

Generalizing Backdoors

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Constraint Satisfaction

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 - Pseudo-Backdoors
 - Heuristic-Backdoors
 - A complex optimization problem
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- 4 Conclusions**
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 - Heuristic-Backdoors
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Sample CSP

- $V = \{x, y\}$
- $D(x) = \{1, 3, 4, 5\}$ $D(y) = \{4, 5, 8\}$
- $C = \{x + 3 = y\}$

A possible solution for the CSP is $x = 1$ and $y = 4$.

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- In Williams et al. [9] discuss a **formal framework** inspired by these techniques.

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- In Williams et al. [9] discuss a **formal framework** inspired by these techniques.
- One of the main contributions in this work is the notion of “**Backdoor**” variables.

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Backdoor

Backdoor Set: a set of variables for which there is a value assignment such that the simplified problem can be solved by a **poly-time algorithm** called the “sub-solver”

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Strong Backdoor

Strong Backdoor Set: a set of variables for which **any assignment** leads to a poly-time solvable subproblem

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- can determine if C is trivially true (has no constraints) or trivially false (has a contradictory constraint)
- if A determines C , then for any variable x and value v , then A determines the **simplified CSP** where x is assigned to v

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$$V \equiv \{X_1, X_2, \dots, X_m, N\},$$

$$D \equiv \{X_1, X_2, \dots, X_m, N \in \{1, \dots, m\}\},$$

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Propagating the `NValue` constraint is NP-hard (Bessiere et al. [2]) and thus its propagator, which we shall call P , **does not achieve** hyper-arc consistency since this would be computationally too expensive

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Nevertheless it is clear that in the given CSP, once constraint $N = m$ is propagated, constraint $\text{NValue}([X_1, X_2, \dots, X_m], N)$ becomes equivalent to $\text{allDiff}([X_1, X_2, \dots, X_m])$

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Let A be the **poly time** algorithm that achieves hyper-arc consistency for `allDiff`, then $N \rightarrow m$ is a Backdoor with respect to A

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In this regard an interesting discussion is carried on in Bessiere et al. [1], where the **parameterized complexity** of global constraints is discussed.

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A given sub-solver A must **run in polynomial time** and must **reject** (in polynomial time) the input if it is not able to either conclude satisfiability or unsatisfiability.

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Backdoor Condition

Given a CSP, C , a *Backdoor Condition* with respect to a sub-solver A is a (global) constraint P on the subset $S \subseteq V$ of the decision variables in C that are currently instantiated, such that if the partial assignment $a_S : S \subseteq V \rightarrow D$ satisfies P , then a_S is a Backdoor in C for A . Determining if a_S satisfies P must be performed in polynomial time.

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- Having an efficient (polynomial) algorithm for handling a subproblem that arises when some of the decision variables are fixed is indeed **desirable**
- Nevertheless, often it may be the case that, after some decision variables have been fixed, the remaining subproblem is still NP-hard, but it has some **additional structure** that the original problem does not have
- If this is the case, it is possible that specialized algorithms, such as dedicated propagators or heuristic procedures, may be able to **exploit this additional structure** in order to either achieve a stronger filtering or quickly produce promising or optimal assignments for all or some of the remaining decision variables

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- Nevertheless the sub-solver should still be able to **reject the input in polynomial time** if satisfiability or unsatisfiability cannot be inferred
- The **key idea** then is that, although a given sub-solver is not guaranteed to produce a solution in polynomial time, it should be able to **produce competitive run times in practice**.

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A nonempty subset S of the variables is a Pseudo-Backdoor in C for \hat{A} if for some $a_S : S \rightarrow D$, \hat{A} returns a satisfying assignment of $C[a_S]$ or concludes unsatisfiability of $C[a_S]$.

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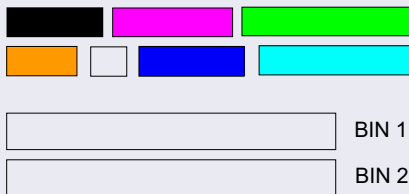
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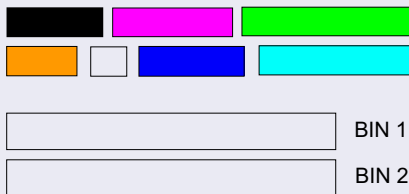
An example: Multiple Knapsack



We consider a **multiple knapsack problem** with two bins into which objects can be fitted. A set of objects is given, for each object a **profit** and a **weight** are also given. Each bin is assigned a certain **capacity**. We want to fit as many objects as possible in the bins in such a way to **maximize profit** and to not exceed the capacity available for each bin.

Pseudo-Backdoors

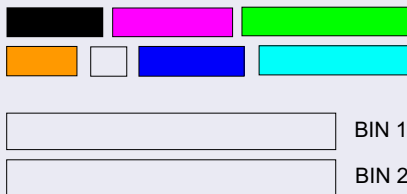
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A simple observation directly leads to an effective **Pseudo-Backdoor Condition**. As soon as *the objects fitted in one of the two containers occupy enough capacity so that none of the remaining objects can be fitted in it*, the remaining problem is then to fit the unassigned objects to a “virtual bin” having a capacity equal to the residual capacity of the other bin.

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Once a given partial assignment a_S satisfies the Pseudo-Backdoor Condition described, the remaining problem is obviously a **simple 0-1 Knapsack**.

Pseudo-Backdoors

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BIN 1



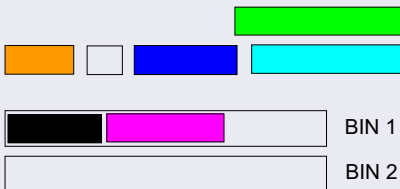
BIN 2

Tree Search



Pseudo-Backdoors

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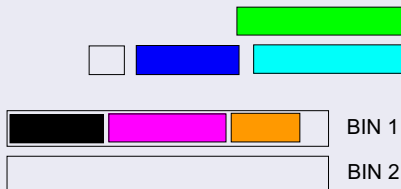


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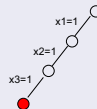


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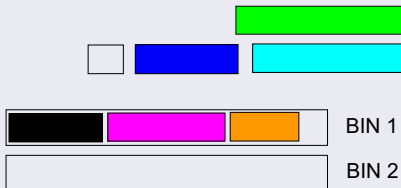


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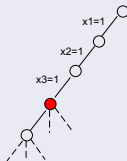


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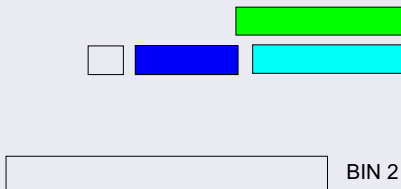


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Items	KP-DFS	KP-DFS-DP
10	0.02	0.03
15	0.45	0.04
20	14	0.100
25	210	0.270

Table: Multiple Knapsack Problem. Comparison between the run times (in seconds) of a pure depth-first search strategy (KP-DFS) and of the hybrid depth-first/dynamic programming search strategy based on the Pseudo-Backdoor discussed (KP-DFS-DP).

Heuristic-Backdoors

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In **CSPs** the former observation leads to the following approach:

- A solution method in which the sub-solver is used for **heuristically produce a feasible assignment** for some or all the remaining decision variables.

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In **COPs** the former observation can lead to two different approaches:

- A complete solution method in which the heuristic sub-solver is used to **generate a near-optimal solution** that provides a **good bound** during the search. This approach is typically used in branch and bound algorithms (Lawler and Wood [7]).
- A heuristic solution method in which the heuristic sub-solver is used for **assigning “promising” values** to some or all the remaining decision variables.

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In order to clarify, “may induce” means that the sub-solver will actually **induce an assignment** if the heuristic strategy employed is able to produce such an assignment **within the given time/runs limit**, otherwise the sub-solver will simply **reject the input**.

Heuristic-Backdoors

Heuristic-Backdoor

A nonempty subset S of the variables is a Heuristic-Backdoor in C for \tilde{A} if for some $a_S : S \rightarrow D$, \tilde{A} may return a feasible assignment for $C[a_S]$.

Heuristic-Backdoors

Strong Heuristic-Backdoor

A nonempty subset S of the variables is a Strong Heuristic-Backdoor in C for \tilde{A} if for all $a_S : S \rightarrow D$, A may return a feasible assignment for $C[a_S]$.

Heuristic-Backdoors

Heuristic-Backdoor Condition

Given a CSP, C , a *Heuristic-Backdoor Condition* with respect to a heuristic sub-solver \tilde{A} is a (global) constraint P on the subset $S \subseteq V$ of the decision variables in C that are currently instantiated, such that if the partial assignment $a_S : S \subseteq V \rightarrow D$ satisfies P , then a_S is a Heuristic-Backdoor in C for \tilde{A} . Determining if a_S satisfies P must be performed in polynomial time.

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- In what follows we will show that using this novel concept it is possible to develop **effective heuristic approaches** to complex combinatorial optimization problems by employing very simple heuristic strategies, such as Hill Climbing procedures.

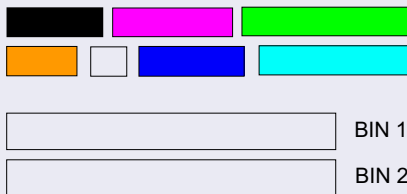
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- (Strong) Heuristic-Backdoors are particularly suitable for developing **structured ways of heuristically solving complex problems**.
- In what follows we will show that using this novel concept it is possible to develop **effective heuristic approaches** to complex combinatorial optimization problems by employing very simple heuristic strategies, such as Hill Climbing procedures.
- The main reason for this is that, by using tree search, the original problem is **split into much smaller problems**. On these smaller problems simple heuristic rules such as iterative improvement often produce high quality assignments in almost no time.

Heuristic-Backdoors

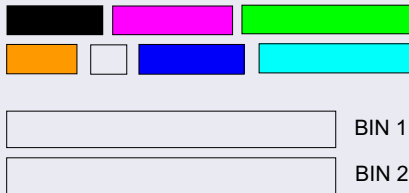
An example: Multiple Knapsack



Let \tilde{A} be a simple **Greedy Algorithm** for solving 0-1 Knapsack problems. In this algorithm objects are **ordered by decreasing profit over weight**. Once ordered, objects are scanned sequentially and put into the knapsack if the residual capacity allows the insertion. This can be seen as a simple **Hill Climbing strategy** in which at each step we perform an “improving” move (insertion of an object in the bin) until a local maximum is achieved (no more objects can be fit in the bin).

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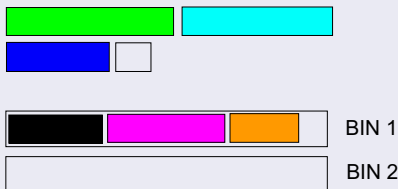
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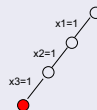
In the former example the Pseudo-Backdoor Condition described incidentally is also a Heuristic-Backdoor Condition with respect to this Greedy algorithm \tilde{A} . Thus as soon as this condition is met by a given partial assignment a_S the remaining subproblem can be solved in a heuristic way by using \tilde{A} .

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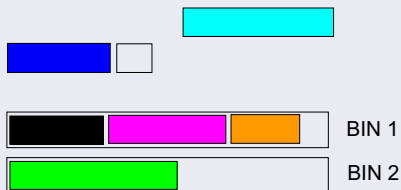


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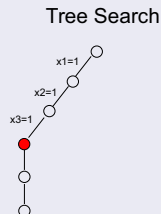
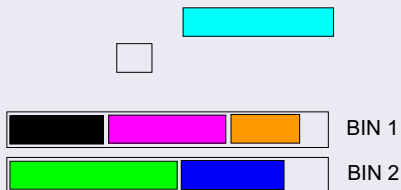
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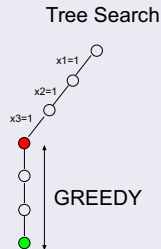
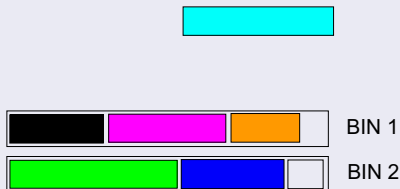
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20	14	0.100	0.01	100
25	210	0.270	0.02	99.2

Table: Multiple Knapsack Problem. Comparison between the run times (in seconds) of a pure depth-first search strategy (KP-DFS), of the hybrid depth-first/dynamic programming search strategy based on the Pseudo-Backdoor discussed (KP-DFS-DP), and of the hybrid depth-first/local search strategy based on the Heuristic-Backdoor discussed (KP-DFS-LS). % of real optimum denotes the fraction (in percentage) of the optimum profit achieved by the heuristic approach.

Inventory Control

An example

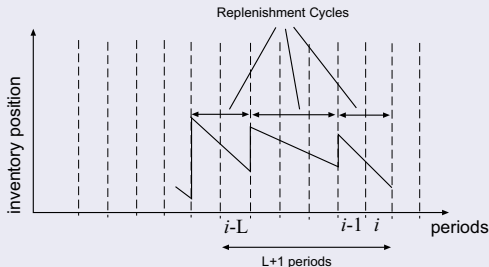


Figure: Replenishment Cycles corresponding to the following partial assignment for replenishment decisions: $\delta_{i-L-1} = 1$, $\delta_{i-L} = 0$, $\delta_{i-L+1} = 1$, $\delta_{i-L+2} = 0$, $\delta_{i-L+3} = 0$, $\delta_{i-1} = 1$, $\delta_i = 0$. Since at least L periods before period i are covered by this set of consecutive cycles it is possible to determine the service level at period i .

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- Nevertheless, to the best of our knowledge, in the literature Backdoors **have not been used** so far for **switching the search strategy** either to a complete or incomplete different strategy **not necessarily polynomial** (such as Dynamic Programming).

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- Nevertheless, operations research techniques are typically employed for **generating valid relaxations** used for performing domain filtering and, with the exception of Bender's Decomposition in Cambazard et al. [3], they are **not employed as alternative search strategies** that can take over the control of the search process when a given condition is met.

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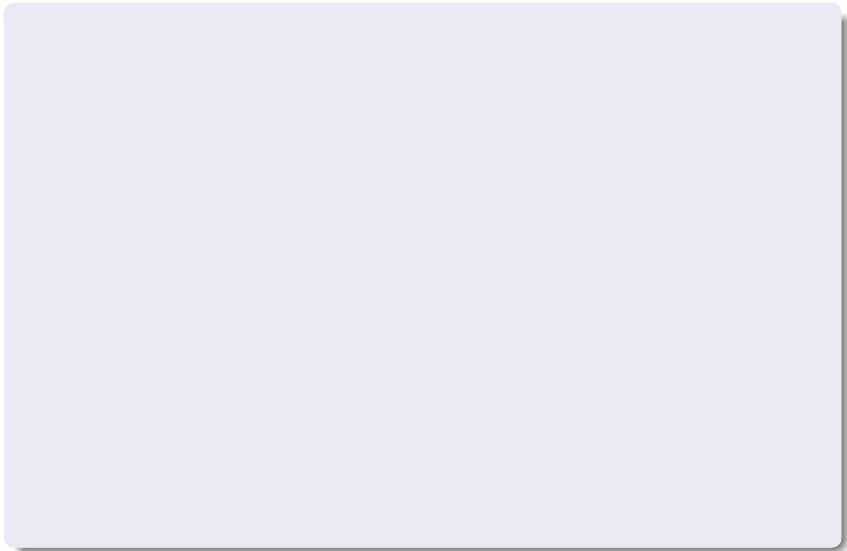
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The technique we propose is of this **second** kind, but the notion of Heuristic-Backdoor makes our approach novel and more general compared to other specialized approaches presented in the literature.

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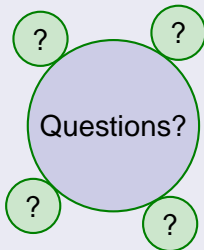
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- We applied both Pseudo-Backdoors and Heuristic-Backdoors to a simple **Multiple Knapsack Problem** taken as running example.
- We have also discussed the effectiveness of Heuristic-Backdoors on a complex combinatorial optimization problem.

The End

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