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About milk collection in a special branch of dairy industry

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About milk collection in a special branch of dairy industry

G.D.H. Claassen and Th.H.B. Hendriks

Abstract

This paper concerns the development of a pilot decision support system for the collection of goat's milk . The annual growth of the sector and the continuous imbalance between milk supply and demand has urged the sector to look for a different approach to the (daily) milk collection problem. We propose an OR-based approach to support the milk collection problem such that the daily vehicle routing problem has become inferior to short- to medium term planning. From a computational point of view it turned out that the application of Special Ordered Sets (SOS) was very useful. This study showed that computational advantage of the SOS-formulation is not necessarily restricted to special cases.

Keywords: Milk collection, vehicle routing, planning, mixed integer linear programming, special ordered sets type 1

1. Introduction

The introduction of the so-called milk quotation system for cow's milk in 1984, implied a strong stimulus for the annual growth of milk goats for professional use. The continuous growth since 1984 was intensified by the favourable profit for the production of goat's milk on a biological and professional scale. Nowadays, Dutch goatherds for professional use produce about 40 million litres of milk yearly. The main part is used for domestic cheese production but export of fresh milk to Belgium, Germany and the UK is also quite common. The remainder of the supply is sold for food (milk powder) of young animals and dairy concentrates.

The annual growth of the sector and the problem of imbalance between milk supply and demand has urged the sector to look for a different approach to the (daily) milk collection problem. Although a lot of literature has been dedicated to (the application of) vehicle routing problems (Toth, 2000; Ghiani, 2003; Gayialis, 2004) and even on milk collection problems in common dairy industry (Basnet, 1996; Gerdessen, 1996), the milk collection problem for milk goats is characterised by rather specific details. The characteristics of this milk collection problem calls for a different approach of the (daily) vehicle routing problem. This paper deals with an approach to support this (daily) milk collection problem such that several, mostly conflicting, goals of the relevant players (i.e. farmers, processing industry and transporters) are taken into consideration. The daily vehicle routing problem has become inferior to an interactive planning system in which the central issue aims at fitting milk supply and -demand. We will discuss how an OR-based planning model, the related optimization techniques, structured data queries and additional analysis tools can support different (conflicting) approaches to planning and vehicle routing. First we describe the problem environment and focus on the main differences between the milk collection problem for cow's milk and the similar problem for collecting goat's milk in Dutch dairy industry.

2. Problem description

Although the yearly supply of goat's milk is of minor importance for the Dutch dairy industry, the market for the related end products is still growing annually. The (exclusive) end products of goat's milk, mainly cheese, are processed by a limited number of dairy factories. Actually, just a few factories are processing goat's milk and their production capacity is mainly based on processing large quantities of (common) cow's milk. As a consequence, set-ups for processing goat's milk on demand level are usually restricted to a limited number of days weekly. However, in view of meeting all (predefined) quality standards the freshness or "age" of the milk at arrival time is of major importance. This implies that the raw material has to be collected before the "age" of the (oldest) milk exceeds three days. This time restriction is fixed and independent of the final destination of the raw material; inside or outside the Netherlands. So on the one hand, dairy factories call for large amounts of raw material and their demand is only scheduled to arrive at fixed days. On the other hand, looking at supply level, the number of goat's for professional use are relatively small compared to common dairy farms, the average milk production yields on individual farms are substantially less and goat's farms are geographically spread over the country. So, from a transportational point of view the complexity of the vehicle routing problem for collecting goat's milk is quite different from collecting cow's milk. Especially if we take into account that (cooled) storage of milk at supply level is restricted to at most three days and the dairy factories only take delivery of goat's milk at a small number of fixed days. This in turn enhances the problem that the transported amount of milk between the supply- and demand level is often out of balance with the capacity of modern transportation vehicles.

These conflicting interests, together with the annual growth of the sector has urged for a different approach of the (daily) milk collection problem in the goatherd sector. It raised the question to develop and evaluate an interactive planning tool in order to support the milk collection problem and attune the imbalance between milk supply on the one hand and the individual demand levels of dairy factories on the other hand. The system should have a major focus on short- to medium term planning rather than solving the daily vehicle routing problem. The daily routing problem might be simplified by using the system but is inferior to planning support.

3. Model formulation

Part of the system is based on a mixed integer linear programming model. This model takes a (rolling) planning horizon of two weeks into account. Milk supply and –demand is exactly known for two weeks in advance. Individual farms are clustered to larger entities. This grouping is primary based on the geographical location of the farms and the (daily) available quantity of milk within a cluster. The central idea is that within each cluster the entire milk production will be collected at days still to be determined within the planning horizon. The available amount of milk after one, two or three days should match with the carrying capacity of (several) transportation vehicles. Now the question is not only which cluster should be visited but also when to visit the farms in a cluster such that the demand levels are served as good as possible. A surplus of milk at supply level can only be sold at an unattractive price level to a selected number of surplus companies. In order to meet the most important quality standards of the collected milk, the period of time between two consecutive visits within a cluster should not exceed three days. In fact this constraint means that the potential number of milk collection schemes or rhythms, for a two weeks planning horizon, is finite and limited (see table 3.1). Collecting milk at Sundays is not allowed.

	← Week 1 →							← Week 2 →						
Rhythm	Mo1	Tu1	We1	Th1	Fr1	Sa1	Su1	Mo2	Tu2	We2	Th2	Fr2	Sa2	Su2
1	✓		✓		✓			✓		✓		✓		
2	✓		✓		✓			✓		✓			✓	
3		✓		✓		✓		✓		✓			✓	
.														
.														
.														
r	✓			✓	✓			✓		✓		✓		
.														
.														
R	✓	✓	✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	

Table 3.1: some milk collection rhythms

The introduction of these so-called milk collection rhythms has reduced the complexity of the problem considerably. After all, the problem is now which milk collection rhythms should be assigned to each cluster such that the individual demand levels are served as good as possible. This problem can be formulated by a mixed integer linear programming model.
Suppose we define:

Indices

$c = 1 .. C$	~	the different clusters at supply level
$b = 1 .. B$	~	the different buyers at demand level
$r = 1 .. R$	~	the available milk collection rhythms
$t = 1 .. T$	~	the relevant days of the planninghorizon

Data

$S_{c,r,t}$	~	the milk supply in cluster *c* on day *t* according to milk collection rhythm *r*
$D_{b,t}$	~	the demand of milk for buyer *b* on day *t*
P_b^+	~	penalty for every unit of milk delivered supplementary to the actual demand for buyer *b*
P_b^-	~	penalty for every unit of milk delivered less than the actual demand for buyer *b*

Variables

$x_{c,r,b,t}$	~	delivered amount of milk from cluster *c* at rhythm *r* for buyer *b* on day *t*
$xd_{b,t}^+$, $xd_{b,t}^-$	~	the surplus or shortage of milk at demand level (buyer *b*) on day *t*
$y_{c,r}$	~	binary variable in order to assign milk collection rhythms to clusters

Model formulation

$$\text{Min}\left\{\sum_b \sum_t (P_b^+ \cdot xd_{b,t}^+ + P_b^- \cdot xd_{b,t}^-)\right\} \tag{1}$$

S.T.

$$\sum_r y_{c,r} = 1 \qquad \text{for all } c \tag{2}$$

$$\sum_b x_{c,r,b,t} \leq S_{c,r,t} \cdot y_{c,r} \qquad \text{for all } c,r,t \qquad (3)$$

$$\sum_c \sum_r x_{c,r,b,t} - xd^{+}_{b,t} + xd^{-}_{b,t} = D_{b,t} \qquad \text{for all } b,t \qquad (4)$$

$$y_{c,r} \in \{0,1\} \qquad \text{for all } c,r \qquad (5)$$

$$x_{c,r,b,t}, xd^{+}_{b,t}, xd^{-}_{b,t}, xs^{+}_{c,r,t} \geq 0 \qquad \text{for all } c,r,b,t \qquad (6)$$

The objective function (1) minimises the total weighted sum of deviations on demand level. Especially the penalty coefficients $P^{+}_{b}\ \forall\ b$ (surplus) are important in order to weight any amount of milk delivered at an unattractive subset of surplus companies. The constraints in (2) ensure that exactly one milk collection rhythm will be assigned to every cluster of farmers. The equations in (3) are classical logical conditions between the continuous variables at the left hand side and the binary variables at the right hand side. They imply that no milk can be transported from a cluster on a day to any buyer if it is not in accordance with the chosen milk collection rhythm. Moreover, the equations in (3) ensure that the total amount of milk to be transported from a cluster to the buyers may not exceed the available quantity on supply level. The equations in (4), together with the objective function, ensure that demand levels of all buyers are (more or less) satisfied.

The difference between the delivered amount of milk and the actual demand level is expressed by the auxiliary variables $xd^{+}_{b,t}$ (surplus) and $xd^{-}_{b,t}$ (shortage).

4. Solving the model

Despite of the limited number of both the predefined milk supply clusters and the milk-collection rhythms, the problem turned out to be quite hard to solve. In most cases we defined ten to twelve different milk supply clusters. An (arbitrary) upper bound of ten cpu-minutes for solving a problem is already reached at six to seven potential milk collection rhythms.

Instead of defining $C * R$ different binary variables and subsequently branch on individual variables in a branch-and-bound tree, the integrality constraints (5) can be relaxed and it is possible to apply the concept of special ordered sets type 1 (Beale and Tomlin 1969).

An SOS1 is a set of variables within which at most one variable may be non-zero. In this case we defined for each milk supply cluster c an SOS1 set $S1_c$ of continuous variables such as

$S1_c := \{y_{c,1}, y_{c,2}, \ldots, y_{cR}\}$ together with the conditions:

at most one of the variables within this set $\{y_{c,1}, y_{c,2}, \ldots, y_{c,R}\}$ can be non-zero for all c (5.1)

Note that it is not necessary to treat the variables $y_{c,r}$ in (5) as binary variables since the $S1_c$ – conditions in (5.1) together with the constraints in (2) ensure that within each $S1_c$-set exactly one continuous variable will get a final value of one.

As an alternative to define the variables $y_{c,r}$ as 0–1 integers for all c,r in (5), it is convenient to regard each $S1_c$ -set as a discrete entity or generalisation of a 0–1 variable.

Conditions (5.1) can be dealt with algorithmically through the method of integer programming (Williams, 1993). Treating each set as an entity rather than as a collection of variables makes it possible to branch in a branch-and-bound algorithm on an entity rather than on individual (integer) variables. The non-zero variable in each $S1_c$ -set of (5.1) will lie either to the left, or to the right, of any marker placed between two consecutive variables within a set (Beale and Tomlin, 1969). So:

either $\{y_{c,1}, y_{c,2}, \ldots, y_{c,j}\}$ are all zero

or $\{y_{c,j+1}, y_{c,j+2}, \ldots, y_{c,R}\}$ are all zero

These two possibilities correspond to a branch in a solution tree as demonstrated in the next figure in which $P_{c,k}$ is defined as a sub-problem P for a $S1_c$ -set in node k of the search-tree (Williams,1993).

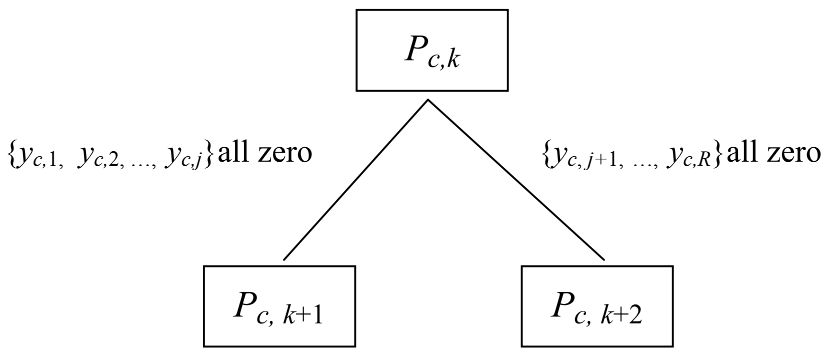

Figure 4.1: The branching procedure in a SOS1 search-tree

In the appendix we proof that the upper bound $\bar{B}$ for the number of branches B in case there are C different S1-sets (milk supply clusters) and R different milk collection rhythms, is defined by:

$$\bar{B} = \sum_{c=1}^{C} R^{c-1}(2R-2) \tag{7}$$

Increasing the number of clusters C will have a larger impact (exponentially) on the potential size of the search-tree than the number of milk collection rhythms R. In the appendix we also

proof that the potential number of branches B for a common branch-and-bound approach, i.e. branching on individual binary variables $y_{c,r}$ for problem (1) to (6), is also equal to (7). However, according to Williams (1990) there is a great advantage to be gained in the SOS-formulation provided that the variables within the sets have a natural ordering.
As the size of an SOS-based search tree is equal to a 'conventional' B&B-tree, any computational advantage of the SOS-formulation has to be based on finding early bounds in the search-tree. Possibly that's why Williams (1990) adds that "the variables should have a natural ordering within the sets". Unfortunately, in our case the variables (i.e. the milk collection rhythms) within the sets can hardly be ordered in a natural way. In the next paragraph we will focus on an alternative procedure for ordering the variables within the sets such that the computational effort will still decrease substantially.

If more than one variable in (5.1) takes a non-zero value, the S1-set is infeasible. In order to measure this infeasibility analogous to the fractionality of an integer variable, the variables in each set of (5.1) have to be associated with a monotonic set of numbers ($a_1, a_2, \ldots, a_R$) known as the reference row. In the formulation of some applications this set of numbers arises from a constraint. In case these constraints are not present, the index numbers can be used in order to associate each variable with its place in the ordering, so a_1=1, a_2=2, …, a_R=R. Now, the fractionality of an infeasible $S1_c$-set in any node of the B&B-tree, can be calculated as follows (Williams, 1993):

$$\sum_{r=1}^{R} a_r \tilde{y}_{c,r} \Big/ \tilde{y}_{c,r} \qquad \text{for all c} \qquad (8)$$

In which $\tilde{y}_{c,r}$ denotes the value of the variables in the current node of the B&B-tree. Since the numbers a_r are monotonic, there will be some a_r such that

$$a_r \leq \sum_{r=1}^{R} a_r \tilde{y}_{c,r} \Big/ \tilde{y}_{c,r} < a_{r+1} \qquad \text{for all c} \qquad (9)$$

indicating that the "centre of gravity" of the set has come out between the index r and r+1 (Williams, 1993). If the set is infeasible the branching marker will be placed between the variables $y_{c,r}$ and $y_{c,r+1}$. Now the problem is how to order the (continuous) variables $y_{c,r}$ within every $S1_c$-set such that the B&B procedure can be executed more efficiently than in case of branching on the individual (binary) variables $y_{c,r}$ in problem (1) to (6).

It is obvious that finding strong bounds in an early stage of the B&B procedure will have a significant effect on the efficiency of the B&B algorithm. However, a general strategy for strong bounds may be hard to find. Nevertheless, we could try to set up the branching-tree in such a way that the chances for fathoming large(r) parts of the search-tree in an early stage of the B&B algorithm are increasing. Within this context we will focus on a sorting procedure for the individual variables within the $S1_c$-sets. According to (9) the position of the branching marker in an infeasible $S1_c$ -set depends both on the values for a_1, a_2, …, a_R in the reference row and on the position of the non-zero variables within the set. Within this study the reference row itself remains unaltered, so a_1=1, a_2=2, …, a_R =R. If the actual position of each decision variables $y_{c,r}$ within each set is such that the value of the corresponding non-zero variables $\tilde{y}_{c,r}$ of an infeasible $S1_c$ -set will be located on the left (or right) side within a set, the position of the branching marker will be placed in the same area. As a result, the subsets corresponding to each of the branches in figure 4.1 will be unequal in size. This in turn means that the potential depth of the branch related to the largest subset will be less than the depth of the opposite branch. So, it is likely to expect that the chances for finding an early solution (i.e. bound) will be larger in a node beneath the branch on the largest subset. After all, according to the constraints in (2), every $S1_c$ -set has to be feasible in the end. Note that for all potential milk collection rhythms a feasible solution for problem (1) to (6) can be found.

Now we will focus on an ordering procedure for the decision variables $y_{c,r}$ within the $S1_c$ -sets such that the value of the corresponding non-zero variables $\tilde{y}_{c,r}$ of an infeasible set will be located on the left (or right) side within the set.

Within this context it is convenient to define some measure of performance for each milk collection rhythm r on supply level. Suppose we define a parameter $D\_S_{c,r}$ for every decision variable $y_{c,r}$ within an infeasible $S1_c$– set. The value of these parameters should be regarded as a fit for applying milk collection rhythm r in cluster c (supply level) to all needs on demand level. The value for $D\_S_{c,r}$ is defined as:

$$D\_S_{c,r} = \sum_{t=1}^{T} \left| \left(\sum_{b=1}^{B} D_{b,t} - S_{c,r,t} \cdot \tilde{y}_{c,r} \right) \right| \quad \forall r \text{ in all infeasible } S1_c \text{ - sets}.$$

Now the actual position from $r=1$ to R of the variables $y_{c,r}$ within an infeasible $S1_c$ –set is based on an increasing (or decreasing) value for $D\_S_{c,r}$. So, in case of an increasing ordering the variables $y_{c,r}$ of the corresponding non-zero solution values $\tilde{y}_{c,r}$ will be placed on the left side in the set and vice versa (right side) for a decreasing ordering.

The results for all computational experiments are summarised in figure 4.2. The four curves represent different strategies. One curve (BIN) is based on a common branch-and-bound approach, i.e. branching on individual binary variables $y_{c,r}$ of problem (1) to (6). All other curves are related to the application of a SOS1 branching procedure. For the S1_MID curve the continuous variables $y_{c,r}$ of the corresponding non-zero solution values $\tilde{y}_{c,r}$ are placed in the middle of the $S1_c$ –set. The S1_LEFT and S1_RIGHT curves are based on a left- or right most ordering procedure within each $S1_c$ –set.

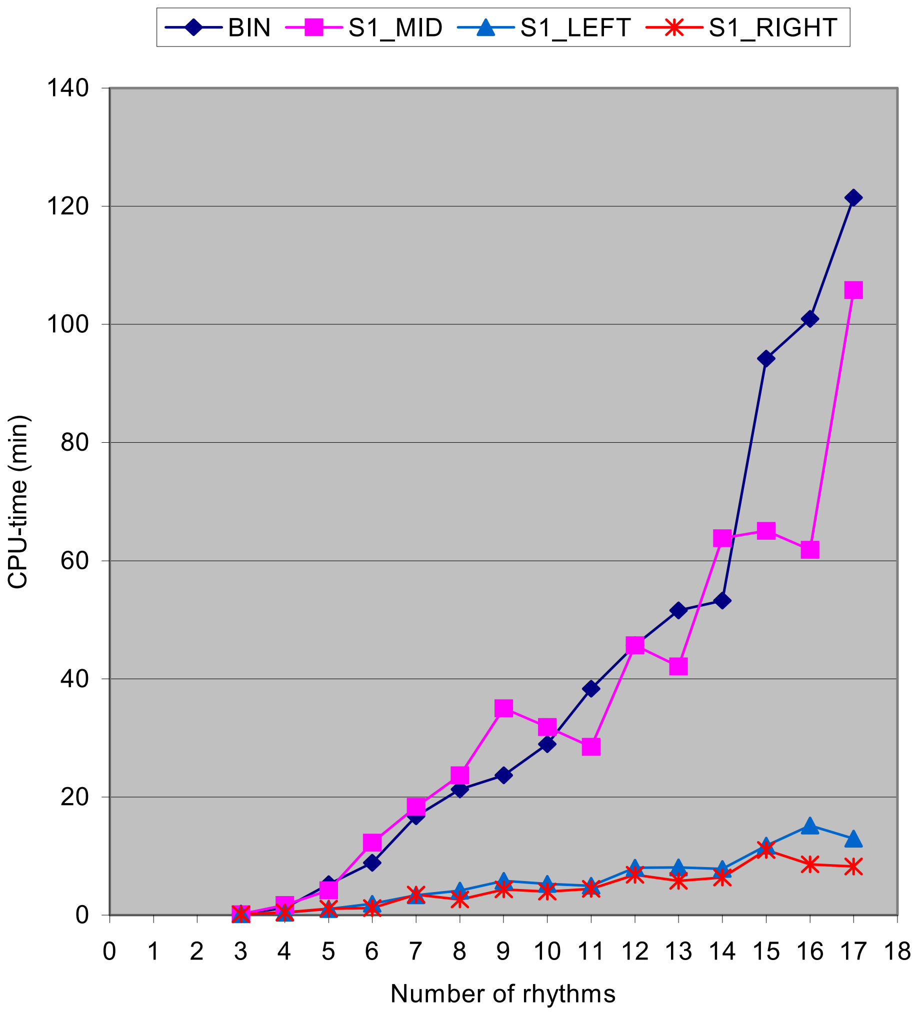

Figure 4.2 Computational results

Concluding remarks

The goal of this study was to develop a pilot system in order to support the milk collection problem and attune the imbalance between milk supply on the one hand and the demand levels of dairy factories on the other hand. The system should have a major focus on short- to medium term planning rather than solving the daily vehicle routing problem.

The optimization module should be able to generate plans within a reasonable amount of time. From a computational point of view it turned out that the application of Special Ordered Sets (Beale and Tomlin, 1969) was quite useful. The numerical experiments confirm that the efficiency of the SOS-formulation strongly depends on the ordering of the variables within the sets. However, we also showed that the computational advantage of the SOS-formulation is not restricted to cases in which the variables within the sets have a natural ordering. A reordering procedure of the variables, based on the solution values of the LP-relaxation of the original MILP-problem (1) to (6), turned out to be very effective too. The values of the numbers in the reference row are of minor importance for the computational efficiency of the SOS-formulation.

The model has been specified and solved by a modelling and solving language. The modelling component also provides for a set of procedures and functions that have been used to access a database. All necessary data for the input of the model has been retrieved directly from the database by an ODBC-interface. In return the output of the optimization routine has been written by an ODBC-interface into the database. Structured (SQL-)data queries and additional analysing tools were developed to analyse and present both the input and output of the model in a user friendly environment. The system should not be considered as an optimizer but rather as a tool for generating high quality plans to be used for further analyses. In this connection the analysing tools in the user-interface are indispensable for a user in order to get a profound insight into the problem and the generated solutions. Several (conflicting) measurements of performance are calculated and presented in the reports. Changing the data, for example moving farms from one cluster to an other, or changing the generated plans manually, is possible. However, the consequences of any (manual) modification will affect the measures of performance too. The visualization of modifications and the possibility to store plans enables the decision maker to 'optimize' his / her own performance with respect to his or her mission. All stored plans and their (daily) information about delivered amounts of milk sold (at unattractive price levels) to so-called surplus companies, can be quite helpful in order to attune the imbalance between milk supply on the one hand and the individual demand levels of dairy factories on the other hand. A profound analysis of these data will be very beneficial for the outcome of the yearly negotiations with dairy factories about the expected amounts of milk to deliver and the desired delivery days weekly.

Although the main goal of the optimization module was to generate plans in order to support short- to medium term planning, these plans can also serve as a starting point for solving the daily vehicle routing problem. Once the farms on the supply level are clustered and assigned to a milk collection rhythm, the daily routing problem has been simplified substantially. Only the human way of reasoning and the practical knowledge of a planner can compensate for day to day based deficiencies in any computer system.

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Appendix

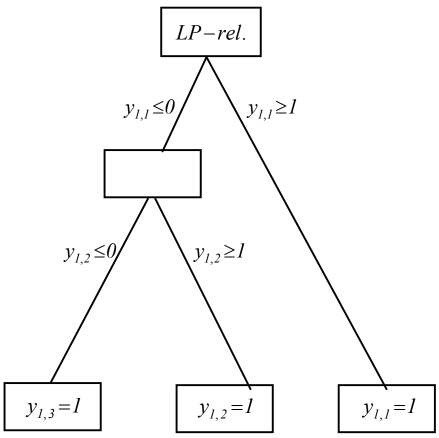
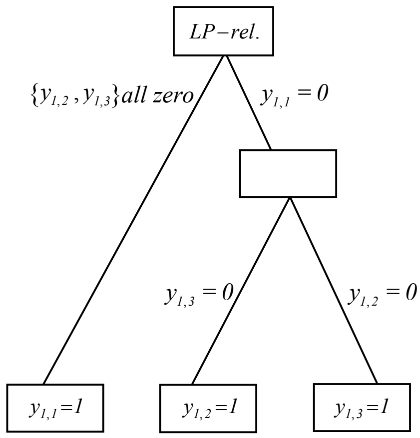
We will proof that the upper bound for the number of branches B in an SOS-based branch-and-bound search tree is defined by:

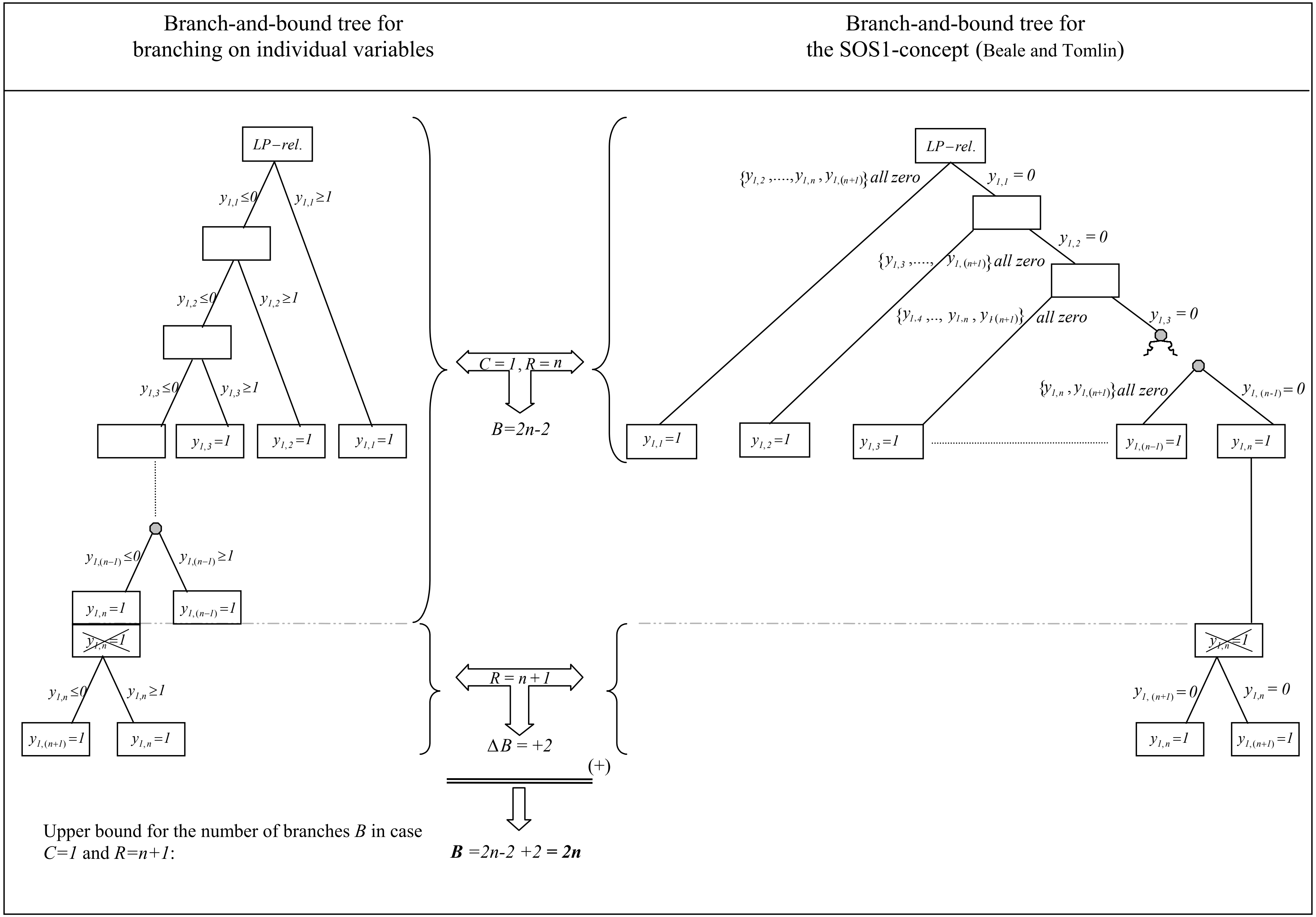
$$B = \sum_{c=1}^{C} R^{c\text{-}1}(2R - 2) \qquad (7)$$

in which C denotes the number of milk supply clusters (S1-sets) and R denotes the number of different milk collection rhythms. We also proof that the potential number of branches B in a common branch-and-bound approach, i.e. branching on individual binary variables $y_{c,r}$ in problem (1) to (6), equals (7) too.

***A*)** First we focus on the impact of R (the number of milk collection rhythms) on the number of branches B and prove that:

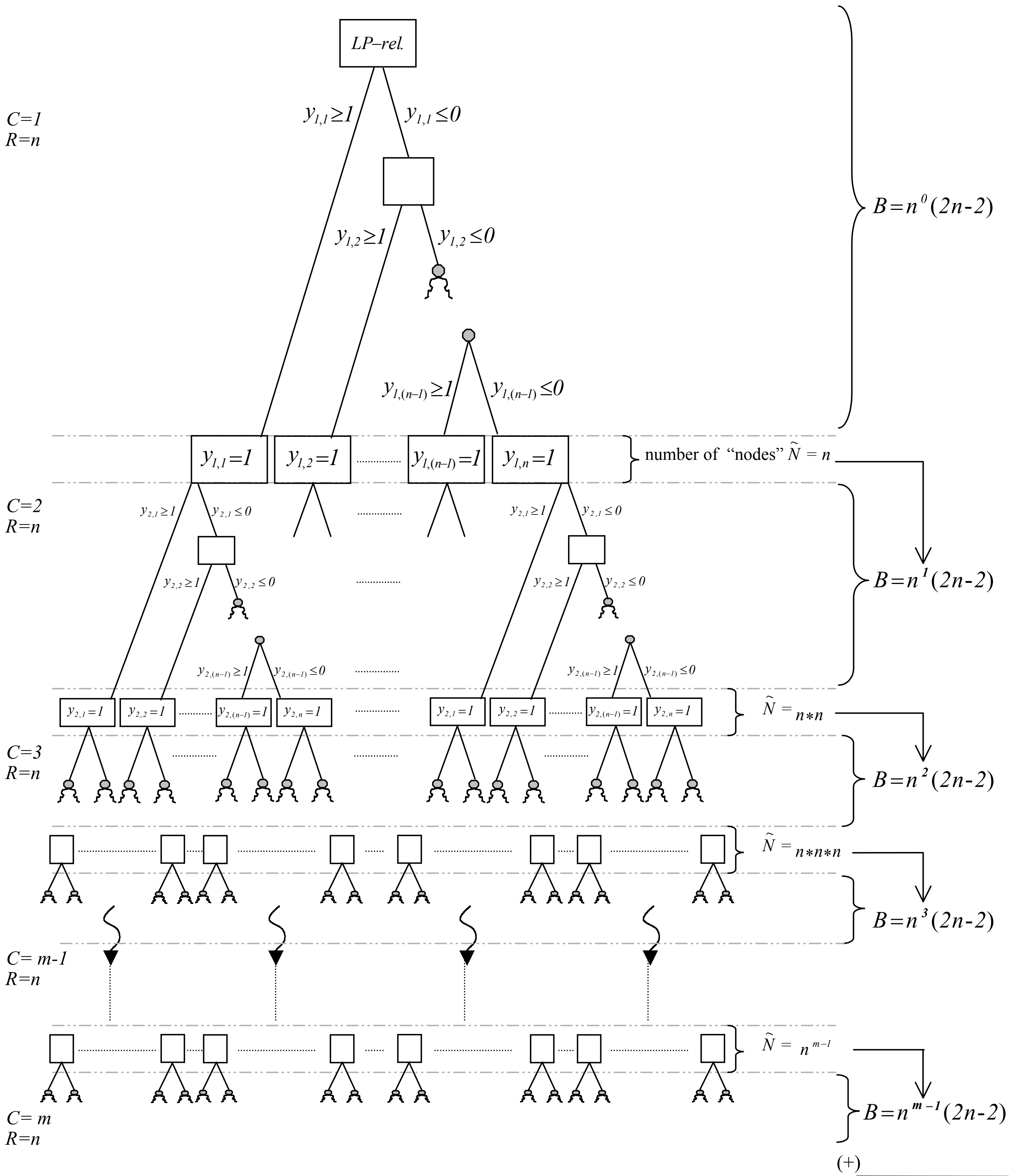
(1) $B = (2R - 2)$ in case we define only one milk supply cluster ($C=1$)

	Classical B&B tree	*B ~ # branches*	*SOS1 B&B tree*
	$\sum_{r=1}^{R} y_{c,r} = 1 \quad \forall c$ $y_{c,r} \in \{0,1\} \quad \forall c,r$	$B = (2R - 2) \quad (1)$	$S1_c := \{y_{1,1}, y_{1,2}, ..., y_{1,R}\} \quad \forall c$ $\sum_{r=1}^{R} y_{c,r} = 1 \quad \forall c$ $y_{c,r} \geq 0 \quad \forall c,r$
C=1 *R=1*	LP−rel. $y_{1,1}=1$	→ $B = 0$ ←	LP−rel. $y_{1,1}=1$
C=1 *R=3*		→ $B = 2*3-2$ ←	
C=1 *R=n*	Suppose relation (1) is correct for both methods. The potential number of branches B in case $R=n$ is equal to $B = (2n\text{-}2)$ Using the assumption that $B=(2n\text{-}2)$ for $C=1$ and $R=n$, we have to prove that relation (1) holds for $C=1$ and $R=n+1$ too.		
C=1 *R=n+1*	For $R=n+1$ the potential number of branches B for both principles should be equal to $\mathbf{B} = (2\ (n+1) - 2)$ **= 2*n*.**		


Branch-and-bound tree for
branching on individual variables
Branch-and-bound tree for
the SOS1-concept (Beale and Tomlin)
LP−rel.
y1,1 ≤0
y1,1 ≥1
y1,2 ≤0
y1,2 ≥1
y1,3 ≤0
y1,3 ≥1
y1,3 =1
y1,2 =1
y1,1 =1
y1,(n−1) ≤0
y1,(n−1) ≥1
y1,n =1
y1,(n−1) =1
y1,n ≤0
y1,n ≥1
y1,(n+1) =1
y1,n =1
C = 1, R = n
B=2n-2
R = n + 1
ΔB = +2
(+)
LP−rel.
{y1,2 ,....,y1,n , y1,(n+1)} all zero
y1,1 = 0
y1,2 = 0
{y1,3 ,...., y1,(n+1)} all zero
{y1,4 ,.., y1,n , y1(n+1)} all zero
y1,3 = 0
{y1,n , y1,(n+1)} all zero
y1,(n-1) = 0
y1,1 =1
y1,2 =1
y1,3 =1
y1,(n−1) =1
y1,n =1
y1,(n+1) = 0
y1,n = 0
y1,n =1
y1,(n+1) =1
Upper bound for the number of branches B in case
C=1 and R=n+1:
B =2n-2 +2 = 2n

By means of complete induction we proved that the relation between the potential number of branches B and the available number of milk collection rhythms R is equal to $B=2R-2$ for both branching principles in case we define only one cluster $C=1$.

B) Next we will prove that the potential number of branches is defined by (2): $B = \sum_{c=1}^{C} R^{c-1}(2R-2)$ for an arbitrary number of clusters $c = 1 \ldots C$.
Note that in each cluster exactly one milk collection rhythm must be chosen.



Total number of branches: $B=\sum_{c=1}^{m} n^{c-1}(2n-2)$