3. 'Partial balance'- regression models for $N_{min,H}$

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3.1 Introduction

In this chapter we take the balance equations 2.8 and 2.11 (Chapter 2) as a starting point for defining a series of regression models. These will include selected terms of the balance equations and can therefore be referred to as 'partial balance' models. Strictly speaking, this term is incorrect because the implementation in the form of regression models defeats, in a way, the balance principle: the regression coefficients that modify the balance terms have no strict physical meaning and may account for various known and unknown processes. But then, this is one of the very advantages of the regression approach: it does allow to include factors the role of which may be unknown beforehand, such as precipitation; and offers objective measures to judge the relative importance of the various factors. Regression models can be parameterized easily, and their coefficients may compensate, to some extent, for deviations that result from ignoring some of the balance terms. In short, the regression approach is attractive in problems where we have only partial understanding of the processes involved, or where only part of the relevant variables can be observed directly. Both conditions apply to the quantification of residual mineral nitrogen in soils. Nevertheless, the theory presented in Chapter 2 provides a guideline for developing and comparing regression models in the next paragraphs.

Prior to the evaluation of the various models in Paragraphs 3.5-3.7, we will introduce the data sets available for this purpose (Par. 3.2), and then first use these data to inspect a few general issues. One is to test, in Par. 3.3, the two propositions used in the derivation of Eq. 2.11 of Chapter 2. The other is to assess variation in the base level of residual mineral N, $N_{min,H,0}$ (Paragraph 3.4).

Paragraph 3.8 is devoted to applications of the QUADMOD concept to the modelling of $N_{min,H}$. Finally, Par. 3.9 deals with the modelling of $N_{min,H}$ under 'steady state' soil conditions, that is, in the case where no net changes occur of soil N pools. This may have crucial implications for the reduction of 'noise' in surplus-based models.

3.2 Data sources

Chapters 3, 4 and 5 compile virtually all data of experiments that were conducted during the last two decades in the Netherlands with the aim to assess the effects of fertiliser management on $N_{min,H}$. In a limited number of those trials, crop N uptake (N-yield) was observed, and often a zero-N treatment was included which yielded observations on N-yield (U_0) and residual mineral soil N ($N_{min,H,0}$) in absence of N-input. The trials meeting all these conditions allow a closer inspection of the principles introduced in Chapter 2, and enable a comparison of a range of regression models of increasing complexity. The datasets used for this purpose are listed in Table 3.1.

The effectiveness ('working coefficient') of N in animal manures was assumed to be equal to 0.60 (injected slurry) and 0.24 (surface applied slurry) and all N-rates were converted into effective N-rates. All N-rates refer to effective N-rates in this study.

Table 3.1. Data sets used in this chapter to analyse relations between residual soil mineral nitrogen ($N_{min,H}$) and selected variables. All sets include observed data on crop N yield (U), N-yield in absence of N-input (U_0), and $N_{min,H}$ in absence of N-input ($N_{min,H,0}$). Regression models for maize and grass were based on sand data only.

Crops	Soil	# Trials	# Nmin observ's	Years	Source
Cauliflower	Clay	4	40	90,92	Everaarts, 1995
Broccoli	Clay	4	40	90-92	Everaarts, 1995
White cabbage	Clay	4	42	92-93	Everaarts, 1995
Potatoes ware	Clay	5	104	87-98	Hengsdijk, 1992; Titulaer, 1997, Van Loon, 1998; Anon., 1999
Potatoes starch	Sand	6	113	91-97	Van Loon, 1995; Wijnholds, 1995, 1996, 1997
	Dal	6	44	89-92	Van Loon, 1995; Postma, 1995; Anon., 1999
Maize silage	Clay	7	84	85-94	Schröder, 1990; Van der Schans, 1995; Van Dijk, 1996
	Loess	4	80	95-98	Geelen, 1999
	Sand	97	423	75-99	Schröder, 1985, 1987, 1989, 1990, 1992, 1993; Van Dijk, 1995, 1996, 1997, 1998; Van der Schans, 1995, 1998; Van der Schoot 2000; Anon., 1999
Sugarbeet	Clay	8	139	88-97	Hengsdijk, 1992; Westerdijk, 1992; Van Dijk, 1999; Anon., 1999
	Sand	1	6	89	Anon., 1999
Cut grass	Sand	27	373	80-84; 87-90; 92-94; 96-98;	Van der Meer *et al.*, 1992; Fonck (1982a,b; 1986a,b,c); Wouters *et al.*,
Cut grass	Peat	5	39	92-94	1992; Van Bockstaele *et al.*, 1996,
Cut grass	Clay	3	21	92-94	1997, 1998; Wadman & Sluijsmans, 1992; Anon., 2000.

3.3 Validation of propositions in Chapter 2

The two key propositions introduced in Chapter 2 are validated here, as far as possible on the basis of available evidence.

The first was that the term $A(1-\rho_{ini})$ represents the sum of (i) applied N absorbed in crop residues, and (ii) the amount of applied N lost from the root zone or fixed in inaccessible form (organic matter) as expressed in Eq. 2.6. The consequence is that no mineral N would accumulate in excess of the base level, $N_{min,H,0}$, as long as applied N rates do not exceed the local value of the critical N rate, A_{crit}. (For a definition see Chapter 2 and Ten Berge *et al.* (2000)). This is so because the second and the third right hand side term of Eq 2.8 are equal - by definition - for subcritical N rates, and so they cancel out. The Figures 3.1 – 3.7 serve to confirm that this approximation is justified in most cases. Negative values of the x-variable in these figures correspond to subcritical doses, and the associated y-values do not differ from zero, or do so only slightly. Broccoli seems to deviate from this pattern.

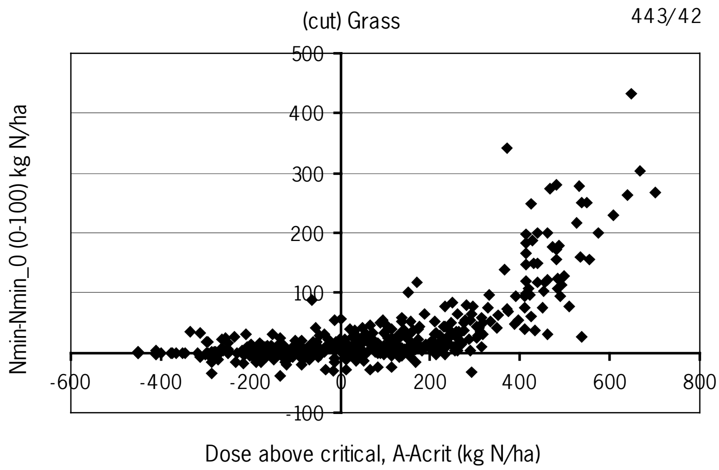


Figure 3.1. *Increment in residual soil N (0-100 cm) at the last cut harvest ($N_{min,H}$) in grass, relative to residual soil N observed at the same time in plots that received no N input ($N_{min,H,0}$), versus the amount by which the applied N rate exceeds the critical N rate, A_{crit}. The values of $N_{min,H,0}$ and A_{crit} are case-specific: they vary with the location and the year of the experiment. All grass data listed in Table 3.1 are included.*

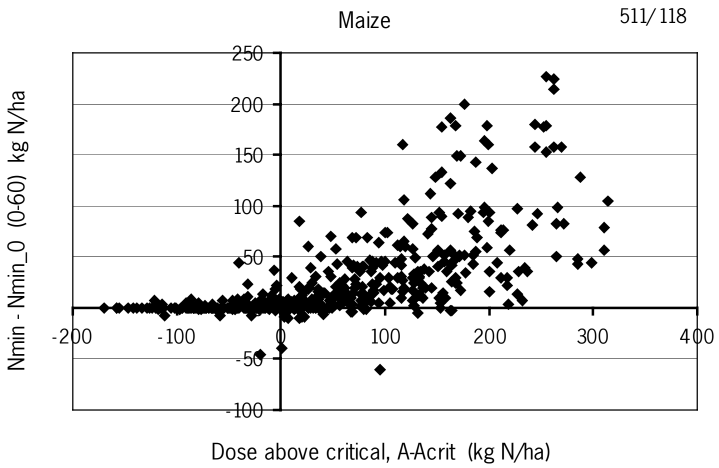


Figure 3.2. *Increment in residual soil N (0-60 cm) at harvest ($N_{min,H}$) in maize, relative to residual soil N observed at the same time in plots that received no N input ($N_{min,H,0}$), versus the amount by which the applied N rate exceeds the critical N rate, A_{crit}. The values of $N_{min,H,0}$ and A_{crit} are case-specific: they vary with the location and the year of the experiment. All maize data listed in Table 3.1 are included.*

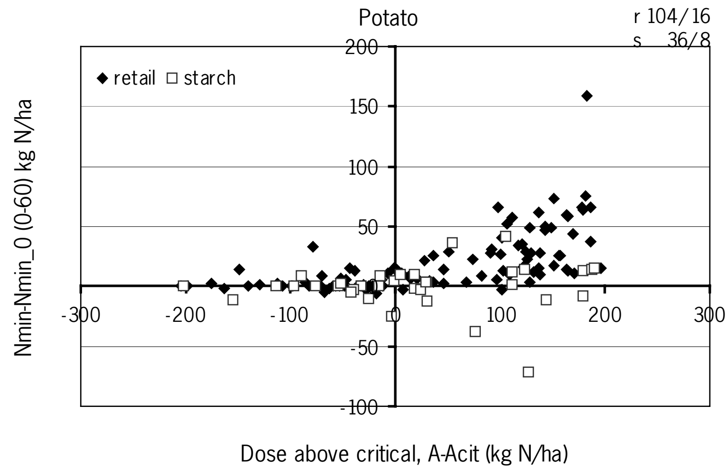


Figure 3.3. *Increment in residual soil N (0-60 cm) at harvest ($N_{min,H}$) in potato, relative to residual soil N observed at the same time in plots that received no N input ($N_{min,H,0}$), versus the amount by which the applied N rate exceeds the critical N rate, A_{crit}. The values of $N_{min,H,0}$ and A_{crit} are case-specific: they vary with the location and the year of the experiment. All potato data listed in Table 3.1 are included.*

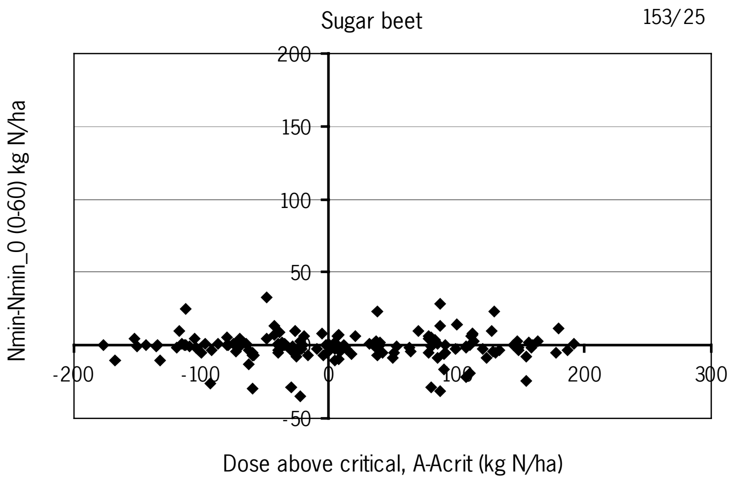


Figure 3.4. *Increment in residual soil N (0-60 cm) at harvest ($N_{min,H}$) in sugar beet, relative to residual soil N observed at the same time in plots that received no N input ($N_{min,H,0}$), versus the amount by which the applied N rate exceeds the critical N rate, A_{crit}. The values of $N_{min,H,0}$ and A_{crit} are case-specific: they vary with the location and the year of the experiment. All sugar beet data listed in Table 3.1 are included.*

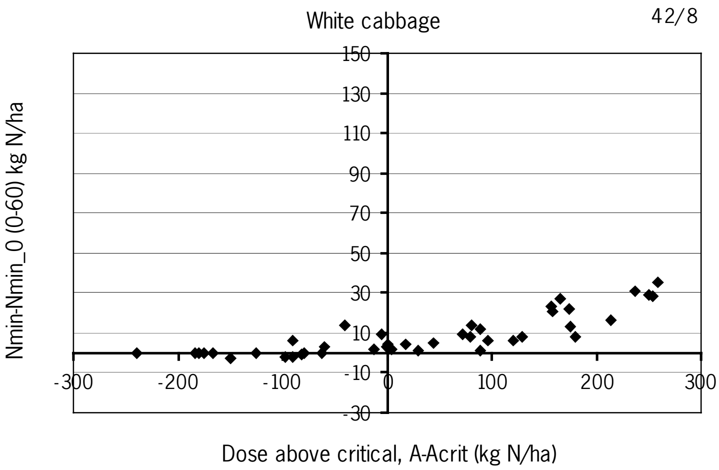


Figure 3.5. *Increment in residual soil N (0-60 cm) at harvest ($N_{min,H}$) in cabbage, relative to residual soil N observed at the same time in plots that received no N input ($N_{min,H,0}$), versus the amount by which the applied N rate exceeds the critical N rate, A_{crit}. The values of $N_{min,H,0}$ and A_{crit} are case-specific: they vary with the location and the year of the experiment. All data refer to clay soils.*

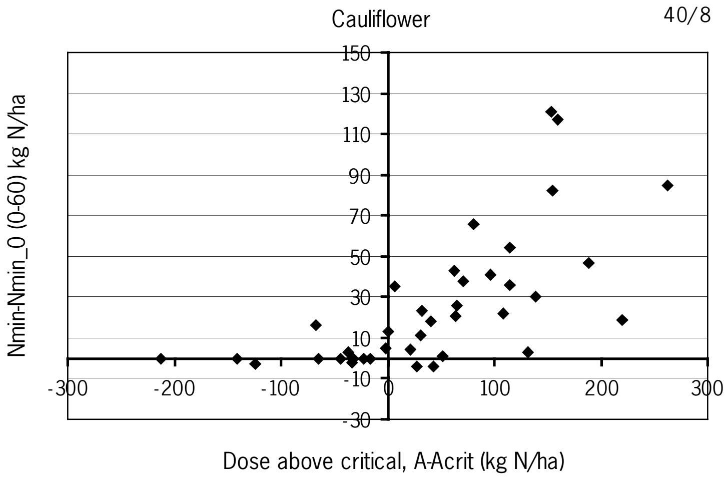


Figure 3.6. *Increment in residual soil N (0-60 cm) at harvest ($N_{min,H}$) in cauliflower, relative to residual soil N observed at the same time in plots that received no N input ($N_{min,H,0}$), versus the amount by which the applied N rate exceeds the critical N rate, A_{crit}. The values of $N_{min,H,0}$ and A_{crit} are case-specific: they vary with the location and the year of the experiment. All data refer to clay soils.*

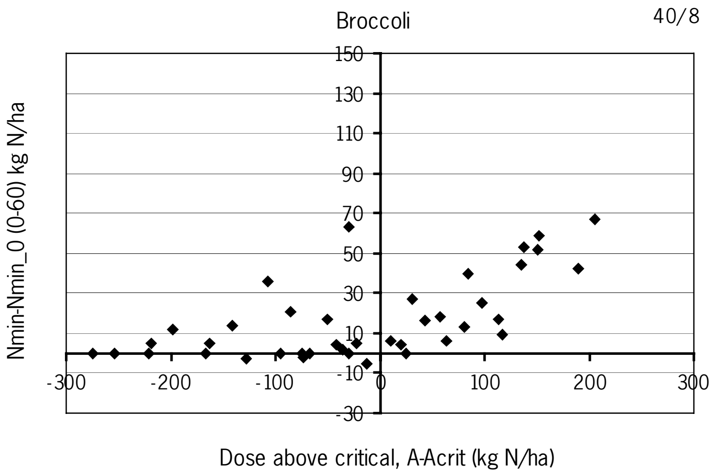


Figure 3.7. *Increment in residual soil N (0-60 cm) at harvest ($N_{min,H}$) in broccoli, relative to residual soil N observed at the same time in plots that received no N input ($N_{min,H,0}$), versus the amount by which the applied N rate exceeds the critical N rate, A_{crit}. The values of $N_{min,H,0}$ and A_{crit} are case-specific: they vary with the location and the year of the experiment. All data refer to clay soils.*

The second proposition is that Eq. 2.6 is not only valid in the N supply range where crop N demand well exceeds N supply, but also in the 'near-saturation' range where a decreasing recovery ρ indicates the reduced absorption capacity of the crop. One could argue against this that there is a limit to the N storage in plant parts designated as 'crop residues'. As a consequence, Eq. 2.6 would not be valid at high N rates unless the limited N sink function of crop residues would be compensated for by an increased losses term, $\Sigma(L_i\text{-}L_{i,0})$. In absence of such compensation, Eq 2.8 would be expected to underestimate $N_{\mathrm{min,H}}$ for the higher N rates. This issue can be inspected on the basis of Figures 3.9 and 3.10, where $N_{\mathrm{min,H}}$ calculated according to Eq. 2.8 is plotted versus observed values. No such systematic deviation appears to occur in grass. In maize, on the other hand, Eq 2.8 indeed appears to underestimate $N_{\mathrm{min,H}}$ at higher N rates. The error introduced by the second proposition is thus likely to depend on the specific capacity of crops to absorb N in their residues.

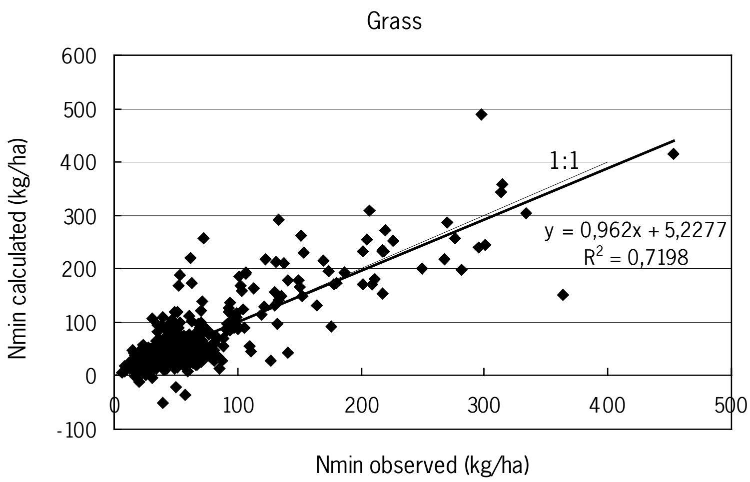


Figure 3.8. *Residual soil N (0-100 cm) at last cut harvest ($N_{min,H}$) in grass, calculated with the help of Eq. 2.8, versus observed values. Values of ρ_{ini}, U_0 and $N_{min,H,0}$ used in this calculation are case-specific: they vary with the location and the year of the experiment.*

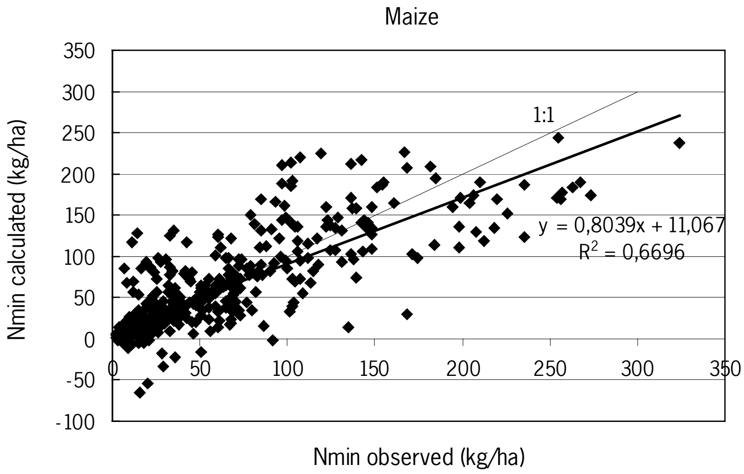


Figure 3.9. *Residual soil N (0-60 cm) at harvest ($N_{min,H}$) in maize, calculated with the help of Eq. 2.8, versus observed values. Values of ρ_{ini}, U_0 and $N_{min,H,0}$ used in this calculation are case-specific: they vary with the location and the year of the experiment.*

3.4 The base level $N_{min,H,0}$ and the Spring value $N_{min,S}$

As shown in Eqs. 2.5 and 2.8 of Chapter 2, the base level $N_{min,H,0}$ sets the value of $N_{min,H}$ that should be expected anyhow, that is, in absence of N application. This base level may vary considerably across sites and also between years at a given site, and between crops. It is therefore, as shown later, an important factor to explain the total variance found in $N_{min,H}$.

Based on the available observations (Table 3.1), typical values of $N_{min,H,0}$ are between 15 and 80 kg N ha-1 in maize, 10 to 45 in grass, 20 to 60 in potato, and 5 to 30 kg N ha-1 in sugar beet. The trials with open field vegetables revealed values of 10-20 kg N ha-1 for white cabbage and broccoli, and 10-40 kg N ha-1 for cauliflower (all on clay soils!).

At first, one might expect that the higher $N_{min,H,0}$-values are found on soils with a relatively high seasonal mineralisation and that, therefore, a positive correlation could be expected between $N_{min,H,0}$ and the corresponding crop N-yield U_0 in zero-input treatments. The existence of such relation would be convenient as it would enable local corrections on observed $N_{min,H}$ in monitoring programs. Figure 3.10 shows, however, that no clear correlation is found between N-yield and $N_{min,H}$ in zero-input plots, except in maize.
At closer inspection, the absence of a correlation can be understood based on Eq. 2.11 (Chapter 2), which states that $N_{min,H}$ does not increase in response to applied N up to a certain input level, A_{crit}. If this is so, then why would $N_{min,H}$ respond to the amount of N liberated from the soil itself, as long as this amount is well below the level A_{crit}? As long as crop N demand is larger than supply, whether derived from the soil or from external inputs, no substantial amounts of $N_{min,H}$ would be expected to build up.

Now the behaviour of $N_{min,H,0}$ in maize provides a clue to understanding the variation found in $N_{min,H,0}$ in general. Maize differs from the other crops inspected here, in that a long period passes between the cessation of crop N uptake and the date of harvest, often up to two months or more. Most likely, $N_{min,H,0}$ is mainly determined by mineralisation during this period. Based on this reasoning, it can now be understood that soils with higher mineralisation potential will only show a higher build-up of $N_{min,H,0}$ if a sufficiently long period of crop 'inactivity' allows this. Such build-up will be enhanced if large doses of animal manures were applied, and this may explain that maize shows relatively high values for $N_{min,H,0}$. So, we presume that in all crops the variation found in $N_{min,H,0}$ across sites and years must be attributed, at least partly, to variations in the duration of the period between the end of crop N uptake and the moment of harvest, to variable weather conditions in this period, and to different – soil dependent - mineralisation rates during this period.

In most crops, farmers can do little in terms of crop management to change $N_{min,H,0}$ for their specific conditions in the short term, other than trying to match crop timing with the natural mineralisation pattern. In maize, however, the use of catch crops is very well possible and even seems imperative to keep $N_{min,H,0}$ within acceptable limits, on soils with high mineralisation.

Consequence of a positive correlation between $N_{min,H,0}$ and U_0 – as in maize - is that ignoring both these terms from the N balance represented by Eqs. 2.7 and 2.8 (Chapter 2) causes relatively large errors in estimates of $N_{min,H}$. This becomes apparent from the poor correlation in maize between the N surplus (*A-U*) and $N_{min,H}$ (Paragraph 3.5.2).

A comment should be made here with respect to the amount of mineral soil nitrogen in Spring, $N_{min,S}$. Other than most balance terms, this variable can be obtained by direct measurement at the start of the season and so it could represent, for farmers, a guide in planning N management. Regression models that account explicitly for $N_{min,S}$ would be attractive for this reason, but in most experiments in arable crops the corresponding values were not recorded. Available data for grass, however, do enable to account for $N_{min,S}$ and this aspect is treated in Chapter 4. A comparison between Eqs 2.1 and 2.5 (Chapter 2) shows that we should use either $N_{min,S}$ or $N_{min,H,0}$ in such regression models, but not both, because $N_{min,H,0}$ incorporates already the effects of $N_{min,S}$. (The latter term is equal in fertilised and non-fertilised plots.) In models expressing $N_{min,H}$ directly as function of N rate *A,* $N_{min,S}$ can be regarded as being part of *A*. In all calculations for arable crops (Chapter 5) it was presumed that $N_{min,S}$ had a fixed value, and this value was subtracted from the crop-specific recommended N rate for which the expected $N_{min,H}$ value is calculated.

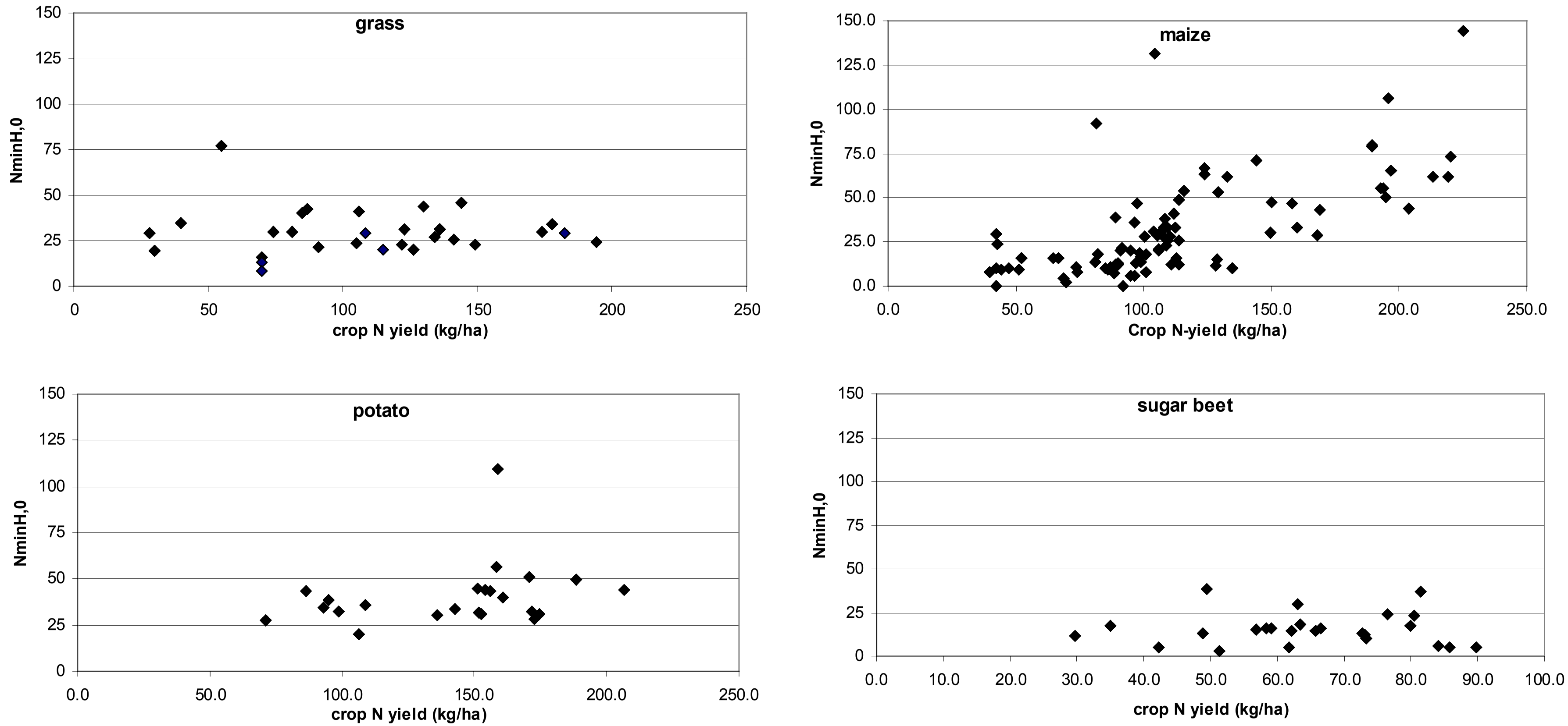


Figure 3.10. Residual mineral soil N at harvest versus crop N yield, observed in treatments with zero N input, in cut grass, maize, potato and sugar beet.

3.5 Linear regression models for $N_{min,H}$

3.5.1 The models

The structure of available data differs between crop species, and crops themselves differ, too, with respect to their $N_{min,H}$ responses. This makes a 'broad' survey of possible regression models desirable. We use Eqs. 2.8 and 2.11 as a starting point to identify suitable regressors (*x*-variates) composed of one or more balance terms, and we will inspect the performance of these 'partial balance regression models'. These models are all linear in the regression coefficients.

Model 1a.	$N_{min,H} = c + b^* A + e^*P$	3.1
Model 2a.	$N_{min,H} = c + b^*(A\text{-}U) + e^* P$	3.2
Model 3a.	$N_{min,H} = c + b^*(A\text{-}(U\text{-}U_0)) + e^* P$	3.3
Model 4a.	$N_{min,H} = c + b^*\mu\ (A\text{-}A_{crit})^2 + e^* P$	3.4
Model 5a.	$N_{min,H} = c + b^*(\rho_{ini}A - (U\text{-}U_0)) + e^* P$	3.5
Model 1b.	$N_{min,H} = a^*N_{min,H,0} + b^* A + e^* P$	3.6
Model 2b.	$N_{min,H} = a^*N_{min,H,0} + b^*(A\text{-}U) + e^* P$	3.7
Model 3b.	$N_{min,H} = a^*N_{min,H,0} + b^*(A\text{-}(U\text{-}U_0)) + e^* P$	3.8
Model 4b.	$N_{min,H} = a^*N_{min,H,0} + b^*\mu\ (A\text{-}A_{crit})^2 + e^* P$	3.9
Model 5b.	$N_{min,H} = a^*N_{min,H,0} + b^*(\rho_{ini}A - (U\text{-}U_0)) + e^* P$	3.10

where *a, b, c*, and *e* are the regression coefficients; *A* is the applied N rate (fertiliser N plus effective N in animal manures), A_{crit} is the critical N rate, *U* is the N yield (N uptake in harvested crop parts), and U_0 is the N yield observed in absence of N application, that is, harvested N that was supplied by the soil. All these variables are expressed in kg ha^{-1}. μ is a crop specific coefficient (see Eq. 2.11 Chapter 2; and Paragraph 3.8) that can be expressed in QUADMOD parameters. The values for μ were fixed *a priori*, based on independent ('generic') values of the QUADMOD crop parameters. (μ =0.000924 ha kg^{-1} for grass and 0.00175 ha kg^{-1} for maize.)
P is the precipitation (mm) accumulated over the entire growing season. For the maize data sets, this variable was assessed as the integral of precipitation between the actual sowing date and actual harvest date corresponding to the particular experiment. For grass, precipitation was integrated over the period from April 1st to October 15th for all data sets. Daily precipitation data from the nearest KNMI weather station were used, both for the grass and the maize experiments. The mean value of *P* was 450 mm for the grass experiments and 345 mm for the maize experiments.

3.5.2 Results for grass and maize

Our analysis focusses (cut) grass and maize, because these are dominant crops and the data sets available for these crops are far more numerous than for other crops, especially with reference to sandy soils. For the purpose of regression analysis, we used for these two crops only data from sandy soils. The results for (ware) potato, sugar beet, cauliflower, broccoli and white cabbage are given in a separate paragraph (3.5.4). Virtually all data on those crops refer to clay soils, and the results are not always consistent with the findings for grass and maize on sand.

The results obtained with the 10 linear models are given in Tables 3.2 (grass) and 3.3 (maize).

In grass, a gradual improvement of the correlation coefficient with increasing complexity of the model is seen, with the percentage of variance accounted for ($R^2_{adjusted}$, here also referred to as R^2) increasing from about 53% to about 76%. The inclusion of $N_{min,H,0}$ in the model does not bring an improvement. Comparison between the model series 1a-5a *versus* 1b-5b shows, however, that the importance of the precipitation term *eP* diminishes when $N_{min,H,0}$ is introduced into the model, suggesting that the negative

effect of rainfall on $N_{\mathrm{min,H}}$ can be largely expressed via the associated variation in $N_{\mathrm{min,H,0}}$. In the 1b-5b series, the effect of rainfall becomes very small and even positive values appear for *e*.
For grass, Models 4 and 5 show the best performance, both with and without the $N_{\mathrm{min,H,0}}$ term, in terms of R^2. These models show values for coefficients *a* and *b* that are reasonably close to unity, suggesting that indeed the associated – more complex -regressors are a better approximation of the overall mineral N balance than those in Models 1-3.

The relation between model complexity and R^2 is less consistent in maize (Table 3.3). Although we do see better fits with the Models 3a-5a than with Models 1a-2a, this ranking vanishes upon including $N_{\mathrm{min,H,0}}$ in the model. The general pattern with maize is that adopting $N_{\mathrm{min,H,0}}$ into the model results in a drastic improvement of the fit. Then, all models perform roughly equally well, and the simplest expression with N rate A as regressor behaves surprisingly well, with 73% of the variance explained. (In grass this was only 53%.) We see again – as in grass – that the importance of the eP term decreases as we move from the 1a-5a to the 1b-5b model series.
Model 2, with A-U as regressor, performs least of all, both with and without the $N_{\mathrm{min,H,0}}$ term, but the fit is really poor if this term is omitted. An explanation for this lack of correlation, specifically in maize, has been suggested in Paragraph 3.4.
In favor of the Models 4 and 5 is – as in grass – that the coefficients *a* and *b* approximate here, better than for other models, the value of 1.

Table 3.2. Percentage of variance accounted for, and estimated regression parameters for Models 1a-5a ($N_{min,H,0}$ not included as regressor) and Models 1b-5b (which include $N_{min,H,0}$) for grass on sandy soils.

Grass Model	estimate c	se	estimate b	se	estimate e	se	R^2_{adjusted}
1a. A	64.3	19.0	0.238	0.015	-0.151	0.042	53.2%
2a. A-U	204.0	17.0	0.604	0.034	-0.219	0.039	59.0%
3a. A-(U-U_0)	95.3	15.5	0.660	0.031	-0.167	0.035	67.0%
4a. $\mu\,(A-A_{\mathrm{crit}})^2$	80.4	13.0	0.720	0.026	-0.110	0.029	77.2%
5a. $\rho_{\mathrm{ini}}A-(U-U_0)$	44.3	13.7	0.806	0.030	-0.019	0.031	75.8%
	estimate a	se	estimate b	se	estimate e	se	R^2_{adjusted}
1b. A	0.97	0.30	0.240	0.015	-0.070	0.021	53.0%
2b. A-U	3.46	0.27	0.638	0.033	0.024	0.018	61.6%
3b. A-(U-U_0)	1.68	0.24	0.670	0.030	-0.062	0.017	68.3%
4b. $\mu\,(A-A_{\mathrm{crit}})^2$	0.95	0.21	0.726	0.027	0.009	0.015	75.5%
5b. $\rho_{\mathrm{ini}}A-(U-U_0)$	0.83	0.21	0.806	0.030	0.027	0.014	76.3%

Table 3.3. Percentage of variance accounted for, and estimated regression parameters for Models 1a-5a ($N_{min,H,0}$ not included as regressor) and Models 1b-5b (which include $N_{min,H,0}$) for maize on sandy soils.

Maize Model	estimate c	se	estimate b	se	estimate e	se	$R^2_{adjusted}$
1a. A	55.0	0.5	0.447	0.032	-0.099	0.028	36.2%
2a. A-U	120.0	11.0	0.353	0.045	-0.099	0.032	15.5%
3a. A-(U-U_0)	60.7	8.1	0.681	0.036	-0.108	0.024	51.9%
4a. $\mu\,(A-A_{crit})^2$	80.7	8.3	1.153	0.066	-0.128	0.025	47.6%
5a. $\rho_{ini}A-(U-U_0)$	70.1	8.0	1.008	0.052	-0.112	0.024	52.7%
	estimate a	se	estimate b	se	estimate e	se	$R^2_{adjusted}$
1b. A	1.44	0.06	0.388	0.021	-0.074	0.009	73.3%
2b. A-U	1.91	0.07	0.360	0.028	0.046	0.010	63.3%
3b. A-(U-U_0)	1.25	0.06	0.531	0.028	-0.037	0.008	74.0%
4b. $\mu\,(A-A_{crit})^2$	1.35	0.07	0.861	0.052	-0.022	0.009	69.9%
5b. $\rho_{ini}A-(U-U_0)$	1.24	0.07	0.745	0.044	-0.013	0.009	70.5%

3.5.3 Patterns of $N_{min,H}$ versus selected regressors

This paragraph illustrates graphically, for the example case of maize, the patterns of $N_{min,H}$ versus the various regressors, each of which represents one or more components of the mineral N balance. (See also Chapter 2.). All figures are based on the same data sets, as analysed in the previous paragraph.

As a general rule we should expect the relation between $N_{min,H}$ and the applied N rate A to be non-linear in A, because the uptake U tends to level off at high N supply and so more N will remain 'unused', per unit applied N, as the crop approaches a state of 'nitrogen-saturation'. Chapter 5 will address a large range crops for which $N_{min,H}$ is described by a regression model that is non-linear in both A and in the parameters, and will confirm this expectation. In Figure 3.11, however, the non-linearity in A is not so evident - as also supported by the good performance of Model 1. This is partly the result of merging data from different experiments.

Models expressing $N_{min,H}$ as just a function of the applied N rate are attractive because they require no additional information. Large residuals, on the other hand, are found due to variation in U, U_0 and $N_{min,H,0}$ that exist across experiments (i.e., locations, years), as explained in Chapter 2.

Figure 3.12 underlines the poor performance of the surplus-based Model 2a in the previous paragraph. With the help of Eq. 2.5 (Chapter 2) it can be understood why the N-surplus (A-U) should be a rather poor indicator for $N_{min,H}$, given the balance terms ignored when taking just (A-U) as regressor. The large variation normally found in U_0 – with values between 50 and 200 kg N ha^{-1} - causes a high level of 'noise' in Figure 3.12, and it is common in all crops to find negative surplusses associated with considerable $N_{min,H}$ values. In maize, 'noise' in the $N_{min,H}$ versus (A-U) relation is especially large because U_0 is large relative to A, to U, and to their difference, but also because $N_{min,H,0}$ and U_0 are positively correlated as was demonstrated in Fig. 3.10, which enhances the error caused by ignoring these terms (See Eq. 2.5). Further, variation in the effectiveness ('working coefficient') of animal manures should be mentioned here as another possible cause of 'noise': we used a fixed value for this coefficient in converting all N-inputs to effective-N doses. As N was often applied in the form of animal manures, in the maize trials, this variation is more expressed in maize than in other crops.

Moreover, late-season mineralisation from manures (beyond 'uptake season') may have contributed to large and variable $N_{\mathrm{min,H}}$ in maize.

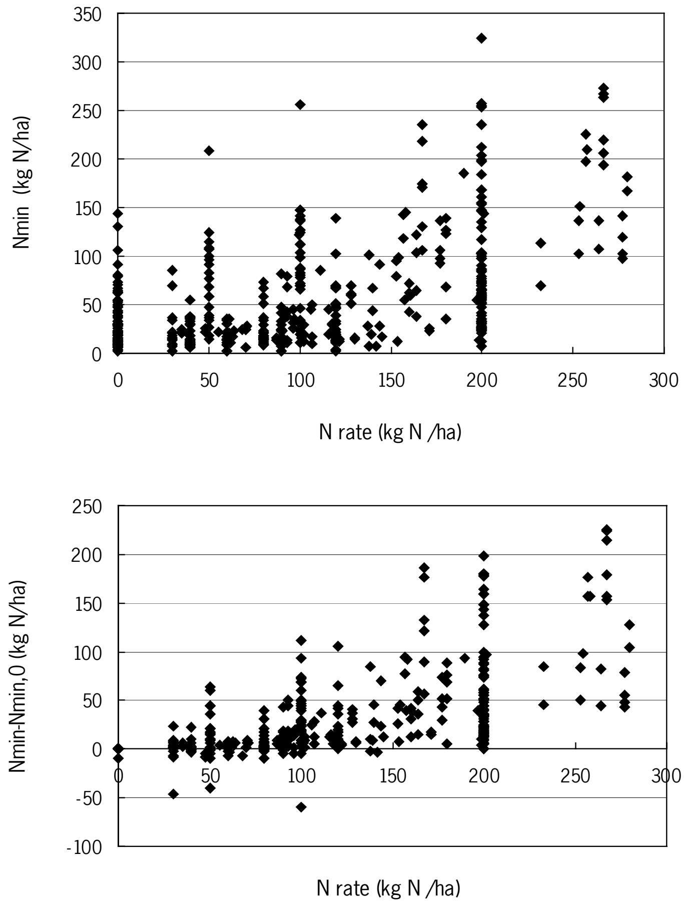


Figure 3.11. *Residual mineral N at harvest ($N_{min,H}$) observed in the 0-60 cm soil layer, versus the rate of effective N applied, in maize on sandy soils in the Netherlands (top). The lower graph shows the difference ($N_{min,H}$.-$N_{min,H,0}$) between treatments that received N input and those that received no N input, versus the rate of effective N applied.*

Note on the definition of N-surplus

It is stressed that N surplus is defined in this chapter as the difference between effective N input (only) and crop offtake. This deviates from the standard definition where the total N input is used in calculating the surplus. We chose the 'narrower' definition because more 'noise' would be introduced in the various relations if total N input were used in calculating the surplus. The long term effects of N that is 'ineffective' for crop nutrition in the short term are discussed in the last paragraph of this chapter.

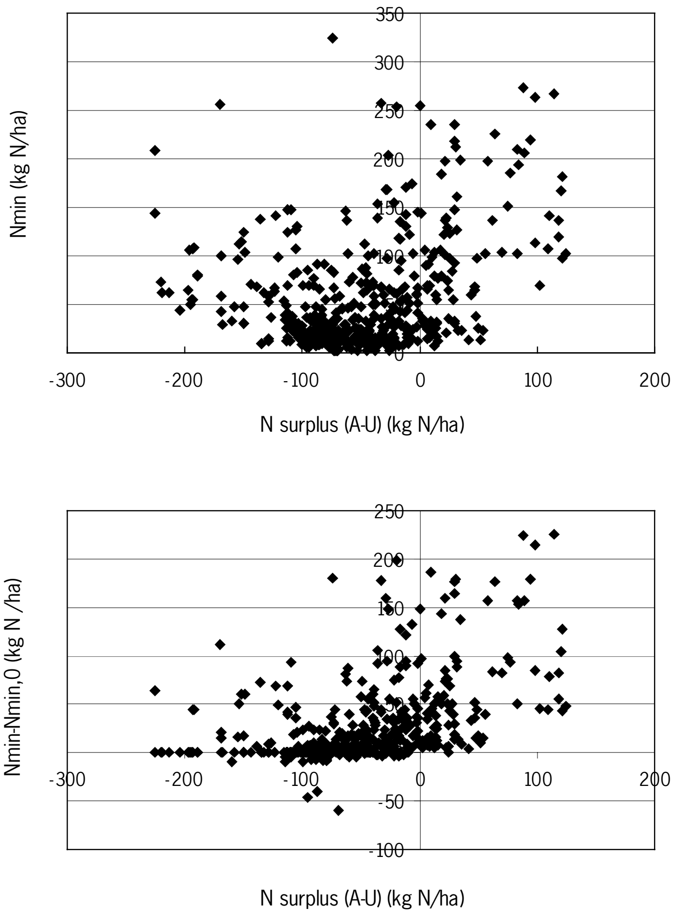


Figure 3.12. Residual mineral N at harvest ($N_{min,H}$) observed in the 0-60 cm soil layer, versus the N surplus, in maize on sandy soils in the Netherlands (top). The lower graph shows the difference ($N_{min,H}$.- $N_{min,H,0}$) between treatments that received N input and those that received no N input, versus the N surplus. In both cases, the N surplus is the rate of effective N applied minus N offtake by the crop.

If we use A-(U-U_0) as regressor, the surplus is corrected for local soil fertility U_0 and this improves the relation with $N_{min,H}$ considerably (Figure 3.13). Obviously there is no problem with negative surplusses here: the corrected surplus is the amount of N that remains from the effective input A after subtracting the N recovered from this input by the crop. The figure shows that high $N_{min,H}$ at A-(U-U_0)=0 seen in the top graph are entirely due to high base values $N_{min,H,0}$; as they disappear in the bottom graph in Figure 3.13.

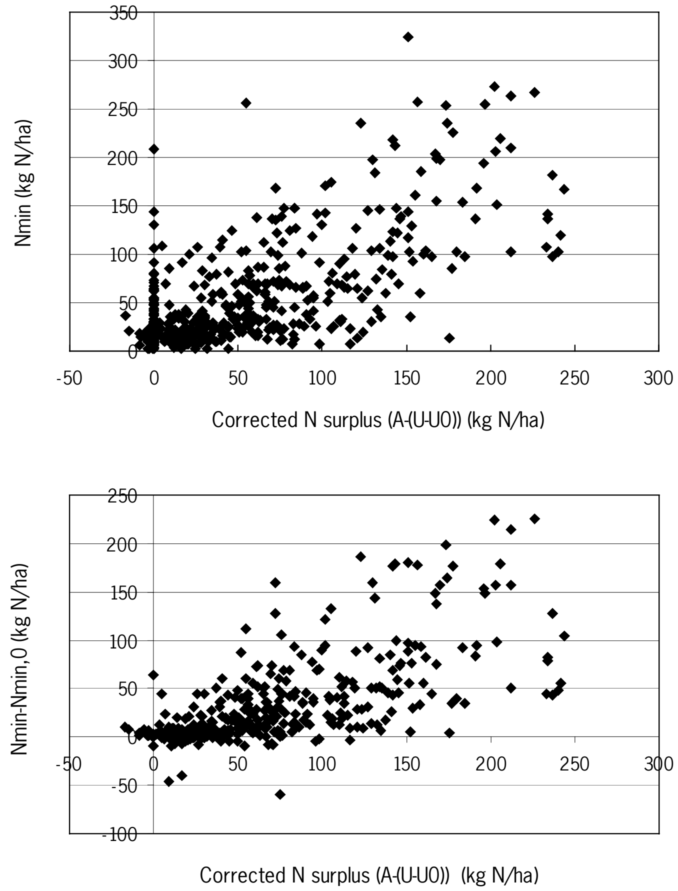
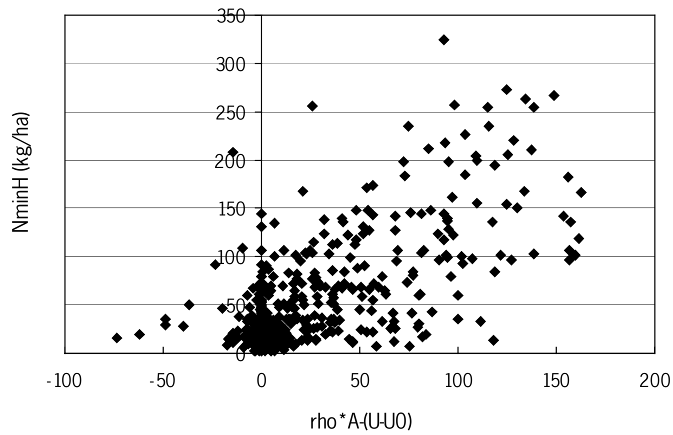


Figure 3.13. *Residual mineral N at harvest ($N_{min,H}$) observed in the 0-60 cm soil layer, versus the U_0-corrected N surplus, or 'surplus from fertiliser', in maize on sandy soils in the Netherlands (top). The lower graph shows the difference ($N_{min,H}$- $N_{min,H,0}$) between treatments that received N input and those that received no N input, versus the corrected N surplus.*



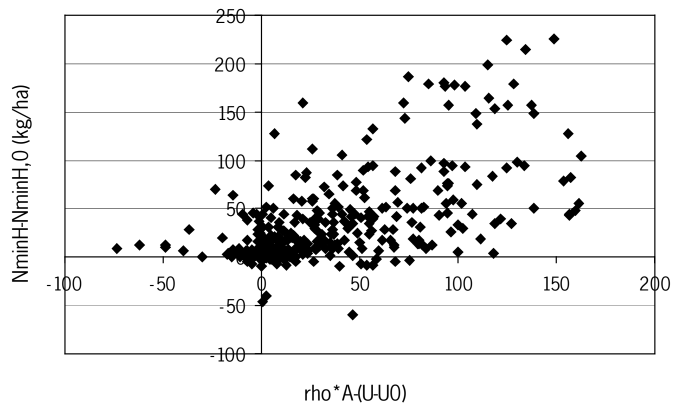


Figure 3.14. Residual mineral N at harvest ($N_{min,H}$) observed in the 0-60 cm soil layer, versus $\rho_{ini}A$-(U-U_0), in maize on sandy soils in the Netherlands (top). The lower graph shows the difference ($N_{min,H}$- $N_{min,H,0}$) between treatments that received N input and those that received no N input, versus the same x-variate.

Introducing $\rho_{ini}A$-(U-U_0) as x-variate does not improve the correlation with $N_{\mathrm{min,H}}$ (cf. Table 3.3). As discussed earlier, the proposition that a fraction (1-ρ_{ini}) of the applied N rate is entirely lost (and thus not found as residual mineral N) apparently does not hold strictly in maize, as it does in grass. Nevertheless, the change in the coefficient b from 0.68 (Model 3a) to 1.0 (Model 5a) suggests that the structure of a model based on $\rho_{ini}A$-(U-U_0) may be more attractive than one based on A-(U-U_0) only.

Finally, Figure 3.15 demonstrates the pattern of the relation with $N_{\mathrm{min,H}}$ when the regressor $\rho_{ini}A$-(U-U_0) is approximated by (A-A_{crit})^2 (see Chapter 2). Though the figure suggests a coalescence of the data points, relative to Figure 3.14, the correlation coefficient hardly changes (Table 3.3).

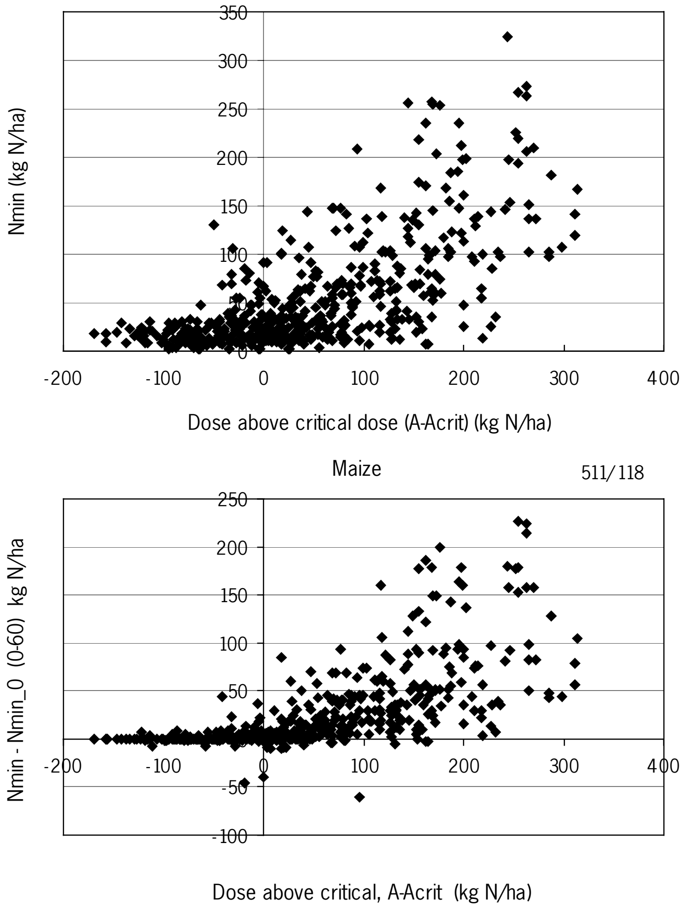


Figure 3.15. Residual mineral N at harvest ($N_{min,H}$) observed in the 0-60 cm soil layer, versus $(A-A_{crit})^2$, in maize on sandy soils in the Netherlands (top). The lower graph shows the difference ($N_{min,H}$ - $N_{min,H,0}$) between treatments that received N input and those that received no N input, versus the same x-variate.

3.5.4 Crops other than grass and maize

The results obtained with the linear regression models of Paragraph 3.5.1 in potato, sugar beet, white cabbage, cauliflower, and broccoli are given in Tables 3.4 to 3.8. Note that virtually all data refer to clay soils (cf Table 3.1).
In analysing the potato data, we omitted all data on starch potato because $N_{min,H}$ values were very scattered and seemed inconsistent with N management. (The data were shown in Figure 3.3.)

The general pattern with these five crops, as with grass and maize, is that models improve consistently by including the term $N_{min,H,0}$ as a regressor, as can be seen by comparing the series 1a-5a with 1b-5b within each of the tables. The only exception is broccoli.

No single model comes out as an overall best model. In potato, Model 4 performs best but Model 5, which has very similar structure (cf Chapter 2), gives very poor results. The straight dose, A, shows a reasonable correlation with $N_{min,H}$ for this crop, compared with the other models. Irrespective of the model, no better values for R^2 than about 60% were obtained in potato.

In sugar beet, correlations are very poor but then all $N_{min,H}$ observations remained low and could hardly be distinguished from the base values (See also Figure 3.4).

The three vegetable crops show variable results. In white cabbage, Model 4b is the best and Model 5 the least, in terms of R^2. In cauliflower, Models 3 and 5 are the best, and 4 is the worst, both with and without inclusion of $N_{\mathrm{min,H,0}}$. In broccoli, all models except Model 5 perform approximately equal, both with and without inclusion of $N_{\mathrm{min,H,0}}$. In broccoli no more than about 60% of the variance could be explained. This fraction was higher in cauliflower (up to 76%, with model 5) and white cabbage (up to 86%, with Model 4).

Of all crops tested, only cauliflower and broccoli show values for the coefficient *b* that are well above 1.0 in Models 4 and 5. This suggests that at higher N doses, more N is left as residual mineral soil N than expected on the basis of Eqs. 2.8 and 2.11, and that the second proposition tested in Paragraph 3.3 is invalid for these crops.

Table 3.4. *Percentage of variance accounted for, and estimated regression parameters for Models 1a-5a ($N_{min,H,0}$ not included as regressor) and Models 1b-5b (which include $N_{min,H,0}$) for ware potato on clay soils.*

Ware potato Model	estimate *c*	se	estimate *b*	se	estimate *e*	se	R^2_{adjusted}
1a. A	66.0	6.6	0.16	0.02	-0.08	0.02	45.6%
2a. A-U	86.9	6.5	0.15	0.03	-0.07	0.02	33.5%
3a. A-(U-U_0)	67.6	6.8	0.20	0.03	-0.08	0.02	41.9%
4a. μ (A-A_{crit})^2	72.3	5.5	0.69	0.07	-0.09	0.01	56.3%
5a. $\rho_{\mathrm{ini}}A$–(U-U_0)	85.2	7.5	0.13	0.09	-0.09	0.02	18.6%
	estimate *a*	se	estimate *b*	se	estimate *e*	se	R^2_{adjusted}
1b. A	1.34	0.11	0.17	0.02	-0.04	0.01	58.8%
2b. A-U	1.82	0.10	0.22	0.02	0.00	0.01	55.1%
3b. A-(U-U_0)	1.37	0.11	0.23	0.03	-0.04	0.01	55.7%
4b. μ (A-A_{crit})^2	1.42	0.10	0.71	0.07	-0.03	0.01	61.1%
5b. $\rho_{\mathrm{ini}}A$–(U-U_0)	1.64	0.13	0.25	0.08	-0.03	0.01	26.8%

Table 3.5. Percentage of variance accounted for, and estimated regression parameters for Models 1a-5a ($N_{min,H,0}$ not included as regressor) and Models 1b-5b (which include $N_{min,H,0}$) for sugar beet on clay soils.

Sugar beet Model	estimate *c*	se	estimate *b*	se	estimate *e*	se	$R^2_{adjusted}$
1a. A	3.31	4.95	0.01	0.01	0.02	0.01	2.6%
2a. A-U	6.33	4.92	0.02	0.01	0.02	0.01	4.5%
3a. A-(U-U_0)	3.19	4.85	0.02	0.01	0.02	0.01	4.3%
4a. μ $(A\text{-}A_{crit})^2$	4.27	4.96	0.003	0.10	0.02	0.01	1.8%
5a. $\rho_{ini}A$–(U-U_0)	3.07	4.86	0.04	0.02	0.02	0.01	4.4%
	estimate *a*	se	estimate *b*	se	estimate *e*	se	$R^2_{adjusted}$
1b. A	0.34	0.08	0.01	0.01	0.02	0.003	14.0%
2b. A-U	0.34	0.08	0.02	0.01	0.02	0.002	15.3%
3b. A-(U-U_0)	0.33	0.08	0.02	0.01	0.02	0.003	14.9%
4b. μ $(A\text{-}A_{crit})^2$	0.35	0.09	0.01	0.09	0.02	0.003	13.3%
5b. $\rho_{ini}A$–(U-U_0)	0.34	0.08	0.04	0.02	0.02	0.003	15.6%

Table 3.6. Percentage of variance accounted for, and estimated regression parameters for Models 1a-5a ($N_{min,H,0}$ not included as regressor) and Models 1b-5b (which include $N_{min,H,0}$) for white cabbage on clay soils.

White cabbage Model	estimate *c*	se	estimate *b*	se	estimate *e*	se	$R^2_{adjusted}$
1a. A	2.48	7.76	0.06	0.01	0.02	0.01	40.6%
2a. A-U	6.85	7.44	0.09	0.02	0.02	0.01	43.1%
3a. A-(U-U_0)	2.31	7.48	0.10	0.02	0.02	0.01	44.7%
4a. $\mu(A\text{-}A_{crit})^2$	16.9	6.8	0.43	0.07	0.00	0.01	52.0%
5a. $\rho_{ini}A$–(U-U_0)	9.84	7.46	0.34	0.06	0.01	0.01	42.0%
	estimate *a*	se	estimate *b*	se	estimate *e*	se	$R^2_{adjusted}$
1b. A	1.04	0.17	0.05	0.01	-0.002	0.005	70.3%
2b. A-U	1.05	0.16	0.08	0.01	0.010	0.005	73.0%
3b. A-(U-U_0)	0.99	0.16	0.08	0.01	0.00	0.01	71.4%
4b. μ $(A\text{-}A_{crit})^2$	1.20	0.11	0.41	0.04	0.00	0.00	86.1%
5b. $\rho_{ini}A$–(U-U_0)	0.98	0.18	0.28	0.05	0.01	0.01	65.2%

Table 3.7. Percentage of variance accounted for, and estimated regression parameters for Models 1a-5a ($N_{min,H,0}$ not included as regressor) and Models 1b-5b (which include $N_{min,H,0}$) for cauliflower on clay soils.

Cauliflower Model	estimate c	se	estimate b	se	estimate e	se	$R^2_{adjusted}$
1a. A	194	28	0.23	0.04	-0.87	0.15	55.0%
2a. A-U	217	29	0.27	0.05	-0.87	0.15	54.6%
3a. A-(U-U_0)	189	25	0.31	0.04	-0.86	0.13	64.7%
4a. μ (A-A_{crit})^2	184	31	1.41	0.34	-0.74	0.16	44.3%
5a. $\rho_{ini}A$–(U-U_0)	161	20	1.20	0.13	-0.68	0.10	75.8%
	estimate a	se	estimate b	se	estimate e	se	$R^2_{adjusted}$
1b. A	2.12	0.22	0.23	0.03	-0.15	0.04	70.0%
2b. A-U	2.36	0.22	0.27	0.04	-0.07	0.03	72.1%
3b. A-(U-U_0)	1.98	0.21	0.28	0.04	-0.14	0.04	74.0%
4b. μ (A-A_{crit})^2	2.04	0.26	1.37	0.29	-0.06	0.04	59.8%
5b. $\rho_{ini}A$–(U-U_0)	1.57	0.21	1.01	0.14	-0.05	0.03	73.6%

Table 3.8. Percentage of variance accounted for, and estimated regression parameters for Models 1a-5a ($N_{min,H,0}$ not included as regressor) and Models 1b-5b (which include $N_{min,H,0}$) for broccoli on clay soils.

Broccoli Model	estimate c	se	estimate b	se	estimate e	se	$R^2_{adjusted}$
1a. A	1.64	5.29	0.15	0.02	0.06	0.03	54.7%
2a. A-U	7.58	4.69	0.17	0.02	0.05	0.03	58.0%
3a. A-(U-U_0)	2.11	5.08	0.17	0.02	0.06	0.03	57.2%
4a. μ (A-A_{crit})^2	19.1	4.5	1.61	0.24	0.01	0.04	55.0%
5a. $\rho_{ini}A$–(U-U_0)	25.0	4.9	1.68	0.29	-0.04	0.04	48.2%
	estimate a	se	estimate b	se	estimate e	se	$R^2_{adjusted}$
1b. A	0.61	0.87	0.15	0.02	0.02	0.07	55.2%
2b. A-U	1.44	0.79	0.17	0.02	-0.03	0.07	58.8%
3b. A-(U-U_0)	0.70	0.84	0.17	0.02	0.01	0.07	57.8%
4b. μ (A-A_{crit})^2	3.36	0.77	1.63	0.24	-0.17	0.07	56.0%
5b. $\rho_{ini}A$–(U-U_0)	4.11	0.87	1.65	0.30	-0.25	0.08	45.6%

3.6 Expo-linear models

3.6.1 The models

The following expo-linear models were fitted to the data:

Model 6	$N_{min,H} = a*N_{min,H,0} + (b/c) * \ln(1+ \exp(c*(A-A_{crit} -d)))$	3.11
Model 7	$N_{min,H} = a*N_{min,H,0} + (b/c) * \ln(1+ \exp(c*(A-(U-U_0) -d)))$	3.12
Model 8	$\Delta N_{min,H} = k + (b/c) * \ln(1+ \exp(c*(A-A_{crit} -d)))$	3.13
Model 9	$\Delta N_{min,H} = k + (b/c) * \ln(1+ \exp(c*(A-(U-U_0) -d)))$	3.14
Model 6p	$N_{min,H} = a*N_{min,H,0} + (b/c) * \ln(1+ \exp(c*(A-A_{crit} -d))) + e P$	3.15
Model 7p	$N_{min,H} = a*N_{min,H,0} + (b/c) * \ln(1+ \exp(c*(A-(U-U_0) -d))) + e P$	3.16
Model 8p	$\Delta N_{min,H} = k + (b/c) * \ln(1+ \exp(c*(A-A_{crit} -d))) + e P$	3.17
Model 9p	$\Delta N_{min,H} = k + (b/c) * \ln(1+ \exp(c*(A-(U-U_0) -d))) + e P$	3.18

where *a, b, e* and *k* are the linear regression coefficients and *c, d* the non-linear coefficients. $\Delta N_{min,H}$ is the difference between the experimental values of $N_{min,H}$ and $N_{min,H,0}$. All other variables are as in the earlier models (Paragraph 3.5.1).
Expolinear models have been introduced to describe crop growth (Goudriaan and Monteith , 1990) and their characteristic is that, after an exponential response of y to the independent x for low x-values, the slope approaches a fixed value (*b*) for larger values of x. Coefficient *c* is the relative rate of change of x in the exponential phase.

The expolinear pattern seems suitable to describe the response of $N_{min,H}$ to the selected 'normalised' N rate-variables, $A-A_{crit}$ and $A-(U-U_0)$. Its major advantage over other models is the constant slope of $N_{min,H}$ at 'saturating' N rates. At the same time, the above forms include the base level $N_{min,H,0}$ which was shown to be important in the previous paragraphs.

3.6.2 Results (non-weighted regression)

The results are given in Tables 3.9 and 3.10. Gaps in these tables indicate that no convergence could be obtained by the (GENSTAT) 'Fit-Non-linear' procedure used for this analysis, which implies that the data set does not allow the estimation of one or more model parameters.

The results show that the ($A-A_{crit}$)-form describes best the response in grass. Including precipitation *P* as extra regressor brings virtually no improvement.
In maize, both the ($A-A_{crit}$)-form and the $A-(U-U_0)$-form perform well, but only after including precipitation *P* in the model. The models with ΔNmin as response variable gave no result if $A-(U-U_0)$ was used as regressor, with or without *P*.

Based on these results, further analyses were restricted to the Models 6p and 7p only, which take $N_{min,H}$ as response variable and include precipitation as an explaining factor.

Table 3.9. Percentage of variance accounted for ($R^2_{adjusted}$) and estimated parameter values for Models 6-9 and Models 6p-9p, for grass on sandy soils.

Grass	R^2_{adj}	*b/c*	*b*	*c*	*d*	*a* or *k*	*e*
6. $N_{min,H}$, A-A_{crit}	74.1	154.5	1.05	0.0068	392.4	0.83	-
7. $N_{min,H}$, A-$(U$-$U_0)$	59.4	58.6	1.00	0.0172	97.6	0.70	-
8. Δ $N_{min,H}$, A-A_{crit}	74.4	72.8	0.77	0.0106	291.1	3.72	-
9. Δ $N_{min,H}$, A-$(U$-$U_0)$	-	-	-	-	-	-	-
6p. + precipitation	75.7	147.0	0.99	0.0068	373	0.83	-0.0034
7p. + precipitation	70.3	59.6	0.88	0.0147	29.9	1.37	-0.092
8p. + precipitation	75.3	77.9	0.78	0.0100	296.3	37.8	-0.081
9p. + precipitation	69.8	23.7	0.83	0.0351	48.8	53.9	-0.124

Table 3.10. Percentage of variance accounted for ($R^2_{adjusted}$) and estimated parameter values for Models 6-9 and Models 6p-9p, for maize on sandy soils.

Maize	R^2_{adj}	*b/c*	*b*	*c*	*d*	*a* or *k*	*e*
6. $N_{min,H}$, A-A_{crit}	-	-	-	-	-	-	-
7. $N_{min,H}$, A-$(U$-$U_0)$	-	-	-	-	-	-	-
8. Δ $N_{min,H}$, A-A_{crit}	53.5	2.7	0.43	0.161	19.0	0.63	-
9. Δ $N_{min,H}$, A-$(U$-$U_0)$	-	-	-	-	-	-	-
6p. + precipitation	73.7	6.96	0.42	0.061	10.8	1.25	-0.036
7p. + precipitation	74.5	20.8	0.58	0.028	-13.0	1.17	-0.077
8p. + precipitation	56.3	4.67	0.45	0.097	21.0	26.97	-0.085
9p. + precipitation	-	-	-	-	-	-	-

3.6.3 Results (weighted regression)

It was investigated how the increase in variance with increasing x-variate could be accounted for. This can be done by assigning weights which are a function of the difference ε between the value of the fitted model (y_m) and the observation. First, a weight proportional to y_m^{-2} was tried, but this proved to overestimate the response of variance to the x-variate. We then chose to take $(y_m + tiny)^{-1}$ as weight, where *tiny* is a small positive real to avoid division by zero. It was thus assumed that the variance is proportional to the function value y_m.

The results are shown in Tables 3.11 and 3.12. No dramatic effects of the regression method (weighted vs non-weighted) on the parameters are seen. The above pattern is confirmed here: the grass data are best decribed by the A-A_{crit}-model, whereas both the A-A_{crit}- form and the A-$(U$-$U_0)$-form are well suited for the maize data.

An emerging difference between grass and maize is the consistently higher value of parameter *b* in grass, which suggests that at 'saturating' N rates a much larger proportion of incremental N doses are left as residual N in grass than in maize. (This occurs, however, at much higher N rates in grass than in maize). We cannot assess whether this difference is an artefact arising from the chosen upper range of N rates in the experiments.

Table 3.11. Percentage of variance accounted for ($R^2_{adjusted}$) and estimated parameter values for Models 6p and 7p, with non-weighted and weighted regression for grass on sandy soils.

Grass	Non-weighted estimate	s.e.	Weighted estimate	s.e.
6p. A-A_{crit}				
b	0.985	0.313	0.915	0.282
c	0.0067	0.0017	0.0080	0.0017
d	373	109	343	89
a	0.83	0.22	1.05	0.14
e	-0.0034	0.017	-0.003	0.009
$R^2_{adjusted}$	75.7%		72.0%	
7p. A-(U-U_0)				
b	0.875	0.115	0.947	0.150
c	0.0147	0.0060	0.0201	0.0083
d	29.9	31.5	67.9	24.4
a	1.37	0.30	1.37	0.19
e	-0.092	0.033	-0.037	0.016
$R^2_{adjusted}$	70.3%		67.7%	

Table 3.12. Percentage of variance accounted for ($R^2_{adjusted}$) and estimated parameter values for Models 6p and 7p, with non-weighted and weighted regression for maize on sandy soils.

Maize	Non-weighted estimate	s.e.	Weighted estimate	s.e.
6p. A-A_{crit}				
b	0.422	0.038	0.419	0.037
c	0.0608	0.0763	0.094	0.122
d	10.8	14.5	29.0	10.3
a	1.25	0.07	1.23	0.059
e	-0.036	0.011	-0.013	0.005
$R^2_{adjusted}$	73.7%		77.8%	
7p. A-(U-U_0)				
b	0.592	0.072	0.580	0.060
c	0.0284	0.0150	0.0496	0.0268
d	-13.0	19.6	14.3	10.3
a	1.17	0.07	1.17	0.06
e	-0.077	0.018	-0.025	0.010
$R^2_{adjusted}$	74.5%		76.8%	

3.7 Confidence intervals of selected models

The 90%-confidence intervals for one linear model (2b) and one non-linear model (6p) were determined and are given in Figures 3.16-3.19. Model 2b was selected because of its practical significance, the surplus A-U representing an easy-to-measure variable. Model 6p was chosen because it gave the best results on grass and maize combined, and because it is attractive for further modelling efforts, given its constant final slope. This property makes the model robust in more complex applications (whole farm modelling; uncertainty analyses), in contrast to models quadratic in the x-variate. Values chosen for the constants required to parameterize the models are given in Table 3.13. The values are considered representative of average conditions for grass and maize, respectively.

Table 3.13. Values adopted for constants in Models 2b and 6p, to enable the calculation of model predictions and confidence intervals.

Variabele	gras	mais
$N_{min,H,0}$ (kg N ha^{-1})	20	30
Precipitation (mm)	450	345
A_{crit} [1]	265	40
N rate (A) [1]	0 to 500	0 to 200
Surplus (A-U) [2]	-200 to +200	-100 to +100
A-A_{crit} [1]	-265 to 235	-40 to +160

[1]. *applies to Model 6p only;* [2] *applies to Model 2b only*

Note that the graphs show both the confidence interval for the regression curve, and for new individual predictions. The latter is, obviously, much wider, and this is a measure of the uncertainty we are dealing with when the model is applied under any new set of conditions.
The confidence interval (new prediction) is slightly larger for the surplus based linear model than for the expolinear model. On average, an $N_{min,H}$-level of 40 kg N ha^{-1} is reached at N rates of about 400 kg ha^{-1} in grass and 100 kg ha^{-1} in maize (bold lines in graphs, Figs. 3.16, 3.17). This corresponds, according to the N-surplus based model, with an N surplus of values of about –70 kg ha^{-1} in grass, and –100 kg ha^{-1} in maize (bold lines in graphs, Figs. 3.18, 3.19). It is stressed again that the surplus is based on effective N input only; the relations should apply, therefore, to mineral fertilisers as well as animal manures. The surplus value based on total N input is obviously higher with animal manures than with fertilisers, at the same N rate.

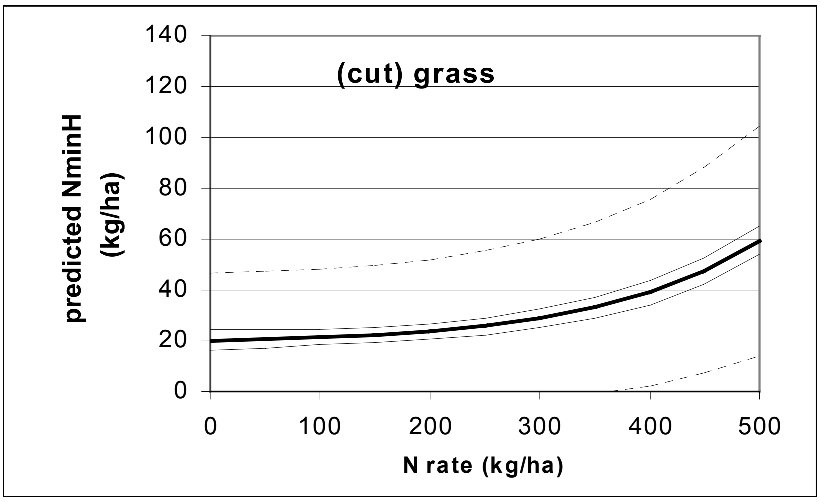


Figure 3.16. Prediction of the response of $N_{min,H}$ to N rate in (cut) grass (bold line), according to the A_{crit}-based expo-linear model (Model 6p). The 90% confidence interval for the regression line is indicated in solid lines; the 90% confidence interval for a prediction under a new set of conditions is given in broken lines. The confidence intervals increase with N rate. Values presumed for constants are listed in Table 3.13.

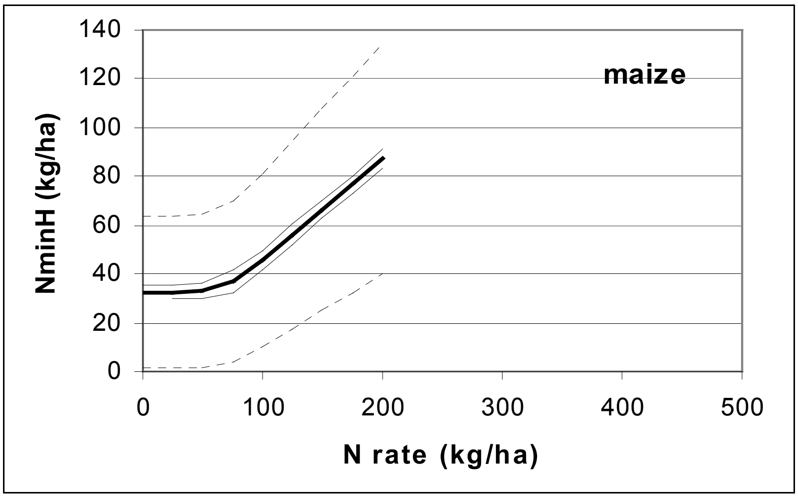


Figure 3.17. Prediction of the response of $N_{min,H}$ to N rate in maize (bold line), according to the A_{crit}-based expo-linear model (Model 6p). The 90% confidence interval for the regression line is indicated in solid lines; the 90% confidence interval for a prediction under a new set of conditions is given in broken lines. The confidence intervals increase with N rate. Values presumed for constants are listed in Table 3.13.

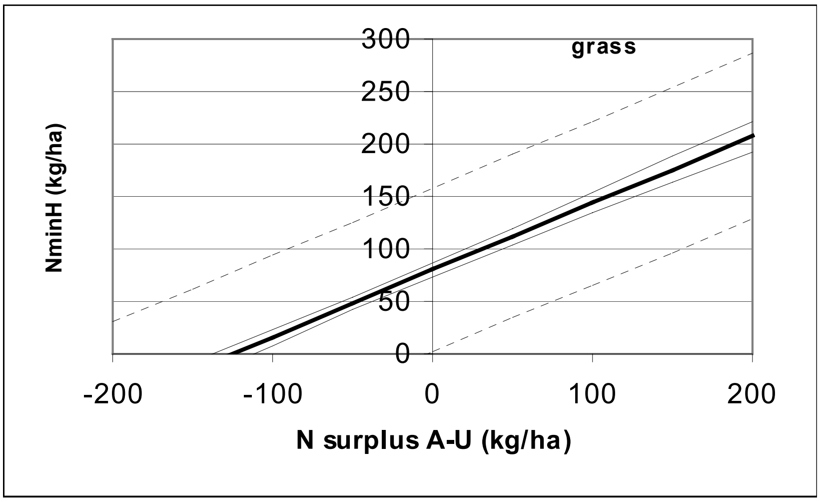


Figure 3.18. Prediction of the response of $N_{min,H}$ to N rate in grass (bold line), based on the linear surplus model (Model 2b). The 90% confidence interval for the regression line is indicated in solid lines; the 90% confidence interval for a prediction under a new set of conditions is given in broken lines. In calculating the surplus, only the effective N rate (not total N input) was taken as N input. The graph therefore applies to both fertiliser and animal manures. Values presumed for constants are listed in Table 3.13.

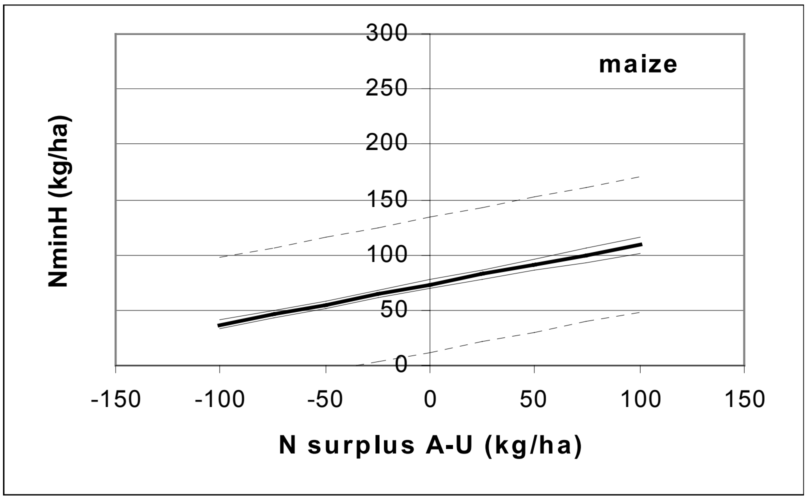


Figure 3.19. Prediction of the response of $N_{min,H}$ to N rate in maize (bold line), based on the linear surplus model (Model 2b). The 90% confidence interval for the regression line is indicated in solid lines; the 90% confidence interval for a prediction under a new set of conditions is given in broken lines. In calculating the surplus, only the effective N rate (not total N input) was taken as N input. The graph therefore applies to both fertiliser and animal manures. Values presumed for constants are listed in Table 3.13.

3.8 QUADMOD parameterisation for $N_{min,H}$ models

3.8.1 Purpose

In this paragraph we apply the QUADMOD concept to the modelling of $N_{min,H}$. QUADMOD is essentially only a parameterisation of crop responses (biomass yield and N yield) to applied N rates, but it provides a useful frame in the present context. As mentioned earlier, this descriptive model is invoked here with the following purposes:

i to quantify crop-specific characteristics that determine N yield in response to N rate; based on these, crop N yields can be estimated for new conditions as is required in any application of surplus-based expressions;

ii to assess for each given data set the critical N rate (see Chapter 2) which is required as parameter in some of the regression models;

iii to evaluate Eq. 2.11 (Chapter 2), on the basis of observed crop responses (biomass yield and N yield) for a larger range of crops

For details on the QUADMOD model, the reader is referred to Ten Berge *et al.* (2000). The model is summarized below in Figure 3.20, and the seven independent model parameters are defined in Table 3.14.

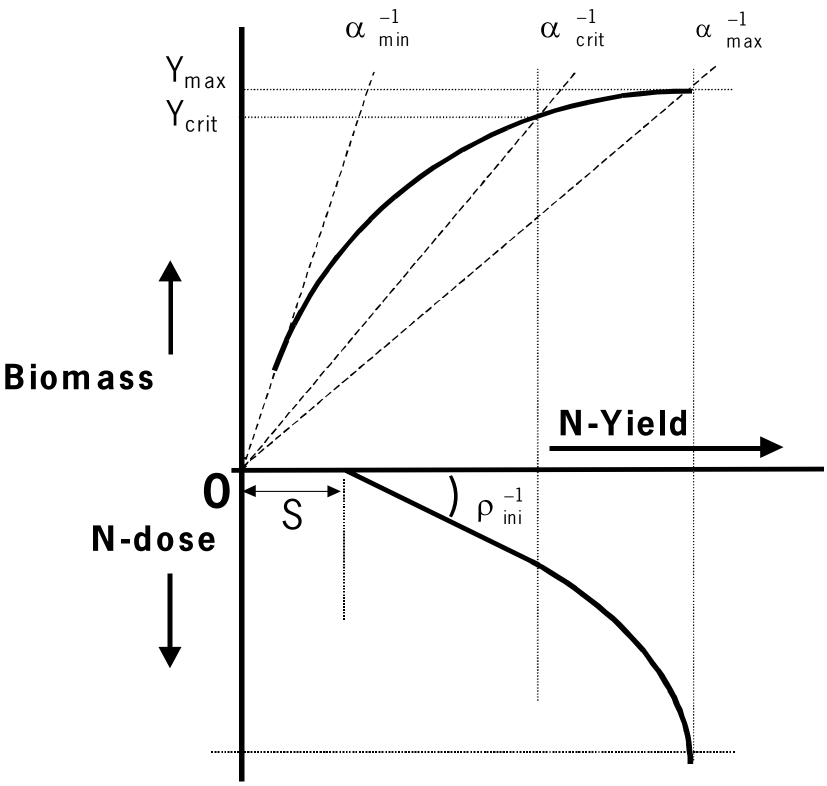


Figure 3.20. Graphical representation of the QUADMOD model.

Table 3.14. The parameters of the QUADMOD core model, describing the response of N-yield (U, kg N ha⁻¹) to N-dose (A, kg N ha⁻¹) and the response of biomass DM-yield Y (kg/ha) to N-yield.

Name	Definition	Unit
Y_{max}	maximum biomass dry matter (DM) yield	kg DM ha^{-1}
γ	relative biomass yield at critical point (= Y_{crit}/Y_{max})	-
α_{min}	minimum N-concentration in biomass	kg N kg_{DM}^{-1}
α_{crit}	N-concentration in biomass at critical point	kg N kg_{DM}^{-1}
α_{max}	maximum N-concentration in biomass	kg N kg_{DM}^{-1}
ρ_{ini}	apparent initial fertiliser-N recovery. i.e., apparent recovery under low N-availability	kg N kg_{N}^{-1}
S	uptake of N supplied from soil, i.e., N-yield on non-fertilised plots	kg N ha^{-1}

3.8.2 Data sets

The model parameters were determined by parameter optimisation, on the basis of the experimental data sets listed in Table 3.15. These data complement, for the purpose of this study, a larger number of sets analysed earlier (Ten Berge *et al.*, 2000). The procedure to assess the parameter values is a numerical random search global optimisation algorithm, which optimizes all parameters to find the best match between the model and observed values of N-yield and biomass yield, simultaneously. Obviously, no information on $N_{min,H}$ is used in optimizing the model parameters.

Table 3.1.5. Data sets used to determine QUADMOD parameters from N-response trials.

Crops	Soil	# Sets	Years	Source
Cauliflower	Clay	8	90,92	Everaarts, 1995
Broccoli	Clay	8	90-92	Everaarts, 1995
Potatoes ware	Clay	16	87-90	Hengsdijk, 1992; Titulaer, 1997, Van Loon, 1998; Anon., 1999
Potatoes starch	Sand	37	91-97	Van Loon, 1995; Wijnholds, 1995, 1996, 1997
	Reclaimed peat	11	88-93	Van Loon, 1995; Postma, 1995; Anon., 1999
Iceberg lettuce	Clay	5	85,86	Anon., 1999
Iceberg lettuce	Sand	15	85-87	Slangen, 1989, Anon., 1999
Leek	Sand	3	90-92	Anon., 1999; Geel, 2000
Spinach	Clay	6	94-96	De Kraker, 1997
Sugarbeet	Clay	24	87-91	Hengsdijk, 1992; Westerdijk, 1992; Van Dijk, 1999; Anon., 1999
	sand	1	89	Anon., 1999
Winter wheat	Clay	5	94-98	Darwinkel, 2000; Anon.,1999; Timmer, 1999
	Loess	4	95-98	Geelen, 1999
Witloof chicory	Clay	2	88-94	Van Kruistum, 1997; Schober, 1998; Anon., 1999
	Loess	2	91,93	Postma, 1995
White cabbage	Clay	8	92-93	Everaarts, 1995
Seed onion	Clay	8	89-94	De Visser, 1996; Anon.,1999
Digitalis	Clay	4	89-92	Anon. 1999
Barley	Clay	3	96-98	Anon. 1999
Brew barley	Sand	3	96-98	Anon. 1999
Brew barley	Clay	6	96-98	Anon. 1999
Summer barley	Clay	4	92-95	Anon. 1999
Miscanthus	Reclaimed peat	2	94-95	Anon. 1999
	Clay	2	94-95	Anon. 1999
	Loess	2	94-95	Anon. 1999

3.8.3 Results : QUADMOD parameters for crops

The results are given in Table 3.16.

Table 3.16. Mean values of QUADMOD parameters per crop. The number of valid datasets (n) varies between parameters because the N-supply range covered in the experiment must include observations at low and high N-supply for sets to qualify for the assessment of all seven parameters. See text for validity criteria. cv refers to the coefficient of variation of the sample. Grass and maize parameters were taken from Ten Berge et al. (2000) and refer to sand soils.

	S			ρ_{ini}		α_{min}			α_{crit}		γ			α_{max}		Y_{max}		
		cv	*n*		*cv*		*cv*	N		*cv*		*cv*	*n*		*cv*		*cv*	*n*
Cut grass	206	*0.15*	*70*	0.90	*0.10*	0.023	*0.10*	*65*	0.030	*0.11*	0.88	*0.11*	*49*	0.041	*0.05*	14020	*0.15*	*54*
Maize	88	*0.28*	*14*	0.72	*0.31*	0.007	*0.28*	*11*	0.010	*0.08*	0.86	*0.08*	*8*	0.013	*0.05*	14080	*0.12*	*9*
Cauliflower	80	*0.24*	*8*	0.34	*0.19*	0.018	*0.11*	*2*	0.026	*0.33*	0.99	*0.001*	*2*	0.040	*0.10*	2959	*0.14*	*8*
Broccoli	26	*0.20*	*8*	0.16	*0.25*	0.033	*0.08*	*8*	0.042	*0.06*	0.93	*0.08*	*6*	0.045	*0.06*	1121	*0.26*	*6*
White cabbage	67	*0.19*	*8*	0.46	*0.07*	0.010	*0.14*	*8*	0.017	*0.14*	0.96	*0.08*	*5*	0.022	*0.16*	8554	*0.12*	*5*
Potato (ware)	129	*0.20*	*16*	0.50	*0.57*	0.008	*0.13*	*5*	0.014	*0.04*	1.00	*0.00*	*2*	0.016	*0.08*	14201	*0.06*	*12*
Potato (starch)	118	*0.30*	*11*	0.54	*0.16*	0.006	*0.07*	*2*	-	-	-	-	*0*	0.012	*0.09*	14092	*0.26*	*7*
Iceberg lettuce	45	*0.46*	*19*	0.61	*0.55*	0.021	*0.23*	*5*	0.026	*0.17*	0.91	*0.09*	*3*	0.036	*0.11*	2154	*0.19*	*15*
Witloof chicory	73	*0.55*	*4*	0.18	-	0.004	-	*1*	0.008	-	0.99	-	*1*	0.014	*0.11*	8617	*0.17*	*3*
Sugar beet[1]	62	*0.26*	*27*	0.54	*0.49*	0.005	*0.26*	*10*	0.0069	*0.37*	0.99	*0.001*	*5*	0.0074[2]	*0.35*	16510	*0.19*	*12*
Barley (brew.)	76	*0.31*	*9*	0.51	*0.17*	0.012	*0.21*	*3*	0.015	*0.16*	0.94	*0.04*	*2*	0.017	*0.10*	6781	*0.09*	*8*
digitalis	75	*0.52*	*4*	0.37	*0.22*	0.013	*0.14*	*2*	0.017	-	0.80	-	*1*	0.025	*0.03*	6729	*0.24*	*3*
Leek	121	*0.46*	*6*	0.38	*0.47*	0.022	*0.63*	*2*	0.025	*0.49*	0.98	*0.03*	*2*	0.035	*0.17*	5241	*0.13*	*3*
Winter wheat[3]	90	*0.26*	*11*	0.68	*0.25*	0.012	*0.12*	*9*	0.019	*0.21*	0.89	*0.18*	*2*	0.024	*0.17*	8726	*0.08*	*4*
Seed onion	105	*0.22*	*8*	0.38	*0.30*	0.009	*0.19*	*3*	0.014[4]	-	0.95[4]	-	*0*	0.021	*0.39*	9042	*0.26*	*3*
Spinach	60	*0.61*	*6*	0.19	*0.45*	0.036	*0.26*	*3*	0.048	-	1.00	-	*1*	0.052	*0.08*	1899	*0.28*	*3*

[1] *parameters refer to only beet root*

[2] *observed N-concentrations upto 0.012 in sugar beet*

[3] *parameters refer to grain only*

[4] *mean of all sets, but no sim.match of both validity criteria*

3.8.4 Evaluation of Eq 2.11

In this paragraph we attempt to model $N_{min,H}$ by straightforward application of Eq. 2.11 – that is, using the straight balance expression without incurring regression on $N_{min,H}$ data. This would enable to predict $N_{min,H}$ responses directly from crop properties, thus avoiding the need to parameterise regression models which require sufficiently large sets of $N_{min,H}$ observations.

QUADMOD parameter values obtained for the respective crops (Table 3.16; also from Ten Berge *et al.*, 2000) were used to quantify the coefficient associated with the quadratic term in Eq. 2.11 (Chapter 2). This coefficient can be expressed in 'primary' QUADMOD parameters according to:

$$\mu \equiv \frac{\rho_{ini}}{2\left[A_{\max} - A_{crit}\right]} = \frac{\rho_{ini}^{2}}{4Y_{\max}\left[\alpha_{\max} - \gamma\alpha_{crit}\right]} \qquad 3.19$$

Note that no information on observed $N_{min,H}$ was used in this modelling attempt. The figures below show the comparison between these calculated curves and the actual $N_{min,H}$ observations. The results for grass and maize are considered reasonablye good. $N_{min,H}$ in the vegetable crops, however, is described with varying success by this simplified approach.

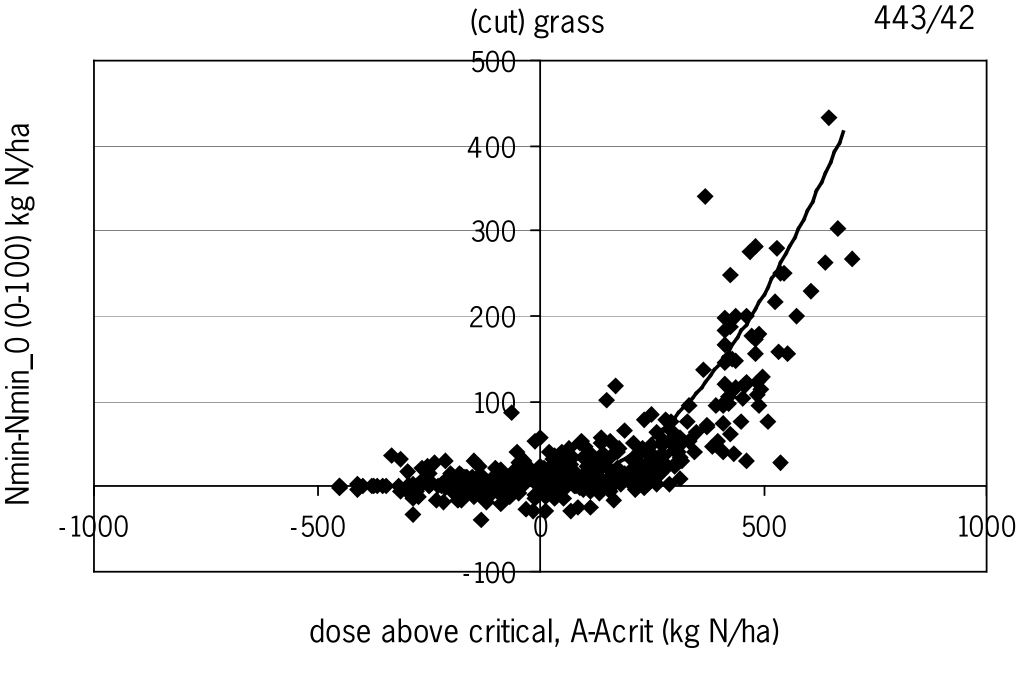


Figure 3.21. Increment in residual soil N (0-100 cm) at the last cut harvest ($N_{min,H}$) in grass, relative to residual soil N observed at the same time in plots that received no N input ($N_{min,H,0}$), versus the amount by which the applied N rate exceeds the critical N rate, A_{crit}. The values of $N_{min,H,0}$ and A_{crit} are case-specific: they vary with the location and the year of the experiment. All grass data listed in Table 3.1 are included. The plotted curve expresses Eq. 2.11 with μ = 0.000924 ha kg^{-1} according to Eq. 3.19 and grass parameters from Ten Berge et al. (2000).

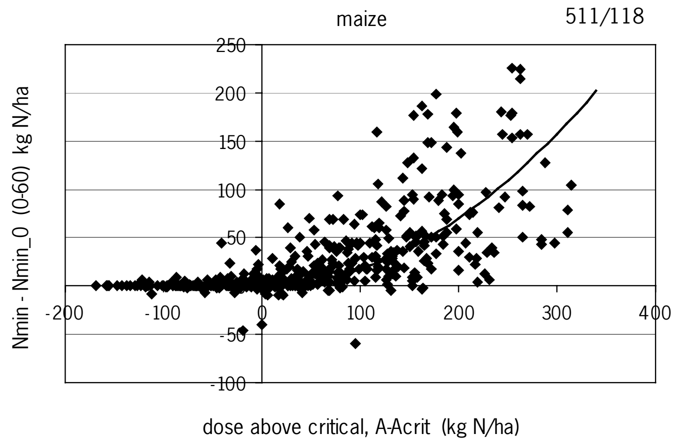


Figure 3.22. Increment in residual soil N (0-60 cm) at harvest ($N_{min,H}$) in maize, relative to residual soil N observed at the same time in plots that received no N input ($N_{min,H,0}$), versus the amount by which the applied N rate exceeds the critical N rate, A_{crit}. The values of $N_{min,H,0}$ and A_{crit} are case-specific: they vary with the location and the year of the experiment. All maize data listed in Table 3.1 are included. The plotted curve expresses Eq. 2.11 with μ = 0.00175 ha kg^{-1} according to Eq. 3.19 and maize parameters from Ten Berge et al. (2000).

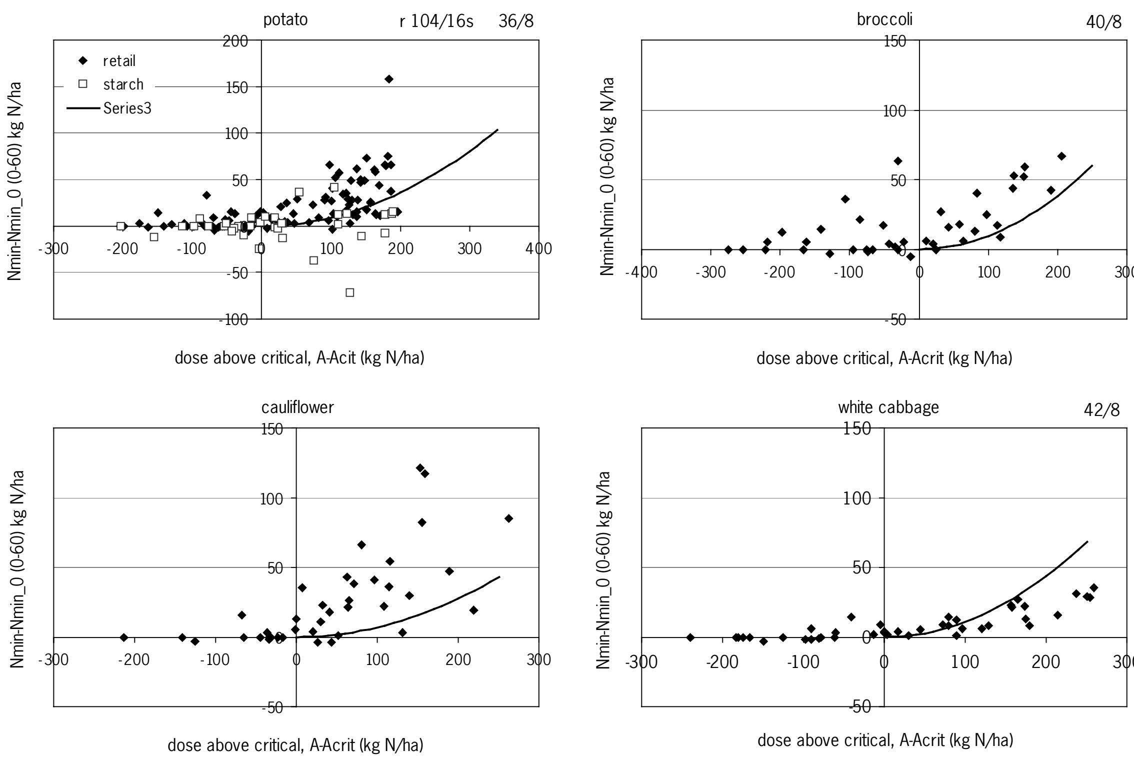


Figure 3.23. Increment in residual soil N (0-60 cm) at harvest ($N_{min,H}$), relative to residual soil N observed at the same time in plots that received no N input ($N_{min,H,0}$) versus the amount by which the applied N rate exceeds the critical rate, A_{crit}. For potato, broccoli, cauliflower and white cabbage. Data for the latter three crops refer to clay soils only. The plotted curves express Eq. 2.11 with μ calculated from Table 3.16 with the help of Eq. 3.19.

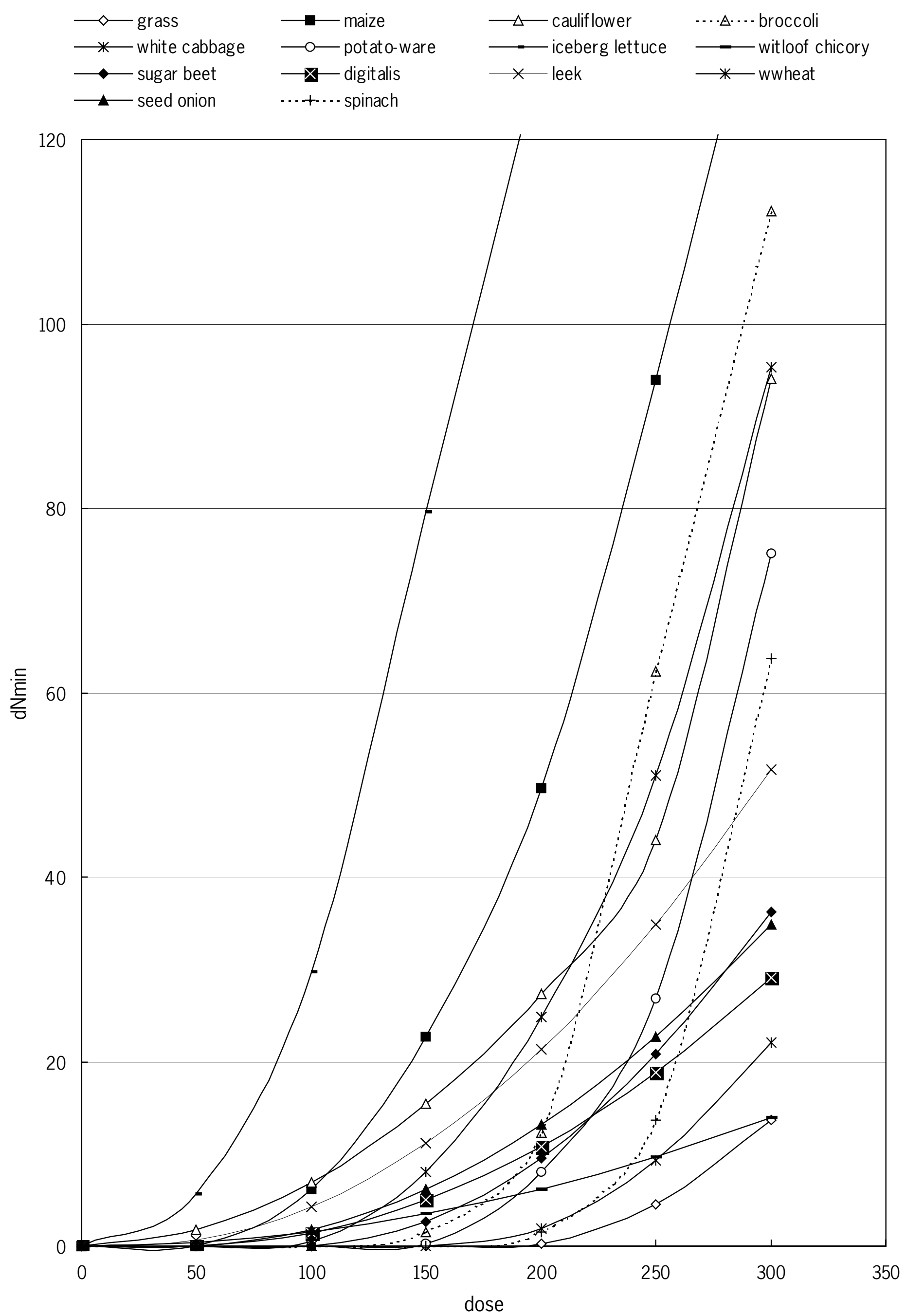


Figure 3.24. Calculated responses of $N_{min,H}$ to N rate, based on Eq. 3.19 and Table 3.16; for a range of crops.

3.8.5 The relation between (*A-U*) and (*A-A*$_{crit}$)

The crop N-uptake observed under zero-N input, U_0, has been included explicitly in some of the composed regressors introduced earlier, while it is 'hidden' in others, for instance in the surplus (*A-U*), because U_0 affects U at given input A. We investigate in this paragraph the role that U_0 plays in relating two important regressors, namely (*A-U*) and (A-A_{crit}), and we will attempt to evaluate how changes in this parameter will affect the relations between these regressors, and the relations with $N_{min,H}$.

The value of U_0 is determined to some extent by inherent soil properties (texture, drainage) and annual weather conditions, but is largely affected by the long term history of management, that is, the annual net inputs of organic matter and nitrogen. This is why considerable variation exists in U_0, both in space and in time.

The effect of U_0 on crop N yield can be expressed by a relation introduced earlier (Chapter 2), and which was adopted from the QUADMOD model:

$$U = U_0 + \rho_{ini} A - \mu (A - A_{crit})^2 \qquad 3.20$$

where the last term vanishes for $A < A_{crit}$; the coefficient μ is defined as in Eq. 3.19

The relation between the applied N rate A and the surplus A-U follows from Eq 3.20

$$A - U = (1 - \rho_{ini}) A - U_0 + \mu (A - A_{crit})^2 \qquad 3.21$$

and this gives also the connection between the surplus and A-A_{crit}.
We suppose here that the accumulation of residual mineral N in soils is indeed directly related to the amount by which the N rate exceeds the critical rate A_{crit}, as suggested by the results presented earlier in this report. Variations in U_0 will then not affect that relation between $N_{min,H}$ and (A-A_{crit}), but will affect the critical rate itself, according to:

$$A_{crit} = \frac{U_{crit} - U_0}{\rho_{ini}} \qquad 3.22$$

where $U_{crit} \equiv \gamma \alpha_{crit} Y_{max}$ is independent of U_0, as is ρ_{ini}.
Eqs. 3.21, 3.22 enable to assess whether the relation between (A-U) and $N_{min,H}$ shares this independence with respect to U_0. That would only be so if changes in the first two right hand side terms in 3.21, resulting from changes in U_0, would offset each other. We maintain in this exercise that the primary determinant for $N_{min,H}$ is A-A_{crit}. Now if, for a given fixed value of A-A_{crit}, U_0 changed by an increment $+\Delta U_0$, A_{crit} would change according to Eq 3.22 by an amount $-\Delta U_0/\rho_{ini}$. At constant A-A_{crit}, this implies that A, too, would change by an amount $-\Delta U_0/\rho_{ini}$ and as a consequence, the term $(1-\rho_{ini})A$ changes by $-\Delta U_0(1-\rho_{ini})/\rho_{ini}$. This would only cancel out the increment $+\Delta U_0$ of the second term in the special case where $\rho_{ini} = 0.5$. The conclusion is that, at given A-A_{crit}, A-U may attain a range of different values depending on U_0. This explains the often poor correlation found between surplus and $N_{min,H}$ if data stem from experiments (years, sites) with different U_0. It also implies that the relation between surplus and $N_{min,H}$ will gradually change if U_0 changes due to long term accumulation or breakdown of organic soil N reserves.

Eq. 3.21 also allows to assess the 'safe' surplus value in function of U_0, if we presume again that positive values of A-Acrit are required to build up $N_{min,H}$. At A=A_{crit}, the surplus follows from Eqs 3.21 and 3.22 as

$$A - U = \frac{(1 - \rho_{ini})\, U_{crit} - U_0}{\rho_{ini}} \qquad 3.23$$

It is obvious that this surplus value can be positive as well as negative, depending on the soil fertility level expressed in U_0, relative to the crop demand U_{crit} which is largely defined by the yield potential under the local circumstances, Y_{max}, and the crop characteristic N-concentration at the critical point, α_{crit}. This is also what we have observed from the empirical analysis earlier in this chapter, and it is indeed a serious drawback of the surplus (A-U) as indicator for potential nitrate losses.

In summary, it is postulated here that relations for $N_{\mathrm{min,H}}$ based on (A-A_{crit}) as regressor will not be altered due to changes in U_0– whether arising from long term developments in U_0 or from (spatial) differences between soils. Instead, the value of A_{crit} changes in function of U_0. The surplus (A-U) is poorly correlated with $N_{\mathrm{min,H}}$, largely because variation exists in the parameter U_0. For an indication of the range of variation that may occur in U_0, we refer to Fig. 3.10 earlier in this chapter.
It can be argued on good grounds, however, that the value of U_0 will adjust in the long run to the chosen N management, and that the variation found in U_0 today across all experimental datasets is much larger - and the correlation between $N_{\mathrm{min,H}}$ and surplus poorer – than after a period of equilibration. An attempt is therefore made in the next paragraph to look at the system in a state of pseudo-equilibrium.

3.9 Long term equilibration of U_0, and its effects on relations between $N_{\mathrm{min,H}}$ and selected regressors

The soil fertility parameter U_0 adjusts itself over the years to the specific management practices, notably N input levels and N species, and after several decades of maintaining these practices will a 'pseudo steady state' be reached when the (upward or downward) drift in U_0 has faded. This equilibrium situation is rarely found under experimental conditions. More than anywhere, this is true of N response trials: the very introduction of graded N rates applied to different plots all sharing one and the same initial U_0 value implies that – at most, and accidentally – only one of the chosen rates might be in steady state equilibrium with the existing soil conditions. The entire database presented and used in this report is subject to this inconsistency. On the one hand, this does not invalidate our analyses, because it requires many years (decades) for soils to adjust to management and hence the relations established upon existing empirical data would remain valid for some time. Moreover, some variation in U_0 will always remain due to annual weather conditions. On the other hand, the noise in some of our relations may be inflated due to the absence of 'consistency' between N input and U_0 in the data sets, especially in view of the extreme N rates usually employed in N response trials.

For a first approximation of the relation between U_0 and N management, we use the simplest possible model of organic matter decay. Since the early 19th century (Thaer, 1809; Von Wulffen 1823; 1830; 1847; summarized by De Wit, 1974) it has been known that the build-up – or decline - of soil fertility is the net result of two opposing fluxes: the annual inputs brought into the soil system, and the liberation and subsequent uptake of nutrients from the stock contained in the soil. Von Wulffen expressed this release as a fixed fraction, annually, of the total stock, and introduced the concept that an equilibrium state should finally be approached if the annual input remained constant during many years. This state was referred to as the 'Beharrungspunkt' and it was also recognized that the corresponding fertility level would be high in systems where the annual fraction released (τ, y^{-1}) was small, and low where this coefficient was large. Von Wulffen's analysis was based on elaboration of numerous empirical long-term records of crop yields, and his first order approach still stands, be it of old age and approximative, as a robust model of organic matter-related soil fertility development. The approach can be formalised as below.

Let I_r denote the annual input of organic nitrogen that is not readily accessible for plant uptake in the first year upon application. This fraction has been referred to as 'resistant' organic nitrogen, N_r, and the total pool of this N-form in the soil system is written as N_r. For the rate of change of this pool we can write

$$\frac{dN_r}{dt} = +I_r - \tau(N_r + I_r) \qquad 3.24$$

with t for time, and τ the relative decay rate (year^{-1}), representing the fraction of the pool that is mineralised every year. Solving this gives

$$N_r = \frac{I(1-\tau)}{\tau} + \left(N_{r,0} - \frac{I(1-\tau)}{\tau} \right) e^{-t\tau} \qquad 3.25$$

$$= N_{r,\infty} - (N_{r,\infty} - N_{r,0})\, e^{-t\tau}$$

with $N_{r,\infty}$ for the pool size at time infinity, and $N_{r,0}$ for the initial pool size at t=0. The annual decay or release rate M is then

$$M = \tau(I_r + N_r) \qquad 3.26$$

Combining Eqs 3.7 and 3.8 gives

$$M = I_r + \tau(N_{r,0} - N_{r,\infty})\, e^{-t\tau} \qquad 3.27$$

This expression shows that the value of M approaches I_r for large values of t , and that the annual release is larger than I_r when the intial state of the soil, $N_{r,0}$, is 'richer' than the state $N_{r,\infty}$ corresponding ultimately to I_r - as is the case in the Netherlands where allowed N-surpluses are increasingly tightened under the implementation of MINAS.

Today it is obvious that Eqs 3.24-3.27 are a simplification, because in reality the breakdown coefficient τ itself is a function of time as it depends on the quality– and thereby the age - of the organic N-compounds contained in the pool. It is therefore difficult to reproduce, with a single constant τ-value, observed time patterns of M or N_r. A few conclusions, however, can safely be drawn which are of direct relevance to our case:

(i) under any given level of annual input I_r, both the total pool of organic soil N and the annual N-release from this pool by mineralisation should in the long run approach constant values, such that the amount of N released per year is almost equal to the net annual N input into the soil pool, that is, N-input after subtraction of crop offtake and N-losses not captured into organic matter;

(ii) for a τ-value of 0.01 y^{-1}, for example, the 'half life' required to bridge half the gap between the initial and final pool sizes, $N_{r,0}$ and $N_{r,\infty}$ respectively, would be 70 years, based on Eq. 3.25. For τ=0.05, this would reduce to 14 years, and most authors presume time coefficients for organic matter decay in between these two values. Depending on the change in input I_r imposed at t=0, this may have a strong bearing on the annual release rate M, according to Eq. 3.27.

The parameter U_0 is not identical with the annual mineralisation M, but is obviously closely related. U_0 includes – in addition to N derived from mineralisation of organic matter – also N derived from atmospheric deposition. On the other hand, not all N that becomes available from these two sources is actually captured in harvested crop parts and would thus be included in the value of U_0. Let this fraction captured be written as q_1, and the annual deposition as d (approximately 50 kg N/ha/y in the Netherlands) of which amount a fraction q_2 is intercepted by the crop. It follows that the parameter U_0, once equilibrated to long-term constant input, is then given by

$$U_{0,\infty} = q_1\, I_r + q_2 d \qquad 3.28$$

The annual input I_r is a function of the applied N rate A but, obviously, also of the forms in which N is supplied, in other words the ratio of N in animal manures to N in mineral fertilisers, and the properties of the manure:

$$I_r = A \cdot \left(\frac{f\beta}{w} + r(1-\rho) \right) \qquad 3.29$$

where f is the fraction of the total effective-N dose A that is supplied in the form of animal manure, β is the fraction of manure-N which is in the form of N_r (estimated at 0.3 in cattle slurry); w is the 'working coefficient' or fraction of N in animal manure that is as effective – in terms of crop uptake - as mineral fertiliser. The first term in brackets thus represents the external N_r-input. In the second term, (1-ρ) represents the amount of effective N (both from fertiliser and manure sources) that is not recovered in harvested crop parts, a fraction r of which is not really lost from the soil system but converted into N_r. Little is known about the fate of non-recovered N, and the two extremes for r would be 0 (no N retained as N_r) to 1 (all N retained as N_r). (For $A < A_{crit}$, ρ would be equal to ρ_{ini}.)

Continued application of large amounts of N_r will, if maintained over decades, increase soil fertility parameter U_0 and thereby reduce the threshold dose A_{crit} which is taken here as the safely permissible N-dose precluding the build-up of $N_{min,H}$. This dose was defined by Eq. 3.22. By substituting A_{crit} for A in Eq. 3.11, and then introducing Eqs 3.28 and 3.29 into Eq 3.22, we can now express the permissible dose in the equilibrium state, $A_{crit,\infty}$, in function of the parameters in Eq 3.29:

$$A_{crit,\infty} = \frac{U_{crit} - U_{0,\infty}}{\rho_{ini}} = \frac{\gamma \alpha_{crit} Y_{max} - q_2 d}{\rho_{ini} + q_1 \zeta} \qquad 3.30$$

where ζ represents the term in brackets in Eq. 3.29. Note that $A_{crit,\infty}$ as defined by Eq 3.30 corresponds to the situation where precisely this same level $A= A_{crit,\infty}$ has been maintained as the annual input.

One could argue, based on the results of the earlier regression analysis, that an N rate A_x somewhat higher than A_{crit} could be allowed before $N_{min,H}$ passes a given threshold. The values of $U_{0,\infty}$ and $A_{crit,\infty}$ that would correspond to this A_x ($>A_{crit}$) can be calculated based on the above expressions but that is slightly more complex and the derivation is omitted here.

From 3.28 and 3.29 with 3.30, it follows that the U_0 value that corresponds in the long run with the annual N-input as defined by Eq 3.30, is given as

$$U_{0,\infty} = q_1 \left[\frac{\gamma \alpha_{crit} Y_{max} - q_2 d}{\rho_{ini} + q_1 \zeta} \zeta\right] + q_2 d \qquad 3.31$$

Likewise, the surplus corresponding to that case is found from combining 3.21, 3.28, 3.29 and 3.30 as

$$A - U = (1 - \rho_{ini}) A_{crit,\infty} - U_{0,\infty} = (1 - \rho_{ini} - q_1 \zeta) A_{crit,\infty} - q_2 d \qquad 3.32$$

where $A_{crit,\infty}$ must be substituted from Eq 3.30.
This surplus can still be negative at A=A_{crit} in the pseudo equilibrium state; but it must be reminded that only effective N was taken as input in the definition used here for N surplus.

With the help of the above, we can now quantify pairs of $A_{crit,\infty}$ and $U_{0,\infty}$ that are consistent in a steady state situation. We take grass as an example and use the mean values of ρ_{ini}, α_{crit}, γ and Y_{max} reported by ten Berge *et al.* (2000), q_1 = q_2 =0.8, and a value of 0.5 for w. Taking A_{crit} as the threshold rate that is still safe with respect to $N_{min,H}$, it follows that this dose ranges between 200 and 225 kg effective N per ha if all N were given as cattle slurry, (the lower value referring to r =1 and the higher to r =0). This is equivalent with 400-450 kg total N in slurry. If 50% of the effective N dose is given as fertiliser-N and 50% as effective slurry-N, the permissible dose would be 240-280 kg effective N per ha. This is equivalent with the same amounts in total slurry N (w=0.5 offsets the fact that half the dose is given as fertiliser). If all N is applied in mineral fertiliser, the doses are between 300 and 360 kg N/ha. The corresponding $U_{0,\infty}$ -values are 150 (r =0) to 170 (r =1) kg N per ha for the case with 100% slurry-N; 105 to 135 kg N per ha for 50% fertiliser-N and 50% slurry-N; and 40-90 kg N per ha if all N is given as mineral fertiliser.
Note that no assumptions on the value of the coefficient τ are required to estimate the equilibrium values $A_{crit,\infty}$ and $U_{0,\infty}$. The time coefficient does affect, however, the rate at which the steady state is approached, and the final size of the organic soil N pool, as noted already by Von Wulffen.