APPLIED MACROECONOMICS

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CONTENTS

PREFACE

This book is meant for students in the second year of their Economics study. It combines the regular macroeconomics theory with spreadsheet applications. The necessary assistance for the use of the spreadsheet programme and a set of data are also provided in the book.[1]

The contributions of Roel Jongeneel from Wageningen University and Frans de Vries from Groningen University are gratefully acknowledged.

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1The spreadsheet programme used here is Excel.

CHAPTER 1: INTRODUCTION[1]

Study objectives
- To learn the distinction between micro- and macroeconomics;
- To learn about the targets and instruments of economic policy;
- To acquire familiarity with Tinbergen's view on economic policy;
- To learn to solve simple economic policy models by means of matrix analysis;
- To learn about the various indicators for macroeconomic activity.

1.1 Background

macro-economics

Macroeconomics is the study of aggregate behaviour in an economy; it is defined as a study of the national economy as a whole and/or of the interaction between economies of various countries. So, the focus is on aggregates like inflation, total unemployment, total consumption et cetera.

partial versus integral analysis

Distinguishing between macroeconomics and microeconomics is easy since microeconomics makes use of partial analysis. For example, in microeconomics the income of the consumer is assumed to be fixed. In macroeconomics, however, the ultimate aim is often towards integral analysis, where the influence of economic variables on each other is studied as thoroughly as possible.[2] As we shall see from the following, in macroeconomics the level of consumption influences the level of national income, while at the same time the level of national income determines the level of consumption. Examples of this integral school of thought will be encountered frequently in the following chapters.

The objective of this book is to emphasize the solving of macroeconomic problems. But we have to bear in mind that macroeconomics is in reality useful not only for *solving* problems. It is also a science which attempts to give fundamental insights into economics. However, the latter aspect is not the central theme of this book.

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1 This chapter, especially Sections 1.2 and 1.3, is partly based upon Chapter 1 in: E.C. van Ierland (ed.), W.J.M. Heijman, E.P. Kroese and E.A. Oskam, 1994. *Grondslagen van de macro-economie*. Stenfert Kroese, Houten.

2 This does not imply that macroeconomics and microeconomics are opponents. There is no conflict between macroeconomics and microeconomics. Instead, they differ by spotlighting different relationships (Gordon, 1993, p.5).

This chapter is organized as follows. In Section 1.2 we start examining the targets of economic policy. Then the instruments of economic policy are discussed in Section 1.3. Section 1.4 is focused on Tinbergen's view on economic policy. Finally, Section 1.5 provides some definitions regarding the valuation of economic activity.

1.2 Targets of economic policy

targets and instruments

If we restrict our attention to solving macroeconomic problems, it is important to formulate the targets of economic policy. We can then examine which policy instruments are available to the authorities in order to achieve these targets. In this section we will discuss the targets of economic policy.

Even though opinions about the targets of economic policy may vary between individuals and between different political parties, a certain agreement has been reached over the years. Almost every economist, government civil servant, and politician agree - roughly speaking - on the following targets:

1. An acceptable level of economic growth;
2. Full and fulfilling employment;
3. A fair distribution of income;
4. A stable price level;
5. A stable exchange rate;
6. Equilibrium on the balance of payments;
7. A good environmental quality.

An acceptable level of economic growth

income

An acceptable level of economic growth is important because it is an indication of how many goods and services are available to us. If the income per capita is too low, then the result is poverty and we are unable to provide for the basic necessities of life. What in fact is an 'acceptable' level of economic growth is difficult to define. Some think the level of income per capita in the industrialized countries is high enough, others would rather see it raised.

Full and fulfilling employment

employment

Full employment is one of the targets because it is important that everyone who wishes to work is able to do so. People can develop their individual talents and earn their own income. The concept 'fulfilling' indicates the quality of employment. It is vital that working conditions are agreeable so that an employee can work with pleasure under optimum conditions. This means, for example, that noise and smell are kept to an absolute minimum and that there is no exposure to dangerous materials.

A fair distribution of income

distribution of income

A fair distribution of income is one of the targets because it is not considered acceptable if someone, through no fault of their own, is unable to work or can only earn very little. A fair distribution of income is achieved in most countries through taxes, supplementary benefits, subsidies, and social security payments.

A stable price level

stable price level

A stable price level is necessary because if prices fall too low (deflation) or rise too high (inflation), the economic process is adversely affected. For the inhabitants of the country and for their trading partners abroad it is important to know that prices are more or less stable and are not going to rise or fall too suddenly. Therefore, the monetary authorities (usually the Central Bank and the Minister of Finance) attempt to maintain a stable price level.

A stable exchange rate

exchange rate

A stable exchange rate is important for international trading. Countries import goods and services from abroad and export goods and services to other countries. During these transactions different currencies need to be exchanged. If these exchange rates fluctuate too much, manufacturers participating in international trade transactions become uncertain about the value of the goods they are selling or buying from abroad. This is of course a serious obstacle for international trade.

Equilibrium on the balance of payments

balance of payments

Equilibrium on the balance of payments roughly means that the value of goods and services exported by a country should, in the long run, be about equal to the value of goods and services imported by the same country. If, for example, a country imports far more goods and services from abroad than it exports, then the foreign debt will continue to rise. This is a situation in which many developed and developing countries are finding themselves. In the long run poor and rich countries must stabilize their balance of payments.

A good environmental quality

environmental quality

Since the 1960s, protection of the environment has been added to the list of targets. Economic activity is accompanied by overcrowding and pollution of soil, water, and air, which cause considerable damage to the environment and to ecosystems. For the present generation as well as for future generations, it is of paramount importance that the environment is sufficiently protected. Environmental measures demand a greater labour effort and extra capital to enable more environmentally friendly products to be manufactured. This generally means an increase in manufacturing costs. Macroeconomics is also concerned with the consequences of environmental policy for employment, balance of payments, and inflation.

conflicting targets

Unfortunately, it is difficult to achieve all the targets mentioned above simultaneously in practice. Targets are often conflicting, that is to say improving one target results in the worsening of another. Here are two examples.

high level of income vs low inflation

If we attempt to achieve a high level of income and low unemployment this often results in tension on the labour market because the work force is too limited to carry out the work necessary to achieve this high level of income. In such cases, employees demand more money from the employer and frequently wages are increased, resulting in higher manufacturing costs and a rise in prices. We now have inflation which according to target 4 (a stable price level) is exactly what we are trying to avoid.

high income vs the environment

If we try to achieve a high level of income, we will use a considerable amount of raw materials and energy, which generally will result in an increase in environmental pollution. Target 1 (an acceptable level of income) improves, however target 7 (a good environmental quality) is adversely affected.

macroeconomic instruments

Having discussed the targets of economic policy, we can now consider the question of policy instruments that are available in order to achieve these targets. In other words, what macroeconomic policy measures can we take to allow the economic process to run in such a way that the targets can be achieved as efficiently as possible? In macroeconomics these possible policy measures are often collectively known as the instruments of economic policy.

1.3 Instruments of economic policy

adjusting the economic process

If the outcome of the economic process is not compatible with the targets of macroeconomic policies as formulated by the government, then they will look for ways of adjusting the economic process. The ministers responsible, together with the monetary authorities, would investigate which policy measures would be most suited to guide the process along the right lines again. For example, suppose that unemployment is too high. In such a case the cause as well as the measures which could be taken in order to lower unemployment would be investigated. Countless policy options would be available, for example:

alternative policies

- to raise the level of government expenditure so that economic activity would expand and more jobs would become available;
- to lower wages in order to reduce production costs which would make the economy more competitive abroad;
- to shorten the average working week so that employment opportunities could be distributed more widely;
- to circulate more money (monetary financing) in order to encourage consumption and investment. In this way economic activity would flourish and new businesses would be set up, thus expanding employment opportunities.

applying the instruments

From this simple example it can be seen that there are various instruments of economic policy that the government can apply in order to solve macroeconomic problems. An economy that is not functioning well is often compared with a sick patient. As in medicine, one first looks at the specific problem (a diagnosis is made), then a decision on how the condition should be treated is taken (the therapy is decided upon). As an analogy, economists decide what is wrong with the economy; which targets are not being achieved.

optimum policy mix

time inconsistency

The time factor is of paramount importance. In economics it is not only essential to choose the right combination of instruments (optimum policy mix), but also the right moment at which to apply them. In this regard time inconsistency is a significant issue. Time inconsistency decribes the temptations of policy makers to deviate from a policy once it is announced and private decision makers have reacted to it.[3] The basic idea is that discretionary policy makers decide on policy *A* because it is optimal at that time, and private decision makers make consumption, investment, and labour supply decisions based on that

3 See Gordon, 1993, p.490.

policy. However, once private decision makers have done so it may be optimal for policy makers to shift to policy *B*, thus invalidating the expectations on which private decision makers acted.

By now the reader will be intrigued to know what the instruments of macroeconomic policy are. Even though the full role of the instruments will not be immediately apparent, we will give a brief summary of those most used in macroeconomic policy. This is intended just as an introduction, because the rest of this textbook is primarily occupied with the question of how the various instruments influence the economic process and which instruments should be used under which conditions.

economic system

Macroeconomic policy can be divided into demand policy and supply policy. In order to describe the differences between the two, it is important to distinguish between the following aspects of the economic system:

- the economic order;
- the economic process;
- the economic structure.

economic order

The economic order describes how decisions are made in an economy and how these decisions are coordinated. Here four ideal typical categories of economic order are distinguished, namely decision making based on:

- the market mechanism;
- the democratic mechanism;
- the bureaucratic mechanism;
- price manipulation.

decentralized decision making

The first two are examples of decentralized decision making, whereby all members of society are involved. If decisions are made by means of the market mechanism, the final result of the economic process will depend on decisions made by consumers and producers about the purchasing of goods and services and about using production factors. If the democratic mechanism is applicable then decisions are made on the basis of policy proposals that are then agreed upon in parliament.

centralised decision making

The bureaucratic mechanism and the system of price manipulation are examples of centralized decision making, whereby decisions are made at central government level and realized by giving orders and having them carried out (bureaucratic mechanism) or by levying taxes and providing subsidies (price manipulation). In reality, there is never any question of a pure form of the ideal typical categories mentioned above, but there is always a blend that contains two or more of the categories of economic order mentioned.

economic process

The economic process is concerned with actual economic acting between people and is related to both the production process and the consumption process. The economic process describes the continuing process of the production and consumption of goods and services during a specific period of time, how large the demand for goods and services is and how many people are taking part in the production process. In particular, the economic process describes the path of the variables within a specific period of time. It is particularly devoted to flows, such as national income, consumption, investment, exports, and imports.

economic structure

The economic structure concerns all the variables that determine the potential of an economic system. A strong economic structure indicates that an economy has numerous possibilities of supplying goods and services to the markets at competitive prices. Essential to the economic structure of a country are the following supply elements: the number and quality of the professional labour force, the range and composition of capital goods, and the supply of raw materials. The climate, geographical situation, and technological capabilities are also often included in the economic structure. The state of technology is especially concerned with the supply side of the economy. The economic structure largely determines whether a country is able to create many or only a limited number of goods and services.

supply side and demand side

In contrast to the supply side of the economy it is the demand side that determines the extent to which the goods and services produced can be sold. If the demand for goods and services is inadequate we speak of a recession. Factories are left with unsold stocks and production capacity in a recession is only partially utilized. We speek of a 'boom' period if the demand for goods and services is so big that the factories can hardly keep up with production to satisfy demand.

demand policy

The instruments of macroeconomic policy include both demand policy and supply policy. Demand policy focuses on how to bring the production of goods and services to such a level that full employment and full utilization of the production capacity can be achieved. This can be done by varying tax levies or altering the level of government spending. Demand policy is especially related to the short and medium-long run.

supply policy

Supply side policy aims at strengthening the productive capacity of an economy (economic structure). Supply policy is, therefore, especially concerned with the education of the professional workforce, the range and composition of capital goods, the detection and exploitation of minerals, and technology. Supply policy is often expressed as a long run policy because the measures only take effect after a prolonged period of time. Important instruments of supply policy include investment subsidies, the reduction of frictions on the labour market (including training programmes), and the promotion of research programmes to develop new technologies. We can divide the instruments of macroeconomic policy as follows:

- fiscal policy;
- monetary policy;
- income and price policy;
- other instruments of macroeconomic policy.

fiscal policy

Fiscal policy is related to income distribution and government spending. On the income side, it is concerned with levying taxes, especially regarding the height of the various levies and what sort of taxes should be implemented (for example income tax, corporate tax and value added tax -VAT-). On the spending side the question is what the level of government spending should be and where the money should be spent. The government has many responsibilities and can influence the economic process by its spending pattern. The government can also provide subsidies or can, for example, invest in infrastructure, coastal defence, or school facilities. A separate feature of fiscal policy concerns the question of what the government should do if its income is higher than its spending or vice versa. In the first case there is a government deficit and in the second a government surplus. Fiscal policy aimed at influencing the demand for goods and services is a typical example of demand policy.

monetary policy

Monetary policy is especially related to the total amount of money circulating in the economy, the value of money (inflation and deflation), the exchange rate, and the interest rate. We make use of money daily. As we shall see, the central bank has various instruments at its disposal for regulating the total amount of money circulating in the economy.

circulation of money

The value of money (measured in the quantity of goods and services that can be bought for a monetary unit) is closely related to the total amount of money being circulated. If the central bank continues to print more bank notes and to circulate them whenever more credit is required (borrowing money from banks by the public), then the total amount of money in circulation would increase. If the number of goods and services being produced do not increase at the same rate, then the value of money would fall rapidly. In other words, we would then have inflation due to the rapid rise in the price of goods and services. By applying a rigourous monetary policy whereby the annual rise in the amount of money available is restricted, the central bank is able to maintain the value of the money.

exchange rate, devaluation and revaluation

The value of a currency compared with other currencies is called the exchange rate. As we have already seen, this exchange rate plays an important role in international trade. The central bank is able to influence the exchange rate (within certain limits), for example by devaluation (lowering the value of its own currency) or revaluation (raising the value of its own currency) compared to other currencies. By doing so, the central bank influences the economic process. The exchange rate is therefore one of the instruments of monetary policy.

interest

If you borrow or lend money, it is usual to either pay or receive interest. The interest rate in an economy is of considerable importance with respect to the strategy of the economic process. As we shall see, by applying monetary policy the central bank is able to influence the interest rate which in turn can either stimulate investment (in the case of low interest rates) or suppress it (in the case of high interest rates).

income and price policy

Income and price policy is an important policy instrument for the government. Labour costs play an important role in the economic process, because these costs are a substantial part of the manufacturing costs of a company.[4] A sharp rise in labour costs will eventually be calculated into the price of the product resulting in a tendency towards inflation. Rising prices are bad for international competition and an income policy, whereby the government curbs the annual rise of nominal wages per employee, can be helpful in order to achieve full employment. If income and prices rise sharply and unions and employers are unable to break through the so-called income and price spiral, a sensible income and price policy is needed.

other instruments

In addition to the instruments of economic policy mentioned above the government has a range of other instruments at its disposal. These include legislation, subsidies, and taxes. By means of legislation the government is able to influence the economic process. In addition to the income and price policy, legislation applied to social security is important to the economic process. Another way of influencing the economic process is to improve education and training. It is also possible for the government, via state owned companies, to play a role in the production process or to expand its role by nationalizing certain

4 In reality this instrument must not be overestimated.

companies (placing under government control). It is clear that a range of instruments is available which can affect the economic process. This will be dealt with extensively in the following chapters.

centrally directed economies,

market economies, and

mixed economies

It is useful here to comment that economies directed by a central government, as was previously the case in eastern Europe, function differently compared to mixed western economies. In centrally guided economies, central planning plays an important role (bureaucratic mechanism) and many prices are fixed by the state (price manipulation). In western economies, free markets play a significant role (market mechanism) and the government adjusts the economic process using instruments of economic policy on the basis of democratic decisions (democratic mechanism). The macroeconomic theory discussed in this textbook is mainly concerned with mixed economies.

1.4 Tinbergen's view on economic policy

normative approach

Tinbergen was the first economist who analyzed the basic theory of economic policy systematically.[5] His approach for analyzing economic policy was to examine how policy makers *should* act. This can be referred to as the normative theory of economic policy. Tinbergen delineated the process of optimal policy making as follows:[6]

1. The policy maker must specify the goals of economic policy assuming that each country has its 'social welfare function' which the policy maker is attempting to maximize;[7]
2. The policy maker identifies the targets to be attained based on the social welfare function;
3. The policy maker must specify the instruments that are available to reach the targets;
4. The policy maker needs a model of the economy which links the instruments to the targets in order to be able to choose the optimal level of the various policy instruments.

5 Tinbergen J., 1970 (1952). *On the theory of economic policy.* North-Holland, Amsterdam.
6 This interpretation stems from: J.D. Sachs and F. Larrain, 1993. *Macroeconomics in the global economy.* Ch. 19. Harvester Wheatsheaf, New York.
7 A social welfare function specifies both the optimal levels of the target variables and the costs to society of deviations from those levels. The economy can, however, deviate from the optimum due to exogenous shocks, for example a change in tastes, a change in the terms of trade, a movement in the international interest rate etcetera. See also Sachs & Larrain (1993), p. 590.

linear model

For his analysis Tinbergen used a linear framework. Consider, for example, two targets T_1 and T_2, and two instruments: I_1 and I_2. Now assume that the desired levels of T_1 and T_2 are T_1^* and T_2^* respectively. It is assumed that the targets can be described as linear functions of the instruments:

$$T_1 = \alpha_1 I_1 + \alpha_2 I_2,$$
$$T_2 = \beta_1 I_1 + \beta_2 I_2. \tag{1.1}$$

We see that both instruments influence each target. The objective is to derive the desired levels of both targets. Substituting T_1^* and T_2^* into (1.1) gives:

$$T_1^* = \alpha_1 I_1 + \alpha_2 I_2,$$

$$T_2^* = \beta_1 I_1 + \beta_2 I_2. \tag{1.2}$$

Subsequently, (1.2) can be solved for I_1 and I_2 in terms of T_1^* and T_2^*:

$$I_1 = \frac{\beta_2 T_1^* - \alpha_2 T_2^*}{\alpha_1\beta_2 - \beta_1\alpha_2},$$

$$I_2 = \frac{\alpha_1 T_2^* - \beta_1 T_1^*}{\alpha_1\beta_2 - \beta_1\alpha_2}. \tag{1.3}$$

linear independency

A solution exists only if $\alpha_1\beta_2 - \beta_1\alpha_2 \neq 0$ or $\alpha_1/\beta_1 \neq \alpha_2/\beta_2$ (linear independency). If the conditions of linear independence are met then an economy reaches its point of maximum happiness (bliss point) if $T_1 = T_1^*$ and $T_2 = T_2^*$. This is referred to as the economy's *bliss point*.

n targets, n instruments

If $\alpha_1/\beta_1 = \alpha_2/\beta_2$, the two instruments have the same proportional effects on the two targets. In this case the policy maker has only one independent instrument with which to try to hit two targets. Mostly, this cannot be accomplished simultaneously. To put it generally: if in an economy with a linear structure, the policy maker has n targets, these targets may be reached as long as there are at least n linearly independent policy instruments.

an example

Let us now apply this to an imaginary example of a macroeconomic problem. Assume that there are two targets: output Q and price level P. Suppose that there are also two instruments: monetary policy M and fiscal policy G. The economy can now be described by:

$$Q = \alpha_1 G + \alpha_2 M,$$
$$P = \beta_1 G + \beta_2 M. \tag{1.4}$$

The coefficients α_1, α_2, β_1, and β_2 measure the quantitative effect of G and M on Q and P.

Assume that in order to reach the economy's bliss point inflation should be 0 $(\Delta P = 0)$ and production should increase by 2 $(\Delta Q = 2)$. Can this optimum be reached?

To analyze this problem we have to restate the problem in terms of deviations from a baseline.[8] If ΔX denotes the necessary deviation of variable X from its initial baseline level then the problem can be written as:

$$\Delta Q = \alpha_1 \Delta G + \alpha_2 \Delta M,$$
$$\Delta P = \beta_1 \Delta G + \beta_2 \Delta M. \tag{1.5}$$

Substituting the target values for ΔP and ΔQ in 1.5 gives:

$$2 = \alpha_1 \Delta G + \alpha_2 \Delta M,$$
$$0 = \beta_1 \Delta G + \beta_2 \Delta M. \tag{1.6}$$

Solving this for ΔG and ΔM gives:

$$\Delta G = \frac{2\beta_2}{(\alpha_1\beta_2 - \alpha_2\beta_1)},$$

$$\Delta M = \frac{-2\beta_1}{(\alpha_1\beta_2 - \alpha_2\beta_1)}. \tag{1.7}$$

Now, assume the following values for the coefficients: $\alpha_1 = 1$, $\alpha_2 = 2$, $\beta_1 = 0.5$, and $\beta_2 = 1.5$. Then the outcome is: $\Delta G = 6$, $\Delta M = -2$. So, in this specific case the two policy targets can be accomplished.

8 The baseline is the starting values of the variables.

matrix algebra We can derive the solution also by using matrix algebra.[9] With this tool it is easier to compute the solution in the case of more than two targets and more than two instruments.[10] We can rewrite the equations (1.6) by matrix notation as:

$$\mathbf{AX} = \mathbf{D},\ \text{where}$$

$$\mathbf{A} = \begin{pmatrix} \alpha_1 & \alpha_2 \\ \beta_1 & \beta_2 \end{pmatrix}, \mathbf{X} = \begin{pmatrix} \Delta G \\ \Delta M \end{pmatrix}, \mathbf{D} = \begin{pmatrix} \Delta Q \\ \Delta P \end{pmatrix}.$$

Rewriting this system yields:

$$\mathbf{X} = \mathbf{A}^{-1}\mathbf{D}. \tag{1.8}$$

In our numerical example, this leads to (see Appendix 1.1):

$$\begin{pmatrix} \Delta G \\ \Delta M \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 0.5 & 1.5 \end{pmatrix}^{-1} \begin{pmatrix} 2 \\ 0 \end{pmatrix} = \frac{1}{|\mathbf{A}|} \begin{pmatrix} 1.5 & -2 \\ -0.5 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

$$= 2\begin{pmatrix} 1.5 & -2 \\ -0.5 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 & -4 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 6 \\ -2 \end{pmatrix}.$$

From this we see that the same results are generated as with (1.7).

1.5 The value of economic activity

aggregation As we pointed out earlier, macroeconomics is the study of aggregate behaviour in an economy. Therefore, it is inevitable to know some key measures of overall economic activity in an economy, such as value added, gross domestic product GDP, gross national product GNP, and national income NI. These aggregate measures can be seen as the building blocks of macroeconomics.[11]

value added When defining an aggregate measure, the starting point is always the firm. One method used to derive the value of *all* final goods and services is the value added approach. In this approach, the total value added is the sum of the value added at each stage of production. The

9 For a concise overview of linear models and matrix algebra in combination with economics see for example A.C. Chiang, 1984.
10 Moreover, when we are dealing with more complicated models we will use a spreadsheet programme (Excel) to solve the models. In that case matrix algebra is unavoidable.
11 See: Sachs and Larrain, 1993, p.19.

value added of a firm equals the value of the firm's output less the value of the intermediate goods that the firm purchases. More generally: value added is the value of labour and capital services that take place at a particular stage of the production process. It is the sum of wages, interest, rents and profits in an economy.

Gross value added, net value added

Value added is divided into *gross value added* and *net value added*. Gross value added is the difference between the value of raw materials and the value of the final product. However, from value added some money must be set aside for the depreciation of machines, buildings, and other capital goods. When this amount is excluded we speak of net value added.

GDP

Next, we come to an important measure of production in the economy: GDP. This statistic measures the total gross value added produced in the geographic boundaries of an economy within a given period of time. Total net value added $(GDP$ minus depreciation) is referred to as Net Domestic Product NDP. Further, the domestic product $(GDP$ and $NDP)$ can be measured at market prices and at factor costs. At market prices, cost reducing subsidies and indirect taxes (VAT) are included. With domestic product measured at factor cost, these taxes are excluded.

GNP

The country's national income or national product is defined as the sum of all incomes earned by a country's residents in a certain period of time. It can be computed from the domestic product by adding up the net primary income (before tax) earned in the rest of the world. National income can be computed with depreciation included: Gross National Product (GNP) and with depreciation excluded: Net National Product (NNP). Furthermore GNP and NNP can be computed both at market prices and factor cost by including or excluding cost reducing subsidies and indirect taxes respectively.

Disposable National Income

If we take into account the net income transfers from the rest of the world we deal with the Disposable National Income. Table 1.1 presents the above mentioned indicators for The Netherlands for the year 1995.

nominal and real indicators

Finally, the distinction between nominal and real indicators is important. For example, the nominal GDP measures the value of goods and services according to their current market prices, while real GDP attempts to measure the physical volume of production. The real value of a particular variable is derived by dividing the nominal value by the deflator, so:

$$\text{real } GDP = \frac{\text{nominal } GDP}{GDP\text{-deflator}}, \tag{1.17}$$

where the $GDP-$deflator is usually the price level (price index divided by 100). For example, nominal GDP (market prices) in the Netherlands is 635.01 billion guilders in 1995 (see Table 1.1). The $GDP-$deflator for that year is 110.7/100 = 1.107 (1990=100). So, real GDP for 1995 is 635.01/1.107 = 573.63 billion guilders.[12] Table 1.2 gives an impression of the development of the real GDP at market prices in some countries.

Table 1.1: Production and income measures for the Netherlands in 1995 (mln guilders).

1	Compensation of employees (wages, salaries)	328,350
2	Taxes on production and imports less subsidies	70,240
3	Consumption of fixed capital (depreciation)	73,600
4	Operating surplus (interest, rents, profits)	162,820
	Domestic product	
5	GDP, market prices (1+2+3+4)	635,010
6	NDP, market prices (1+2+4)	561,410
7	GDP, factor costs (1+3+4)	564,770
8	NDP, factor costs (1+4)	491,170
9	Net primary income from the rest of the world	1,040
	National income (= National product)	
10	Gross, market prices (5+9)	636,050
11	Net, market prices (6+9)	562,450
12	Gross, factor costs (7+9)	565,810
13	Net, factor costs (8+9)	492,210
14	Net current transfers from the rest of the world	-8,410
	Disposable national income	
15	Gross, market prices (10+14)	627,640
16	Net, market prices (11+14)	554,040

Source: Statistics Netherlands (CBS), National Accounts 1995.

12 Statistics Netherlands (CBS), 1997. *Statistical Yearbook of the Netherlands: National Accounts.* Voorburg.

Table 1.2: GDP (real, market prices) in some countries (indices, 1990=100).

	1993	1994	1995
The Netherlands	105.1	108.7	111.0
Austria	105.3	108.5	110.5
Belgium	102.4	104.7	106.7
Denmark	103.1	107.6	110.4
Finland	88.6	92.5	96.4
France	100.6	103.4	105.7
Germany	113.1	116.4	118.9
Greece	103.0	104.5	106.6
Ireland	106.5	116.9	126.9
Italy	100.5	102.7	105.7
Luxembourg	105.0	108.4	111.9
Portugal	102.1	102.8	104.8
Spain	101.8	103.9	107.0
Sweden	95.3	97.8	100.8
United Kingdom	99.7	103.5	106.0
EU 15	103.4	106.3	108.9
Japan	105.2	105.7	106.6
United States of America	104.0	107.6	109.8
OECD total	104.1	106.8	108.8

Source: Statistics Netherlands (CBS), 1997. *Statistical Yearbook of the Netherlands 1997.* Voorburg.

Questions and exercises

1.1 Give two clear examples of conflicting macroeconomic targets.

1.2 Assume the following economic system:

$$Q = \alpha_1 G + \alpha_2 M,$$
$$P = \beta_1 G + \beta_2 M.$$

$\alpha_1 = 1$, $\alpha_2 = 2$, $\beta_1 = 2$, $\beta_2 = 3$. Further, the macroeconomic targets are: $\Delta Q = 3$, $\Delta P = 0$.

a. Formulate the system in terms of deviations from the baseline.
b. Formulate the system in terms of matrices.
c. Solve the problem numerically.

1.3 Assume that the following data for an economy are given: $GDP = 600$, indirect taxes minus subsidies: 40, depreciation: 40. price index number: 110. Compute the real NNP at factor costs.

Appendix 1.1

Assume that $\mathbf{A}$ is a two by two matrix:

$$\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix},$$

Then the determinant of $\mathbf{A}$, $|\mathbf{A}|$, equals $ad - bc$. Further the inverse of $\mathbf{A}$, $\mathbf{A}^{-1}$, equals:

$$\mathbf{A}^{-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{|\mathbf{A}|}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} = \frac{1}{ad - bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}.$$

CHAPTER 2: BASIC MACROECONOMIC MODELS

Study Objectives
- To solve basic macroeconomic models
- To learn the difference between *ex ante* and *ex post* investment
- To solve a simple dynamic macroeconomic model
- To study the Haavelmo effect
- To study the various effects of taxation
- To learn to compute the 'wedge'
- To acquire familiarity with matrix operations in a spread sheet
- To be able to solve simple macroeconomic policy models with a spread-sheet
- To show the interdependency between economies

2.1 The models

basic issues

In this chapter the central theme is to examine some basic issues regarding macroeconomic models. Therefore, we will provide a short review of standard macroeconomic Keynesian models and some of the effects of macroeconomic policy measures. Section 2.2 is concerned with the assumptions of a closed economy and fixed prices. Section 2.3 discusses the model of a closed economy with fixed prices and a government budget. In section 2.4 international trade is incorporated. Finally, in section 2.5 the interdependency between countries is studied in the framework of a two-country model.

2.2 Closed economy, fixed prices

closed economy

The simplest model with which we can start the analysis is that of a closed economy in which prices are fixed. Within a Keynesian framework this case can be modelled as follows:[1]

$$ED = C + I,$$
$$C = \gamma Y + \underline{C},$$
$$I = \underline{I},$$
$$Y = C + S,$$
$$I = S.$$

ED	Effective demand
C	Consumption
I	Private investment
Y	National income
S	National savings
γ	Marginal propensity to consume

1 Exogenous variables are underscored.

equilibrium condition

The equilibrium condition implies $Y = ED$. Solving this model we get:

$$Y = \frac{1}{1-\gamma}(\underline{C} + \underline{I}). \qquad (2.1)$$

multiplier

with $1/(1-\gamma)$ for the so-called *multiplier*. Figure 2.1 gives a geometrical presentation of the model.

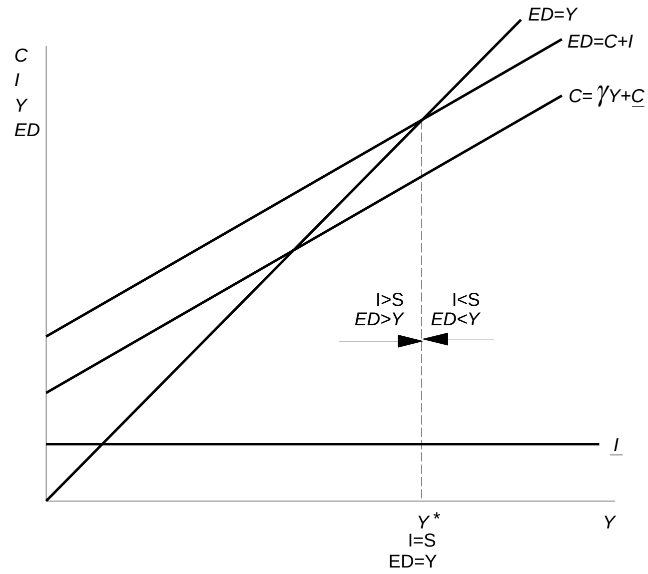

Figure 2.1: Closed economy without government.

income equilibrium

In figure 2.1 Y^* represents the income equilibrium. At this point effective demand equals production or income. To the left of Y^* effective demand ED exceeds production Y, or investments I exceed savings S. In this case producers observe that their stocks on hand decrease. They will respond to this by increasing production. To the right of Y^* effective demand is less than production. In this case producers observe an increase of their stocks on hand (reflected in increased investments) and will respond to that by lowering their production. From this it can be concluded that Y^* represents a stable equilibrium. The essential feature of a stable equilibrium is that when

stability versus unstability

it gets distorted, after a certain amount of time, it will be restored by some mechanism.[2] In this case the mechanism works as follows (with V for stocks on hand):

$$ED > Y(I > S) \Rightarrow V \downarrow \Rightarrow Y \uparrow,$$

$$ED < Y(I < S) \Rightarrow V \uparrow \Rightarrow Y \downarrow,$$

$$ED = Y(I = S) \Rightarrow V \text{ constant } \Rightarrow Y \text{ constant.}$$

With an unstable equilibrium the equilibrium is not restored after distortion.

simultaneous equations model

The above model is the most simple case of a simultaneous equations model. That is, the model consists of a system of equations. In such models variables are classified as endogenous and exogenous. Endogenous variables are determined by the model itself, while exogenous variables are determined from outside the model. So exogenous variables are in fact predetermined while endogenous variables are not.

reduced form

In the model outlined above both C and Y are endogenous. In fact, C affects Y and Y in turn affects C. However, this problem of endogeneity can be avoided if we rewrite the model in reduced form. This means that each endogenous variable is expressed in terms of exogenous variables. The reduced form of the model above becomes:

$$Y = \frac{\underline{C}}{1-\gamma} + \frac{\underline{I}}{1-\gamma},$$

$$C = \frac{\underline{C}}{1-\gamma} + \frac{\gamma \underline{I}}{1-\gamma}. \quad (2.2)$$

ex ante and ex post investments

Considering the model above and contrasting savings with investment, it is important to make a distinction between *ex ante* and *ex post* investments. This approach focuses on the fact that people who help create the gross national product inevitably earn an equivalent amount of income. In the model discussed above, people can spend this income on consumption goods or they can save it $(Y = C + S)$. To the extent that people buy consumption goods, a portion of the GNP that firms have produced is sold and out of the hands of the producing firms. To

2 This will be examined later on in this chapter.

the extent that people save, a portion of the GNP that firms have produced is not sold to households, but remains in the hands of the firms and is invested $(Y = C + I)$. So household saving is matched by the ex post investment of firms $(S = I)$.

Whether the situation is one of equilibrium depends on what portion of GNP firms want to retain. So it depends on their ex ante (desired) investment. If ex post investment equals ex ante investment then there are no unplanned changes in the inventories of firms.[3]

S=I or I=S

However, the relationships $I = S$ and $S = I$ seem to be equal, but actually there can be two different interpretations. In the former the central issue is the factors which determine investments. This can be called the classical view of investment and savings, with the capital market equalizing investment and savings. The latter can be called the Keynesian view. Keynes sees investment I as the independent variable: savings adjust. Investments do not depend entirely on savings but on other things like future expectations and new inventions.[4]

dynamics

Dynamics is of most importance as we see from the above. Therefore, we shall now give an illustration of how to solve a simple model in a continuous time framework. In this case the model can be written as:

$$ED = C + I,$$
$$C = \gamma Y + \underline{C},$$
$$I = \underline{I},$$
$$Y = C + S,$$
$$\frac{dY}{dt} = \alpha(ED - Y).$$

ED	Effective demand
C	Consumption
I	Private investment
Y	National income
γ	Marginal propensity to consume
$\frac{dY}{dt}$	Change in income

differential equation

Compared to the first model we have added the first-order differential equation dY/dt to it. This equation describes the changes in income over time. To find the solution of this equation we need to apply some mathematics. By means of substitution we get:

$$\frac{dY}{dt} = \alpha(\gamma Y + \underline{C} + \underline{I} - Y) = -\alpha(1-\gamma)Y + \alpha(\underline{C} + \underline{I}). \tag{2.3}$$

3 See Kohler, 1992.
4 See Pen, 1977, p. 83.

general solution The general solution for this differential equation is:[5]

$$Y(t) = \left(Y(0) - \frac{\underline{C} + \underline{I}}{1-\gamma} \right) e^{-\alpha(1-\gamma)t} + \frac{\underline{C} + \underline{I}}{1-\gamma}. \tag{2.4}$$

where $(1-\gamma) \neq 0$ because $0 < \gamma < 1$. Figure 2.2 presents (2.4) when: $Y(0) = 100$, $\underline{I} = 150$, $\underline{C} = 50$, $\alpha = 0.75$, and $\gamma = 0.8$.[6] When $t \to \infty$, $Y \to 1000$, which is the equilibrium value of income. The equilibrium value is the asymptotic value of the function, where income Y equals effective demand ED.

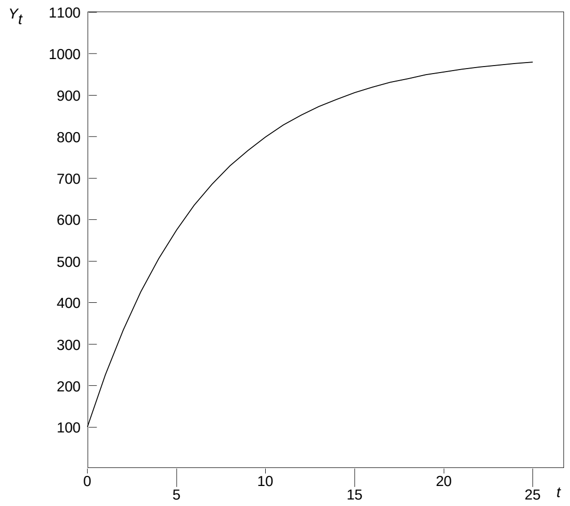

Figure 2.2: The dynamics of national income.

2.3 Closed economy, fixed prices, government budget

function of government The main function of the government in the economy is to levy taxes and to spend it for the benefit of the society. Taking that into account the model has to be adapted as follows:

$ED = C + I + G,$	ED	Effective demand
$C = \gamma(Y - T) + \underline{C},$[7]	C	Consumption

5 See Appendix 2.1.
6 The equation then becomes: $Y_t = -900e^{-0.15t} + 1000$.
7 The term $Y - T$ is also called disposable income.

$I = \underline{I},$	I	Private investment
$Y = C + S + T,$	Y	National income
$T = \underline{T},$	T	Taxes
$I + G = S + T,$	S	National savings
$G = \underline{G},$	G	Government spending
$B = T - G.$	B	Government surplus
$N_d = \alpha Y,$	N_d	Demand for labour
$N_s = \underline{N}_s,$	N_s	Supply of labour
$U = N_s - N_d.$	U	Unemployment
	γ	Marginal propensity to consume
	α	Labour coefficient

In the model the demand for labour N_d equals the coefficient α times production Y. The coefficient alpha is called the labour coefficient: the amount of labour needed to produce one unit of production. Rewriting the model gives:

$$Y = \frac{1}{1-\gamma}(\underline{C} + \underline{I} + \underline{G} - \gamma \underline{T}). \tag{2.5}$$

stimulating the economy

Now, assume that the government wants to stimulate the economy while leaving the budget surplus B unchanged. If the surplus is not to be changed, then:

$$\Delta B = \Delta \underline{T} - \Delta \underline{G} = 0 \quad \Rightarrow \Delta \underline{T} = \Delta \underline{G}. \tag{2.6}$$

Further:

$$\Delta Y = \frac{1}{1-\gamma}(\Delta \underline{C} + \Delta \underline{I} + \Delta \underline{G} - \gamma \Delta \underline{T}). \tag{2.7}$$

Assume that $\Delta \underline{C} = \Delta \underline{I} = 0,$ then:

$$\Delta Y = \frac{1}{1-\gamma}(\Delta \underline{G} - \gamma \Delta \underline{T}). \tag{2.8}$$

And:

$$\Delta Y = \frac{1}{1-\gamma}(\Delta \underline{T} - \gamma \Delta \underline{T}) = \Delta \underline{T} = \Delta \underline{G}. \tag{2.9}$$

Haavelmo effect This implies that income Y increases by the same amount as the increase in taxation and government spending. This effect is called the Haavelmo effect.

endogenous taxation In the last model taxation was completely exogenous. If the taxation is partly endogenous the taxation equation becomes:

$$T = \tau Y + \underline{T}. \qquad \tau \quad \text{tax rate}$$

Rewriting the model and assuming that $\Delta\underline{I} = \Delta\underline{C} = 0$ gives:

$$Y = \frac{1}{1-\gamma+\gamma\tau}(\underline{C} + \underline{I} + \underline{G} - \gamma\underline{T}),$$

$$\Delta Y = \frac{1}{1-\gamma+\gamma\tau}(\Delta\underline{G} + \gamma\Delta\underline{T}) \tag{2.10}$$

Moreover:

$$B = \tau Y + \underline{T} - \underline{G},$$

$$\Delta B = \tau\Delta Y + \Delta\underline{T} - \Delta\underline{G} \tag{2.11}$$

Equation (2.10) substituted in equation (2.11) gives:

$$\begin{aligned}\Delta B = \tau\Delta Y + \Delta\underline{T} - \Delta\underline{G} &= \frac{\tau}{1-\gamma+\gamma\tau}(\Delta\underline{G} - \gamma\Delta\underline{T}) + \Delta\underline{T} - \Delta\underline{G} \\ &= \left(\frac{\tau}{1-\gamma+\gamma\tau} - 1\right)\Delta\underline{G} - \left(\frac{\gamma\tau}{1-\gamma+\gamma\tau} - 1\right)\Delta\underline{T} \\ &= \frac{1-\gamma}{1-\gamma+\gamma\tau}\Delta\underline{T} - \frac{(1-\gamma)(1-\tau)}{1-\gamma+\gamma\tau}\Delta\underline{G}.\end{aligned} \tag{2.12}$$

So, if we assume that stimulating the economy does not change the budget surplus $(\Delta B = 0)$, then from (2.12) it follows:

$$\Delta\underline{T} = (1-\tau)\Delta\underline{G}. \tag{2.13}$$

And from (2.10) and (2.13) it follows:

$$\Delta Y = \frac{1-\gamma(1-\tau)}{1-\gamma+\gamma\tau}\Delta\underline{G} = \Delta\underline{G}. \tag{2.14}$$

So, also in this case the Haavelmo effect occurs.

two targets, two instruments

Now, assume that the government has two targets: full employment and a balanced government budget. Indicating the targets with a hat: $\Delta B = \Delta\hat{B}$, $\Delta U = \Delta\hat{U}$. In this case we also have two instruments (indicated with a tilde): public spending $\Delta\underline{G} = \Delta\tilde{G}$ and taxation $\Delta\underline{T} = \Delta\tilde{T}$. The Tinbergen condition for solving this problem is fulfilled. The solution procedure is as follows. We know the reduced form equation for the case of a partly endogenous tax revenue (2.10). We also know that $\Delta\hat{B} = \Delta T - \Delta\tilde{G} = \tau\Delta Y + \Delta\tilde{T} - \Delta\tilde{G}$, and $\Delta\hat{U} = \Delta\underline{N}_s - \alpha\Delta Y$. Writing the target variables and exogenous variables on the right hand side and the endogenous and instrument variables on the left hand side of the equal sign, the problem can now be formulated as follows (with all variables written as deviations like ΔY, $\Delta\tilde{G}$ etc.):

$$\Delta Y - \frac{1}{1-\gamma+\gamma\tau}\Delta\tilde{G} + \frac{\gamma}{1-\gamma+\gamma\tau}\Delta\tilde{T} = \frac{1}{1-\gamma+\gamma\tau}(\Delta\underline{C} + \Delta\underline{I}),$$

$$\tau\Delta Y + \Delta\tilde{T} - \Delta\tilde{G} = \Delta\hat{B},$$

$$\alpha\Delta Y = \Delta\hat{U} - \Delta\underline{N}_s.$$

In matrix notation the above set of equations becomes:

$$\begin{pmatrix} 1 & -\frac{1}{1-\gamma+\gamma\tau} & \frac{\gamma}{1-\gamma+\gamma\tau} \\ \tau & -1 & 1 \\ \alpha & 0 & 0 \end{pmatrix} \begin{pmatrix} \Delta Y \\ \Delta\tilde{G} \\ \Delta\tilde{T} \end{pmatrix} = \begin{pmatrix} \frac{1}{1-\gamma+\gamma\tau}(\Delta\underline{C} + \Delta\underline{I}) \\ \Delta\hat{B} \\ \Delta\hat{U} - \Delta\underline{N}_s \end{pmatrix},$$

Rewriting yields:

$$\begin{pmatrix} \Delta Y \\ \Delta\tilde{G} \\ \Delta\tilde{T} \end{pmatrix} = \begin{pmatrix} 1 & -\frac{1}{1-\gamma+\gamma\tau} & \frac{\gamma}{1-\gamma+\gamma\tau} \\ \tau & -1 & 1 \\ \alpha & 0 & 0 \end{pmatrix}^{-1} \begin{pmatrix} \frac{1}{1-\gamma+\gamma\tau}(\Delta\underline{C} + \Delta\underline{I}) \\ \Delta\hat{B} \\ \Delta\hat{U} - \Delta\underline{N}_s \end{pmatrix}.$$

If we know the values of the coefficients, the exogenous variables and the target variables on the right hand side of the equal sign, it is possible to solve the problem with the help of a spread-sheet programme.[8]

spread sheet

8 In this book we use the spreadsheet programme Excel. For matrix operations in Excel, see Appendix 2.2.

crowding out

A possible effect of the government stimulating the economy is that the interest rate goes up, because the government borrows money to finance its extra expenses. This means that private investments are discouraged. This is called the crowding out effect. This effect will be dealt with more extensively in Chapter 3.

An adaptation of the tax function is based on the idea that when the taxation rate τ increases, tax payers tend to pay less tax.[9] The function which then arises is:

$$T = \tau Y - \tau^{a} Y. \tag{2.13}$$

Laffer curve

In this case, taxation T is a function of the taxation rate, with Y standing for the maximum (official) income that a society can obtain and α for the "tax avoidance coefficient". Figure 2.3 presents a picture of this function. The curve shown is the so-called Laffer curve. The implication of this is that it is unwise for the government to set the taxation rate τ too high. The tax rate that generates the maximum tax revenue is called the optimum tax rate.[10]

optimum tax rate

(sub) tax rates

In general one can decompose the taxrate τ into three different (sub) taxrates:

- premiums and contributions (for social insurances) on wages paid by employers (τ_1);
- taxes and premiums on wages paid by employees (τ_2);
- taxes on products paid by consumers (*e.g.* a value added tax) (τ_3).

consumption wages

Employers are confronted with gross wages W plus employer premiums $\tau_1 W$ deflated by production price P. So the real wage cost equals $\frac{W + \tau_1 W}{P} = \frac{W(1+\tau_1)}{P}$. The supply of labour depends on the consumption wages. This is defined as net wages $W(1-\tau_2)$ deflated by the price of consumption $P(1+\tau_3)$.

wedge

Given this the wedge (the difference between real wage costs and net wage) can be derived. Relative to the real wage costs the wedge ω equals:

9 The basic assumption here is that the higher the tax rate the more people try to avoid paying taxes. This will result in a growth of the "unofficial" economy at the expense of the "official" economy. Further, high taxation may reduce the motivation to work.

10 This taxrate can be computed as follows:
$dT/d\tau = Y - a\tau^{a-1} = 0$, so: $\tau = (Y/a)^{1/(a-1)}$.

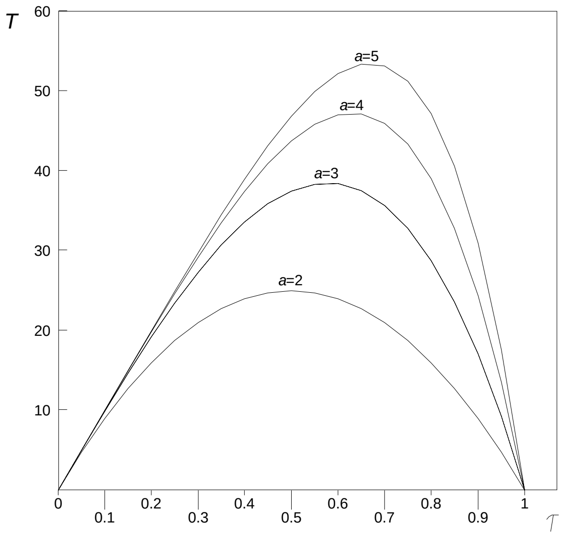

Figure 2.3: Laffer curve with $Y = 100$, and different values for a.

$$\omega = \frac{\frac{W(1+\tau_1)}{P} - \frac{W(1-\tau_2)}{P(1+\tau_3)}}{\frac{W(1+\tau_1)}{P}} = \frac{(1-\tau_3)(1-\tau_1)-(1-\tau_2)}{(1-\tau_3)(1-\tau_1)}. \tag{2.14}$$

For example. take 0.2, 0.4 and 0.2 for τ_1, τ_2 and τ_3 respectively. Then the wedge ω equals:

$$\frac{(1+0.2)(1+0.2)-(1-0.4)}{(1+0.2)(1+0.2)} = \frac{0.84}{1.44} = 0.583..$$

2.4 International trade, fixed prices, government budget

international trade

In this model economic interaction with foreign countries is taken into account. We assume that exports X are exogenous, while imports F depend on national income. Balance of trade BB can now be defined as the difference between exports and imports. The full model can now be described as follows:

$Y = C + I + G + X - F,$ Y National income

$C = \gamma(Y - T) + \underline{C},$ C Consumption

$$I = \underline{I},$$
$$G = \underline{G},$$
$$T = \tau Y,$$
$$B = T - G,$$
$$X = \underline{X},$$
$$F = \mu Y,$$
$$BB = X - F.$$
$$N_d = \alpha Y$$
$$N_s = \underline{N_s},$$
$$U = N_s - N_d.$$

I	Private investment
G	Government spending
T	Taxes
B	Government surplus
X	Exports
F	Imports
BB	Surplus trade balance
N_d	Demand for labour
N_s	Supply of labour
U	Unemployment
γ	Marginal propensity to consume
τ	Tax rate
μ	Imports coefficient
α	Labour coefficient

Rewriting the model gives:

$$Y = \frac{1}{1-\gamma+\gamma\tau+\mu}(\underline{C}+\underline{I}+\underline{G}+\underline{X}).$$

It is clear that making the model more complex leads to a smaller multiplier. Table 2.1 shows the multipliers for different models.

Table 2.1: Comparing multipliers.

Type of model	Multiplier
Closed economy	$\frac{1}{1-\gamma}$
Closed economy, government budget	$\frac{1}{1-\gamma+\gamma\tau}$
International trade, government budget	$\frac{1}{1-\gamma+\gamma\tau+\mu}$

Further we can deduce from the above model that:

$$\Delta Y = \frac{1}{1-\gamma+\gamma\tau+\mu}(\Delta \underline{C} + \Delta \underline{G} + \Delta \underline{I} + \Delta \underline{X}),$$

$$\Delta U = \Delta \underline{N}_s - \alpha \Delta Y,$$

$$\Delta B = \tau \Delta Y - \Delta \underline{G},$$

$$\Delta BB = \Delta \underline{X} - \mu \Delta Y.$$

policy in an open economy

With the help of the formerly sketched solution procedure it is possible to compute the necessary values of the policy instruments to reach the policy determined macroeconomic targets. In this model we can distinguish three policy targets: budget equilibrium $(B = 0)$, full employment $(U = 0)$, and equilibrium on the balance of trade $(BB = 0)$. This implies that we would need three instruments to reach them simultaneously. At this stage we only have two instruments: public spending and taxation. The third instrument will be dealt with in Chapter 3.

2.5 A two-country model of international trade

2 countries model

To give some notion about macroeconomic interdependency in a multi-country world, we now examine a simple model of two countries (1 and 2). We assume that country 1 imports the quantity exported by country 2 and that country 2 imports the quantity that country 1 exports. Furthermore, the propensities to spend γ and the imports coefficients μ are equal for both countries.[11] The model can be expressed mathematically by the following equations:

$Y_1 = A_1 + X_1,$	Y	Income
$Y_2 = A_2 + X_2,$	A	Spending on home produced goods
$X_1 = \mu Y_2,$	X	Exports
$X_2 = \mu Y_1,$	γ	Marginal propensity to spend
$A_1 = \gamma Y_1 + \underline{A}_1,$	μ	Marginal propensity to import
$A_2 = \gamma Y_2 + \underline{A}_2.$	B	Trade balance surplus
$B_1 = X_1 - X_2,$		
$B_2 = X_2 - X_1.$		

11 For simplicity reasons no distinction is being made between consumption and investment. It is assumed that part of the spending depends on income and part of it is exogenous.

Rewriting yields:

$$\begin{aligned}
&(1-\gamma)Y_1 = \mu Y_2 + \underline{A}_1 \\
&\underline{(1-\gamma)Y_2 = \mu Y_1 + \underline{A}_2} \quad - \\
&(1-\gamma)(Y_1 - Y_2) = \mu(Y_2 - Y_1) + (\underline{A}_1 - \underline{A}_2),
\end{aligned} \tag{2.16}$$

and

$$\begin{aligned}
&(1-\gamma)Y_1 = \mu Y_2 + \underline{A}_1 \\
&\underline{(1-\gamma)Y_2 = \mu Y_1 + \underline{A}_2} \quad + \\
&(1-\gamma)(Y_1 + Y_2) = \mu(Y_2 + Y_1) + (\underline{A}_1 + \underline{A}_2),
\end{aligned} \tag{2.17}$$

This means that:[12]

$$Y_1 - Y_2 = \frac{1}{1-\gamma+\mu}(\underline{A}_1 - \underline{A}_2),$$

$$Y_1 + Y_2 = \frac{1}{1-\gamma-\mu}(\underline{A}_1 + \underline{A}_2). \tag{2.18}$$

equalizing and maximizing autonomous spending

From this result we can draw the following conclusion: if $\underline{A}_1 \neq \underline{A}_2 \Rightarrow Y_1 \neq Y_2 \Rightarrow \mu Y_2 \neq \mu Y_1 \Rightarrow X_2 \neq X_1 \Rightarrow B_1, B_2 \neq 0$. Generally the conclusions to be drawn from (2.18) are that in order to achieve the highest possible income while maintaining equilibrium on the trade balance simultaneously, the two countries should agree on equalizing and maximizing their autonomous spending $\underline{A}$. [13]

Now we have reviewed some basic issues regarding macroeconomic models, the next step is to provide a more extended framework to analyze macroeconomic policy. These topics are included in (Chapters 3 and 4).

12 With $A_1, A_2 > 0$ and $1-\gamma-\mu > 0$.
13 This model can be expanded to more countries and can be made more complex by allowing the coefficients γ and μ to differ between countries.

Questions and exercises

2.1 When would a macroeconomic model of a closed economy without a government budget be unstable? When is there no equilibrium at all? Explain why it is unlikely that instability and the absence of equilibrium occur.

2.2 Average propensity to consume equals C/Y. When does the average propensity to consume equal the marginal propensity to consume?

2.3 Explain the Haavelmo effect.

2.4 Compute the optimum tax rate if the tax function is: $T = \tau 2000 - \tau^5 2000$.

2.5 Why is the multiplier getting smaller when government and international trade are introduced in the model?

2.6 Why is the multiplier in the case of the two country model bigger than in the regular case of one country with international trade?

Case 2.1

Assume the following model:

$$Y = C + I + G + X - F,$$
$$C = \gamma(Y - T) + \underline{C},$$
$$I = \underline{I},$$
$$G = \underline{G},$$
$$T = \tau Y + \underline{T},$$
$$B = T - G,$$
$$X = \underline{X},$$
$$F = \mu Y,$$
$$BB = X - F.$$
$$N_d = \alpha Y$$
$$N_s = \underline{N_s},$$
$$U = N_s - N_d.$$

Y	National income
C	Consumption
I	Private investment
G	Government spending
T	Taxes
B	Government surplus
X	Exports
F	Imports
BB	Surplus trade balance
N_d	Demand for labour
N_s	Supply of labour
U	Unemployment
γ	Marginal propensity to consume
τ	Taxation rate
μ	Imports coefficient
α	Labour coefficient

Solve the following questions given that: $\gamma = 0.8$, $\tau = 0.4$, $\mu = 0.4$, $\alpha = 0.01$, $\underline{C} = 80$, $\underline{I} = 50$, $\underline{G} = 200$, $\underline{T} = 80$ $\underline{X} = 225$, $\underline{N_s} = 6.5$.

1. Determine the reduced form equation.
2. Compute the income equilibrium.
3. What is the level of unemployment?
4. What is the budget result B of the government?
5. What is the surplus BB of the balance of payments?
6. What mix of measures should the government take to solve both the problems of unemployment and the government surplus? To solve this question, use the spreadsheet programme and apply the following steps:
   a. Take the reduced form equation and formulate the macroeconomic targets mathematically;
   b. Write the target variables and the exogenous variables on the right hand side of the equal sign. Write the instrument variables and endogenous variables on the left hand sign of the equal sign;
   c. Write the system in matrix notation;
   d. Write the solution in matrix notation;
   e. Solve the problem in Excel.

Appendix 2.1

Differential equation (2.3):[14]

$$\frac{dY}{dt} = -\alpha(1-\gamma)Y + \alpha(\underline{C} + \underline{I}). \tag{2.3}$$

The homogenous equation of this differential equation is:

$$\frac{dY}{dt} = -\alpha(1-\gamma)Y,$$

$$\frac{dY}{dt}\frac{1}{Y} = -\alpha(1-\gamma), \quad \Rightarrow Y = Ae^{-\alpha(1-\gamma)t}.$$

With A as a constant. The particular solution is:

$$\frac{dY}{dt} = -\alpha(1-\gamma)Y + \alpha(\underline{C} + \underline{I}) = 0, \quad \Rightarrow Y = \frac{\underline{C} + \underline{I}}{1-\gamma}.$$

This means that the general solution is:

$$Y = Ae^{-\alpha(1-\gamma)t} + \frac{\underline{C} + \underline{I}}{1-\gamma}.$$

14 See for example A.C. Chiang (1984) for methodological details regarding the subject of continuous time first-order differential equations.

If $t = 0$, then:

$$Y(0) = A + \frac{C+I}{1-\gamma}, \quad \Rightarrow A = Y(0) - \frac{C+I}{1-\gamma}.$$

So:

$$Y(t) = \left(Y(0) - \frac{C+I}{1-\gamma} \right) e^{-\alpha(1-\gamma)t} + \frac{C+I}{1-\gamma}.$$

Appendix 2.2: Matrix operations in Excel[15]

In general, the most used operations on matrices are:

I Calculation of the determinant of a matrix;
II. Transposing matrices;
III. Deriving the inverse of a matrix;
IV. Multiplication of matrices.

Below you will find some instructions of how these operations can be done in Microsoft Excel. Before one actually can start to do matrix operations one first needs to put the various coefficients of the matrix into the cells of the spreadsheet program. We make the above four points more clear by using the matrices A and B: [16]

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, B = \begin{pmatrix} 2 & 1 & 3 \\ 4 & 1 & 0 \end{pmatrix}.$$

Before we start to discuss the different procedures we assume that the coefficients of matrix A are in the cells A1, A2, B1, and B2 of Excel. (Thus the cells A1 and B1 include the first row of matrix A and in the cells A2 and B2 of Excel we find the second row of the matrix). Moreover, we presume that the coefficients of matrix B are in the cells D1, E1, F1, D2, E2, F2 respectively. Given this, we can start to discuss the different operations.

I. Calculation of the determinant of a matrix

The determinant of a matrix can be used in order to ascertain whether a *square* matrix is non-singular. In our case we can only determine the determinant of matrix A because this is the only square matrix. To do so, follow the next steps:[17]

– Go to an empty cell;

15 See: Person R., 1993. *Using Excel 5 for Windows: Special Edition.* Que.
16 Any matrix X is of the form X_{ij}, where *i* indexes the number of rows and *j* the number of columns.
17 We use capital letters for commands, operations, etc.

– Insert the formula: =MDETERM(A1:B2);[18]
– Press: ENTER.

You will find the value -2 in the cell. This is the determinant of the matrix A. More generally: the determinant of a square matrix can be calculated by the command: MDETERM(array). In this case "array" is a numeric array with an equal number of rows and columns. An array can be given as a cell range (in our case A1:B2); as an array constant, such as {1,2,3;4,5,6;7,8,9}; or as a name to either of these. If any cells in array are empty or contain text, MDETERM returns the #VALUE! error value. MDETERM also returns #VALUE! if array does not have an equal number of rows and columns.

II. Transposing matrices

When the rows and columns of a matrix are interchanged, we get the transpose of that particular matrix. Thus, the first row becomes the first column and the first column becomes the first row, etc. For example, to obtain the transpose of matrix B one has to do the following:

– Select a range of empty cells (3x2). In the case of matrix B you select three rows and two columns (because B was in the first instance of the form B_{23});
– Insert the formula: =TRANSPOSE(D1:F2);
– Press: CTRL+SHIFT+ENTER (simultaneously).

In the selected cells you will find the transpose of matrix B. This is: $\begin{pmatrix} 2 & 4 \\ 1 & 1 \\ 3 & 0 \end{pmatrix}$. The general command for determining the transpose of a matrix is: TRANSPOSE(array).

III. Deriving the inverse of a matrix

The inverse of a matrix is only defined if that specific matrix is a square matrix. In our case only matrix A has an inverse denoted by A^{-1}. To derive A^{-1} one needs to:

– Select a range of empty cells with the same features: the same number of rows and colomns as the matrix from which you want to determine its inverse. In the case of matrix A you select two rows and two columns;
– Insert the formula: =MINVERSE(A1:B2);
– Press: CTRL+SHIFT+ENTER (simultaneously).

18 In Excel, before one enters a formula into a cell one first needs to put an equal sign "=" in front of the formula statement.

In the selected cells you will find the inverse of matrix A, *viz.* $\begin{pmatrix} -2 & 1 \\ 1.5 & -0.5 \end{pmatrix}$. The general command is: MINVERSE(array). If any cells in the array are empty or contain text, MINVERSE returns the #VALUE! error value. Moreover, it returns this message also if the array does not have an equal number of rows and columns.

IV. Multiplication of matrices

Before one is actually going to multiply matrices with each other, be aware of the fact that $AB \neq BA$. That is, matrix multiplication is not commutative. It is even possible that BA is not defined while AB is. Multiplication between two matrices is defined if the number of columns of the first matrix equals the number of rows of the second. We see that this is also true in the case of the matrices as specified above. In fact, AB is defined, *i.e.* the number of columns of A is equal to the number of rows of B. Multiplying AB results in a 2x3 matrix. To calculate the product between these two matrices AB within Excel one needs to do the following:

- Select a range of 2x3 empty cells (the number of rows of A and the number of colums of B);
- Insert the formula: =MMULT(A1:B2;D1:F2);
- Press: CTRL+SHIFT+ENTER (simultaneously).

The general command regarding the operation of multiplying matrices is: MMULT(array1,array2). Array1 and array2 are arrays you want to multiply. The number of colums in array1 must be the same as the number of rows in array2, and both arrays must contain only numbers. If any cells are empty or contain text, or if the number of columns in array1 is different from the number of rows in array2, MMULT returns the #VALUE! error value. If there still remain uncertainties or strange outcomes it might be useful to search in the HELP-file of Excel on this topic.

CHAPTER 3: IS-LM ANALYSIS

Study objectives

- to study the demand for and supply of money
- to study the Hicks-Hansen diagram
- to study the transmission mechanism between real and monetary sector
- to understand what the 'liquidity' trap is
- to solve IS-LM models under different assumptions
- to derive the money multiplier
- to understand the influence of the trade balance on the economy
- to understand the effects on the economy of revaluation and devaluation
- to solve policy models under different assumptions

3.1 Introduction

The aim of this chapter is to discuss the IS-LM framework, which includes both the real sector of the economy, and the monetary sector. Section 3.2 examines a closed economy with fixed prices and with a monetary sector. In Section 3.3 the government budget is included in this model. In Sections 3.4 and 3.5 we study an open economy instead of a closed one, which means that international economic relations are taken into account.

3.2 Closed economy, fixed prices, monetary sector

As in Chapter Two we start our analysis with the presumption of a closed economy and fixed prices. However, the extension which we make is to include a monetary sector in the economy.

demand for and supply of money

The monetary sector consists of the demand for money L and the supply of money M. The demand for money depends on the level of income Y and the rate of interest R. The demand for money is positively related to production, because when production is high the amount of money needed to pay for all kinds of inputs and finished products is also high and vice versa. This is called the *transaction motive*.

transaction motive

The demand for money is negatively related to the rate of interest, because when the rate of interest is high having a lot of cash money is expensive. Further, the market price of shares is relatively low with in this situation. This will stimulate the buying of shares because the

speculative motive

buyers expect a rise in the price of it (*speculative motive*).[1] Further there is the autonomous demand for money $\underline{L}$ because of the so called *precautionary motive*. This means that actors prefer to dispose of an amount of money to deal with unexpected expenses.

precautionary motive

The supply of money is determined by the monetary authorities (for example the Central Bank), so this is determined autonomously. Finally, there is equilibrium in the monetary sector when the supply of money equals demand. This monetary theory can be presented with a model consisting of three equations: the demand for money, the supply of money and the equilibrium equation:

$L = \xi Y - \chi R + \underline{L}$,	L	Demand for money
$M = \underline{M}$,	M	Supply of money
$L = M$.	Y	National income
	R	Rate of interest
	ξ	coefficient
	χ	coefficient

Rewriting gives:

$$R = \frac{\xi}{\chi} Y + \frac{\underline{L} - \underline{M}}{\chi}. \tag{3.1}$$

LM function

Equation (3.1) is called the LM function. It represents all combinations of R and Y for which there is equilibrium in the monetary sector.

dichotomy

As long as there is no connection between the real sector and monetary sector there is a so called *dichotomy*. The connection can be made by taking into account the rate of interest in the investment function. Investment is negatively related to the rate of interest. When the interest rate is high, credit will be expensive and therefore investment will be relatively low. If we take that into account we get the following model:

$Y = C + I$,	Y	National income
$C = \gamma Y + \underline{C}$,	C	Consumption
$I = -\iota R + \underline{I}$.	I	Private investment

1 For a more extended examination of the three motives (transaction motive, speculative motive and precautionary motive) see for example Kohler (1994) or Burda and Wyplosz (1993).

R	Rate of interest
γ	Marginal propensity to consume
ι	coefficient

Rewriting the model gives:

$$R = \frac{-(1-\gamma)}{\iota} Y + \frac{\underline{C} + \underline{I}}{\iota}. \tag{3.2}$$

IS function

Equation (3.2) is called the IS function. It represents all combinations of R and Y for which there is equilibrium in the real sector. Figure 3.1 shows the IS as well as the LM function.

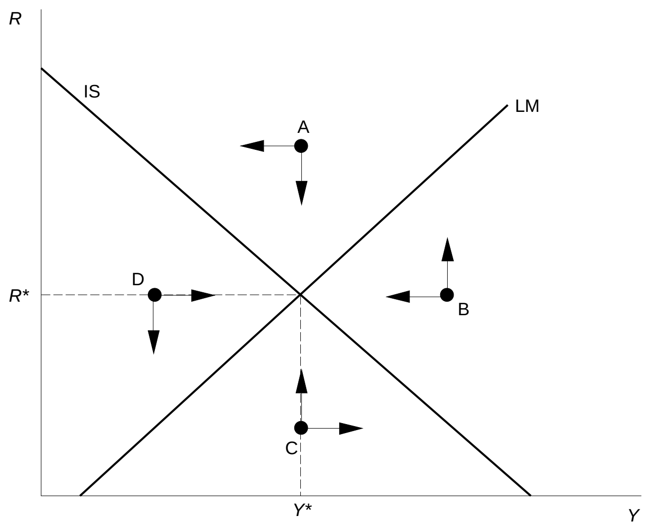

Figure 3.1: Hicks-Hansen diagram.

Hicks-Hansen diagram

Figure 3.1 is called the Hicks-Hansen diagram. The only combination of R and Y for which there is equilibrium in the real sector as well as in the monetary sector is R^* and Y^*. The stability analysis for this model is as follows. At point A, the interest rate is relatively high so supply of money M exceeds demand for money L. This will lower the interest rate as is indicated by the arrow. Point A is to the right of the IS curve. This implies that production Y exceeds effective demand ED, or investment I is smaller than savings S, so that stocks V are increasing. This causes a decrease in production. In other terms:

$$M > L \Rightarrow R \downarrow,\ ED < Y(I < S) \Rightarrow V \uparrow \Rightarrow Y \downarrow.$$

The same analysis carried out for B, C and D gives respectively:

B: $M < L \Rightarrow R \uparrow,\ ED < Y(I < S) \Rightarrow V \uparrow \Rightarrow Y \downarrow$;
C: $M < L \Rightarrow R \uparrow,\ ED > Y(I > S) \Rightarrow V \downarrow \Rightarrow Y \uparrow$;
D: $M > L \Rightarrow R \downarrow,\ ED > Y(I > S) \Rightarrow V \downarrow \Rightarrow Y \uparrow$.

transmission mechanism

The mechanism which determines the relation between the real and monetary sector is called the *transmission mechanism*. When discussing this mechanism we also enter directly the field of monetary economics. Figure 3.2 gives an illustration of the transmission mechanism.

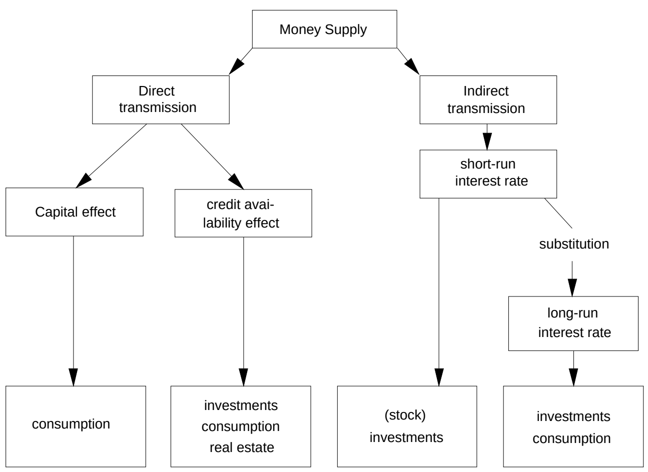

Figure 3.2: The transmission mechanism.

We can say that, eventually, the objectives of monetary policy are to influence the real sector in a certain way. In general, there are two different kinds of changes in the monetary sector that influence the real sector: direct and indirect transmission.

direct transmission:
- wealth effect
- real balance effect

Firstly, let us consider direct transmission. This mechanism works by way of the capital (wealth) effect and the possible credit availability effect. The *wealth effect* arises from a change in the monetary sector (for example, a change in the interest rate influences people's wealth). If for any reason people get richer, they will consume more. This is also known as the *real balance effect*.

- credit availability effect

The real sector can also be influenced by the so-called *credit availability effect*. After all, besides the price of money (interest for loans) the availability of credit is also relevant. In practise, the banks are only willing to issue limited amounts of credit irrespective of the price clients want to pay (credit rationing). If the central bank enforces a restricted policy, then, in principle, the credit availability effect will diminish without a change in the credit interest rate.

indirect transmission

Secondly, we come to the indirect transmission. The indirect transmission mechanism operates by way of an adjustment of the interest rate. An induced change in the interest rate by the central bank leads to an adjustment in the real sector. If the central bank enforces an expanded monetary policy then banks will also enlarge their credit policy. Subsequently, the short run interest rate will fall. Moreover, people will demand more bonds and stocks. This demand leads to an increase of the price of these bonds and stocks, hence the long-run interest rate will decrease. In other words, there is substitution. Firstly, the short run interest rate decreases and secondly the long run interest rate also decreases.

As noted, these changes will have an impact on the real sector. A decrease in the short run interest rate will probably lead to additional inventory investments.[2] On the other hand, a decrease in the long run interest rate has a positive impact on the demand for capital goods.

classical view

The question is now how important these different transmission channels are. Economists have different opinions about this. Classical economists emphasize direct transmission. In fact, an increase in the money supply would in the first instance lead to an increase in the real money balances. As long as prices do not rise then the real money balances exceed the desired money balances. The 'surplus' will find its way out in terms of consumption. The marginal utility of the last unit of money in the money balance is smaller than the marginal utility of the last unit spent on consumption. Equilibrium can be established through extra consumption.

2 Inventory investments imply all changes in the stock of raw materials, parts, and finished goods held by business (Gordon, 1993, p.34).

Keynesian view

According to Keynes indirect transmission prevails. A monetary incentive leads to adjustments in the capital market. Subsequently, the demand for goods will change. According to Keynes the effect of a change in the interest rate on investments must not be overestimated. Keynes mainly focused on profit and sales expectations. These are the main aspects which determine the attractiveness of new investments. So, according to the Keynesian view there is a very small indirect relationship between monetary variables and effective demand. Therefore, the best instrument to stabilize the business cycle is fiscal policy rather than monetary policy.

exogeneity of money supply

In standard Keynesian theory money supply is seen as an exogenous variable. The assumption is made that the monetary authority (Central Bank) can influence the supply of money in order to reach the desired level under all circumstances. Behind this vision of exogeneity of money supply we find the theorem of the mechanical money multiplier.

money multiplier

This theorem states that a simple algebraic relationship exists between the supply of money and the so-called monetary base $M_{0.}$[3]

endogeneity of money supply

The above outlook does not take into account that the process of money supply is a result of continuous interaction between banks, government, and public. Monetarists share the opinion that the supply of money is also determined by endogenous factors (for example the supply of money can also depend on profit maximization objectives of banks). However, empirical research shows that the monetary authorities can also control the supply of money.

We will now examine a simple algebraic model of the money multiplier in order to give some more insight into the process of money supply. The money supply can be defined in three ways:

- M_1: Currency in circulation plus sight deposits;
- M_2: M_1 plus time deposits;
- M_3: M_2 plus fixed term deposits plus accounts at non-bank institutions.

money supply

If we talk about the money supply we usually refer to M_1. The money supply is based on the so-called monetary base M_0 consisting of the sum of currency held by the non-bank public and bank reserves. The total amount of M_0 is controlled by the central bank.

3 The monetary base or M_0 is also called *high-powered money*. It is the sum of currency held by the nonbank public and bank reserves. For further details see for example Gordon (1993).

monetary base

The monetary base is partly in hands of the public: $M_0^p = cM_0$ and partly in the hands of the bank: $M_0^b = (1-c)M_0$. The money multiplier m can be defined as the ratio between M_1 and M_0:

money multiplier

$$m = \frac{M_1}{M_0}.$$

It indicates the increase of money supply M_1 if the central bank decides to increase the monetary base M_0 by one monetary unit. M_1 consists of the part of the monetary base in hands of the public cM_0 plus the amount of sight deposits M_s:

$$M_1 = cM_0 + M_s.$$

Furthermore we assume that the part of the monetary base in hands of the bank M_0^b covers a fixed part r of the sight deposits:

$$M_0^b = rM_s \Rightarrow M_s = \frac{1}{r}M_0^b.$$

Now we can deduce:

$$m = \frac{M_1}{M_0} = \frac{M_0^p + M_s}{M_0} = \frac{cM_0 + \frac{1}{r}M_0^b}{M_0}$$

$$= \frac{cM_0 + \frac{1}{r}(1-c)M_0}{M_0} = \frac{1+cr-c}{r} = \frac{1-c(1-r)}{r}. \quad (3.3)$$

money multiplier exceeds 1

The money multiplier exceeds one. This can be proved as follows:

$$0 < r < 1 \;\Rightarrow\; 1-c > r(1-c) \;\Rightarrow\; 1-c > r - rc$$

$$\Rightarrow\; 1 + rc - c > r \;\Rightarrow\; \frac{1-c(1-r)}{r} > 1.$$

Questions and exercises

3.1 Draw a Hicks-Hansen diagram
   a. What happens when the supply of money and public spending decreases?
   b. Indentify the sector in which effective demand exceeds production and where at the same time the demand for money is less than its supply.

c. Show how the equilibrium situation will be reached eventually.

3.2 a. Explain how the transmission mechanism works in a Keynesian model.
b. What is the difference between direct and indirect transmission?

3.3 Assume that $M_0 = 100$. Further we assume that the $c = 0.25$, and $r = 0.10$. Compute the money supply M_1.

3.3 Closed economy, fixed prices, government budget, monetary sector

real sector model

Taking into account the rate of interest, which is correlated negatively with investments and assuming endogenous taxation, the real sector model is as follows:

$Y = C + I + G,$	Y	National income
$C = \gamma(Y - T) + \underline{C},$	C	Consumption
$I = -\iota R + \underline{I},$	I	Private investment
$T = \tau Y,$	T	Taxes
$G = \underline{G},$	G	Government spending
$B = T - G.$	B	Government surplus
	R	Rate of interest
	γ	Marginal propensity to consume
	τ	Taxation rate
	ι	coefficient

Rewriting yields a decreasing IS function:

$$R = \frac{-(1 - \gamma + \gamma\tau)}{\iota} Y + \frac{\underline{C} + \underline{I} + \underline{G}}{\iota}. \tag{3.4}$$

The monetary sector does not change:

$$R = \frac{\xi}{\chi} Y + \frac{\underline{L} - \underline{M}}{\chi}. \tag{3.1}$$

policy goals

Now assume that the government wants to reach two goals: full employment and equilibrium on the government account. To study this, we make use of Figure 3.3.

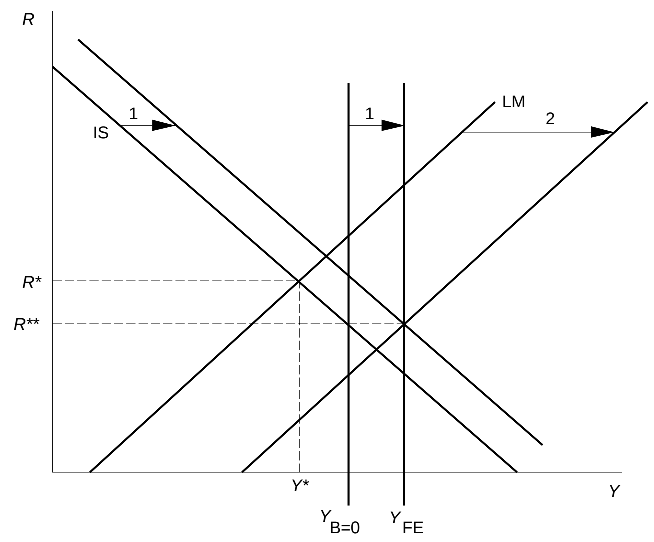

Figure 3.3: Reaching full employment and equilibrium on the government budget.

$Y_{B=0}$ – curve

In the initial situation (R^*, Y^*) of Figure 3.3 there is unemployment and a budget deficit. The measures to be taken are first increase government spending (1) and second to increase the money supply (2). The increase in government spending has two effects: the IS curve moves to the right and the $Y_{B=0}$ -curve[4] also moves to the right. This occurs because the equation of the last curve is:

$$\tau Y = \underline{G}, \ \Rightarrow \ Y_{B=0} = \frac{\underline{G}}{\tau}. \tag{3.5}$$

increasing money supply

Increasing the supply of money causes a decrease in the rate of interest which in turn stimulates investment and national income. In the end the new equilibrium is reached (Y_{FE}, R^{**}).

4This curve refers to the situation at which there is equilibrium on the government budget.

A more realistic shape of the LM curve is shown in Figure 3.4.

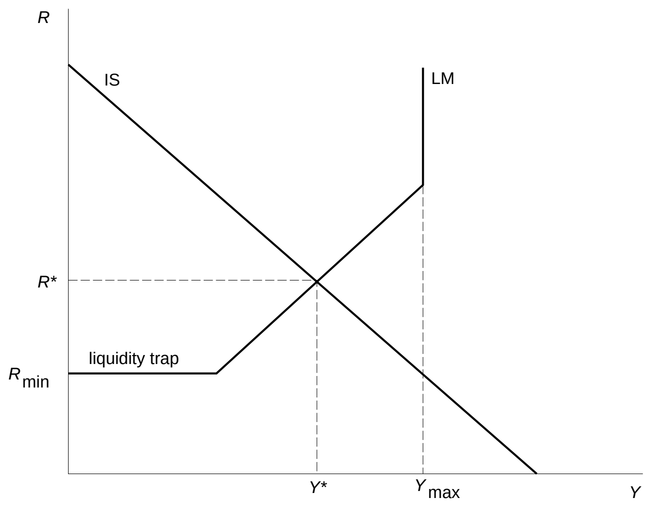

Figure 3.4: The liquidity trap.

liquidity trap

It shows a minimum rate of interest $R_{\min}$ and a maximum production $Y_{\max}$. The reason for this is that the elasticity of the demand for money becomes infinite at this rate. This means that on the one hand every extra unit supply of money will be encashed. On the other hand production cannot grow beyond $Y_{\max}$ because the production capacity is fully used. The horizontal part of the LM curve is called the liquidity trap. An important characteristic of this part of the curve is that monetary policy is not effective while budget policy is. Monetary policy is not effective because it is based upon a decrease in the rate of interest. But such a decrease is impossible. Past $Y_{\max}$ neither budgetary policy nor monetary policy can be effective.

Questions and exercises

3.4 Draw a situation where the $Y_{B=0}$ – curve is situated to the right of the Y_{FE} – curve and where Y^* lies to the left of Y_{FE}. Make a suggestion for the policy mix in order to achieve equilibrium on the labor market as well as equilibrium on the government budget.

3.5 a. Explain the relationship between ’crowding out’ (see Section 2.3) and ’transmission’.
b. Why is there no ’crowding out’ during the stage of the ’liquidity trap’?

3.4 Open economy, fixed prices, government budget

trade balance

In this section we take the economic foreign relations into account. The trade balance consists of exports X on the one hand and imports F on the other hand. Trade balance B equals $X - F$. The whole model is described by:

$Y = C + I + G + X - F,$	Y	National income
$C = \gamma(Y - T) + \underline{C},$	C	Consumption
$I = \underline{I},$	I	Private investment
$T = \tau Y,$	T	Taxes
$G = \underline{G},$	G	Government spending
$B = T - G,$	B	Government surplus
$X = \underline{X},$	X	Exports
$F = \mu Y,$	F	Imports
$BB = X - F,$	BB	trade balance
$N_d = \alpha Y,$	N_d	Demand for labour
$N_s = \underline{N}_s.$	N_s	Supply of labour
	γ	Marginal propensity to consume
	τ	Taxation rate
	μ	coefficient
	α	labour coefficient

The model yields:

$$Y = \frac{1}{1 - \gamma + \gamma\tau + \mu}(\underline{C} + \underline{I} + \underline{G} + \underline{X}). \tag{3.6}$$

The equation for the balance of payment is:

$$\begin{aligned} BB = \underline{X} - \mu Y &= \underline{X} - \frac{\mu}{1 - \gamma + \gamma\tau + \mu}(\underline{C} + \underline{I} + \underline{G} + \underline{X}), \\ &= \frac{1 - \gamma + \gamma\tau}{1 - \gamma + \gamma\tau + \mu}\underline{X} - \frac{\mu}{1 - \gamma + \gamma\tau + \mu}(\underline{C} + \underline{I} + \underline{G}). \end{aligned} \tag{3.7}$$

If the government wants to stimulate the economy then the following effects can be computed:

$$\Delta Y = \frac{1}{1-\gamma+\gamma\tau+\mu}\Delta \underline{G},$$

$$\Delta BB = -\frac{\mu}{1-\gamma+\gamma\tau+\mu}\Delta \underline{G}. \tag{3.8}$$

effects of increasing public spending

The conclusion from this model is that increasing government spending has a positive influence on production and employment, but is affecting the balance of payment BB negatively.

3.5 Open economy, fixed prices, government budget, monetary sector

We will now integrate a monetary sector into the model we have just developed. The monetary sector consists of the LM function. Furthermore, we will have to insert the rate of interest into the function for private investment and with respect to international relations a capital balance will be developed. The real sector then looks as follows:

capital balance

$Y = C + I + G + X - F,$	Y	National income
$C = \gamma(Y-T) + \underline{C},$	C	Consumption
$I = -\iota R + \underline{I},$	I	Private investment
$T = \tau Y,$	T	Taxes
$G = \underline{G},$	G	Government spending
$B = T - G,$	B	Government surplus
$X = \underline{X},$	X	Exports
$F = \mu Y,$	F	Imports
$BB = X - F.$	R	Rate of interest
	BB	Surplus trade balance
	γ	Marginal propensity to consume
	τ	Taxation rate
	ι	coefficient
	μ	coefficient

Rewriting gives the IS function:

$$R = \frac{-(1-\gamma+\gamma\tau+\mu)}{\iota}Y + \frac{1}{\iota}(\underline{C}+\underline{I}+\underline{G}+\underline{X}). \tag{3.9}$$

The LM function does not change and is thus again:

$$R = \frac{\xi}{\chi}Y + \frac{\underline{L}-\underline{M}}{\chi}. \tag{3.1}$$

The balance of payments including the capital balance K is:

$$BB = X - F + K,$$
$$K = \varepsilon R + \underline{K},$$

where ε represents a coefficient and $\underline{K}$ equals the exogenous flow of capital. substitution yields:

$$BB = \underline{X} - \mu Y + \varepsilon R + \underline{K}. \tag{3.10}$$

K represents net inflow of capital

Here, it is assumed that K represents here the net *inflow* of capital. This depends on the interest rate. If the interest rate is increasing it is attractive for foreign investors to invest their capital *ceteris paribus*. The capital balance is part of the balance of payments. It is the part that records capital flows consisting of purchases and sales of foreign assets by domestic residents and purchases and sales of domestic assets by foreign residents. Moreover, the balance of payments also includes the current account. This is the part that incorporates exports, imports, investment income, and transfer payments to and from foreigners. Thus the balance of payments is the sum of the current account balance and the capital account balance.

If $BB = 0$, we can write:

$$R = \frac{\mu}{\varepsilon}Y - \frac{\underline{X}+\underline{K}}{\varepsilon}. \tag{3.11}$$

BB=0-function

This function is called the BB=0-function. It represents all combinations of R and Y for which there is equilibrium on the balance of payments. The three functions are presented graphically in Figure 3.5.

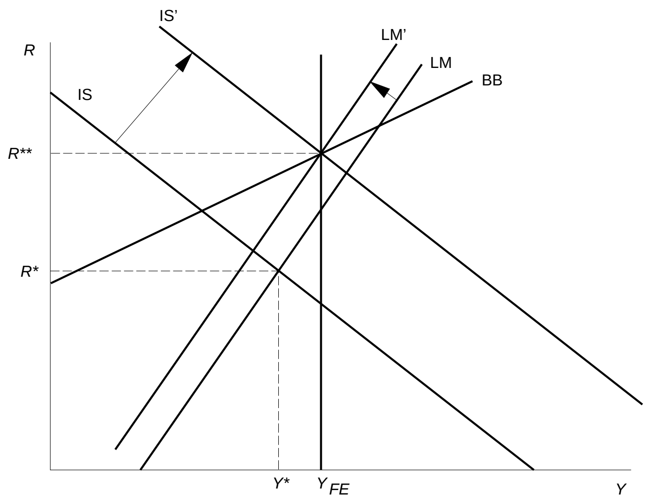

Figure 3.5: The IS, LM and BB curve.

Figure 3.5 shows that in the initial stage (R^*, Y^*) the labour market is not in equilibrium; there is unemployment. Moreover, there is a deficit on the balance of payment. The policy to be carried out is to increase government spending (IS curve moves to IS') and to decrease the money supply (LM curve moves to LM'). Eventually the target level of full employment with equilibrium on the balance of payments has been reached. So, the equilibrium point is then (R^{**}, Y_{FE}).

We can extend the model further by including the exchange rate.[5] Both exports and imports depend on the real exchange rate.[6] Exports will increase if the exchange rate increases. An increase of the exchange rate is called *devaluation*, a decrease of the exchange rate is called *revaluation*.[7] Imports are negatively related to the exchange rate and are positively related to production. In our linear model this works as follows:

devaluation and revaluation

5 An exchange rate is the value of a foreign currency expressed in the home currency. Example: Am $1.-= DFl 2.05.

6 Real exchange rate E_r equals $(EP^*)/P$, in which E represents the nominal exchange rate, P^* the foreign price level, and P the domestic price level. Because, it is assumed that the domestic price level equals the foreign price level the nominal exchange rate equals the real exchange rate here.

7 Under a free floating system revaluation is called *appreciation* and devaluation is called *depreciation.*

$X = gE + \underline{X}$,	E	Exchange rate
$F = -hE + \mu Y.$		
$K = \varepsilon R + \underline{K}$,		

We can now deduce:

$$BB = X - F + K = \pi E - \mu Y + \underline{X} + \varepsilon R + \underline{K},$$

where: $\pi = g + h$. The $BB = 0-$ function becomes now:

$$BB = 0, \text{ so: } 0 = X - F + K = \pi \underline{E} - \mu Y + \underline{X} + \varepsilon R + \underline{K},$$

$$R = \frac{\mu}{\varepsilon} Y - \frac{\pi}{\varepsilon} \underline{E} - \frac{\underline{X} + \underline{K}}{\varepsilon}. \quad (3.12)$$

From the $BB = 0-$ function it can be concluded that a revaluation of the home currency shifts the BB=0-curve upward and a devaluation shifts the BB=0-curve downward.

The trade balance $(X - F)$ is also part of the IS function. From the analysis above we can observe that:

$$X - F = \pi \underline{E} - \mu Y + \underline{X}.$$

In IS function (3.9) the trade balance (without the influence of the exchange rate $\underline{E}$) is written as:

$$X - F = \underline{X} - \pi Y.$$

So, to adapt the IS function (3.9) for the exchange rate, the only thing we have to do is to replace $\underline{X}$ by $\pi \underline{E} + \underline{X}$:

$$R = \frac{-(1 - \gamma + \gamma\tau + \mu)}{\iota} Y + \frac{\pi}{\iota} \underline{E} + \frac{1}{\iota} (\underline{C} + \underline{I} + \underline{G} + \underline{X}). \quad (3.13)$$

three targets,three instruments

With the supply of money, government spending, and the exchange rate, we now have three instruments of economic policy.[8] This means that we are in a position to reach three goals simultaneously: full employment, equilibrium on the government account and equilibrium on the balance of payment. Figure 3.6 illustrates this situation.

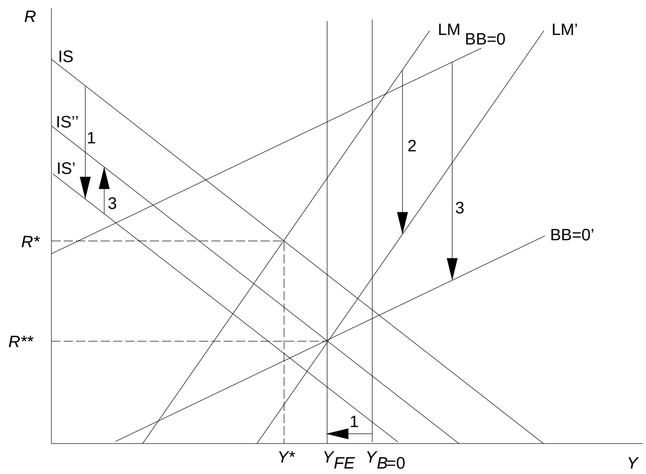

Figure 3.6: Reaching three goals of economic policy.

geometrical solution procedure for a policy problem

In Figure 3.6 there is an initial equilibrium at $Y*$ and $R*$. The situation is characterised by Unemployment, because Y_{FE} lies to the right of $Y*$, a government's deficit because $Y_{B=0}$ lies to the right of $Y*$, and a deficit on the balance of payment because the initial equilibrium $(R*, Y_{FE})$ lies below the BB=0-curve. The measures to be taken to redress the disequilibria in this situation are the following:

– Decrease government spending. This will cause a shift of the $Y_{B=0}$ curve to the left and a downward shift of the IS curve to IS' (1);
– Increase the money supply causing the LM curve to shift downward (2);
– Devaluation of the home currency will cause a downward shift of the BB=0 curve to BB=0' and an upward shift of the IS curve to IS''(3).

8 We are assuming here that the exchange rate is fixed but adaptable. With a free floating system the exchange rate is completely free. A third system is the so called managed float, in which the exchange rate is free, but the Central Bank is influencing the exchange rate by buying and selling foreign currency to avoid too heavy shocks.

fixed and flexible exchange rates

Looking at the LM curve we have to make a distinction between fixed and flexible exchange rates. In a system of fixed exchange rates the supply of money may become endogenous. In this case a balance of payments surplus will cause an increase in the money supply. So the central bank would increase the reserves in order to stabilize the exchange rate. The balance of payments position influences the supply of money via these reserves. In our simple model, however, we assume that the central bank can compensate for this by influencing other parts of the monetary base or by restricting the banker's lending capacity. The conclusion is that even in a situation of fixed exchange rates the supply of money is exogenous (at least in the short run).

IS together with LM

The policy model of the above situation looks as follows. From the IS and LM function, (3.13) and (3.1) respectively, we derive:

$$\frac{\xi}{\chi}Y+\frac{\underline{L}-\underline{M}}{\chi}=\frac{-(1-\gamma+\gamma\tau+\mu)}{\iota}Y+\frac{\pi E}{\iota}+\frac{(\underline{C}+\underline{I}+\underline{G}+\underline{X})}{\iota},$$

$$Y\left(\frac{\xi}{\chi}+\frac{1-\gamma+\gamma\tau+\mu}{\iota}\right)=\frac{\underline{M}-\underline{L}}{\chi}+\frac{\pi E}{\iota}+\frac{(\underline{C}+\underline{I}+\underline{G}+\underline{X})}{\iota},$$

$$Y=\frac{\iota\underline{M}}{\iota\xi+\chi(1-\gamma+\gamma\tau+\mu)}+\frac{\chi\underline{G}}{\iota\xi+\chi(1-\gamma+\gamma\tau+\mu)}$$

$$-\frac{\iota\underline{L}}{\iota\xi+\chi(1-\gamma+\gamma\tau+\mu)}+\frac{\chi\pi E}{\iota\xi+\chi(1-\gamma+\gamma\tau+\mu)}+\frac{\chi(\underline{C}+\underline{I}+\underline{X})}{\iota\xi+\chi(1-\gamma+\gamma\tau+\mu)}, \quad (3.14)$$

or

reduced form

$$Y=\mathrm{a}\underline{M}+\mathrm{b}\underline{G}+\mathrm{c}E+\mathrm{d}, \quad (3.15)$$

where

$$\mathrm{a}=\frac{\iota}{\iota\xi+\chi(1-\gamma+\gamma\tau+\mu)},$$

$$\mathrm{b}=\frac{\chi}{\iota\xi+\chi(1-\gamma+\gamma\tau+\mu)},$$

$$\mathrm{c}=\frac{\chi\pi}{\iota\xi+\chi(1-\gamma+\gamma\tau+\mu)},$$

$$\mathrm{d}=\frac{-\iota\underline{L}}{\iota\xi+\chi(1-\gamma+\gamma\tau+\mu)}+\frac{\chi(\underline{C}+\underline{I}+\underline{X})}{\iota\xi+\chi(1-\gamma+\gamma\tau+\mu)}.$$

The LM function (3.1) substituted in the BB-function (3.10) yields:

$$BB = \pi E + \left(\frac{\varepsilon\xi}{\chi} - \mu\right)Y - \frac{\varepsilon}{\chi}(\underline{M} - \underline{L})\underline{X} + \underline{K}. \tag{3.16}$$

From this it follows:

$$E = \frac{BB}{\pi} + \left(\frac{-\varepsilon\xi}{\chi\pi} + \frac{\mu}{\chi}\right)Y + \frac{\varepsilon}{\pi\chi}\underline{M} - \frac{\varepsilon}{\pi\chi}\underline{L} - \frac{1}{\pi}(\underline{X} + \underline{K}).$$

or

$$E = \frac{BB}{\pi} + \mathrm{e}Y + \mathrm{f}\underline{M} + \mathrm{g}, \tag{3.17}$$

where

$$\mathrm{e} = \left(\frac{-\varepsilon\xi}{\chi\pi} + \frac{\mu}{\chi}\right),\ \mathrm{f} = \frac{\varepsilon}{\pi\chi},\ \mathrm{g} = -\frac{\varepsilon}{\pi\chi}\underline{L} - \frac{1}{\pi}(\underline{X} + \underline{K}).$$

After we have substituted (3.17) in (3.15) we arrive at:

$$Y = \frac{a + cf}{1 - ce}\underline{M} + \frac{b}{1 - ce}\underline{G} + \frac{c}{(1 - ce)\pi}BB + \frac{cg + d}{1 - ce}. \tag{3.18}$$

or

$$Y = \mathrm{A}\underline{M} + \mathrm{B}\underline{G} + \mathrm{C}BB + \mathrm{D}. \tag{3.19}$$

where[9]

$$\mathrm{A} = \frac{a + cf}{1 - ce},\ \mathrm{B} = \frac{\mathrm{b}}{\mathrm{1\text{-}ce}},\ \mathrm{C} = \frac{\mathrm{c}}{\mathrm{(1\text{-}ce)}\pi},\ \mathrm{D} = \frac{\mathrm{cg+d}}{\mathrm{1\text{-}ce}}.$$

The equation for the government account is:

$$B = \tau Y - \underline{G}, \tag{3.20}$$

Finally, the equation for the labour market is:

$$U = \underline{N}_s - \alpha Y. \tag{3.21}$$

9 This equation is called the Aggregate demand function for an open economy (see Chapter 4).

the model in deviations

In deviations the full model can now be written as:[10]

$$\Delta Y - \mathrm{A}\Delta \underline{M} - \mathrm{B}\Delta \underline{G} = \mathrm{C}\Delta BB + \Delta \mathrm{D},$$

$$-\mathrm{e}\Delta Y + \Delta E - \mathrm{f}\Delta \underline{M} = \frac{\Delta BB}{\pi} + \Delta \mathrm{g},$$

$$\tau\Delta Y - \Delta \underline{G} = \Delta B,$$

$$\alpha\Delta Y = \Delta \underline{N}_s - \Delta U. \qquad (3.22)$$

When we write this system in matrix form we get:

$$\begin{pmatrix} 1 & \text{-A} & \text{-B} & 0 \\ \text{-e} & \text{-f} & 0 & 1 \\ \tau & 0 & -1 & 0 \\ \alpha & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \Delta Y \\ \Delta \underline{M} \\ \Delta \underline{G} \\ \Delta E \end{pmatrix} = \begin{pmatrix} \mathrm{C}\Delta BB + \Delta \mathrm{D} \\ \Delta BB/\pi + \Delta \mathrm{g} \\ \Delta B \\ \Delta \underline{N}_s - \Delta U \end{pmatrix}. \qquad (3.23)$$

So, the conclusion is:

$$\begin{pmatrix} \Delta Y \\ \Delta \underline{M} \\ \Delta \underline{G} \\ \Delta E \end{pmatrix} = \begin{pmatrix} 1 & \text{-A} & \text{-B} & 0 \\ \text{-e} & \text{-f} & 0 & 1 \\ \tau & 0 & -1 & 0 \\ \alpha & 0 & 0 & 0 \end{pmatrix}^{-1} \begin{pmatrix} \mathrm{C}\Delta BB + \Delta \mathrm{D} \\ \Delta BB/\pi + \Delta \mathrm{g} \\ \Delta B \\ \Delta \underline{N}_s - \Delta U \end{pmatrix}. \qquad (3.24)$$

Questions and exercises

3.6 a. Explain why a decrease in the exchange rate implies a revaluation (appreciation) of the home currency and vice versa.
b. Why does a devaluation of the home currency stimulate exports and disencourage imports?
c. Explain why the $BB = 0 -$ curve is a rising linear curve.

3.7 a. Explain why the balance of payments is always in equilibrium when there is free float of the exchange rates.
b. Assume flexible exchange rates. Draw a situation with a deficit on the government account and unemployment. Make a suggestion for the right policy mix (aiming at full employment and equilibrium on the government account).

10 Here we take into account the solution rule: dependent and instrument variables to the left side of the equal sign and target variables and exogenous variables to the right side of the equal sign.

3.6 Estimation of coefficients

solution procedure

Before we can solve the matrix equation (3.23) we have to estimate the coefficients in the matrix. We will estimate them row by row. The unknown coefficients in the first row are A, B, C, and D. The method of estimation is simple linear regression. With this method we can see what the statistical relationship is between a dependent variable and a number of explanatory variables. To estimate A, B, C, and D we need equation (3.19):

$$Y = \mathrm{A}\underline{M} + \mathrm{B}\underline{G} + \mathrm{C}BB + \mathrm{D}.$$

dependent and explanatory variables

In (3.19) there is one dependent variable Y and three explanatory variables: $\underline{M}$, $\underline{G}$, and BB. Data for these variables are available in Appendix 3.2. With the help of these data and the linear regression procedure described in Appendix 3.1, it is possible to estimate the four coefficients.

In the second row we see the coefficients e, f, g and $1/\pi$. The estimation of these coefficients can be done with equation (3.17), Appendix 3.2 and the procedure described in Appendix 3.1. In the third row we find τ as the only coefficient. This can be determined with the equation for the taxation $T = \tau Y$. Again the data needed for this can be found in Appendix 3.2. Finally the value of α has to be estimated. This coefficient can be estimated with equation $N_d = \alpha Y$. See Appendix 3.2 for the necessary data.

target variables and exogenous variables

At the right hand side of (3.23) we find the target variables and the exogenous variables. Normally we assume that in the end there should be equilibrium on the labour market $(U = 0)$, the balance of payment $(BB = 0)$, and the government account $(B = 0)$. So, if, for example in the starting situation $U = 0.5$, $BB = 1$, and $B = 0.75$, then the values of the target variables are: $\Delta U = -0.5$, $\Delta BB = -1$, and $\Delta B = -0.75$.[11] Further the exogenous variables should also have a value. If no further information is available we can safely assume that $\Delta \mathrm{D} = \Delta \mathrm{g} = \Delta \underline{N}_s = 0$.

After the values of all coefficients and exogenous variables are determined, equation (3.24) can be solved following the procedure in Appendix 2.2.

11 In Appendix 3.2 you will find data till 1992. For our analysis 1992 is the starting year. So, the targets for the economic policy should be derived from the data of that year.

Case 3.1

Suppose that the economy can be modelled by:

$Y = C + I + G,$	Y	National income
$C = \gamma(Y - T) + \underline{C},$	C	Consumption
$I = -\iota R + \underline{I},$	I	Private investment
$G = \underline{G},$	G	Government spending
$T = \tau Y,$	T	Taxes
$N_d = \alpha Y,$	N_d	Demand for labour
$N_s = \underline{N}_s,$	N_s	Supply of labour
$L = \xi Y - \chi R + \underline{L},$	L	Demand for money
$M = \underline{M}.$	M	Supply of money
	E	Exchange rate
	R	Interest rate
	γ	coefficient
	π	coefficient
	ι	coefficient
	τ	coefficient
	α	coefficient
	ξ	coefficient
	χ	coefficient

1. Derive both the IS and LM function. Derive the equilibrium.
2. Formulate the targets of full employment and equilibrium on the government budget and write the model in deviations from the optimum.
3. Write the policy model in matrix notation, assuming that government expenditures and money supply are instrument variables.
4. Solve the model using Excel and using the results from question 3. Assume that $\gamma = 0.4$, $\iota = 105$, $\xi = 0.32, \tau = 0.32$, $\chi = 4.6$, $\alpha = 0.012$ million jobs per DFL 1 billion production value, $\underline{N}_s = 6.5$ milion workers. The rest of the data is in billions of guilders. Moreover $\underline{C} = 10$, $\underline{I} = 75$, $\underline{G} = 210$, $\underline{L} = -20$, $\underline{M} = 130$.

Case 3.2

Consider the economy modelled as:

$Y = C + I + G + X - F,$	Y	National income
$C = \gamma(Y - T) + \underline{C},$	C	Consumption
$I = -\iota R + \underline{I},$	I	Private investment
$G = \underline{G},$	G	Government spending
$T = \tau Y,$	T	Taxes
$X - F = \pi E - \mu Y + \underline{X},$	X	Exports
$K = \varepsilon R + \underline{K},$	F	Imports
$BB = X - F + K,$	K	Capital
$B = T - G,$	BB	Surplus balance of payments
$L = \xi Y - \chi R + \underline{L},$	B	Government surplus
$M = \underline{M},$	L	Demand for money
$N_d = \alpha Y,$	M	Supply of money
$N_s = \underline{N}_s.$	N_d	Demand for labour
	N_s	Supply of labour
	R	Interest rate
	V	Exchange rate
	γ	coefficient
	ι	coefficient
	τ	coefficient
	μ	coefficient
	ε	coefficient
	ξ	coefficient
	χ	coefficient
	α	coefficient

1. Derive the IS, LM, and BB function and the equilibrium.
2. Formulate the targets of full employment, equilibrium on the government budget, and equilibrium on the balance of payments. Write the model in deviations and in matrix form, assuming that the the supply of money, government spending and the exchange rate are instrument variables.
3. Estimate the necessary coefficients with Excel, using Appendix 3.1 and Appendix 3.2.
4. Solve the policy model using Excel (see Appendix 2.2).

Appendix 3.1: Linear regression with Excel[12]

This appendix shows how one can apply linear regression (OLS) with Excel.[13] Before we can estimate an equation by OLS we first need to put all the relevant data into the cells. Mostly the data on the different variables are all in separate colums or rows. We will show you what steps must be taken in order to estimate an equation. This will be done by using a numerical example. Assume that we want to estimate a relationship between the dependent variable Y and an explanatory variable X. Lets presume that the data on these variables are respectively:

Y	X
1	6
2	3
3	9
4	8

Moreover, we assume that all these data are in the cell range A1:B5 (the names of the variables Y and X are in the cells A1 and B1 respectively). After you have put the relevant data into the cells do the following:

- Go to the menu TOOLS
- Choose DATA ANALYSIS
- Choose REGRESSION

The cursor is now in the cell with INPUT Y RANGE.

- Give the range of cells which include the data on the dependent variable. Thus, in our case A2:A5. (You can also select the range with the mouse; the range is then directly indicated in the cell with INPUT Y RANGE).
- Go to the cell with INPUT X RANGE by pressing TAB or by using the mouse
- Give the range of cells which include the data on the explanatory variables. Thus, in our case B2:B5.
- Select LABELS only if you selected the names of the variables in the previous steps. This is not done in our case. So, we do not have to select this option.
- You can select whether the CONSTANT has to be zero or not. In our case: no.
- You can select the confidence level. In our case we do not want this.

12 See Person R., 1993. *Using Excel 5 for Windows: Special Edition.* Que.
13 OLS stands for Ordinary Least Squares.

- In the menu OUTPUT OPTIONS you can select an option regarding the regression results. If you select OUTPUT RANGE you must specify the range in which the results must be placed. Selecting NEW WORKSHEET PLY implies that the results are put in a new worksheet. Selecting NEW WORKBOOK implies that the results are included in a new workbook. We choose NEW WORKSHEET.
- In the menu RESIDUALS you can specify if the program must also provide the RESIDUALS, STANDARDIZED RESIDUALS, RESIDUAL PLOTS or LINE FIT PLOTS. We do not select one of these options.
- In the menu NORMAL PROBABILITY you can select the option NORMAL PROBABILITY PLOTS if you want to have these plots. In our case we do not select this option.
- Press: OKE

Now, if you go to the menu FORMAT, choose COLUMN, and choose AUTOFIT SELECTION, you can read the output better. We see that the estimated coefficients of the intercept and the X-variable are 0.642857 and 0.285714 respectively.

In the HELP-file you can also find more information on regression within Excel.

Appendix 3.2: Time series of macroeconomic variables

In the table below there are deflated data for the Dutch economy for the period 1970-1992. Consumption and taxes are deflated with the Consumer Price Index, while the other monetary values are deflated with the GDP deflator. Government expenditures include government consumption, government investment and premiums paid. Taxes consist of direct taxes and premiums.

Year	Y (GDP) Bln DFL	$\underline{M}$ Bln DFL	$\underline{G}$ Bln DFL	T Bln DFL	E DFL/US$	BB Bln DFL	U $\times$ 1 Mln	N_d $\times$ 1 Mln	P Index	$\dot{P}$ %/100
1970	313.9	67.2	109.5	84.7	3.62	1.5	0.05	4.55	100	
1971	327.4	71.6	119.6	94.2	3.50	-2.4	0.06	4.58	104	0.040
1972	337.5	76.9	121.1	101.1	3.21	-6.7	0.11	4.54	112	0.077
1973	353.5	70.6	129.1	112.6	2.78	-8.5	0.11	4.55	128	0.143
1974	367.9	72.6	139.5	120.9	2.69	-4.7	0.14	4.55	139	0.086
1975	367.2	78.8	149.1	125.1	2.52	-4.1	0.20	4.64	154	0.108
1976	386.4	77.5	156.2	127.9	2.64	-7.8	0.21	4.65	167	0.084
1977	395.0	83.0	157.8	131.7	2.45	-11.2	0.20	4.70	179	0.072
1978	405.2	82.1	167.2	142.4	2.16	-10.9	0.22	4.76	188	0.050
1979	414.6	81.2	175.0	143.3	2.01	-10.0	0.23	4.82	194	0.032
1980	418.3	81.5	175.4	143.5	1.99	-5.4	0.27	4.97	206	0.062
1981	415.6	75.4	176.8	144.2	2.48	0.9	0.41	5.07	217	0.053
1982	409.4	80.2	173.4	144.5	2.67	-0.2	0.58	5.01	231	0.065
1983	415.1	86.8	178.5	149.9	2.85	6.0	0.80	4.95	234	0.013
1984	428.1	90.9	174.9	143.9	3.20	-0.1	0.82	4.98	240	0.026
1985	439.4	93.8	174.6	147.0	3.29	-0.4	0.76	5.15	244	0.017
1986	448.0	99.5	171.0	150.8	2.44	-11.6	0.71	5.15	245	0.004
1987	462.8	109.4	182.5	160.1	2.02	-1.9	0.69	5.86	243	-0.008
1988	475.0	114.8	187.0	166.3	1.97	9.5	0.66	6.03	246	0.012
1989	497.1	122.1	183.4	164.0	2.12	20.3	0.60	6.16	249	0.012
1990	516.3	123.9	193.3	169.1	1.82	1.7	0.53	6.36	250	0.004
1991	528.2	126.1	201.5	183.4	1.86	-4.1	0.49	6.52	257	0.028
1992	537.4	128.9	208.1	181.6	1.75	-24.9	0.51	6.60	264	0.027

CHAPTER 4: AGGREGATE DEMAND AND AGGREGATE SUPPLY ANALYSIS

Study objectives

- to understand the difference between real values and nominal values
- to understand the difference between IS-LM models and AS-AD models
- to derive the Aggregate demand curve and to understand its meaning
- to derive the Aggregate supply curve and to understand its meaning
- to understand the Phillips curve
- to understand Okun's law
- to study the relationship between the AS curve and the Phillips curve
- to estimate coefficients for an open economy AS-AD model
- to solve AS-AD policy models for an open economy under different assumptions with the help of a data set

4.1 Introduction

flexible prices

In the previous chapter we discussed the general IS-LM framework. In this chapter we make some extensions to these models and make the transition to aggregate demand (AD) and aggregate supply (AS). This means that we relax the assumption of fixed prices and will consider flexible prices instead.

general price level

Before discussing the AD-AS model it is important to realize that the AD curve is actually not a 'regular' demand curve which is often used in partial analysis. In the latter case the partial price level is important, whereas for the AD curve the general price level is considered. This will be made clear when we discuss AD-AS in a closed economy in Section 4.2. Subsequently Section 4.3 examines AD-AS in an open economy.

4.2 AD-AS in a closed economy

basic IS-LM framework

We will start by showing how from the basic IS-LM framework the AD and AS curve can be derived. Assume that the economy is modelled like:

$Y = C + I + G,$	Y	National income
$C = \gamma(Y - T) + \underline{C},$	C	Consumption
$I = -\iota R + \underline{I},$	I	Private investment
$T = \tau Y,$	T	Taxes
$G = \underline{G},$	G	Government spending

$L = \xi Y - \chi R + \underline{L}$,	L	Demand for money
$M = \underline{M}$.	R	Interest rate
	γ	Marginal propensity to consume
	τ	Taxation rate
	ι	coefficient
	ξ	coefficient
	χ	coefficient

real and nominal amount of money

To explain the AD curve, we make a distinction between the nominal and real amount of money. The real amount of money M equals the nominal amount of money M_n divided by the price level: $M = M_n/P$, or $M_n = MP$. Starting point is the IS curve, which for the above model is:

$$Y = \frac{1}{1-\gamma(1-\tau)}[-\iota R + \underline{C} + \underline{I} + \underline{G}]. \tag{4.1}$$

The LM curve is:

$$R = \frac{\xi}{\chi}Y + \frac{1}{\chi}\left(\underline{L} - \frac{M_n}{P}\right). \tag{4.2}$$

Substituting (4.2) into (4.1) yields:

$$Y = \frac{1}{1-\gamma(1-\tau)}\left[-\iota\left[\frac{\xi}{\chi}Y + \frac{1}{\chi}\left(\underline{L} - \frac{M_n}{P}\right)\right] + \underline{C} + \underline{I} + \underline{G}\right], \tag{4.3}$$

which is equivalent to

aggregate demand

$$Y = \frac{1}{1-\gamma(1-\tau)+\iota\xi/\chi}\left[-\frac{\iota}{\chi}\left(\underline{L} - \frac{M_n}{P}\right) + \underline{C} + \underline{I} + \underline{G}\right]. \tag{4.4}$$

aggregate demand in a closed economy

This equation represents the AD curve. It shows combinations of the price level and real output at which the money and commodity markets are both in equilibrium (see Figure 4.1). The term $\iota\xi/\chi$ reflects the feedback from the monetary sector which reduces the traditional multiplier $1/(1-\gamma(1-\tau))$. If government spending increases then national income will increase. However, from the model discussed in Section 3.3 we also saw that when government spending rises, the interest rate tends to rise. Due to this, investments have a tendency to decrease which in its turn affects national income negatively.[1] Furthermore, from the AD curve we see that

crowding out

$$\frac{\partial Y}{\partial \underline{L}} < 0, \frac{\partial Y}{\partial M} > 0, \frac{\partial Y}{\partial M_n} > 0, \frac{\partial Y}{\partial P} < 0.$$

Regarding the real amount of money, we see that when the price level rises together with a constant nominal amount of money, the real amount of money decreases and the LM curve moves to the right. This is illustrated in Figure 4.1.

In the upper diagram the normal IS-LM diagram can be observed. In the lower diagram the AD curve is constructed. LM_1 is given for a certain price level P_1 and a certain income level Y_1. If the price level decreases, real amount of money increases (LM_1 curve moves to LM_2). In this situation price P_2 is lower than P_1, while output increases toward Y_2. In this way one can construct the AD curve.

AD-curve

aggregate supply

Now we will make the step to the AS curve. The AS curve shows the alternative quantities of real output that producers are able and willing to supply at all possible price levels. At this point we have to be aware of a crucial distinction which depends on the time horizon under consideration. There are two types of the AS curve: a long run and a short run AS curve.

Let us start with AS in the long run. Consider the classical model where it is assumed that the amount of output produced depends on the fixed amounts of capital and labour and on the available technology, i.e.

$$Y = F(\overline{K}, \overline{L}) = Y_{FE}, \tag{4.5}$$

1 This is called the *crowding out* effect, see Ch. 3.

long run aggregate supply

where Y_{FE} is the long run level of output.[2] According to this model we see that output does not depend on the price level P. This implies a vertical shape of the long run AS curve (LRAS), which is shown in Figure 4.2.

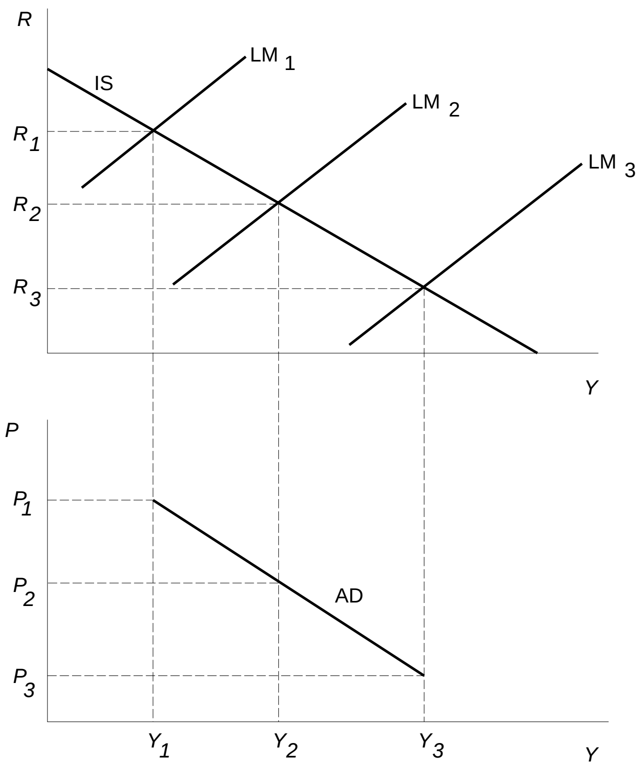

Figure 4.1: The aggregate demand curve.

full employment level of output

The price level is determined by the intersection of the AD curve with the LRAS curve. If the AS curve is vertical then changes in AD affect only prices but not output.[3] The long run level of output, Y_{FE} is called the natural level or full employment level of output. It is the level of output at which the economy's resources are fully employed or, at which unemployment is at its natural level.[4]

2 The suffix FE stands for 'full employment'.
3 In a first approximation, the monetary and real sectors of the economy do not affect each other in the long run. This is the principle of *dichotomy* (Burda & Wysplosz, 1993, p.181).
4 The natural rate of unemployment is the steady state rate of unemployment: the rate of unemployment towards which the economy gravitates in the long run. We call this full employment.

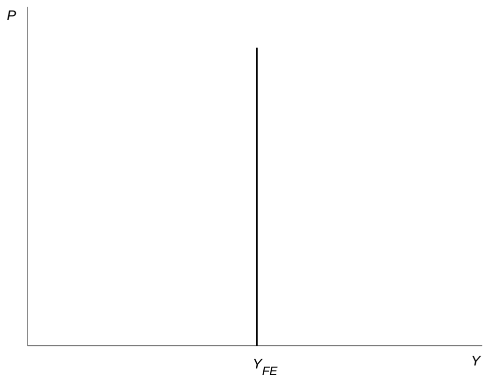

Figure 4.2: Aggregate supply in the long run.

short run aggregate supply

The AS curve in the short run shows the amount of output that business firms are willing to produce at different price levels. The AS curve in the long run shows that equilibrium in the labour market can be achieved at many different price levels but only at a single level of output.[5] In the short run, some prices are sticky.[6] In this case they do not adjust to changes in demand. This price stickiness implies that the short run AS curve (SRAS) is not perfectly vertical. Figure 4.3 illustrates this.

The SRAS curve is expressed by:

$$P = \beta(Y - Y_{FE}) + \underline{P}, \tag{4.6}$$

expected price level

where P is the price level, $\underline{P}$ is the expected price level, Y is actual real output, and Y_{FE} is the full employment output level. The equation relates the price level to the output level of the economy. The greater the extent to which the full employment level of the economy exceeds the output level, the lower the price level will be, and vice versa. Only in the case where the full employment level of the economy equals real output the price level P in the economy equals the expected price level $\underline{P}$.

5 The LRAS might shift due to investment expenditure changes. So fiscal policy may lead to both shifts of the AD and LRAS curve simultaneously. Within the Keynesian model only the expenditure effects of changes in government expenditure are taken into account. But now capacity effects also play a role.
6 Think of wages fixed by contracts.

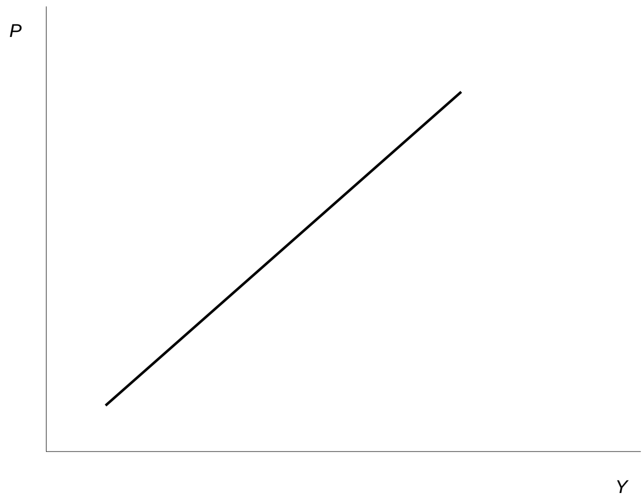

Figure 4.3: Aggregate supply in the short run.

Phillips curve

From AS it is a small step to derive the Phillips curve. The Phillips curve indicates the negative relationship between inflation and unemployment. The curve is illustrated in Figure 4.4.

Okun's law

To derive the Phillips curve we need Okun's law which states the negative relationship between unemployment and real GNP and is expressed as:

$$N_d - N_s = \alpha Y - \alpha Y_{FE},$$

$$-U = \alpha(Y - Y_{FE}),$$

$$Y - Y_{FE} = \frac{-U}{\alpha}, \tag{4.7}$$

where N_d is labour demand, N_s is labour supply, U is unemployment, and α is the labour output ratio. Equation (4.7) substituted in (4.6) yields:

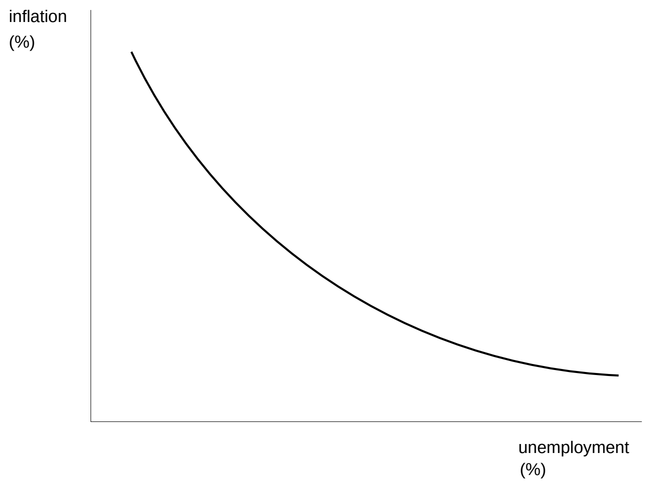

Figure 4.4: The Phillips curve.

$$P = \frac{-\beta}{\alpha} U + \underline{P}. \tag{4.8}$$

The variables in equation (4.8) are denoted in levels. The variables of the Phillips curve are in percentages. Therefore, (4.8) must be rewritten as follows:

$$\Delta P = \frac{-\beta}{\alpha} \Delta U + \Delta \underline{P},$$

$$\frac{\Delta P}{P} = \frac{-\beta}{\alpha} \frac{\Delta U}{P} + \frac{\Delta \underline{P}}{\underline{P}},$$

$$1 + \dot{P} = \frac{-\beta}{\alpha} \frac{N_s}{P} \frac{U}{N_s} + 1 + \underline{\dot{P}},$$

$$\dot{P} = \frac{-\beta}{\alpha} \frac{N_s}{P} \dot{U} + \underline{\dot{P}}, \tag{4.9}$$

expectations-augmented Phillips curve

where $\dot{P}$ for the rate of inflation, $\dot{U}$ is the unemployment rate, and $\underline{\dot{P}}$ is the expected rate of inflation. This curve is also called the expectations augmented Phillips curve.

Until now we have examined the IS-LM, AD, and AS curves in a more or less separate way. Now, we will relate the three concepts.

labour market

In Figure 4.5, the IS-LM diagram is in the upper panel and both the AS and AD curve are in the lower panel. Furthermore, the full employment output level Y_{FE} is indicated. At the output level Y_1 there is unemployment. In a free labour market this implies that wages will decrease. In turn this will cause decreasing prices. This means that the LM function will shift to the right. At the same time the AS curve will also shift to the right. Eventually full employment will be reached.[7]

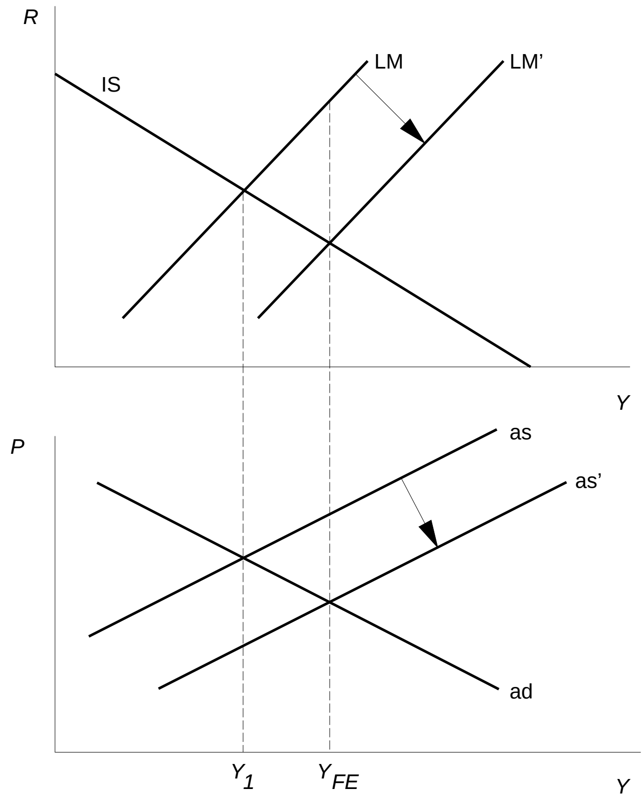

Figure 4.5: Aggregate demand and aggregate supply.

stagflation

oil shocks

This system functions with flexible wages and flexible prices. In the case of stagflation there is an upward shift of the AS curve, for example caused by an increase of wages. In this case prices will increase and at the same time output and employment tend to decrease. Another example why the SRAS curve may shift is due to sharp increases in

7 Because $M \uparrow \Rightarrow R \downarrow \Rightarrow I \uparrow \Rightarrow ED \uparrow \Rightarrow Y \uparrow$. It is assumed here that the capacity effect of investments is zero. So the LRAS remains fixed.

raw materials or energy prices. The oil shocks in the beginning of 1970s and the 1980s caused firms to produce less output at each price level. This implied a leftward shift of the SRAS curve.[8]

government budget included

What happens if we also take a government budget into account? When prices and wages are flexible government does not play an important role in the economy. The only thing it has to take care of is equilibrium on its budget. This is illustrated in Figure 4.6.

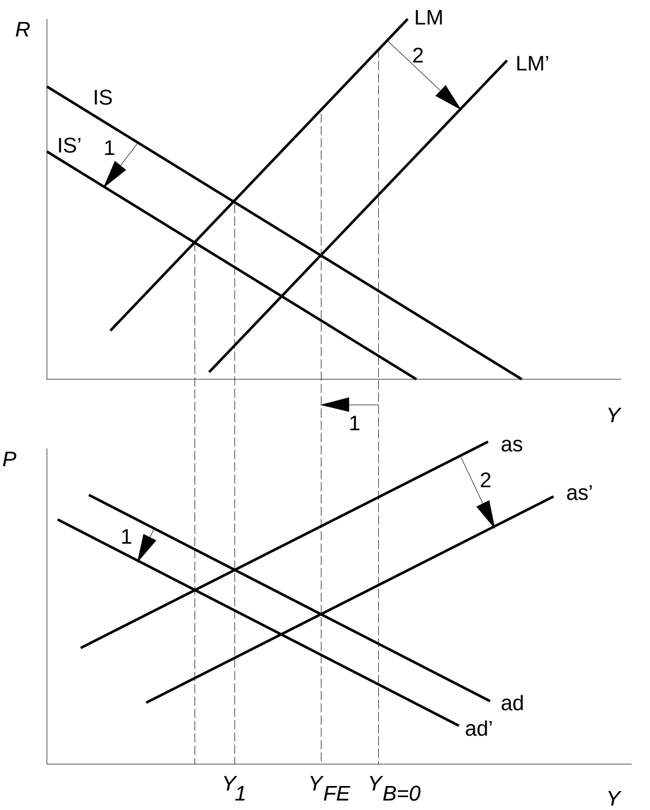

Figure 4.6: Government and flexible prices.

In Figure 4.6 government has to decrease its spending (1). In turn the IS curve and the AD curve shift to the left. Then, the mechanism

8 See Gordon, 1993.

equilibrium on government budget

works the same as in the previous case. Wages and prices decrease and the LM curve and AS curve move to the right (2). Finally we have full employment together with equilibrium on the government budget.

Questions and exercises

4.1 Why is the AD curve a decreasing curve?

4.2 Why is the AS curve an increasing curve?

4.3 What is the relationship between the AD curve and the Phillips curve? what is the difference between the two?

4.4 Why do you need Okun's law to explain the relationship between the AS curve and the Phillips curve?

4.5 Draw a Hicks-Hansen diagram together with a corresponding AS-AD diagram reflecting a labour deficit and a surplus on the government budget. Assuming flexible markets, what should the government do to arrive at equilibria both on the government budget and the labour market.

4.3 Aggregate Demand and Aggregate Supply in an open economy

the full model: open economy with government budget

We now consider an open economy with flexible prices. We recall from Section 3.5 the aggregate demand function (3.19), the function for the government budget B (3.20), the function reflecting unemployment U (3.21), and the function for the exchange rate E (3.17). To that model we add the price function, which gives the relationship between the inflation percentage $\dot{P}$, the exchange rate, and unemployment:[9]

$$\dot{P} = -\beta U + \nu E + \underline{\dot{P}}, \quad \beta < 0, \ \nu > 0 \tag{4.10}$$

inflation rate, unemployment, and exchange rate

where $\dot{P}$ is the rate of inflation, U is the level of unemployment, E is the exchange rate, and $\underline{\dot{P}}$ is the part of the inflation rate that is exogenous. This equation implies that the rate of inflation is negatively related to unemployment and positively to the exchange rate. The latter can be explained as follows. If the exchange rate rises imported goods become more expensive. This causes a rise of the general price level.

9 For practical reasons this function is not identical with the Phillips function, because it is the relationship between a percentage (price) and a level (unemployment), while the Phillips curve is a relationship between the inflation percentage and the unemployment percentage. Further the exchange rate is included here.

expectations of the public and government policy

The exogenous part $\underline{\dot{P}}$ of the inflation rate $\dot{P}$ reflects price expectations of the public. To keep things simple we assume that the expectations closely reflect the deliberate price and wage policy of the government, which means that the government now has an extra instrument to regulate the economy. The whole real model reads as follows (see also Chapter 3):

eq. (3.19)	$Y = \mathrm{A}\underline{M} + \mathrm{B}\underline{G} + \mathrm{C}BB + \mathrm{D},$	Y	Real income
eq. (3.20)	$B = \tau Y - \underline{G},$	$\underline{M}$	Real money supply
eq. (3.21)	$U = \underline{N}_s - \alpha Y,$	E	Exchange rate
eq. (3.17)	$E = \dfrac{BB}{\pi} + \mathrm{e}Y + \mathrm{f}\underline{M} + \mathrm{g},$	B	Government budget (real)
eq. (4.11)	$\dot{P} = -\beta U + \nu \underline{E} + \underline{\dot{P}}.$	$\underline{G}$	Real public spending
		BB	Balance of payments
		U	Unemployment
		$\underline{N}_s$	Labour supply
		$\dot{P}$	Inflation rate[10]

Written in deviations this is:

$$\Delta Y - \mathrm{A}\Delta\underline{M} - \mathrm{B}\Delta\underline{G} = \mathrm{C}\Delta BB + \Delta\mathrm{D},$$

$$-\mathrm{e}\Delta Y - \mathrm{f}\Delta\underline{M} + \Delta E = \frac{\Delta BB}{\pi} + \Delta\mathrm{g},$$

$$\tau\Delta Y - \Delta\underline{G} = \Delta B,$$

$$\alpha\Delta Y = \Delta\underline{N}_s - \Delta U,$$

$$\nu\Delta E + \Delta\underline{\dot{P}} = -\Delta\dot{P} + \beta\Delta U.$$

real value of variable

To compute the total real value of a variable at the end year we take the real value x_s at the starting year and add the computed real deviation. The total value x_e at the end year equals x_s plus Δx. To compute the nominal amounts from real amounts we have to inflate

10 $\dot{P} = \dfrac{P_t - P_{t-1}}{P_{t-1}}.$

nominal value of variable

the real numbers. This is done by multiplying the real numbers of the end year by the price level in the end year. The price level in the end year P_e is determined as follows:[11]

$\dot{P}_s = \dfrac{P_s - P_{s-1}}{P_{s-1}},$	$\dot{P}_s$	Inflation rate in the starting year[12]
$\dot{P} = \dot{P}_s + \Delta\dot{P},$	P_s	Price level (index/100) in starting year
$P_e = P_s(1+\dot{P}),$	P_{s-1}	Price level of the year before the starting year
	$\Delta\dot{P}$	Absolute change in inflation rate
	$\dot{P}$	Inflation rate
	P_e	Price level (index/100) in end year

So, the nominal value x_n of real variable x equals xP_e. For example, the nominal values of income, money supply and public spending in the end year are computed as follows:

$Y_n = Y_e P_e,$	Y_n	Nominal income in end year
$M_n = M_e P_e,$	M_n	Nominal supply of money in end year
$G_n = G_e P_e.$	G_n	Nominal public spending in end year

Questions and exercises

4.6 a. Assume that $\Delta M = 50$, and $M_s = 350$. Compute M_e.
b. Assume that $P_s = 250$, and $P_{s-1} = 235$. Compute $\dot{P}_s$.
c. Assume that $\Delta\dot{P} = 0.02$. Compute $\dot{P}$, taking into account the result of 4.6b.
d. Compute P_e, taking into account the value of P_s (see 4.6b) and the result of 4.6c.
e. Compute M_n, taking into account the value of M_e (see 4.6a.) and the result of 4.6d.

Case 4.1

1. Estimate the values of the coefficients of the real model in Section 4.3 with the help of Excel and Appendix 3.2;
2. Write the model in matrix form;

11 The end year is normally the year $s+1$. If our starting year is 1992, then our end year is 1993.
12 'Starting year' refers to 1992. To compute the inflation rate we also need P_{s-1}, the price level of the year before the starting year, in this case that is 1991.

3. Formulate the policy targets numerically (use the year 1992 as the starting point of the analysis) and assume that it is decided that the inflation rate should be 2% less than in the starting year 1992 $(\Delta\dot{P} = -0.02)$, and the level of unemployment should be 200.000 less than in 1992 $(\Delta U = -0.2)$. Further, the government deficit should be 0 in the end year.
4. Solve the model with the help of Excel and Appendix 2.2;
5. Compute the inflation rate and the price level in the end year;
6. Compute the nominal values of the variables from the real values with the help of the price level in the end year.

CHAPTER 5: THEORY OF THE BUSINESS CYCLE[1]

Study objectives:

- to be able to describe Schumpeter's theory of cyclical growth
- to know the difference between 'inventions' and 'innovations'
- to know what 'creative destruction' means
- to be able to describe the accelerator mechanism
- to be able to describe the multiplier-accelerator model
- to be able to indicate, on the basis of the values of the coefficients, what type of movement the multiplier accelerator model will make after a boost in the economy.
- to be able to give a definition of investments
- to know the difference between 'replacement investment' and 'extension/development investment'
- to know the difference between 'economic life span' and 'technical life span'
- to be able to distinguish between capital widening and capital deepening
- to be able to describe the effect of investments on both the business cycle and the production capacity
- to be able to make computations with the M-A-C model
- to be able to predict with the business cycle model what effects an economic impulse will have on structural and cyclical unemployment.

5.1 Long-term and medium-term fluctuations

On studying the development of the economic variables over time, it is clear that they are determined by long-term, medium-term and short-term fluctuations. In this chapter we explain the fluctuations in economic development in the medium and long term.

In the explanation of long-term development (Kondratieff fluctuations), Schumpeter's viewpoint is at the centre. In Section 5.2 his theory is discussed. Section 5.3 highlights the multiplier-accelerator model, which describes medium-term fluctuations, the so-called Juglars. In Section 5.4 and 5.5 the relation between capital, investment,

1 This chapter is a revised version of Chapters 5.1, 5.2 and 5.3 of: E.C. van Ierland, W.J.M. Heijman, E.P. Kroese and E.A. Oskam, 1990 (2nd ed.). *Leerboek Algemene Economie: Macro-economie.* Stenfert Kroese, Leiden.

and the business cycle is described. In Section 5.6 the multiplier-accelerator model is extended to the multiplier-accelerator-capital model. Finally, in Section 5.7, this model is applied to the labour market, where the phenomenon of cyclical unemployment is explained.

5.2 The long term: Schumpeter's theory on cyclical growth

cyclical growth

Cyclical growth refers to fluctuations of the national income, with cyclical fluctuations coupled to an upward trend. In this situation the cyclical fluctuations are coupled to an upward trend. It means that each peak is higher than the previous one and, also, that each slump is less deep than the previous one. This means that the upward movement lasts longer than the downward movement. Figure 5.1 shows the trend and the business cycle in a graph. The upward phase of the Kondratieff is characterized by endogenous innovations in the production process.

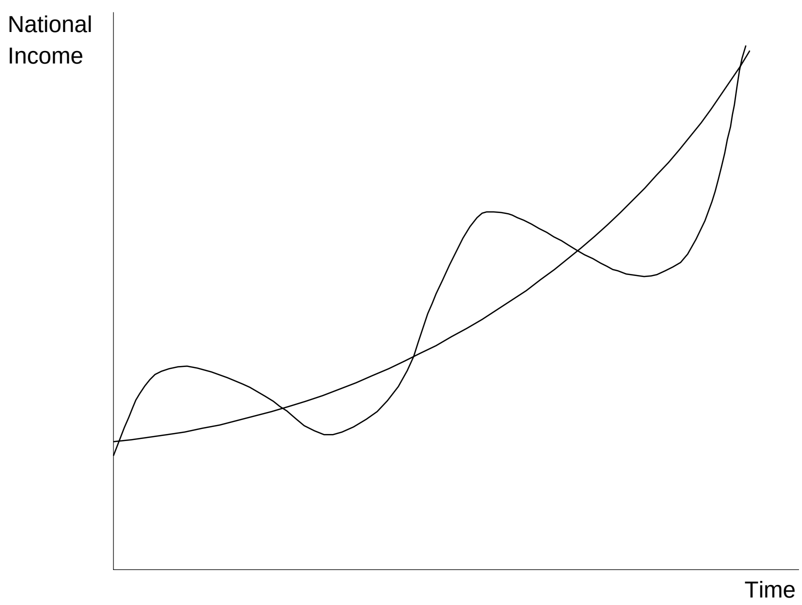

Figure 5.1: Long run cyclical development.

invention and innovation

In the view of Schumpeter, innovations play an important role. He differentiates between the invention itself, and its application in the economic process, the innovation. For instance, the invention of the steam engine has led to the adoption of numerous innovations in the production process.

In essence, innovations are a change in the production function that follows from ’basic innovations’: the adoption of new products and novel ways of production. An example is the use of the steam engine in the steam locomotive, the steamship and the steam pumping station. A more contemporary example is the use of the computer in the production process. From these ’major innovations’ spring ’minor innovations’, i.e. small adaptations of the production process. Innovations, according to Schumpeter, precede a period of economic boom and create the long- run fluctuation, the Kondratieff.

the role of the entrepreneur

Central in Schumpeter’s reflections is the role of the entrepreneur. Innovative entrepreneurs try to get a headstart on their competitors in order to realise a so-called ’advantage premium’ or a ’pioneer’s profit’. Because of the competitive rivalry, the pioneer’s advantage premium will slowly disappear. Entrepreneurs who refrained from introducing innovations will go bankrupt, which will partly detract from the initially positive effects of the innovation process on the national income. However, the initial expansion will exceed the subsequent contraction, so the income level will become higher. This is the phenomenon of cyclical growth according to Schumpeter.

An explanation for the regularity of waves of innovation is that they are related to the life cycle of capital goods. In a period of economic boom, innovations in the production of capital goods are implemented. In a period of economic decline this capital technically or economically wears out. However, in the absence of application opportunities, innovation opportunities accumulate, thus creating the climate for a new boom.

creative destruction

Through the innovation process, characterized by Schumpter as a process of ’creative destruction’, innovations are continuously implemented, thereby eliminating obsolete production methods. The new is built up and the old is demolished. The chance that newly adopted means of production will rapidly become economically obsolete makes investing a risky business. This risk can be diminished by acquiring as large a market share as possible. Thus, the competition mechanism in the innovation process, with its inherent uncertainties for entrepreneurs, leads to the formation of conglomerates.

Questions and exercises

5.1 What causal function, according to Van Duijn, do innovations have in the development in the economic trend? Describe how this process leads to a situation of economic boom.

5.2 What does Schumpeter consider ’inventions’ and what ’innovations’?

5.3 What types of innovation does Schumpeter distinguish?

5.4 Describe the development of the economic trend (business cycle) according to Schumpeter.

5.5 What does Schumpeter mean by 'creative destruction'?

5.3 The medium-term business cycle: the multiplier-accelerator (M-A) model

interaction between multiplier and accelerator

An important model explaining medium-term economic cycles is the dynamic model developed in 1939 by Samuelson and later elaborated on by Hicks. The model became known as the Samuelson & Hicks multiplier-accelerator model. It describes the interaction between the workings of the multiplier and the so-called acceleration mechanism.

investments partly endogenously determined

In the multiplier-accelerator model (M-A model), investment plays an important role in explaining the economic cycle. Investment in the model is partly determined endogenously by the increase in the national income. This acceleration principle is based on the consideration that entrepreneurs base their decisions on investment (with regard to extending the stock of capital goods) on the increase in production. Table 5.1 shows this principle. From this table it can be seen that the net national income in years 2, 3 and 4 show an increase of 50 billion.

Table 5.1: The acceleration principle

Year	Net National Income	Increase Net National Income	Net Investment
1	250	0	0
2	300	50	0
3	350	50	25
4	400	50	25

The end result is a relationship between the increase in net national income and net investment. In our example, net investment is assumed to equal half the increase in net national income (of last period) at a one year time lag. In the M-A model the multiplier effects of an increase in investment is enhanced by the accelerator mechanism. In this process an interaction takes place between multiplier and accelerator, which can be shown as follows: increase in income → investments (through accelerator) → increase in income (through multiplier) → investments (through accelerator). It is obvious that the system also works in the opposite direction, in which case the accelerator also enhances the workings of the multiplier mechanism. We will now give an exact description of the model.

In the model the effective demand ED_t equals consumptive spending C_t plus the net investments I_t:

Effective Demand

$$ED_t = C_t + I_t. \tag{5.1}$$

Consumption consists of an endogenous and an exogenous part. The endogenous part depends on the national income of the last period Y_{t-1}. The macroeconomic consumption function is then:

consumption function

$$C_t = \gamma Y_{t-1} + \underline{C}_t, \tag{5.2}$$

in which γ is the marginal propensity to consume. Endogenous investment depends on the increase in the national income for a one period time lag (ΔY_{t-1}). The increase in the national income for a one period time lag is per definition equal to: $Y_{t-1} - Y_{t-2}$. Investment depends on the increase in national income, since we assume that an increase in production will lead to an increase in the production capacity desired. So the stock of capital increases, which leads to an increase in production capacity. The one-period delay refers to the time it takes entrepreneurs to react to changing circumstances. Including the exogenous part of the investment, the investment function now is indicated as

investment function

$$I_t = a(Y_{t-1} - Y_{t-2}) + \underline{I}_t. \tag{5.3}$$

In equation (5.3) the reaction coefficient a indicates the accelerator. The autonomous part of the investment $\underline{I}_t$ can be explained by the more or less optimistic outlook of the producers. If they are optimistic the autonomous investment will be relatively high. If they are pessimistic they will be relatively low, and can even be negative.

Lastly, equation (5.4) reflects the equilibrium condition. As we know, in a Keynesian model there is equilibrium in income if production equals the effective demand.

equilibrium condition

$$Y_t = ED_t. \tag{5.4}$$

We can now solve the model for the production Y_t by substitution:

reduced form equation

$$Y_t = (\gamma + a)Y_{t-1} - aY_{t-2} + \underline{C}_t + \underline{I}_t. \tag{5.5}$$

the discriminant of the characteristic equation

Equation (5.5) is a second-order difference equation. If we know the value of Y_{t-1} and Y_{t-2} and also that of $\underline{C}_t$ and $\underline{I}_t$ for each year the value of Y_t can be calculated. Depending on the value of the marginal propensity to consume γ and the accelerator a, the time path of the production shows different forms. The sequence shows oscillations if the discriminant of the characteristic equation is smaller than 0, or if: $(\gamma + a)^2 - 4a < 0$ or $\gamma < 2\sqrt{a} - a$. [2] Damped oscillations result if the accelerator is smaller than one. An unbounded pattern occurs if the accelerator is larger than one. The sequence is a monotone if $a = 1$.

We now assume that the economy is in equilibrium in period 0. We define equilibrium as the situation when all variables are stationary. In other words: $Y_t = Y_{t-1} = Y_{t-2}$. Substituting this equilibrium condition in the reduced form equation gives an expression for Y_0:

$$Y_0 = \frac{1}{(1-\gamma)}(\underline{I}_t + \underline{C}_t). \tag{5.6}$$

Now let's assume that $\underline{I}_t$ is equal to 10 and $\underline{C}_t$ is equal to 20. Setting the marginal propensity to consume γ to 0.8, in period 0 production equals 150. Then, from period 1, we assume a constant investment impulse of 10. Different movement patterens are created depending on the value of the accelerator. Table 5.2 shows movement patterns for specific values of a.

As an example, we now illustrate in Table 5.3 variation in output over time if a is equal to one. This table shows the business cycle associated with this value of a. The average increase in income can be calculated with the help of equation (5.6). Assume autonomous investment of 20 instead of 10, which gives a production of 200. This is 50 more than in the original situation, so that a constant investment impulse of 10 from period 1 will lead to an average increase in output of 50.

---

2 For the characteristic equation, see Appendix 5.1.

Table 5.2: Five possible values of the accelerator with its related movement patterns ($\gamma = 0.80$).

a	Presence of oscillations	Boundedness
0.25	$\gamma > 2\sqrt{a} - a$, so: no oscillations	$a < 1$, so: damped
0.80	$\gamma < 2\sqrt{a} - a$, so: oscillations	$a < 1$, so: damped
1.00	$\gamma < 2\sqrt{a} - a$, so: oscillations	$a = 1$, so: monotone
1.30	$\gamma < 2\sqrt{a} - a$, so: oscillations	$a > 1$, so: unbounded
3.00	$\gamma > 2\sqrt{a} - a$, so: no oscillations	$a > 1$, so: unbounded

Table 5.3: The development of production with a continuous investment push of 10 from period 1 onwards ($a = 1$, $\gamma = 0.8$).

t	$Y_t = 1.8Y_{t-1}$	$-Y_{t-2}$	$+\underline{C}_t$	$+\underline{I}_t$
0	150.0 = 270.0	- 150.0	+ 20.0	+ 10.0
1	160.0 = 270.0	- 150.0	+ 20.0	+ 20.0
2	178.0 = 288.0	- 150.0	+ 20.0	+ 20.0
3	200.4 = 320.0	- 160.0	+ 20.0	+ 20.0
4	222.7 = 360.7	- 178.0	+ 20.0	+ 20.0
5	240.5 = 400.9	- 200.4	+ 20.0	+ 20.0
6	250.2 = 432.9	- 222.7	+ 20.0	+ 20.0
7	249.8 = 450.4	- 240.5	+ 20.0	+ 20.0
8	239.5 = 449.6	- 250.2	+ 20.0	+ 20.0
9	221.3 = 431.1	- 249.8	+ 20.0	+ 20.0
10	198.8 = 398.3	- 239.5	+ 20.0	+ 20.0
11	176.6 = 357.8	- 221.3	+ 20.0	+ 20.0
12	159.0 = 317.9	- 198.8	+ 20.0	+ 20.0
13	149.7 = 286.2	- 176.6	+ 20.0	+ 20.0
14	150.4 = 269.5	- 159.0	+ 20.0	+ 20.0
15	161.0 = 270.7	- 149.7	+ 20.0	+ 20.0

With Table 5.3 and equations (5.2) and (5.3) the development of consumption and investments may be determined. Table 5.4 shows the results over a 4-year period.

accelerator disturbs equilibrium

To conclude this section, we show the oscillation pattern of output in Figure 5.2 for all the accelerator values of Table 5.2. As the marginal propensity to consume holds the same value in all cases (0.8), it can be concluded that the accelerator disturbs equilibrium. As a increases the more vehement is the pattern of output.

Table 5.4: The development of production, consumption and investment with a continuous investment push of 10 from period 1 onwards ($a = 1, \gamma = 0.8$).

t	$Y_t = C_t + I_t$	$C_t = 0.8Y_{t-1} + 20$	$I_t = (Y_{t-1} - Y_{t-2}) + \underline{I}_t$
0	150.0	140.0	10.0
1	160.0	140.0	20.0
2	178.0	148.0	30.0
3	200.4	162.4	38.0
4	222.7	180.3	42.4
5	240.5	198.2	42.3
etc.	*etc.*	*etc.*	*etc.*

Questions and exercises

5.6 Explain the meaning of the accelerator.

5.7 Describe the implications of the accelerator to the movement pattern of the variables in the M-A model.

5.8 Consider the following model:

$$Y_t = C_t + I_t,$$

$$C_t = 0.7Y_{t-1} + 40,$$

$$I_t = 0.5(Y_{t-1} - Y_{t-2}) + 30.$$

a. Calculate the reduced form equation for the production.
b. What type of movement pattern will production show as a result of a consumption impulse?
c. Calculate the value of the production, consumption and investments for the first 5 periods in the case of a constant consumption impulse of 10 in period 1.

5.4 Capital and the business cycle

In Section 5.3 we dealt with a simple theory of the business cycle. In our discussion the supply side was almost fully neglected. For instance, an important variable like production capacity is not included in the multiplier-accelerator model. Yet, the utilization of the production capacity is an important business cycle indicator.

over- and underutilization

Situations of over-utilization of the production capacity usually alternate with situations of under-utilization. In the case of over-utilization the production capacity is in fact too small to meet the demand for end-products, and in the latter case the production capacity is too large. This remaining of this chapter is especially devoted to the

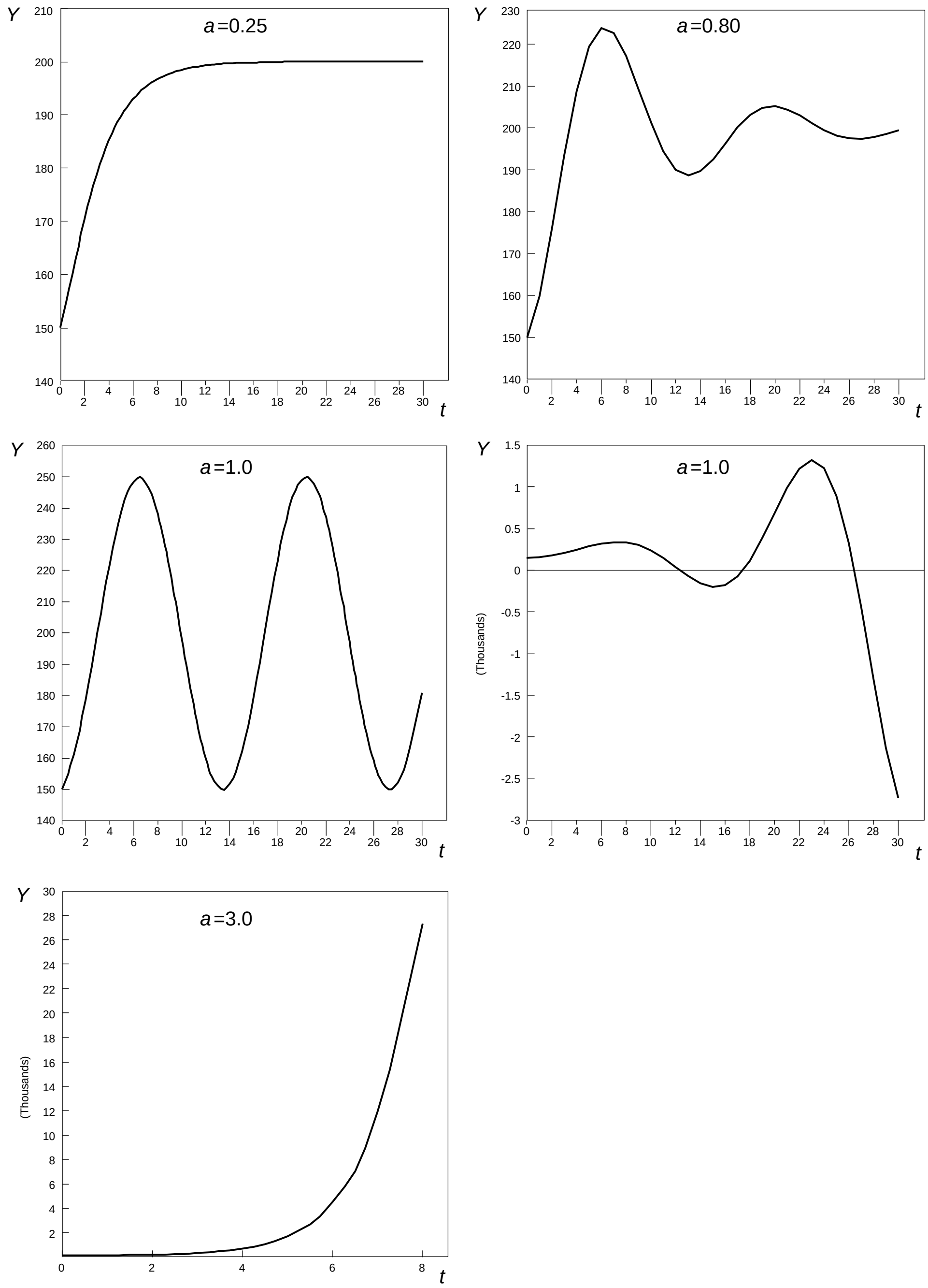

Figure 5.2: Development patterns at different values of a.

interaction between the demand side and the supply side of the economic process. The production capacity is mainly determined by the volume of the capital stock, which is dependent on investment. So, investment plays a role in both the supply and the demand side of the economy.

5.5 Investments

Investment is the production of capital goods in order to maintain and expand the production process. Capital goods consist, for instance, of machines and buildings. Also stocks of raw materials and manufactured goods are considered as capital goods.

depreciation

Annual investment is divided into replacement and expansion investments. The replacement investments are intended to replace obsolete capital goods. If a machine is fully worn there must be enough money available, through depreciation for wear and tear, to be able to purchase new machinery.

wear and obsolescence

When we talk about the wear of capital goods we do not just mean the technical wear but also the so-called obsolescence. The latter term indicates that machines for example can become obsolete because of changes in the production process. Technological innovations can force an entrepreneur - for reasons of competition - to buy new machines before the old ones have become technically worn. In that case we speak of the *economic* life span of a capital good, in contrast with its *technical* life span. Naturally, the economic life span is always shorter, but never longer than the technical life span.

economic and technical lifespan

If we decrease total annual investment, also known as gross investment, by replacement investments, then what is left is the expansion investment or net investment. The latter are considered to be the annual increase in the stock of capital goods.

capital widening and capital deepening investment

degree of mechanization

Besides the division into gross and net investments we can also distinguish between capital widening and capital deepening. Capital widening means that the stock of capital goods is augmented by adding the same type of capital goods as already in use. This implies that the quantity of capital per unit of labour (the capital intensity or degree of mechanization) remains constant. Capital deepening, on the other hand, refers to investments necessary to save on the quantity of labour per unit of capital. So, by capital deepening the quantity of capital needed per unit of labour increases. This is also termed the capital intensity increase or mechanization rate. If, for instance, a manufacturer extends his existing machinery of in total ten machines – each of which is served by two operators – by two identical machines, and in the event creating four new jobs, one can speak of capital deepening. Clearly, in the short term capital widening is more effective for the creation of jobs than capital deepening.

capital deepening and employment

Whether or not in macro-economic terms this is also true, largely depends on the international competitiveness of the branch of industry invested in. If, by capital deepening a company's competitiveness has improved, then it is likely that after some time more workers will be employed in this branch of industry than would have been in the case of capital widening. It may even be true that an entrepreneur who opts solely for capital widening, in the long run will do employment a disservice, because he is likely to collapse under the pressure of international competitiveness.

effects on incomes and output of investments

Investments are part of the effective demand and influence, by way of (the workings of) the multiplier, the volume of the national income. This effect is also known as the business cycle effect or demand effect of investments. Like investments, naturally consumptive spending has a similar income effect. In addition, investments have also a capacity effect. This means that investments influence the volume of the stock of capital goods and, hence, the production capacity. Production capacity refers to the quantity of product that can be produced with the given stock of capital goods. To what extent this potential stock of capital goods is made use of is decided upon on the demand side of the economy. The capacity effect is also called the structural effect of investments.

Questions and exercises

5.9 Assume that a manufacturer has purchased a certain machine that, technically speaking, will function for 10 years. The manufacturer assumes that the machine will become obsolete after 5 years, because by then a new manufacturing process will have been developed for his product. Which statement is correct?

a. A machine's economic life span is 10 years and its technical life span is also 10 years.
b. The economic life span is 5 years and the technical life span is also 5 years.
c. The economic life span is 10 years and the technical life span is 5 years.
d. The economic life span is 5 years and the technical life span is 10 years.

5.10 Which statement is correct?

a. The difference between gross investment and capital widening is depreciation.
b. Expansion investment and the depreciation together form capital-intensive investment.
c. Net investment is equal to gross investment minus depreciation.
d. Net investment and replacement investment together form depreciation.

5.11 Briefly explain why capital deepening (capital-intensive investment) is often necessary for maintaining long-term employment.

5.6 Multiplier-Accelerator-Capital (M-A-C) Model

difference between the simple M-A model and the M-A-C model.

The capacity effect of investments is the effect of investments on the production capacity. One could argue against the multiplier-accelerator model from Section 5.3 that it does not take into account the capacity effect of investments. The M-A-C model does. This model shows some affinity with the views of the Polish economist M. Kalecki (1899-1970), who put his ideas in writing before Keynes' 'General Theory' (1936) was published. Like in the M-A model, in this model the effective demand ED_t equals consumption plus net-investment.

effective demand

$$ED_t = C_t + I_t. \tag{5.7}$$

Further we assume that consumption is partly induced by (net) income, and partly autonomous. For simplicity, we have taken income in the consumption function without a lag, in contrast with the M-A model.

consumption function

$$C_t = \gamma Y_t + \underline{C}_t. \tag{5.8}$$

We also assume in the business cycle model that income is always in equilibrium, which means that production Y_t always equals effective demand ED_t.

income equilibrium

$$Y_t = ED_t. \tag{5.9}$$

From equations (5.7), (5.8) and (5.9) the equation of the demand-side in the model can be deduced.

demand equation

$$Y_t = \frac{1}{1-\gamma}(\underline{C}_t + I_t). \tag{5.10}$$

We will now discuss the supply-side of the economic process. If the production capacity Y_t^* is smaller than the actual production Y_t, then there is over-utilization of the production capacity. Entrepreneurs will pursue to counteract this situation by way of investment. In a situation of under-utilization, however, when production capacity exceeds actual production, investment will be curbed. The above consideration leads to the investment function:

investment function

$$I_t = b(Y_{t-1} - Y^*_{t-1}) + \underline{I}_t. \tag{5.11}$$

Coefficient b indicates the extent of the entrepreneurs' reaction to the utilization situation. As in the M-A model, the autonomous investment $\underline{I}_t$ depends on the entrepreneur's more or less optimistic view of future developments. The production capacity is increased by way of the net investments. We call this the capacity effect of the investments.

the capacity effect of investments

The production of an additional unit of output requires the input of a certain quantity of capital. This quantity is called the capital coefficient κ. So, the increase in production capacity ΔY^*_t is equal to the net investments divided by the capital coefficient.

$$\Delta Y^*_t = Y^*_t - Y^*_{t-1} = \frac{I_t}{\kappa}. \tag{5.12}$$

The equations (5.10), (5.11) and (5.12) together form the M-A-C model. With these equations we can determine, by substitution, the reduced form equation for the production capacity (see Appendix 5.2). This reduced form equation is as follows:

the reduced form equation

$$Y^*_t = \left(1 + \frac{b}{(1-\gamma)} - \frac{b}{\kappa}\right) Y^*_{t-1} - \frac{b}{(1-\gamma)} Y^*_{t-2} + \\ + \frac{1}{\kappa}\underline{I}_t + \frac{b}{\kappa(1-\gamma)}\underline{C}_{t-1}. \tag{5.13}$$

We demonstrate by means of an example how the model works. We assume the following values of the coefficients: $\kappa = 2$, $\gamma = 0.8$, $b = 0.2$. Equation (5.13) will then change into:

$$Y^*_t = 1.9Y^*_{t-1} - Y^*_{t-2} + 0.5\underline{I}_t + 0.5\underline{C}_{t-1}. \tag{5.14}$$

Assume further that in the starting situation (period 0), the autonomous components of the consumption and investments are respectively 20 and 2, and also that in this situation the model is in equilibrium, implying that all variables are stationary $(Y^* = Y^*_{t-1} = Y^*_{t-2})$.

effects of a boost in investments

With equation (5.14) we can now calculate that the production capacity Y_0^* in period 0 equals 110. With this given and with equations (5.10) and (5.12) we now determine the income level Y_0 in period 0. In a stationary situation ΔY_t^* equals zero. Therefore it follows from (5.12) that in period 0 the net investments equal zero. Given that $\underline{C}_t$ equals 20, we can determine with equation (5.10) the level of income in the starting situation. This appears to be 100.

We now assume that from period 1 a maintained investment impulse of 2 is given. Now the average production capacity in the periods following the starting situation is to be calculated in the same manner as the production capacity in period 0. Instead of the value of 2 for the autonomous investments we now substitute a value of 4. The average production capacity appears to have increased from 110 to 111. Also the average value of the income in the periods to follow (subsequent periods) are to be determined in the same way as for period 0. With the difference equation (5.14) it can be calculated how the production capacity develops. With equations (5.14), (5.10) and (5.12) the production value for each period can be found. Figure 5.3 shows the development of both variables.

As already known, we define ’structure’ as the volume and composition of the production capacity as determined by the stock of capital goods. Hence, the development of the production capacity, as shown in Figure 5.3, is to be regarded as the development of the economic structure. The state of the business cycle is to be measured from the degree of utilization of the existing production capacity. A standard of the development of the business cycle is the degree of capacity utilization q_t. The latter is equal to the production Y_t divided by the production capacity Y_t^* times 100:

$$q_t = \frac{Y_t}{Y_t^*}100. \tag{5.15}$$

Table 5.5 shows the development of the degree of capacity utilization in the event of a generated investment impulse for 9 periods.

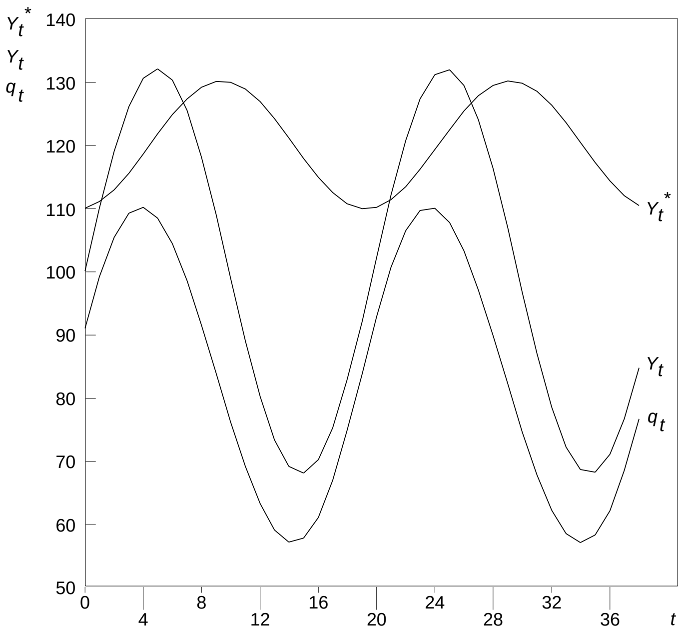

Figure 5.3: Production and production capacity in the M-A-C model.

Table 5.5: Development of the capacity utilization as a result from an investment impulse in the M-A-C model.

t	0	1	2	3	4	5	6	7	8	9
Y_t	100,0	110,0	119,0	126,0	131,0	132,0	130,0	125,0	119,0	108,0
Y_t^*	110,0	111,0	112,9	115,5	118,6	121,8	124,8	127,3	129,2	130,0
q_t	91,0	99,0	105,0	109,0	111,0	108,0	104,0	98,0	92,0	83,0

boom and recession

If the degree of utilization increases, the market is booming. In the instance of a diminishing degree of utilization we speak of a recession. Figure 5.3 shows the different stages of the business cycle.

Questions and exercises

5.12 Describe the concept of "production capacity".

5.13 When does one speak of over-utilization?

5.14 How will entrepreneurs react to a situation of under-utilization?

5.15 Describe accurately how a consumption impulse on the demand side of the business cycle model will affect the supply-side.

5.16 In what respect does parameter b in the business cycle model deviate from the accelerator in the multiplier-accelerator (M-A model)?

5.17 Indicate with the help of the reduced form equation for the production capacity from the business cycle model when an explosive situation in the system occurs. Explain also why in our example of the investment impulse a monotonous fluctuation occurs.

5.7 Cyclical unemployment

An important issue in modern day society is that of unemployment. Unemployment U_t is defined as the difference between labour supply L_t^s and labour demand L_t^d, which is employment. We consider labour supply to be exogenous $L_t^s = \underline{L}_t^s$. Hence:

the concept of unemployment

$$U_t = \underline{L}_t^s - L_t^d. \tag{5.16}$$

Labour demand is linked to the production Y_t, because for one unit of product, a certain quantity of labour is required. This quantity of labour is denoted the labour coefficient α. This relationship between labour demand and production is therefore to be formulated as:

labour demand

$$L_t^d = \alpha Y_t. \tag{5.17}$$

labour demand employment capacity

In addition to labour demand, the *employment capacity* L_t^* also plays a role. The difference between employment and employment capacity could be drawn as follows. For example, a company has enough capital (buildings, equipment) to employ 100 workers. In this case the employment capacity equals 100 jobs. However, the demand for the product produced may be so low that only 50 workers can be employed by this particular company. In that case employment equals 50. So, generally speaking the employment capacity equals the number of jobs when the production capacity is fully utilized, where the employment equals the actual number of jobs.

Employment capacity depends on production capacity Y_t^*. If a company can produce 1000 units of product a year and the labour coefficient α equals 0.5, then the employment capacity is: 0.5 x 1000 = 500. The relationship between production capacity Y_t^* and the employment capacity L_t^* can be shown as:

$$L_t^* = \alpha Y_t^*. \quad (5.18)$$

cyclical and structural unemployment

On the basis of the discussion above, unemployment can be divided into a structural and a cyclical component. Structural unemployment U_t^* is to be characterised as unemployment caused by a lack of employment capacity. Cyclical unemployment U_t^c can be characterized as unemployment caused by a lack of effective demand. In the case of cyclical unemployment, effective demand is too low to utilize total employment capacity. Equations (5.19) and (5.20) reflect the two types of unemployment.

$$U_t^* = \underline{L}_t^s - L_t^*, \quad (5.19)$$

$$U_t^c = L_t^* - L_t^d. \quad (5.20)$$

Further it holds that:

$$U_t = U_t^* + U_t^c. \quad (5.21)$$

The addition of the right hand terms of (5.19) and (5.20) indeed results in the right hand term of equation (5.16). With equations (5.17) and (5.18) we rewrite equation (5.19) as:

$$U_t^* = \underline{L}_t^s - \alpha Y_t^*. \quad (5.22)$$

effects of an investment impulse on unemployment

Table 5.6 shows the development of unemployment in the example of an investment impulse we gave in the previous paragraph, and the results of which are partially shown in table 5.5. In this table we assume that labour supply is 60, and that the labour coefficient is 0.5.

In Table 5.6 unemployment is sometimes negative. This can be interpreted as a tight labour market: labour supply is smaller than labour demand. Figure 5.4 reflects the development of the different types of unemployment in a graph.

Table 5.6: Development in structural and cyclical unemployment over time.

$t \rightarrow$	0	2	4	6	8	10	12
Y_t	100.0	119.0	131.0	130.0	119.0	98.0	79.0
Y_t^*	110.0	112.9	118.6	124.8	129.2	129.8	126.5
$L_t^s = \underline{L}_t^s = 60$	60.0	60.0	60.0	60.0	60.0	60.0	60.0
$L_t^d = 0.5Y_t$	50.0	59.5	65.5	65.0	59.5	49.0	39.5
$L_t^* = 0.5Y_t^*$	55.0	56.5	59.3	62.4	64.6	64.9	63.3
$U_t^* = \underline{L}_t^s - L_t^*$	5.0	3.5	0.7	-2.4	-4.6	-4.9	-3.3
$U_t^c = L_t^* - L_t^d$	5.0	-3.0	-6.2	-2.6	5.1	15.9	23.8
$U_t = U_t^* + U_t^c$	10.0	0.5	-5.5	-5.0	0.5	11.0	20.5

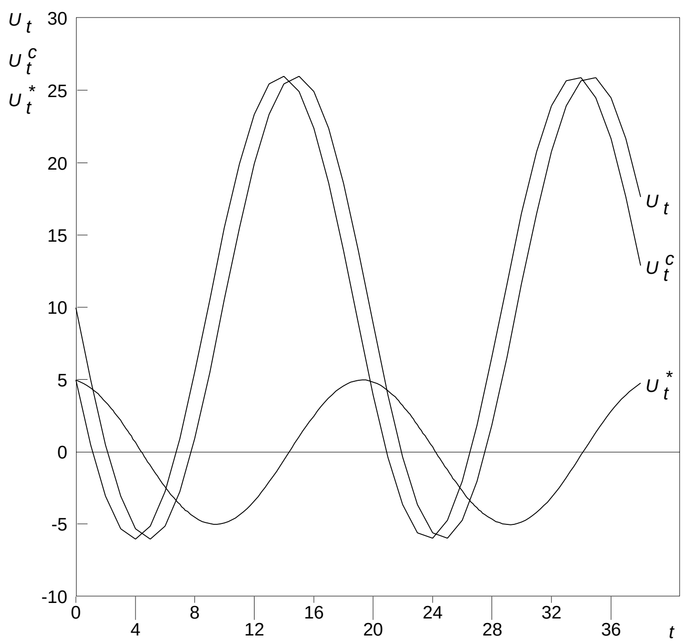

Figure 5.4: Cyclical and structural unemployment.

Questions and exercises

5.18 Calculate for 5 years what the effects would be on both employment capacity and employment if in the M-A-C model, a permanent consumption impulse of 5 had been given instead of an investment impulse from period 1. Calculate further what effects this intervention has on the cyclical and structural unemployment.

Appendix 5.1

The characteristic equation belonging to the homogenous part of the second-order differential equation described, reads as follows:

$$\lambda^2 - (\gamma + a)\lambda + a = 0.$$

The characteristic roots are:

$$\lambda_1 = \frac{(\gamma + a) + \sqrt{(\gamma + a)^2 - 4a}}{2},$$

$$\lambda_2 = \frac{(\gamma + a) - \sqrt{(\gamma + a)^2 - 4a}}{2}.$$

Depending on the value of the characteristic roots, and so depending on the marginal propensity to consume and the accelerator, the time path of net production Y_t takes different forms. Here we limit ourselves to this short explanation of the characteristic equation. For an extensive description of how to solve second-order differential equations we refer to the existing literature in this field.

Appendix 5.2

The deduction of equation (5.13) runs as follows. From equation (5.12) it follows:

$$I_t = \kappa(Y_t^* - Y_{t-1}^*).$$

With this expression we can now eliminate I_t from (5.10) and (5.11):

$$Y_t = \frac{1}{1-\gamma}\{C_t + \kappa(Y_t^* - Y_{t-1}^*)\},$$

$$\kappa(Y_t^* - Y_{t-1}^*) = b(Y_{t-1} - Y_{t-1}^*) + \underline{I}_t. \qquad (\#)$$

From this it follows:

$$Y_{t-1} = \frac{1}{1-\gamma}\{\underline{c}_{t-1} + \kappa(Y^*_{t-1} - Y^*_{t-2})\}.$$

The last equation substituted in (#) gives (5.13).

CHAPTER 6: GROWTH THEORY[1]

Study objectives

- To know what the meaning is of steady state growth
- To understand the concept of decoupling
- To be able to compute the ’doubling’ time of a variable
- To be able to explain the Harrod-Domar model
- To know the concept of growth equilibrium
- To understand the relationship between economic development and savings in the Harrod-Domar model
- to understand the relationship between (un)employment and economic growth in the Harrod-Domar model
- To be able to explain the neoclassical growth model
- To be able to indicate the differences between the Harrod-Domar model and the neoclassical model
- To understand the concept of ’steady state’ in the neoclassical growth model
- To understand the relationship between steady growth and technological change in the neoclassical model
- To be able to explain the relationship between population growth and economic growth in the neoclassical model

6.1 Introduction

economic growth

Economic growth has been the subject of study for almost as long as economic science exists, albeit to a greater or lesser extent in different periods of time. For example, during the period 1870-1920 economic growth received hardly any attention, as the focus was predominantly on marginalistic theory. Most growth theories have been developed after the second World War and this is undoubtedly related to the rapid economic growth that occurred after the war. In this chapter we discuss the Harrod-Domar growth model, developed in the 1940s by the British economist Harrod and the American Domar.

1 This chapter is a revised version of Chapters 5.4 and 5.5 of: E.C. van Ierland, W.J.M. Heijman, E.P. Kroese and E.A. Oskam, 1990 (2nd ed.). *Leerboek Algemene Economie: Macro-economie.* Stenfert Kroese, Leiden, and Chapter 21 of E.C. van Ierland, W.J.M. Heijman, E.P. Kroese and E.A. Oskam, 1994. *Grondslagen van de macro-economie,* 1994. Stenfert Kroese, Houten.

Keynesian growth model

The model has a Keynesian character. The key variable in a Keynesian growth model is the effective/actual demand. In the 1950s and 1960s, as a reaction to the relative limitations of the Keynesian growth model, the neoclassical growth model was developed (mainly) by the American Robert Solow. Contrary to the Harrod-Domar growth model, the neoclassical model has the price mechanism as equilibrium variable.

Section 6.2 first discusses the phenomenon of the steady growth rate. The concept is based on the assumption that the most important economic variables annually increase by a fixed percentage. An urgent question in the face of a growing world population is whether or not steady growth remains possible. If the answer is no, then a steadily growing world population will unavoidably lead to a reduction of the per capita income and, hence, to an increase in poverty in the world. Section 6.3 discusses the Harrod-Domar growth model. Section 6.4 deals with the conditions for a growth equilibrium. In section 6.5 the neoclassical growth model is discussed. Section 6.6 discusses the possibility of steady growth in the neoclassical model. The effects of technological change are dealt with in section 6.7. Section 6.8 gives the conclusions of this chapter.

6.2 Steady state growth

steady state growth

If the economic variables (production, income) increase annually by a constant rate (growth rate), then we speak of steady state growth. If the growth rate is equal to zero, then we speak of a stationary situation. This is not a state of rest, but a situation in which the economic cycle remains unchanged from year to year. Paradoxical though it may sound, what we have here is an undynamic process. It can be compared to the situation with the railways: 'if there are no changes in the time table, one never needs to buy a new one'.[2] Summarizing, in a stationary state the variable itself is constant; in a state of steady growth the growth rate of the variable concerned is constant.

stationary situation

exponential growth

Steady growth can also be termed as even, proportional or exponential growth. To what unexpected effects exponential growth can lead, can be learned from the following (well-known) story. The inventor of the chess game, commissioned by a rich and mighty king, requested to be paid in kind, in the form of a quantity of corn grains. The quantity was to be calculated as follows: one grain corn for the

2 See S. Huisman, 1981. *Conjunctuur- en structuurtheorie 1: dynamisering van het keynesiaanse model.* Wolters-Noordhoff, Groningen.

first square, two corns for the second, four for the third, and so on. The king, relieved by what seemed a modest request, agreed immediately, but had a rude awakening. For, before the chessboard was half filled, the granaries of the king were empty. What had happened? A chessboard consists of sixty-four squares. The sixty-fourth square alone supplied the inventor with more than nine trillion (2^{63}) grain corns, which equals a layer of grain with a thickness of 4,5 metres covering the total surface of the Netherlands.

growth rates

Although a variable increasing by 100 per cent with each successive square, like in the above example, is quite exceptional, and such large growth rates are hardly ever seen in economics, a relatively small growth rate does lead in a relatively short period of time to a considerable increase of the variable concerned.

doubling time

This is illustrated by the 'doubling time' of each growth rate: the time it takes for a steadily growing variable to double in value. The doubling time of variable x can be derived with the following equation:

$$x(1+d)^T = 2x,$$

$$(1+d)^T = 2,$$

$$T\ln(1+d) = \ln 2,$$

$$T = \frac{\ln 2}{\ln(1+d)} = \frac{\ln 2}{d} \quad \text{if } d \text{ is small,} \tag{6.1}$$

in which T represents doubling time and d the steady rate of growth. For example, assume that the annual growth rate of the world population is 1.7%. Then the doubling time is about 40 years. Table 6.1 clearly illustrates the growth phenomenon.

Table 6.1 clearly shows that not everything grows at the same speed. For instance, the world income in the period 1970-1990 increased by approximately 100%, whereas world population increased by almost 50%. This means an increase of the per capita income during this period. The annual oil consumption increased by a little more than 40%, which means a decrease in the oil consumption per unit of product. The annual consumption of some other raw materials, however, increased by more than the income growth rate.

decoupling

The quality of life in the future can only be sustained if the growth of the world income on the one hand, and the growth of pollution and the growth in the consumption of raw materials on the other, are decoupled. In other words, if the world production is to grow

Table 6.1: Growth of world population and growth of some economic activities 1970-1990.

	1970	1990
World population	3.6 billion	5.3 billion
Estimated world income (GDP in billions of US dollars 1980)	8.000	16.000
Number of cars	250 million	560 million
Number of kilometres (mileage) (OESO-countries only)		
– covered by private cars	2585 billion	4489 billion
– covered by lorries	666 billion	1536 billion
Annual oil consumption	17 billion barrels	24 billion barrels
Annual natural gas consumption	880 billion m^3	1980 billion m^3
Annual coal consumption	2.3 billion tons	5.2 billion tons
Power generated by electric power plants	1.1 billion kWh	2.6 billion kWh
Electricity generated by nuclear power plants	79 billion kWh	1884 billion kWh
Annual consumption of soft drinks	18 billion litres	44 billion litres
Annual beer consumption	15 billion litres	23 billion litres
Annual consumption of aluminium for the production of beer can or cans for soft drinks	72.700 tons	1.251.900 tons
Annual quantity of household refuse (OESO-countries only	302 million tons	420 million tons

Source: D.H. Meadows, D.L. Meadows and J. Randers, 1992. *De grenzen voorbij: een wereldwijde catastrofe of een duurzame wereld,* p.28. Het Spectrum, 1992.

continuously, then pollution and the consumption of raw materials are to grow less than the growth rate of world production. In time it will even become necessary that pollution and the consumption of raw materials decrease at a constant or ever-increasing world production. Yet, it will remain necessary that world production keeps on growing, if at least the current average income level is to be maintained at an ever-increasing world population rate. As things stand now, the world population is expected to grow in the next century to at least 10 billion people. If the per capita income remained constant, this would mean a world income of about 32.000 billion dollars (value 1980). National and international environmental policies should prevent this doubling of the world production from leading to a doubling of pollution and use of raw materials.

6.3 The Harrod-Domar growth model

income equilibrium

This section describes a steadily growing economy with the help of the Harrod-Domar growth model. This model is based on the important assumption that income remains constantly in equilibrium. So the ex ante net investments, I_t are considered to be equal to the savings S_t continuously,

$$I_t = S_t. \tag{6.2}$$

net investments

The net investments I_t are, per definition, equal to the expansion of the stock of capital goods ΔK_t:

$$I_t = K_t - K_{t-1} = \Delta K_t. \tag{6.3}$$

savings

In the Harrod-Domar growth model the savings ratio σ is considered to be constant. The savings are then equal to the savings ratio times the net national income Y:

$$S_t = \sigma Y_t. \tag{6.4}$$

stock of capital goods

The stock of capital goods at time t is equal to the capital output ratio κ times the production capacity Y_t^*. The second important assumption of the model is that spending remains in equilibrium, which means that the production is considered to be continuously equal to production capacity: $Y_t = Y_t^*$. In other words: there is neither under nor over-utilisation. Therefore, the stock of capital goods as a function of production, given the capital output ratio, is obtained as follows:

$$K_t = \kappa Y_t^* = \kappa Y_t. \tag{6.5}$$

From the equations (6.2) up to (6.5) we can derive the steady growth rate of capital. From equation (6.3) and (6.4) it follows that:

$$\Delta K_t = \sigma Y_t. \tag{6.6}$$

growth rate of capital

Then we continue dividing both terms of this equation by the stock of capital goods K_t. After this operation the left term is transformed into the growth rate of the stock of capital goods, which hereafter is indicated as k. After these operations we can write:

$$k = \frac{\sigma Y_t}{K_t}. \tag{6.7}$$

As will be shown later, the growth rate of the stock of capital goods in the Harrod- Domar model is equal in all periods. Where this is the case, the suffix t can be suppressed.

capital productivity

The term Y_t/K_t at the right hand side of equation (6.7) represents the average capital productivity. It is easy to see that the average capital productivity is the reverse of the average capital output ratio K_t/Y_t. With this as given, equation (6.7) can be rewritten into:

$$k = \frac{\sigma}{\kappa}. \tag{6.8}$$

steady growth path

On the steady growth path, the growth rate of the stock of capital goods appears to be equal to the savings ratio divided by the capital output ratio.

steady growth rate of capital

Because in the model both σ and κ are assumed to be constant, what we have here is a steady, i.e. constant, growth rate of the stock of capital goods. If, for instance, the savings ratio is 10% and κ is equal to 2, then the growth rate of the stock of capital goods is 5% per year.

The steady growth rates of both production and production capacity are to be determined as follows. From equation (6.5) we derive: $\Delta K_t = \kappa \Delta Y_t$. We divide both sides of the equation by K_t, so that the left hand side equals the growth rate of the stock of capital goods. The latter, according to equation 4, is equal to κY_t. After substituting this for K_t in the right hand side of the equation we arrive at:

$$k = y = y^* = \frac{\sigma}{\kappa}, \tag{6.9}$$

with y and y^* for the growth rates of production and production capacity respectively. This means that the steady growth rate of both production and production capacity is equal to the steady growth rate of the stock of capital goods.

investment and consumption

Next we investigate how investments and consumption develop on the path of steady growth. Because the Harrod-Domar growth model assumes the savings ratio to be constant and because investments are equal to savings, the following condition holds:

$$I_t = S_t = \sigma Y_t. \tag{6.10}$$

investment output ratio

From this it follows that investments are a constant fraction of income, so that the annual growth rate of the investments i is equal to the annual growth rate of the national income y:

$$i = y. \tag{6.11}$$

consumption output ratio

The consumption output ratio in the Harrod-Domar growth model is equal to $1-\sigma$. Hence, consumption C_t is:

$$C_t = (1-\sigma)Y_t. \tag{6.12}$$

From equation (6.12) it appears that consumption is also a constant fraction of the national income. The annual growth of consumption c is therefore equal to the annual growth rate of national income y:

$$c = y. \tag{6.13}$$

growth speed

From the previous operations it can be concluded that on the path of steady growth it holds that:

$$k = y^* = y = i = c = \frac{\sigma}{\kappa}. \tag{6.14}$$

savings output ratio and capital output ratio

This is a significant result. According to the Harrod-Domar model, the growth rate of an economy is fully determined by the saving output ratio and the capital output ratio (the growth rate is equal to the savings ratio divided by the capital output ratio). As both the savings ratio and the capital output ratio are considered to be constant, there exists a constant growth rate on the steady growth path, whereby the level of both income and spending remains in equilibrium.

example of Harrod-Domar growth model

We will illustrate the conclusion drawn above by way of an example. Let us assume that the average savings ratio σ in an economy is equal to 0.2, and that the capital output ratio κ is equal to 2. Then the steady growth rate is: 0.2 : 2 = 0.1, which implies that the economy annually grows by 10%. The value of each of the variables Y_t, K_t, Y_t^*, I_t and C_t as compared to the value in the starting position of equilibrium, therefore annually increases by 10%. The solutions to this example are given in table 6.2. We assume national income and production capacity in period 0 to be 100. Naturally, the stock of capital goods in the year concerned equals the production capacity multiplied by the capital output ratio (= 2). This means that the stock of capital goods in period 0 is equal to 200. The savings ratio is equal to 0.2, which means that in the starting position investments and consumption are 20 and 80 respectively.

verification

For verification purposes, two more equations have been added: (1) the accumulation function of capital (i.e. the function reflecting the growth of the stock of capital goods), and (2) the equation of the national income. By definition, the stock of capital goods in period t is equal to the stock of capital goods $t-1$ plus the net investments in period $t-1$. Table 6.2 clearly shows that, according to the Harrod-Domar model, a steady growth path is indeed possible.

Table 6.2: The steady growth path in the Keynesian growth model.

Period	0	1	2	3	4	5	6
K_t	200.0	220.0	242.0	266.2	292.8	322.1	354.3
Y_t	100.0	110.0	121.0	133.1	146.4	161.1	177.2
I_t	20.0	22.0	24.2	26.6	29.3	32.2	35.4
C_t	80.0	88.0	96.8	106.5	117.1	129.8	141.7
Y_t^*	100.0	110.0	121.0	133.1	146.4	161.1	177.2
Check							
$K_t = K_{t-1} + I_{t-1}$	200.0	220.0	242.0	266.2	292.8	322.1	354.3
$Y_t = C_t + I_t$	100.0	110.0	121.0	133.1	146.4	161.1	177.2

So, the savings ratio σ and the capital output ratio κ are clearly the two determining agents in the growth of production and income. Now we distinguish three groups of countries: the NIC's, the developing countries and the developed countries. The group of NIC's (newly industrializing countries) consists of countries with a relatively high savings ratio and a relatively low capital output ration. They are the fastest growers. Two examples are South-Korea and Taiwan.

developing countries

In contrast, most developing countries have a low economic growth rate. Not only do they have a relatively low capital coefficient, but also a very low savings ratio. The latter is the result of the low average income in these countries. And, people living at a subsistence level can hardly save money. The low savings ratio causes a low growth rate of production. In the case of a rapidly growing population, this will lead to a decrease in per capita income, which in turn leads to a further decrease in the savings ratio.

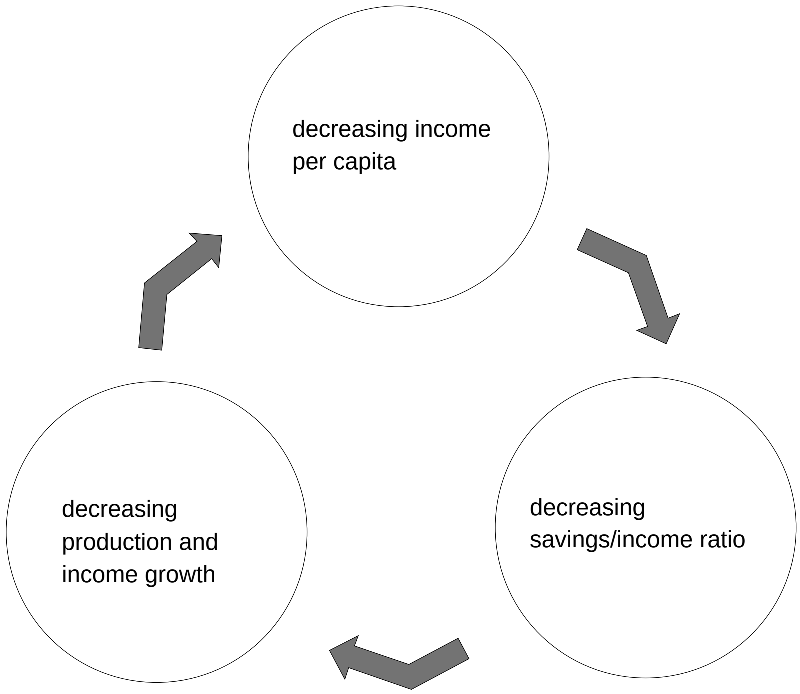

Figure 6.1: Vicious circle of poverty.

vicious circle of poverty

developed countries

This vicious circle of poverty has been drawn in figure 6.1. As a result of the high capital output ratio, many developed countries are being confronted with a slow growth rate. However, the effects are less dramatic than they are for the developing countries because the relatively high standard of living and the low population growth rate in the developed countries have a stabilizing effect on average income. Finally, it must be noted that the composition of the groups of NIC's, and the groups of developing and developed countries is by no means fixed. Developing countries with a high growth rate sometimes succeed in entering the group of developed countries. Japan is a striking example.

6.4 Growth equilibrium

full employment In the Harrod-Domar growth model, a steadily growing economy does not necessarily guarantee that there is full employment. The demand for labour L also depends on the volume of the national product. If the labour output ratio (the quantity of labour necessary to produce one unit of product) is constant, the demand for labour grows at the same rate as production. According to our numerical example of section 6.3 this means that, because the annual production growth is 10%, the annual labour demand also grows by 10%. In figure 6.2 the labour demand is represented by the middle curve.

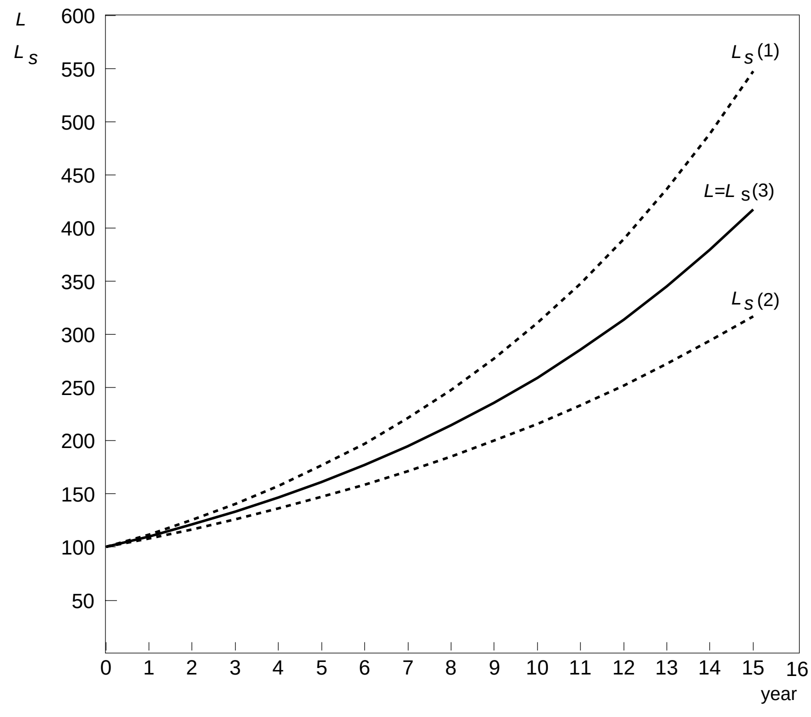

Figure 6.2: Unemployment or a tight labour market.

fixed labour output ratio Because the labour output ratio is fixed, labour demand increases at the same rate as national income does in table 6.2. Let us assume that in period 0 labour supply L_s equals labour demand (i.e. 100 units). However, from period 0 onward the labour supply does not grow by 10% annually, like the labour demand, but by 12%.

The labour supply in this situation $L_s(1)$ is represented by the top

unemployment

curve. Because labour supply exceeds labour demand, unemployment will develop. It is also possible that labour supply increases by less than 10%, for example by 8%. Then the labour supply in the situation $L_s(2)$ develops according to the bottom curve in Figure 6.2. Because the growth of labour demand exceeds that of labour supply, the labour market tightens. Only in the situation represented by the middle curve does the labour supply $L_s(3)$ grow at the same speed as the labour demand. The latter situation is called growth equilibrium.

growth equilibrium

decreasing labour output ratio

The above analysis is only true if the labour output ratio is constant. In reality this is not so. Because of labour saving technology less and less labour is needed per unit of product. If, for instance, production increases by 10%, and the amount of labour required per unit of product decreases by 4%, then the labour demand does not increase by 10%, but only by 6%. In general it could be said that the growth rate e of labour demand E increases by the rate of the production growth y plus the relative change a of the labour coefficient α. This can be proved as follows:

$$E_t = \alpha_t Y_t,$$

$$E_0(1+e)^t = \alpha_0(1+a)^t Y_0(1+y)^t. \tag{6.15}$$

Starting from an equilibrium at time 0, so $E_0 = \alpha_0 Y_0,$ we can derive:

$$1+e = (1+a)(1+y). \tag{6.16}$$

Disregarding second order effects, this gives:

$$e = a + y. \tag{6.17}$$

labour saving technological development

When, like in the real world, the labour-output ratio α is decreasing, $a < 0.$ Therefore, steady growth in conjunction with labour-saving technological development means that labour supply grows at a speed less than that production does, of where it is reduced by the rate of labour-saving technological change. This line of thought is reflected in Figure 6.3, which also illustrates the development of production, the supply of and demand for labour, and unemployment in the OECD-countries in the 1980s.

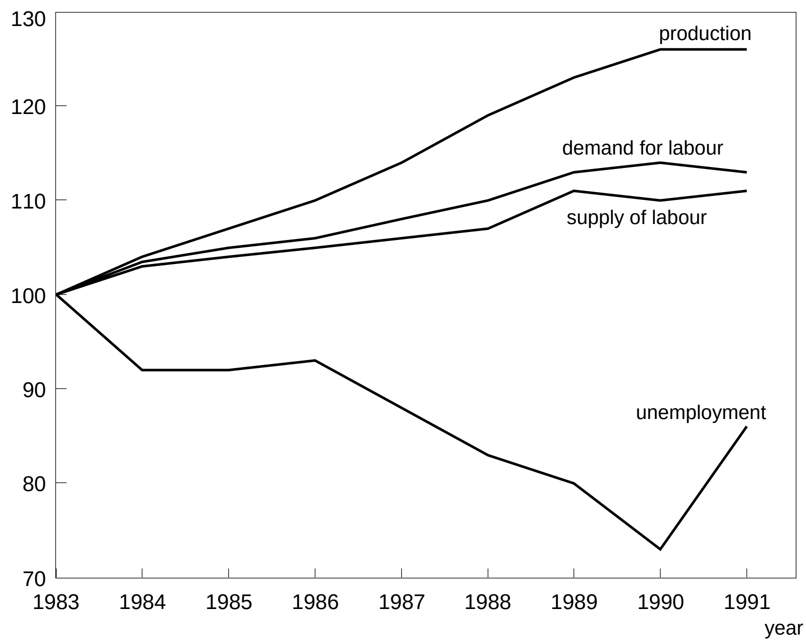

Figure 6.3: Production (real GDP), demand for and supply of labour and unemployment in the OECD-countries 1983-1991 (1983 = 100).

unemployment in the 80's

From Figure 6.3 it appears that, owing to labour-saving technological progress, production in the period considered increases faster than labour demand, and labour supply increases even slower. In general, this resulted in the 1980s in a reduction in the number of unemployed. Summarizing, it could be said that economic growth was more or less stable in the 1980s.

Questions and exercises

6.1 Explain the concept of steady growth.

6.2 Calculate how much time it takes for the value of a variable, that is growing by 2% annually, to double.

6.3 Derive the mathematical expression (terms) for the steady growth rate in the Harrod-Domar growth model.

6.4 Explain the difference in growth between the NIC's and the developed countries.

6.5 When does the labour market tighten in the Harrod-Domar growth model?

6.6 Assume that labour-saving technological development is equal to -5%, the savings rate is 15%, and the capital output ratio is equal to 3. Labour supply increases by 2%. In the reference year the labour market is in equilibrium. Describe the expected development of the labour market.

6.7 Assume that in a certain year there is equilibrium on the labour market. In addition both incomes and spending are in equilibrium. The savings rate of the economy concerned is equal to 0.4%, while the capital output ratio equals 8. Labour supply increases by 10% annually. Which statement is correct? According to the Keynesian growth model described, in time
a. unemployment will develop;
b. the labour market will tighten;
c. the equilibrium in spending will be broken;
d. the equilibrium in incomes will be broken.

6.5 The neoclassical growth model

production function

In addition to the Keynesian growth models like the Harrod-Domar model, there are the neoclassical growth models. The key concept in a neoclassical growth model is the production function. The production function describes the relationship between the input of the production factors and production or income Y. If there are two production factors, labour L and capital K, the production function can be defined as follows:

$$Y = Y(K, L). \tag{6.18}$$

Cobb-Douglas

A production function often used in economics is the Cobb-Douglas production function. It reads as follows:

$$Y = \beta K^{\delta} L^{1-\delta}, \quad 0 < \delta < 1. \tag{6.19}$$

technological development

Coefficient β is an indicator of technological development. If, with the same input of production factors, β increases, so does production. In this case both capital productivity and labour productivity will increase.

increase of per capita income

We will now investigate, on the basis of the production function, how per capita income can increase. To this effect we divide both sides of the equal sign by L. If we assume, as is usually done in the neoclassical growth theory, that the factor labour is fully employed, this operation will provide the average per capita income:

$$\frac{Y}{L} = \beta K^{\delta}\frac{L^{1-\delta}}{L} = \beta\left(\frac{K}{L}\right)^{\delta}. \tag{6.20}$$

rising capital-labour ration

This answers the question of how the per capita income can be increased. Firstly, by raising the capital-labour ratio K/L. This ratio is sometimes called the capital intensity of labour. Secondly, by technological development. This is reflected by an increase in coefficient β or δ. In Figure 6.4 this relation is drawn between the per capita income Y/L and the capital intensity of labour K/L at a relatively low value of β (10) and a relatively high value of β (15). For both curves it holds that $\delta = 0.25$.

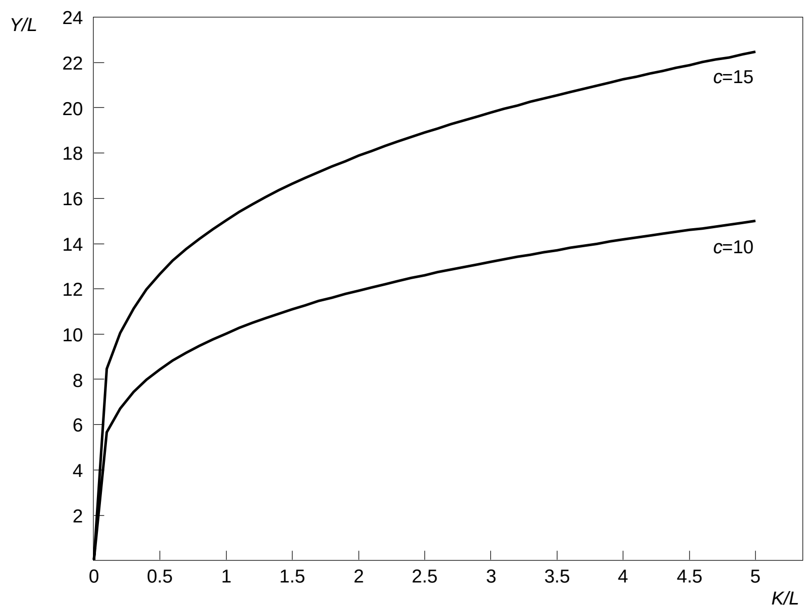

Figure 6.4: The interrelationship between per capita income, the capital intensity of labour, and the state of technology.

capital intensity

Figure 6.4 illustrates that the more the capital intensity of labour increases at an otherwise steady state of technology, the more the per capita income increases. The per capita income also increases if the technology coefficient β increases despite the capital intensity of labour remaining constant.

labour productivity

The relationship between capital intensity of labour and per capita production is illustrated in Figure 6.5, in which the points on the x-axis represent the capital intensity of labour of a number of countries and the y-axis gives the per capita gross domestic product. The figure clearly shows that, in general, a high capital intensity coincides with a high labour productivity rate. This can also be reflected by a linear regression equation:

$$\frac{Y}{L} = 0.64 + 0.94\frac{K}{L}. \tag{6.21}$$

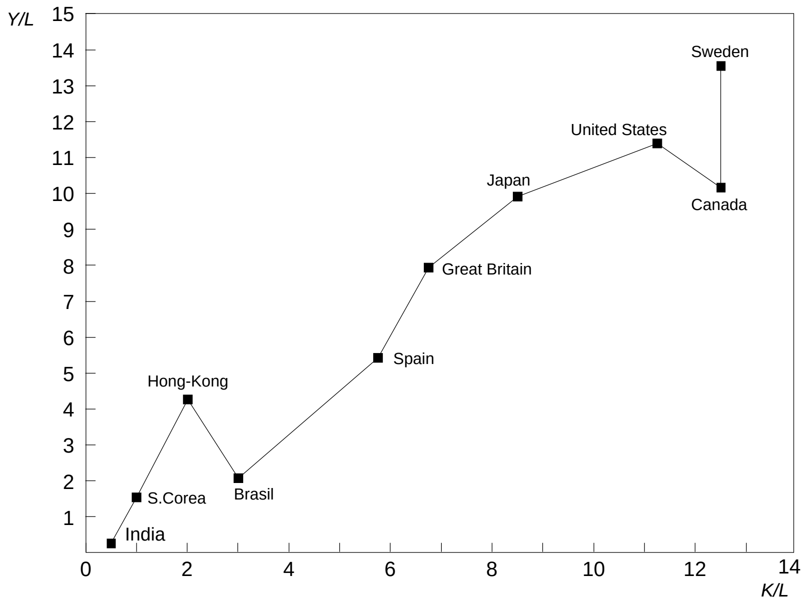

Figure 6.5: The interrelationship between capital intensity (K/L) (measured in 1000 dollars per labour year) and the per capita gross domestic product (measured in 1000 dollars) in the year 1980.
Sources: World Bank, 1982. *World Development Report*. Washington; D. Begg, S. Fischer and R. Dornbusch, 1991. *Economics*. McGraw-Hill, London.

stock of capital goods

This equation implies that if the capital intensity increases by one dollar, the per capita gross domestic product increases by 0.94 dollar. The connection has, of course, a statistical value only, so that it is not simply applicable to each individual country. We are dealing here with a rather specific definition of the stock of capital goods. For instance, should we consider private property also to belong to the stock of capital goods? Questions like this will have to be answered before we can determine the capital volume.

effective demand and fixed prices

In the Keynesian growth model, discussed in chapter 6.4, establishing and maintaining full employment is a matter of chance. If labour supply grows faster than production, adjusted for labour-saving technological progress, then unemployment will arise. If labour supply grows slower than production (adjusted for labour-saving technological progress), this will create tension on the labour market. Factors such as price level and interest rate do not play a role in the Keynesian model. Income stability is maintained by adjusting production to effective demand; spending will remain stable because the production capacity will also adjust to effective demand. However, the steady growth rate resulting is not a sufficient condition for a growth equilibrium but a necessary one. For, one can speak of growth equilibrium only if steady growth is coupled to full employment.

the price mechanism

In the neoclassical growth model, prices rather than effective demand play an important role. The price mechanism is considered to work so well that all markets are in equilibrium. To achieve this, investments are made equal to savings by way of changes in the exchange rate. If savings exceed investments, the exchange rate will go down (fall) making investing more and saving less attractive. Labour supply is equalled to labour demand via income changes. This means that unemployment does not exist in the neoclassical growth model.

employment of production factors

The price mechanism operates in such a way that all production factors remain fully employed. Thereby two conditions for stable growth are automatically met, *viz*. existence of equilibrium in both spending and income.

production function

Another difference between the neoclassical growth model and the Keynesian model is the production function used. In the Keynesian model, production capacity is solely determined by the size of the input of capital goods, whereas in the neoclassical model the labour input is an equally important factor in the production volume. In the following sections we will deal with the following questions:

– Is steady growth possible in the neoclassical model? From the previous chapter we know that steady growth is achieved if all factors, such as income, stock of capital goods, consumption and investments, grow at an equally constant rate. Although in the neoclassical situation there is always income equality and spending equality this does not necessarily guarantee that there is stable growth, since this also requires there to be steady growth.

- In the neoclassical growth model, how is an increase of income per head to be realised? In the Keynesian growth model this can be accomplished, while maintaining stable growth, through labour-saving technological progress.
- If stable growth is distorted, is there a mechanism in the neoclassical model that automatically restores it? In other words: how stable is the stable growth path?

Questions and exercises

6.14 Prove that in the absence of technological development on the path of stable growth it is true that

$$y = y^* = l = l_s = \frac{\sigma}{\kappa}.$$

6.15 What connection is there between the capital intensity of labour and the production per head?

6.16 Describe the role of the price mechanism in both the Keynesian growth model and the neoclassical growth model.

6.17 Describe the role of effective demand in both the Keynesian model and the neoclassical model.

6.18 Why is it that in the Keynesian model a situation of full employment only happens by accident, and why is it that in the neoclassical model full employment is guaranteed?

6.6 Steady state in the neoclassical growth model

investment

As we know, gross investments consist of depreciation together with net investments. Depreciation equals a fixed fraction π times the capital stock K. Net investments equal the growth rate of capital k times capital stock K. Savings S equal gross investments. With this in mind, we can write:

$$S = kK + \pi K, \Rightarrow S = (k + \pi)K. \quad (6.22)$$

savings

Savings S are assumed to be a fixed fraction σ of production Y:

$S = \sigma Y,$ so:

$$\sigma Y = (k + \pi)K \text{ and } \frac{\sigma Y}{L} = (k + \pi)\frac{K}{L}. \quad (6.23)$$

production function

The production function is the same as expressed by (6.18) Because Y is homogenous of the first degree, we can write:

$$Y(K,L) = LY\left(\frac{K}{L},1\right), \text{ so: } \frac{Y}{L} = Y\left(\frac{K}{L},1\right) = Y\left(\frac{K}{L}\right). \qquad (6.24)$$

steady state

There is a 'steady state' sitation when Y/L is constant. This implies that K/L is constant too. If K/L is constant, then the growth rate k of capital stock equals the growth rate l of labour:

$$k = l. \qquad (6.25)$$

So, in the steady state:

$$\sigma\frac{Y}{L} = (l+\pi)\frac{K}{L}. \qquad (6.26)$$

This is illustrated by Figure 6.6. For the production function a Cobb Douglas production function is used,

$$Y = 10K^{0,25}L^{0,75},$$

$$\frac{Y}{L} = 10\left(\frac{K}{L}\right)^{0,25}. \qquad (6.27)$$

Further we assume that $\pi = 0,20$, $l = 0,05$ en $\sigma = 0,20$.

stable equilibrium

To the left of point S gross investments exceed the necessary investments for keeping K/L constant. In this case K/L will increase. To the right the opposite is the case: gross investment are less than the necessary investments to keep K/L constant, so now K/L will decrease. This means that S is a stable equilibrium or a 'steady state'. In Figure 6.6 the steady state is found when $Y/L = 20$, $S/L = 4$ and $K/L = 16$.

growth rate of population

In the steady state both Y/L and K/L are constant. This also includes a stable capital output ratio $\kappa = K/Y$. We can conclude that in the steady state the growth rates of capital, labour and output are equal. From figure 6.6 we can also derive that the growth rate of production, capital and labour do not depend on the savings rate σ, but is determined entirely by the growth rate of the population. An increase of the savings rate causes an increase of the K/L and Y/L ratios.

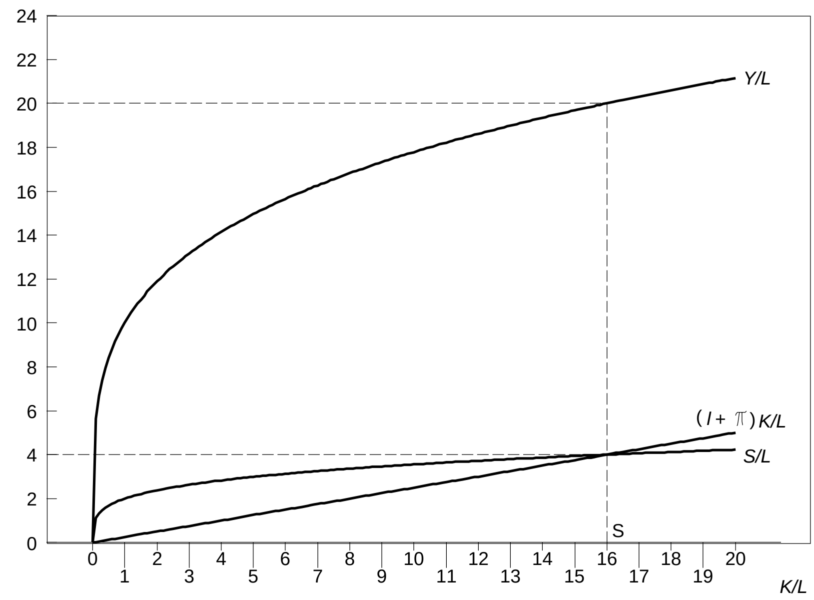

Figure 6.6: Steady state in the neoclassical growth model.

Questions and exercises

6.19 What is the difference between the production function used in the Keynesian model and the one used in the neoclassical model?

6.7 Steady growth and technological change

the production function

As we already know, in the neoclassical growth model a production function is used with production Y_t and two inputs: capital K_t and labour L_t, which in a general form can be written as follows:

$$Y_t = Y_t(\beta_t, K_t, L_t), \tag{6.28}$$

level of know how

with β_t as an exogenous indicator for the level of technological know how. With Euler's rule[3] we can derive:

3Euler's rule states that if a production function $Q = f(K, L)$ is linear homogeneous, we can write: $K\frac{\partial Q}{\partial K} + L\frac{\partial Q}{\partial L} = Q$.

$$y = \varepsilon_\beta^y b + \varepsilon_k^y k + \varepsilon_l^y l, \tag{6.29}$$

elasticity of production

with ε_β^y for the elasticity of production Y with respect to technological know how β, ε_k^y for the production elasticity with respect to capital and ε_l^y for the production elasticity with respect to capital. For the sake of simplicity we assume the earlier mentioned Cobb-Douglas production function:

$$Y = \beta K^\delta L^{1-\delta}, \quad 0 < \delta < 1. \tag{6.30}$$

growth rates

Then $\varepsilon_\beta^y = 1$, $\varepsilon_k^y = \delta$, and $\varepsilon_l^y = 1 - \delta$. The function written in terms of growth rates is:

$$y = b + \delta k + (1-\delta) l. \tag{6.31}$$

Extension of the stock of capital goods takes place through the net investments I_t, which are equal to the savings S. The savings are, as in the Harrod-Domar model, equal to the savings ratio σ times the national income Y_t. From this it follows:

investment

$$I_t = \sigma Y_t. \tag{6.32}$$

The growth rate k of the stock K_t of capital goods can be defined as follows:

growth rate of capital

$$k = \frac{dK}{dt}\frac{1}{K} = \frac{I}{K}. \tag{6.33}$$

From the previous two equations we can derive that:

$$k = \sigma \frac{Y_t}{K_t}. \tag{6.34}$$

Differentiation with respect to t gives:

$$\frac{dk}{dt} = \frac{\sigma \frac{dY}{dt} K - \sigma Y \frac{dK}{dt}}{K^2}. \tag{6.35}$$

Finally, dividing this form by k yields:

$$\frac{dk}{dt}\frac{1}{k} = \overline{k} = \frac{\sigma \frac{dY}{dt} K - \sigma Y \frac{dK}{dt}}{K^2} \frac{1}{\sigma} \frac{K}{Y} = y - k, \tag{6.36}$$

relative change of the growth rate of capital

with $\overline{k}$ for the relative change of the growth rate k of the capital stock. With a steady growth rate k of capital stock, $\overline{k}$ must be 0, so, in the steady state: $y = k$. This equilibrium condition together with (6.31) have been depicted in Figure 6.7, with $b = 0.02$, $l = 0.04$ and $\delta = 0.25$, so:

$$y = 0.02 + 0.25k + 0.75 \times 0.04 = 0.05 + 0.25k.$$

equilibrium

To the right of k^* $y < k$. This implies that $\overline{k} < 0$ and so k will decrease. To the left of k^* $y > k$. This implies that k will increase. This proves that k^* is a stable equilibrium. In this equilibrium $k = y$, so the capital output ratio κ is constant. Further, we see the influence of technological progress here. A change in b gives a change in the equilibrium. For example, when b increases, the equilibrium moves to the right, causing a higher rate of growth for capital and income.

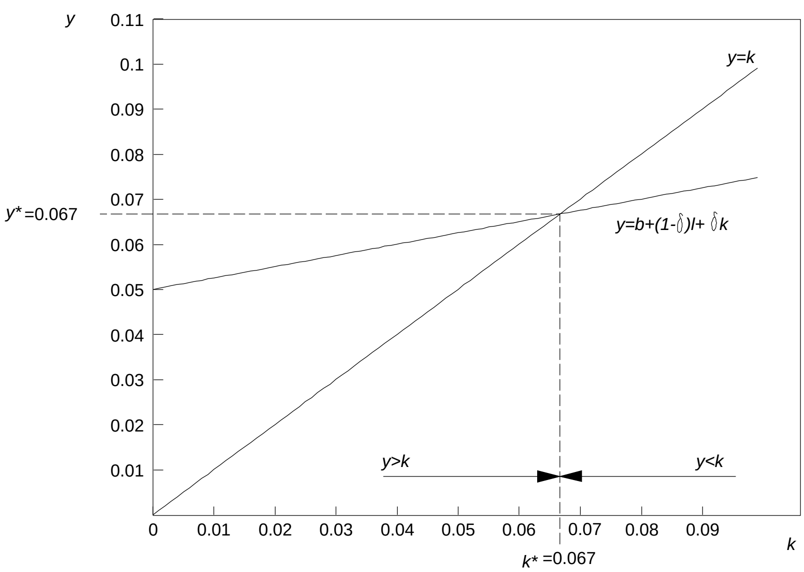

Figure 6.7: Steady state and technological change in the neoclassical model.

In the neoclassical growth model δ is considered to be constant. This means that an increase in production per capita can only be achieved by an increase in coefficient β. In this form of technological progress the capital output ratio remains constant, while production per head increases. This can be proved as follows:

$y = b + \delta k + (1 - \delta) l,$

$y = k,$ so:

$$y - l = \frac{b}{1 - \delta}. \tag{6.37}$$

Here we deal with a pure form of labour-saving technological change, called in the literature 'Harrod-neutral technological change'.

endogenous technological change

In modern growth theory technological change is not exogenous, but endogenous. It is linked to the development of the capital stock by the knowledge-capital elasticity ν. So:

$$b = \nu k. \tag{6.38}$$

Now we can derive:

$$y = (\nu + \delta) k + (1 - \delta) l. \tag{6.39}$$

Combining this with the steady state condition $y = k$ we can now conclude that:

- There is no steady state growth when $\nu \geq 1 - \delta$.
- There is a steady state growth when $\nu < 1 - \delta$.

This is illustrated by Figure 6.8. Still, the possibility of a steady state with positive growth rates for production and capital depends on a positive population growth rate. When population growth is zero, three possibilities exist:

- $\nu > 1 - \delta$. In this case the growth rate of capital will rise continuously.
- $\nu = 1 - \delta$. The growth rate of production will always be equal to the growth rate of capital (this is not taken into consideration).
- $\nu < 1 - \delta$. Steady state exists where $y = k = 0$.

An alternative is another function of technical change, for example:

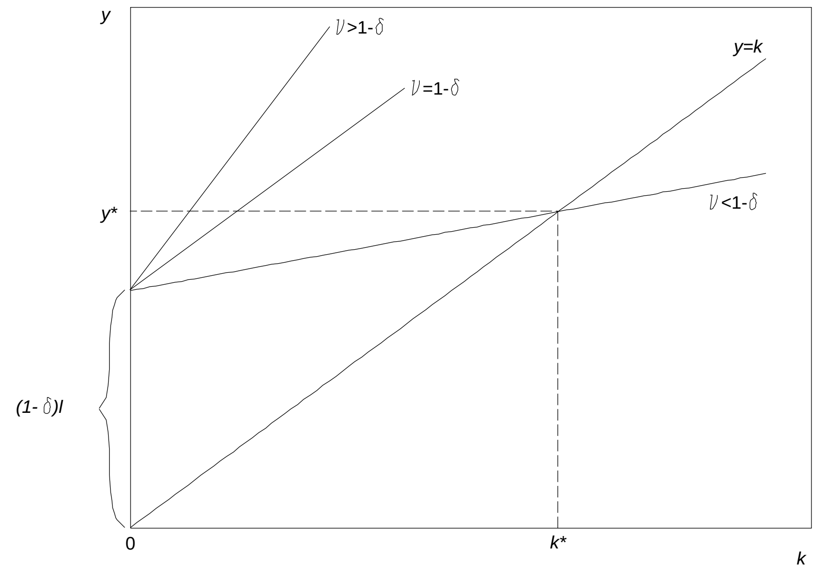

Figure 6.8: Endogenous technological change.

$$b = \varepsilon k^{\psi}. \qquad (6.40)$$

If $0 < \psi < 1$, a steady state economy will occur with positive growth rates for production and capital stock, even when $l = 0$. This is illustrated in Figure 6.9.

The possibility of a steady state in this situation is based on a decreasing knowledge-capital elasticity. In that case the production curve is concave. This can be proved as follows:

$$y = \varepsilon k^{\psi} + \delta k,$$

$$\frac{dy}{dk} = \psi \varepsilon k^{\psi - 1} + \delta > 0,$$

$$\frac{d^2 y}{dk^2} = (\psi - 1)\psi \varepsilon k^{\psi - 2} < 0, \quad \text{if } 0 < \psi < 1. \qquad (6.41)$$

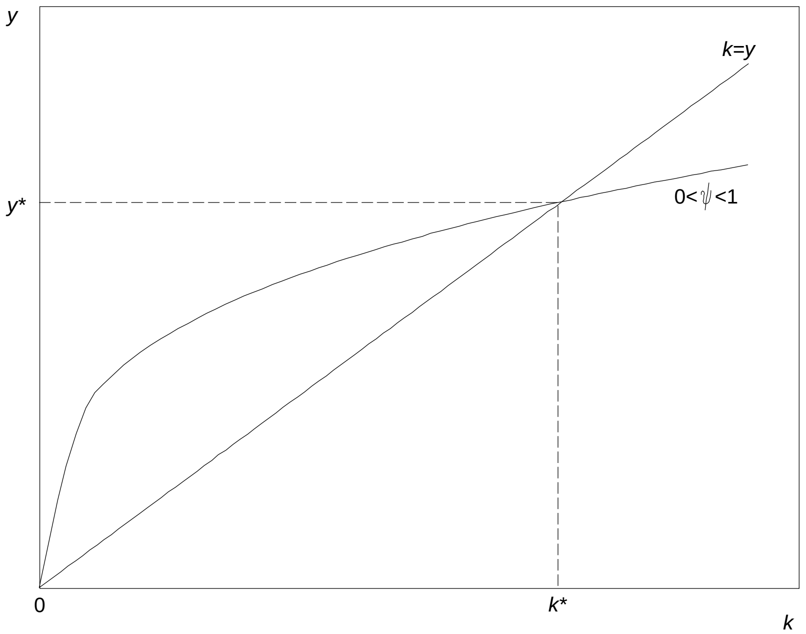

Figure 6.9: Steady state in the situation of endogenous technological change and zero population growth.

Questions and exercises

6.20 Assume the following production function:

$$Y = 10K^{0.5}L^{0.5}.$$

The growth of the labour force equals 2% per year. The depreciation rate equals 8% per year. The savings rate is 15%.

a. Compute the K/L ratio in the steady state.
b. Why is it that in the neoclassical growth model the growth rate of national income does not depend on the savings rate?
c. Prove that the output/labour ratio and the capital/labour ratio do depend on the savings rate.

6.21 When is there steady growth in the neoclassical model?

6.22 Why can the path of steady growth in the neoclassical model be called stable? Does this also hold for the Keynesian growth model?

6.23 Suppose the following data are known of a national economy: $\sigma = 0.2$, $y = 0.1$, $k = 0.1$, $K_0 = 100$, $Y_0 = 100$. Prove that, according to the neoclassical growth model, these data contradict one another.

6.24 Suppose the following data are known for a national economy: $\sigma = 0.1$, $K_0 = 200$, $L_0 = 50$. The labour force grows by 5% per year. The production function for this national economy is as follows:

$$Y_t = K_t^{0.5} L^{0.5}.$$

There is no technological progress. Now compute, on the basis of the neoclassical growth model, for five periods, the steady growth path of: income Y_t, stock of capital goods K_t, investments I_t, and consumption C_t and employment L_t. Verify the outcome with the accumulation function for capital and the production function.

6.25 What is Harrod-neutral technological progress?

6.26 How does an increase of income per head come about in the neoclassical growth model, and in the Keynesian growth model?

6.27 Under what circumstances does endogenous technological change lead to a stable steady state with positive growth for capital and production?

6.28 How can a steady state come about without population growth and with endogenous technological change?

6.8 Conclusions

growth models

This chapter dealt with growth models. Contrary to the Keynesian growth model, where the effective demand is the most important variable, the neoclassical model has the price mechanism as its main variable. This implies that in the neoclassical model labour and capital are fully employed and investments are equated to savings by means of changes in the interest rate. This is in contrast to the Keynesian model, where prices and, hence, the interest rate, are fixed. If, in the latter, ex ante investments exceed savings and consequently effective demand exceeds production, then production will increase as a result of undesirable stock mutations. This mechanism will eventually lead to an income equilibrium.

neoclassical model

As we already mentioned, there is always full employment in the neoclassical model. This is in contrast to the Keynesian model, where full employment is purely coincidental. Only if labour supply grows at the same rate as production, corrected for labour saving technological progress, can there be stable growth in the Keynesian model.

steady growth

An important matter in the context of the neoclassical model is that of steady growth. One can speak of steady growth if the stock of capital goods grows annually by a constant rate. In the previous paragraphs it was clear that in the neoclassical model there is steady growth if the growth rates of income and capital are equal. Hence, on the steady growth path, a rise in income per head can only take place if there is labour-saving technological progress.

stability

As to the stability of the path of steady growth in the neoclassical growth model, it can be said that it is stable by nature. For if the stock of capital goods grows faster than production, this will result in a decline in the growth rate of the stock of capital goods. Conversely, the growth rate of the stock of capital goods will increase. Finally, when technological change is endogenous, under certain conditions a steady state with positive growth rates for capital and production can be achieved.

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