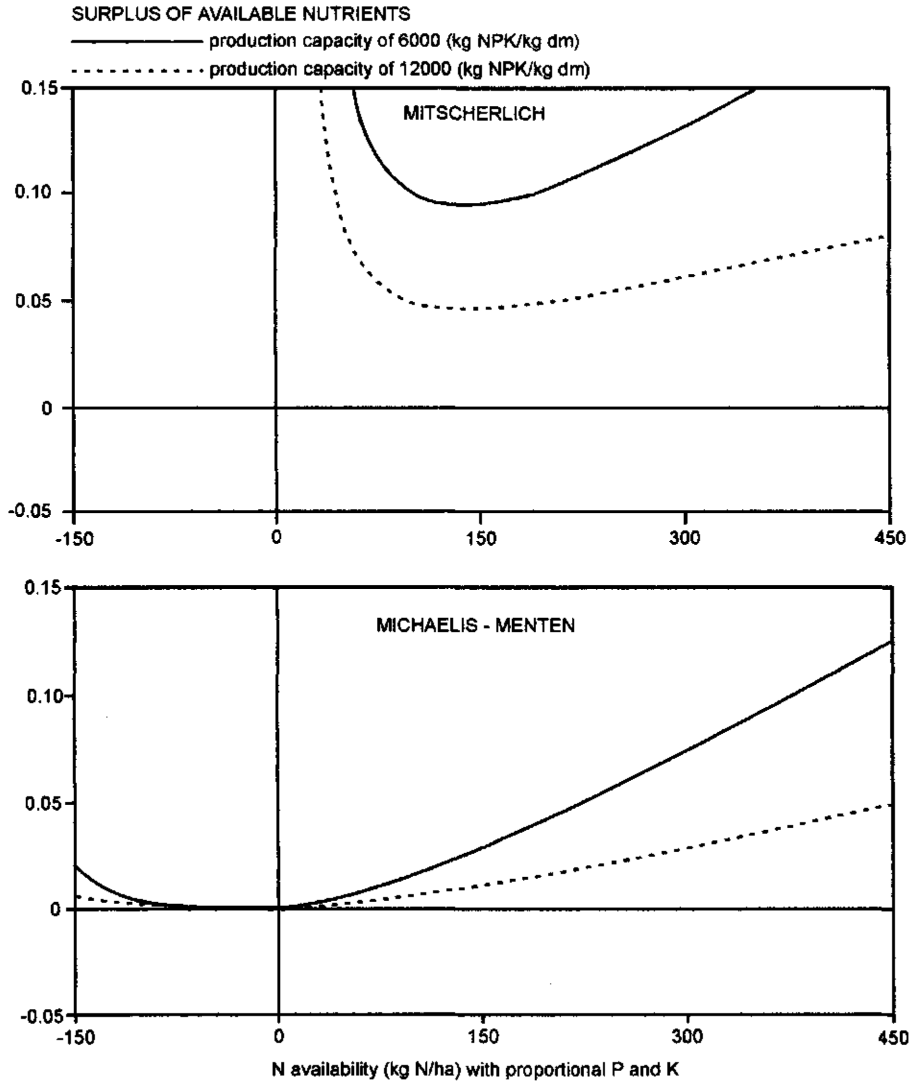


Figure 8.4.2 Relation between surplus of available nutrients (kg NPK/kg dm) and available nitrogen (kg N/ha) with proportional N, P and K and at production capacities of 6000 and 12000 kg dry matter per ha, in the Mitscherlich model (above) and the Michaelis-Menten model (below). For coefficient values see Appendix 12.4.

The optimum availability is expressed as units of Nϕ. In the case of the Michaelis-Menten model, the nutrient surplus per ha as well as the nutrient surplus per kg dm has its minimum at a proportionate N, P and K availability of zero (Figure 8.4.1).

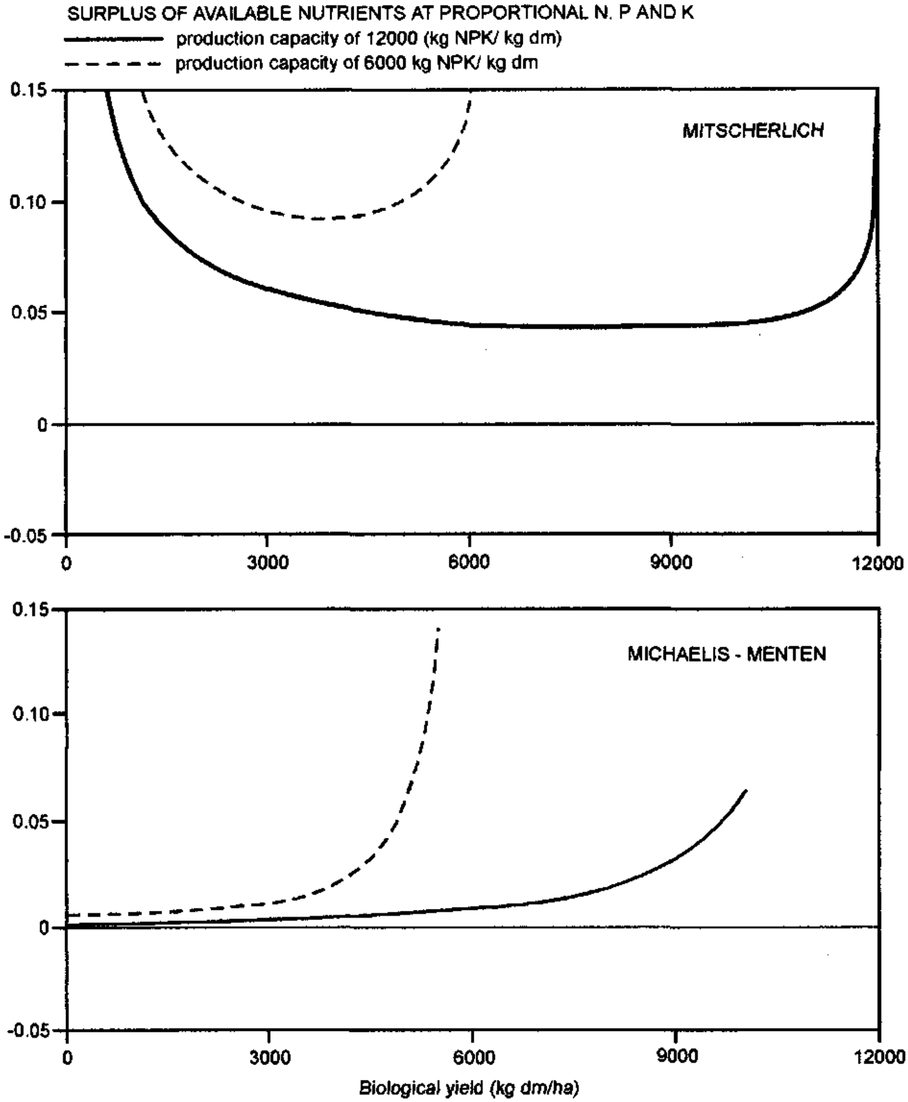


Figure 8.4.3 Relation between surplus of available nutrients (kg NPK/kg dm) and biological yield (kg dm/ha) at proportional N, P and K and at production capacities of 6000 and 12000 kg dry matter per ha, in the Mitscherlich model (above) and the Michaelis-Menten model (below). For coefficient values see Appendix 12.4.

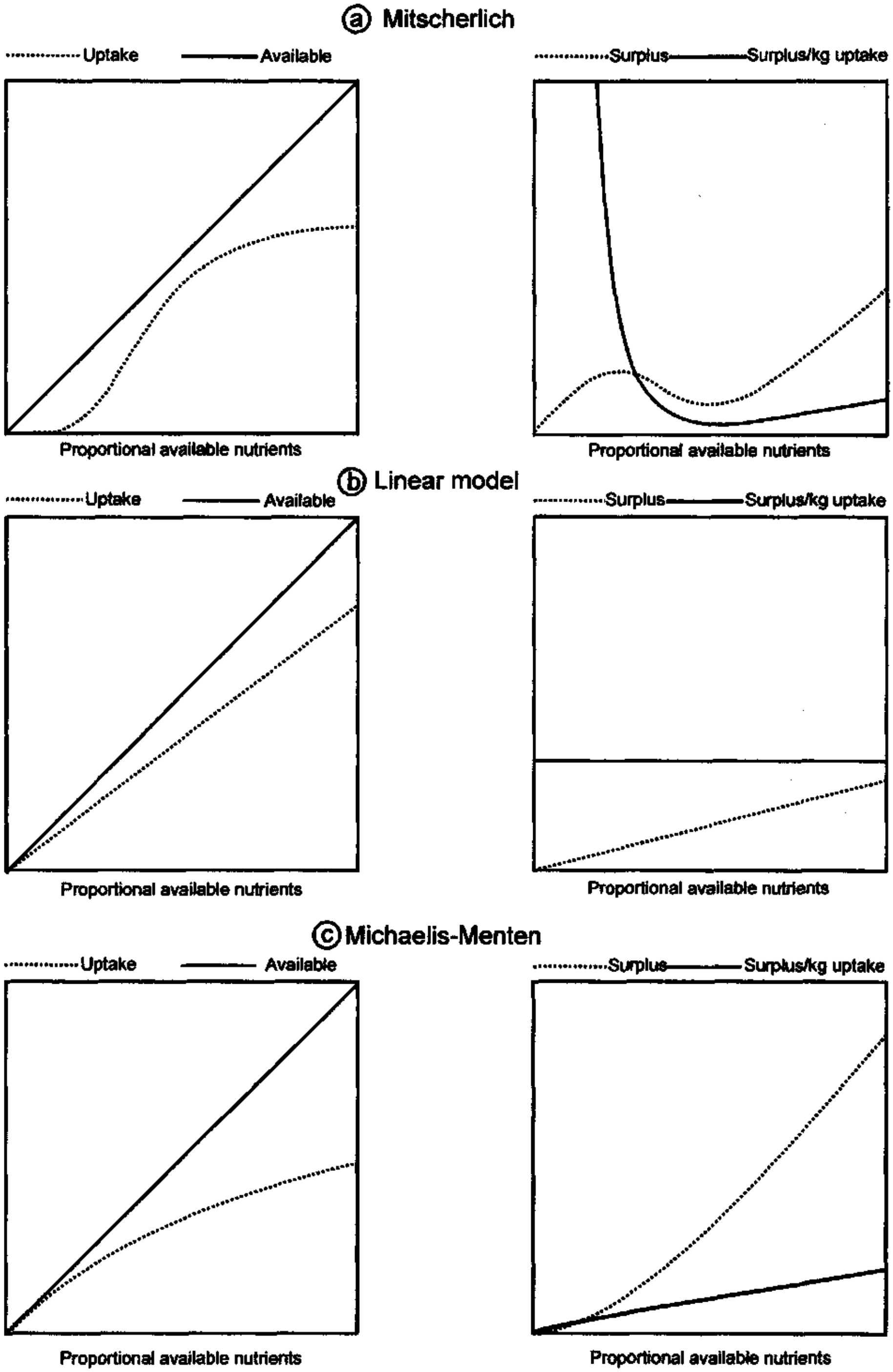
8.5 Comparing linear, Mitscherlich and Michaelis-Menten models

In previous paragraphs (8.1, 8.2, 8.3, 8.4 and Appendix 12.11) it was demonstrated theoretically (and or by simulation), that in the trajectory from zero to medium nutrient availability:

a) if the relations between availability, uptake and yield are proportional, the surplus increases also linearly with increasing proportional availability, and the surplus per kg of uptake is a constant. The Von Liebig model may be considered to be a proportional linear model with a maximum yield plateau, so if the different nutrients are available (in the correct mutual ratios), the surplus per kg of yield will be constant between availability zero and the availability that just gives the maximum yield.
b) in the case of a Michaelis-Menten model, a more than linearly increasing surplus and also an increasing surplus per kg product results from increasing proportional availability of nutrients (Figure 8.4.1 and 8.4.2).
c) in the case of Mitscherlich relations, with increasing nutrient availability, initially the surplus per ha is zero at zero availability, then increases until a local maximum is reached, the deceases until a local minimum and ultimately increases again and remains increasing. So the curve is a sigmoid with a local maximum and a local minimum. The surplus per kg uptake is infinitely high at zero availability, decreases until a minimum is reached, and ultimately remains increasing (Figure 8.4.1 and 8.4.2).

These cases are simulated with the respective models and the behavioral modalities are compared schematically in Figure 8.5.1.

Figure 8.5.1 Nutrient availability, with proportional N, P and K, and nutrient uptake (left parts of the figure) and nutrient surplus per ha and nutrient surplus per kg nutrient uptake (right part of the figure) as a function of available nutrients in (a) a Mitscherlich, (b) a Linear and (c) a Michaelis-Menten model (schematic).


(a) Mitscherlich
Uptake
Available
Surplus
Surplus/kg uptake
Proportional available nutrients
Proportional available nutrients
(b) Linear model
Uptake
Available
Surplus
Surplus/kg uptake
Proportional available nutrients
Proportional available nutrients
(c) Michaelis-Menten
Uptake
Available
Surplus
Surplus/kg uptake
Proportional available nutrients
Proportional available nutrients

8.6 Inferences

- In all investigated models (except the Mitscherlich model, see Figure 8.4.1) the surplus of nutrients/ha increases with increasing nutrient availability.
- In a linear model, the surplus of proportionally available nutrients per kg of yield (or per kg uptake) does not appear to be dependent on the level of available nutrients (see Figure 8.5.1 b). The surplus of applied nutrients per kg of yield increases with the level of proportionally available nutrients (see § 8.3, B.2.).
- In the case of the Von Liebig model, at proportional availability of different nutrients, the surplus of available nutrients per kg nutrient uptake (or per kg of yield) is constant in the range between a nutrient availability of zero and an availability that just assures the maximum uptake (and yield). Above that level the nutrient surplus per kg uptake (or per kg of yield) increases (§ 8.3). Just as with the linear model the surplus of applied nutrients per ha as well as the surplus of available nutrients per ha increases with the proportional availability.
- In case of a Michaelis-Menten model, the lowest nutrient surplus per kg nutrient uptake is found at the lowest levels of proportionally available nutrients (Figure 8.5.1 c). The same holds for the nutrient surplus per kg of yield, even at non-constancy of nutrient concentrations in this model.
- In the case of a Mitscherlich model, the minimum nutrient surplus per kg nutrient uptake is not situated at the nutrient availability that just gives maximum production, nor at a nutrient availability of zero, but is somewhere in between (Figure 8.5.1 a).

9 Nutrient surplus and set aside policy at regional level

9.1 Introduction

In Chapter 7 it was shown that with regard to nutrient **productivity**, the distribution of fields over production capacity (in case of a Michaelis-Menten model) has no significance for the question if land should be discarded from production or not. We found that then all fields (independent of the distribution over soil quality) remained in production, with different intensities of applied nutrients. In case of the Mitscherlich model this was different because in that case the distribution of fields over soil quality appeared relevant for the question which part of the production decrease should be realized through input reduction and which part through discarding fields. In this chapter it will be examined by simulation **if this also applies using the criterion of nutrient surplus**. Considering production surpluses the question has remained whether **for scaling up to the regional level** the same conclusions will be reached as with the productivity criterion in the Chapters 3 and 4. We assume that the production capacity of the soil is related to its retention capacity for nutrients and that the distribution of the fields within a region over retention capacity resembles very much the distribution of the same fields over maximum production capacities. Thus for retention capacity a similar figure as Figure 7.2.1 may be drawn. It must be stated, however, that there may be exceptions to this correlation (some soils have P, and/or K fixing properties, resulting in a much more complicated relation (even an inverse relation) between nutrient retention capacity and production capacity), so that the correlation becomes more complicated. Moreover, nutrient surplus in case of high retention may have a different effects (environmentally) than surplus in case of small retention. Even if both distribution curves are congruent, this does not imply that the bad fields with respect to production capacity are also the bad fields with respect to nutrient retention. The hypothesis to be tested will be that the differentiating quality of the Michaelis-Menten model (as a "landmark" between models with constant or increasing marginal **use of nutrients per kg dm** on the one hand and decreasing marginal nutrient use per kg dm on the other hand) is also valid with regard to **surplus of nutrients per kg dm**. The decision of land discarding versus input reduction is of course not only a matter of minimalisation of nutrient surpluses, but also is also very dependent on farm economic arguments for instance reduced fixed costs fixed costs.

9.2 One variable nutrient, other factors constant, linear model

In § 8.3 it was concluded that for a linear model, at the plot level of aggregation and for one nutrient, when the availability increases, the surplus of applied nutrients per kg dm and the gross input-output balance per kg dm and the surplus of available nutrients per ha increases, while the surplus of available nutrients per kg dm remains constant. How does this work out at regional level of aggregation? Nitrogen (N) is the specific nutrient taken as an example. Imagine that a total regional production of REGY is desired. Then the area needed (A) equals:

$$\mathbf{A = REGY\ /\ (Y_{N0} + \alpha . N_E)}$$

in which $Y_{N0} + \alpha . N_E$ is the yield per ha.

The surplus of available, of applied N and of the input-output balance per h a, respectively, are expressed as functions of N, resp. N_E, as given in § 8.3. Now all the surplus measures for the whole region may be calculated by multiplying the indicators by the area of the region: $\mathbf{REGY\ /\ (Y_{N0} + \alpha . N_E)}$, or in terms of N: $\mathbf{REGY\ /\ (\alpha\ .\ N)}$

<u>Surplus of available nutrients for the whole region</u>:

$$\mathbf{REGSN = \{N - \alpha . N_C . N\}\ .\ \{REGY\ /\ (\alpha\ .\ N)\} = (REGY)/\alpha - N_C . REGY}$$

This equation is of the form: REGSN = constant.

So the regional surplus of available nutrients is constant at increasing N availability.

<u>Surplus of applied nutrients for the whole region</u>:

$$\mathbf{REGSN_E = \{ N - Y_{N0}/\alpha - \alpha . N_C . N + N_C . Y_{N0}\}\ .\ \{REGY\ /\ (\alpha\ .\ N)\} =}$$

$$\mathbf{= REGY/\alpha - (Y_{N0} . REGY/\alpha^2)/N + \{(N_C . Y_{N0} . REGY)/\alpha\}/N - N_C . REGY =}$$

$$\mathbf{= REGY/\alpha - N_C . REGY + Y_{N0} . REGY . (N_C/\alpha - 1/\alpha^2)/N}$$

This equation is of the form: $REGSN_E = constant_1 - constant_2/N$

$N_C/\alpha - 1/\alpha^2$, the same as $(N_C . \alpha - 1)/\alpha^2)$, which is always negative, because $N_C . \alpha < 1$. So $constant_2$ is always negative and the regional surplus of applied N increases at increasing N availability.

Table 3 gives an example of the calculation of these regional variables, based on data from a figure of De Wit (see our Figure 5.2.1).

Table 3 Values of crop ecological variables as a function of variable nitrogen application in a linear production function of a specific example (see text).

Independent variables and coefficients:			
N application (kg N/ha)			
N_E (arbitrary values)	0	80	160
Internal N availability (kg/ha)			
N_I (estimated from data)	62.45	62.45	62.45
Regional production target (kg)			
REGY (arbitrary value)	10^6	10^6	10^6
N % in dry matter (dimensionless)			
N_C (assumption)	0.018	0.018	0.018
Response coefficient (kg dm/kg N)			
α (estimated from data)	34.35	34.35	34.35
Dependent variables (all derived from the equations)			
Total N availability (kg/ha)			
$N = N_I + N_E$	62.45	142.5	222.5
Dm Production (kg/ha)			
$Y = \alpha \cdot N$	2145	4895	7643
N uptake (kg/ha)			
$UN = Y \cdot N_C$	38.61	88.11	137.6
Area needed for yield target (ha)			
REGA = REGY / Y	466	204	131
Surplus of available N (kg/ha)			
SN = N - UN	23.84	54.39	84.9
Surplus of external N (kg/ha)			
$SN_E = SN - SN_I$	0	30.55	61.06
N output - input balance (kg/ha)			
$BAN = N_E - UN$	-38.61	-8.11	+22.4
Surplus of available N (kg/kg dm)			
SN/Y	0.011	0.011	0.011
N Output - N input (kg/kg dm)			
BAN/Y	-0.018	-0.0017	+0.003
Regional surplus of available N (kg)			
$REGSN = SN \cdot REGA$	$11.1 \cdot 10^3$	$11.1 \cdot 10^3$	$11.1 \cdot 10^3$
Regional surplus of external N (kg)			
$REGSN_E = SN_E \cdot REGA$	0	$6.2 \cdot 10^3$	$8.0 \cdot 10^3$
Regional N balance (kg)			
$REGBAN = BAN \cdot REGA$	-17992	-1654	+2934

Table 3 is based on a linear relation of yield and applied nitrogen. The results of the abovementioned model are given for three different levels of N per ha, assuming a target for regional production of: REGY = 10^6 kg dm and values of the other coefficients of: Y_{N0} = 2145 kg dm/ha, α = 34.35 kg dm/kg N, N_C = 0.018 (proportion of dm).

<u>Gross balance of N for the whole region</u>:

$$\mathbf{REGBAN = \{N - Y_{N0}/\alpha - \alpha.N_C.N\} . \{REGY / (\alpha . N)\} =}$$

$$\mathbf{= REGY/\alpha - N_C.REGY - \{(Y_{N0}.REGY)/\alpha^2\}/N}$$

This equation is of the form:

$$\text{REGBAN} = \text{constant}_3 - \text{constant}_4/\text{N}$$

So the regional gross balance of N increases at increasing N availability. This also applies to proportional availability of nutrients in the linear model, as the linearity of the model is maintained (see Appendix 12.11).

9.3 Several proportional nutrients; non-linear models

From the derivation and the calculation above, we may conclude that, at the regional level, assuming a **linear model** and one separate nutrient (N in this case), an increasing surplus of available N per ha, a constant surplus of available N in the region and consequently a constant surplus of available N per kg of yield is found with increasing N. In Appendix 12.11 it was elaborated that these inferences will be unchanged when the linear model with one nutrient is extended to several proportional nutrients or when the model is transformed into a Von Liebig model.

We expect that the hypotheses of increasing nutrient surplus at increasing availability will be the more endorsed if the relations between availability and uptake and between uptake and production are **not linearly** related, but with decreasing slope. Even when nutrient concentrations in the dry matter would increase at the higher levels of availability, that would not lead to a higher recovery per unit of nutrient. Thus, at high availability, increasing surplus per ha and per kg dm will be the result, differing from the case of a linear relation (§ 8.4). This will be the more so for the regional level with its greater influence of heterogeneity. Only in the Mitscherlich model increasing yield per kg nutrient surplus may be expected in the lower range of increasing availability.

Not in all theoretical models, is maximizing nutrient productivity inseparably connected to minimizing nutrient surplus. Because the nutrient uptake is not always linearly related to the production, especially for high nutrient availabilities, nutrient concentration of the product may increase considerably.

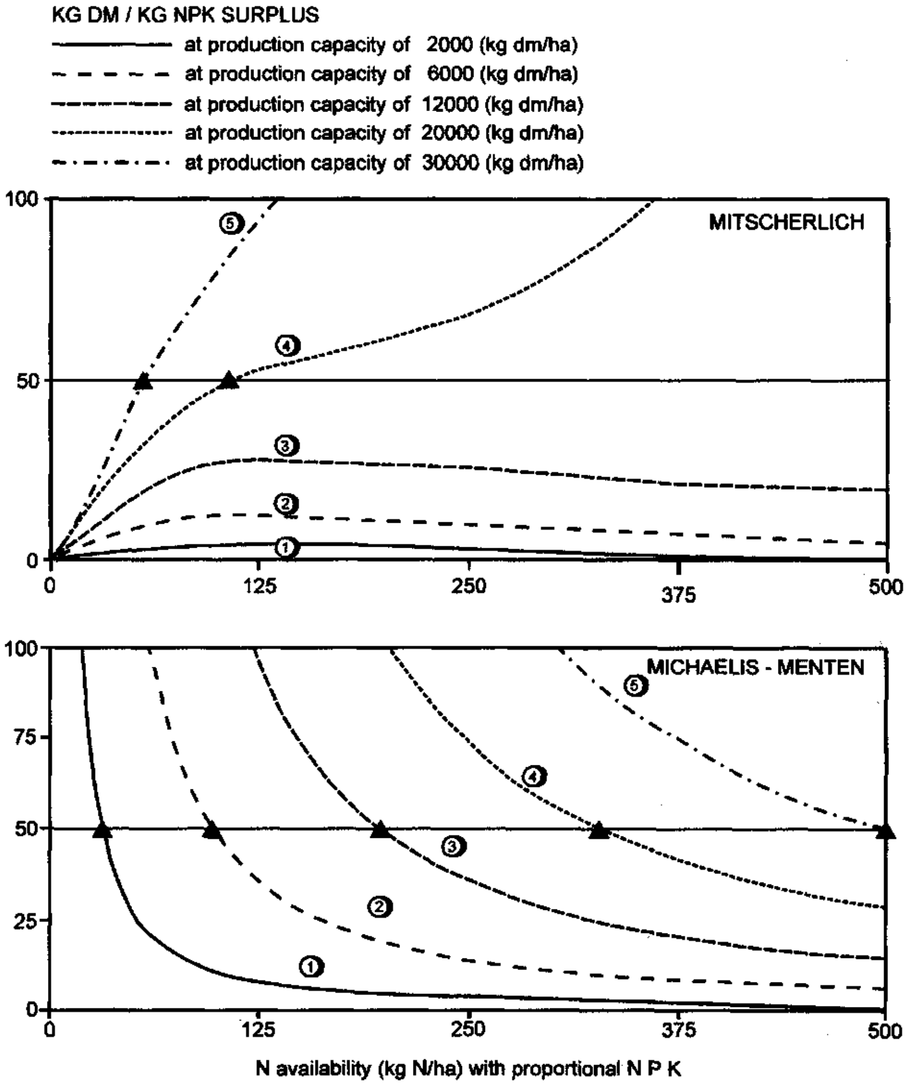


Figure 9.3.1 Relation between the nutrient surplus productivity (quotient between production and N+P+K surplus = reciprocal of nutrient surplus per kg dm yield) and available N (with P, and K availability proportional to that of N), for five levels of maximum production for two models (the numbered lines represent five soil qualities). The horizontal lines connect points of equal average productivities of the production curves. For coefficient values see Appendix 12.4. For the meaning of ▲ see the text.

Despite this feature, simulation with the models illustrates that the conclusion for (nutrient) use productivity and (nutrient) surplus productivity in case of the Michaelis-Menten model is the same: (Nutrient) use productivity and (nutrient) surplus productivity (the latter being the reciprocal of nutrient surplus per kg dm yield), both have their maximum at minimum nutrient availability in the Michaelis-Menten model. In Figure 9.3.1 we show that in the Mitscherlich model these maxima are situated at higher levels of nutrient availability.
The encircled numbers in the figures represent the order of most efficient reduction of nutrient application and/or reduction of land area of increasing qualities, if a minimum nutrient surplus for the total region is aimed at. From Figure 9.3.1 it follows that, to reduce nutrient surplus per kg product, the intensification on good land and the setting aside of bad land tends to be recommended **if the Mitscherlich model holds**, while keeping the total area in production and reducing the nutrient availability on the area tends to be recommended **if the Michaelis-Menten model applies**. But in all situations nutrient availability should be reduced more on bad land than on good land. Ratios between nutrient availabilities should be kept proportional. Reducing the nutrient availabilities on all fields to a level such that equal ratios between production and kg nutrient surplus are found, until the regional production goal is just reached, will reduce the total nutrient surplus maximally, as it appears from the Michaelis-Menten model.

Reduction policy is, however, much more complicated than the inferences drawn in this chapter imply. This is because (apart from relevant criteria other than nutrient use productivity and nutrient surplus productivity), the relations (see introduction § 9.1) underlying the assumption of coincidence of the distribution of fields over production capacity and over retention capacity may be more complicated and important than assumed here.

9.4 Inferences

- In case of the Michaelis-Menten model, at regional level, the nutrient surplus will be least at the lowest external application. As in this model also the highest productivity is obtained at the lowest application, it may be stated that in this model **high productivity and environmental cleanliness go together**. However, these performances do not go together with maximum production and maximum profit.
- At the regional level, in the case of one variable nutrient (other factors constant) and assuming a linear production function, the surplus of available nutrients per kg product is constant at different nutrient availabilities. In case of several proportionally available nutrients, the same conclusion is valid, if the relation between production and proportionally

available nutrients is linear and if there are no interaction effects (Appendix 12.11).

- In the case of the Von Liebig model, at a regional level, and at proportional availability of nutrients, a reduction of the target yield may be just as well reached by reducing input of nutrients as with reduction of acreage, since both the productivity and surplus per kg dm is constant over the whole activity range of the specific nutrient. If other criteria, e.g. fixed costs, are also considered, conclusions may be different.
- In the case of the Mitscherlich model the conclusion is different: Fields have to be taken out of production until the level of regional production goal is just reached and the ratios of production per kg nutrient surplus are just equal. This will be partly attained by taking out of production the fields in sequence of increasing production capacities, and partly by reducing the levels of available nutrients on the remaining fields.
- In the case of the Michaelis-Menten model the situation of equity between yield and regional yield target, under conditions of equal marginal ratios between production and surplus, will be attained by reducing the nutrient availability on all fields, and discarding none of the fields.
- The modelling exercise in this chapter gives only a first approach, with regard to the specific characteristics of the Mitscherlich and Michaelis-Menten models, to calculations regarding the extensification / intensification policy. For more than one criterion variable, more than one limiting condition, a complicated distribution of fields over land qualities, more extended optimization models are needed to calculate how much land to discard, and/or nutrients to apply, in each category of land quality.

10 Application, surplus and residue of nutrients (data)

10.1 Introduction

In this chapter a number of data sets from literature were reanalysed in order to test empirically the hypotheses of increased nutrient surplus per kg product, at increased nutrient availability, as predicted by the Michaelis-Menten model. In fact no data could be found in which the condition of proportional availability of different nutrients was met exactly. Moreover in empirical and experimental observations theoretical concepts like nutrient availability or nutrient surplus cannot be observed. Variables which often appear in observations are: "nutrient residue (after harvest)", "nutrient uptake with the crop", not such theoretical variables as "internally available nutrients" and "nutrient losses". The experimental data in this chapter were taken from publications of Chaney (1990), Dilz (1971), Schröder (1993, 1996) and Neeteson (1995).

10.2 Analysis of the data

Looking at actual data, where surplus can be derived from available data on uptake and application, it appears from data of Dilz (1971) that the Michaelis-Menten model holds, except that at the medium range of application a plateau occurs (Figure 10.2.1).

Part of the explanation may be that at a higher amount of nitrogen a relatively large amount of straw is produced (see the decreasing harvest index with increasing N application) and relatively more nitrogen turns up in the straw (see the decreasing N harvest index). This may have caused an increased uptake at applications of 80 and 120 kg N, even such that the surplus per kg dry matter (or, per kg total N, or, per kg nitrogen in the grain component) is hardly more than at 40 kg N.

Considering the N concentration (Figure 10.2.2), even a significant decrease shows up at 40 kg N as compared with zero kg N. For N concentration of dry matter the t_{10} of that difference is -6.5; $P < 0.0001$. For N concentration of grain the t_{10} is -3.82; $P < 0.05$.

This provides circumstantial evidence that the crop grew under conditions of very low N, and so a large need for N prevailed. At higher nitrogen applications the extra nitrogen is taken up for the most part, so the surplus per kg product remains about the same. Larger plants may have deeper roots and thus a larger nitrogen absorption capacity.

The calculated surplus does not necessarily equal the observed residue after harvest: Immobilization, denitrification and leaching may provoke great differences between both variables.

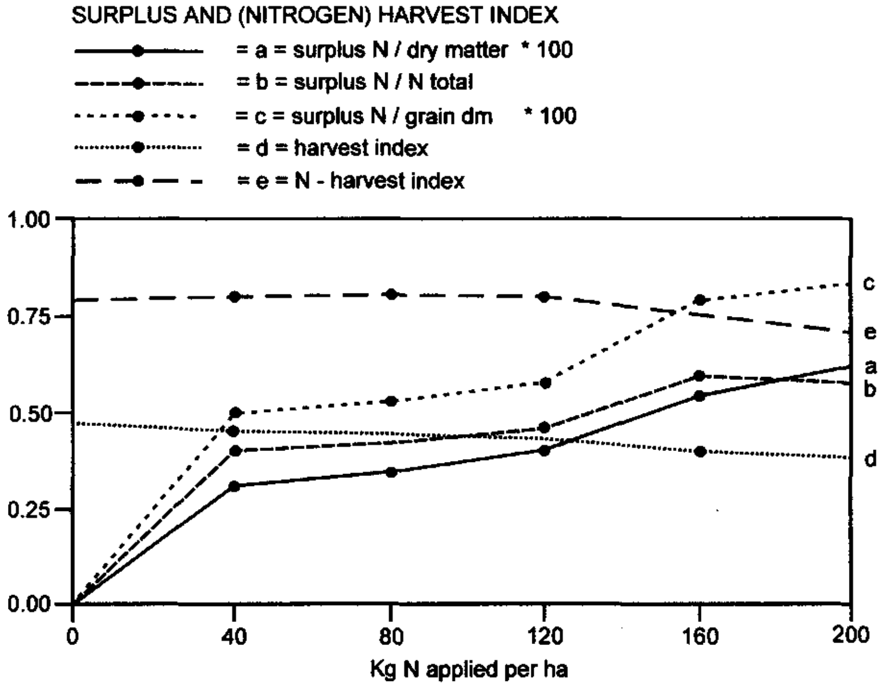


Figure 10.2.1 Relation of nitrogen application and surplus of applied N per kg dry matter (a), surplus of applied N per kg total N in crop (b), surplus of applied N per kg grain dry matter (c), harvest index (d) and harvest index of N (e). (Dilz, 1971).

Comparing surplus and residue in the literature reveals that the residue shows about the same course as the surplus. In the data from Chaney (1990) the residue per kg of yield even slightly drops at increasing application (Figure 10.2.3), but the drop is not significant (t_{30} = 0.55; n.s.).

Chaney observed in the parcel "Writtle 1445" a higher amount of soil nitrate when no fertilizer was applied than when 80 or 120 or 160 or 200 or 240 kg N was used. Soil residue is the result of a complex interaction process of soil, crop and fertilizer, with feedback among these three factors. E.g.

- Crops desiccate the soil, and the low water concentrations inhibits mineralisation, growth and activity of roots.
- Nitrogen starvation results in small plants, so roots grow less deep and mineralised nitrogen is not explored by the roots.
- Leaching is different with different crops.

In maize (Schröder, 1993, 1996) and potatoes (Neeteson, 1996) an increasing nitrogen residue per kg of yield at increasing application was found. In sugar beet and grass Lantinga (1996, personal communication) found decreasing residue per kg product. The latter two crops have a very long growing period and a relatively extended root system. This may be the explanation of a higher utilization of nitrogen at a medium application than at a low application.

In general, when increasing nitrogen, a decreasing residue per kg product seems an exception rather than a rule. Does this exception occur, it is very difficult, because of soil heterogeneity, to apply the exact amount of fertilizer, in order to get a smaller residue per kg product at higher application than at lower application.

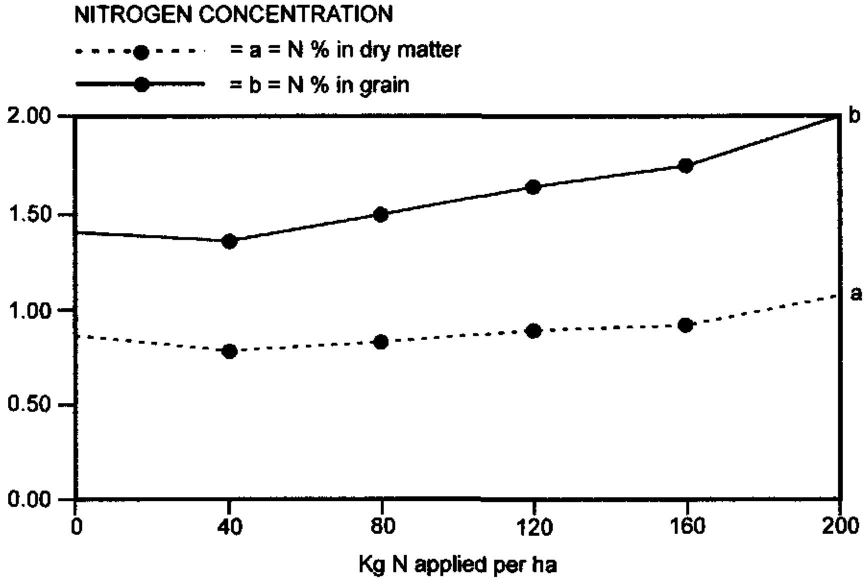


Figure 10.2.2 Nitrogen application and nitrogen concentration in dry matter (a), nitrogen concentration in grain (b). (Dilz, 1971).

Farmers cannot (because of ignorance) and will not (because of the man on the machine or the machine being not accurate or ready) take into account the differences between different parts of the field and apply more than the optimal rate in some fields. In farming, it is impracticable and difficult to achieve the minimal residue per kg product, if such a minimal residue occurs in specific crops.

Scaling up this exception from the plot to the field level will result in an almost constant residue per kg of yield. Precision farming is (still) not practicable.

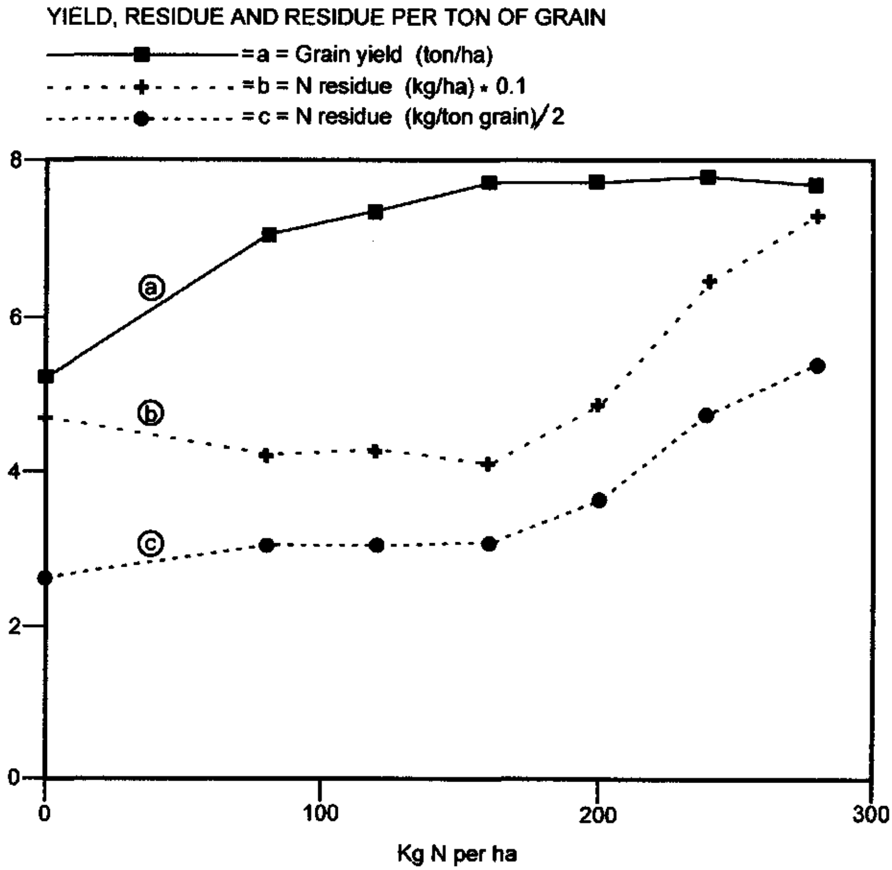


Figure 10.2.3 Nitrogen application and grain yield (a), N residue in kg per ha (b) and N residue per ton of grain (c) (Chaney, 1990).

That means that in general higher residues per kg product arise at increasing applications, but with a somewhat lower slope in the lower part of the application range. It should be stressed that neither surplus nor residue are the variables that really matter. It is the **amount** of nutrients **emitted** per kg dry matter that is more relevant. The nutrients not taken up by the crop, should only partly be regarded as lost nutrients. The other part will be accumulated, especially in ecological well-managed production systems. In this study the focus is on surplus, because emission is a variable difficult to formalize in a static model. Clearly, a higher surplus increases the chances on higher emission.

10.3 *Inferences*

- Because the available experimental data are not the exact equivalents of the theoretical concepts (assumptions of nutrient proportionality and static soil stocks are not met), the inferences may only be regarded as tentative.
- Neither the hypothesis of decreased nor that of increased nutrient surplus per kg dm yield at increased application appear to be empirically validated in general. Some data seem to corroborate increased surplus, others decreased surplus or constant surplus per kg dm yield at increased application in the trajectory of zero up to medium nutrient availability. In the range of higher applications the hypothesis of increased nutrient surplus per kg dm yield at increased application was always, and in the range of the lower application mostly, corroborated.

11 Inferences and possible implications

In this study we concluded that, both at field and regional level of aggregation, the intuitive expectation that high production with high application of nutrients is associated with low nutrient productivity and high nutrient surpluses (per ha as well as per kg product) was confirmed. It was argued that there may be other arguments in favour of high external applications, but there are also several arguments in favour of low external applications. Thus for land use practice a trade off is necessary between different interests.

In reconsidering our total study, in this chapter the major conclusions in the chapters before are recapitulated. Hypotheses which may be the subject of further discussion, of systems analysis studies, or of experimentation are added. Further considerations for policy formation, for which the reasonings in the earlier chapters are relevant, are given.

Definition of concepts

We argued, that it is of major importance to make one's reasoning very explicit by sharply defining objectives, concepts, variables, coefficients, theories, mechanisms, aggregation levels and time horizons. Confusion between agronomists, ecologists, economists and politicians may thus be prevented.

It is rather different whether one talks about biological production or agronomical production, about productivity or profit, about production from external, internal or total inputs, about marginal or average productivity, about short term or long term sustainability, about productivity at the field, or regional level, about nutrient productivity or nutrient losses, about land, labour or resource productivity. It is further important to differentiate, in historical data, between increases of applications and technological leaps. Above all it is important to which final objectives the study is directed.

Theoretical validity of the Liebscher theory

Considering the optimal rate of nutrient application, the focus was not especially on the relation between production and the increase in inputs separately applied, but on the relation between production and the increase in **proportionally available** inputs.

In § 3.4 the hypothesis was put forward, that for input-output relations at the aggregation level of the field and higher, the Liebscher theory is theoretically more valid than the Mitscherlich theory, on one hand, and the Von Liebig theory, on the other hand. It was reported that the Liebscher theory is supported by research from the past (Van der Paauw, 1938; De Wit, 1992b), and it was argued that the Liebscher theory may be regarded as a Von Liebig model scaled up in space and time.

The Michaelis-Menten model as a formalization of the Liebscher theory

The Michaelis-Menten model, without interaction terms, appeared to be a good formalization of the Liebscher theory, not only for the relation between nutrient application and uptake, but also for the relation between uptake and production and for the relation between application and production, provided that adequate internal availabilities of nutrients were estimated.

The consequences of spatial heterogeneity, and the imperfect correlations in the occurrence of nutrients in the soil, are theoretically accounted for in the Michaelis-Menten model (see Appendix 12.6) where we derived the Michaelis-Menten model from soil heterogeneity assumptions). The Von Liebig and Mitscherlich models do not consider field variability in their assumptions.

It was concluded that in most empirical situations the effects of spatial and temporal heterogeneity (at macro-level) **overrule** the positive interaction effects of nutrients. So, at macro-level the Michaelis-Menten model is more appropriate than the Mitscherlich model.

The Michaelis-Menten model covers part of the interaction effects between different nutrients. From its structural form the model gives positive interaction effects between nutrients, without the need for any explicit interaction terms. In case of yield depressing effects from large applications of nutrients, an extra correction factor may be needed (the Greenwood extension of the Michaelis-Menten model (Greenwood, 1971)). A hypothesis, which has still to be tested more extensively (we have only one example, see Figure 4.5.1), is that **at the field scale (and higher level)** the model is also applicable for other growth factors (water, radiation, carbon dioxide, crop protection chemicals). This does not imply that **at the level of the individual plant** different types of relations for different processes give a more valid description. However the higher the aggregation level, the more different situations may be included in the same type of production function. See for example the Cobb-Douglas function in economics, where such very different production factors as labour, land and capital all have the same formal position in the model structure.

Other formalizations of the Liebscher theory

A chemical analogon formalization of De Wit (1992, unpublished to be reported soon (Goudriaan, 1997, personal communication)) of the Von Liebig, the Liebscher and the Mitscherlich model assumes a homogeneous substrate (perfect mixture of nutrients). The formalization of De Wit has the advantage that it puts the three models of Von Liebig, Mitscherlich and Liebscher in one integrated system dynamics theory of the underlying processes. The difference between the three model variants boils down to different values for only one model coefficient. In the formalization of De Wit the differences among the Mitscherlich, the Von Liebig and the Liebscher models are attributed to differences in reaction velocities and accumulation. However, unlike the Michaelis-Menten formalization, the model does not account for substrate heterogeneity.

Empirical validity of the production functions

At the aggregation level of the field the Michaelis-Menten model - which is a good representation of the Liebscher theory gave a better description of most of our reinvestigated empirical nutrient-production relations than the Von Liebig or Mitscherlich theories (§ 4.4). Additional empirical research is needed to test whether the Michaelis-Menten model is also valid for combinations of other production factors than nutrients; we have shown this for CO_2 only.

The model appears to fit with empirical data not only for one nutrient, but for several proportionally available nutrients as well (§ 4.4 and Appendix 12.8).

The Michaelis-Menten equation not only gives a satisfactory description for the empirical relationship between application of nutrients and the resulting production, but mostly also for the intermediate relations: the relation between application and uptake of nutrients, and also - as a mathematical consequence - for the relation between uptake and production (§ 4.4 and Appendix 12.9).

Pragmatic relevance of the production functions

Even if the Mitscherlich model would be theoretically and empirically valid, the hypothetical increasing productivity characteristic at the lower range will, in many instances, be situated below the range of available nutrients, furnished by internal delivery (fixation, deposition, flooding) of the system. So, even if the Mitscherlich model would be appropriate, the availability where nutrient surplus is minimal will be situated close to the level of nutrients internally available. In practice, agriculture is not in a situation of "tabula rasa", where one can start from scratch. If there is no history of farming, then there is a history of natural biological vegetation. This implies a certain level of internal nutrients, which is mostly to the right of the point of inflexion of the compound sigmoid Mitscherlich production curve and not to the left. How theoretically and empirically valid Liebschers' law of the optimum may be it has a low pragmatic validity. Zoebl (1996, p. 419) mentions this disadvantage where he states that the lack of knowledge of the farmers is a restriction for use in practice of this model.

Some important consequences of validity of the Michaelis-Menten model

If the Michaelis-Menten equation (as a formal model of the Liebscher theory) describes adequately nutrient-soil-production relations, the inference is that, when several nutrients are applied proportionally (e.g. application of organic manure), no increasing productivity is to be expected, even when the number of nutrients in the model increases (Appendix 12.8). Consequently the maximum productivity (kg product per kg available nutrient) is found at zero availability (§ 3.4). By contrast in the Mitscherlich model the optimum availability is situated at a rather high level (§ 3.3) and in the Von Liebig model even at the level which gives the maximum production (§ 3.2).

Implications of different measures of productivity

If productivity is expressed as kg harvestable yield per kg applied nutrient, for some crops (e.g. cereals), the availability at which maximum harvestable productivity is attained will move from zero to a low value (close to the value of zero application), because the harvest index increases concomitantly with production (Table 2).

If the hypothesis of maximum productivity at low nutrient availability holds when productivity is expressed as kg yield per kg **available nutrients**, (total productivity) it holds even more if productivity is expressed as kg total yield per kg **applied nutrients** (system productivity). We argued that an appropriate measure for the nutrient productivity at the system boundaries of an agro-ecosystem (provided that it remains in steady state !) is the "system productivity". The indicator "total productivity", is physiologically more relevant. The results of ecologization in which external nutrients are replaced by internal nutrients will be measured by this measure, and not by others [17]).

Internal and external nutrients

Important in our discourse is the distinction between external and internal nutrients. As long as the system remains in steady state, the quotient between external production and external nutrient gives a fair measure of "(system) productivity", better than the quotient between total yield and available nutrients, in which the achievements of reducing the external input are less rewarded. If, however, soil stocks are being depleted by production, the output factor in the quotient must be corrected by subtracting that part of the production which has been generated out of soil stocks. The yield generated from nutrient input by deposition, biological nutrient fixation, nutrient mobilization from rock minerals and/or flooding, however, should be included in the output/input quotient, because this yield is generated from internal nutrients without depleting the soil stock. For instance small amounts of rock minerals, permanently and regularly becoming available from layers of subsoil are not regarded as depletion, as well as N fixation from the atmosphere is not regarded as depletion. However, the yield generated from finite stocks of nutrients in the soil **does deplete** the reserves of nutrients in the soil and so is not regarded sustainable. However, when the reserves in the soil are excessive (as is the case for P in the Netherlands), depletion to a certain level may be regarded as sustainable.

Possible consequences of different models for farm practice
An overoptimistic hypothesis of "increasing nutrient productivity by means of increasing the availabilities of several nutrients proportionally" may suggest that an increase in the external application of nutrients is rational. Even when a Mitscherlich relation would be appropriate for homogeneous situations such as prevailing in water cultures, in pot experiments and at small trial fields, care should be taken not to generalize the observed relations too easily from plant, and plot, to field, farm, and regional, or even to European level. Another, probably still more important, factor for the nowadays in Western Europe usual high nutrient application is the relatively low price of nutrients, as has been stated in Chapter 6.

The role of system diversity
As the diversity in growing conditions becomes more predictable or manageable, the empirical validity of the models with increasing productivities at increasing application would improve. However, the question is, to what extent can the heterogeneity in space and time be made predictable or manageable? How far can heterogeneity in future be predicted from heterogeneity in the past? How much "new" heterogeneity is introduced when equalizing "old" heterogeneity? From a nearly homogeneous initial condition, a little better provision with nutrients at one location may give the crop a small initial advantage, which will be reinforced by positive feedback. This may give rise on neighbouring locations to an environment which may constrain the crop, from producing at a maximum level. The other option is to make heterogeneity manageable, by means of site-specific monitoring and associated fertilization. Mostly the poor spots are also the spots with the lowest production potential, and (even partly because of) lower retention capacity for nutrients of the soil. What makes more sense, applying more fertilizer on poor spots to compensate for the low internal supply? or applying less fertilizer on poor spots to maintain the proportional (harmonious) ratios of nutrients? In the first case the surpluses may increase because the nutrient applications are not adapted to the potential of the poor spots. In the second case, would not the differentiation in soil fertility cause differentiation in ripening of the crop? Does this also imply the necessity of site specific harvesting dates?

Causes of increasing or decreasing nutrient use productivity
Possible causes of increasing nutrient use productivity at increased nutrient availability were noted:

a) positive interactions between nutrients in the production process,
b) increasing soil qualities of remaining soil, in case of selective volume reduction,
c) a more extended root system, when applying more nutrients.

Possible causes of decreasing nutrient use productivity at increased nutrient availability are:

d) spatial variability within and between fields,
e) increasing proportions of nutrient loss with increasing nutrient concentrations in the soil,
f) decreasing activity of the crop to mobilize (deeper) internally available nutrients, when external nutrients are superficially abundantly available,
g) the decreasing proportion of internal nutrients of total available nutrients.

It was noted (§ 8.2) that increasing nutrient concentration of product with increasing application (luxury consumption of nutrients) tends to cause a decrease of nutrient surplus, but at the same time a decrease of productivity (kg dry matter per kg nutrient).

Comparing productivities of low and high nutrient availability levels and good and bad production technologies

It may be concluded that low external input agriculture is at least as productive as medium external input agriculture and more productive than high external input agriculture. This applies for production per kg (of the weighted sum of different) nutrients, and for production per kg (of the weighted sum of different) nutrient surpluses). Remains that systems with a higher production capacity and a better technology have higher productivity than systems with a lower production capacity and a worse technology. An assumption is that the high inputs are not needed for the application of advanced technology.

Productivity of nutrients, compared to those of land and labour

The largest productivity of nutrients does not coincide with maximum productivity of labour and land. With low availabilities, nutrient productivity is high, but in this condition the production per ha of land and per unit of work force tends to be low. This is only a disadvantage if there is shortage of labour and land. In case of abundance of land and labour, and from an environmental standpoint, the substitution of environmentally unfriendly material inputs by labour may become desirable because it reduces not only the use of material input, but raises also the material resource productivity.

From an overloaded market point of view, the exchange of land productivity by resource productivity has advantages as well. For a better integration of all these aspects some researchers tend to express all inputs and outputs in terms of energy, or in terms of money. It may be questioned if this is sensible because all inputs are connected to different social values which must be traded off against each other and quantified in their own units. Even within the domain of "labour" it is sensible to differentiate between human, animal and mechanical labour and between family labour and hired labour.

Meanwhile, we state that during the last decades productivity of resources has risen by means of technological and biological innovations, but that this effect hardly compensates for the loss in productivity that results from raising application levels (Chapter 5). Just as the increase of external inputs is usually accompanied by an amelioration of "best technological means", so is decrease of external inputs accompanied by an amelioration of "best ecological means". In the case of a sigmoid production curve, both of these correlations would decrease the curvature of the production function, so a more constant nutrient productivity would result.

Incompatibility of "best ecological means" with high external input

Best ecological means often appear to be not compatible with high external inputs. Biotic N fixation decreases as N fertilization is raised. Phosphate mobilization by mycorrhiza does not take place in situations of abundant phosphate fertilization.

Productivity and farm economic profit

Obviously, all other conditions equal, a higher output from a certain input is generally preferable. Thus, productivity as a criterion is an interesting indicator. Generally at the amount of nutrient giving the highest productivity, surpluses will be the least. However, productivity is not the only decisive criterion in farming. High gross margin, high yield and low emissions are the important yardsticks at the farm level. A farmer, which has negative or relatively low gross margins will not survive. So his decisions will be based not on a biological criterion, e.g. productivity, but on an economic criterion, e.g. gross margin. Thus productivity is a rather academic criterion.

Furthermore we observed, that using a sigmoid curve as the model for the effects of different amounts of nutrients, productivity is highest and surplus is lowest at a rather high amount. In the case of a Michaelis-Menten model that situation occurs at zero application. So productivity is a criterion which is very dependent on the assumptions behind the problem definition.

The optimum nutrient application from the perspective of farm economics remains at a **very high** level, at the current price ratios between input and output in Western countries. The level of application would be lower when nutrient productivity (kg product/kg available nutrient) and/or nutrient surplus (kg surplus/ha, or kg surplus/kg product), would be optimized. If the environmental costs are included as production costs, and/or the prices of the nutrients would be higher, and/or the product prices would be lower and/or labour or land (as a substitute for nutrients) would be (made) cheaper (or some of these together), then, according to the economists oriented in the free-market

(Kol & Kuijpers, 1996), the farm economic optimum nutrient application would drop. Small farmers' organisations and some agricultural sociologists and social economists, however, argue that for those very reasons profits would drop, marginal farms would be liquidated and marginal land would be taken out of production. This process would result in an increase in scale and a technological shift to megafarms presumably using high material inputs (biocides + nutrients) and low labour input. This is accompanied by liquidation of the "laggards". It would results in exactly the opposite of a substitution of nutrients by land and labour, as derived above from the reasoning of free market economy (Koning, 1991). Such ambiguity, especially within the socio-economic analysis, makes it also difficult to decide about the desirability of extensification versus intensification.

Different optima for nutrient productivity and nutrient surplus

When yield is expressed in terms of nutrient uptake and yield is proportional to nutrient uptake, and uptake to availability too, the optimum nutrient availabilities (for maximizing the yield per kg available nutrients and for maximizing the yield per kg surplus of available nutrients) would coincide. It depends also on the type of production function.

From the Michaelis-Menten production function it was concluded that not only the nutrient surplus per ha is smaller at low nutrient availability, but also the surplus per kg product. In that model the mathematical analysis results in the maximum productivity, the minimum surplus/ha and the minimum surplus/kg product, lying at different levels below zero. So, despite the theoretical non-coincidence of the optima in this model, in practice the optima regarding productivity and surplus coincide at an availability of zero (Figure 8.4.2). If the nutrient use productivity and the nutrient surplus productivity are defined differently (e.g. as yield/applied nutrients and yield/surplus of available nutrients), then this is not necessarily the case.

Some considerations for regional policy

Some conclusions of this analysis at a regional aggregation level are:

- As the Liebscher theory appeared empirically to be the most valid of the three models, it is better, from a nutrient productivity point of view, to lower the nutrient availability per ha, than to decrease the area of cultivated land, if a reduction of total regional production is desired.
- From the perspective of nature conservation, energy, farm economics, national economics, or other mondial criteria, the conclusions may be different, dependent on the exchange values among (combinations of) these goal variables. Additional research is needed to answer these questions.
- One effect of intensification that has not been taken into account in the model studies, and that is in favour of a policy of extensification is the following: When extending the production by increasing the land area

and/or decreasing the external application per ha, the supply of nutrients from internal sources will form a greater part of the total nutrient use. But when intensifying, the area decreases and the external application increases and so the relative contribution of internal nutrients decreases. So, as intensifying proceeds, less use is made of the freely available resources, such as microbiological fixation of nutrients, nutrient input by deposition and nutrient mobilization from soil minerals by mycorrhiza, and from all other free factors that are proportional to the area used, such as radiation, water and space. In this way, intensification has a negative effect on the total productivity of (external + internal) nutrients (TZP), and a still more negative effect on the system nutrient productivity (SZP) of the **external** nutrients.

The latter mentioned effect also implies that even if the Mitscherlich model would be valid (which we deny) it would not be very likely that, in current high external input agriculture, when reducing the regional production goal, taking out of production would be a more efficient way than the reduction of external inputs.

Reflections with regard to other aggregation levels

The reasoning above seems very much in line with the perspective of European food self-sufficiency. Van der Woude (1992, p. 53) estimates that during the next 50 years the European demand for food will decrease by 25 percent (10 percent, due to a decrease in population, 5 percent due to the decreased need of an ageing population, and 10 percent due to improved dietary habits). Together with an estimated 25 percent increase of the land productivity, Van der Woude estimates that 50 percent of the land area may be taken out of production during the next half century. So from this European point of view there seems little reason to aim for very high productions per ha.

From a perspective of world food need, this matter is quite different (FAO, 1992, p. 4). It is expected that during the next 60 years the world population will increase from 6 to 12 billion. The doubled food need will mainly have to be attained by an increase of the production per ha, for in many countries no increase of agricultural area is possible. A quarter of all soils is degraded leaving no room to increase crop acreage. At present the number of people suffering from food shortage is estimated at 500 million. However, people suffer from food shortage because of lack of purchasing power. Food shortage is not primarily a technical problem of production, but rather a poverty problem and a logistical problem.

For intensive agriculture with high external resource inputs, many arguments may be made in its favour, such as nourishing the (future) world population, the desirability to utilize the cleared land and labour for other social needs, to furnish an acceptable farm income, or to create larger areas of "pure" nature. Of course it is necessary to take these social targets also into consideration.

But, as has been demonstrated in the foregoing chapters, economic goals and ecological goals do not automatically coincide. It is not that simple.

Short and long term and transient state problems

In addition to having different kinds of criteria, the pros and cons of intensification and extensification with regard to these criteria may differ in the long and the short term. An answer to such questions needs dynamic simulation studies, which can be a useful bridge not only between short and long run reasonings but also between scientific disciplines and between science and policy (Rabbinge, 1986). The most appropriate level for such studies may be the farm or regional level, and not the field level (too low) nor the European or world-wide level (too high). As has been remarked before, care must be taken not to loose the advantages of integration of disciplines through unjustified application of a relation found at a lower to a higher level of aggregation (the scaling up problem).

One of the arguments against applying low external inputs to attain maximum productivity and minimum residues is that (even when the hypothesis behind this practice may be valid in the short run), such a situation is not sustainable in the long run, because soils become depleted at low applications and high productions. It is not the ecological sustainability itself which will be endangered by low external applications (the production will adapt to the lower applications until a new equilibrium has been reached). Rather the total physical production and the profit (so the economic sustainability) will be endangered. Low, but harmonious, nutrient levels give a sustainable and high productivity, but may not give enough total physical production and/or profit. **Achievement** and **sustainability** of achievement of goals should for analytical reasons not be entangled; they are quite different concepts from a system theoretical view.

Existence of optimum levels of nutrient availability

De Wit (1992b, p. 147) postulated that "... Research should not be so much directed towards the search for marginal returns of variable production resources, as towards the search for the minimum of each production resource that is needed to allow maximum utilization of all other production resources ...". Do such optima exist? And if so, how relevant are they?

If the theory of Von Liebig is our premise, then this proposition of De Wit seems to be valid. It even holds that the availability giving the maximum productivity gives the maximum profit too: the volume of the barrel in the "barrel with staves" model is determined by the shortest stave. It has no use that any stave is longer than the shortest. So all staves could be as long as the level of the roof of the room in which the barrel is positioned. The potential yield determines the optimum availability (= minimum availability required to attain maximum yield). This optimum seems to be a purely production ecology optimum (not dependent on other factors). But what makes this maximum more preferable than any input between zero and the optimum? The fertilizers may be so expensive that it is decided not to apply any external nutrients, and only make use of internal nutrients, or they may be so detrimental to the environment that it is decided to use only 75% of the so called "optimum", and be satisfied with 75% of the possible yield. But these are other arguments than physiological ones. What does optimal mean?

With non linear production functions (like Mitscherlich and Michaelis-Menten) this may be different. Maximum productivity of production factor N is realized at maximum values of other factors P, K etc.

In case we depart from the assumptions of the Mitscherlich model, a point exists at which the decrease of availability of N just outweighs the increase of the availability of P, K etc. Regarding the S-curve, a N availability (with proportional P and K) exists where the yield per kg N is maximal. Also for P (with proportional N and K) and for K (with proportional N + P) such points exist and those optimal availabilities coincide at the same proportional combination of N, P and K. This is however a purely mathematical optimum, which is not connected to any explicit criteria that seem socially relevant. As soon as the use of 1 kg N is not equivalent to 1 kg P, because of different prices, and/or different environmental loads, the optimum becomes dependent on those prices and/or loads. The optimal technical productivity cannot meaningfully be separated from other relevant disciplines and from social values. A purely independent technical optimum does not exist in the Mitscherlich model. In the Mitscherlich model the optimal rates of application are dependent on economic as well as ecological and other social factors. And the optimum is situated somewhere between zero application and some very high input. Not only the optimal rates but also the optimal ratios between the various inputs are dependent on actual input and output prices (such as nutrient and product prices) and on latent input and output prices (such as environmental damage and resource exhaustion).

If the Liebscher theory is valid (and if the Liebscher theory is formalized adequately by the Michaelis-Menten model), then, along the whole nutrient availability range the productivity of one specific factor increases as another factor increases, but the productivity of the other factor itself decreases as this latter factor increases. With regard to nutrient productivity there is no specific optimal level of nutrient availability for proportionally available nutrients, apart from the lowest. However this level may not be optimal from the standpoint of farmers' income. Here, too, we have no general optimum, except when accepting values, most of them imposed by society.

Adequacy of the concept of productivity

Productivity, no matter which variant of it, seems not to be a very adequate criterion for evaluating a production system. Also other **variables which are**

quotients of variables from different societal interest (e.g. surplus per kg product), are inadequate. It would be much better to use indicators which measure the absolute amounts of system performance, originating directly from more ultimate specific goals, such as: total profit, number of birds, full time equivalents employment, kg (harmful) emission, kg use of resources, joules of food, and not to hide all criteria in quotients or in one common (monetary) variable. Policy dilemmas and compromises are not obscured so easily then. They are formulated in their own pay-off terms (e.g.:Y kg extra food production needs N kg more nitrogen, and P kg more phosphate application, it gives EN kg more N emission and R_F guilder extra profit, but is likely to be accompanied by a decrease of S plant species and the phosphate emission will rise by EP kg).

Ratios between nutrients

The **ratios** between nutrients seem much more independent of the values accorded by society. The ratios which do not change the steady state levels of resources required to realize the target production level, in the long run, are agronomically the optimal ratios. But the target is also a societal value.

We emphasize, however - for low as well as for high nutrient levels - that production cannot be optimal when nutrients are not available in the correct ratios (see Appendix 12.10). And consequently, when in the past a nutrient has been applied at a very high rate, accumulating a large stock in the soil (e.g. P), a high external application of other nutrients seems to be "needed" to prevent loss of the high nutrient stocks (e.g. phosphorus) from the past. But practising such optimization may result in an "arms race" between the application of different resources. One should strive for a situation in which reserves of soluble nutrients are minimal, and reserves of poorly soluble nutrient compounds are sufficiently to gradually supply the nutrients needed.

Decision making

Summarizing the different decision measures (see Table 4) it becomes very clear, that both the choice of the model and the choice of the decisive variable makes an essential difference for the final decision. The different variables do have a different relevance. The farmer will give most attention to the financial productivity and profit, the environmentalist to surplus per ha and the agricultural politician to the regional measures. However, a consistent use of the same model is preferable for a consistent activity on the different aggregation levels.

Table 4 Response to increase of proportionally available nutrients in the low to medium range of a number of decision variables in a Linear, Mitscherlich and Michaelis-Menten model. N.B. Low = a bit above zero, medium = the availability at the inflexionpoint of the sigmoid curve.

	Linear	Sigmoid curve derived from Mitscherlich	Michaelis-Menten
Total nutrient productivity (TZP) (kg dm/kg available NPK)	constant (§3.2)	increasing (§3.3)	decreasing (§3.4)
Financial productivity (FINP))(f product/f nutrient)	constant	increasing (§6.3)	decreasing (§6.3)
System productivity (SZP) (kg dm/kg external nutrients)	decreasing (Figure 12.2.1)	increasing (Figure 6.3.2)	decreasing (Figure 12.2.1)
Surplus of available nutrients per ha (SNP) (kg NPK/ha)	constant (Appendix 12.11)	increasing (Figure 8.4.1)	increasing (Appendix 12.12)
Surplus of available nutrient per kg dm(RSN) (kg NPK per kg dm)	constant (Appendix 12.11)	decreasing (Figure 8.4.1)	increasing (Appendix 12.12)
Regional surplus of available nutrients with target output (REGSN)	constant (§ 9.2)	not studied	increasing (§9.3)
Regional surplus of **applied** nutrients with target output ($REGSN_E$)	increasing (§ 9.2)	not studied	increasing (§9.3)
Regional balance (REBAN)	increasing (§9.2)	not studied	increasing (§9.3)

Continued research

For a better foundation of the relation between nutrient inputs and yield new attention should be directed to the very low inputs. Especially the interaction between low inputs and improvement of ecological means on one hand, and the interactions between high inputs and improvement of technological means on the other, deserves more attention so as to enable comparisons. Further research should indicate whether the relations between nutrient input and production may be generalized to other material inputs as well, and how the micro relations can be properly scaled up to field, and regional level. Other aspects are also very relevant for an integrated understanding of the sustainability problem at the regional level, such as susceptibility to diseases, risk management, local employment, exchange between crop and animal production and absorption capacity for urban waste products.

12 Appendices

Appendix 12.1 Concepts, symbols and units

General concepts (without symbols)

- # Theory: a representation of phenomena by means of cause-effect relations, mostly expressed in verbal language.
- # Model: a formal (often mathematical) representation of (a part of) a theory.
- # Production function: a mathematical model representing the relation between the input and output of a production system.
- # Intensification: increase of (proportionally available) nutrients (resources) per ha.
- # Extensification: decrease of (proportionally available) nutrients (resources) per ha.
- # Proportional availability of nutrients: availability of different nutrients, such that the ratios between the availabilities of the nutrients are constant.
- # Proportional application of nutrients: application of nutrients, such that the ratios between the amounts of available nutrients are constant.
- # "Proportional increase of nutrients" or "Proportional application of nutrients" are synonymous expressions for: "increase of nutrients such that the available amounts of the different nutrients are proportional".
- # Returns to scale: increase of yield when several (or all) production factors are proportionally increased together (De Wit, 1994, p. 46).
- # Marginal returns to scale: extra yield of the last unit increase of a production factor when the other production factors are also simultaneously and proportionally increased.
- # Marginal nutrient productivity: increase of productivity resulting from the last unit increase of available nutrients (kg dm/kg NPK).
- # Production factor: any factor influencing production; may be specified in nutrients, labour, materials for crop protection, land, temperature, soil acidity, diseases, toxic substances, etc..
- # Input: synonym for part of the production factors (e.g. resources, radiation, temperature, water, labour, crop protection).
- # Resources: production factors which are constituents of the crops; may be specified in nutrients, carbon dioxide and water.
- # Nutrients: resources which are taken up by the roots of a crop.

#	Available nutrients: amount of nutrients which can be taken up by the crop; may be differentiated in internally available nutrients and externally applied nutrients.
#	Output: performance of an agricultural system; may be specified in dry matter yield, and other functions e.g. aesthetic, recreational and waste absorption functions.
#	Yield: (= production) = total biotic yield.
#	Surplus: without further specification the **calculated** difference of available nutrients (internal + applied) and total nutrients taken up by the crop during a year; specified in N, P and K surplus.
#	Productivity (general definition): quotient between output and input. See more specifically at the subparagraphs with concepts: nutrient productivity, agronomic productivity etc. When "productivity" is used without further specification, biotic dry matter yield per kg available nutrient (total productivity) is meant.
#	Marginal nutrient productivity: increase of production resulting from the last unit increase of available nutrients (kg dm/kg nutrients).
#	Average nutrient productivity: quotient of total production and total available nutrients.
#	Efficiency: any achievement of a system, expressed as a proportion of the theoretical (or empirical) maximum achievement (dimensionless); for specification see below.
#	Productivity efficiency: productivity expressed as a proportion of the maximum theoretically derived productivity, or, the maximum empirically observed productivity (dimensionless).
#	Recovery efficiency: nutrient recovery expressed as proportion of the maximum theoretical nutrient recovery, or, the maximum empirically observed nutrient recovery (dimensionless).
#	Retaining efficiency: nutrient surplus retained in the soil, expressed as a proportion of the maximum theoretically derived retention, or, the maximum empirically observed retention (dimensionless).
#	Nutrient residue: **observed** stock of nutrients in the soil; for specification see below.
#	Nutrient residue before sowing (kg Z/ha).
#	Nutrient residue after harvest (kg Z/ha).
#	Nutrient surplus: the **calculated** difference between nutrient availability and nutrient output in a production system; for specification see below.

Gross balance of nutrients: the difference between applied nutrients and nutrients carried off by the crop during a year (may be negative).
Surplus of external nutrients: the difference between external nutrients plus nutrient uptake at zero application on the one hand and the nutrient carried off by the crop during a year on the other hand (cannot be negative).
Surplus [18]) of available nutrients: the difference between externally + internally generated nutrients and nutrients carried off by the crop (cannot be negative) (kg NPK/ha).
Nutrient loss: (synonym nutrient emission) nutrients which are "definitively lost" [19]) from the reach of the crop (kg NPK/ha).
Mining: uptake of internal nutrients resulting in decrease of soil stock.

<u>Subscripts of symbols</u>

C	=	refers to concentration or content
E	=	refers to external (applied)
F	=	refers to financial (or economic)
H	=	refers to harvestable
HET	=	refers to soil heterogeneity
I	=	refers to internal
-I	=	refers to minus internal
i,j,k	=	refers to different production factors
K	=	refers to potassium
M	=	refers to the Mitscherlich model
MY	=	refers to maximum productivity
MIN	=	refers to minimum
N	=	refers to nitrogen
N0	=	refers to zero nitrogen application
P	=	refers to phosphorus
PHY	=	refers to plant physiological
RE	=	refers to regional
S	=	refers to surplus of nutrient
U	=	refers to uptake of nutrient
Y	=	refers to yield
1,2,3	=	refers to different **values** of variables or coefficients (e.g. input levels, production capacities or response coefficients)

<u>Production measures</u>

Y	=	Yield (synonym = production): primary biological production at prevailing values of N, P and K, expressed in (kg dm/ha).
MY	=	Maximum production: attainable production if nitrogen, phosphorus and potassium do not limit the production (kg dm/ha).
$MY_{P,K}$	=	Maximum attainable production when N is not a limiting factor and at current levels of P, K and all other factors except N.
REGY	=	Desired production for a given area or region (kg dm).
Y_H	=	Harvestable production: harvestable yield (kg dm/ha).
Y_F	=	Financial production (synonym = economic return): production in monetary units (*f*/ha). (F refers to financial.)
Y_{N0}	=	Production at nitrogen application of zero (kg dm/ha).
REGA	=	Area needed for a certain target production for a region.

<u>Nutrient supply measures</u>

NPK	=	Available nutrients for the crop: available (internal plus external) nutrients (kg NPK/ha); not synonymous with nutrient uptake.
NPK_E	=	External nutrients: nutrients which are applied intentionally from outside the system boundary (kg NPK/ha).
NPK_I	=	Internal nutrients: nutrients originating from weathering of rock minerals, nutrients from mineralized crop residues, symbiontic nutrient production, deposition and flooding, and in case of mixed farm and regional system level also nutrients from internally recycled nutrients (kg NPK/ha).
N,P,K	=	1) Nitrogen, Phosphorus, Potassium as nutrient names.
	=	2) Amounts of available nitrogen, phosphorus and potassium (kg/ha); (meaning 1 or meaning 2 are dependent on the context).
NP	=	N+P
NK	=	N+K
PK	=	P+K
NPK	=	N+P+K
Z	=	A certain nutrient
N_{MY}	=	N availability at which the (average) productivity is maximal.
X_i, X_j	=	Different formal production factors in a production function.
N_E,P_E, K_E	=	Externally applied nitrogen, phosphorus and potassium (kg Z/ha).
N_I,P_I, K_I	=	Internally available nitrogen, phosphorus and potassium, generated during one year (kg Z/ha). *(This is not exactly the same as the initial amount of mineral nutrients in the soil at planting (N_m, a quantity defined by Vos et al. (1997), which quantity is an instantaneous state variable).*

$N\phi$ = Available N, if the availabilities of P and K are proportional to that of N (kg N/ha).

Nutrient response coefficients Mitscherlich

α_M = Coefficient of response of **production to N availability** expressed as the production increase, as a proportion of the maximum attainable production per kg of available nitrogen th at has not yet been realized by means of nitrogen.

β_M, τ_M = Coefficients of response of production on availability of phosphorus and potassium (same definition as for nitrogen).

$\alpha\alpha_M$ = Coeff. of response of N uptake to N availability (prop. N/kg N).

$\beta\alpha_M$ = Coeff. of response of N uptake to P availability (prop. N/kg P).

$\tau\alpha_M$ = Coeff. of response of N uptake to K availability (prop. N/kg K).

$\alpha\beta_M$ = Coeff. of response of P uptake to N availability (prop. P/kg N).

$\beta\beta_M$ = Coeff. of response of P uptake to P availability (prop. P/kg P).

$\tau\beta_M$ = Coeff. of response of P uptake to K availability (prop. P/kg K).

$\alpha\tau_M$ = Coeff. of response of K uptake to N availability (prop. K/kg N).

$\beta\tau_M$ = Coeff. of response of K uptake to P availability (prop. K/kg P).

$\tau\tau_M$ = Coeff. of response of K uptake to K availability (prop. K/kg K).

Nutrient response coefficients Von Liebig

$\alpha_V, \beta_V, \tau_V$ = Coefficients of response of **production** to the **availability** of nitrogen, phosphorus and potassium, respectively, expressed as production increase per kg increase of available nutrient (kg dm/kg nutrient).

Nutrient response coefficients Michaelis-Menten

α, β, τ = Coefficients of response of **production** to the **availability** of nitrogen, phosphorus and potassium, respectively, expressed as production increase per kg increase of available nutrient (kg dm/kg nutrient).

$\alpha\alpha$ = Coeff. of response of N uptake to N availability (kg N/kg N).

$\beta\alpha$ = Coeff. of response of N uptake to P availability (kg N/kg P).

$\tau\alpha$ = Coeff. of response of N uptake to K availability (kg N/kg K).

$\alpha\beta$ = Coeff. of response of P uptake to N availability (kg P/kg N).

$\beta\beta$ = Coeff. of response of P uptake to P availability (kg P/kg P).

$\tau\beta$ = Coeff. of response of P uptake to K availability (kg P/kg K).

$\alpha\tau$ = Coeff. of response of K uptake to N availability (kg K/kg N).

$\beta\tau$ = Coeff. of response of K uptake to P availability (kg K/kg P).

$\tau\tau$ = Coeff. of response of K uptake to K availability (kg K/kg K).

$\alpha\alpha\alpha$ = Coeff. of response of yield to N uptake (kg dm/kg N).

ßßß = Coeff. of response of yield to P uptake (kg dm/kg P).
τττ = Coeff. of response of yield to K uptake (kg dm/kg K).
μ = Compound coefficient of response of **N uptake to availability of N** (kg N/kg N), if the availabilities of P and K are proportional to the availability of N (μ is a function of the original coefficients αα, ßα and τα and of the coefficients qq and rr).
σ = Compound coefficient of response of **production to N availability** (kg dm/kg N), if the availabilities of P and K are proportional to the availability of N (σ is a function of the original coefficients α, ß, τ, qq and rr).
Ω = Compound coefficient of response of **production to N uptake** (kg dm/kg N), if the availability of P and K is proportional to that of N (Ω is a function of the original coefficients ααα, ßßß, τττ, αα, ßα, τα, αß, ßß, τß, ατ, ßτ, ττ, qq, rr).
ε = Positive constant (ε is a function of the original coefficients MY, MUN, MUP, ßßß, τττ, αα, ßα, τα, αß, ßß, τß, ατ, ßτ, ττ, qq, rr, but not of ααα).
Θ = Compound coefficient of response of **production to N-availability** (in case of proportional co-availability of P and K) the coefficient is a function of the original coefficients α, ß and qq.
α_{PHY} = Plant physiological component (factor, multiplier) of the response coefficient α of **yield to N availability**, in the Michaelis-Menten model.
α_{HET} = Soil heterogeneity component (factor, multiplier) of the response coefficient α of **yield to N availability**, in the Michaelis-Menten model.

Ratios

qq = Ratio of P to N in the total available NPK (kg P/kg N).
rr = Ratio of K to N in the total available NPK (kg K/kg N).
N_C = Amount of N per unit dry matter of yield (kg N/kg dm).
P_C = Amount of P per unit dry matter of yield (kg P/kg dm).
K_C = Amount of K per unit dry matter of yield (kg K/kg dm).

Nutrient uptake measures

MUN = Maximum uptake of N (kg/ha).
MUP = Maximum uptake of P (kg/ha).
MUK = Maximum uptake of K (kg/ha).
UNPK = Nutrient uptake: nutrients taken up by the crop during the growing season (kg NPK/ha).
$UNPK_I$ = Uptake from internal nutrients (kg NPK/ha).

$UNPK_E$ = Uptake from applied nutrients (kg NPK/ha).
UN_{N0} = Nitrogen uptake at an N application of zero.
UN = Uptake of nitrogen (internal + external) (kg N/ha).
UP = Uptake of phosphorus (internal + external) (kg P/ha).
UK = Uptake of potassium (internal + external) (kg K/ha).
UN_I = Uptake from internal N (kg N/ha).
UN_E = Uptake from applied N (kg N/ha).

Measures of nutrient residues and nutrient surpluses

SN = Surplus of available N (= amount of available N, which was n ot taken up by the crop (kg N/ha)); **so a calculated value to be distinguished from the amount of applied N which was not taken up**.
SN_I = Surplus of internal nutrients.
SN_I = Surplus of external nutrients.
RSN = SN/Y: the surplus of available N per kg product.
REGSN = The nitrogen surplus for a total given area.
NPK_{-I} = Total available nutrients minus surplus of internal nutrients.
SNPK = NPK - UNPK = Surplus of available nutrients (if N, SN).
$SNPK_I$ = NPK - NPK_{-I} = Surplus of internal nutrients (if N, SN_I).
$SNPK_E$ = Surplus of applied nutrients (if N, SN_E). **(The subscript E refers to external nutrients.)**
BANPK = NPK_{-I} - UNPK = Input - output balance from applied nutrients (may be negative).
BAN = N_{-I} - UN = Input - output balance from applied nitrogen at the system boundary (may be negative). This measure should not b e confused with the total N balance: Total N input (application, fixation, deposition) minus total N output (yield, leaching, volatilization, denitrification) at the system boundary.
BAN/Y = Nutrient balance per kg dm.

Nutrient loads on the environment

EN = Polluting load of nitrogen (units per kg N).
EP = Polluting load of phosphorus (units per kg P).

Productivity and efficiency

# = Nutrient productivity: production per unit of nutrients. This ter m may be differentiated as follows:
IZP = Internal nutrient productivity: kg dry matter from internal nutrient per kg of internally available nutrient.

EZP	=	External nutrient productivity: kg dry matter from external nutrient per kg of applied nutrient.
TZP	=	1) Y_H/NPK1: total nutrient productivity: kg total harvestable dr y matter yield per kg of available nutrient. If in the text the ter m "productivity", or "nutrient productivity" is used, then "total nutrient productivity" is meant. (TZP is the same as NP_r of Vos et al. (1997).)
BIZP	=	2) Y/NPK: biological productivity: kg dry matter yield per kg of available nutrient.
HAZP	=	3) Y_H/NPK: harvestable productivity: kg harvestable dry matter yield per kg of available nutrient.
PhZP	=	4) Y/UNPK: physiological productivity, or nutrient use productivity: kg total biomass production per kg uptake of nutrient (reciprocal of the nutrient concentration in dry matter).
SZP	=	5) Y/NPK_E: system nutrient productivity: kg biomass from external + internal nutrients per kg applied nutrients. Note that system productivity is almost the same as financial productivity. Only in the latter measure input and output are expressed in monetary units.
SSZP	=	5) $(Y-Y_{miner})/NPK_E$: sustainable system productivity: kg biomas s from external + internal nutrients (mining excluded) per kg applied nutrients.
MZP	=	Marginal nutrient productivity (kg dm/kg Z).
NSUP	=	6) Y/SNPK: nutrient surplus productivity: kg product per kg of nutrient surplus (reciprocal of nutrient surplus per kg product).
FINP	=	8) Y_F/C_F: financial productivity: monetary return divided by monetary expenditure (*f*/*f*). See also the definition of SZP.
RU	=	Nutrient use (kg Z/kg dm). (Not equal to NPK/Y but to UNPK /Y).
NPK_R	=	Apparent recovery of nutrients: nutrients <u>extra</u> taken up by the crop because of nutrient application, above the uptake from unfertilized soil, expressed as a proportion of applied nutrients (kg/kg).
AY	=	Area per kg dm yield (reciprocal of yield per ha) at given values of the production factors (ha/kg dm).
AY_{MIN}	=	Minimum area per kg dm yield = reciprocal of maximum yield per ha (ha/kg dm), if the production factors do not limit the production (ha/kg dm).
f_H	=	Net harvestable yield as a proportion of total dry matter yield.
RP	=	Nutrient productivity: Y/NPK or Y/NP.

Economic variables

PRI_Y	=	Price of product (*f*/kg dm).
PRI_N	=	Price of nitrogen (*f*/kg N).
PRI_P	=	Price of phosphorus (*f*/kg P).
PRI_K	=	Price of potassium (*f*/kg K).
C_F	=	Variable nutrient costs (*f*/ha).
PR_F	=	Financial productivity: monetary return of product divided by monetary expenditure of nutrients (*f*/*f*).
R_F	=	Profit (financial revenue, gross margin); in general output minu s input (*f*/ha), specified for nutrient input: yield minus external nutrient application expressed in (*f*/ha).
Y_F	=	Financial production (synonym = economic return): production in monetary units (*f*/ha).

Miscellaneous

cm^2	=	Square centimetre
ha	=	Hectare
J	=	Joule
t	=	Ton
kg	=	Kilogram
h	=	Hour
g	=	Gram
mm^3	=	Cubic millimetre
dm	=	Dry matter
EN	=	Nitrogen emission (kg)
EP	=	Phosphorus emission (kg)
S	=	Number of plant species
EXP(.)	=	$e^{(.)}$ (in which e = 2.7... = base of natural logarithms).
MIN(.)	=	A logical function selecting the minimum outcome of any expr ession, separated by commas, in the brackets.
MAX(.)	=	A logical function selecting the maximum outcome of any expr ession, separated by commas, in the brackets.
LN(.)	=	Natural logarithm (^{e}Log(.)).
n!	=	n faculty = n * (n-1) * (n-2) * .... * ..... * 1
5!	=	5 faculty = 5 * 4 * 3 * 2 * 1
^	=	Continuation symbol for truncated equations.
δ	=	A small increase (approaching zero) of a model variable in case of differentiation.
$\triangle$	=	A value relatively close to zero.

Appendix 12.2 Productivity and surplus measures

Definition and comparison of different productivity, surplus and recovery measures.

Dependent on the research question different definitions of the concepts of productivity, surplus and recovery are relevant. In all of the three quadrants of the well known application resp. availability / uptake / production diagrams (Frankena & De Wit, 1958; Van Noordwijk & Wadman, 1992), different relations between the quantities on the axes may be distinguished. Based on Figures 12.2.1 and 12.2.2 the specific quotients concerning uptake and application / yield and uptake / yield and application / surplus and yield are now defined and discussed.

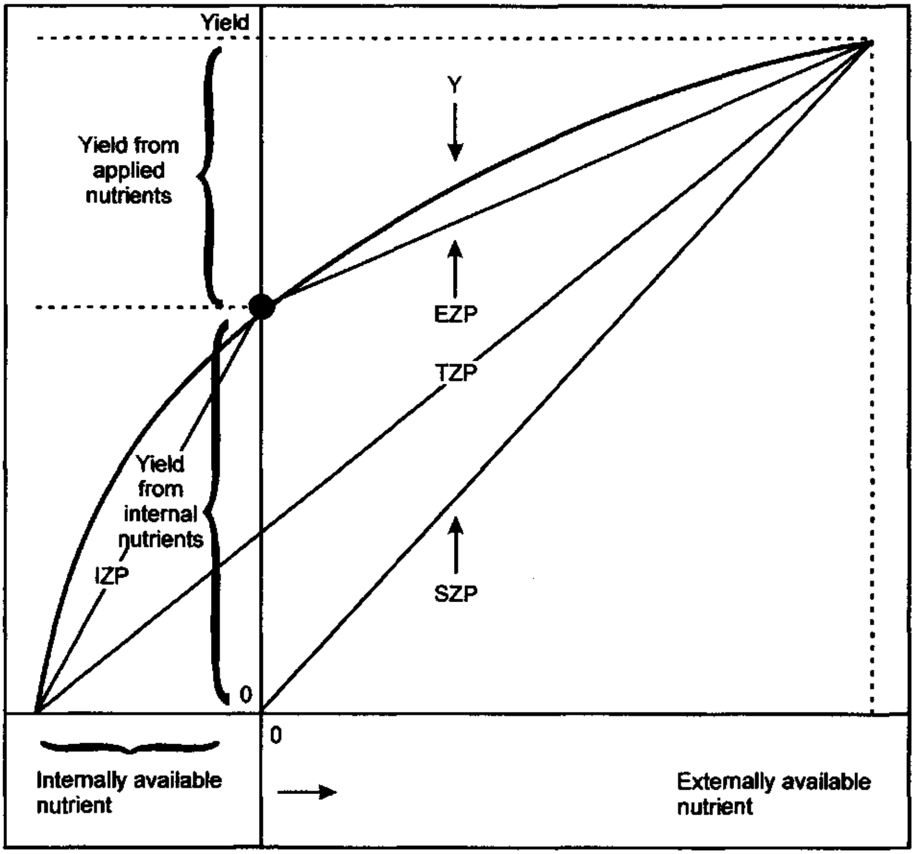


Figure 12.2.1 Relation between available nutrients and yield (curve), and four quotients (slopes of the lines) representing four different measures of productivity in production systems. Y = Production function, EZP = External nutrient productivity, TZP = Total nutrient productivity, SZP = System nutrient productivity, IZP = Internal nutrient productivity.

A. Yield and availability:

The quotients below refer to nutrient productivities at the crop system level. Line IZP may be denoted as the "internal nutrient productivity" which is relevant for forms of agriculture (e.g. organic agriculture), in which external nutrients are not at all used. The quotient between yield from applied nutrients and applied nutrient represented in line EZP refers to the "external nutrient productivity". De Wit (1992b) apparently used this measure where he stated that, after the Second World War, the nutrient productivity remained constant or even increased with increased nutrient application. Line TZP (TZP is the same concept as NP_r of Vos et al. (1997)) may be called here the "total nutrient productivity", which is the most relevant quantity for the theoretical production ecologist and crop physiologist. Line SZP denotes the "system nutrient productivity", which is especially relevant for sustainability questions of ecological agronomy, with low external input. Ecological agronomists (e.g. Besson et al., 1995) use this measure. They consider the total yield from the system (in steady state, so under conditions of retaining the nutrient stocks) mainly as a result of the **internal** nutrient generation complemented by only low external applications. The external achievement of the total system is best characterized as the quotient of this **total** yield and **external** resources. This indicator gives mostly higher values for productivity than the measures EZP and TZP, especially as the part of the yield originating from internal nutrients is bigger and that from external nutrients smaller and as the production function is more convex. In case of mining these higher values of SZP are not justified and a correction of the quotient is necessary then. It is exactly because of this that some authors (Vos et al., 1997) oppose against the use of the indicator SZP.

Apart from the productivity variants defined above one may distinguish between:

- "average productivity", being the productivity of the total input of nutrients and "marginal productivity", being the productivity of the last applied unit of nutrients,
- biological productivity based on total biological yield, agronomic productivity based on harvestable yield and financial productivity based on input and yield in terms of money.

Depending on the shape of the production functions, and on the occurrence of internal nutrients and mining some of the lines will coincide. If the production function is linear, lines Y, TZP and EZP will coincide, which means that marginal and average productivity will have the same values then, and external and total productivities are also the same. The smaller the ratio between internal nutrients and application, the more lines EZP and TZP will coincide.

B. Uptake and availability:

These measures refer to the nutrient recovery, also called nutrient utilization efficiency (Vos et al., 1997). The interpretation of the different variants parallels the different measures for productivity above. Line NPK gives the available nutrients; this is the nutrient uptake if all nutrients were taken up (45 degrees line). Line NPK_{-I} denotes the total available nutrients minus the surplus of internal nutrients. This line has been drawn parallel to NPK, but dependent on the situation the line may be slightly diverging or converging, because the surplus of internal nutrients (NPK - NPK_{-I}) may be different at high external application and low external application. Line UNPK gives the total nutrient uptake and the difference (NPK_{-I} - UNPK) the surplus of applied nutrients. NPK_I gives the uptake from internal nutrients. Analogically to surplus of internal nutrients the nutrient uptake from internal nutrients may not be constant. The difference (UNPK - NPK_I) gives the uptake from external nutrients. Given the above defined quantities one may derive from them different measures of nutrient surplus per kg dm, each of them in the range (0 - 1):

a) (NPK_{-I} - UNPK) / NPK_E: Surplus of applied nutrients per kg dm,
b) (NPK - UNPK) / NPK: Surplus of available nutrients per kg dm, a measure, together with measure a) much used as criterion in this report,
c) (NPK - NPK_{-I}) / NPK_I: Nutrient surplus of internal nutrients per kg dm.

One may also derive different measures of recovery, each of them in the range (0 - 1):

d) Apparent nutrient recovery, denoting the quotient (UNPK - NPK_{-I}) / NPK_E is appropriate when the recovery of especially the applied nutrients is of interest.
e) The quotient UNPK / NPK may be called the "total nutrient recovery" or the recovery of available nutrients (a measure much used in this report).
f) The quotient $UNPK_I$ / NPK_I gives the recovery of internal nutrients.

C. Yield and uptake:

The indicators refer to the physiological nutrient productivity, also called nutrient use efficiency (Vos et al., 1997), being the reciprocal of nutrient concentration in dry matter. Of the four possible variants the quotient between total yield and total nutrient availability is most relevant, as it gives an indication of the productivity of the plant physiological processes.

D. Nutrient surplus and yield:

If one wants to relate the nutrient surplus with the achievement of productivity it may be appropriate to divide the surplus by output instead of by input. These quotients (analogically to the productivity quotients) give different possible measures of "nutrient surplus per kg dm".

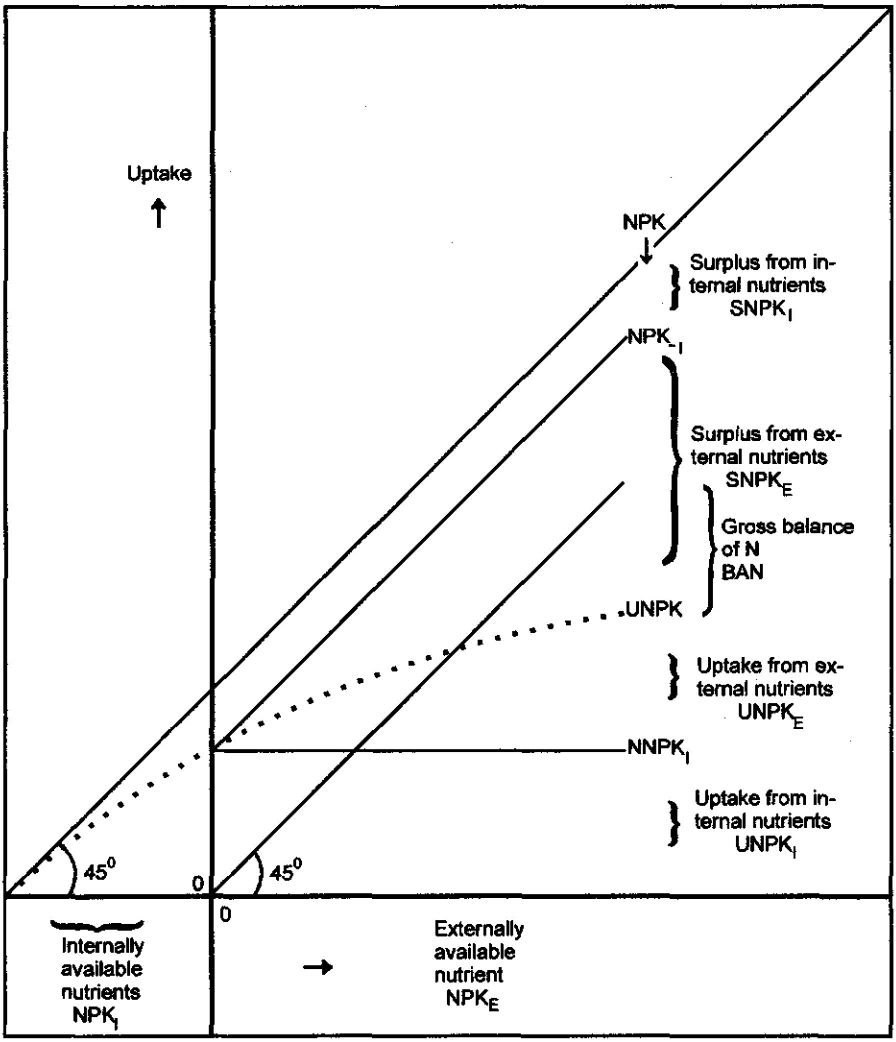


Figure 12.2.2 Relation between availability, uptake and surplus of nutrients

NPK_I	= Internal nutrients,
NPK_E	= External nutrients,
$NPK = NPK_I + NPK_E$	= Total available nutrients,
UNPK	= Nutrient uptake,
NPK_{-I}	= Available NPK minus surplus of internal NPK,
SNPK = NPK-UNPK	= Surplus of available nutrients,
$SNPK_I = NPK-NPK_{-I}$	= Surplus of internal nutrients,
$SNPK_E = NPK_{-I}-UNPK$	= Surplus of applied nutrients,
$UNPK - NPK_I$	= Uptake from applied nutrients.

Sometimes the reciprocals of the measures are given if they are compared with other indicators and also to prevent division by zero. The quotient refers to the "nutrient surplus per kg dm" or the nutrient surplus per kg dm yield, and is used as a theoretical approximation of the nutrient emission per kg product. Of the four variants the quotient "total nutrient surplus per kg total production" seems most relevant, since it produces a measure of pollution at a certain output in the specific situation. With regard to environmental criteria it seems irrelevant whether emission (surplus) originates from internal or applied nutrients, however for farm economic profit it is also important whether production stems from cheap, internal or expensive, applied nutrients.

Appendix 12.3 The simulation models

Models for simulation of the relation among nutrient application, uptake, surplus and yield, for non-correlated and for proportionally related nutrients

The definition of the variables and coefficients (left from the = sign) is given in capitals, on the line following the equation.

MICHAELIS MENTEN

VARIABLE N, CONSTANT P AND K
for my=m1,m2,m3,m4,m5
    FIVE LEVELS OF PRODUCTION CAPACITY (KG DM/HA)
NPKapplic(p,k)=Napplic+Papplic(p)+Kapplic(k)
Napplic=0 up to 500; Papplic(p)=ph(p); Kapplic(k)=ka(k)
    APPLICATION OF P AND K CONTINUOUS RANGE OF NITROGEN (KG N,P,K/HA)
for p=p1,p2,p3
    THREE APPLICATION RATES OF PHOSPHORUS (KG P/HA)
for k=k1,k2,k3
    THREE APPLICATION RATES OF POTASSIUM (KG K/HA)
Naprecover(p,k)=(Nuptak(p,k)-iNuptak)/Napplic
    RECOVERY FRACTION OF APPLIED NITROGEN (KG N/KG N)
Navrecover(p,k)=Nuptak(p,k)/Navai
    RECOVERY FRACTION OF AVAILABLE NITROGEN (KG N/KG N)
Ncontent(p,k)=Nuptak(p,k)/bioyield(p,k)
Pcontent(p,k)=Puptak(p,k)/bioyield(p,k)
Kcontent(p,k)=Kuptak(p,k)/bioyield(p,k)
    NUTRIENT-CONTENT IN BIOMASS (KG N,P,K/KG DM)
PNratio(p,k)=Puptak(p,k)/Nuptak(p,k)
    P/N-RATIO IN BIOMASS (KG P/KG N)
agroyield(p,k)=bioyield(p,k).harvestindex
    HARVESTABLE YIELD (KG DM/HA)
harvestindex=xharvestind(m3)
    HARVESTINDEX (KG DM/KG DM)
avcNUsurprod(p,k)=avNPKsurplus(p,k)/agroyield(p,k)
    SURPLUS OF AVAILABLE NPK PER KG HARVESTABLE DM
avNPKsurplus(p,k)=NPKavai(p,k)-NPKuptak(p,k)
avNsurplus(p,k)=Navai-Nuptak(p,k)
avPsurplus(p,k)=Pavai(p)-Puptak(p,k)
avKsurplus(p,k)=Kavai(k)-Kuptak(p,k)
    SURPLUS OF AVAILABLE NUTRIENTS (KG N,P,K/HA)
apNPKsurplus(p,k)=NPKapplic(p,k)-NPKuptak(p,k)+iNPKuptak
apNsurplus(p,k)=Napplic-Nuptak(p,k)+iNuptak
apPsurplus(p,k)=Papplic(p)-Puptak(p,k)+iPuptak
apKsurplus(p,k)=Kapplic(k)-Kuptak(p,k)+iKuptak
    SURPLUS OF APPLIED NUTRIENTS (KG NPK/HA)
NPKupapbal(p,k)=apNPKsurplus(p,k)-iNPKuptak
Nupapbal(p,k)=apNsurplus(p,k)-iNuptak
Pupapbal(p,k)=apPsurplus(p,k)-iPuptak
Kupapbal(p,k)=apKsurplus(p,k)-iKuptak
    UPTAKE BALANCE OF APPLIED NUTRIENTS (KG N,P,K/HA)
bioyield(p,k)=1/((1/maxyield)+(1/(α.Navai))+(1/(ß.Pavai(p)))+(1/(τ.Kavai(k))))

```
  BIOTIC YIELD (KG DM/HA)
Nuptak(p,k)=MIN(1/((1/mNuptak)+(1/(αα.Navai))+(1/(ßα.Pavai(p)))+(1/(τα.Kavai(k)))),Navai)
Puptak(p,k)=MIN(1/((1/mPuptak)+(1/(αß.Navai))+(1/(ßß.Pavai(p)))+(1/(τß.Kavai(k)))),Pavai(p))
Kuptak(p,k)=MIN(1/((1/mKuptak)+(1/(ατ.Navai))+(1/(ßτ.Pavai(p)))+(1/(ττ.Kavai(k)))),Kavai(k))
     NUTRIENT UPTAKE (KG N,P,K/HA)
iNuptak=MIN(1/((1/mNuptak)+(1/(αα.Nsoil))+(1/(ßα.Psoil))+(1/(τα.Ksoil))),Nsoil)
iPuptak=MIN(1/((1/mPuptak)+(1/(αß.Nsoil))+(1/(ßß.Psoil))+(1/(τß.Ksoil))),Psoil)
iKuptak=MIN(1/((1/mKuptak)+(1/(ατ.Nsoil))+(1/(ßτ.Psoil))+(1/(ττ.Ksoil))),Ksoil)
     NUTRIENT UPTAKE AT EXTERNAL N-INPUT = 0 (KG N,P,K/HA)

NPKavai(p,k)=Navai+Pavai(p)+Kavai(k)
Navai=Napplic+Nsoil
Pavai(p)=Papplic(p)+Psoil
Kavai(k)=Kapplic(k)+Ksoil
     NUTRIENTS AVAILABLE (KG N,P,K/HA)
NPKsoil=Nsoil+Psoil+Ksoil
Nsoil=xNsoil(m3); Psoil=xPsoil(m3); Ksoil=xKsoil(m3)
     INTERNALLY AVAILABLE NUTRIENTS (KG N,P,K/HA)
maxyield=xmaxyield(m3)
     DM-PRODUCTION CAPACITY (KG DM/HA)
mNPKuptak=xmNPKuptak(m3)
mNuptak=xmNuptak(m3); mPuptak=xmPuptak(m3); mKuptak=xmKuptak(m3)
     NUTRIENT-UPTAKE CAPACITY (KG N,P,K/HA)

PROPORTIONAL NUTRIENTS AND DIFFERENT LEVELS OF PRODUCTION CAPACITY
xbioyield(my)=1/((1/xmaxyield(my))+(1/(α .xNavai(my)))+(1/(ß. xPavai(my)))+(1/(τ.xKavai(my))))
     BIOTIC YIELD (KG DM/HA)
xNuptak(my)=MIN(1/((1/xmNuptak(my))+(1/(αα.xNavai(my)))+^
  (1/(ßα.xPavai(my)))+(1/(τα.xKavai(my)))),xNavai(my))
xPuptak(my)=MIN(1/((1/xmPuptak(my))+(1/(αß.xNavai(my)))+^
  (1/(ßß.xPavai(my)))+(1/(τß.xkavai(my)))),xPavai(my))
xKuptak(my)=MIN(1/((1/xmKuptak(my))+(1/(ατ.xNavai(my)))+^
  (1/(ßτ.xPavai(my)))+(1/(ττ.xKavai(my)))),xKavai(my))
     NUTRIENT UPTAKE (KG N,P,K/HA)
xiNuptak(my)=MIN(1/((1/xmNuptak(my))+(1/(αα.xNsoil(my)))+^
  (1/(ßα.xPsoil(my)))+(1/(τα.xKsoil(my)))),xNsoil(my))
xiPuptak(my)=MIN(1/((1/xmPuptak(my))+(1/(αß.xNsoil(my)))+^
  (1/(ßß.xPsoil(my)))+(1/(τß.xKsoil(my)))),xPsoil(my))
xiKuptak(my)=MIN(1/((1/xmKuptak(my))+(1/(ατ.xNsoil(my)))+^
  (1/(ßτ.xPsoil(my)))+(1/(ττ.xKsoil(my)))),xKsoil(my))
     NUTRIENT UPTAKE AT EXTERNAL N-INPUT = 0 (KG N,P,K/HA)
xapNrecover(my)=(xNuptak(my)-ixNuptak(my))/xNapplic
     RECOVERY FRACTION OF APPLIED N WITH PROP. P AND K (KG N/KG N)
xavNrecover(my)=xNuptak(my)/xNavai(my)
     RECOVERY FRACTION OF AVAILABLE N WITH PROP. P AND K (KG N/KG N)
xNcontent(my)=xNuptak(my)/xbioyield(my)
xPcontent(my)=xPuptak(my)/xbioyield(my)
xKcontent(my)=xKuptak(my)/xbioyield(my)
     NUTRIENT-CONTENT IN DRY MATTER (KG N,P,K/KG DM)
xprofit(my)=xfinyield(my)-xNPKcost
     PROFIT (ƒ DM MINUS ƒ NPK)
xeconpro(my)=xfinyield(my)/xNPKcost
```

FINANCIAL PRODUCTIVITY OF NUTRIENTS(*f* DM/*f* NPK)
xphysiopro(my)=xbioyield(my)/xNPKuptak(my)
PHYSIOLOGICAL PRODUCTIVITY (KG DM/KG NPK)
xagronpro(my)=xagroyield(my)/xNPKavai(my)
TOTAL AGRONOMIC PRODUCTIVITY (KG DM/KG NPK)
xbiopro(my)=xbioyield(my)/xNPKavai(my)
BIOLOGICAL YIELD PER KG AVAILABLE NPK (KG DM/KG NPK)
xshortecolpro(my)=xagroyield(my)/xNPKapplic
SYSTEM PRODUCTIVITY (KG DM/KG NPK)
xsusecolpro(my)=(xagroyield(my)-xmining(my))/xNPKapplic
SUSTAINABLE SYSTEM PRODUCTIVITY (KG DM/KG NPK)
xfinyield(my)=xagroyield(my).Yprice
FINANCIAL YIELD (*f*/HA)
xNPKcost=xNapplic.Nprice+xPapplic.Pprice+xKapplic.Kprice
VARIABLE COSTS OF NUTRIENTS (*f*/HA)
xavNPKsurprod(my)=xavNPKsurplus(my)/xagroyield(my)
HARVESTABLE YIELD PER SURPLUS OF AVAILABLE NPK (KG DM/KG NPK)
xapNPKsurprod(my)=xapNPKsurplus(my)/xagroyield(my)
HARVESTABLE YIELD PER SURPLUS OF APPLIED NPK (KG DM/KG NPK)
xuaNPKsurprod(my)=xNPKupapbal(my)/xagroyield(my)
HARVESTABLE YIELD PER SURPLUS OF APPLIED NPK (KG DM/KG NPK)
xavNPKsurplus(my)=xNPKavai(my)-xNPKuptak(my)
xavNsurplus(my)=xNavai(my)-xNuptak(my)
xavPsurplus(my)=xPavai(my)-xPuptak(my)
xavKsurplus(my)=xKavai(my)-xKuptak(my)
SURPLUS OF AVAILABLE NUTRIENTS (KG N,P,K/HA)
xapNPKsurplus(my)=xNPKapplic-xNPKuptak(my)+ixNPKuptak(my)
xapNsurplus(my)=xNapplic-xNuptak(my)+ixNuptak(my)
xapPsurplus(my)=xPapplic-xPuptak(my)+ixPuptak(my)
xapKsurplus(my)=xKapplic-xKuptak(my)+ixKuptak(my)
SURPLUS OF APPLIED NUTRIENTS (KG N,P,K/HA)
xNPKupapbal(my)=xapNPKsurplus(my)-ixNPKuptak(my)
xNupapbal(my)=xapNsurplus(my)-ixNuptak(my)
xPupapbal(my)=xapPsurplus(my)-ixPuptak(my)
xKupapbal(my)=xapKsurplus(my)-ixKuptak(my)
EXTERNAL INPUT-OUTPUT BALANCE OF NUTRIENTS (KG N,P,K/HA)
xagroyield(my)=xbioyield(my).xharvestind(my)
HARVESTABLE YIELD (KG DM/HA)
xharvestind(my)=TABLE(txharvestind(.),xbioyield(my)/xmaxyield(my),0,1,.1)
HARVESTINDEX (KG DM/KG DM)
xNPKavai(my)=xNavai(my)+xPavai(my)+xKavai(my)
xKavai(my)=xKapplic+xKsoil(my)
xPavai(my)=xPapplic+xPsoil(my)
xNavai(my)=xNapplic+xNsoil(my)
AVAILABLE NUTRIENTS (KG N,P,K/HA)
xNPKsoil(my)=xNsoil(my)+xPsoil(my)+xKsoil(my)
xKsoil(my)=(cr/cp).xNsoil(my)
xPsoil(my)=(cq/cp).xNsoil(my)
xNsoil(my)=xmaxyield(m3)..0033
INTERNAL AVAILABLE NUTRIENTS (KG N,P,K/HA)
xNPKapplic=xNapplic+xPapplic+xKapplic
xKapplic=(cr/pp).xNapplic; xPapplic=(cq/pp).xNapplic; xNapplic=Napplic

```
    EXTERNAL APPLICATION OF NUTRIENTS (KG N,P,K/HA)
xmNPKuptak(my)=xmNuptak(my)+xmPuptak(my)+xmKuptak(my)
xmNuptak(my)=xmaxyield(my).maxNfractionindm
xmPuptak(my)=xmaxyield(my).maxPfractionindm
xmKuptak(my)=xmaxyield(my).maxKfractionindm
    UPTAKE CAPACITIES (KG N,P,K/HA)
```

MITSCHERLICH

```
VARIABLE N, CONSTANT P AND K
bioyield(p,k)=maxyield.(1-EXP(-α.Navai)).(1-EXP(-ß.Pavai(p))).(1-EXP(-τ.Kavai(k)))
    BIOTIC YIELD (KG DM/HA)
NPKuptak(p,k)=Nuptak(p,k)+Puptak(p,k)+Kuptak(p,k)
Nuptak(p,k)=MIN(mNuptak.(1-EXP(-αα.Navai)).^
(1-EXP(-ßα.Pavai(p))).(1-EXP(-τα.Kavai(k))),Navai)
Puptak(p,k)=MIN(mPuptak.(1-EXP(-αß.Navai)).^
 (1-EXP(-ßß.Pavai(p))).(1-EXP(-τß.Kavai(k))),Pavai(p))
Kuptak(p,k)=MIN(mKuptak.(1-EXP(-ατ.Navai)).^
 (1-EXP(-ßτ.Pavai(p))).(1-EXP(-ττ.Kavai(k))),Kavai(k))
    NUTRIENT UPTAKE (KG N,P,K/HA)
PROPORTIONAL NUTRIENTS AND DIFFERENT LEVELS OF PRODUCTION CAPACITY
xbioyield(my)=xmaxyield(my).(1-EXP(-α.xNavai(my))).^
 (1-EXP(-ß.xPavai(my))).(1-EXP(-τ.xKavai(my)))
    BIOTIC YIELD (KG DM/HA)
xNPKuptak(my)=xNuptak(my)+xPuptak(my)+xKuptak(my)
xNuptak(my)=.01+MIN(xmNuptak(my).(1-EXP(-αα.xNavai(my))).^
 (1-EXP(-ßα.xPavai(my))).(1-EXP(-τα.xKavai(my))),xNavai(my))
xPuptak(my)=.01+MIN(xmPuptak(my).(1-EXP(-αß.xNavai(my))).^
 (1-EXP(-ßß.xPavai(my))).(1-EXP(-τß.xKavai(my))),xPavai(my))
xKuptak(my)=.01+MIN(xmKuptak(my).(1-EXP(-ατ.xNavai(my))).^
 (1-EXP(-ßτ.xPavai(my))).(1-EXP(-ττ.xKavai(my))),xKavai(my))
    NUTRIENT UPTAKE (KG N,P,K/HA)
ixNPKuptak(my)=ixNuptak(my)+ixPuptak(my)+ixKuptak(my)
ixNuptak(my)=.01+MIN(xmNuptak(my).(1-EXP(-αα.xNsoil(my))).^
 (1-EXP(-ßα.xPsoil(my))).(1-EXP(-τα.xKsoil(my))),xNsoil(my))
ixPuptak(my)=.01+MIN(xmPuptak(my).(1-EXP(-αß.xNsoil(my))).^
 (1-EXP(-ßß.xPsoil(my))).(1-EXP(-τß.xKsoil(my))),xPsoil(my))
ixKuptak(my)=.01+MIN(xmKuptak(my).(1-EXP(-ατ.xNsoil(my))).^
 (1-EXP(-ßτ.xPsoil(my))).(1-EXP(-ττ.xKsoil(my))),xKsoil(my))
    NUTRIENT UPTAKE AT EXTERNAL N-INPUT = 0 (KG N,P,K/HA)
```

Appendix 12.4 Coefficient values

Unless stated otherwise the standard values of the coefficients of the models are as follows (given values or values calculated from other coefficients).

The Mitscherlich model:	The Michaelis-Menten model:
α_M = 0.014 (prop. of max. yield/kg N)	α = 469 (kg dm/kg N)
β_M = 0.115 (prop. of max. yield/kg P)	β = 3750 (kg dm/kg P)
τ_M = 0.029 (prop. of max. yield/kg K)	τ = 938 (kg dm/kg K)
$\alpha\alpha$ = 0.0015 (prop. of max. UN per kg N)	$\alpha\alpha$ = 2.7 (kg N/kg N)
$\beta\alpha$ = 0.0117 (prop. of max. UN per kg P)	$\beta\alpha$ = 21.6 (kg N/kg P)
$\tau\alpha$ = 0.0029 (prop. of max. UN per kg K)	$\tau\alpha$ = 5.4 (kg N/kg K)
$\alpha\beta$ = 0.0015 (prop. of max. UP per kg N)	$\alpha\beta$ = 0.34 (kg P/kg N)
$\beta\beta$ = 0.0117 (prop. of max. UP per kg P)	$\beta\beta$ = 2.7 (kg P/kg P)
$\tau\beta$ = 0.0029 (prop. of max. UP per kg K)	$\tau\beta$ = 0.68 (kg P/kg K)
$\alpha\tau$ = 0.0015 (prop. of max. UK per kg N)	$\alpha\tau$ = 1.35 (kg K/kg N)
$\beta\tau$ = 0.0117 (prop. of max. UK per kg P)	$\beta\tau$ = 10.8 (kg K/kg P)
$\tau\tau$ = 0.0029 (prop. of max. UK per kg K)	$\tau\tau$ = 2.7 (kg K/kg K)

(UN, UP, UK means: uptake of N, P, K; prop. of max. means: proportion of maximum.)

The Von Liebig model:

α_V = 125 (kg dm/kg N); β_V = 625 (kg dm/kg P); τ_V = 250 (kg dm/kg K)

All three the models [20]):

N_{Cmax} = 0.064	cp = 0.615 (kg N/kg NPK)	PRI_N = 1.60 (*ƒ*/kg N)
P_{Cmax} = 0.008	cq = 0.077 (kg P/kg NPK)	PRI_P = 2.50 (*ƒ*/kg P)
K_{Cmax} = 0.032	cr = 0.308 (kg K/kg NPK)	PRI_K = 1.80 (*ƒ*/kg K)

f_H = 0.60	PRI_Y = 0.47 (*ƒ*/kg dm)
N_I = 0.0	MUN = 128, 384, 768, 1280, 1920 (kg N/ha)
P_I = 0.0	MUP = 16, 48, 96, 160, 240 (kg P/ha)
K_I = 0.0	MUK = 64, 192, 384, 640, 960 (kg K/ha)

N_E = from 0 to 500 kg N/ha
P_E = 4, 20 and 100 kg P/ha [21])
K_E = 8, 40 and 200 kg K/ha
M = 2/6/12/20/30 $\cdot 10^3$ (kg dm)

Appendix 12.5 Features of the Mitscherlich model

A. Feature of increasing and decreasing marginal and average productivity at proportional availabilities of different nutrients.

The marginal productivity of the Mitscherlich model may be obtained as the first derivative of the general model. The general model is [eq. 6]:

$$Y = MY * \{1\text{-}EXP(-\alpha_M * X_i)\} * \{1\text{-}EXP(-\beta_M * X_j)\} * \{..\}$$

For meaning of the symbols see Appendix 12.1.

This mathematical model is found in some publications dealing with the Mitscherlich theory e.g. Meyer (1926/1927, p. 150-151); Harmsen (1993, p. 297). For two nutrients (e.g. N and P) the Mitscherlich model has the following mathematical form:

$$Y = MY * \{1\text{-}EXP(-\alpha_M * N)\} * \{1\text{-}EXP(-\beta_M * P)\}$$

If P is put equal to $qq * N\phi$ (P and N are proportionally related then), the function converts to:

$$(Y/MY) = \{1\text{-}EXP(-\alpha_M * N\phi)\} * \{1\text{-}EXP(-\beta_M * qq * N\phi)\}$$

Multiplication of both factors gives:

$$(Y/MY) = 1 + EXP\{-(\alpha_M+\beta_M * qq) * N\phi\} - EXP(-\alpha_M * N\phi) - EXP(-\beta_M * qq * N\phi)$$

The marginal productivity of $N\phi$ ($N\phi$ = N if proportionally related with P) is represented by the first derivative of Y with respect to $N\phi$:

$$(1/MY) * (\delta Y/\delta N\phi) = -(\alpha_M+\beta_M * qq) * EXP\{-(\alpha_M+\beta_M * qq) * N\phi\} \wedge \\ + \alpha_M * EXP(-\alpha_M * N\phi) + \beta_M * qq * EXP(-\beta_M * qq * N\phi)$$

At $N\phi$=0 all parts "EXP(.)" get the value 1 and the first derivative is equal to $-(\alpha_M+\beta_M * qq)+\alpha_M+\beta_M * qq = 0$. If $N\phi$ approaches infinity then all parts EXP(.) approach zero and the first derivative also approaches zero. So the productivity approaches zero at available amounts (N and P) equalizing zero as well as at amounts of N and P becoming extremely high. As the yield has the value zero at availability zero and approaches MY at very high availabilities, and as the curve of Y against $N\phi$ increases, there is a point of inflection, where the increasing productivity changes into a decreasing productivity. This point of inflexion may be found by putting the second derivative zero.

The second derivative is:

$$(1/MY).(\delta^2 Y/\delta N\phi^2) = (\alpha_M+\beta_M.qq)^2 . EXP\{-(\alpha_M+\beta_M.qq).N\phi\} \wedge$$
$$- \alpha_M^2 . EXP(-\alpha_M.N\phi) - (\beta_M.qq)^2 . EXP(-\beta_M.qq.N\phi)$$

If this second derivative is zero then the following expression holds:

$$(\alpha_M+\beta_M.qq)^2 . EXP\{-(\alpha_M+\beta_M.qq).N\phi\} =$$
$$= \alpha_M^2 . EXP(-\alpha_M.N\phi) + (\beta_M.qq)^2 . EXP(-\beta_M.qq.N\phi)$$

From this equation $N\phi$ cannot be analytically solved in general. This means that the value of $N\phi$, at which the curve shows an inflection point, may only be approximated by means of numerical methods. Meyer (1926/1927, p. 150-151) showed that the point of inflection can be found analytically if a special condition $\alpha_M.N = \beta_M.P$ is assumed. This assumption is not too unrealistic, because both N and P are constituents of the plant, which occur in rather constant proportions and ratios [22]). The production with proportional availability of two nutrients under that condition is:

$$Y = MY . \{1-EXP(-\alpha_M.N\phi)\}^2$$

and the first derivative is:

$$\delta Y/\delta N\phi = -2 . MY . \alpha_M . \{1-EXP(-\alpha_M.N\phi)\} . EXP(-\alpha_M.N\phi)$$

en the second derivative:

$$\delta^2 Y/\delta N\phi^2 = -2.MY.\alpha_M^2. \{2 . EXP(-2.\alpha_M.N\phi) - EXP(-\alpha_M.N\phi)\}$$

The second derivative becomes zero if:

$$N\phi = (1/\alpha_M) . LN(2) = 0.69/\alpha_M$$

in which LN(.) = natural logarithm = (^{e}Log(.)).

For three nutrients N+P+K the point of inflection will be found at

$$N\phi = (1/\alpha_M) . LN(3) = 1.1/\alpha_M$$

Because of the occurrence of a point of inflexion (Figure 12.5.1) the curve is a sigmoid curve and the point of inflection moves to the right as more production factors are interacting. So as a rule of thumb one may formulate that the availability giving maximum marginal productivity is reciprocally proportional to the response coefficients and proportional to the logarithm of the number of nutrients.

The proportional availability at maximum average productivity (Y/Nϕ) (does not coincide with the point of inflection) is still more difficult to determine.

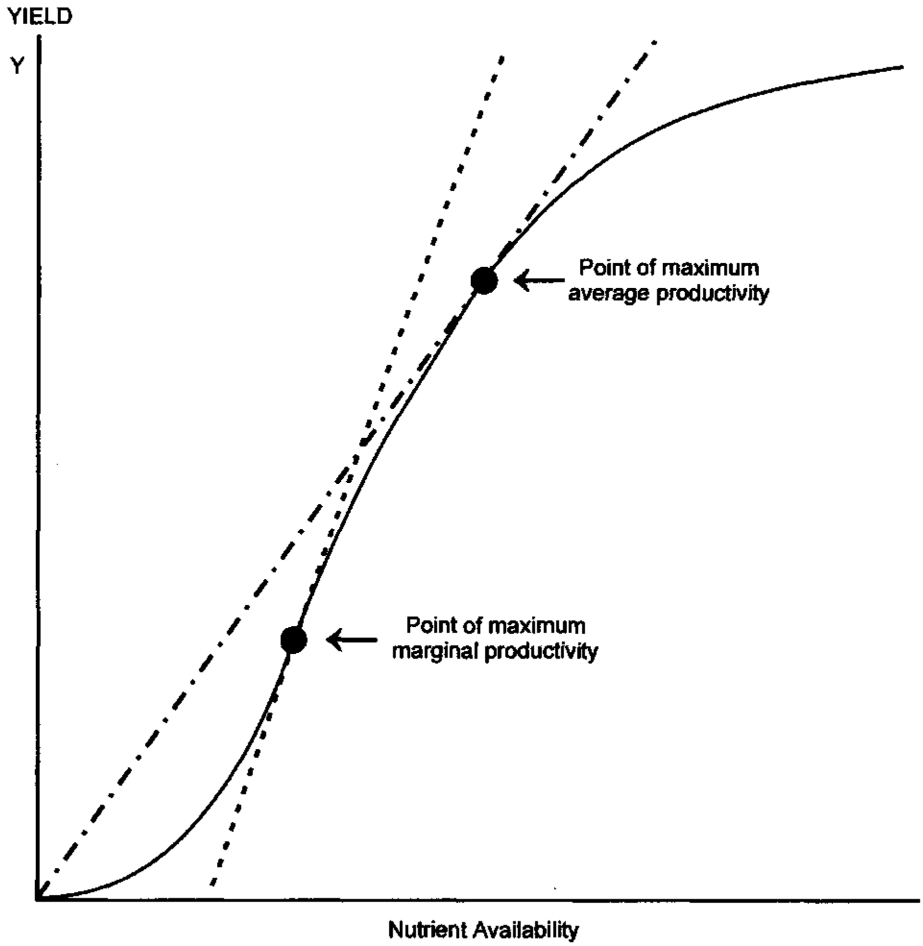


Figure 12.5.1 Maximum marginal productivity (line through the point of inflexion) and maximum average productivity (tangent through the origin) in their dependence of the relation between the dry matter production and nutrient availability in the Mitscherlich model (schematic).

For 2 nutrients the average productivity (expressed as a fraction of the maximum) is AP = Y/Nϕ is:

$$AP = (1/N\phi) \cdot (1 - EXP(-\alpha_M \cdot N\phi))^2$$

The maximum of the average productivity cannot easily be derived analytically. As the availability giving maximum **average** productivity is larger than the availability giving maximum **marginal** productivity, we formulate here as a rule of thumb that the availability giving maximum average productivity is (just as the availability giving maximum marginal productivity) approximately proportional to the logarithm of the number of nutrients and proportional to the reciprocals of the response coefficients. The optimum point may be more accurately approached by numerical simulation. A formalization of a sigmoid curve which is simpler to handle analytically, because derivatives are more easily derived (but which is more difficult ecologically to interpret) is:

$$Y = MY / \{1 + (j/N\phi)^z\}$$

in which j and z are constant coefficients. In that case the maximum average productivity is found at:

$$N\phi = j \cdot \{z-1\}^{(1/z)}$$

B. <u>Feature of constant ratio between productions.</u>

The feature that the marginal production $\delta Y/\delta N$ of the Mitscherlich function is proportional to the part of the maximum attainable production that has not yet been achieved can be made transparent by substituting the appropriate parts of eq. 3 and 1 of § 3.3 for $MY_{P,K}$ and Y in the equation: $\delta Y/\delta N = \alpha_M \cdot (MY_{P,K} - Y)$:

$$\delta Y/\delta N = \alpha_M \cdot [MY \cdot \{1\text{-EXP}(-\beta_M \cdot P)\} \cdot \{1\text{-EXP}(-\tau_M \cdot K)\} \wedge$$

$$- MY \cdot \{1\text{-EXP}(-\alpha_M \cdot N)\} \cdot \{1\text{-EXP}(-\beta_M \cdot P)\} \cdot \{1\text{-EXP}(-\tau_M \cdot K)\}]$$

or:

$$\delta Y/\delta N = \alpha_M \cdot MY \cdot \{1\text{-EXP}(-\beta_M \cdot P)\} \cdot \{1\text{-EXP}(-\tau_M \cdot K)\} \cdot [1-\{1\text{-EXP}(-\alpha_M \cdot N)\}]$$

and after removing the parts between { } in the last factor, this equation is equivalent to eq. 2 of § 3.3:

$$\delta Y/\delta N = \alpha_M \cdot MY \cdot \{1\text{-EXP}(-\beta_M \cdot P)\} \cdot \{1\text{-EXP}(-\tau_M \cdot K)\} \cdot \text{EXP}(-\alpha_M \cdot N)$$

Referring to Figure 12.5.2:

$$Y_1 = MY_1 \cdot \{1\text{-EXP}(-\alpha \cdot X)\}$$

$$Y_2 = MY_2 \cdot \{1\text{-EXP}(-\alpha \cdot X)\}$$

the first derivatives are:

$$\delta Y_1/\delta X = \alpha \cdot MY_1 \cdot EXP(-\alpha \cdot X)$$

$$\delta Y_2/\delta X = \alpha \cdot MY_2 \cdot EXP(-\alpha \cdot X)$$

thus:

$$\frac{\delta Y_1/\delta X}{\delta Y_2/\delta X} = \frac{\alpha \cdot MY_1 \cdot EXP(-\alpha \cdot X)}{\alpha \cdot MY_2 \cdot EXP(-\alpha \cdot X)} = MY_1/MY_2$$

The ratio between responses equals the ratio between the maxima.

This feature is derived from the Mitscherlich model in the form of eq. 2 (§ 3.3).

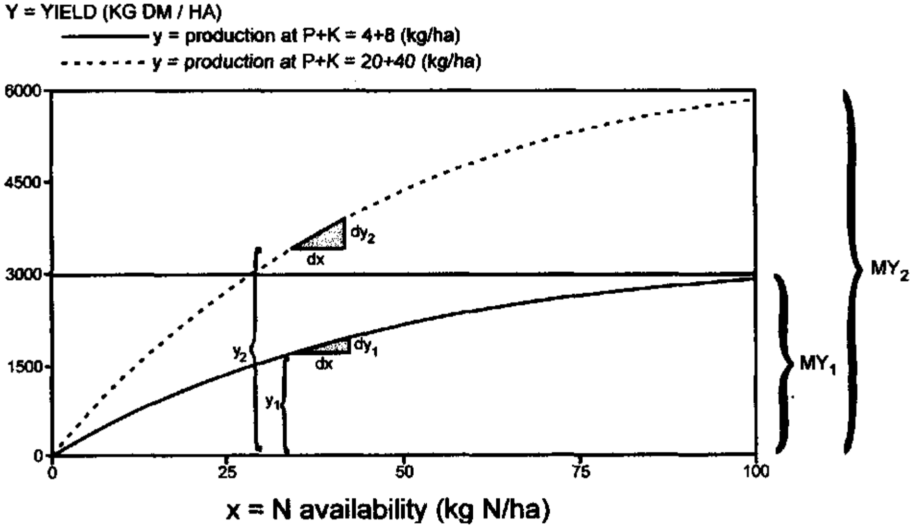


Figure 12.5.2 Relation between yield and availability at two combinations of P and K, demonstrating that, in the Mitscherlich model, for different combinations of other factors, the ratio of responses (dy_1 and dy_2) equals both the ratio between the maximum productions (MY_1 and MY_2) and the ratio between the actual productions (Y_1 and Y_2).

C. <u>Feature of proportionality between marginal production and the part of potential production that has not yet been achieved.</u>

For one variable production factor, and keeping the others constant, the ratio between productions with two different sets of those other production factors is **independent of the value of the variable production factor.** This can be demonstrated by dividing the expression for a given set of constant values

(MY_1, P_1, K_1) by the expression for another set of constant values (MY_2, P_2, K_2), leaving one factor (here N) variable:

$$\frac{Y_1}{Y_2} = \frac{[MY_1 \cdot \{1\text{-}EXP(-\alpha_M \cdot N)\} \cdot \{1\text{-}EXP(-\beta_M \cdot P_1)\} \cdot \{1\text{-}EXP(-\tau_M \cdot K_1)\}]}{[MY_2 \cdot \{1\text{-}EXP(-\alpha_M \cdot N)\} \cdot \{1\text{-}EXP(-\beta_M \cdot P_2)\} \cdot \{1\text{-}EXP(-\tau_M \cdot K_2)\}]}$$

As both factors, $\{1\text{-}EXP(-\alpha_M \cdot N)\}$, above and below the horizontal line cancel each other out in the division, the remaining quotient contains only constants, and therefore is a constant itself.

Referring to Figure 12.5.3:

$$Y_1 = MY \cdot \{1\text{-}EXP(-\alpha \cdot X_1)\} \text{ or: } MY_1 \cdot EXP(-\alpha \cdot X_1) = MY - Y_1$$

$$Y_2 = MY \cdot \{1\text{-}EXP(-\alpha \cdot X_2)\} \text{ or: } MY_2 \cdot EXP(-\alpha \cdot X_2) = MY - Y_2$$

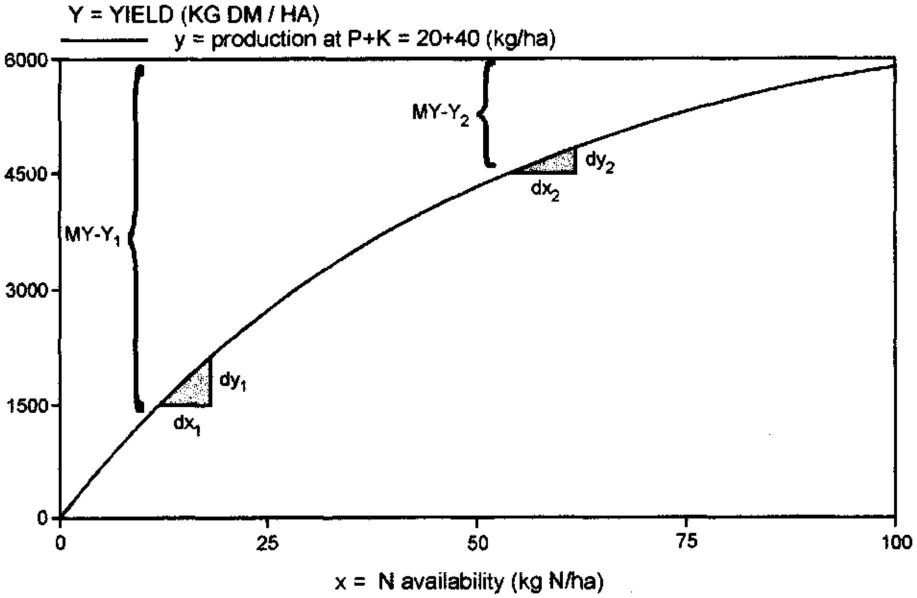


Figure 12.5.3 Relation between yield and availability, demonstrating that for different values of the variable production factor (x) holds that the response (dy_1 and dy_2) is proportional to the parts of the productions that have not yet been realized ($MY - Y_1$) and ($MY - Y_2$).

$$\delta Y/\delta X_1 = \alpha \cdot MY \cdot EXP(-\alpha \cdot X_1) = \alpha \cdot (MY - Y_1)$$

$$\delta Y/\delta X_2 = \alpha \cdot MY \cdot EXP(-\alpha \cdot X_2) = \alpha \cdot (MY - Y_2)$$

$$\frac{\delta Y/\delta X_1}{\delta Y/\delta X_2} = \frac{\alpha * (MY - Y_1)}{\alpha * (MY - Y_2)}$$

Thus the quotient between responses equals the ratio between the parts of the yield that have not yet been realized. If there are two different curves, then the quotient equals the ratio between the proportions (MY-Y)/Y of the yields that have not been realized.

A theoretical agronomic elaboration of the Michaelis-Menten equation.

In this discourse the aim is to derive the Michaelis-Menten relation as a plausible agronomical theoretical relation between the yield, nutrient availability and spatial or temporal heterogeneity.
For three nutrients the Michaelis-Menten model is formulated as [eq. 5]:

$$1/Y = 1/MY + 1/(\alpha.N) + 1/(\beta.P) + 1/(\tau.K)$$

This model is a variant of the Michaelis-Menten model for more than one substrate with only **implicit** structural interaction between the substrates, but without any **explicit** interaction terms [23]). For the full variant with explicit interaction terms see Fell (1997, p. 58-59).

$$AY = AY_{MIN} \qquad \text{or:} \qquad 1/Y = 1/MY$$

in which as new variables:

AY = Area per kg dm yield (reciprocal of yield per ha) at given values of the production factors (ha/kg dm).
AY_{MIN} = Minimum area per kg dm yield = reciprocal of maximum yield per ha (ha/kg dm), if the production factors do not limit the production (ha/kg dm).

If one of the nutrients (e.g. N) is sub-optimal, then the area required for 1 kg dm yield will be larger than the area required for the maximum production The crop needs a larger area of land to collect with its roots the N required.

$$AY = AY_{MIN} + \textbf{area for N deficiency}$$

or:

$$1/Y = 1/MY + 1/(\alpha.N)$$

It is plausible that the extra area needed for collecting N, concerns the area on which N is available and that this area is proportional to the area on which the N is available.
This extra area $(1/\alpha).(1/N)$ is, by definition, inversely proportional to the concentration of N in that area, by which α equals the kg dm yield per kg available N (= N present; not N absorption).

Consequently, as $\alpha.N$ is larger, the extra area needed for 1 kg yield will be closer to zero. In that case, N is sufficiently present and only slightly limiting. If $\alpha.N$ approaches infinity, N will not be limiting at all [24]).
Different combinations of α and N give the same product $\alpha.N$. This means that if the response of N would be twice, half the amount of N would be sufficient for the same yield.
On the other hand, as $\alpha.N$ approaches zero, the extra area needed will approach infinity, making the area needed for one kg yield infinite. This is the case if $N = 0$ or if $\alpha = 0$.

Next we imagine that a second nutrient (for example P) is sub-optimal, the area needed for 1 kg of yield will still be larger (the crop needs a still larger area of land to produce 1 kg yield, unless the P occurs at exactly the same location and time as the N):

$$\mathbf{AY = AY_{MIN} + 1/(\alpha.N) + area\ for\ P\ deficiency}$$

or:

$$\mathbf{1/Y = 1/MY + 1/(\alpha.N) + 1/(\beta.P)}$$

This may also be written as:

$$\mathbf{1/Y = 1/MY + (1/\alpha).(1/N) + (1/\beta).(1/P)}$$

Analogically the derivation may be extended to three (or more) nutrients:

$$\mathbf{1/Y = 1/MY + (1/\alpha).(1/N) + (1/\beta).(1/P) + (1/\tau).(1/K)}$$

The Michaelis-Menten model may be schematically presented as an image (Figure 12.6.1) consisting of four concentric circles with different diameters, representing ecological subspaces, needed for the minimum physical space required, and the spaces needed for obtaining N, P and K respectively N, P and K. It may be expected that the extra area needed for N and P is in part the same area. N and P may occur in the soil (dependent on the degree of similarity of the spatial distribution) at about the same locations. Therefore the factors $(1/\alpha)$ and $(1/\beta)$ will be smaller (α and β larger) if the nutrients in the soil occur more uniformly similarly (are better correlated).
The response coefficients α and β should in fact be regarded as the product of a plant-physiological factor (α_{PHY}) and a soil heterogeneity factor. The product (α_{HET}). α_{PHY} indicates the yield per kg available N in case of a normal (average) heterogeneity of the soil. α_{HET} is a multiplier of α_{PHY}. With normal heterogeneity α_{HET} has the value 1. As the heterogeneity of the soil increases

the value of the coefficient α_{HET} becomes smaller. At a (theoretical) value of zero, the soil is extremely heterogeneous and the area needed for 1 kg yield becomes infinite.

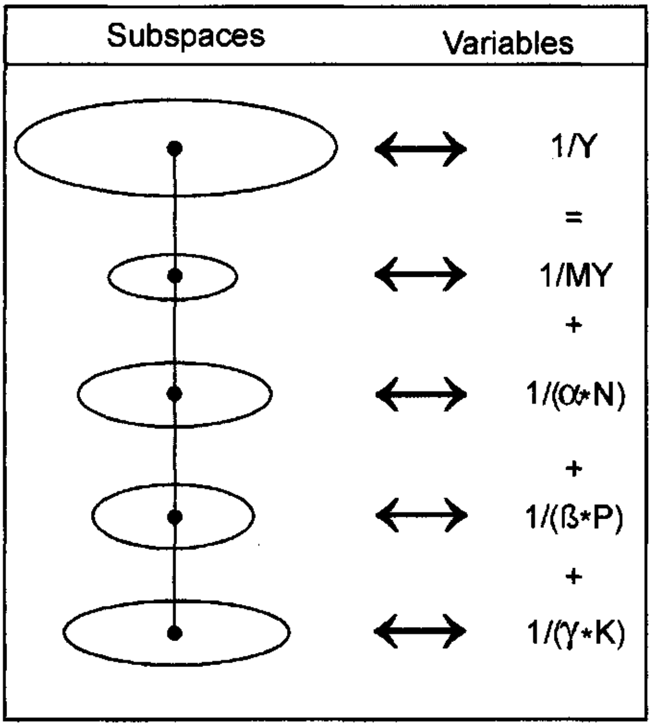


Figure 12.6.1 Schematic representation of the Michaelis-Menten model.

Though all nutrients may be in sufficient supply, their co-occurrence will be poor because of the heterogeneity of the soil. The highest value of α_{HET} is determined by the least possible variance of nutrients in the soil. At the maximum empirical value of α_{HET} the product of α_{HET} and α_{PHY} will possibly take the value of α in a water culture. Greater values of α_{HET} probably do not have any empirical significance, because the distribution of nutrients is unlikely to be more homogeneous in the soil than in an aqueous solution.
It is obvious that the theoretical values of $\alpha = 0$ and ∞ (∞ = infinity) are only of mathematical interest.

The area needed for 1 kg of yield will increase when a larger number of nutrients become limiting or, when they become limiting more strongly or when they are found at more separate locations in the soil (thus heterogeneity of the soil is very important with respect to the response of crop production to nutrient availability).

In the "ecological space" interpretation of the Michaelis-Menten model two extreme and one intermediate case may be distinguished:

1 The ecological subspaces needed for collecting the different nutrients overlap totally:

$$\mathbf{1/Y = MAX\ \{\ 1/MY\ ,\ 1/(\alpha.N)\ ,\ 1/(\beta.P)\ ,\ 1/(\tau.K)\ \}}$$

Inverting this equation gives the Von Liebig function:

$$\mathbf{Y = MIN\ \{\ MY,\ \alpha.N\ ,\ \beta.P\ ,\ \tau.K\ \}}$$

2 The ecological subspaces do not overlap at all:

$$\mathbf{1/Y = 1/MY + 1/(\alpha.N) + 1/(\beta.P) + 1/(\tau.K)}$$

This is an extreme variant of the Michaelis-Menten model in which the response coefficients α, β and τ have their minimal values.

3 The ecological subspaces overlap partially:

$$\mathbf{1/Y = 1/MY + f1\ .\ \{1/(\alpha.N)\} + f2\ .\ \{1/(\beta.P)\} + f3\ .\ \{1/(\tau.K)\}}$$

This is the most common case. The response of yield to nutrients is greater than in case 2 because of the multipliers f1, f2 and f3, which are all less than 1. The multipliers f1 and α, f2 and β, f3 and τ give new constant coefficients alfa$_*$, beta$_*$ and gamma$_*$. So the basic Michaelis-Menten relation remains. Unlike the graphical and qualitative derivation in Figure 12.6.1, this is a formal and mathematical derivation but also departing from the concept of soil variability. The Michaelis-Menten function is also supported by empirical data (elaborated in Chapter 4), at least on the interval of low applications where no damage occurs because of excessively high applications.

Appendix 12.7 Test on constant activity in yield-nutrient data

Analysis of empirical data to test the Mitscherlich model.

It may be examined how far, at two different levels of one nutrient, the ratio between productions remains constant if the level of another nutrient varies. From the Mitscherlich model it follows that this ratio remains constant (see Appendix 12.5).
For different data sets the applicability of the Mitscherlich model was tested, by means of this calculation. Data from publications of Penning de Vries & Van Keulen (1982, p. 196-226) and Mitscherlich (1923, p. 201) were analyzed. The results are represented in Table 5.

Table 5a Relative production (as a proportion of production at P application=30) at different N, and P applications (kg/ha).

N application	Relative production at different P applications	
	P = 0	P = 10
0	0.60	0.90
30	0.68	0.94
100	0.42	0.73

Source: Penning de Vries & Van Keulen (De Wit, 1992, Figure 11 p. 138).

Table 5b Relative production (as a proportion of production at N application=100) at different P and N applications (kg/ha).

P application	Relative production at different N applications	
	N = 0	N = 30
0	0.48	0.84
10	0.42	0.68
30	0.34	0.52

Source: Penning de Vries & Van Keulen (De Wit, 1992, Figure 11 p. 138).

Table 5c Relative production (as a proportion of production at K_2O application=0.73) at different N and K_2O applications (kg/container).

N application	Relative production at different K_2O applications		
	K_2O = 0	K_2O = 0.136	K_2O = 0.324
0	0.95	-	-
0.533	0.67	0.84	0.93
1.33	0.46	0.77	0.93

Source: Mitscherlich (1923, p. 201).

Table 5d Relative production (as a proportion of production at N application=1.33) at different K_2O and N applications (kg/container).

K_2O application	Relative production at different N applications	
	N = 0	N = 0.553
0	0.17	0.14
0.136	-	0.85
0.324	-	0.78
0.730	0.08	0.78

Source: Mitscherlich (1923, p. 201).

The ratio did not appear to be a constant value (Table 5a, 5b, 5c and 5d), but decreased as the application rate of the other nutrient increased. This means that the observed positive interaction between N and P is less than may be expected by reasoning from the Mitscherlich model. The effect of extra nutrient on the production (expressed as a proportion of the maximum production) is smaller the more of the other nutrient is available. This points to response curves according to Liebscher (see Figure 3.4.2), and not according to Mitscherlich (constant response factors) or Liebig (discontinuities in the interaction).
Other authors (e.g. Von Boguslawski, 1958) did similar tests and obtained the same results.

Appendix 12.8 Proportional nutrients in Michaelis-Menten

Inference of the equation of the relation between yield and proportionally available nutrients from the equation of the relation between yield and non correlated availabilities in case of the Michaelis-Menten production function.

For the sake of simplicity here we start with the original equation in which only two nutrients N and P occur [eq. 8]:

$$1/Y = 1/MY + 1/(\alpha . N) + 1/(\beta . P)$$

Suppose the fertilizers N and P are available in the proportional ratio of 1/qq, and the available amount of P is expressed in that of N. So with proportionally increasing availability the production function will be [eq. 9]:

$$1/Y = 1/MY + 1/(\alpha . N\phi) + 1/(\beta . qq . N\phi)$$

Or:

$$1/Y = 1/MY + (1/\alpha) . (1/N\phi) + \{1/(\beta . qq)\} . (1/N\phi)$$

Or:

$$1/Y = 1/MY + [(1/\alpha) + \{1/(\beta . qq)\}] . (1/N\phi)$$

And since the total nutrient availability NP equals:

$NP = (1+qq) . N\phi$ and: $1/N\phi = (1+qq)/NP$

This may be written as:

$$1/Y = 1/MY + [\{(1+qq)/\alpha\} + \{(1+qq)/(\beta . qq)\}] . (1/NP)$$

The basic structure of the equation between yield and the proportional available nutrients, is therefore the same as that of the relation between yield and the availability of one of the separate nutrients:

$1/Y = 1/MY + (1/\Theta . NP)$ or:

$$Y = (\Theta . NP . MY) / (\Theta . NP + MY)$$

The constant Θ is a function of the original coefficients α, β and qq from eq. 8 and 9, and will have a value > 0.

The productivity of nutrients is expressed by the quotient Y/NP (or by the quotient Y/Nϕ, which is proportional to Y/NP). The average productivity is relevant here, not the marginal productivity:

$$\mathbf{RP = Y/NP = (MY.\Theta) / (\Theta.NP+MY)}$$

An extreme high value of RP will be reached if:

Θ.NP+MY approaches 0

This is the case if:

$$\mathbf{\Theta.NP = -MY} \qquad \text{or:} \qquad \mathbf{NP = -MY/\Theta}$$

This is an extreme value of productivity; this domain of the equation however has no ecological meaning. It is the vertical asymptote (NP = - MY/Θ) in the third quadrant of the diagram. Also the extreme low values of RP for (Θ.NP+MY) approaching zero from the negative side, in this hyperbolic function, is not ecologically relevant.

When NP approaches ∞, then RP approaches zero.:

$$\mathbf{\Theta.NP + MY \rightarrow \infty} \qquad \text{Or:} \qquad \mathbf{NP \rightarrow \infty}$$

So PR decreases at increasing NP. Further this demonstrates that the maximum productivity of the Michaelis-Menten production function lies at the minimum availability of N and P, if the availabilities of the nutrients are proportional. When generalizing the model to more than two nutrients, in principle the inference runs analogue and the productivity decreases at increasing proportional availabilities.
Also if the different nutrients cannot be regarded as equivalent, but have to be weighted when summing up (for example because they have different environmental loads), the relation between weighted nutrient availabilities and yield still remains a Michaelis-Menten equation.

Appendix 12.9 Michaelis-Menten and the four quadrants

<u>Evidence that if both the relations between nutrient availability and uptake on the one hand and the relation between nutrient uptake and yield on the other, are of the Michaelis-Menten type, the relation between nutrient availability and yield is also of Michaelis-Menten type.</u>

The relation between nutrient uptake (UN and UP) and yield (Y) is a Michaelis-Menten equation [eq. 10]:

1/Y = 1/MY + 1/(ααα.UN) + 1/(ßßß.UP)

And the relations between nutrient availability and nutrient uptake are [eq. 11], and [eq. 12]:

1/UN = 1/MUN + 1/(αα.N) + 1/(ßα.P)

1/UP = 1/MUP + 1/(αß.N) + 1/(ßß.P)

with as new symbols:

MUN	=	maximum uptake of N (kg/ha).
MUP	=	maximum uptake of P (kg/ha).
αα	=	Coeff. of response of N uptake to N availability (kg N/kg N).
ßß	=	Coeff. of response of P uptake to P availability (kg P/kg P).
ßα	=	Coeff. of response of N uptake to P availability (kg N/kg P).
αß	=	Coeff. of response of P uptake to N availability (kg P/kg K).
ααα	=	Coeff. of response of yield to N uptake (kg dm/kg N).
ßßß	=	Coeff. of response of yield to P uptake (kg dm/kg P).

Now we will derive below that the Michaelis-Menten model is necessarily also valid for the relation between available nutrients and yield [eq. 13]:

1/Y = 1/MY + 1/(α.N) + 1/(ß.P)

To realize this, eq. 11 and eq. 12 are substituted in eq. 10, which gives:

1/Y = 1 /MY + 1/ααα . {1/MUN + (1/αα).(1/N) + (1/ßα).(1/P)} ^

+ 1/ßßß . [1/MUP + 1/{(αß).(1/N)} + {(1/ßß).(1/P)}]

This may also be expressed as:

$$
\begin{aligned}
\mathbf{1/Y} &= \mathbf{(1/MY) + \{(1/\alpha\alpha\alpha) \cdot (1/MUN) + (1/\beta\beta\beta) \cdot (1/MUP)\}\ \hat{}} \\
&\quad \mathbf{+ \{(1/\alpha\alpha\alpha) \cdot (1/\alpha\alpha) + (1/\beta\beta\beta) \cdot (1/\alpha\beta)\} \cdot 1/N\ \hat{}} \\
&\quad \mathbf{+ \{(1/\alpha\alpha\alpha) \cdot (1/\beta\alpha) + (1/\beta\beta\beta) \cdot (1/\beta\beta)\} \cdot 1/P}
\end{aligned}
$$

The general form of this equation is:

$$\mathbf{1/Y = 1/constant_1 + (1/constant_2) \cdot (1/N) + (1/constant_3) \cdot (1/P)}$$

And the form of the latter equation is identical to eq. 13, in which:

$$\mathbf{1/constant_1 = \{1/MY + (1/\alpha\alpha\alpha)\cdot(1/MUN) + (1/\beta\beta\beta)\cdot(1/MUP)\}}$$

$$\mathbf{1/constant_2 = \{(1/\alpha\alpha\alpha)\cdot(1/\alpha\alpha) + (1/\beta\beta\beta)\cdot(1/\alpha\beta)\}}$$

$$\mathbf{1/constant_3 = \{(1/\alpha\alpha\alpha)\cdot(1/\beta\alpha) + (1/\beta\beta\beta)\cdot(1/\beta\beta)\}}$$

For more than two nutrients the derivations runs in analogous fashion.

Appendix 12.10 Optimum nutrient ratios in Michaelis-Menten

Deriving the optimum ratio between different nutrients for maximizing of the productivity of the (weighted) sum of the nutrient availability in the case of the Michaelis-Menten model.

Let the relation between yield and availability of nutrients be:

$$\mathbf{1/Y = 1/MY + (1/\alpha) * (1/N) + (1/\beta) * (1/P)}$$

If:

$\mathbf{P = qq * N\phi}$ and: $\mathbf{NP = (1+qq) * N\phi}$ and: $\mathbf{NP = N\phi + P}$

Then: $\mathbf{N\phi = 1/(1+qq) * NP}$ and: $\mathbf{P = qq/(1+qq) * NP}$

Substitution gives:

$$\mathbf{1/Y = 1/MY + \{(1+qq)/\alpha + (1+qq)/(\beta * qq)\} * (1/NP)}$$

The nutrient use RU (kg N + kg P) per kg dm is represented by eq. 14:

$$\mathbf{RU = NP/Y = NP/MY + 1/\alpha + qq/\alpha + 1/(\beta * qq) + 1/\beta}$$

Differentiation with respect to qq reveals at which qq-value RU has its minimum. In general this minimum will have little significance, because implicitly 1 kg N is put equal to 1 kg P, while from the economic viewpoint P may be more expensive per kg than N, and from an ecological viewpoint N may be much more environmentally harmful than P. However, for the sake of simplicity, weighing factors are initially omitted, and subsequently the changes in conclusions are examined if these factors are introduced. We now show that at qq-value for which the first derivative of RU with respect to qq becomes zero and the second derivative becomes positive RU will have its minimum.

The first derivative of eq. 14 is:

$$\mathbf{\delta RU/\delta qq = 1/\alpha - 1/(\beta * qq^2)}$$

this becomes zero if for qq it holds that [eq. 15]:

$$\mathbf{qq = (\alpha/\beta)^{0.5}}$$

The second derivative of RU with respect to qq equals:

$$\delta^2 RU/\delta qq^2 = 2 / (\beta \cdot qq^3)$$

For any positive qq value this second derivative is positive if α and ß are positive. So, for a value of zero of the first derivative a minimum consumption of NP (kg N + kg P)/kg yield is found.
What changes if the environmental load of N and P are not equal?
It may be demonstrated that if nitrogen pollutes the environment with EN units and phosphorus with EP units per kg availability, eq. 15 should be replaced by:

$$qq = \{(\alpha \cdot EP)/(\beta \cdot EN)\}^{0.5}$$

Likewise, from an economic point of view, it holds that if nitrogen costs PRI_N monetary units per kg and phosphorus PRI_P monetary units per kg, the equation for qq becomes:

$$qq = \{(\alpha \cdot PRI_N)/(\beta \cdot PRI_P)\}^{0.5}$$

And for three nutrients N, P and K with response coefficients α, ß and τ it may be elaborated that the optimal ratios pp, qq and rr are:

$$pp = \sqrt{(\beta \cdot \tau)} / \{\sqrt{(\beta \cdot \tau)} + \sqrt{(\beta \cdot \alpha)} + \sqrt{(\alpha \cdot \tau)}\}$$
$$qq = \sqrt{(\alpha \cdot \tau)} / \{\sqrt{(\beta \cdot \tau)} + \sqrt{(\beta \cdot \alpha)} + \sqrt{(\alpha \cdot \tau)}\}$$
$$rr = \sqrt{(\beta \cdot \alpha)} / \{\sqrt{(\beta \cdot \tau)} + \sqrt{(\beta \cdot \alpha)} + \sqrt{(\alpha \cdot \tau)}\}$$

Some inferences:
In the Michaelis-Menten production function the ratio between nutrients which gives the maximum total productivity, is not dependent upon the availability level, but only upon the response coefficients (and in case of ecologic or financial optimal productivity also upon the specific prices and environmental loads of the nutrients). In the Mitscherlich model this is different; in that model the ratio between nutrients which maximizes productivity depends on the level of availability as well.

If P is more expensive than N, then the ratio of N to P at which productivity is maximal is larger when the calculation is done in monetarily weighed units instead of in physical units.

And if N is more harmful for the environment than P, then the ratio of N to P, for which the productivity is maximal, will be smaller when calculated in environmentally weighed units, than when calculated in physical units.

Appendix 12.11 Nutrient surplus; extension of the linear model

For several nutrients in a linear additive model the yield (Y) is:

$$\mathbf{Y = Y_{N0} + \alpha{*}N_E + \beta{*}P_E + \tau{*}K_E}$$

or in terms of available nutrients:

$$\mathbf{Y = \alpha{*}N + \beta{*}P + \tau{*}K}$$

in which: $N = N_I + N_E$, $P = P_I + P_E$ and $K = K_I + K_E$

Proportional availability implies that $P=qq{*}N$ and $K=rr{*}N$, so that the yield is:

$$\mathbf{Y = \alpha{*}N + \beta{*}qq{*}N + \tau{*}rr{*}N = (\alpha+\beta{*}qq+\tau{*}rr) * N}$$

Uptake of NPK:

$$\mathbf{UNPK = Y * (N_C+P_C+K_C) = (\alpha+\beta{*}qq+\tau{*}rr) * N * (N_C+P_C+K_C)}$$

in which:
N_C = Amount of N per unit dry matter of yield (kg N/kg dm).
P_C = Amount of P per unit dry matter of yield (kg P/kg dm).
K_C = Amount of K per unit dry matter of yield (kg K/kg dm).

The total available nutrients N+P+K are denoted by NPK:

$$\mathbf{NPK = N + qq{*}N + rr{*}N = (1+qq+rr) * N}$$

The surplus of available N+P+K:

$$\mathbf{SNPK = N+P+K - Y{*}(N_C+P_C+K_C) =}$$

$$\mathbf{= (1+qq+rr){*}N - (\alpha+\beta{*}qq+\tau{*}rr){*}N{*}(N_C+P_C+K_C) = constant_4 * N}$$

As with one variable nutrient and other factors constant, the surplus of available NPK is linearly related to the available N (with proportional co-availability of P and K). If one refuses to add apples and oranges, a much more laborious derivation may be worked out using instead of the concepts N+P+K availability and N+P+K surplus, separately N availability (and N surplus) with proportional co-availability of P and K. This derivation gives formally the same result, because of the linearity of the model.

In case of a Von Liebig model the yield is:

$$\mathbf{Y = MIN(MY\ ,\ \alpha_V{*}N\ ,\ \beta_V{*}P\ ,\ \tau_V{*}K)}$$

Taking the special case of proportional availability of N, P and K, such that the proportionality is a harmonious proportionality ($\alpha_V{*}N = \beta_V{*}P = \tau_V{*}K$), then $qq = \alpha_V/\beta_V$ and $rr = \alpha_V/\tau_V$. Substituting these values gives for Y:

$$\mathbf{Y = MIN(MY\ ,\ \alpha_V{*}N\phi\ ,\ \alpha_V{*}N\phi\ ,\ \alpha_V{*}N\phi) = MIN(MY,\ \alpha_V{*}N\phi)}$$

As it is of no use that $\alpha_V{*}N$ is greater than MY, the relation can be simplified by:

$\mathbf{Y = \alpha_V{*}N\phi}$ ($N\phi$ = N with harmonious proportional P+K)

N+P+K uptake is:

$$\mathbf{UNPK = (N_C + P_C + K_C) * Y = (N_C + P_C + K_C) * (\alpha_V{*}N\phi)}$$

and N+P+K availability:

$$\mathbf{NPK = N\phi + (\alpha_V/\beta_V){*}N\phi + (\alpha_V/\tau_V){*}N\phi = (1+\alpha_V/\beta_V+\alpha_V/\tau_V) * N\phi}$$

The N+P+K surplus (SNPK) is the difference of N+P+K availability and N+P+K uptake:

$$\mathbf{SNPK = (1+\alpha_V/\beta_V+\alpha_V/\tau_V) * N\phi - (N_C + P_C + K_C) * \alpha_V{*}N\phi}$$

this formula again has the simple structure:

$$\mathbf{SNPK = constant_5 * N\phi}$$

The NPK surplus per kg dm is:

$$\mathbf{SNPK/Y = \{(1+\alpha/\beta+\alpha/\tau) * N\phi - (N_C + P_C + K_C) * \alpha{*}N\phi\} / \{\alpha{*}N\phi\}}$$

Or:

$$\mathbf{SNPK/Y = 1/\alpha_V + 1/\beta_V + 1/\tau_V - N_C - P_C - K_C}$$

As with the linear model with one variable nutrient and constant other factors, we see that the NPK surplus per kg dm is a constant (independent on available proportional NPK). However, beyond the availability giving maximum possible yield the surplus increases, the increase being partly dependent on the degree of luxury consumption.

At the end of this section we draw attention to the fact that these models, unlike the non-linear models in the next sections, assume constancy of nutrient concentrations in dry matter.

Appendix 12.12 Nutrient surplus in Michaelis-Menten

<u>Increasing nutrient surplus per ha and nutrient surplus per kg dm at increase of proportionally available nutrients, if the Michaelis-Menten equation is valid for the relation between availability and uptake and for the relation between uptake and production.</u>

In the Michaelis-Menten model the following relations between available nutrients (N), nutrient uptake (UN) and production (Y) are tenable, when availabilities are proportional. The derivation for one nutrient (nitrogen) is given, under the assumption of proportional availability of P and K:

$$\mathbf{UN = (MUN_*\mu_*N\phi) / (MUN+\mu_*N\phi)}$$

and:

$$\mathbf{Y = (\epsilon_*\Omega_*UN) / (\epsilon+\Omega_*UN)}$$

and also:

$$\mathbf{Y = (MY_*\sigma_*N\phi) / (MY + \sigma_*N\phi)}$$

and the surplus of available N per ha (SN):

$$\mathbf{SN = N\phi - UN}$$

and the surplus of N per kg product (SN/Y):

$$\mathbf{SN/Y = SN / Y}$$

in which as new symbols:

SN = N surplus (= amount of available N, which was not taken up by the crop (kg N/ha)). To be distinguished from the amount of applied N which was not taken up.
μ = Compound coefficient of response of N uptake to N availability, if P, (or P, and K) availability is (are) proportional to that of N (μ is a function of the original coefficients αα, ßα and τα of the Michaelis-Menten model and of the coefficients qq and rr).
Ω = Compound coefficient of response of **production** to **N uptake** (kg dm/kg N), if P, (or P and K) availability is (are) proportional to that o f N (Ω is a function of the original coefficients ααα, ßßß, τττ, αα, ß α, τα, αß, ßß, τß, ατ, ßτ, ττ, qq, rr).

ε = Positive constant [25]) (ε is a function of the original coefficients MY, MUN, MUP, ßßß, τττ, αα, ßα, τα, αß, ßß, τß, ατ, ßτ, ττ, qq, rr, but not of ααα).

Expressing the N surplus/ha (SN) in N availability with proportional P and K (Nϕ) gives [eq. 16]:

$$SN = (N\phi . MUN + \mu . N\phi^2 - \mu . MUN . N\phi) / (MUN + \mu . N\phi)$$

SN approaches to ∞ (or - ∞) at a value of Nϕ = - MUN/μ. This value of Nϕ is always negative, because μ and MUN are always positive. So a vertical asymptote exists in the negative domain of Nϕ which is an ecologically irrelevant domain of the function. SN approaches ∞ if Nϕ approaches ∞ or -∞ if Nϕ approaches -∞. An oblique asymptote exists, equalling:

$$SN = N\phi + MUN.(1/\mu - 1)$$

In eq. 16 SN is zero if Nϕ is zero. This does have ecological significance, fo r it means that with N availability = zero the surplus is of course zero too. I n eq. 16 SN is also zero if μ.Nϕ equals μ.MUN - MUN, or, if Nϕ equals N ϕ = MUN.(1-1/μ) (the numerator of the equation then becomes zero). Because the coefficient μ has values which are greater than, or equal to, zero and smaller than, or equal to, 1, (1-1/μ) is always negative or zero. And because MUN is always positive, a negative value of Nϕ, at which SN is zero, exists as well. This negative value of Nϕ however is not ecologically relevant. Important are the values of Nϕ where the surplus SN becomes minimal. Therefore the first derivative has to be taken, and put equal to zero.
The first derivative of SN with respect to Nϕ is:

$$\delta SN/\delta N\phi = \frac{MUN^2 + 2.\mu.N\phi.MUN + \mu^2.N\phi^2 - \mu.MUN^2}{(MUN + \mu.N\phi)^2}$$

The first derivative becomes zero if the numerator becomes zero. In that case a quadratic equation of Nϕ is obtained, of which the roots equal:

$$N\phi_1 = -(MUN/\mu) + (MUN/\mu) . (\sqrt{\mu})$$

For this Nϕ value SN reaches a minimum because the second derivative is positive. The other root is:

$$N\phi_2 = -(MUN/\mu) - (MUN/\mu) . (\sqrt{\mu})$$

For this $N\phi$ value SN reaches a maximum because the second derivative is negative. Because MUN is positive and μ can not be greater than 1 (N absor ption of N cannot exceed N availability) and not be smaller than zero (N absorption of N cannot be negative), both extremes are situated at $N\phi$-values lower than zero; for at values of μ between zero and 1, $\sqrt{\mu}$ is greater than μ.

SN as a function of $N\phi$ has two values for which the first derivative is zer o, both of which are situated in the irrelevant domain of negative values of $N\phi$. The function has a vertical asymptote in the irrelevant negative domain of $N\phi$ and a oblique asymptote SN = $N\phi$ - MUN. The oblique asymptote means that at extreme values high values of available N the surplus approaches the difference between availability and maximum uptake. The equation of the oblique asymptote may be derived by division of the numerator by the denominator in eq. 16. For $N\phi$ = zero, SN reaches a value of zero too. That means that the N surplus (kg/ha) between a N availability of zero and infinity increases, which had to be demonstrated.

Surplus per kg dry matter.

It has to be proved now that the surplus per kg dm is also minimal at the lowest possible nutrient availability. We do this for nitrogen and express SN/Y in $N\phi$. As demonstrated in Appendix 12.8 the dm production (Y) and the uptake (UN) of nitrogen can - when the availability of N is linearly correlated with the availability of P - be expressed as Michaelis-Menten functions of the nitrogen available (with proportional P):

$$Y = (MY \cdot \sigma \cdot N\phi) / (MY + \sigma \cdot N\phi)$$

and:

$$UN = (MUN \cdot \mu \cdot N\phi) / (MUN + \mu \cdot N\phi)$$

in which as new coefficient:

σ = Compound coefficient of response of production to N availability (kg dm/kg N), if P, and K availabilities are proportional to that of N (σ is a function of the coefficients α, ß, τ, qq and rr).

Now following eq. 16 the surplus of nitrogen not taken up (SN) is:

$$SN = (N\phi \cdot MUN + \mu \cdot N\phi^2 - \mu \cdot MUN \cdot N\phi) / (MUN + \mu \cdot N\phi)$$

And the surplus per kg product RSN (= SN/Y):

$$\mathrm{RSN} = \frac{\{(\mathrm{N}\phi.\mathrm{MUN} + \mu.\mathrm{N}\phi^2 - \mu.\mathrm{MUN}.\mathrm{N}\phi) . (\mathrm{MY} + \sigma.\mathrm{N}\phi)\}}{\{(\mathrm{MUN} + \mu.\mathrm{N}\phi) . (\mathrm{MY}.\sigma.\mathrm{N}\phi)\}} =$$

$$= \frac{\mathrm{MY}.\mathrm{MUN}.\mathrm{N}\phi + \mu.\mathrm{MY}.\mathrm{N}\phi^2 - \mu.\mathrm{MY}.\mathrm{MUN}.\mathrm{N}\phi}{\sigma.\mathrm{MY}.\mathrm{MUN}.\mathrm{N}\phi + \sigma.\mu.\mathrm{MY}.\mathrm{N}\phi^2} \wedge$$

$$+ \frac{\sigma.\mathrm{MUN}.\mathrm{N}\phi^2 + \sigma.\mu.\mathrm{N}\phi^3 - \sigma.\mu.\mathrm{N}\phi^2.\mathrm{MUN}}{\sigma.\mathrm{MY}.\mathrm{MUN}.\mathrm{N}\phi + \sigma.\mu.\mathrm{MY}.\mathrm{N}\phi^2}$$

The numerator and denominator divided by Nϕ gives:

$$\mathrm{RSN} = \frac{\mathrm{MY}.\mathrm{MUN} + \mu.\mathrm{MY}.\mathrm{N}\phi - \mu.\mathrm{MY}.\mathrm{MUN}}{\sigma.\mathrm{MY}.\mathrm{MUN} + \sigma.\mu.\mathrm{MY}.\mathrm{N}\phi} \wedge$$

$$+ \frac{\sigma.\mathrm{MUN}.\mathrm{N}\phi + \sigma.\mu.\mathrm{N}\phi^2 - \sigma.\mu.\mathrm{N}\phi.\mathrm{MUN}}{\sigma.\mathrm{MY}.\mathrm{MUN} + \sigma.\mu.\mathrm{MY}.\mathrm{N}\phi}$$

Differentiation with respect to Nϕ then gives eq. 17:

$$\frac{\delta \mathrm{RSN}}{\delta \mathrm{N}\phi} = \frac{\sigma.\mathrm{MUN}^2 + 2.\sigma.\mu.\mathrm{MUN}.\mathrm{N}\phi - \sigma.\mu.\mathrm{MUN}^2}{\sigma.\mathrm{MY}.(\mathrm{MUN}+\mu.\mathrm{N}\phi)^2} \wedge$$

$$+ \frac{\sigma.\mu^2.\mathrm{N}\phi^2 + \mu^2.\mathrm{MUN}.\mathrm{MY}}{\sigma.\mathrm{MY}.(\mathrm{MUN}+\mu.\mathrm{N}\phi)^2}$$

At which availability of N does RSN become minimal? At one in which the first derivative of RSN with respect to Nϕ is zero and the second derivative is positive.

δRSN/δNϕ may equal zero in different ways; there are two solutions of the eq. 17:

a. The denominator is a quadratic equation in Nϕ, putting this equal to zer o gives the roots:

$$N\phi_1 = [- MUN - MUN . \sqrt{\{\mu - (\mu^2 . MY) / (\sigma . MUN)\}}]/\mu$$

and:

$$N\phi_2 = [- MUN + MUN . \sqrt{[\mu - (\mu^2 . MY) / (\sigma . MUN)\}}]/\mu$$

Because μ is in the order of $\mu=1$ and σ is in the order of σ=MY/MUN the discriminant of the quadratic equation equals approximately zero. For maximum production $\approx \sigma$. maximum uptake. But also if (because of non-linearity of the uptake function σ and μ are both about an equal fraction of their maximum values the discriminant will be zero. Then the first and second root are approximately equal to:

$$N\phi_1 = (-MUN - \triangle)/\mu \approx -MUN/\mu \approx MUN$$

and:

$$N\phi_2 = (-MUN + \triangle)/\mu \approx -MUN/\mu \approx MUN$$

in which $\triangle$ = a value relatively close to zero.

The root $N\phi_1$ has a minimum in the negative interval of $N\phi$. In this domain the function has mathematical significance but no ecological significance.
For values of $\mu < 1$ the discriminant of the quadratic equation becomes negative. In that case there are no solutions. The root of the equation can only be positive if μ is greater than 1. But such a value of the coefficient μ is physically impossible, because the uptake of a nutrient cannot be greater than the available amount. Moreover, the value of σ should at the same time be much larger than MY/MUN, and that is not probable either, because the magnitude of maximum production will be not very different from the magn itude of the mathematical product of maximum uptake and the response coefficient.
This demonstrates that the surplus per kg product for nitrogen is minimal a bit lower than the value of zero availability (solution $N\phi_1$), and when the available nutrient is raised the surplus per kg product will increase further (solution $N\phi_2$). This derivation for nitrogen may be produced for the surplus of each nutrient separately.

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Notes

1. (Chapter 2) Proportional availability is very difficult to realize in field experiments. A near zero situation is mostly easier and faster to realize for N than for nutrients which are fixed or absorbed to the soil like P, Mg, Ca and K.

2. (§ 3.2) The economist Leontieff also developed a production function from the same assumption as Von Liebig (Paris & Knapp, 1989).

3. (§ 3.3) Lissman developed this production function at the same time with Mitscherlich, whereas the extension of the Mitscherlich model to more than one nutrient is mostly referred to as the Mitscherlich-Baule model (Langeveld, 1997, personal communication).

4. (§ 3.3) For another value of the factor on the X-axis the proportion has a different value.

5. (§ 3.3) The definition of the concept of marginal production means that maximum marginal productivity - in case of a sigmoid production function (see figure 3.3.1) - is situated at a lower availability than the availability at the maximum (average) productivity. The former coincides with the availability value at the inflexion-point of the s-curve; the latter with the tangent of the sigmoid curve through the origin (see the sigmoid Mitscherlich function in Appendix 12.6).

6. (§ 3.3) In the literature we did not find any report on research about the shape of a production curve with proportional application of nutrients. Many publications, however, stress the importance of harmonious proportions of the different nutrients. A sigmoid curve has hardly ever been observed (Van Keulen, personal communication). In case of one nutrient, occasionally a sigmoidal curve was found. Van Diest (1977) showed a sigmoid potassium response curve in a potassium yield experiment, where at very low rates potassium was fixed by the soil and at somewhat higher rates plants took up a greater proportion of the increasing amounts of potassium, thus creating a more than proportional increasing uptake.

7. (§ 3.4) In the Michaelis-Menten model the values of the response coefficients α, β and τ are in a quite different order of magnitude than in the model of Mitscherlich. In Mitscherlich α_m, β_m and τ_m are yield increases (at the availability value of zero) per kg available nutrient, expressed as **proportions** of the maximum yield, while in Michaelis-Menten they are expressed as **kg production** per kg available nutrient. In both cases the coefficients refer to the slopes of the production function **near the origin**.

8. (§ 3.5) Interaction and substitution often go together, but not necessarily. Possible is substitution without interaction (additive linear model without interaction terms), but also interaction without substitution is possible (Von Liebig model).

9. (§ 4.3) Here it should be stressed that the curve of the relation between production and proportional increase of N and P uptake (quadrant 1) is not automatically represented in the right way, because N-uptake and P-uptake in quadrant 1 are not necessarily related strictly linear if N-application and P-application are linearly related in quadrant 4. The superimposed **(dotted)** curve is nevertheless based on that assumption.

10. (§ 4.4) Greenwood himself added a parameter for this depression in production due to over-fertilization with nitrogen.

11. (§ 5.4) For the sake of better communication, different alternative definitions of the concepts of nutrient productivity and of nutrient recovery were defined (see Appendix 12.1 and Appendix 12.2). As an alternative to the conventional agronomic definition, productivity may be defined as "system productivity": production derived from external + internal nutrients divided by the external nutrient application. Internal nutrients are defined as nutrients which can be disposed of without depleting the nutrient stocks below the sustainable equilibrium level needed to maintain the desired production.

12. (§ 6.2) A matter to consider may be if not the costs of total internal nutrients, but only the costs of nutrients from mining should be added to the costs of applied nutrients. Because the models in this study are not dynamic, we can only indirectly touch at this point, but not really analyze it.

13. (§ 6.3) De Wit (1981, p. 248) mentions literature from the eighteenth century in which nutrient stocks are regarded as buffers for price fluctuations of products and resources. The farmer may temporarily deplete the system dependent on the prices in the market.

14. (§ 8.1) The term surplus, if not used as a generalizing term for all surplus indicators, or if not defined otherwise, means surplus of available nutrients.

15. (§ 8.2) We take the productivity per kg available, since under the condition of sustainability this equals the productivity per kg applied in the long run.

16. (§ 8.3) UN_I is defined as the uptake from internal N at other external N-applications and may be dependent on the latter. UN_{N0} is the N uptake at application zero. We assume that UN_{N0} is equal to UN_I, which is not necessarily so.

17. (Chapter 11) To prevent the mathematical artefact of approaching to very high values when application approaches zero a better measure for the low range of applications may be the reciprocal of system productivity. This measure "consumptivity" gives the use of external nutrient per kg production.

18. (Appendix 12.1) For a static analysis, such as in this paper three concepts should be differentiated carefully: surplus, residue and emission. All kind of situations may occur. It is possible that there is:

- nutrient residue without nutrient emission because nutrients may not yet have been lost, but have been immobilized and temporarily accumulated.
- emission without residue, in case all emerging surpluses are immediately lost.
- residue without surplus, because input and output were in balance but there was mineralisation from stocks of insoluble nutrients.
- surplus without residues, because all surpluses are immediately immobilized or lost (Neeteson, 1963).

A better type of analysis is clearly dynamic modelling with stocks and rates.

19. (Appendix 12.1) The term "definitely lost" is of course also relative, for volatilized nitrogen may recycle within a few days by means of deposition. Leached phosphate may recycle by means of consumption of plants from ditches by cattle.

20. (Appendix 12.4) The coefficients cp, cq and cr in the simulation model (Appendix 12. 4) are not the same as the coefficients pp, qq and rr in the text and in Appendix 12.10. The sum of cp, cq and cr is 1 and the sum of pp, qq and rr is 1/cp. The coefficients pp, qq and rr may be calculated from cp, cq and cr as follows: pp = cp/cp = 1; qq = cq/cp; rr = cr/cp

21. (§ 12.4) In case of simulations with proportional nutrients P_E and K_E are not constants but continuous variables proportional with N_E.

22. (Appendix 12.5) Note that this condition is a special case of the condition of proportional availability of N and P, namely if the coefficient qq equals α/β.

23. (Appendix 12.6) The Michaelis-Menten model without explicit interaction terms corresponds holds for a situation of a relative large substitutability of nutrients, the Michaelis-Menten model with explicit interaction terms with a situation of moderate substitutability and the Von Liebig model with a situation of substitution of zero.

24. (Appendix 12.5) Actually, with extreme high values of N plasmolysis will occur, indicating that the theoretical model is only valid within certain limits.

25. (Appendix 12.12) ε is the maximum yield in the case of proportional nutrient availability.