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Thermal profiles and thermal runaway in microwave heated slabs

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The microwave heating of slabs of water bound with a gel is modeled and analyzed without any restriction to the Biot number regime. Despite the fact that the temperature distribution over the slab is not uniform at all, the phenomenon of thermal runaway is basically caused by resonance of the electromagnetic waves within the object, combined with heat loss. A plot of the steady-state temperature at any position within the slab, versus the microwave power, is an S-shaped or a multi S-shaped curve. With respect to thermal runaway there is a strong similarity between isothermal and nonisothermal slabs. Using the average temperature of the nonisothermal slab, regardless of its Biot number, yields a reasonable approximation to describe the runaway. This is caused by the specific characteristic of the dielectric loss factor of water, which decreases with increasing temperature. This results in an almost constant absorption of energy over the whole slab without disturbing the wave character of the absorption. It turned out that this smoothing of the absorbed power plays a dominant role in the calculations of the temperature profiles. Any calculation where the temperature dependence of the permittivity is omitted, will not only pass the phenomenon of thermal runaway, but its temperature profiles will differ substantially from the ones where the temperature dependence has been taken into account. © *1999 American Institute of Physics.* [S0021-8979(99)02107-6]

I. INTRODUCTION

The application of microwave heating (frequency range 2450 MHz) is seriously hampered by two problems, both having their roots in the basic physics of the heating process. The first difficulty is the uneven spatial absorption of energy within the irradiated object. The second difficulty is the catastrophic phenomenon of thermal runaway in which a slight change of microwave power causes the temperature of the object to increase rapidly. The aim of this investigation is to find the physical origin of the runaway process and hope that it leads to a general rule in preventing runaway. Temperature profiles of the irradiated slab are necessary in order to achieve this. These temperature profiles give a good insight in the uneven spatial absorption of energy. Both problems, the uneven spatial absorption and the thermal runaway, are related to each order. The process of thermal runaway is a nonlinear problem and can be explained by taking the temperature dependence of the permittivity ϵ into account. This means that the temperature profiles, as shown in this article, will also reflect this temperature dependence. Many studies of microwave heated slabs of foodstuffs (see f.i., Dolande *et al.*[1]). have been performed, but the majority ignores the temperature dependence of the permittivity arguing that this is a second order phenomenon. As will be shown in this study, the process of thermal runaway is caused by resonance within the irradiated medium due to the temperature dependence of ϵ. Resonance is always a very strong phenomenon and it must not be regarded as a second order effect. It has a major impact on the absorption of energy, and by this, as will be shown on the temperature profiles. The study of thermal runaway in microwave heated objects usually starts by formulating the equation of the absorbed power D.

$$D=\tfrac{1}{2}\omega\epsilon''|E|^2, \tag{1}$$

where ω is the angular frequency, ϵ'' is the imaginary part of the permittivity, and E is the electric field within the object. Instead of ϵ'' one can also formulate Eq. (1) with the dielectric loss factor, which equals ϵ'' divided by ϵ_0, the permittivity of vacuum. Equation (1) is a very powerful equation because it is a general equation and does not depend on the geometry of the irradiated object. All problems involved in the geometry of the object are expressed by the field E. The absorbed power is always directly proportional to ϵ'' or the dielectric loss factor. The microwave sintering of ceramics especially is seriously hampered by the phenomenon of thermal runaway. The dielectric loss factor of ceramics strongly increases with increasing temperature. Looking at Eq. (1) the explanation of thermal runaway in ceramics is almost evident. While the temperature increases, the absorbed power will increase, resulting in a stronger increase of the temperature, which then results in more absorbed power, etc. This ''hand-waving'' argument can often be heard. Kriegsmann[2] formulated a plausible explanation of thermal runaway in ceramics in the small Biot number regime. Strangely enough the hand-waving argument does not play a major role in his explanation. Kriegsmann did not mention it but, in fact, he describes the phenomenon of resonance in an almost isothermal slab of ceramics. That the phenomenon of resonance within an isothermal slab results in thermal runaway became clear by the study of demineralized water (Stuerga *et al.*[3]). The dielectric loss factor of water hardly depends on temperature as compared to ceramics. It decreases a little bit with increasing temperature. It is obvious that in the case of water the hand-waving argument is not appropriate in explaining runaway. All of this does not mean that the argument is complete nonsense. Probably the hand-waving argument, combined with resonance, plays an important role in the description of runaway of ceramics in the large Biot regime. Resonance as the origin of thermal runaway in isothermal objects has been studied in earlier articles.[4,5] The aim of this study is to investigate the phenomenon of thermal runaway in nonisothermal slabs of foodstuffs without any limitations with respect to the Biot number.

Water is a major constituent of many foodstuffs and its Biot number can be quite large ($\mathrm{Bi}\approx 14$). This is why water (bound with a gel) has been used to illustrate the theory. Because the dielectric loss factor of ceramics strongly increases with increasing temperature, which is completely opposite to water, the results of this investigation in the large Biot number regime cannot be used to explain the behavior of ceramics. From time to time some remarks will be made about the impact of certain parts of the theory in relation to ceramics. An analysis of this study probably makes it clear why it is suggested that the hand-waving argument in combination with resonance plays an important role in explaining thermal runaway in ceramics in the case of large Biot numbers.

Attention has to be paid to the notation. In contravention of the usual notation[6] the symbol α will be used as the phase constant and β as the attenuation constant. Both are real numbers. This notation is easier to read and it fits in better with the theory of wave propagation as described in classic books. The runaway phenomenon, as described in this article, is a result of the application of the wave propagation theory.

II. THEORY

Consider a layer of material specimen, irradiated from one side by microwave radiation with a frequency of 2450 MHz. The wave is a plane, harmonic one and impinges normally upon the material. In order to explain the principles of thermal runaway, the simplest possible system was conceived. The slab was located in free space, so no other waves than the incident one would be involved. Solving Maxwell's equations (see, e.g., Stratton[7] or Ayappa *et al.*[8]) yields the classical wave equation in one dimension

$$\frac{d^2E}{dx^2}+k^2(T)E=0, \tag{2}$$

where the electric field E is a function of position x and temperature T at that position. The $k(T)$ stands for the temperature dependent complex wavenumber, which is connected to the permittivity ϵ of the medium by the following equation:

$$k^2=\omega^2\mu_0\left(\epsilon+\frac{i}{\omega}\frac{\partial\epsilon}{\partial t}+\frac{i\sigma}{\omega}\right). \tag{3}$$

This equation takes into account that the permeability of the irradiated object almost equals μ_0, being the permeability in vacuum. Many materials treated by microwaves fulfill this requirement. The symbol σ is the ohmic electric conductivity, caused by the free charges of the object (free electrons, ions, etc.). The presence of free charges is not without consequences. The electric field causes surface charges and surface currents, which have their impact on the electromagnetic boundary conditions. The analytical description of a mixture of an isolator (dielectric) and conductor is nearly impossible without any numerical assumption of the ohmic conductivity. For this reason, the ohmic conductivity is neglected, as is usually done in this kind of research, at least with respect to the electromagnetic boundary conditions. Therefore, this article deals with pure dielectrics. With this assumption the electromagnetic boundary conditions read

$$E_1=E_2;\quad \frac{dE_1}{dx}=\frac{dE_2}{dx} \tag{4}$$

at the boundaries $x=0$ and $x=L$ (L is the thickness of the slab). The subscript 1 refers to vacuum or air and the subscript 2 is related to the irradiated medium.

The second term of Eq. (3) is very interesting because it can be written as the product of $d\epsilon/dT$ times dT/dt. In the case of thermal runaway the temperature increases rapidly and this could result in a significant contribution of the second term during the temperature jump. On the other hand, this term is small compared to the first term ϵ of Eq. (3), because it is divided by ω ($2\pi\times 2450\times 10^6\ \mathrm{s}^{-1}$). Including the second term in Eq. (1) of the absorbed power D results in

$$D=\frac{1}{2}\omega\left(\epsilon''+\frac{1}{\omega}\frac{d\epsilon'}{dT}\frac{dT}{dt}\right)|E|^2, \tag{5}$$

where ϵ is written as the difference of a real and an imaginary part, according to $\epsilon=\epsilon'-i\epsilon''$. For foodstuffs and also for ceramics, the temperature dependence of ϵ' is so small, that Eq. (5) might be replaced by the familiar Eq. (1) without any loss of generality. The second term has no consequences for the absorbed power. The interesting impact of the second term is found in the expression of the real wavenumber, or phase constant α. Omitting the small terms it yields

$$\alpha=\omega\sqrt{\frac{\mu_0}{2}\left(\epsilon''+\epsilon'-\frac{1}{\omega}\frac{d\epsilon''}{dT}\frac{dT}{dt}\right)}. \tag{6}$$

For water ϵ'' decreases with increasing temperature, resulting in a decreasing wavenumber or increasing wavelength. At certain temperature the wavelength ''fits'' within the slab, causing resonance. Exactly at that moment the temperature will rise strongly and the time dependent term in Eq. (6) will become significant in an interesting way. It will resist the decrease of α, keeping the system in the resonant mode. This effect has been investigated with the aid of computer simulations applied on water. The conclusion is that compared to ϵ'', the time dependent term is just too small to have any influence. Perhaps in the case of ceramics, where the temperature dependence of the permittivity is much larger, there will be some effect. To calculate the temperature within the slab as a function of position and time the three electromagnetic Eqs. (1), (2), and (4) have to be combined with Fourier's law.

$$\rho C_p\frac{\partial T}{\partial t}=K\frac{\partial^2 T}{\partial x^2}+D, \tag{7}$$

where ρ is the density, C_p is the thermal capacity, and K is the thermal conductivity. Although these parameters depend upon the temperature T, they are assumed constant for the following reason. The phenomenon of runaway is caused by resonance due to the temperature dependence of the wavelength within the slab, as will be shown later in this article. The temperature dependence of the three parameters mentioned above has no influence on the appearance of the resonance. The only consequence which can be found, is a small shift of the average temperature for which resonance occurs. The heat balance in Eq. (7) has to be accomplished by its boundary conditions. For small temperature differences between the surface of the slab and the ambient read

$$K\frac{\partial T}{\partial x}=h(T-T_a),\quad x=0, \tag{8}$$

$$K\frac{\partial T}{\partial x}=-h(T-T_a),\quad x=L, \tag{9}$$

where h is some effective heat transfer coefficient related to the convective and radiative heat loss. T_a is the ambient temperature and also the initial temperature of the slab at $t=0$ s.

III. STEADY STATE SOLUTIONS

The heating process evolves as follows: while the temperature T increases, the electrical properties k and ϵ'' of the slab will change. This, in turn influences the absorbed power D, resulting in a new temperature. A complete analytical solution of this nonlinear problem is nearly impossible. Only a partially analytical solution can be formulated. In order to formulate this solution the slab has to be divided into a large number of small subslabs. The heating process starts with heating of an isothermal slab at $t=0$ s. The analytical description and the solutions of this process is well known. Thus at $t=\Delta t$ the temperature at each position x within the slab is known, where Δt is small. Assuming each subslab n is isothermal it is possible to calculate the electric field E_n in each subslab n with these data

$$E_n(x,T)=A_n e^{ik_n(T)x}+B_n e^{-ik_n(T)x}. \tag{10}$$

This is the solution of Eq. (2). The integration constants A_n and B_n depend on the temperature of subslab n and the temperature of its neighbors, the subslabs $n+1$ and $n-1$. With the aid of the electromagnetic boundary conditions it is possible to write (A_n,B_n) as a function of (A_{n+1},B_{n+1}) or as a function of (A_{n-1},B_{n-1}). This process of replacing the integration constants by its neighbors goes on until the surfaces of the slab has been reached. At $x=0$ and $x=L$ the boundary conditions are numerically known. This means that the value of A_n and B_n for each subslab n is known. Substituting field (10) in the expression of the absorbed power, generates new temperatures for each subslab at $t=2\Delta t$, and so on. This is how the computer program has been set up.

This computer program has been applied to demineralized water, bounded with a gel. See Kaatze[9] for the dielectric properties. Other data which have been used are: $C_p=4186$ J/kgK, $K=0.7$ W/mK, $\rho=1000$ kg/m^3, and $h=100$ W/Km^2. The thickness of the slab must be chosen. It is the only degree of freedom. In the case of an isothermal slab the thickness L plays a very important role. There is a kind of standing wave within the slab if $L=n\pi/\alpha$ $(n=1,2,3...)$. However, not every standing wave causes thermal runaway. Thermal runaway is possible, but only if the temperature at which the standing wave occurs has been preceded by a lower temperature at which the waves more or less cancel each other. These conditions, combined with an approximated formula for α as a function of temperature ($\alpha=470-0.9T$; T in °C) results in thicknesses where the phenomenon of thermal runaway should be possible. The minimum thickness for runaway in an isothermal slab is about 1.6 cm $(n=2)$. In the case of $L=4$ cm, the effect of runaway is very pronounced $(n=5)$. The behavior of a thick slab, $L=10$ cm, where two temperature jumps $(n=13,14)$ are expected, is also interesting.

For these reasons nonisothermal slabs with a thickness of 1.6, 4.0, and 10.0 cm have been investigated. The steady-state temperature profiles as a function of the microwave

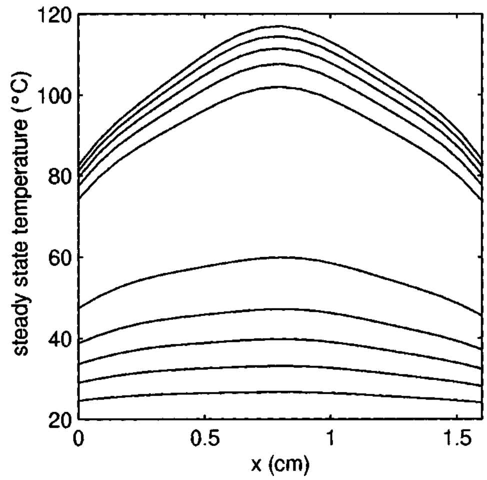

FIG. 1. Steady-state temperature profiles in a 1.6 cm slab of water. The microwave power increases from 4 to 40 kW/m^2 in steps of 4 kW/m^2. Notice the large gap, illustrating the phenomenon of thermal runaway.

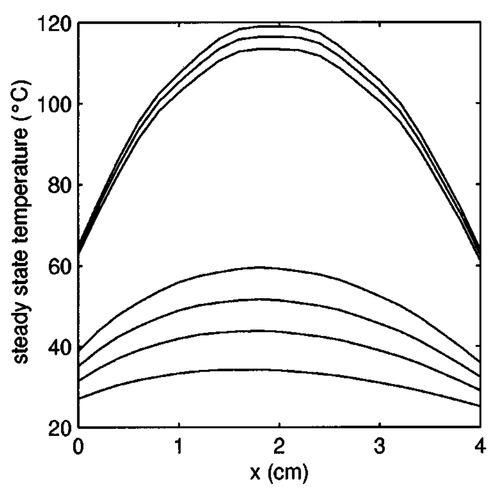

FIG. 2. Steady-state temperature profiles in a 4 cm slab of water. The microwave power increases from 4 to 28 kW/m^2 in steps of 4 kW/m^2. Notice the large gap, illustrating the phenomenon of thermal runaway.

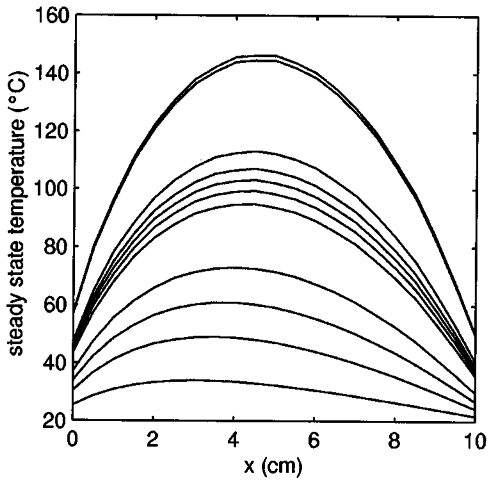

FIG. 3. Steady-state temperature profiles in a 10 cm slab of water. The microwave power increases from 2 to 22 kW/m^2 in steps of 2 kW/m^2. Notice the two gaps, illustrating the phenomenon of thermal runaway.

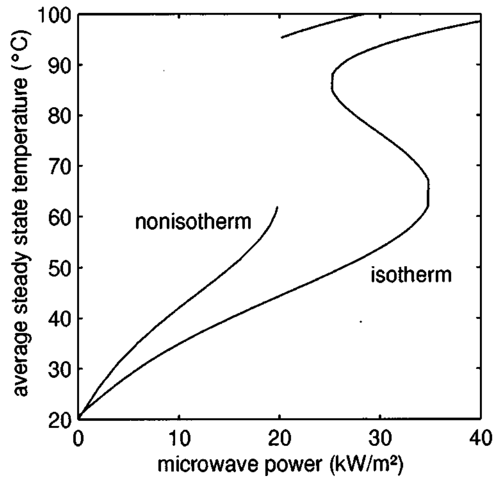

FIG. 4. Steady-state response curves for an isothermal and nonisothermal slab of water ($L=4$ cm). The graph of the nonisothermal slab is in fact an S-shaped curve.

power have been plotted in Figs. 1, 2, and 3. The jumps in temperature can be seen clearly and the conclusion is obvious. The behavior of the nonisothermal slab with regard to thermal runaway is the same as the behavior of the isothermal slab. The Biot number does not play a role. Even in the case of the relatively thick slab of 10 cm ($\mathrm{Bi}=hL/K=14{,}3$) the origin of thermal runaway is resonance once again. A plot of the steady-state temperature at every position x within the slab versus the microwave power will show an S-shaped curve or a multi S-shaped curve. The behavior of the complete slab can be described with the average temperature. To have a standing wave the only thing which counts is the way the wave fits within the slab. It makes no difference if the wavelength is large in the middle and small at the sides of the slab due to the temperature differences within the slab. If the wave fits then, there is resonance. This is one of the reasons why the slab, with respect to runaway, can be described with the average temperature. Replacing the local temperature T by the average temperature $\overline{T}$ in Eq. (7), and integrating over x yields

$$\rho C_p L \frac{d\overline{T}}{dt} = K\left(\frac{d\overline{T}}{dx}\right)_L - K\left(\frac{d\overline{T}}{dx}\right)_0 + D_{\mathrm{tot}}, \tag{11}$$

where D_{tot} is the total amount of absorbed power within the slab. Substitution of the thermal boundary conditions (8) and (9) results in

$$\rho C_p L \frac{d\overline{T}}{dt} = -2h(\overline{T}-T_a) + D_{\mathrm{tot}}. \tag{12}$$

This is the heat balance of an isothermal slab in free space. Figure 4 shows the S-shaped curve of the approximated solution of Eq. (12), compared to the computational analysis of the correct Eq. (7) for $L=4$ cm. The correct curve shows the upper and lower branch of a S-shaped curve. If one starts the heating with the initial condition that the temperature of the slab equals the ambient temperature Eq. (7) has no physical or mathematical solution in the instable region. It does demonstrate the dramatic jump in temperature. The complete correct S-shaped curve will be found if one equals Eq. (7) to zero. The smaller the Biot number, the better the approximation, as has been proven by Kriegsmann, but the main conclusion, that the origin of thermal runaway in a nonisothermal slab (regardless of the value of the Biot number), is caused by resonance, still remains.

IV. TIME DEPENDENT SOLUTIONS

That the phenomenon of thermal runaway can be described so successfully by regarding it as an isothermal slab with one temperature $\overline{T}$, while the real slab is not isothermal at all, needs a more thorough explanation. The reason can be found in the specific character of water. The dielectric loss factor of water decreases by increasing temperature, while the penetration depth increases. These two factors are responsible for the fact that the absorbed power becomes constant in the average very soon. Only the small temperature dependence of the wavelength remains and this can change the absorbed power significantly because of the resonance. These phenomena have been illustrated in Fig. 5. In the case of ceramics the dielectric loss factor increases and the penetration depth decreases by increasing temperature. The behavior of ceramics will be opposite to that of water. While water tends to smooth the electromagnetic energy over the whole slab, ceramics will concentrate the energy on the hot spot at the start of the heating process. Here it becomes clear why the hand-waving argument mentioned in the introduction cannot be ignored and replaced by the argument of resonance only.

The steady-state solutions of the former chapter are interesting because they give a good insight into the origin of thermal runaway. On the other hand, in the industrial processing of foodstuffs, one does not usually wait until the steady state has been reached. It just takes too long; if one heats a 4 cm slab of water with a microwave power of 18 kW/m^2 under the circumstances as described in this article, it will take about 2.2 h before the steady state ($\overline{T}=55\,°\mathrm{C}$) has been reached. Using 200 kW/m^2 instead of 18 kW/m^2 will

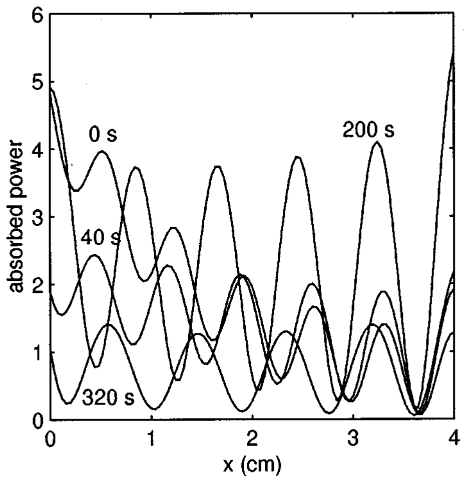

FIG. 5. Evolution in time of the absorbed power $(\mathrm{MW/m^3})$ in a 4 cm slab of water. The exponentially decreasing function $(t=0\ \mathrm{s})$ evolves to a constant oscillation $(t=40\ \mathrm{s}$ and $t=320)$. The amplitude of this oscillation increases strongly at $t=200$ s, because of the resonance. The microwave power is 200 $\mathrm{kW/m^2}$.

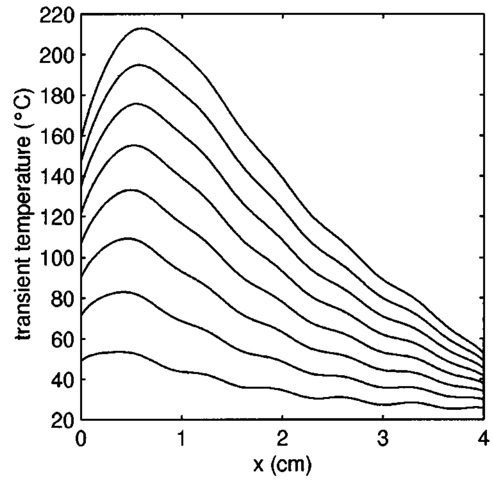

FIG. 7. Same as Fig. 6, except the temperature dependence of the permittivity has been omitted.

result in an average temperature of 55 °C within 2 min. Figure 6 shows the temperature profiles as a function of the heating time. As could be expected, resonance is still present. At a certain moment the temperature rises strongly in a very short time. Suppose this jump in temperature is not wanted because it will overtreat the foodstuff, then the only way to prevent this kind of thermal runaway is by stopping the microwave heating at the right moment. Figure 7 shows the temperature profiles of water neglecting the temperature dependence of the dielectric constant, assuming that the value of the dielectric constant at 20 °C will result in a rather correct plot of the profiles. As can be seen, this is not the case. Not only the jump in temperature is missing, but the smoothing of the absorbed power by increasing temperature has also not been described. This demonstrates that the method of neglecting the temperature dependence of the dielectric constant yields a very poor approximation. The investigation of the uneven spatial absorption of the electromagnetic energy within the irradiated object (one of the main problems in the application of microwave heating) by omitting the temperature dependence of the permittivity has little value, at least with respect to watery objects.

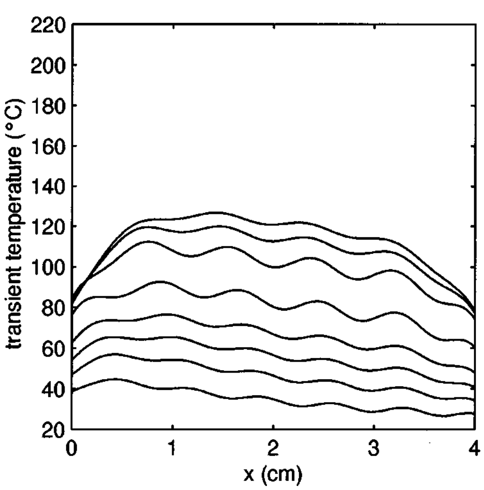

FIG. 6. Transient temperature profiles in a 4 cm slab of water. The time increases from 0 to 320 s in steps of 40 s. Notice the gap between $t=200$ and $t=240$ s. The microwave power is 200 $\mathrm{kW/m^2}$.

V. CONCLUSIONS

The behavior of the nonisothermal slab in relation to thermal runaway is analogous to the behavior of the isothermal slab. The Biot number does not play a role. Even in the case of relatively large Biot numbers, the physical origin of thermal runaway is still the phenomenon of resonance. A plot of the steady-state temperature at any position within the slab, versus the microwave power, will be an S-shaped or a multi S-shaped curve.

It is possible to approximately describe the complete nonisothermal slab regarding thermal runaway by its average temperature similar to the description of the isothermal slab. This means that a nonisothermal slab with thickness L, irradiated from one side by microwaves, will never be overtreated or damaged by the phenomenon of thermal runaway if L is smaller than $\pi/4\Delta\alpha$, where $\Delta\alpha$ is the difference between the maximum and the minimum value of the phase constant α within the temperature interval of the heating process. For water, this results in a dimension L of about 1 cm. The main reason for the similarity between isothermal and nonisothermal slabs can be found in the specific character of water, where the permittivity decreases by increasing temperature. The absorbed power within the slab will be small at hot spots and large at cold spots. The result is an almost constant absorption of energy over the whole slab. Only the wave character remains, causing runaway. This smoothing of the absorbed power during the heating process is very dominant. The calculations where the temperature dependence of the dielectric constant has been omitted, yield temperature profiles which substantially differ from the profiles where the temperature dependence has been taken into account.

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