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A state-dependent parameterization of saturated-unsaturated zone interaction

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Abstract

[1] The relevance of groundwater as an important source of root zone moisture by means of capillary rise is increasingly being recognized. This is partly reflected in many current land surface schemes, which increasingly replace a one-way (i.e., downward) drainage of water by a two-way interaction flux between the root zone and a groundwater system. A fully physically correct implementation of this two-way saturated-unsaturated interaction flux requires transient simulations using the highly nonlinear Richards' equation, which is a computationally demanding approach. We test a classic simple approximation that computes the root zone–groundwater interaction flux as the net effect of a downward drainage flux and an upward capillary rise flux against the Darcy equation for quasi steady state conditions. We find that for a wet root zone and/or shallow groundwater, the errors within this approximation are significant and of the same magnitude as the interaction flux itself. We present a new closed-form parameterization of the Darcy equation–based fluxes that accounts both for root zone soil moisture and depth to the water table. Parameter values for this parameterization are listed for 11 different, widely applied soil texture descriptions. The high numerical efficiency of the proposed method makes it suitable for inclusion into demanding applications, e.g., a Monte Carlo framework, or high spatial resolution.

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1. Introduction

[2] An accurate description of the water and energy budgets at the land surface depends on a correct representation of soil moisture availability for evapotranspiration. The presence of groundwater can affect the soil moisture budget in the root zone [*Beldring et al.*, 1999], as well as vegetation and evapotranspiration, especially in (riparian zones within) semiarid areas [*Cooper et al.*, 2006; *Newman et al.*, 2006; *Ridolfi et al.*, 2006]. However the modeling of the fluxes between the groundwater and the

root zone, and the accurate modeling of groundwater dynamics in relation to both vertical and horizontal redistribution of water in the landscape, is complicated because of the high nonlinearities involved. The potential impact of groundwater on evapotranspiration and climate is now widely recognized [e.g., *Bierkens and van den Hurk*, 2007], and groundwater modules are currently being implemented in existing land surface models [e.g., *Koster et al.*, 2000; *Liang et al.*, 2003; *Maxwell and Miller*, 2005; *Yeh and Eltahir*, 2005; *Fan et al.*, 2007; *Gulden et al.*, 2007; *Miguez-Macho et al.*, 2007; *Niu et al.*, 2007]. It is important that these modules have a realistic and consistent representation of the saturated-unsaturated zone coupling.

[3] Different types of land surface models can be distinguished on the basis of their treatment of groundwater. A possible classification is the following.

[4] Type 1 is root zone controlled and assumes no or very deep groundwater. This is the “classical” land surface model where flow from the root zone downward is driven by gravity and is only a function of the soil moisture content in the lower part of the root zone. Capillary effects are ignored.

[5] Type 2 is groundwater controlled and assumes an equilibrium soil moisture profile (i.e., a balance between gravitational and capillary forces) extending from the groundwater upward. The soil moisture content in the root zone is directly controlled by the depth of the groundwater. Since simple analytical expressions exist for the equilibrium profile, this type of models is computationally efficient. Examples of this type of models are found in work by *Koster et al.* [2000] and *Hilberts et al.* [2005].

[6] Type 3 is Richards' equation based. In this type of models, the Richards' equation is solved over the full extent of the unsaturated-saturated zone (including groundwater). This approach is most realistic, but is computationally more demanding because of the high number of soil layers. Examples of this type of models are found in work by *Maxwell and Miller* [2005], *Yeh and Eltahir* [2005], and *Niu et al.* [2007].

[7] Under specific conditions the Richards' equation can be linearized, and vertical fluxes, including capillary rise, can be computed analytically [*Wang and Dooge*, 1994; *Pullan*, 1990, and references therein]. The main requirement for this “quasi-linear approximation”, however, is that the unsaturated hydraulic conductivity is an exponential function of soil matric potential. See section 5.3 for a further discussion.

[8] Type 4 is mixed. This type of models apply the gravitational flux from the root zone downward as in type 1 models, but in addition there is a capillary flux from the groundwater upward based on the (semi) analytical steady state expressions of *Gardner* [1958] and *Eagleson* [1978]. An example of this type of models is TOPLATS [*Famiglietti and Wood*, 1994]. Although this type of models is computationally efficient, the underlying assumptions are not internally consistent (see discussion below). More modern parameterizations of root zone–groundwater interaction [e.g., *Liang et al.*, 2003] use a more sophisticated yet physically based approach, but are less numerically efficient.

[9] Models of types 1 and 2 have no real coupling between the saturated and unsaturated zone. In the following we will focus on type 4 models since they are relatively simple, and yet they allow for a full coupling. We will use the Darcy-Buckingham equation to develop a simple and consistent parameterization for capillary fluxes that does not only depend on the depth of the groundwater table, but also on the soil moisture content of the root zone.

[10] A classic solution to the type 4 formulation is due to *Gardner* [1958] and *Eagleson* [1978], who presented simple, physically based, analytical approximations for the capillary rise from a groundwater at given depth (explained in detail in section 2.2). The coupling between root zone and groundwater is then obtained by taking the net effect of both this (upward) capillary rise flux (which is a function of depth Z_g to the groundwater), and the (downward) gravity drainage flux, which is a function of root zone moisture content, expressed here as degree of saturation s_r, or equivalently root zone pressure head ψ_r.

Root zone moisture is assumed to be uniform within the root zone $-Z_r \leq z \leq 0$.

[11] The Gardner-Eagleson capillary rise formulation has a number of underlying assumptions, one of which is the assumption that the soil surface is completely dry, i.e., $s|_{z=0} = 0$ or $\psi|_{z=0} = -\infty$. This assumption clearly is in contradiction with the assumption of the gravity drainage model, where $0 \leq s_r \leq 1$. Hence, a parameterization with consistent underlying assumptions would be desirable.

[12] The purpose of this paper is twofold: (1) to assess the effects of the conflicting assumptions underlying the above described Gardner-Eagleson saturated-unsaturated interaction flux against a physically based (Darcy equation) approach and (2) to present a simple self-contained analytical parameterization of Darcy equation derived flux predictions, that fits the same boundary conditions as the Gardner-Eagleson approach without sharing its problematic underlying assumption, and is considerably faster than numerical integration of the Richards' equation.

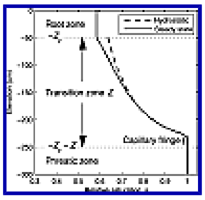

Figure 1. Idealized moisture profile. A root zone of fixed thickness Z_r and uniform relative moisture content s_r is connected to a phreatic zone by a transition zone with thickness Z.

[13] We consider the problem as sketched in Figure 1. Immediately below the land surface there is a root zone, with thickness Z_r. In this paper we assume Z_r = 50 cm, although our parameterization does not depend on the value of Z_r. Moisture content (quantified here as the relative degree of saturation s_r) in this zone is assumed to be vertically uniform. Groundwater is below the phreatic level at depth $Z_g = Z + Z_r$. The intermediate zone, of thickness Z, is called the transition zone. This transition zone includes the capillary fringe, if present. Plotted in Figure 1 are (for a unspecified soil) the hydrostatic moisture profile within the transition zone, and a possible actual moisture profile.

2. Gardner-Eagleson Parameterization

[14] This section outlines the Gardner-Eagleson [*Gardner*, 1958; *Eagleson*, 1978] approach to saturated-unsaturated interaction. Although the equations can be found in the above cited references, they are repeated here to present a self-contained description of the approach.

[15] It is assumed that all flow is vertical, and that flow is governed by the one-dimensional Darcy-Buckingham equation

$$q = -k\frac{\partial h}{\partial z} = -k\left(\frac{\partial \psi}{\partial z} + 1\right) \qquad (1)$$

where q is the water flux (positively upward), k is the hydraulic conductivity, $h = \psi + z$ is the hydraulic head, ψ is the pressure head, and z is the elevation.

2.1. Gravity Drainage

[16] If one assumes that the moisture distribution within the root zone is homogeneous, then

$$\partial \psi / \partial z = 0 \qquad (2)$$

that is, there are no capillary effects, and equation (1) reduces to

$$q_{\text{grav}} = -k \qquad (3)$$

Usually k is computed as a function of volumetric water content θ or pressure head ψ. In this paper we adopt the Campbell [1974] water retention characteristic and k parameterization, because of its parsimoniousness, and because the parameters of this model are well documented for many soil types [e.g., Clapp and Hornberger, 1978]. It should be noted, though, that calibrated, pedon-scale effective soil parameters are preferable above laboratory-determined sample-scale parameters, to account for the effects of soil heterogeneities.

[17] The Campbell [1974] water retention characteristic is given by

$$\psi = \psi_{ae} s^{-b} \quad (4)$$

where ψ_{ae} is the air entry pressure, $s = \theta/\theta_s$, is the relative saturation, θ_s is the saturated water content or porosity, and b is a pore size distribution index. The corresponding k parameterization is given by

$$k(s) = k_s s^c \quad (5)$$

where k_s is the saturated hydraulic conductivity, and $c = 2b + 3$ is a pore disconnectedness index.

2.2. Capillary Rise Flux

[18] Gardner [1958]
assumed that long-term evaporation rate from a soil column is controlled by the steady capillary rise flux from the groundwater, which is assumed to be at a fixed depth Z_g. Assuming quasi steady state, and solving the Darcy-Buckingham equation (1) for z yields [Gardner, 1958]

$$z = -\int \frac{d\psi}{q/k + 1} \quad (6)$$

assumed that the soil water retention characteristic can be approximated by

$$k = \frac{a}{(-\psi)^{\beta} + b} \quad (7)$$

which enables analytical solutions for (6) for values of β = 3/2, 2,3,4. Assuming that the maximum evaporation rate or capillary rise flux q_{cap} corresponds to the upper boundary condition $\psi \rightarrow -\infty$ at the soil surface $z = Z_g$ and the lower boundary condition $\psi = 0$ at the water table $z = 0$, Gardner [1958] then proceeds showing that his solutions to (6) yield the approximations

$$q_{\text{cap}} = \begin{cases} 3.77\, a Z_g^{-3/2} & \text{for} \quad \beta = 3/2, \\ 2.46\, a Z_g^{-2} & \text{for} \quad \beta = 2, \\ 1.76\, a Z_g^{-3} & \text{for} \quad \beta = 3, \\ 1.52\, a Z_g^{-4} & \text{for} \quad \beta = 4. \end{cases} \quad (8)$$

[19] Eagleson [1978] proposed a slightly different version of equation (8), using the Campbell [1974] retention characteristics, equation (4) and (5) instead of equation (7). This resulted in

$$q_{\text{cap}} = k_s \alpha \left(\frac{\psi_{ae}}{Z_g}\right)^{\beta} \quad (9)$$

[20] The constants in equation (8) were generalized to allow arbitrarily values of β, by fitting the empirical function

$$\alpha \approx 1 + \frac{3/2}{\beta - 1} \quad (10)$$

Exponent β is related to b (equation (4)) by

$$\beta = 2 + 3/b \tag{11}$$

A complete formulation for the two-way interaction flux between saturated and unsaturated stores can be obtained by taking the net effect of gravity drainage q_{grav} (3) and capillary rise q_{cap} (4):

$$q_{net} = q_{grav} + q_{cap} \tag{12}$$

which is the approach that has been used by, e.g., *Famiglietti and Wood* [1994] within the TOPLATS land surface scheme.

3. Darcy Approach

[21] The validity of equation (12)
is checked by comparing the predicted fluxes with the fluxes that are associated with quasi steady state moisture profiles, constrained by an upper boundary condition of specified root zone moisture s_r (or equivalently, root zone pressure head ψ_r) and specified thickness of the transition zone Z. Apart of this procedure, we replace Z_g in equation (9) by Z. The original meaning of equation (9) is to predict the flux between two boundaries where pressure is defined, originally the phreatic surface and the soil surface, and in our case the phreatic surface and the bottom of the root zone.

[22] We do not consider a more dynamic approach (i.e., transient moisture profiles) because that would increase the number of degrees of freedom beyond what could be justified given the boundary conditions.

[23] The Darcy-Buckingham equation (1) is used to compute the steady state moisture profile. Using $k = k(\psi)$, this can be rewritten as

$$\frac{\partial \psi}{\partial z} = -\frac{q}{k(\psi)} - 1 \tag{13}$$

which can be used to compute the steady state moisture profile $s(z)$ for a given flux q by numerical integration over the domain from $z = -Z_r - Z$ to $z = -Z_r$ and converting the resulting $\psi(z)$ profile to an equivalent $s(z)$ profile. To this end, the *Campbell* [1974] water retention characteristic (4) and k parameterization (5) are used.

[24] Finding that value for q that results in the required s_r is achieved by means of a simple golden section search [*Press et al.*, 2002]. This approximation of $q(s_r,Z)$ will be labeled q_{darcy} hereafter.

[25] The above routine can be applied to numerically approximate q_{darcy}, which is interpreted here as the 'true' value for the root zone–groundwater interaction flux. In the next few sections, we both test the validity of q_{net} against q_{darcy}, and provide simple parameterizations of the q_{darcy} approximations (that is, removing the need for the inefficient iterative numerical routine outlined above).

[26] This procedure is broken up into two steps. First, for a given value of Z, $q(s_r)$ approximations are evaluated and new parameterizations are developed. In the second step Z variability is taken into account.

[27] Alternatively, one may precompute q_{darcy} for many combinations of Z and s_r and build a lookup table with them. This approach is not being discussed here.

4. Results

4.1. Step 1: Determination of $q(s_r)$

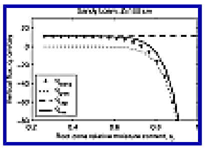

Figure 2. Vertical flux q as a function of relative root zone moisture content s_r for a sandy loam soil with a transition zone thickness of $Z = 100$ cm as computed by four different methods: q_{darcy}, using the Darcy equation; q_{grav}, gravity drainage from the root zone; q_{cap}, capillary rise flux according to the Gardner-Eagleson approximation; q_{net}, the net effect of q_{grav} and q_{cap}.

[28] For a given transition zone thickness Z, the dependence on the net flux q on root zone moisture s_r can be computed using either equation (12) or using the methodology outlined above in section 3. An example of this relationship for the sandy loam soil of *Clapp and Hornberger* [1978] (see Table 1 for the soil hydraulic parameters used) with a transition zone thickness of $Z = 100$ cm is given in Figure 2.

[29] Several phenomena are apparent in Figure 2 with respect to the numerical solutions q_{darcy} compared to the analytical solution q_{net}. First, the analytically derived fluxes q_{grav} and q_{cap} act as limiting cases for q_{net}. q_{net} approaches q_{grav} if the root zone moisture content approaches saturation ($s_r \rightarrow 1$). This is the result of the small vertical pressure gradients under saturated conditions. For dry root zone conditions (low s_r) q_{net} approaches q_{cap}. The analytically derived flux q_{net} is a good approximation of the numerically obtained flux q_{darcy}, for these limiting cases.

[30] Second, the numerical solution q_{darcy} deviates significantly from q_{net} for intermediate values of q_{net}. The interpretation for this behavior is that the assumptions underlying the q_{cap} model (esp. the assumption of a dry soil surface) are not met.

[31] Note that the maximum rate of capillary rise in Figure 2 is high, with a magnitude of $\approx$10 cm/d. This is an artifact of the static model setup, where the combination of a relatively dry root zone, combined with a shallow groundwater level produce a very steep hydraulic gradient, resulting in a large flux. In a more realistic, dynamic, setting, the water balance within the root zone will adapt to the boundary conditions, such that this combination will not occur, and the capillary rise flux will be constrained by evaporation (see also Figure 10, below).

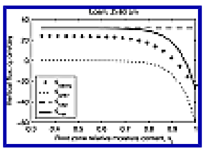

Figure 3. Vertical flux q as a function of relative root zone moisture content s_r for a loam soil with a transition zone thickness of $Z = 80$ cm.

[32] A second example is shown in Figure 3, where the same procedure is applied to the loam soil of *Clapp and Hornberger* [1978], for a transition zone thickness of $Z = 80$ cm. Here it can be seen that the analytically derived flux q_{net} no longer is a good approximation of the numerically obtained flux q_{darcy} for low values of s_r. The asymptote of q_{darcy} is significantly lower than the asymptote of q_{net}. The explanation for this behavior is that for very shallow groundwater tables, i.e., low transition zone thickness Z, the capillary fringe is relatively thick. The impact of the capillary fringe on the shape of the pressure and relative moisture content profiles becomes significant, and this affects the corresponding fluxes, resulting in a lower capillary rise flux.

4.2. Step 2: Parameterizing $q(s_r)$

[33] As expressed in the introduction, the aim of this paper is not only to test the validity of the Gardner-Eagleson approximation with respect to the Darcy equation, but also to construct a parameterization of the steady state Darcy solution obtained flux rates. This requires two additional steps: (1) to generate a parameterization of the Darcy equation obtained flux rates for constant Z, as shown in Figures 2 and 3, and (2) to include variable Z in such a parameterization.

[34] A pragmatic way to numerically approximate the predictions of Darcy equation fluxes q_{darcy} with respect to q_{grav}, q_{cap}, and q_{net} is to subtract q_{grav} from q_{darcy}, and normalize by q_{cap}. This results in a range 0…1, with 1 indicating that q_{darcy} is well approximated by q_{cap}, which should hold for low s_r, and 0 indicating that q_{darcy} is well approximated by q_{grav}, which should hold for high s_r.

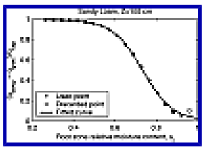

Figure 4. Transformed and normalized vertical flux rates as a function of root zone moisture s_r for a sandy loam soil with a transition zone thickness of Z = 100 cm.

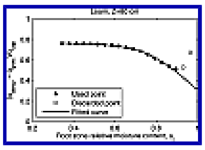

Figure 5. As in Figure 4 but for a loam soil with Z = 80 cm.

[35] The results of this approach are shown in Figures 4 and 5.

[36] The pattern of transformed and normalized flux rates, plotted against relative root zone moisture s_r strongly suggest a sigmoid-type relationship

$$y = \frac{\sigma_\alpha}{1 + \exp(\sigma_\beta(s_r - \sigma_\gamma))} \qquad (14)$$

where σ_α is the maximum value ($\sigma_\alpha = 1$ if the effect of a capillary fringe is small, $0 < \sigma_\alpha < 1$ otherwise), σ_β is a scale parameter, and σ_γ is a shift parameter. The result of equation (14) fitted to the transformed and normalized flux rates is also shown in Figures 4 and 5.

[37] Note that not all numerically obtained q_{darcy} points are used for fitting. When s_r approaches 1, the capillary fringe effects can cause q_{darcy} to increase with respect to q_{grav}. These points are detected and excluded from the fitting procedure.

4.3. Including the Dependence on *Z*

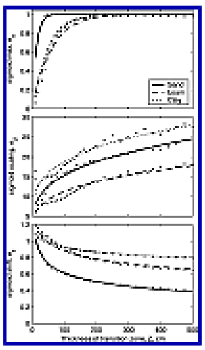

Figure 6. Fitted sigmoid parameters σ_α (max), σ_β (scale), and σ_γ (shift) as a function of transition zone thickness Z. Dots are numerical results, and lines are analytical approximations.

[38] The $q - s_r$ relationship is thus defined by combining the analytical predictors q_{grav} and q_{cap} with the three parameters σ_α, σ_β, σ_γ that define a sigmoid, for a given soil type, and a single value for Z. A practical way of bringing Z into the parameterization is to find out how, for a given soil type, σ_α, σ_β, σ_γ depend on Z. This is achieved by varying Z in steps of 25 cm within the range 25–500 cm and fitting a sigmoid for each of these steps. The resulting dependence of σ_α, σ_β, σ_γ on Z is shown in Figure 6.

[39] The dependence of the sigmoid parameters on z displays a relatively simple relationship. A satisfactory approximation is given by the analytical functions

$$\sigma_\alpha = 1 - \exp(-k_1 Z) \quad (15)$$

$$\sigma_\beta = k_2 Z^{k_3} \quad (16)$$

$$\sigma_\gamma = k_4 \exp\left(-Z^{k_5}\right) \quad (17)$$

Sigmoid metaparameters $k_1 \ldots k_5$ have been obtained for all 11 soils described by *Clapp and Hornberger* [1978], and are listed in Table 2.

5. Evaluation

[40] In this section, we compare the vertical fluxes as predicted by our proposed parameterization with the Gardner-Eagleson approximation, both in a static (i.e., constant boundary conditions) and dynamic (i.e., time-variable boundary conditions) framework. We also compare with the quasi-linear approximation.

5.1. Static Evaluation

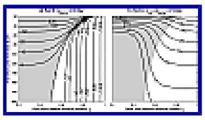

Figure 7. (a) Root zone–groundwater interaction flux q_{darcy} as function of transition zone thickness Z and root zone relative saturation s_r for a sand soil. Gray area indicates upward flow. (b) Difference between q_{net} and q_{darcy}.

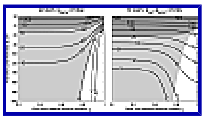

Figure 8. As in Figure 7 but for a loam soil. The dark gray area indicates where $q > k_s$ for either $q = q_{\text{darcy}}$ or $q = q_{\text{net}}$.

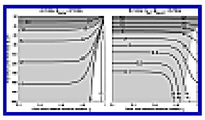

Figure 9. As in Figure 8 but for a clay soil.

[41] Figures 7, 8, and 9 show the differences in predicted fluxes according to our q_{darcy} parameterization, and the Gardner-Eagleson approximation q_{net}. Because predicted fluxes have both negative and positive values, it is not possible to express the flux difference as a relative fraction of, e.g., q_{darcy}. Therefore, both the reference flux q_{darcy}, and the flux difference $q_{\text{net}} - q_{\text{darcy}}$ are presented, for three soils (sand, loam, clay) from the *Clapp and Hornberger* [1978] database. It should be noted that for the loam and clay soils $q > k_s$
for shallow groundwater tables. This is the effect of the unrealistic high-pressure gradient associated with this forcing.

[42] It can be seen in Figures 7–9 that $q_{\text{net}} - q_{\text{darcy}} > 0$ for all soils and boundary conditions, thus q_{net} never under predicts with respect to q_{darcy}. It can be further seen that the difference $q_{\text{net}} - q_{\text{darcy}}$ is largest for high s_r and for small Z. Both these observations are in line with the findings presented in Figure 3 and discussed in section 4.1. Note that the difference $q_{\text{net}} - q_{\text{darcy}}$ can be of the same order as the reference flux q_{darcy} for shallow groundwater.

5.2. Dynamic Evaluation

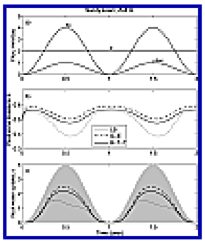

Figure 10. Temporal evolution of a simplified one-dimensional root zone model with groundwater depth fixed at $Z = 5$ m. See main text for model setup. (a) Climate forcing, defined by time series of potential evapotranspiration (Ep), precipitation (P), and vegetation phenology (phen). (b) Evolution of relative soil moisture, as modeled by a leaking bucket (LB) approach without capillary rise, and the Gardner-Eagleson (G-B) or our proposed Bogaart-Teuling-Troch (B-T-T) parameterizations. (c) Evolution of the root water uptake (i.e., transpiration) flux for the three models. The gray background represents the potential evapotranspiration flux.

[43] The dynamic effects of using the proposed parameterization instead of the Gardner-Eagleson flux depends on many factors, but will be largest when the landscape system studied will be wet for a significant amount of time. Figure 10 shows the temporal evolution of hydrological states and fluxes for a highly conceptualized setting, representing a humid climate in which precipitation is constant throughout the year, while potential evapotranspiration and vegetation phenology fluctuate in a seasonal fashion. Annual average potential evapotranspiration rate equals average precipitation rate. Root zone soil water balance is modeled with the lumped model presented by *Teuling and Troch* [2005], while capillary rise is either not modeled (representing the classic "leaky bucket" model approach), or modeled with the Gardner-Eagleson approach, or the parameterization presented here. Phreatic groundwater level is kept at a fixed depth, assuming that capillary rise is balanced by lateral supply. This assumption may describe the case where groundwater within a riparian area or floodplain is fed by lateral supply from either hillslopes bordering the riparian area, or from a loosing stream within the floodplain. Soil parameters are: sandy loam soil, thickness of transition zone $Z = 5$ m. Additional model parameters for the *Teuling and Troch* [2005] model are: max. Leaf area index 4.0, root zone thickness $L = 0.5$ m, root fraction within L $f_r = 0.9$, light efficiency $c = 0.5$.

[44] It can be seen from Figure 10 that including the capillary rise process has, in this case, a strong effect of moisture fluctuations and transpiration. The dynamic range in root zone water content is halved, because water supply by capillary rise mitigates the drying out during the summer season. Summertime transpiration is increased by 30–50% because of the higher available moisture within the root zone. Note, though, that in this simplified simulation the groundwater level is fixed, while in many real applications depletion of the groundwater reservoir will occur. For the forcing and model parameters used, the difference in relative moisture content between the Gardner-Eagleson parameterization and ours is small, with an absolute value of ≈0.025. However, this is ≈25% of the total dynamic range in relative soil moisture of ≈0.1. A more complete assessment of the differences between the Gardner-Eagleson parameterization and ours, for a more realistic setting, is beyond the scope of the current paper.

5.3. Comparison With the Quasi-linear Approximation

[45] As outlined above, in section 1, analytical solutions to the Richards' equation can be derived provided the unsaturated hydraulic conductivity–soil matric potential relation is expressed as an exponential function, i.e.,

$$k(\psi) = k_s e^{\alpha\psi} \tag{18}$$

After applying the Kirchhoff transformation

$$\phi = \int_{-\infty}^{\psi} k(\psi) d\psi = k/\alpha \tag{19}$$

where ϕ is known as the matric flux potential, the nonlinear Richards' equation becomes a linear differential equation, that can be solved to yield the steady state solution for root zone matric flux potential ϕ_r

$$\phi_r = \frac{q}{\alpha} e^{\alpha Z} - \frac{q}{\alpha} + \phi_0 e^{\alpha Z} \qquad (20)$$

where $\phi_0 = k_s/\alpha$ is the saturated matric flux potential [*Brandyk and Romanowicz*, 1989]. Solving for flux q yields

$$q = \frac{\alpha \phi_r - e^{\alpha Z} k_s/\alpha}{e^{\alpha Z} - 1} \qquad (21)$$

[46] In order to apply equation (21), using soil hydraulic data that are available as parameters for the Campbell conductivity model, one has to convert c in equation (5) to parameter α of the exponential model (18). Although values for α can be found that result in the same vertical fluxes as predicted by our proposed parameterization, or the Gardner-Eagleson parameterization within its range of validity, these value for α differ significantly from those that result from fitting the exponential conductivity model (18) to the Campbell model (5). Apart from that, the sensitivity of q to α is large. Further research to the applicability of the quasi-linear approximation in case soil hydraulic data is available for k parameterizations other than the exponential one is required.

6. Discussion and Conclusions

[47] We have tested the classic Gardner-Eagleson analytical approximation of the root zone–groundwater interaction flux, which is computed as the net effect of a downward gravity drainage flux, and an upward capillary rise flux, against numerical solutions using the Darcy equation, and assuming steady state within the transition zone. We find that errors in the Gardner-Eagleson flux q_{net} can be significant with respect to the Darcy flux q_{darcy}, especially for high root zone moisture content and/or a shallow groundwater level.

[48] We have developed a simple approximation of q_{darcy}, consisting of a few analytical functions which parameterize the effects of groundwater depth and root zone moisture. We suggest that our new approximation should be selected above the classic Gardner-Eagleson flux for applications where the assumptions underlying the classic approximation are not met. This is most likely the case in landscapes where convergent topographies, riparian areas or wetlands are abundant. For small-scale applications the spatial extent of these areas is highly variable, and depends on the local climatology, lithology and geomorphic setting.

[49] A rough estimate of the large-scale extent where this conditions are met can be made by analyzing global maps of soil moisture. Results of this paper indicate that our proposed parameterization becomes relevant at least for relative root zone moisture of $s > 0.6$. When considering the 0.25° resolution daily surface soil moisture products of *Owe et al.* [2008], we find, after computing monthly averages to eliminate high-frequency variability, that on average ≈10% of the land surface has a relative root zone moisture of $s = 0.6$ or higher.

[50] It should be noted that current generation of land surface schemes increasingly take capillary rise into account, using Richards' equation solved for 10 or more layers [*Yeh and Eltahir*, 2005; *Miguez-Macho et al.*, 2007; *Niu et al.*, 2007], or a simplified yet physically based approach, e.g., variable infiltration capacity–ground [*Liang et al.*, 2003]. However, these schemes are still much more numerically demanding than the proposed parameterization, making them less suitable for use in a high spatial resolution or Monte Carlo setting, especially when desktop rather than mainframe equipment is used. It it this type of applications where the proposed parameterization would be in place, provided that the underlying assumptions are met, and the flexibility that is inherent to the Richards' equation is not required.

[51] One assumption is that the vertical movement of water can adequately be described within a

Darcy framework, i.e., where the underlying soil physics, as expressed in the soil water retention characteristics and the conductivity model hold. That is, preferential flow, soil heterogeneities etc. either have a limited effect, or these effects can be cast into the deviation of pedon-scale “effective” hydraulic properties from sample-scale derived properties. Analysis of water particle travel time distributions on the hillslope scale suggest that lateral hydrologic fluxes on this scale can still be adequately described from a Darcian perspective [*McGuire et al.*, 2005]

[52] One other assumption is that, when combining the Gardner-Eagleson capillary rise model with a lumped root zone model [e.g., *Famiglietti and Wood*, 1994; *Teuling and Troch*, 2005] it is more important to adjust the capillary rise model to the root zone model than vice versa. For example, in some cases (e.g., coarse sands with a low organic matter content) the assumption that the soil surface will be dry during interstorm periods is more realistic than the assumption of a uniform soil moisture content within the root zone. Consequently the proposed methodology would be less logical for those cases.

[53] A further assumption is that the quasi steady state approach is a realistic approximation of a transient process. Recently, the plausibility of this assumption was tested by comparing a quasi steady state model (using a lookup table approach, rather than an analytical approximation as we do) with a fully dynamic Richards' equation–based model, for the 21 most characteristic soil types of the Netherlands, using 2 years of observed meteorological forcing. For this purpose, the SWAP land surface model was employed [*van Dam et al.*, 2008]. It was concluded that the quasi steady state approach yields satisfactory (Nash-Sutcliffe >0.9) results for ≈75% of the total area of the Netherlands, provided that the root zone is thin (≈33 cm) and groundwater is shallow (within 2 m of the soil surface) [*van Walsum and Groenendijk*, 2008].

[54] Finally, it has been noted that the quasilinear approximation to the Richards' equation is an alternative and potentially interesting approach to surface–groundwater fluxes, but more research here is required, especially because of the specific (exponential) conductivity model required by that approach.

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