Methodologies for design, analysis and interpretation of fertilizer tests performed in the CATALIST program

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1. Introduction

Agronomists and farmers of CATALIST carry out various fertilizer tests in the Great Lakes countries. Most simple are the ‘tests participatifs’ consisting of one treatment only. Another group of testes deal with soil acidity and attempts to improve the acid soils. They consist of one to three treatments. Both, ‘tests participatifs’ and ‘test d’acidité’, offer some information on the best way to manage soil fertility. More conclusions can be drawn from the two other tests carried out: ‘tests comparatifs’ and ‘Essais soustractifs’. The ‘tests comparatifs’ usually consists of four treatments: control, N, NP, NPK; sometimes another treatment is added, for instance NPK + micro-elements. In 2009 some ‘minus one’ experiments (‘Essais soustractifs’) were carried out in Burundi and RDC. They contained five fertilizer treatments: control; -N (= PK); -P (= NK); -K (= NP); and NPK. These ‘essais soustractifs’ provide more and better information than the ‘tests comparatifs’.

In this document it is discussed what information can be derived from the various types of tests. The paper starts with the ‘essais soustractifs’ because they present the clearest picture. In this document it only is show how the effects and agronomic efficiencies can be calculated, for the separate nutrients N, P and K, as well as for all three nutrients together. Next the tests ‘comparatifs’ are considered. The simple ‘tests participatifs’ are difficult to interpret; it is tried to apply some recently developed concepts for the understanding of their results. The scientific background and justification is presented in another document, dealing with the interpretation of factorial experiments.

The ‘tests d’acidité’ deal with an other problem than the optimum nutrient application and require a different approach.

2. Interpretation of agronomic efficiency (EA) of nutrients

In Section 3 and 4 it is shown how the effects (= additional yields) of individual nutrients can be assessed. Once the effects of N, P_2O_5 and K_2O are known, the agronomic efficiencies, or in French l’efficacité agronomique (EAN, EAP_2O_5, EAK_2O) are calculated as the ratio of the additional yield (effect) to the applied amount of the nutrient (DN, DP_2O_5, DK_2O, where D stands for dose). So, we get:

EAN	=	N effect/DN	kg/kg
EAP_2O_5	=	P_2O_5 effect/DP_2O_5	kg/kg
EAK_2O	=	K_2O effect/DK_2O	kg/kg

Agronomic nutrient use efficiency is the multiplication of uptake efficiency and physiological efficiency. Uptake efficiency of applied (input) nutrients is synonymous to recovery fraction (REC), which is the portion of the applied nutrient that is taken up by the crop. Physiological efficiency (PhE) relates the yield (Y) of the economic plant components (*e.g.* grains, tubers) to uptake by the whole crop. So: $EA = REC \cdot PhE$.
For several crops the maximum values of PhE are known. The values of REC do vary considerably. In this paper, standard values of REC were used: 0.5 for fertilizer N and K, and 0.1 for fertilizer P. With these standard values of REC and the maximum values of PhE, as derived from literature (Van Keulen and Van Heemst, 1982; Boxman and Janssen, 1990; Janssen et al., 1990; Ojiem, 2006; Zingore, 2006, Witt et al., 1999)., maximum values of EA can be assessed.

The calculated EA of the individual nutrients can be interpreted in terms of response to fertilizer nutrient or in terms of nutrient availability in the soil. A high EA of a nutrient indicates that there is a strong response by the crop to the application of that nutrient. That happens when the soil supply of the nutrient is low, and REC and PhE of the applied nutrient are high. A low EA indicates that there is a weak response of the crop to the application of the nutrient. That may have several causes: (i) the soil supply of the nutrient is great so that there is no need to apply that nutrient, (ii) the crop can only take up a small portion of the applied nutrient (the recovery fraction of the applied nutrient is small), for instance because of leaching, (iii) PhE is low because other growth factor (water, sunshine) are limiting so that the crop cannot efficiently use the nutrient taken up. A more detailed explanation is given in Annex 1 on Response to applied nutrients.

Table 1 gives tentative maximum values of agronomic efficiency (EA_{max}) for several crops. It is explained in Annex 1 that values of EA smaller than 0.5 EA_{max} point to too high nutrient applications with regard to other growth factors such as water availability, genetic potential of the crop cultivar, or to insufficient crop management. In such cases it is recommended to apply smaller doses of nutrients or to improve crop management. The optimum value of EA depends also on the prices of nutrients and produce. In general optimum EA is about 0.55 times EA_{max}. So if the measured EA is more than 0.55 EA_{max}, it is advised to increase the amounts of nutrient inputs.

The crops distinguished in Table 1 are cereals crops, potatoes, and legumes. There are some differences among cereal crops, *e.g.* irrigated rice has higher EA_{max} values than maize and wheat. For potatoes the yield data refer to fresh weight which is about four times as high as dry weight. The values for potatoes may, for the time being, also be used

Table 1. Tentative values of maximum agronomic efficiencies (EA_{max}, in kg kg^{-1}) of N, P_2O_5 and K_2O for cereal crops, potatoes and legumes. EA_{max} for potatoes refers to fresh weight of tubers. It is assumed that tubers contain 25% dry matter.

Crops	$EA_{max}N$	$EA_{max}P_2O_5$	$EA_{max}K_2O$
Potatoes, tubers (fresh weight)	180	105	100
Cereals (maize, wheat), grains	35	21	36
Rice, irrigated	48	27	48
Legumes (beans, peas), seeds	15	13	21
Groundnuts (seeds)	13	22	50

for cassava. The EA values for legumes (beans, peas, groundnuts) refer to seed yields, and hence they cannot be applied to green beans (haricots verts); their values are really tentative and need further confirmation from experiments.

The differences in EA_{max} among the crops are caused by differences in nutrient needs by the crops. Root and tuber crops, and so potatoes need relatively more K than cereals and legumes, and therefore $EA_{max}K_2O$ is low compared to $EA_{max}N$ and $EA_{max}P_2O_5$. In the case of legumes, the values of $EA_{max}N$ and $EA_{max}P_2O_5$ are almost equal while for cereals and potatoes $EA_{max}N$ is almost twice as high as $EA_{max}P_2O_5$; it reflects the high N fraction of legumes.

3. The 'Essais soustractifs'

3.1. Fertilizer treatments

The five fertilizer treatments of the 'essais soustractifs' of the CATALIST program are: control, NP, NK, PK, and NPK. The treatments control, NP, NK, and PK are used for the calculation of the effects of the individual nutrients. They form half a replicate of a 2^3 factorial design. Treatment NPK is used for the calculation of the overall effect and of the NPK interaction.

The treatments may consist of applications of

Control	no fertilizers
NP	urée and DAP
NK	urée and KCl
PK	TSP and KCl
NPK	urée, DAP and KCl

In Rwanda, TSP is not available and hence it would be impossible to make the treatment PK. A compromise for PK might be a combination of DAP and KCl. The composition of DAP is 18-46-0. The treatment PK has then the meaning of low N + PK. The control

treatment should receive a same amount of N. So, if 100 kg DAP is applied for low N + PK, the control should receive 18 kg N, so 39 kg urée (urée contains 46% N).
The treatments may consist of applications of

"Control"	=	low N	urée
NP	=	high N + P	urée and DAP
NK	=	high N + K	urée and KCl
PK	=	low N + P + K	DAP and KCl
NPK	=	high N + P + K	urée, DAP and KCl

In the text below, we always use the terms control and PK. If TSP is not available, they refer to 'low N', and 'low N + P + K'.

3.2 Calculation of the effects of the individual nutrients

Two of the four treatments used for the calculation of the effects of the individual nutrients contain N, namely NP and NK, and two are without N, namely control and PK. For the calculation of the N effect (= additional yield by N), the yields of the treatments without N are subtracted from the yields of the treatments with N. Similarly, the treatments NP and PK are the two containing P, while control and NK are the two without P The treatments containing K are NK and PK and those without K are control and NP.
The effects of (the additional yields by) the individual nutrients are calculated as follows:

N effect = 0.5 · (NP + NK – PK – Control)

P_2O_5 effect = 0.5 · (NP + PK – NK – Control)

K_2O effect = 0.5 · (NK + PK – NP – Control)

The factor 0.5 in these equations is used because the expressions between the brackets represent twice the yield difference between the plus and the minus treatments (see Annex II on the interpretation of factorial experiments). So, all four treatments are used for the calculation of each of the effects of the individual nutrients. This very efficient use of the experimental data is a great advantage of factorial designs.

3.3. Calculation and interpretation of NPK effect and interaction

The NPK effect is calculated as the difference in yields of the NPK and the control treatment:

NPK effect = NPK – Control

If there are no interactions, the NPK effect is equal to the sum of the effects of N, P_2O_5 and K_2O. In other words, the yield of treatment NPK should equal the sum of (control yield + N effect + P_2O_5 effect + K_2O effect). In practice, differences between this sum and the NPK yield often are found. Causes of the differences may be variation in soil fertility in the field and (positive or negative) interactions. The difference between the NPK effect and the sum of (N effect + P_2O_5 effect + K_2O effect) is called the NPK interaction (Table 2).

If there is a substantial positive (negative) interaction, it means that the response to the three nutrients together is better (less) than the sum of the individual responses. If the value of the calculated interaction is not great, likely variation in the soil fertility of the plots with the different treatments has caused the difference between the NPK effect and the sum of the effects of the individual nutrients.

In principle, the agronomic efficiency of NPK could be calculated in a similar way as explained above for EAN, EAP_2O_5 and EAK_2O:

EANPK = NPK effect/D(NPK)

There is a problem, however, in assessing D(NPK), the dose of NPK. The assessment is difficult because we cannot simply add together the kilograms of DN, DP_2O_5 and DK_2O. The same problem is met in the 'tests participatifs'. The trouble is circumvented when N, P and K are expressed in so-called fertilizer crop nutrient equivalents, as explained in Section 5.

3.4. An example of the interpretation of an 'essai soustractif'

Table 2 presents data on yields of rice obtained in Burundi, season 2009B. Also the calculations of the effects and the NPK interaction are shown. The effect of P_2O_5 is great; the ratio of EA/EA_{max} is well above 0.55 indicating that the optimum application of P_2O_5 is higher than the actual application of 80 kg. The ratio EA/EA_{max} of K_2O is low pointing to a high K-availability in the soil, and the EA/EA_{max} of N is somewhat below 0.5 pointing to a somewhat too high application of N.

The NPK effect is hardly more than the sum of the individual N, P_2O_5 and K_2O effects. The NPK interaction is small (only 100 kg), compared to the other effects, and most likely must be ascribed to variability in soil fertility.

Table 2. Yields of rice obtained in an 'essai soustractif' in Burundi, the calculation of effects, calculation and interpretation in terms of response of EA of N, P_2O_5 and K_2O.

NPK	Yields (kg/ha) obtained with treatments				
application	Control	PK (-N)	NK (-P)	NP (-K)	NPK
120-80-70	5200	7300	8200	9500	10000

	Effects individual nutrients		*EA*	*EA/EA*
	Calculation	Value		
N	(9500 + 8200 – 7300 – 5200)/2	2600	2600/120 = 21.7	0.45
P_2O_5	(9500 + 7300 – 8200 – 5200)/2	1700	1700/80 = 21.3	0.79
K_2O.	(8200 + 7300 - 9500 – 5200)/2	400	400/70 = 5.7	0.12
	Effect and interaction of NPK			
Effect	10000 - 5200			= 4800
Interaction	4800 – 2600 – 1700 -400			= 100

4. The ‘tests comparatifs’

4.1. Fertilizer treatments

The four fertilizer treatments of the ‘tests comparatifs’ are: control, N, NP, and NPK. The reasoning behind this design is that the major limiting nutrient is nitrogen, that the need for phosphorus will show up only when N has been applied, and that the soil is so rich in potassium that a response to K can only be expected in case also N and P are applied. The disadvantage of a design with these treatments, however, is that the effects of the individual nutrients can be calculated only once, and not twice as in the ‘essais soustractifs’. That makes the ‘tests comparatifs’ far less efficient than the ‘essais soustractifs’.

A ‘test comparatif’ may consist of applications of the following fertilizers:

Control	no fertilizers
N	urée
NP	urée and DAP
NPK	urée, DAP and KCl

4.2 Calculation of the effects of the individual nutrients

Only one of the four treatments does not contain N, namely the control. The N effect is found as the difference in yields of the treatments N and control. The P effect is calculated as the difference between NP and N, and the K effect as the difference between NPK and NP:

N effect	=	N – Control
P_2O_5 effect	=	NP – N
K_2O effect	=	NPK – NP

So, only two treatments are used at a time for the calculation of the effect of an individual nutrient. This implies that one cannot make the most out of all the labor involved. Moreover, the methods of the calculation of the effects lead to an underestimation of the effect of N, and an overestimation of the effect of K. This is because the effect of N likely would be stronger when it was calculated as the difference between NPK and PK. Similarly the effect of P and K would be smaller if calculated as the differences (P – control) and (K – control).

One may argue that there are two treatments with P and two without P, and hence the P_2O_5 effect could be calculated in s similar way as for the 'essais soustractifs', so as: P_2O_5 effect = 0.5 x (NPK + NP – N – Control). Such a procedure, however, is statistically not correct. The reasons are: (i) both treatments with P also contain N, while of the treatments without P only one contains N; (ii) of the treatments with P one contains K and of the treatments without P none contains K. For more details Annex II on the interpretation of factorial experiments.

However debatable the calculation method of the effects of N, P_2O_5 and K_2O, the agronomic efficiencies (EAN, EAP_2O_5, EAK_2O) can again be found by dividing the effect by the amount of the nutrient applied (DN, DP_2O_5, DK_2O, where D stands for dose). So, we get:

EAN	=	N effect/DN	kg/kg
EAP_2O_5	=	P_2O_5 effect/DP_2O_5	kg/kg
EAK_2O	=	K_2O effect/DK_2O	kg/kg

Also here it holds that the procedure underestimates EAN, and overestimates EAK_2O. For the interpretation of EA, again Table 1 may be used.

4.3. An example of the interpretation of a 'test comparatif'

Table 3 presents data on yields of potatoes obtained in Nyabihu, Rwanda, season 2009A. The calculations of the effects, of EA and of EA/EA_{max} are shown as well. The EA/EA_{max} of N is a little below 0.5 pointing to somewhat too high rate of application. For P_2O_5 and K_2O the values of EA/EA_{max} are about equal and indicate that their rates of application should be a little higher than the actual rates of about 50 kg ha^{-1}.

Table 3. Yields of potatoes obtained in a 'test comparatif' in Rwanda, the calculation of effects, calculation and interpretation in terms of response of EA of N, P_2O_5 and K_2O.

NPK application	Yields obtained with treatments			
	Control	N	NP	NPK
98-51-48	12130	20850	24650	28100

Effects individual nutrients			*EA*	*EA/EA$_{max}$*
	Calculation	Value		
N	20850 - 12130	8720	8720/98 = 89	0.49
P_2O_5	24650 - 20850	3800	3800/51 = 75	0.71
K_2O	28100 - 24650	3450	3450/48 = 72	0.72

5. Fertilizer Crop Nutrient Equivalents (FCNE)

5.1. Definitions of CNE and FCNE

It is always difficult to compare the effects of different nutrients, because we cannot simply weigh one kg of N with one kg of P_2O_5 or one kg of K_2O. It is also meaningless to add together kilograms of N, P_2O_5 and K_2O. As a result it was not possible to calculate EA of NPK in Section 3.3.

These troubles are circumvented when N, P and K are expressed in units that have similar meanings in relation to crop growth and nutrient efficiency. Such units are the so-called crop nutrient equivalents (CNE). The definition is: a (k)CNE of a nutrient is the amount of the nutrient taken up by the crop that in a situation of balanced nutrition has a same effect on yield as the uptake of 1 (k)g of N has (Janssen, 2009; 2010) . The concept of crop nutrient equivalent (CNE) is a spin-off of the model QUEFTS (Janssen et al. 1990). Balanced plant nutrition implies that equal quantities of N, P and K are taken up when the quantities are expressed in CNE. Balanced nutrition is optimum from the physiological as well as from the environmental point of view (Janssen, 1998).

Table 4. Tentative values of the conversion factors (CF) for the translation of 1 kg fertilizer N, P_2O_5 and K_2O into fertilizer crop nutrient equivalents (FCNE) for the types of crops of Table 1.

Crops	1 kg is equivalent to CF kFCNE		
	N	P_2O_5	K_2O
Potatoes, tubers (fresh weight)	1.00	0.58	0.56
Cereals (maize, wheat), grains	1.00	0.60	1.03
Rice, irrigated	1.00	0.56	1.00
Legumes (beans, peas), seeds	1.00	0.87	1.40
Groundnuts (seeds)	1.00	1.69	3.85

In this document the concept of so-called fertilizer crop nutrient supply equivalents (FCNE) is applied. The definition of FCNE is: a (k)FCNE is that quantity of a fertilizer nutrient that has, under conditions of balanced supply of nutrients, the same effect on yield as one (k)g of fertilizer N has.
Fertilizer nutrients expressed in kilograms can be translated into kFCNE. For that purpose conversion factors are used. They can be derived from the values of EA_{max} in Table 1. The translation for cereals, for instance, goes as follows :

1 kg fertilizer N	= 35/35	=	1	kFCNE of fertilizer N
1 kg fertilizer P_2O_5	= 21/35	=	0.60	kFCNE of fertilizer P_2O_5
1 kg fertilizer K_2O	= 36/35	=	1.04	kFCNE of fertilizer K_2O

Table 4 presents the conversion factors for cereal crops, potatoes and legumes of which EA_{max} values are given in Table 1. Using these conversion factors NPK formulas can be expressed a a single value of kFCNE applied. Example: the formula 100-50-80 corresponds for cereals to:

$100 \cdot 1$ (for N) + $50 \cdot 0.60$ (for P_2O_5) + $80 \cdot 1.04$ (for K_2O) =
213 kFCNE of fertilizer NPK

5.2. *Calculation and interpretation of agronomic efficiency (EA) based on FCNE*

The agronomic efficiency of NPK based on FCNE can be calculated as:

EA(NPK, kFCNE) = NPK effect/D(NPK, kFCNE)

where D(NPK, kFCNE) stands for the sum of applied quantities of fertilizer N, P_2O_5 and K_2O expressed in kFCNE. The maximum agronomic efficiency, EA_{max}(NPK, kFCNE) has the same value as $EA_{max}N$, shown in Table 1, because all nutrients are expressed in units equivalent to 1 kg of fertilizer N.

The ratio of EA(NPK, kFCNE) to EA_{max}(NPK, kFCNE) facilitates the interpretation of the outcomes of the tests called 'formule générale' in a semi-quantitative way.

Three levels of the ratio may be distinguished. In Table 5 only general advises can be given. It is impossible to arrive at conclusions about the composition, so about the proportions of N, P_2O_5 and K_2O of the 'formule générale'. When EA/EA_{max} is valued high or optimum, the ratios of N : P_2O_5 : K_2O likely are appropriate. When the ratio EA/EA_{max} is low, it is well possible that the formula of NPK is not correct because it contains too much of a nutrient that is not really growth limiting.

Table 5. Semi-quantitative evaluation of the ratio EA(NPK, kFCNE)/EA_{max}(NPK, kFCNE) in tests of the 'formule générale'.

EA(NPK, kFCNE)/ EA_{max}(NPK, kFCNE)	Valuation of EA/EA_{max}	Fertilizer application rate is	Fertilizer application should be
> 0.6	high	too low	increased
0.5 – 0.6	optimum	optimum	maintained
< 0.5	low	too high	decreased

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Annex I. Response to applied nutrients

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1. Introduction

The purpose of applying nutrients to crops is to increase yields. The crop's response is the greater the greater the nutrient input is, but the relation between crop yield and nutrient input usually follows the law of diminishing returns. This means that the yield increase brought about by an unit of input is less at high input than at low input. Above a certain application rate the yield often does not increase further and stays at a plateau level. Field trials have been used to find out at what rate of nutrient application the maximum yield is reached, or more general what the relation is between crop yield and nutrient input, and what the optimum application rate would be. The trial designs must contain at least three application levels, by preference zero, below, and around the rate corresponding to maximum yield, for each of the nutrients N, P_2O_5 and K_20. That makes these trials complicated and expensive, and such trials are not present among the fertilizer tests of CATALIST. In the CATALIST's tests, only two application levels are found: zero and a level expected to be remunerative to the farmer. Nevertheless, the CATALIST's tests still can be used to find the optimum rate of nutrient application. In this annex it is tried to explain how that can be done. The pivot tool is the use of maximum agronomic nutrient use efficiencies. For some groups of crops these values are known and generally valid, depending only on crop type and genetic properties, provided crop and fertilizer management is good.

In the following sections, the relation between yield and nutrient input is discussed, a method to find economically optimum fertilizer rates is explained, and the consequences of this approach are shown for the recommendations on fertilizers in the CATALIST program.

2. Relations between yield and nutrient input

2.1. An example of the relations between yield and nutrient input

Table A I, 1 presents data on nutrient input and yield. The data are invented but they could be real. They may for instance refer to application of fertilizer N and yield of wheat. The yield response (ΔYield) is calculated as the difference between the yield and the control yield. Fertilizer costs are found as the product of fertilizer N rate and the price per kg N (900 RwF), gross return is the product of ΔYield and the price per kg of wheat

Table A I, 1. Yields, yield responses (ΔYield), fertilizer costs, gross and net return, Ratio of Value to Costs (RVC), agronomic efficiency (EA), and the ratio of EA to maximum agronomic efficiency (EA/EA_{max}), all in relation to increasing quantities of applied fertilizer N. Prices per kg are 900 RwF for N, and 200 RwF for wheat. Costs and returns in kRwF (1 kRwF = 1000 RwF). EA_{max} is 35 kg wheat grains per kg N applied.

N applied	Yield	ΔYield	Fertilizer costs	Gross return	Net return	RVC	EA	EA/EA_{max}
	kg ha^{-1}			kRwF ha^{-1}			kg kg^{-1}	
0	1200							
1	1235	35	0.9	7	6.1	7.8	35.0	1.00
5	1366	166	4.5	33.2	28.7	7.4	33.2	0.95
10	1515	315	9	63	54	7.0	31.5	0.90
20	1760	560	18	112	94	6.2	28	0.80
30	1935	735	27	147	120	5.4	24.5	0.70
40	2040	840	36	168	132	4.7	21	0.60
50	2075	875	45	175	130	3.9	17.5	0.50
70	2075	875	63	175	112	2.8	12.5	0.36
100	2075	875	90	175	85	1.9	8.75	0.25

(200 RwF), and net return is gross return minus fertilizer costs. RVC is the ratio of net return to fertilizer costs. Yields increase from 1200 to 2075 at an application rate of 50 kg N; at higher N rates there is no further increase in yield. The highest net return for the application rates shown in Table A I, 1 is found at 40 kg N per ha. RVC decreases with increasing application of fertilizer N and is less than 2 at a rate of 100 kg of N. On the basis of the data of Table A I, 1, the best advise to farmers would be to apply 40 kg of fertilizer N per ha.

Also shown in Table A I, 1 is the ratio of ∆yield to the quantity of the nutrient applied, so the yield increase per kg of N applied. This ratio is known as the agronomic nutrient use efficiency (AE) or in French l'efficacité agronomique (EA). The maximum value of EA is found at very low application rates; EA_{max} is 35 kg grain per kg N applied (see Table 1 in this paper).

Table A I, 1 is an example of a common relation between crop yield and nutrient input. It follows the law of diminishing returns, which means that the yield increase brought about per unit of input (EA) is less at high input than at low input. Above a certain application rate the yield reaches a plateau level and does not further increase.

2.2. Graphic and mathematic presentation of the relations between yield and nutrient input

In Figure A I, 1, where the yield data of Table A I, 1 have been plotted against the application rates of fertilizer N, a distinction is made between the two parts of the relation between yields and inputs, the part of increasing yields and the plateau. The part of increasing yields can be described by a parabolic expression (Fig. 2):

$$y= a + bx - cx^2 \qquad \text{Eq. 1}$$

where y stands for yield, x for the quantity of nutrient applied, and a, b and c are regression constants. The values of the regression constants in Figure A I, 2 are: a = 1200, b = 35, c = 0.35. When x = 50 kg ha^{-1}, y reaches its maximum of 2075 kg ha^{-1}. At

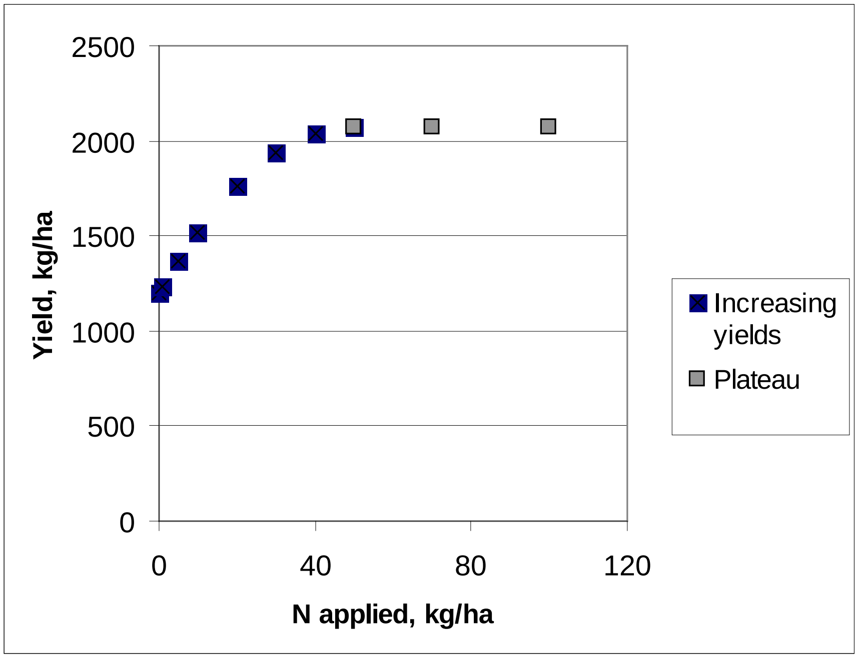

Figure A I, 1. Subdivision of the relation between yield and input of N into an ascending part and a plateau. Data from Table A I, 1.

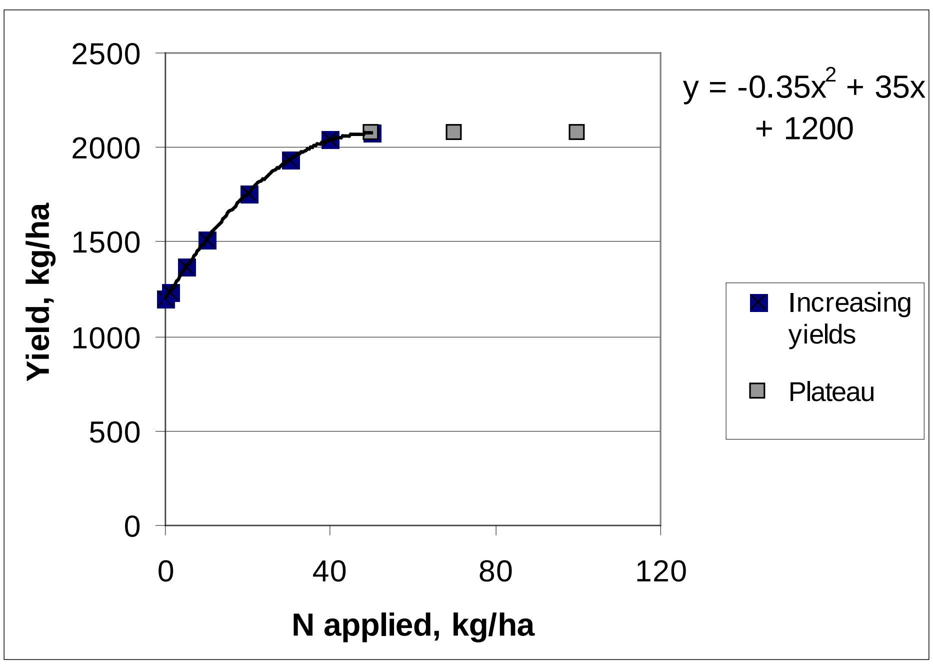

Figure A I, 2. Regression line for the ascending part of the relation between yield and input of N, till x = 50. Above x = 50, y = 2075. Data from Table A I, 1.

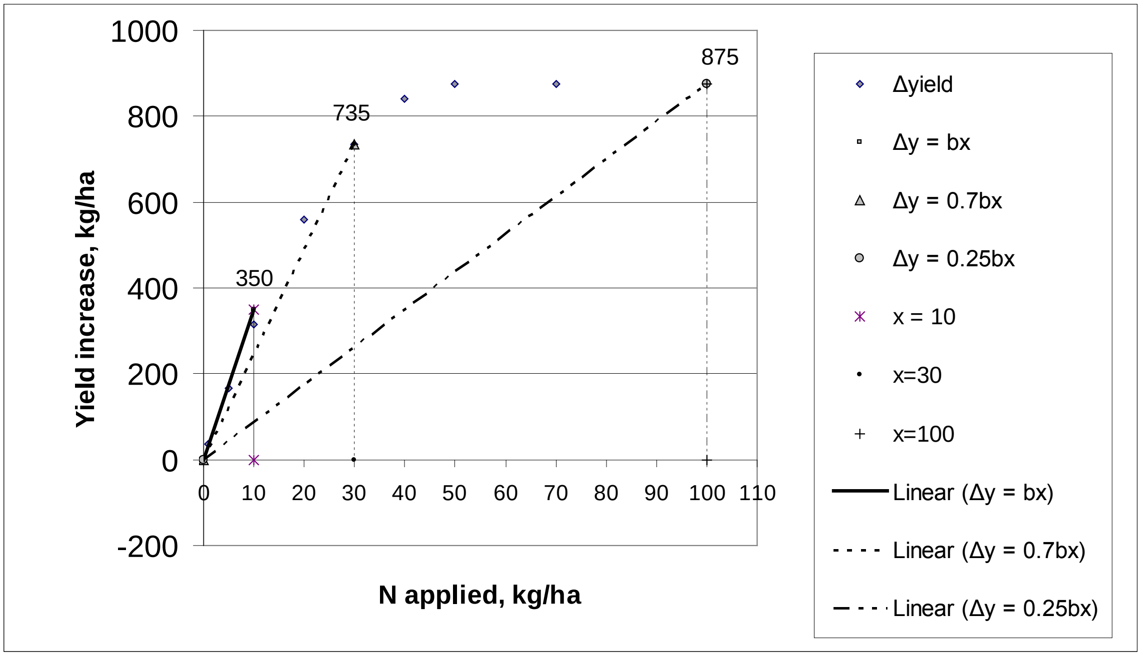

Figure A I, 3. Relation between yield increase (Δyield from Table A I, 1) and the application of N. The slopes of the lines Δy = bx, Δy = 0.7 bx, and Δy = 0.25 bx, represent agronomic nitrogen use efficiency (EA) for some data of Table A I, 1.

higher levels of x, y does not change but remains at 2075 kg ha^{-1}. When x is less than 50 kg ha^{-1}, the yield increase is (Δy) is $bx – cx^2$, so in the used example $35x – 0.35x^2$. The ratio of yield increase to quantity of the nutrient applied (EA) is $(35x – 0.35x^2)/x = 35 – 0.35x$. In general, it holds:

$EA_x = (bx – cx^2)/x = b – cx = \Delta y/x$ Eq. 2.

EA_x is the slope of the lines in Figure A I, 3. The line Δy = bx represents the maximum Δyield that can be obtained at a particular x. In practice, it is found at very low x, so at very low application rates of N. The other lines that are shown in Figure A I, 3 refer to Δyield obtained with N applications of 30 and 100 kg ha^{-1}. The slope of the line is calculated in Table A I, 2 as the tangent of the angle between the particular line and the X-axis. It is equal to EA in Table A I, 1. At x = 50, Δyield is 875, and hence $EA_{x=50}$ is

Table A I, 2. Calculation of the slopes of the lines $\Delta y = bx$, $\Delta y = 0.7\ bx$, and $\Delta y = 0.25\ bx$ of Figure A I, 3. Values of ΔYield at x = 30 and x = 100 are from Table A I, 1.

Line	EA	Calculation	Value
$\Delta y = bx$	EA_{max}	$\Delta y = 35 \cdot 10$	35
$\Delta y = 0.7bx$	$EA_{x=30}$	735/30	$24.5 = 0.7 \cdot 35$
$\Delta y = 0.25bx$	$EA_{x=100}$	875/100	$8.75 = 0.25 \cdot 35$

875/50 = 17.5, equal to half the value of EA_{max}. At $x > 50$, Δyield remains 875 and EA_x turns into values < 0.175 and EA_x/EA_{max} turns into values < 0.5. In other words, if EA measured in the field is smaller than 0.5 times EA_{max}, it is likely that a larger quantity of nutrient has been applied than is needed for maximum production. It points to a waste of nutrients.

2.3. Interpretation of the agronomic nutrient use efficiency (EA)

From the foregoing it follows that EA can be physically interpreted. It is obvious that fertilizer rates should be lowered when EA is less than half the value of EA_{max}. This insight can be used in the analysis of the fertilizer field tests carried out in the CATALIST program. Requirements are that the values for EA_{max}, the control yield and at least one yield of a fertilized crop are known. In such cases, one value of Δyield and hence of EA_x and of the ratio EA_x / EA_{max} can be calculated. For some crops, values of EA_{max}, derived from literature, are given in Table 1 of the paper. Values of EA_x can be derived from the CATALIST program.

Agronomic nutrient use efficiency is the multiplication of uptake efficiency and physiological efficiency. Uptake efficiency of applied (input) nutrients is synonymous to recovery fraction (REC), which is the portion of the applied nutrient that is taken up by the crop. Physiological efficiency (PhE) relates the yield (Y) of the economic plant components (*e.g.* grains, tubers) to uptake by the whole crop. It is also called internal nutrient efficiency (Witt et al., 1999). So:

$$EA = REC \cdot PhE \qquad \text{Eq. 3}$$

For the assessment of REC the application rate as well as the uptake of nutrients must be known, and for the assessment of PhE data on yield and on uptake of nutrients are required. For the assessment of EA, however, yield data alone do suffice.

The highest values of PhE (PhE_{max}) are obtained when the nutrient is maximally diluted in the crop, and lowest values of PhE (PhE_{min}) when the nutrient is maximally accumulated. Values of PhE_{max} and of PhE_{min} have been established for various crops (Van Keulen and Van Heemst, 1982; Janssen et al., 1990; Witt et al., 1999).

Such values of PhE_{max} were used for the estimation (with Equation 3) of EA_{max} in Table 1 of the paper. For REC, the recovery fraction of applied nutrients, standard values were used: 0.5 for fertilizer N and K, and 0.1 for fertilizer P. When in reality REC is higher than such a standard value, EA may be higher than the value of EA_{max} presented in Table 1 of the paper. On the other hand, a low value of EA may be the result of a low value of PhE, or a low value of REC, or low values of both. A low value of PhE may be caused by unfavorable weather conditions or too little uptake of the other nutrients, or by poor crop management. A low value of REC may be caused by high availability of the nutrient in the soil, by too small availability of the other nutrients, by unfavorable soil properties (e.g. P fixation), unfavorable weather conditions (too much rain and hence leaching of applied nutrients), , or improper methods and rates of fertilizer application. In case the soil can supply sufficient available nutrient for maximum crop growth, REC even may be zero. In general, REC is the lower the larger the application rates, and the higher the native (inherent) soil fertility level. As a consequence, also EA decreases with increasing application rates and increasing soil fertility. As shown before, when EA is less half the value of EA_{max} further addition of nutrients will not lead to higher yields and hence is to be considered as a waste, which is disadvantageous to farmer and environment.

3. Optimum nutrient inputs

3.1 Optimum nutrient applications derived from parabolic yield equations

The ascending part of the relation between yield and nutrient applied reaches its maximum, so the maximum value of Equation 1, when the first derivative equals zero. The first derivative (dy/dx) of Eq. 1 is:

$$dy/dx = b - 2\,cx \qquad \text{Eq. 4}$$

and dy/dx is 0 when $b - 2\,cx = 0$, or $x = b/2c$

Substitution of $x = b/2c$ in Eq. 1, results in:

$$y = y_{max} = a + b^2/2c - cb^2/4c^2 = a + b^2/4c \qquad \text{Eq. 5}$$

Substitution of the regression constants a = 1200, b = 35, c = 0.35 in Equation 5 results in $1200 + 35^2/(4 \cdot 0.35)$ which is 1200 + 35/0.04 = 2075, as is seen in Table A I, 1.

Equation 5 shows that the maximum yield response (ΔYield) to nutrient application is equal to $b^2/4c$, and that it is obtained when the applied quantity equals b/2c. The corresponding value of EA_x is denoted by EA_{ymax} and its value is:

$$EA_{ymax} = (b^2/4c)/(b/2c) = 0.5b = 0.5\,EA_{max} \qquad \text{Eq. 6}$$

Equations 5 and 6 confirm the findings of Table A I, 1 and Figure A I, 2.
Farmers, however, do not strive at maximum physical yields but at maximum economic yields. The economic yield (y_{econ}) is the difference between the value of the increased yield (V) obtained with nutrient input (x), and the costs (C) of the nutrient input. It is the same as the net return in Table A I, 1. The value of the increased yield equals:

$V = \Delta\text{yield} \cdot PRHP = (b\,x - c\,x^2) \cdot PRHP$ Eq. 7

where Δyield is the increase in harvested product expressed in kg ha^{-1}, and PRHP the price per kg of harvested product (HP), e.g. in RwF; so V is in RwF ha^{-1}.

The costs (C) of the nutrient input equal:

$C = x \cdot PRNUT$ Eq. 8

where PRNUT stands for the price per kg nutrient (N, P_2O_5, or K_2O). As x is expressed in kg ha^{-1} , C is in RwF ha^{-1}.

The difference between Eq. 7 and Eq. 8 is

$(y_{econ}) = (b\,x - c\,x^2) \cdot PRHP - x \cdot PRNUT$

$(y_{econ}) = (b \cdot PRHP - PRNUT)\,x - c \cdot PRHP \cdot x^2$ Eq. 9

The maximum economic yield, so the maximum value of Eq. 9 is obtained when its first derivative equals zero. The first derivative (dy_{econ}/dx) is:

$dy_{econ}/dx = b \cdot PRHP - PRNUT - 2\,c \cdot PRHP \cdot x$

It follows that dy_{econ}/dx is 0 when $b \cdot PRHP - PRNUT = 2\,c \cdot PRHP \cdot x$, or

$x_{econopt} = (b - PRNUT/PRHP)/2c$

$x_{econopt} = (b - PR)/2c$ Eq. 10

where $x_{econopt}$ is the economically optimum application rate, and PR is the ratio of the prices of nutrient and produce.

A somewhat different approach to find the economic optimum is the demand that the marginal gross return equals the marginal costs, in other words that the first derivative of Equation 7 equals the first derivative of Equation 8. The first derivatives of Equation 7 and 8 are:

$dV/dx = (b - 2cx) \cdot PRHP$

and

$dC/dx = PRNUT$

They are equal, so $x = x_{econopt}$, if $(b - 2cx) \cdot PRHP = PRNUT$ or $2cx = b - PRNUT/PRHP$ again resulting in

$x_{econopt} = (b - PRNUT/PRHP)/2c = (b - PR)/2c$

Figure A I,4 gives a graphical presentation of the assessment of the economic optimum fertilizer application for the values of b = 35 and c = 0.35, as used before (Table A I, 1, Figure A I, 2). Substitution of these values and of PRHP = 0.2 kRwF in Equation 7 results in $V = 7x - 0.07 x^2$, which is the curve of gross return (value) in Figure A I, 4. Substitution of PRNUT = 0.9 kRwF in Equation 8 results in C = 0.9x, which is the straight line fertilizer costs in Figure A I, 4. The ratio of the prices of nutrient and produce (PR) is 0.9/0.2 = 4.5. Substitution of b, c and PR in Equation 10 gives:

$x_{econopt} = (35 - 4.5)/(2 \cdot 0.35) = 43.57$.

The tangent along the gross return curve in Figure A I, 4 runs parallel to the costs line, and it is the first derivative of the equation for gross return at x = 43.57, which is $x_{econopt}$, as is shown by substitution of x = 43.57 in dV/dx = 7 – 0.14x. It results in dV/dx = 0.9, which is equal to the slope of the cost line.

In Figure A I, 4, the distance between the curve of gross return and the costs line is the net return; it is maximum at x = 43.57. Substitution of x = 43.57 in $V = 7x - 0.07x^2$ gives V = 172.1 kRwF, and in C = 0.9x gives C = 39.2 kRwF. The net return is 172.1 - 39.2 = 132.9 kRwF. This net return is higher than but close to 132 that was obtained with 40 kg N which is the best net return in Table A I, 1.

Substitution of x = 43.57 in Equation 1, results in a yield of 2061, and hence a Δyield of 861, which is a little below the maximum Δyield of 875 kg ha^{-1}.

From Equation 10 it follows that the regression parameters b and c of the parabolic response curve must be known as well as the price ratio of nutrient and produce, to be able to determine the economically optimum nutrient application rate. It is also obvious that the optimum fertilizer rate always is smaller than the fertilizer rates corresponding to the plateau yield in Figure A I, 2.

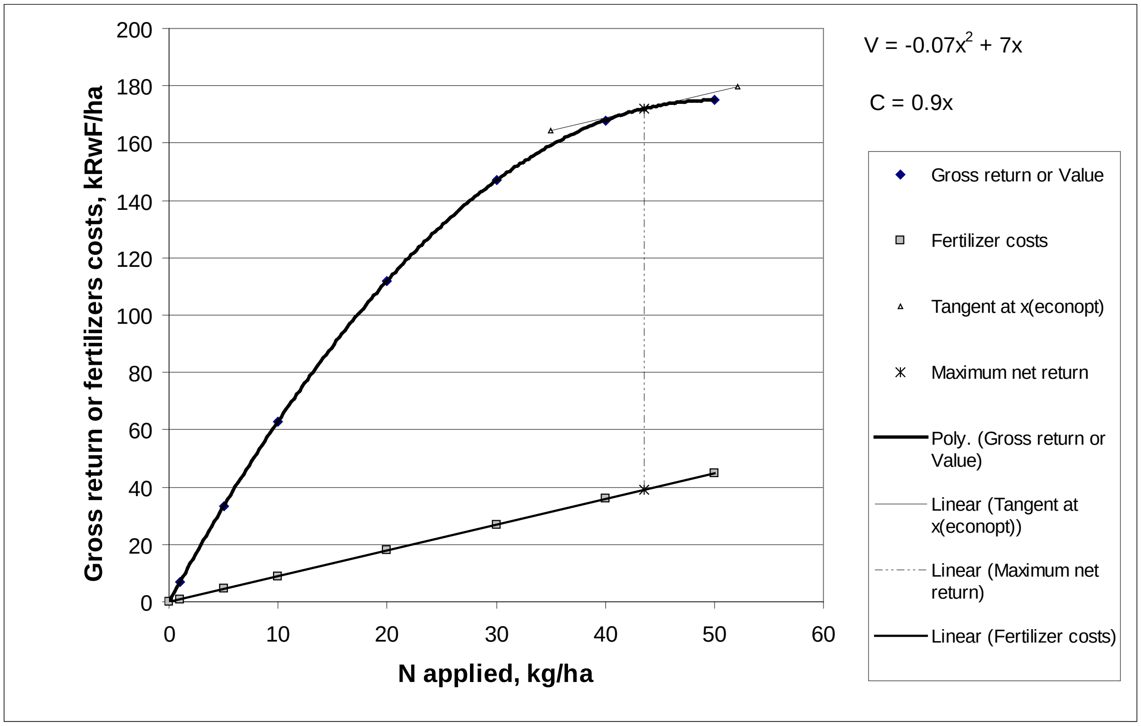

Figure A I, 4. Graphical presentation of the assessment of the economic optimum fertilizer application

3.2 Calculation of economically optimum nutrient application in the CATALIST tests

In the CATALIST tests, only two nutrient rates are used: zero and a level expected to be remunerative to the farmer. For the assessment of the three regression parameters (a, b and c) of the parabolic yield curve, at least three nutrient levels are needed. This implies that the optimum fertilizer rate cannot be assessed with the CATALIST tests as such. The only possibility to find the values of the regression coefficients b and c is with the help of Equation 2 and the use of EA_{max} data. Equation 2 is rewritten as:

$$EA_x = b - cx = EA_{max} - cx \qquad \text{Eq. 11}$$

EA_x is found as ΔYield/x in a CATALIST test with application rate x, and EA_{max} is given in Table I of the paper. Two situations must be distinguished:

(i) EA_x is in between $0.5 \cdot EA_{max}$ and EA_{max}, implying that x is in the part of ascending yields in Figure A I, 2.

(ii) EA_x is smaller than $0.5 \cdot EA_{max}$, implying that x is in the part of plateau yields.
In situation (i), c can be calculated by application of Equation 11 in a reverse direction:

$$c = (EA_{max} - EA_x)/x \qquad \text{Eq. 12}$$

Next, $x_{econopt}$ is found after substitution of PR, $b = EA_{max}$, and c (from Equation 12) in Equation 10.

In situation (ii), however, where EA_x is less than $0.5 \cdot EA_{max}$, it does not make sense to use Equation 12, as follows from the discussion in Section 2.2. The measured yield that is then obtained is not in the ascending part of Figure A I, 2, but somewhere beyond it. It is supposed to be on the plateau, so to equal the maximum yield (y_{max}) that is possible. The lowest value of x at which y reaches the value of y_{max} is x corresponding with the top (optimum) of the parabola, but $x_{econopt}$ is still lower and somewhere in the ascending part of the relation between yield and nutrient level. The value of y_{max} is described in Equation 5, as a function of the regression constants a, b and c. Because y_{max}, a (= control yield) and b (= EA_{max}) are known, Equation 5 can be used for the assessment of the regression parameter c which is needed for the calculation of $x_{econopt}$ with Equation 10. Reversing Equation 5, c can be calculated by

$$c = b^2/(4 \cdot (y_{max} - a)) = b^2/(4 \cdot \Delta y_{max}) \qquad \text{Eq. 13}$$

Next, c is used in Equation 10 to calculate the optimum rate of x.

3.3 Some examples of the calculation of economically optimum application of nutrients to wheat in the CATALIST program

In Table A I, 3, some examples are presented of the results of the *'tests comparatifs'* in Rwanda. The examples were chosen because they show some of the problems encountered when trying to interpret the CATALIST tests. The first problem is that the effect of N cannot be estimated because the difference between treatments T1 and T0 always is the combined effect of N and OM. The effect of P_2O_5 (= T2-T1) can be calculated but it refers to the situation that OM and N already have been applied, while the effect calculated for K_2O (= T3-T2) refers to the situation that OM, N and P_2O_5 already have been applied. Hence, also the economically optimum applications rates calculated from these tests suppose that (unknown) amounts of OM are applied. If no OM

Table A I, 3. Example of the calculation of economically optimum application of nutrients for wheat in Nyabihu ,and Musanze. ΔYield refers to P_2O_5 (= T2-T1) and to K_2O (=T3-T2). EA_{max} = 21 for P_2O_5, and 36 for K_2O. The price ratios (PR) are 4.2 for P_2O_5, and 4.9 for K_2O.

Treatment	Formula	Yield	ΔYield	EA	EA/ EA_{max}	c	$x_{econopt}$ [c]
				Nyabihu, 2008A			
T0	0-0-0	1250					
T1	65-0-0 + OM	1750					
T2	65-60-0 + OM	2100	350	5.83	0.28	0.315[a]	26.7
T3	65-60-36 + OM	2650	550	15.28	0.42	0.589[a]	26.4
				Musanze, Kinigi, 2008B			
T0	0-0-0	1000					
T1	65-0-0 + OM	1400					
T2	65-60-0 + OM	2400	1000	16.67	0.67	0.072[b]	116.4
T3	65-60-36 + OM	2000	-400	-11.11	-0.28	unrealistic	

[a] c = $b^2/(4 \cdot \Delta Yield)$ (Equation 13)

[b] c = $(EA_{max} - EA_x)/x$ (Equation 12)

[c] $x_{econopt}$ = $(b - PR)/2c$ (Equation 10)

is applied the optimum rates of P_2O_5 and of K_2O P probably are higher than those shown in Table A I, 3.

In Nyabihu, the agronomic efficiencies (EA) of P_2O_5 and of K_2O are small (Table A I, 3). The ratios EA/EA_{max} are less than 0.5, indicating that the applied quantities of P_2O_5 and of K_2O were too high. For the calculation of the regression parameter c, Equation 13 must be used. The values found for $x_{econopt}$ are considerably lower than the quantities applied.

In Musanze, Kinigi, the response to P_2O_5 was good, but that to K_2O was negative. This may be a result of the big variability in soil fertility of the plots. Anyhow, only for P_2O_5, it is possible to calculate the economically optimum application rate. Because EA/EA_{max}

is 0.67, so in between 0.5 and 1.0, Equation 12 can be used for the calculation of c. The value found for $x_{econopt}$ is 75, higher than the quantity of 60 kg ha^{-1} that was applied.

Annex II. Design and interpretation of 2 · 2 · 2 factorial NPK trials

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Introduction

In 2009 some 'minus one' experiments ('Essais soustractifs') were carried out in the CATALIST program. They contain five fertilizer treatments: control; -N (= PK); -P (= NK); -K (= NP); and NPK. This design offers an interesting interpretation, in terms of the separate effects of N, P and K, and the NPK interaction. One may consider the treatments as five units of a complete 2N · 2P · 2K experiment with 8 treatments. Such experiments are called 2^3 factorials. In the following sections, first the setup of a complete 2N · 2P · 2K experiment is shown, and the methods of calculation of the effects and the agronomic efficiencies (EA) of N, P and K are demonstrated. Next, it is illustrated that both, the 'Essais soustractifs' and the 'tests comparatifs' form different parts of a complete 2N · 2P · 2K experiment. The methods of calculation of the separate effects and the separate agronomic efficiencies of N, P and K in these tests have applied in Sections 3.2 and 4.2 of this report.

Setup of a complete 2N · 2P · 2K experiment

A complete 2N · 2P · 2K design consists of 8 treatments. Each nutrient is applied at two levels. The lower level usually is zero application (0), while the higher level is commonly indicated by 1. Hence, the codes for the eight treatments are: 000, 001, 010, 011, 100, 101, 110 and 111. Table AII, 1 shows the setup with treatment codes and fancy data of yields. The yield data form a regular and classical pattern the effects and interactions. The fertilizer applications at level 1 in this table are 120 kg for N, 80 kg for P_2O_5 and 60 kg for K_2O. The treatment 101, for instance, consists of 120 N, 0 P_2O_5 and 60 K_2O.

Table A II, 1. Yields (fancy data) obtained in a 2N · 2P · 2K experiment and signs for the calculation of main effects and interactions.

	N0				N1			
	P0		P1		P0		P1	
	K0	K1	K0	K1	K0	K1	K0	K1
Effects and	*Yields*							
interactions	1200	1320	1980	2160	1440	1680	2460	2880
of	*Main effects*							
N	-	-	-	-	+	+	+	+
P_2O_5	-	-	+	+	-	-	+	+
K_2O	-	+	-	+	-	+	-	+
	Two-factor interactions							
NP	+	+	-	-	-	-	+	+
NK	+	-	+	-	-	+	-	+
PK	+	-	-	+	+	-	-	+
	Three-factor interaction							
NPK	-	+	+	-	+	-	-	+

Table AII,2. Main effects and interactions calculated from the data of Table AII,1.

Main effects		Two-factor interactions		Three-factor interactions	
N	450	NP	150	NPK	30
P_2O_5	960	NK	90		
K_2O	240	PK	60		

The effect of N is found as the difference in yield between the treatments N1 and N0, so between the plots receiving 120 and 0 N. In Table AII, 1, the corresponding treatments have plus (+) and minus (-) signs, respectively. The effect of P_2O_5 is found as the difference in yield between the treatments P1 and P0, *i.e.* between plots receiving 80 and no P_2O_5. Similarly, the effect of K_2O is found as the difference in yield between K1 and K0, so between the plots receiving 60 and no K_2O, again denoted by plus (+) and minus (-) signs, respectively. Table A II,2 presents the thus calculated main effects and interaction effects for the yields given in Table A II,1. The thus found totals were divided by 4, because there are four pairs of plus and minus signs. It is essential that for the correct estimation of the effect of one nutrient the average level of the other nutrients is the same in the treatments with a positive and a negative sign. For instance, the treatments with a positive sign for the calculation of N effect are N1P1K1, N1P1K0, N1P0K1 and N1P0K0; the average levels of P and K are P0.5 and K0.5. The treatments with a negative sign for the calculation of N effect are N0P1K1, N0P1K0, N0P0K1 and N0P0K0; also here the average levels of P and K are P0.5 and K0.5.

In Table AII, 3, it is explicitly shown that the effects of each of the nutrients (N, P_2O_5 and K_2O) can be found four times. The effects and the agronomic efficiencies (EA) of each of the nutrients (N, P_2O_5 and K_2O) are calculated in the left-hand side of Table AII, 3 at each of the four combinations of the two other nutrients. Also the general averages are shown. The effects are stronger for P_2O_5, than for N and K_2O. In the right-hand side of the table the effects of one nutrient are calculated at each of the two levels of one of the two other nutrients. The effects are the greater the higher the levels the other nutrients are indicating positive two-factor interactions. The N effect is more influenced by the level of P than by the level of K. The P effect is more affected by the level of N than by the level of K. The effect of K is more influenced by the level of N than by the level of P. In other words the NP interaction is greater than the NK interaction, and the NK interaction is stronger than the PK interaction.

Half replicates a 2N · 2P · 2K experiment

A 2^3 factorial design can be subdivided into two half replicates. They are shown in Table A II, 4. The upper half replicate shows the treatments that have a negative sign for the calculation of the NPK interaction in Table AII, 1, and the lower half replicate shows the treatments that have a positive sign. The NPK interaction is called the *defining contrast.*

Table A II, 3. Calculation of the effects and the agronomic efficiencies (EA) of each of the nutrients (N, P_2O_5 and K_2O), at each of the two levels of each of the two other nutrients, and the general average effects and EA.

Level of other nutrients	N effect		EAN = Effect/120	Level of other nutrients	N effect	EAN
	Calculation	Result				
P0K1	1440 – 1200	240	2	P0	300	2.5
P0K1	1680 – 1320	360	3	P80	600	5
P0K0	2460 – 1980	480	4	K0	360	3
P1K1	2880 – 2160	720	6	K70	540	4.5
Average		450	3.75	Average	450	3.75
	P_2O_5 effect		EAP_2O_5 =		P_2O_5 effect	EAP_2O_5
	Calculation	Result	Effect/80			
N0K0	1980-1200	780	9.75	N0	810	10.125
N0K1	2160-1320	840	10.5	N120	1110	13.875
N1K0	2460-1440	1020	12.75	K0	900	11.25
N1K1	2880- 1680	1200	15	K70	1020	12.75
Average		960	12	Average	915	12
	K_2O effect		EAK_2O =		K_2O effect	EAK_2O
	Calculation	Result	Effect/70			
N0P0	1320 – 1200	120	2	N0	150	2.5
N0P80	2160 - 1980	180	3	N120	330	5.5
N120P0	1680 - 1440	240	4	P0	180	3
N120P80	2880 - 2460	420	7	P80	300	5
Average		240	4	Average	240	4

In each of the two half replicates, the main effects of the individual nutrients can be calculated, but not in a ‘pure’ way, as the main effects of Nutrient 1 are always confounded with two-factor interactions of Nutrients 2 and 3. For instance, the treatments

Table AII, 4. Subdivision of a 2^3 factorial design into two half replicates using the NPK interaction as *defining contrast*. Yields (fancy data) and signs for the calculation of main effects and interactions.

First half	N0		N1					
replicate	P0	P1	P0	P1				
	K0	K1	K1	K0				
Effects and	*Yields*				Average level of the other nutrients			
interactions	1200	2160	1680	2460	At minus sign		At plus sign	
of					*Main effects*			
N	-	-	+	+	P0.5	K0.5	P0.5	K0.5
P_2O_5	-	+	-	+	N0.5	K0.5	N0.5	K0.5
K_2O	-	+	+	-	N0.5	P0.5	N0.5	P0.5
					Two-factor interactions			
NP	+	-	-	+	K0		K1	
NK	+	-	+	-	P0		P1	
PK	+	+	-	-	N0		N1	
	Three-factor interaction							
NPK	-	-	-	-	Impossible			
Second half	N0		N1					
replicate	P0	P1	P0	P1				
	K1	K0	K0	K1				
Effects and	*Yields*				Average level of the other nutrients			
interactions	1320	1980	1440	2700	At minus sign		At plus sign	
of					*Main effects*			
N	-	-	+	+	P0.5	K0.5	P0.5	K0.5
P_2O_5	-	+	-	+	N0.5	K0.5	N0.5	K0.5
K_2O	+	-	-	+	N0.5	P0.5	N0.5	P0.5
					Two-factor interactions			
NP	+	-	-	+	K1		K0	
NK	+	-	+	-	P1		P0	
PK	+	+	-	-	N1		N0	
					Three-factor interaction			
NPK	+	+	+	+	Impossible			

Table A II, 5. Main effects of N, P and K as calculated in the complete 2^3 factorial design of Table 1 and in the upper and lower half replicate of Table 3, and the two-factor interactions for the complete 2^3 factorial design of Table 1.

Effect of	Replicate			Difference complete and half replicates		Interaction in Complete 2^3 factorial	
	Complete	Upper half	Lower half	Upper half	Lower half		
N	450	390	510	60	-60	PK	60
P_2O_5	960	870	1050	90	-90	NK	90
K_2O	240	90	390	150	-150	NP	150

that have a positive sign for the calculation of the main effect of N are the same as the treatments that have a negative sign for the calculation of the interaction of PK. As a consequence the main effect of N is confounded with the PK interaction. Similarly, the main effect of P is confounded with the NK interaction, and the main effect of K is confounded with the NP interaction. This shows up in Table AII, 4.

The values of the main effects calculated in a half replicate differ from the values of the main effects calculated for the complete design (table A II,5). The values of the upper half replicate are smaller, and the values of the lower half replicate are greater than those of the complete design. The upper half replicate corresponds to the negative signs of the NPK interaction in Table A II, 1, and the lower half replicate corresponds to the positive signs of the NPK interaction in Table A II, 1. The main effects of the complete scheme are equal to the main effects in the upper half replicate plus the interaction values, as well as to the sum of the main effects in the lower half replicates minus the interaction values . Table A II,4 also shows that the average levels of Nutrients 2 and 3 are the same for the treatments with a positive and negative sign in the calculation of the main effects of Nutrient 1. In the calculation of the two-factor interactions of Nutrients 1 and 2, however, the levels of Nutrient 3 are not the same for the treatments with a positive and negative sign. This is in line with the already mentioned confounding of the main effect of Nutrient 3 and the interaction of Nutrient 1 and 2. Hence, two-factor interactions cannot be calculated in the half replicates of a 2^3 factorial.

'Essais soustractifs' or minus-one designs

The fertilizer treatments of minus-one designs as used in CATALIST are control, NP, NK, and PK and NPK. The first four form the upper half replicate of Table A II, 3, and are the treatments of a 2^3 factorial that have a negative sign for the calculation of the NPK interaction in Table AII, 1. In Table A II,6 the effects are calculated in three ways: (i) As in the first half replicate of Table A II,4. They consist of effect of Nutrient 1 minus the interaction of Nutrients 2 and 3, as explained above.

(ii) As the difference in yield between NPK and each the two-factor treatments.

(iii) As the best estimate calculated as one third of the sum of the effect found sub (ii) plus two times the effect found sub (i).

The effect found sub (i) is based on two pairs of yields, and hence it gets a higher weight in the calculation sub (iii) than the effect calculated sub (ii). The differences between the values found sub (ii) and (i) are equal to the gross NPK interaction, consisting of the sum of the two-factor interactions NP, NK, PK and the net NPK interaction. It follows from Table A II,2 that this sum is: 150 + 90 + 60 + 30 = 330.

Table A II, 6. 'Essais soustractifs'. Effects of N, P and K as calculated A. in the upper half replicate of a 2^3 factorial, or B as the yield difference between NPK and two-factor treatments. Best estimate is (B + 2A)/3. Gross NPK interaction is B – A.

A. Effect of Nutrient 1 minus interaction of Nutrients 2 and 3		B. NPK minus two-factor treatments		Best estimate	Gross NPK interaction
N – PK interaction	390	NPK - PK	720	500	330
P_2O_5 – NK interaction	870	NPK - NK	1200	980	330
K_2O – NP interaction	90	NPK - NP	420	200	330

Tests comparatifs

In the 'tests comparatifs' the effect of N is underestimated, that of P_2O_5 a little overestimated, while the effect of K_2O is considerably overestimated, as is shown in Table A II,7. Hence the 'tests comparatifs' cannot be recommended. The poor correctness of the 'tests comparatifs' has the following reasons.

The 'tests comparatifs' consist of the treatments: control (= N0P0K0), N (= N0P1K1), NP (= N1P1K0) and NPK(= N1P1K1). These are not well balanced as is seen from the average levels of Nutrients 2 and 3, at the plus and minus signs of the treatments in Table A II,8. used in a 2^3 factorial for the calculation of the main effect of Nutrient 1. These levels are different for the minus and the signs of the effects and interactions that are to be calculated. Calculation of the interactions is impossible.

Therefore only half of the treatments can be used for the calculation of the main effects (Table A II,8 bottom). As mentioned before (Setup of a complete 2N · 2P · 2K experiment), the average levels of Nutrients 2 and 3 must be 0.5 for the correct and balanced calculation of the main effect of Nutrient 1. In Table A II,8 bottom, however, the N effect is calculated at P0K0, the P_2O_5 effect at N1K0, and the K_2O effect at N1K1. This is the cause of the respective underestimation of the N effect, and the overestimation of the K_2O effect. Because for the given fancy yield data, the influence of N is stronger than the influence of K (see Table A II,2), the P_2O_5 effect is overestimated in the 'tests comparatifs'.

Table AII, 7. Comparison of the main effects as calculated with the complete 2^3 factorial design, as the final best estimate in the 'Essais soustractifs', and as calculated in the 'tests comparatifs'. Yields (fancy data) are as in Table A II,1.

	2^3 factorial	Final best estimate	'Tests comparatifs'
N	450	500	240
P_2O_5	960	980	1020
K_2O	240	200	420

Table AII, 8. Treatments a 2^3 factorial design used in the ‘tests comparatifs’. Yields (fancy data) and signs for the calculation of main effects and interactions are as in Table A II,1.

	N0		N1					
	P0	P0	P1					
	K0	K0	K0	K1				
Effects		*Yields*			Average level of the other nutrients			
and inter	1200	1440	2460	2880	At minus sign		At plus sign	
actions of					*Main effects*			
N	-	+	+	+	P0	K0	P0.67	K0.33
P_2O_5	-	-	+	+	N0.5	K0	N1	K0.5
K_2O	-	-	-	+	N0.67	P0.33	N1	P1
					Two-factor interactions			
NP	+	-	+	+	K0		K0.33	
NK	+	-	-	+	P0.5		P0.5	
PK	+	+	-	+	N1		N0.67	
					Three-factor interaction			
NPK	-	+	-	+	N0.5 P0.5 K0		N1 P0.5 K0.5	
					Final calculation of main effects			
N	-	+			P0	K0	P0	K0
P_2O_5		-	+		N1	K0	N1	K0
K_2O			-	+	N1	P1	N1	P1