

Optimal long-term stocking rates for livestock grazing in a Sahelian rangeland

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Abstract

Economic modelling of semi-arid rangelands has received much attention in recent literature. A major outstanding issue is how stochastic rainfall and the feedback effect of heavy grazing pressures on vegetation productivity can be accounted for in these models. This paper presents a model for calculating the optimal livestock stocking rate in a semi-arid rangeland that accounts for stochastic rainfall, the ecological feedback effect and variable prices. The model is developed for rangelands dominated by annual rather than perennial grasses, such as the African Sahel. The feedback effect is modeled on the basis of an ecological study, conducted in northern Senegal, that analyzes the impact of different grazing pressures on vegetation productivity. The paper presents both a general model and an application of the model to the Ferlo, a semi-arid rangeland in northern Senegal.

JEL codes: Q24; D24

Keywords: Semi-arid rangelands; optimal stocking rate; modeling rangeland ecology; Sahel

On a accordé beaucoup d'attention, dans les récentes publications, à la modélisation économique des pâturages semi-arides. La principale question en suspens consiste à savoir comment, dans ces modèles, prendre en compte les précipitations stochastiques et les effets rétroactifs de la pression du pacage intensif sur la production de la végétation. Cet article présente un modèle capable de calculer la charge moyenne optimale du bétail dans un pâturage semi-aride qui inclue les précipitations stochastiques, les effets rétroactifs écologiques et les prix variables. Ce modèle est destiné aux pâturages dominés par les herbages annuels plutôt que vivaces comme le Sahel africain. On évalue l'effet rétroactif en se basant sur une étude écologique menée dans le nord du Sénégal, qui analyse l'impact des différentes pressions du pacage sur la productivité de la végétation. Cet article présente à la fois un modèle général ainsi qu'une application du modèle au Ferlo, pâturage semi-aride du nord du Sénégal.

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1. Introduction

Semi-arid rangelands are the vast tracks between deserts and the agricultural zones where rainfall is generally too low or unreliable for cropping, and where livestock keeping is the most important source of income (Walker & Noy-Meir, 1982; Walker & Abel, 2002). In the last decades, a range of models have been developed that aim to provide guidance on how to maximize income from livestock keeping in semi-arid zones.

The initial rangeland models comprised simple single-period models that implicitly assume exogenous forage production of the rangeland (e.g. Dillon & Burley, 1961; Hildreth & Riewe, 1963; Walter, 1971). Subsequently, a range of more sophisticated models has been developed. For instance, McArthur & Dillon (1971) present a stochastic single-period model with a risk averse manager, and Karp & Pope (1984) and Rodriguez & Taylor (1988) present dynamic models that consider the effects of the stocking rate on forage production. More recently, a number of rangeland models have been developed that encompass both feedback effects and stochasticity. For instance, Perrings (1997) presents a model that includes stochasticity and the impact of grazing based on a Clementsian approach to rangeland productivity, where the impacts of high grazing pressure on the vegetation can be reversed in the short term by reducing the stocking rate. Perrings & Walker (2004) present a stylized model for rangelands driven by fire or grazing that allows for multiple states in rangeland vegetation. Janssen et al. (2004) model the productivity of a rangeland dominated by shrubs and perennial grasses as a function of two control variables, stocking rate and fire.

In spite of the recent progress in semi-arid rangeland modeling, a major remaining issue is how to account for the joint impacts of stochastic and erratic rainfall, and the feedback effect of livestock grazing on vegetation production (Fernandez-Gimenez & Allen-Diaz, 1999; Briske et al., 2003). Annual rainfall determines the year-to-year variation in rangeland productivity, whereas the long-term stocking density determines the composition and density of the plant cover, which, in turn, determines biomass production under certain rainfall conditions (Le Houérou, 1989). Most of the Sahel is dominated by annual rather than perennial grasses, and the existing rangeland models are not applicable. A crucial difference between annual and perennial vegetation is that the standing biomass stock in a certain year is not strongly related to the biomass of the previous year, as all annual plants die during the dry season. Pastoralists make their stocking decisions for a particular year at the end of the rainy season when they have full information on the amount of fodder available in that year – but no information on the biomass available in the coming years. To account for this effect, optimization needs to account for the stochastic nature of the rainfall and production conditions prevalent in the Sahel.

In this paper, we develop a general model that accounts for stochasticity in rainfall, as well as the feedback of high grazing pressures on the vegetation. The decision variable of our model is the long-term stocking density applied by the pastoralist society (cf. Walker, 1993; Batabyal et al., 2001). The model is developed for a rangeland dominated by annual grasses, and it is applied to the Ferlo pastures in northern Senegal. In line with recent ecological insights, we assume that the impacts of high grazing pressure on the vegetation in the Sahel cannot be reversed by a few years of low grazing pressure (cf. Le Houérou, 1989; Walker, 1993, Ludwig et al., 2001; Walker & Abel, 2002). Instead, we examine how the livestock grazing regime will influence the long-term capacity of the land to produce animal feed.

Note that the model calculates the efficient stocking rate for the pastoralist society as a whole. We are aware that, in reality, the problem of the commons is a key factor in rangeland management. Stocking decisions in northern Senegal are made by several tens of thousands of pastoralists who do not act as one profit maximizing entity. In addition, profit maximization is only one of their decision-making criteria; their strategy will also be aimed, for instance, at reducing risks. Hence, the main thrust of the paper is to examine how the ecological feedback can be included in a simple profit maximization model, rather than to present a detailed model of pastoralist rangeland management strategies.

An important asset of our model is that all its parameters have been, or can readily be, measured in ecological studies, which enhances its potential applicability. The model and the case study are based on ecological data from a ten-year grazing experiment conducted in the Ferlo, Senegal (1981–1990), see Hein (2006). These data reveal the joint impact of rainfall variability and long-term stocking rate on rangeland productivity. Our study does not address the issue of multiple states in semi-arid rangelands (Friedel, 1991; Walker & Abel, 2002). The grazing experiments show that the currently applied grazing regimes in the Ferlo are unlikely to lead to a switch to a new rangeland state. This is further elaborated in the discussion section.

2. An enhanced model for optimizing grazing strategies in semi-arid rangelands

In this section we develop a model for optimizing the livestock stocking rate in a semi-arid rangeland dominated by annual grasses, such as the Sahel. The model is built in three steps: (i) a stochastic model to calculate the efficient long-term stocking rate in the absence of an ecological feedback effect and assuming constant livestock prices; (ii) a model with an ecological feedback effect and constant prices; and (iii) a model with feedback effect and variable prices. The model is geared around detailed grazing density – grass production data that were available for the Widou Thiengoly research station in the Ferlo, Senegal (Klug, 1982; Miede, 1992, 1997; André, 1998; Hein, 2006). The general structure of the model is, however, more general and also applies to other rangelands dominated by annual grasses.

The models are based on a static, stochastic optimization procedure, as motivated by the lack of continuity in biomass production between years. There is a strong variation in biomass between years because of the overarching effect of rainfall, which is highly variable between years. Recent ecological insights into the dynamics of semi-arid rangelands show that the impact of high grazing pressures, i.e. the ecological

feedback effect, is only noticeable in the long term (Le Houérou, 1989; Illius & O'Connor, 1999; Sullivan & Rohde, 2002; Briske et al., 2003). The fact that the large interannual variations in biomass and the impacts of overgrazing become apparent only in the long term restricts the possibilities of capturing grass biomass production through a dynamic optimization procedure based on differential equations. Instead we model a grazing regime where the pastoralist community decides on the long-term stocking rate. The model assumes that the stocking rate is reduced only in drought years, specifically to the maximum livestock density that can be sustained in that year. Grazing (the ecological feedback) is included in the model through its impact on the rain-use efficiency of the vegetation (cf. Le Houérou, 1989; Snyman, 1998; Hein, 2006). Rain-use efficiency is an important indicator for the functioning and productivity of semi-arid ecosystems, and expresses the amount of herb biomass produced per unit of rainfall (Le Houérou, 1984; Illius & O'Connor, 1999).

A simplification of the model is that fire is not included as an exogenous driver but accounted for through its average impact on standing biomass. The reason for this simplification is that we wanted to base the model as much as possible on existing ecological data, and the Ferlo grazing experiments did not reveal the long-term impacts of fire on vegetation productivity.

a) The stochastic rangeland model

In its simplest form, a standard model for rangeland management takes the annual forage production of a pasture as fixed (Hildreth & Riewe, 1963). This implies a given annual rainfall. The model assumes no ecological impact of the stocking rate s , and given prices. A pasture's forage production F can be translated into its grazing capacity s_{\max} . Let φ be the amount of plant biomass required to allow the subsistence of a livestock unit. Then

$$s_{\max} = \frac{1}{\varphi} F. \quad (1)$$

The growth of the livestock herd is assumed to follow a logistic growth process (cf. Perrings & Walker, 1997, 2004):

$$\Delta s = \beta \left(1 - \frac{s}{s_{\max}}\right) \cdot s, \quad (2)$$

where s is livestock (expressed as livestock units per hectare), Δs is the gain in livestock, s_{\max} is the grazing capacity of the rangeland and $\beta > 0$ is a scaling parameter capturing the potential natural growth in livestock. We assume that the objective of

the rangeland manager is to maximize profits, and that livestock can be sold at price p per livestock unit. Furthermore there is a variable cost c per livestock unit (e.g. interest and veterinary services) and a fixed cost c_0 (e.g. land tax and fencing). The pastoralist society determines the long-term stocking rate s which is the decision variable in the optimization. The profit function can be written as

$$\pi(s) = p\Delta s - cs - c_0. \quad (3)$$

In order to optimize rangeland management, the rangeland manager chooses the stocking rate s to maximize profits π :

$$\max_s \pi(s). \quad (4)$$

From the first order condition of the maximization problem (4) we derive that

$$c = p\beta - \frac{2p\beta}{s_{\max}} \cdot s^*, \quad (5)$$

where s^* is the optimal stocking rate. Equation (5) is a standard efficiency condition stating that marginal costs equal marginal revenues. Rearranging (5), the optimal stocking rate is written as

$$s^* = \frac{p\beta - c}{2p\beta} \cdot s_{\max}. \quad (6)$$

In order to examine the impact of stochastic rainfall on the optimal stocking rate we modify the assumption that forage production is given and fixed. Instead we assume that forage production is a function of effective rainfall r ($0 < r_{\min} \leq r \leq r_{\max}$). The effective rainfall is the amount of rain left after loss through evaporation or run-off (Haan et al., 1994). It represents the rainfall available to plants. Hence, we write forage production as

$$F = F(r). \quad (7)$$

In line with the characteristics of annual grasses (and excluding for the time being the ecological feedback effect), F is a function of effective rainfall only, and not of the amount of biomass present in the preceding year. We assume here that rainfall is productive, $F(r) > 0$, but its marginal productivity is decreasing, $d^2F/dr^2 \leq 0$, for the relevant range of rainfall. Note that we do not require $dF/dr > 0$ and allow the possibility of a reduction in rain-use efficiency at rainfall levels that are substantially above the average annual rainfall. Hence, applying (1), the grazing capacity is (in the relevant range) a concave function of the effective rainfall. This is in line with e.g. O'Connor et al. (2001). Consider a case where the stocking rate must be chosen at the beginning of the season before rainfall and grazing capacity are known. Let $g(r)$ be the probability density function of effective rainfall. The rangeland manager maximizes expected profits, π^e :

$$\pi^e = \int_{r_{\min}}^{r_{\max}} g(r) \cdot \pi(s_{\max}(r), s) dr. \quad (8)$$

The optimal stocking rate in a stochastic setting will be lower than the stocking rate for a given average effective rainfall (cf. equation (6)); see the appendix for further analysis of the first order conditions of the rangeland manager's maximization problem.

b) The stochastic model inclusive of an ecological feedback

In this section we develop an enhanced model for rangeland management that accounts for stochasticity in rainfall and the ecological feedback of high grazing pressures on rangeland vegetation. The model comprises the assessment of (1) grass production, (2) livestock production and (3) income. These three steps are described below.

Step 1. Assessment of the grass production. The grass production depends on the effective rainfall, i.e. rainfall minus evaporation and run-off, and the rain-use efficiency. The model requires the input of both the average effective rainfall per year and the annual variation in rainfall in the form of a rainfall distribution function. Rain-use efficiency expresses the net primary production per unit of effective rainfall. On the basis of analysis of the grazing data of the Ferlo, we assume that the rain-use efficiency depends on (i) the effective rainfall and (ii) the long-term stocking rate. Our data, as well as studies from other sites (e.g. O'Connor et al., 2001), suggest that the rain-use efficiency is an inverted U-shaped function of the rainfall. The rain-use efficiency is highest at the average rainfall in the Ferlo, which reflects the adaptation

of the plant community to the most common rainfall conditions. At a high stocking rate, the rain-use efficiency decreases, but the inverted U-shape is maintained.

Based on the Ferlo grazing experiment, we use a quadratic equation to capture these effects. Note that a quadratic relation has also been found in a range of other studies, including Tadmor et al. (1974), Zanan (1997), O'Connor et al. (2001) and Hein and De Ridder (2006). First consider a situation without grazing. Let ρ denote rain-use efficiency and r effective rainfall. We assume that rain-use efficiency is positive in the relevant range of rainfall (r_{\min} , r_{\max}). For the case without an ecological feedback of grazing on the ecosystem or in the case of no grazing, a simple relationship capturing the main features is

$$\rho_0 = \alpha(r - r_{\min})\left(1 - \frac{r}{r_{\max}}\right), \quad (9)$$

where ρ_0 refers to the rain-use efficiency in the no grazing case and $\alpha > 0$ is a scaling parameter. On the basis of an analysis of long-term ecological data of the Ferlo rangeland, it is assumed that there are three main general characteristics of the relation between stocking rate and rain-use efficiency: (i) grazing reduces rain-use efficiency: $\partial\rho/\partial s < 0$; (ii) the marginal reduction is increasing in the stocking rate: $\partial\frac{|\partial\rho/\partial s|}{\partial s} > 0$; and (iii) the reduction of the rain-use efficiency due to grazing is the lowest at the level of rainfall that generates maximum forage production. Given a benchmark situation *without* grazing, the rain-use efficiency *with* grazing can be written as follows:

$$\rho(r, s) = \rho_0 - d(r, s), \quad (10)$$

where $d(r, s)$ captures the reduction of rain-use efficiency due to grazing. The reduction function d depends upon the specific ecosystem. We use a simple format for d that maintains the quadratic form of $\rho(r)$ and captures the main features (i)–(iii).

$$d(r, s) = s^\theta(\mu r^2 - 2\mu\bar{r}r + \nu), \quad (11)$$

where $\mu > 0$, $\nu > 0$ and $\theta > 1$ are parameters and \bar{r} is the level of rainfall generating maximum forage. Observe that if $s = 0$, then $d = 0$. Also, the reduction d has a minimum at \bar{r} as required by condition (iii). It is easy to check that conditions (i) and (ii) are also satisfied.

Grass production is determined by the amount of effective rainfall and by the rain-use efficiency of the pasture. It is the product of annual effective rainfall and rain-use efficiency. Accordingly, F is a third power function of the rainfall (cf. Le Houérou et al., 1988; Palmer, 2000). Hence, (7) can be specified as

$$F = \rho r . \quad (12)$$

Step 2. Assessment of the livestock production. This step proceeds analogous to the basic model. The grazing capacity s_{\max} depends on the plant biomass production and the minimum amount of plant biomass required to maintain one livestock unit as stated in (1). The growth of the livestock herd is assumed to follow a logistic growth process described in equation (2). Hence, substituting (1) and (12) into (2), we obtain

$$\Delta s = \beta \left(1 - \frac{s\phi}{\rho r}\right) \cdot s . \quad (13)$$

Step 3. Assessment of the pastoralist's income. The main source of income for pastoralists in the Ferlo is the sale of animals (Guerin et al., 1993). Milk and wool provide some additional income, but this is not considered in our model. Income depends on the amount of surplus livestock that can be sold annually on the market, as well as on the livestock price p . The amount of livestock that can be sold or bought equals the stock in the previous year, s_{t-1} , plus the growth in livestock, Δs , minus the stocking density that is maintained, s_t . The assumption of fixed prices is relaxed below. It is important to note that this approach assumes that pastoralists stock to the preferred long-term stocking rate s when rainfall is sufficient, but that they reduce their herd sizes in years of drought. In drought years (when $s_{\max} < s$), the pastoralists stock up to the maximum possible stocking rate s_{\max} . Pastoralists are able to do so because they can make the stocking decision by the end of the rainy season, when they know the grass resources and stocking capacity of the dry season ahead.

In order to determine the optimal stocking rate we use the standard profit function (3). Substituting (2) into (3) we obtain

$$\pi(r, s) = p\beta \left(1 - \frac{s}{s_{\max}(r, s)}\right) s - c \cdot s - c_0 . \quad (14)$$

The forage production F determines the grazing capacity s_{\max} and is determined by the rain-use efficiency which is, in turn, a function of the long-term stocking rate s . Thus we have

$$s_{\max} = s_{\max}(F(\rho(r, s), r)). \quad (15)$$

Given the ecological feedback of grazing on rangeland productivity, the profit maximization problem is

$$\pi^e = \int_{r_{\min}}^{r_{\max}} g(r) \cdot \pi(s_{\max}(r, s), s) dr, \quad (16)$$

where (16) differs from (8) because s_{\max} is now also a function of the stocking rate. The first order condition is as follows:

$$\frac{\partial \pi^e}{\partial s} = 0 = \int_{r_{\min}}^{r_{\max}} g(r) \cdot \frac{\partial \pi(r, s)}{\partial s} dr. \quad (17)$$

We provide further analysis in the Appendix.

c) The stochastic model with the ecological feedback and with variable prices

So far we have assumed constant prices. However, during a drought many pastoralists will foresee feed shortages and will try to sell animals while the capacity of local markets to absorb the supply is limited. Relatively few local people will buy and prices tend to decrease, in particular in the absence of export markets, which could mitigate regional volatility of supply and demand (Turner & Williams, 2002).

Hence the third model variant accounts for both an ecological feedback and variable prices. When the constant prices assumption does not apply, a drought causes additional losses, as a part of the stock, $s - s_{\max}$, has to be sold at low prices during drought, while high prices prevail during restocking. This price effect is built into the profit function. In years with normal or high rainfall, the profit function is equivalent to the profit function of the previous model (equation 14). However, during a drought, when s_{\max} is lower than the long-term stocking rate s , the profit function changes. The parameter p_{before} reflects the low price at which farmers have to sell during a drought,

whereas p_{after} reflects the prices immediately after a drought when farmers need to restock.

The profit function (14) also has to be adjusted for the impact of sequential drought years; in a second year of drought farmers do not have to sell livestock at low prices, and restocking is delayed. Hence, the occurrence of sequential drought years reduces the price effect. This effect is included by means of the adjustment factor η which equals the fraction of drought years that are consecutive to another drought year. A second year of drought delays restocking and reduces the price effect. The profit function becomes:

$$\pi(r, s) = \begin{cases} -c \cdot s_{\max} - c_0 + \eta(p_{before} - p_{after})(s - s_{\max}) & \text{if } s > s_{\max} \\ p\beta\left(1 - \frac{s}{s_{\max}(r, s)}\right)s - c \cdot s - c_0 & \text{if } s \leq s_{\max} . \end{cases} \quad (18)$$

The impact of variable prices will be further illustrated in the case study.

3. A case study for the Ferlo, Senegal

3.1 The case study area

The Ferlo is in the northern part of Senegal. It is part of the Sahel biogeographic zone, which extends from Senegal in the west to Somalia in the east. The area has a very modest, undulating, relief. Annual rainfall varies between around 120 and 450 mm, with an average of 290 mm (André, 1998). Rain falls predominantly during the short rainy season from July to September. The natural vegetation consists of dry grassland with scattered trees and bushes. The grazing experiment that provided the data that inspired the modeling of the ecological feedback mechanism was conducted in the Widou-Thiengoly study site (Klug, 1982; Miede, 1992, 1997; André, 1998). The coordinates of the study site are 15°20' N, 16°21' W, as shown in Figure 1.



Figure 1: Location of the Ferlo and Widou-Thiengoly study sites, Senegal

Livestock keeping is the main economic activity in the Ferlo. The principal animals kept are cattle (zebu), sheep and goats. Traditionally, the pastoral population, mostly Peuhls, migrated on an annual basis between the Ferlo and the more humid Sudan zone to the south. Since the 1950s, there has been a large increase in livestock density, related to large increases in the local population and the enhanced availability of veterinary aid. At the same time, expanding agricultural activities in the Sudan zone have limited the pastoralists' migration possibilities (FAO, 2001). In combination with the sinking of numerous boreholes, which provide perennial drinking water, this has led to increased sedentarization in the Ferlo (Sinclair & Fryxell, 1985; Guerin et al., 1993). Currently, the total rural population of the Ferlo can be estimated at around 110,000 people (Direction de la prévision et de la statistique, 1997), and the average stocking rate in the Ferlo is in the order of 0.15 to 0.20 tropical livestock units per hectare (De Leeuw & Tothill, 1990; Mieke, 1997). A tropical livestock unit (TLU) corresponds to 250 kg of animal weight (Boudet, 1975; Whiteman, 1980).

3.2 Modeling and data

To illustrate our ecological-economic model for rangelands, we calculate the optimal long-term grazing pressure for the Ferlo Sahelian rangeland in northern Senegal using three different models. We will subsequently use: a) a basic stochastic model – assuming no impact of stocking rate on the ecosystem and constant prices; b) the enhanced ecological-economic model, which includes the long-term impact of stocking rate on the forage production; and c) a further enhanced model that also accounts for the fact that prices are contingent on the occurrence of a drought. The efficient long-term stocking rates are identified by maximizing the profit function for each model. The decision variable of the three models is the long-term stocking rate.

In the models, we assume that pastoralists seek to optimize rangeland management by selecting the efficient long-term stocking rate. It is assumed that all animals that would lead to stocking above this long-term rate are sold. However, we allow for lower stocking rates during dry years that have insufficient animal feed production to

maintain the stocking rate. We assume that, during drought, pastoralists try to keep as many livestock as possible on the available feed resources. In other words, in a year of drought, the actual stocking rate s equals the grazing capacity s_{\max} . This type of stocking strategy has been suggested for rangelands where pastoralists are not able to supply supplementary feed during drought and where they face difficulties in restocking after a drought (FAO, 1988; De Leeuw & Tothill, 1990).

The rainfall probability density function for the Ferlo is constructed on the basis of long term rainfall data reported in André (1998). This data series covers a 50-year period (1947–1997). Mieke (1992) reports the relation between rainfall and effective rainfall. Ecological data are derived from a grazing experiment conducted in the Widou-Thiengoly catchment, northern Senegal (see Figure 1), as reported in Klug (1982), Mieke (1992, 1997) and André (1998). The experiment included the monitoring of the plant biomass production under three different grazing regimes, for a period of ten years. The data have been re-analyzed for this paper in order to reveal the joint impacts of rainfall variability and stocking rate on biomass productivity. The average prices per animal and the herd composition in the Ferlo are from Thébaud et al. (1995). To evaluate the impact of drought on livestock prices, long-term livestock price data for Western Niger are analyzed, as reported in Turner and Williams (2002). In the absence of such data for the Ferlo, it is assumed that the relative price fluctuations in the Ferlo correspond to the price changes in western Niger, but we are aware that in reality a discrepancy may exist.

a) The stochastic model without feedback effect

We first apply the basic, stochastic, single-period model to the case of the Ferlo. The model assumes no ecological feedback and fixed prices for livestock. There are three basic steps: (i) assessment of the grass production; (ii) assessment of the livestock production; and (iii) calculation of the income derived from livestock keeping.

Step 1. Assessment of the grass production. The grass production is calculated as a function of effective rain and the rain-use efficiency. Analysis of the rainfall and the effective rainfall in the Ferlo during the period 1981–1990 has shown that, on average, the effective rain is 65% of the total annual rainfall (Mieke, 1992). The estimate obtained is

$$r = 0.65 r_{\text{tot}} ; \quad (n = 10, R^2 = 0.98, F = 448).$$

where, as before, r is the effective rainfall (mm) and r_{tot} is the total annual rainfall (mm). The production of forage is calculated by multiplying the effective rain with the rain-use efficiency. In the basic model, the rain-use efficiency is a function of the effective rainfall only. On the basis of regression analysis, the following relation between rain-use efficiency ρ_0 (kg/mm) and effective rainfall is derived:

$$\rho_0 = -0.00021 r^2 + 0.0756 r - 1.254. \quad (9^*)$$

$$(n = 10, R^2 = 0.62, F = 5.7)$$

For this regression analysis, only non-grazed sample plots have been used, because we want to examine the relation between effective rainfall and RUE (rain use efficiency) in the absence of an ecological feedback effect. For plots that are being grazed, it is likely that grazing has affected the plant community (Breman & De Ridder, 1991), and inclusion of grazed plots in the regression analysis would lead to a bias. The regression analysis is based on ten years of data, and each year represents the average biomass production in five different plots. Whereas the availability of a consistent ten-year time series on vegetation productivity as a function of effective rainfall and grazing is unique for the West African Sahel, the regression is nevertheless prone to uncertainty because of the still limited number of regression points ($n=10$). The general quadratic shape of the relation between RUE and effective rainfall, however, is confirmed by other research, for instance O'Connor et al. (2001) and Hein & De Ridder (2006). The forage production F (kg) is straightforwardly calculated from $F = \rho_0 \cdot r$ (see equation 12).

Step 2. Assessment of the livestock production. The annual grazing capacity s_{\max} is calculated by dividing the annual forage production F by the amount of plant biomass production required to maintain one TLU, ϕ . In order to calculate this amount, three factors have to be taken into account. The first is the minimum amount of feed that the animals need to maintain themselves, which has been estimated at 4.3 kg/TLU/day – based on the local livestock mix (Thébaud et al., 1995) and the energy requirements per animal (Bayer & Waters-Bayer, 1998). Second, not all herb biomass is available for grazing, due to decomposition, fire, or the unpalatability of certain plants. Estimates of the rates of plant material actually available to livestock range from 35% (Penning de Vries & Djitéye, 1982) to 50% (Breman & De Ridder, 1991). Because these rates tend to be higher for drier areas (Bayer & Waters-Bayer, 1998), we assume in our models that 50% of the plant biomass is available for grazing. Third, it needs to be taken into account that animals supplement their diet with woody plants, particularly in the dry season. According to Breman and De Ridder (1991), the dietary contribution of woody plants can be estimated at 20% in the Ferlo. On the basis of the above, we estimate that ϕ equals $4.3 \cdot 365 \cdot 2 \cdot 0.8 = 2511$ kg herb biomass / TLU / year.

Following the conceptual model, the livestock population grows according to a logistic growth curve as a function of the livestock population and the grazing capacity. Boudet (1975) and Mortimore and Adams (2001) estimate a maximum natural growth of herd size of around 20% per year for the western Sahel. It is assumed that this also holds for the Ferlo. This growth rate corresponds to a logistic growth factor $\beta = 0.6$. Using this estimate, the growth of livestock Δs as given in (13) is fully specified.

Step 3. Calculation of the income derived from livestock keeping. We assume that the aim of the pastoralist society is to maintain the long-term stocking rate, except during drought, when this rate cannot be maintained and the stocking rate will be reduced to

the level of the grazing capacity s_{\max} . Under such a stocking regime, sales are Δs in a normal year, and $s - s_{\max}$ during a drought. After a drought, $s - s_{\max}$ animals are bought to restock, as the growth of the herd is insufficient to stock up to the long-term stocking rate. Under the assumption of constant prices, destocking and restocking affects profits only indirectly, as a reduced herd size has lower opportunity costs of capital. Hence, profits during drought are $-cs_{\max}$.

In the calculations, it is assumed that the main source of income for pastoralists is the sale of animals for meat (cf. Guerin et al., 1993). Hence, for reasons of simplicity, income from milk and wool is not accounted for. Based on the average price per animal and the local livestock mix (Thébaud et al., 1995), the average livestock price is 24,750 CFA/TLU.

Regarding the costs of livestock herding in northern Senegal, we assume that all costs are variable costs, related to the capital and labor inputs required to maintain the herd. It is assumed that the *capital* costs per livestock unit amount to the local real interest rate times the price of a livestock unit. Currently, the average local interest rates are around 18% (Ndour & Wané, 1998), and the annual inflation in Senegal is some 2% (IMF, 1999). Hence, we have capital costs of $0.16 \cdot 24750$ CFA/TLU. The average *labor* costs in rural Senegal can be estimated at 100,000 CFA per person per year (Direction de la prévision et de la statistique, 1997). These costs are incurred only during the period January to June, when the herds are moved south (Guerin et al., 1993), as during the rainy season herds are taken care of by various family members, including children, at no cost. With an average herd size of 44 TLU per family (Thébaud et al., 1995), the annual labor costs amount to $100,000 / 44 / 2 = 1140$ CFA/TLU. We do not have data on fixed costs. We ignore these in the remainder because they do not affect the stocking decision. Therefore, the profit function is specified as follows:

$$\pi = 24750 \cdot \Delta s - (0.16 \cdot 24750 - 1140) \cdot s . \quad (3^*)$$

Results. Based on the simple grazing model that assumes no ecological feedback and constant prices, the average annual income (in CFA) that can be obtained from one hectare of rangeland has been calculated for a range of fixed maximum stocking densities. The optimal long-term stocking rate is 0.11 TLU/ha. The corresponding annual income is 525 CFA/ha.

b) The enhanced model including both stochasticity and feedback effect

Next, we include the ecological feedback of grazing pressure on long-term forage production in our model. Analysis of the grass production data of the Ferlo under different rainfall conditions and grazing regimes shows that a high grazing pressure causes a reduction of the rain-use efficiency and, hence, a reduction of the productivity of the grassland. This effect is strongest in dry years. It reflects a loss of

resilience to drought as a consequence of a high grazing pressure (Pimm, 1984). Using regression analysis (ordinary least squares) we estimate from the available data that the rain-use efficiency is

$$\rho = -0.00021r^2 + 0.0756r - 1.254 - s^2(0.00504r^2 - 2.016r + 210.8) \quad (10^*)$$

($n = 30$, $R^2 = 0.76$, $F = 28$).

Note that we pose the restriction that for $s = 0$ equation 10* should equal equation 9* (the latter representing ρ_0 , the rain-use efficiency in the absence of an ecological feedback effect). We do this because one of our objectives is to examine the implications of the ecological feedback effect for optimizing grazing strategies, and we need to compare the 'with' and 'without' ecological feedback cases. Hence, the first three terms of the equation are equal to equation 9*. Note also that 30 plots have been included in this regression analysis. These cover ten years of data, with three data points for each year. For each year, there is a regression point representing a grazing pressure of respectively 0, 0.10 and 0.15 TLU/ha. Hence n is larger than for equation 9*, which considered ten years of non-grazed plots (0 TLU/ha) only. Whereas the equation is significant at $p = 0.05$, the number of data points ($n=30$) is still modest, and, as with equation 9*, this equation must also be considered with caution in view of the associated uncertainties.

Results. We find that the optimal long-term stocking rate is 0.10 TLU/ha, corresponding to an annual net income of 492 CFA/ha. The optimal stocking rate is about 10% lower than the case in which the ecological feedback effect is not accounted for. The maximum long-term expected annual income is also lower than the maximum income in the basic model because the inclusion of the feedback mechanism in the model accounts for a reduction of the average annual feed production in the rangeland.

c) Rangeland management with stochastic rainfall, ecological feedback and variable prices

The third model variant accounts for both an ecological feedback and variable prices. In drylands, livestock prices tend to decrease during a drought, as many farmers want to sell livestock that they cannot feed. Immediately after a drought, livestock prices increase substantially as farmers want to restock (Turner & Williams, 2002). For the Ferlo, data on price fluctuations are not available. Therefore, as the best proxy available, data on price fluctuations from western Niger have been used. This area has a slightly higher average rainfall but otherwise represents a comparable physical, economic and social environment. On the basis of these data, it is assumed in the model that prices drop to 43% during drought years and that they increase to 146% subsequent to a drought. Part of the stock, $s - s_{\max}$, has to be sold at low prices, while high prices prevail during restocking.

In our application to the Ferlo, the adjusted factor for the impact of sequential dry years, η , is based on rainfall data for the period 1947 to 1997 (as reported in André, 1998). These data show that, on average, around 20% of the drought years are consecutive to another drought year. Since, in our model, the farmer does not sell or restock between two drought years, the price effect does not occur in 20% of the drought years. Hence, the adjustment factor $\eta = 0.8$.

Results. Including variable prices in the grazing model further reduces the optimal long-term stocking rate. As a drought causes additional losses, the vulnerability to drought can be decreased by further decreasing the long-term stocking rate. With an adverse price effect, the optimal long-term stocking rate is 0.09 TLU/ha, with a net annual income of 456 CFA/ha.

3.3 Comparison of results

The outcomes of the three models are summarized in Table 1. Inclusion of the ecological feedback effect reduces the economic efficient stocking rate from 0.11 to 0.10 TLU/ha. With variable prices, the optimal stocking rate is further reduced to 0.09 TLU/ha. For comparison, we also calculated the approximate annual income derived from the Ferlo at the *current* stocking rate. Model c) predicts that, for a stocking rate of 0.15, the average annual income is 237 CFA/ha/year – significantly below the income that could be obtained at a lower stocking rate.

Table 1: Efficient long-term stocking rates for the Ferlo

Model	Optimal long-term stocking rate (TLU/ha)	Annual income (CFA/ha/year)
a) The stochastic model (one period)	0.11	525
b) The stochastic model inclusive of the ecological feedback	0.10	492
c) The stochastic model with ecological feedback and variable prices	0.09	456

4. Discussion

4.1 The reliability and limitations of the modeling approach

The key relations underlying our model are that (i) rain-use efficiency varies with effective rainfall according to a quadratic function (cf. O'Connor et al., 2001); and (ii) grazing affects the rain-use efficiency of the vegetation, in particular in the long term (cf. e.g. Le Houérou, 1989). Consequently, rainfall determines yearly fluctuations in productivity, and grazing pressure affects the long-term productivity. Both relations are non-linear and the impact of grazing on productivity is most pronounced in years with low rainfall.

For our case study, the relations indicating the development of the rain-use efficiency as a function of effective rainfall and stocking rate have been tested with a t-test. The two equations that establish respectively the relation between rainfall and rain-use efficiency in the absence of grazing (equation 9*) and with grazing (equation 10*) are both significant at the $p=0.05$ level. Hence, it is anticipated that the approach may also be relevant to the modeling of other semi-arid rangelands subject to a long-term feedback from high grazing pressures, specifically those rangelands dominated by annual grasses.

The model presented in this study is subject to four main constraints. First, we do not consider the occurrence of multiple states in rangeland dynamics. The existence of multiple equilibria has been demonstrated in relation to the ratio grass : woody plants cover (Friedel, 1991; Laycock, 1991) and in relation to the ratio grass cover : bare soil (Van de Koppel et al., 1997; Scheffer et al., 2001). Our models do not include the dynamics of such transitions and hence are applicable only to systems that are dominated by herbaceous plants, such as the Ferlo. Note that in the Ferlo high pressure on woody species for fuel wood prevents a shift to a state dominated by trees and shrubs (Sinclair & Fryxell, 1985), and continuous tracts of bare soil occur in particular around ponds and boreholes where livestock gathers to drink (De Leeuw & Tothill, 1990).

Second, the paper does not explicitly address the impacts of fire, which is an additional important driver in many rangelands (e.g. West, 1971; Walker, 1981; Perrings & Walker, 1997; Snyman, 1998). In particular, we ignore in our model the fact that fire may have a strong impact on the availability of woody plant biomass for browsing. However, we do account for the impact of fire on the availability of herbaceous biomass for grazing (through modification of the parameter ϕ). Since in the Ferlo only 20% of animal feed is obtained through browsing (Breman & De Ridder, 1991), we expect that this simplification has only a modest effect on the results of our case study.

Third, the paper does not account for the spatial heterogeneity of rangelands. Rainfall patterns in the Sahel are not only highly variable per year, but also show strong spatial variability. Pastoralists adapt to this heterogeneity by adjusting their grazing strategy to the local availability of plant biomass. The importance of spatial heterogeneity for calculating optimal stocking rates is elaborated in, for instance, Turner (1991) and McPeak (2003). In our model, accounting for spatial heterogeneity would probably lead to an increase in the optimal stocking density, as the impacts of dry years in one site may be moderated by higher rainfall in other sites. However, in this respect, it is important to note that the two years with the lowest available plant biomass in the period during which the data supporting our model was collected, 1983 and 1984, count as extremely dry throughout the Sahel (Nicholson et al., 1998). This indicates that animal feed availability was very low in most parts of the Sahel, reducing the opportunity to mitigate for drought by adjusting grazing patterns.

Fourth, we have not addressed the pastoralists' attitude to risk. In general, pastoralists tend to be risk-averse, and prefer to avoid years with below-average income (Anderson & Dillon, 1992; Hardaker, 2000). Accounting for the pastoralists' risk-aversion may lead to a reduction of the calculated optimal stocking rate.

4.2 Implications for rangeland management

In the Ferlo, as in the Sahel at large, transhumance is the dominant management system. Pastoralists migrate according to specific seasonal patterns, complemented by more permanent settlements for the less mobile part of the population. The current average stocking rates in the Ferlo are around 0.15 to 0.20 TLU/ha (De Leeuw & Tothill, 1990; Miede, 1997). The stocking rate is high relative to other parts of the Sahel with comparable rainfall, owing to the presence of a large number of boreholes that provide year-round drinking water for the animals (De Leeuw & Tothill, 1990). Our paper shows that, assuming fixed prices, the efficient stocking rate is 0.10 TLU/ha. With variable prices, this further reduces to about 0.09 TLU/ha. Consequently, the model indicates that the optimal long-term stocking rate in the Ferlo is substantially lower than the current stocking rate. Note, however, that the pastoralists' actual motivations for stocking are likely to include other factors besides income maximization. Pastoralists in remote areas have limited access to banks and livestock performs the functions of insurance and savings account. Nevertheless, the results of this paper indicate that government agencies and development institutes should promote reductions rather than increases in livestock densities in the Ferlo, as any further increases in herd sizes are economically counterproductive. They may also consider improving the functioning of livestock markets, which would enhance the pastoralists' capacity to adjust stocking rates to the annual grazing capacities (see e.g. Holtzman & Kulibaba, 1994).

5. Conclusions

This paper offers a conceptual approach to account for rainfall stochasticity and the long-term impact of grazing in a model that can be used to calculate the optimal long-term stocking rate for a rangeland. The model is applicable to rangelands dominated by annual grasses. A static, stochastic approach was followed in order to deal with the lack of carry-over of biomass between years for such rangelands. A key variable in the model is the rain-use efficiency that expresses plant production per mm of effective rainfall, and which is affected by both rainfall and grazing pressure.

A limitation of the model is that it assumes that the long-term stocking density is fixed over time, and that deviations only take place during drought when pastoralists reduce livestock numbers. Whereas this assumption is realistic compared to current grazing practices in the Ferlo (Le Houérou et al., 1989), it can nevertheless be expected that optimal rangeland management is more flexible in terms of adjusting to variations in rainfall conditions. However, this assumption enabled us to capture the main drivers of the rangeland in a model that could be calibrated with actual data available for the Sahel. Further research would be required to include more flexible management strategies in models for rangelands dominated by annual grasses.

The model is applied to the Ferlo rangeland in northern Senegal. Our analysis shows that the impact of high grazing pressures on rangeland productivity is most pronounced in years with low rainfall, when rangelands subject to a high grazing pressure show a strong decrease in herbaceous biomass production compared to rangelands subject to low grazing pressures. The analysis also shows that current stocking rates in the Ferlo are considerably higher than the efficient stocking rates.

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Appendix

According to equation (8), the rangeland manager’s *expected* profits are:

$$\pi^e = \int_{r_{\min}}^{r_{\max}} g(r) \cdot \pi(s_{\max}(r), s) dr .$$

From integration by parts we obtain

$$\pi^e = \pi(s_{\max}(r_{\max}), s) - \int_{r_{\min}}^{r_{\max}} \frac{\partial \pi(s_{\max}(r), s)}{\partial r} G(r) dr , \tag{A1}$$

where $G(r) \equiv \int_{r_{\min}}^r g(r) dr$. Maximization of expected profits yields the first order condition:

$$s^* = \frac{p\beta - c}{2p\beta} \cdot s_{\max}(r_{\max}) \frac{1}{\underbrace{1 + s_{\max}(r_{\max}) \int_{r_{\min}}^{r_{\max}} \frac{\partial s_{\max} / \partial r}{s_{\max}^2} G(r) dr}_{\Omega}} . \tag{A2}$$

When comparing (6) and (A2), an additional term, denoted Ω , shows up in (A2). In general the denominator of Ω is larger the larger the impact of rainfall on the grazing capacity and the larger the variance of the distribution $g(r)$. Hence, the optimal stocking rate decreases with the increasing impact of rainfall and rainfall variability. Note that (A2) collapses to (6) if there is no impact of rainfall on the grazing capacity ($\partial s_{\max} / \partial r = 0$) or if $g(r)$ is a degenerated distribution such that $r_{\min} = r_{\max}$ (i.e. in the case of certainty).

We further analyze the first order condition (17) for the case of stochastic rainfall and an ecological feedback of grazing on rain-use efficiency.

Integration by parts gives

$$\begin{aligned} \frac{\partial \pi^e}{\partial s} = 0 &= \left[G(r) \frac{\partial \pi}{\partial s} \right]_{r_{\min}}^{r_{\max}} - \int_{r_{\min}}^{r_{\max}} G(r) \cdot \frac{\partial^2 \pi}{\partial r \partial s} dr \\ &= \frac{\partial \pi}{\partial s} - \int_{r_{\min}}^{r_{\max}} G(r) \cdot \frac{\partial^2 \pi}{\partial r \partial s} dr . \end{aligned} \quad (\text{A3})$$

Differentiating (14) we obtain

$$\frac{\partial \pi}{\partial s} = p\beta - c - \frac{2p\beta s}{s_{\max}} + p\beta s^2 \frac{\partial s_{\max} / \partial s}{s_{\max}^2} . \quad (\text{A4})$$

Differentiating (A4) we obtain

$$\frac{\partial^2 \pi}{\partial r \partial s} = 2p\beta s \frac{\partial s_{\max} / \partial r}{s_{\max}^2} + p\beta s^2 \left(\frac{(\partial^2 s_{\max} / \partial r \partial s)}{s_{\max}^2} - \frac{2(\partial^2 s_{\max} / \partial r \partial s)}{s_{\max}^3} \right) . \quad (\text{A5})$$

Combining (A3), (A4) and (A5) gives the following first order condition:

$$\begin{aligned}
 \frac{\partial \pi^e}{\partial s} = 0 = & p\beta - c - \frac{2p\beta s}{s_{\max}(r_{\max})} + \underbrace{\frac{p\beta s^2}{s_{\max}^2} \frac{\partial s_{\max}}{\partial s}}_{\text{ecological feedback}} - \underbrace{2p\beta s \int_{r_{\min}}^{r_{\max}} G(r) \frac{\partial s_{\max} / \partial r}{s_{\max}^2} dr}_{\text{stochastic effect}} \\
 & - \underbrace{p\beta s^2 \int_{r_{\min}}^{r_{\max}} G(r) \left(\frac{\partial^2 s_{\max} / \partial r \partial s}{s_{\max}^2} - \frac{2\partial^2 s_{\max} / \partial r \partial s}{s_{\max}^3} \right) dr}_{\text{stochastic ecological feedback effect}}. \tag{A6}
 \end{aligned}$$

Table A1: Comparison of first order conditions

	Deterministic (given) rainfall	Stochastic rainfall
	$r_{\min} = r_{\max}$	$r_{\min} < r_{\max}$
Single period		
$\frac{\partial s_{\max}}{\partial s} = 0$	$\frac{\partial \pi}{\partial s} = 0 = p\beta - c - \frac{2p\beta s}{s_{\max}}$	$\frac{\partial \pi^e}{\partial s} = 0 = p\beta - c - \frac{2p\beta s}{s_{\max}(r_{\max})} - \underbrace{2p\beta s \int_{r_{\min}}^{r_{\max}} G(r) \frac{\partial s_{\max}}{\partial r} dr}_{s_{\max}^2} \underbrace{\frac{\partial s_{\max}}{\partial r}}_{\text{stochastic effect}} dr$
Long-term		
$\frac{\partial s_{\max}}{\partial s} < 0$	$\frac{\partial \pi}{\partial s} = 0 = p\beta - c - \frac{2p\beta s}{s_{\max}} + \underbrace{\frac{p\beta s^2}{s_{\max}^2} \frac{\partial s_{\max}}{\partial s}}_{\text{ecological feedback}}$	$\frac{\partial \pi^e}{\partial s} = 0 = p\beta - c - \frac{2p\beta s}{s_{\max}(r_{\max})} + \underbrace{\frac{p\beta s^2}{s_{\max}^2} \frac{\partial s_{\max}}{\partial s}}_{\text{ecological feedback}} - \underbrace{2p\beta s \int_{r_{\min}}^{r_{\max}} G(r) \frac{\partial s_{\max}}{\partial r} dr}_{s_{\max}^2} \underbrace{\frac{\partial s_{\max}}{\partial r}}_{\text{stochastic effect}} - \underbrace{p\beta s^2 \int_{r_{\min}}^{r_{\max}} G(r) \left(\frac{\partial^2 s_{\max}}{\partial r \partial s} - \frac{2\partial^2 s_{\max}}{s_{\max}^2} \frac{\partial r \partial s}{\partial r \partial s} \right) dr}_{s_{\max}^3}}_{\text{stochastic ecological feedback effect}}$

