# Inventory control for a perishable product with non-stationary demand and service level constraints

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**Abstract** We study the practical production planning problem of a food producer facing a non-stationary erratic demand for a perishable product with a fixed life time. In meeting the uncertain demand, the food producer uses a FIFO issuing policy. The food producer aims at meeting a certain service level at lowest cost. Every production run a set-up cost is incurred. Moreover, the producer has to deal with unit production cost, unit holding cost and unit cost of waste. The production plan for a finite time horizon specifies in which periods to produce and how much.

We formulate this single item – single echelon production planning problem as a stochastic programming model with a chance constraint. We show that an approximate solution can be provided by a MILP model. The generated plan simultaneously specifies the periods to produce and the corresponding order-up-to levels. The order-up-to level for each period is corrected for the expected waste by explicitly considering for every period the expected age-distribution of the products in stock. The model assumes zero lead time and backlogging of shortages. The viability of the approach is illustrated by numerical experiments. Simulation shows that in 95.8% of the periods the service level requirements are met with an error tolerance of 1%.

**Keywords** Perishable product; Non-stationary stochastic demand; Service-level constraint; Periodic review.

## 1 Introduction

Food supply chains of processed fresh products generally include primary production (farmers), food processing industry, distribution centres of the producer or a retail organisation, retail stores and consumers (e.g. (van der Vorst et al., 2000)). In this paper we study the practical production/inventory control problem faced by a food producer. After processing fresh ingredients into a final product and packing the product, the producer prints a best-before-date on the package of the product. Products can be meat, dairy products, fresh fruit juices and produced fresh meals. If the product is stored and handled under the required conditions, the product is presumed to have a fixed lifetime; the best-before-date is determined by adding a fixed number of days to the production date. In practice, a food producer often faces a non-stationary stochastic demand for his products, caused by, for instance, promotional activities of the retail organisation, or weather conditions. The producer has to decide at any given period (e.g. a week) whether to produce or not, and if so, how

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much to produce. This decision depends on the forecast of the demand, on the age-distribution of the items in stock and is influenced by factors such as the setup cost of a production run and the perishability of the product.

Food producers often have contracts with their customers, regarding delivery performance including service level and remaining shelf life. In order to meet these requirements and to determine production quantities, the producer has to balance product waste (as a result of too much inventory) and out-of-stock (as a result of too little inventory). Due to the perishability of the product, it is likely that the inventory of final products at the producer consists of items of different ages, that is, with different production dates. The producer sells the products to the customers (e.g. supermarkets) with a guaranteed remaining shelf life on the time of delivery. We define *internal shelf life* as the maximum time span between production and distribution. To guarantee a minimum remaining shelf life at the customer, one sets a maximum on the internal shelf life. For an internal shelf life of just 1 period, one can follow the order policy of the so-called Newsboy Problem (Silver et al., 1998) that produces every period with an order quantity that takes the perishability into account. When the internal shelf life is longer than 1 period, the order policy depends on the setup cost and holding cost and the aging of the products (Fries, 1975). When setup cost is relatively high, the optimal order policy for a product, disregarding perishability, may lead to a time between two production runs that exceeds the internal shelf life. In that case a part of the production quantity may become waste. Waste may lead to out-of-stock in the periods before a new production run is planned.

Service level is a way to measure product availability. Chopra and Meindl (2010) define *service level* as *the probability of not having a stock-out in a replenishment cycle.* There are two reasons to study a service level approach. First, food producers often have contracts with their customers, regarding delivery performance including service level and remaining shelf life, as already mentioned. Second, stock-out penalty cost are difficult to quantify (Minner and Transchel, 2010). Requiring a certain service level and therefore a certain level of safety stock, can be seen as a cost component by having extra items in stock.

The demand for products of a food producer is not only stochastic, but may also be non-stationary. This leads to an irregular pattern of the decision when to produce, and how much. Periodic review (*R, S*) or (*R, s, S*) order policies with a fixed order-up-to level typically deal with a stationary stochastic demand. Using such a policy in case of non-stationary demand either leads to a very high production level to fulfil peaks in demand, causing waste when demand is low, or to a low production level causing out-of-stock when demand is very high (Tunc et al., 2011). Such a policy may also cause extra production runs, which will lead to higher costs. Therefore it is interesting to investigate order policies with time-dependent order-up-to levels and replenishment cycle lengths. Furthermore, uncertainty in demand leads to new production runs, while the inventory level is not zero yet. This gives items of different ages in stock. The age-distribution of the items in stock has to be monitored and should be considered in the issuing policy. A food producer has control over its issuing policy and often distributes its inventory according to First In First Out (FIFO). A fixed issuing policy, such as FIFO, is favoured in practise, because it is easy to use and keeps wastage due to outdating low.

The practical problem discussed in this paper is the finite time horizon single-product − single-echelon production/inventory control problem for a perishable product with a fixed lifetime, under a service-level constraint. The product has a non-stationary stochastic demand. The decision problem deals with a fixed setup cost for every production run and a FIFO issuing policy. We consider the age-distribution of the items in stock in a specific theoretical Stochastic Programming (SP) problem that deals with the service level as a chance constraint. The model uses zero lead time and in case of out-of-stock, demand is backlogged. The question addressed in this paper is whether it is possible to construct practical solutions using commercial solvers for business use rather than custom made solution procedures. We approach this question with a Mixed Integer Linear Programming (MILP) model that generates approximate solutions of the problem.

The paper is structured as follows. In Section 2, a literature review on the problem is presented. Section 3 describes the SP problem. The waste compensating replenishment cycle policy is defined in Section 4. In Section 5, a deterministic MILP model for perishable products is formulated. In Section 6 we investigate how well the solutions of the MILP model fulfil the desired service levels using a sensitivity analysis combined with simulation runs. The paper ends with conclusions and topics for future research in Section 7.

## 2 Literature review

In order to construct a model for the practical problem under consideration, we review literature that deals with a combination of the key characteristics of the practical problem: perishability with a fixed lifetime, fixed setup or ordering cost, non-stationary demand, periodic review and a service-level constraint.

Nahmias (1982), Goyal and Giri (2001), Karaesmen et al. (2011) and Bakker et al. (2012) reviewed the literature on inventory models for perishable products with a fixed lifetime. Almost all papers surveyed assume stationary demand, i.e. demand in successive periods is an independent identically distributed random variable. Tekin et al. (2001) formulated an age-based control policy with a continuous review for perishable products with a fixed lifetime, under service-level constraints. The aging starts after unpacking the batch for consumption. As long as the items are packed in stock, the lifetime is virtually infinite. In early works e.g. Nahmias (1975) and Fries (1975) observe that in general an optimal order policy for perishables with a fixed life time should take the ages of the products in stock into account. Even when all perishable items are of the same age, base stock polices are not optimal, as argued by Tekin et al. (2001) and Haijema et al. (2007). Broekmeulen and Van Donselaar (2009) suggest a replenishment policy for perishable products at a retailer, which takes the quantity and the age of the items in inventory into account. They assume negligible fixed ordering cost. The demand is assumed to be stochastic, with a weekly demand pattern per day, but stationary expected demand per week. They apply the same safety stock for each weekday. Haijema et al. (2007) developed an optimal policy for the periodic production and inventory of blood platelets. They combine two types of demand, each of which requires a different issuing policy. The demand distributions they consider have a weekly demand pattern per day, but are stationary across weeks. In Haijema et al. (2009) the approach is extended for non-stationary demand considering holidays and other events. Any fixed production cost is neglected. In Haijema (2011) fixed order cost are studied and a new class of order policies is presented. In none of these papers service-level constraints are included. Minner and Transchel (2010) present a numerical approach to determine replenishment quantities for perishable products in retail dynamically, using a weekly demand pattern. They consider service-level constraints varying for different intra-period time points and for different periods. Fixed ordering cost is assumed to be negligible.

In our investigation, the combination of non-stationary demand and a service level approach in inventory models was mainly found in literature about non-perishable products. Neale and Willems (2009) argue that non-stationary demand is very common nowadays. Therefore they developed a non-stationary supply chain inventory model, by formulating a single-stage inventory model that serves as a component of a multistage system, using service-level constraints to calculate safety stocks. The model is based on Graves and Willems (2000) and closely related to Graves and Willems (2008). Every stage has a base-stock policy with a review period of 1 time unit. The base-stock level is an order-up-to level to cover demand in upcoming periods. The safety stock is calculated as a function of demand over the preceding periods. In the multi-stage system Neale and Willems (2009) minimise the total holding cost of the safety stock in all stages and periods. They do not consider setup cost, which is an important cost component in practice. Bookbinder and Tan (1988) studied single-stage probabilistic lot-sizing problems, where they included setup cost, and service-level constraints. They developed a “static-dynamic” uncertainty model, splitting the problem in two stages. The first stage determines when to order, the second how much to order. Tarim and Kingsman (2004) considered the Bookbinder and Tan approach as a basis for the formulation of a mixed integer programming model for non-stationary stochastic demand for the simultaneous determination of the number and timing of the replenishment orders. In contrast to Bookbinder and Tan’s heuristic approach, Tarim and Kingsman’s approach provides an optimal solution. Several extensions of Tarim and Kingsman’s model exist. Rossi et al. (2011b) and Tarim et al. (2011) proposed efficient and complete special purpose algorithms. Tempelmeier (2007) used Tarim and Kingsman’s model as a basis to formulate different types of service-level constraints. Rossi et al. (2010) and Rossi et al. (2011a) incorporated a stochastic delivery lead time and developed both complete and fast heuristic approaches. Tempelmeier (2011) incorporated supplier capacity constraints. Pujawan and Silver (2008) proposed a novel and effective heuristic approach. However, to the best knowledge of the authors, no paper deals with all aspects of the practical planning problem under consideration: the combination of perishability with a fixed lifetime, fixed setup or ordering cost, non-stationary demand and a service level approach. In this paper we extend the model of Tarim and Kingsman towards a model that includes non-stationary stochastic demand for a perishable product under a FIFO issuing policy.

## 3 Stochastic Programming model for a perishable product

The problem of determining a production plan for a perishable product under non-stationary stochastic demand consists of deciding when to produce and how much to produce for a finite time horizon of $T$ periods, such that the expected total costs are minimised. Periods can be hours, days, weeks or months, whatever is applicable in the practical situation. We adopt a minimum service-level criterion for meeting customer demand. Consider a single-product – single-echelon model where the product has a fixed maximum integer (internal) shelf life $M \geq 2$ periods. A replenishment arrives instantaneously at the beginning of a period, i.e. lead time is zero. Demand $d_t$ is a non-stationary independent stochastic process with probability density function $g_t(\cdot)$ and cumulative distribution function $G_t(\cdot)$. Demand is never negative; food cannot be returned due to food safety regulations.

We consider a FIFO issuing policy in which the first produced items are issued first. Let the ages be indexed by $b = 1,.., M$. Variable $I_{bt}$ denotes the inventory level of items with age $b$ at the end of period $t$. Items that are delivered at the beginning of period $t$ have age $b = 1$ at the end of period $t$. Items of age $M$ at the end of a period are not carried over to the next period, because they are out-dated; inventory $I_{Mt}$ of age $M$ at the end of period $t$ is considered waste. Demand that cannot be fulfilled in one period is backlogged in the next period. Further costs are a fixed setup cost $k$ for every production run and a variable production cost $c$ per item produced. We assume that $k$ and $c$ are independent of the production period, but the model can be generalised with a period-dependent setup cost and production cost. For items that are carried over from one period to the next, a holding cost $h$ per item is incurred. There is a cost $w$ per item of waste, on top of the unit production cost $c$. The case $w > 0$, describes a situation with additional cost to discard the wasted items. Situation $w < 0$ reflects that the wasted items still have a salvage value of $-w$. All costs remain constant within the time horizon. For convenience and without loss of generality, the initial inventory level is set to zero. An overview of the used symbols is presented in Appendix A. The resulting problem can be formulated as a stochastic programming model:

$$\text{Min } E(TC) = \int_{d_1} \ldots \int_{d_T} \sum_{t=1}^{T} \left( kY_t + h \max\{I_{1t}, 0\} + h \sum_{b=2}^{M-1} I_{bt} + cQ_t + wI_{Mt} \right) g_1(d_1) \ldots g_T(d_T) \mathrm{d}d_1 \ldots \mathrm{d}d_T \quad (1)$$

subject to

$$Y_t = \begin{cases} 1 & \text{if } Q_t > 0 \\ 0 & \text{otherwise} \end{cases} \qquad t = 1,..,T \quad (2)$$

$$\sum_{b=1}^{M} I_{bt} = \sum_{b=1}^{M-1} I_{b,t-1} + Q_t - d_t \qquad t = 1,..,T \quad (3)$$

$$\mathrm{P}\left( \sum_{b=1}^{M} I_{bt} \geq 0 \right) \geq \alpha \qquad t = 1,..,T \quad (4)$$

$$I_{bt} = \max\left\{ I_{b-1,t-1} - \max\left\{ d_t - \sum_{j=b}^{M-1} I_{j,t-1},\ 0 \right\},\ 0 \right\} \quad t = 1,..,T;\ b = 2,..,M \quad (5)$$

$$I_{1t} = Q_t - \max\left\{ d_t - \sum_{b=1}^{M-1} I_{b,t-1},\ 0 \right\} \qquad t = 1,..,T \quad (6)$$

$$I_{b0} = 0 \qquad b = 1,..,M \quad (7)$$

$$I_{bt} \geq 0 \qquad t = 1,..,T;\ b = 2,..,M \quad (8)$$

$$I_{1t} \in \mathbb{R} \qquad t = 1,..,T \quad (9)$$

$$Y_t \in \{0,\ 1\},\ Q_t \geq 0 \qquad t = 1,..,T \quad (10)$$

The objective function (1) of the model minimises the expected total costs, comprising fixed setup cost for every production run, holding cost over every item in stock, unit production cost and cost of wasted items. The binary variable $Y_t$ takes value 1 if there is a production run in period $t$, and 0 otherwise. In Eq. (3), the inventory levels of all ages are balanced. Items of age $M$ cannot be used in the next period, so period $t$ starts with the inventory levels at the end of period $t - 1$ of ages $b = 1,.., M - 1$. The inventory at the end of period $t$ equals the starting inventory increased by an amount $Q_t$ that is produced in period $t$ minus the demand in period $t$. Service-level constraint (4) states that the inventory levels of all ages together at the end of period $t$ should be nonnegative with probability α. This type of service level is known as α-service level. Eq. (5) and (6) are the FIFO constraints. They make sure that demand is fulfilled first by the oldest items in stock and then successively by the younger items. Possible shortages only occur for the youngest items (Eq.(6)). Notice that adding up all equations of (5) and (6) results in Eq. (3). The starting inventory level of all ages is 0 (Eq. (7)), and the inventory levels of all ages in all other periods are nonnegative (Eq. (8)) except for the inventory level of age 1 in all periods, which can be negative when stock is too small to fulfil demand (Eq. (9)). Compared to the model of (Bookbinder and Tan, 1988), the SP model also considers FIFO constraints (5) and (6), and included the age of the items to the variable for the inventory level.

## 4 The waste-compensating replenishment cycle policy

The rest of this paper discusses solution strategies for the theoretical SP model that follow a "waste-compensating" replenishment cycle policy. This policy is structured as follows. At the beginning of the planning horizon, it simultaneously determines in which periods to produce, and the associated order-up-to levels which aim at fulfilling the prescribed service level. To determine the order-up-to level for period $t$, there are two aspects that should be taken into account. First, one should determine when the next production run will take place, say in period $t + i + 1$. That means that the production run of period $t$ should cover demand and safety stocks of periods $t$ to $t + i$, this is the basic order-up-to level. Second, there may be a need to increase the order-up-to level to compensate for (expected) waste throughout the replenishment cycle. Therefore, the inventory level at the end of period $t - 1$ and the age-distribution of the inventory should be determined. If the inventory on hand that can be used in period $t$ and (partly) later is unlikely to fulfil demand before it is out-dated, the basic order-up-to level in period $t$ should be increased by the expected amount of waste during periods $t$ to $t + i - 1$. The production quantity is then determined as the increased order-up-to level minus the inventory on hand at the end of in period $t$. In case the inventory on hand exceeds the order-up-to level, the excess stock will be carried forward. We name this order policy "waste-compensating" replenishment cycle policy.

## 5 Deterministic Mixed Integer Linear Programming approximation

We show how a deterministic MILP model can generate a waste-compensating replenishment cycle policy as an approximate solution of the SP model for perishable products. Therefore, we first discuss in Sections 5.1 to 5.3 the following ingredients of the SP model: the objective function (1), the service-level constraint (4) and the FIFO constraints (5) and (6). In Section 5.4, the complete model is presented.

### 5.1 Objective function

Consider the objective function (1). The holding cost is calculated as

$$\int_{d_1}\ldots\int_{d_T}\sum_{t=1}^{T}\left(h\max\left\{I_{1t},0\right\}+h\sum_{b=2}^{M-1}I_{bt}\right)g_1(d_1)\ldots g_T(d_T)\mathrm{d}d_1\ldots\mathrm{d}d_T \tag{11}$$

where no holding cost is paid over negative inventory. The service level α is usually chosen to be large, i.e. the probability of out-of-stock 1– α is small. The approximation assumes that the occurrence and amount of shortage is small enough to be neglected in the calculation of the holding cost (Bookbinder and Tan, 1988). That gives the following objective function:

$$\text{Min } E(TC)=\int_{d_1}\ldots\int_{d_T}\sum_{t=1}^{T}\left(kY_t+h\sum_{b=1}^{M-1}I_{bt}+cQ_t+wI_{Mt}\right)g_1(d_1)\ldots g_T(d_T)\mathrm{d}d_1\ldots\mathrm{d}d_T \tag{12}$$

which is equivalent to

$$\text{Min } E(TC)=\sum_{t=1}^{T}\left\{kY_t+h\sum_{b=1}^{M-1}E(I_{bt})+cE(Q_t)+wE(I_{Mt})\right\} \tag{13}$$

Eq. (13) is the objective function of the deterministic MILP model. Expressions for the expected values of $\sum_{b=1}^{M}I_{bt}$, $X_{bt}$, $S_t$ and $Q_t$ as function of the expected demand $d_t$ are straightforward. Expected values for the separate variables $I_{bt}$ are more complicated as we will specify when considering the FIFO constraints.

## 5.2 Service-level constraint

We start with a reformulation of the service-level constraint, by introducing a variable $S_t$ denoting the order-up-to level or starting inventory level at the beginning of period $t$. $S_t$ is defined by:

$$S_t = \sum_{b=1}^{M-1} I_{b,t-1} + Q_t \qquad t = 1,..,T \qquad (14)$$

When no order is placed, the variable $Q_t = 0$, and $S_t$ is just the ending inventory level of period $t - 1$. When an order is placed, $S_t$ is the order-up-to level. Now Eq. (3) can be rewritten as

$$\sum_{b=1}^{M} I_{bt} = S_t - d_t \qquad t = 1,..,T \qquad (15)$$

Eq. (4) requires the inventory level at the end of every period to be nonnegative with a probability of service level α. Using Eq. (15), Eq. (4) can be rewritten as

$$\mathrm{P}(d_t \le S_t) \ge \alpha \qquad t = 1,..,T \qquad (16)$$

where we require that the order-up-to level or starting inventory level of every period should be greater than the demand of that period, with a probability higher than the service level. Now consider period $t$, when the last order prior to period $t$ took place in period $t - j + 1$, to fulfil demand of $j$ periods. The next order is in period $t + 1$, with $j \in \{1,..,M\}$. So, for example, set $M \ge 3$. When $j = 3$ then $Y_{t-2} = 1$, $Y_{t-1} = 0$, $Y_t = 0$, $Y_{t+1} = 1$. When $j = 2$ then $Y_{t-1} = 1$, $Y_t = 0$, $Y_{t+1} = 1$. Let $G_{t-j+1,t}(.)$ be the cumulative probability distribution function of $d_{t-j+1} + d_{t-j+2} + \cdots + d_t$. To meet the desired service level we need

$$\mathrm{P}\left( \sum_{n=t-j+1}^{t} d_n \le S_{t-j+1} \right) \ge \alpha \qquad t = 1,..,T \qquad (17)$$

which implies

$$G_{t-j+1,t}(S_{t-j+1}) \ge \alpha \text{ or } S_{t-j+1} \ge G^{-1}_{t-j+1,t}(\alpha) \qquad t = 1,..,T \qquad (18)$$

such that

$$\sum_{b=1}^{M} I_{bt} \ge G^{-1}_{t-j+1,t}(\alpha) - \sum_{n=t-j+1}^{t} d_n \qquad t = 1,..,T \qquad (19)$$

Eq. (19) specifies that the safety stock at the end of period $t$, depends on the probability distribution of the demand of the previous periods, that the most recent order was meant to fulfil. The inventory level at the end of period $t$ may consist of items of different ages, including items of age $M$ that cannot be used in period $t + 1$. The required safety stock is known, given the period the order takes place. In this model, a finite planning horizon of $T$ periods is considered. Therefore, $G^{-1}_{t-j+1,t}(\alpha)$ can be calculated in advance for all relevant combinations of $j$ and $t$. Let $Z_{tj}$ be a binary variable that is equal to 1 if the most recent order prior to period $t$ was in period $t - j+1$, then we have that

$$\sum_{b=1}^{M} I_{bt} \ge \sum_{j=1}^{M} \left( G^{-1}_{t-j+1,t}(\alpha) - \sum_{n=t-j+1}^{t} d_n \right) \cdot Z_{tj} \qquad t = 1,..,T \qquad (20)$$

There can only be one most recent order period prior to period $t$, so

$$\sum_{j=1}^{M} Z_{tj} = 1 \qquad t = 1,..,T \qquad (21)$$

and

$$Z_{tj} \geq Y_{t-j+1} - \sum_{n=t-j+2}^{t} Y_n \qquad t = 1,..,T;\ j = 1,..,M \qquad (22)$$

Eq. (22) specifies that if $Z_{tj} = 1$, then the most recent order prior to period $t$ was in period $t - j+1$, so $Y_{t-j+1} = 1$, and $Y_{t-j+2}$ to $Y_t$ should all be 0. Otherwise, $Z_{tj}$ will be 0. Eq. (20) can be interpreted as the calculation of the safety stock needed to fulfil demand from periods $t - j + 1$ to $t$, when there is an order in period $t - j + 1$. The safety stock, or the inventory level at the end of period $t$ may consist of items of different ages, including items of age $M$, which can be used in period $t$, but not in period $t + 1$. These items are considered waste at the end of period $t$. Using expected values, Eq. (20) becomes

$$\sum_{b=1}^{M} E(I_{bt}) \geq \sum_{j=1}^{M} \left( G^{-1}_{t-j+1,t}(\alpha) - \sum_{n=t-j+1}^{t} E(d_n) \right) \cdot Z_{tj} \qquad t = 1,..,T \qquad (23)$$

## 5.3 FIFO constraints

Constraints (5) and (6) make sure that items are issued according to a FIFO policy. To study the effect of constraints (5) and (6), consider $M = 3$, with different values for the index $b$. In this setting Eq. (5) implies equations (24) and (25), and Eq. (6) becomes Eq. (26):

$$I_{3t} = \max\left\{ I_{2,\,t-1} - \max\left\{ d_t - 0,\, 0 \right\},\, 0 \right\} = \max\left\{ I_{2,\,t-1} - d_t,\, 0 \right\} \quad t = 1,..,T \qquad (24)$$

$$I_{2t} = \max\left\{ I_{1,\,t-1} - \max\left\{ d_t - I_{2,\,t-1},\, 0 \right\},\, 0 \right\} \qquad t = 1,..,T \qquad (25)$$

$$I_{1t} = Q_t - \max\left\{ d_t - I_{1,\,t-1} - I_{2,\,t-1},\, 0 \right\} \qquad t = 1,..,T \qquad (26)$$

To construct a deterministic MILP model, the expected values of these constraints are needed. Since the function $\max\{\cdot\}$ is a convex function, Jensen Inequality (Mood et al., 1974) applies. In the deterministic model we use the following equalities to approximate the stochastic programming model.

$$E(I_{3t}) = \max\left\{ E(I_{2,\,t-1}) - E(d_t),\, 0 \right\} \qquad t = 1,..,T \qquad (27)$$

$$E(I_{2t}) = \max\left\{ E(I_{1,\,t-1}) - \max\left\{ E(d_t) - E(I_{2,\,t-1}),\, 0 \right\},\, 0 \right\} \qquad t = 1,..,T \qquad (28)$$

$$E(I_{1t}) = E(Q_t) - \max\left\{ E(d_t) - E(I_{1,\,t-1}) - E(I_{2,\,t-1}),\, 0 \right\} \qquad t = 1,..,T \qquad (29)$$

According to Jensen Inequality the expected waste in Eq. (27) is underestimated and the expected inventory level of the freshest items in Eq. (29) is overestimated. Due to the nested function $\max\{\cdot\}$ in Eq. (28), Jensen Inequality does not apply and the approximation could be an under- or overestimation.

Consider Eq. (27). If $E(I_{2,\,t-1}) - E(d_t) \geq 0$ then $E(I_{2,\,t-1}) - E(d_t) = E(I_{3t})$. If $E(I_{2,\,t-1}) - E(d_t) \leq 0$ then $E(d_t) - E(I_{2,\,t-1}) \geq 0$. The value of $E(d_t) - E(I_{2,\,t-1}) \geq 0$ can be seen as the residual demand for the oldest items. This amount has to be fulfilled by fresher items as can be seen in Eq. (28). Let the auxiliary variable $E(X_{bt})$ denote the residual demand for items of age $b$ with $b = 1,..,M-1$ in period $t$. If $E(X_{bt})$ has a positive value, then fresher inventory is used to fulfil demand:

$$E(X_{2t}) = E(d_t) - E(I_{2,\,t-1}) \geq 0 \qquad (30)$$

Using $E(X_{2t})$ , Eq. (27) becomes

$$I_{2,\,t-1} - d_t = I_{3t} - X_{2t} \qquad (31)$$

and Eq. (28) becomes

$$E(I_{2t}) = \max\left\{ E(I_{1,\,t-1}) - E(X_{2t}),\, 0 \right\} \text{ or}$$

$$E(I_{1,\,t-1}) - E(X_{2t}) = E(I_{2t}) - E(X_{1t}). \qquad (32)$$

Finally, Eq. (29) can be formulated as

$$E(Q_t) - E(X_{1t}) = E(I_{1t}), \qquad (33)$$

where $E(I_{1t}) \geq 0$, because the deterministic model assumes there are no out-of-stocks. More generally, Eq. (5) and (6) are equivalent to:

$$\begin{cases} E(I_{M-1,t-1}) - E(d_t) = E(I_{Mt}) - E(X_{M-1,t}) \\ E(I_{M-2,t-1}) - E(X_{M-1,t}) = E(I_{M-1,t}) - E(X_{M-2,t}) \\ E(I_{M-3,t-1}) - E(X_{M-2,t}) = E(I_{M-2,t}) - E(X_{M-3,t}) \\ : \\ E(I_{1,t-1}) - E(X_{2t}) = E(I_{2t}) - E(X_{1t}) \\ E(Q_t) - E(X_{1t}) = E(I_{1t}) \end{cases} \quad t = 1,..,T \tag{34}$$

This set of equations handles the age-distribution of the items in stock. Adding up the equations of Eq. (34) results into Eq. (35).

$$\sum_{b=1}^{M} E(I_{bt}) = \sum_{b=1}^{M-1} E(I_{b,t-1}) + E(Q_t) - E(d_t) \quad t = 1,..,T \tag{35}$$

Eq. (35) is equivalent to Eq. (3) of the SP model. In Section 5.4, the complete set of FIFO constraints are written in Eq. (43) to (47).

## 5.4 MILP model for Perishable products

The complete deterministic MILP model that generates approximate solutions of the SP model is presented below.

$$\text{Min } E(TC) = \sum_{t=1}^{T} \left\{ kY_t + h\sum_{b=1}^{M-1} E(I_{bt}) + cE(Q_t) + wE(I_{Mt}) \right\} \tag{36}$$

$$\sum_{b=1}^{M} E(I_{bt}) = E(S_t) - E(d_t) \quad t = 1,..,T \tag{37}$$

$$E(Q_t) \leq \boldsymbol{M} Y_t \quad t = 1,..,T \tag{38}$$

$\boldsymbol{M}$ is a sufficiently large number, for instance $\boldsymbol{M} = \sum_{t=1}^{T} E(d_t)$. Because of the perishability, one will never order in the first period up to period $T$, so this amount will also cover for the necessary safety stocks.

$$E(S_t) \geq \sum_{j=1}^{M} \left( G_{t-j+1,t}^{-1}(\alpha) - \sum_{n=t-j+1}^{t} E(d_n) \right) \cdot Z_{tj} + E(d_t) \quad t = 1,..,T \tag{39}$$

Eq. (39) is a reformulation of Eq. (23) using the variable $E(S_t)$, instead of $E(I_{bt})$, to obtain the desired order-up-to levels to meet the α-service level requirement.

$$\sum_{j=1}^{M} Z_{tj} = 1 \quad t = 1,..,T \tag{40}$$

$$Z_{tj} \geq Y_{t-j+1} - \sum_{n=t-j+2}^{t} Y_n \quad t = 1,..,T;\ j = 1,..,M \tag{41}$$

$$E(S_t) - \sum_{b=1}^{M-1} E(I_{b,t-1}) = E(Q_t) \quad t = 1,..,T \tag{42}$$

Eq. (42) specifies the expected required production quantity. Waste is considered by leaving out items $E(I_{M,t-1})$ , because they cannot be used in period $t$. Let the auxiliary variable $E(X_{bt})$ denote the residual demand for items of age $b$ with $b = 1,..,M-1$ in period $t$. If $E(X_{bt})$ has a positive value, then fresher inventory is used to fulfil demand.

$$E(I_{M-1,t-1}) - E(d_t) = E(I_{Mt}) - E(X_{M-1,t}) \quad t = 1,..,T \tag{43}$$

$$E(I_{b,t-1}) - E(X_{b+1,t}) = E(I_{b+1,t}) - E(X_{bt}) \qquad t = 1,..,T;\ b = 1,..,M-2 \qquad (44)$$

$$E(Q_t) - E(X_{1t}) = E(I_{1t}) \qquad t = 1,..,T \qquad (45)$$

Eq. (43), (44) and (45) keep track of the age-distribution of the items in stock, under a FIFO-issuing policy. Eq. (43) imposes the oldest inventory to be used first to fulfil demand. What is left over has the maximum shelf life and will become waste, or there will be a residual demand for the oldest items. In the latter case Eq. (44) is appropriate. The residual demand has to be fulfilled by items of intermediate ages, until the demand is fulfilled by the freshest items that are produced in the current period, according to Eq. (45).The right-hand-sides of equations (43) and (44) can each contain at most one variable with a positive value. The other variable needs to have a value of 0. Equations (46), and (47) impose that, using the binary variable $BX_{bt}$.

$$\boldsymbol{M} \cdot BX_{bt} \geq E(X_{bt}) \qquad t = 1,..,T;\ b = 1,..,M-1 \qquad (46)$$

$$\boldsymbol{M} \cdot (1 - BX_{bt}) \geq E(I_{b+1,t}) \qquad t = 1,..,T;\ b = 1,..,M-1 \qquad (47)$$

$$E(I_{b0}) = 0 \qquad b = 1,..,M \qquad (48)$$

$$E(I_{bt}),\ E(S_t),\ E(Q_t) \geq 0 \qquad t = 1,..,T;\ b = 1,..,M \qquad (49)$$

$$E(X_{bt}) \geq 0 \qquad t = 1,..,T;\ b = 1,..,M-1 \qquad (50)$$

$$Y_t,\ Z_{tj} \in \{0, 1\} \qquad t = 1,..,T;\ j = 1,..,M \qquad (51)$$

$$BI_{bt} \in \{0, 1\} \qquad t = 1,..,T;\ b = 1,..,M \qquad (52)$$

$$BX_{bt} \in \{0, 1\} \qquad t = 1,..,T;\ b = 1,..,M-1 \qquad (53)$$

The starting inventory is zero (Eq.(48)). Eq. (49) to (53) are definition constraints. The required safety stocks are part of the inventory level. This model is an extension of the MILP model formulated by (Tarim and Kingsman, 2004), considering the age-distribution of the items in stock and the FIFO constraints. Their model provides an optimal solution for an SP model for non-perishables. Specifically, the addition of the FIFO constraints makes that the MILP model for perishable products generates approximate solutions for the SP model.

## 6 Numerical illustration of the MILP model

In Section 6.1, we illustrate the waste-compensating replenishment cycle policy of the model with a numerical example. The chosen parameter values are extreme to demonstrate the behaviour of the model. In Section 6.2, we relax the FIFO constraints for the same numerical example, to show that FIFO is not necessarily an optimal issuing policy. In Section 7, where the results of the MILP model are presented, the parameter values are chosen closer to practice.

### 6.1 Replenishment cycle policy with FIFO issuance

A numerical example illustrates the waste-compensating replenishment cycle policy of the model for a product with a fixed (internal) shelf life of 3 periods. We assume that the demand in each period is normally distributed, with a Coefficient of Variation (CV) of 0.333. Demand forecasts $E(d_t)$ are given in Table 1. The fixed setup cost is set to $k = 3000$, the unit production cost to $c = 2$ and the holding cost to $h = 1$. The cost of waste or markdown of the product $w$ is 4. We use a service level of $\alpha = 95\%$.

**Table 1** Forecasts and standard deviations of demand with a constant CV = 0.333

| Period $t$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $E(d_t)$ | 1900 | 950 | 40 | 80 | 30 | 150 | 800 | 950 | 1100 | 350 | 150 | 700 |
| $St.dev.(d_t)$ | 632.7 | 316.4 | 13.32 | 26.64 | 9.99 | 49.95 | 266.4 | 316.4 | 366.3 | 116.6 | 49.95 | 233.1 |

The safety stocks in Eq. (39) to meet a 95% service level are given in Table 2. For example, the safety stock at the end of period $t = 3$ is 521 (highlighted in Table 2) when the most recent order prior to period 3 was in period $t - j+1 = 3 - 2 + 1 = 2$, for $j = 2$ periods.

**Table 2** Safety stocks when the most recent order prior to period $t$ was in period $t - j+1$: ordering for $j$ periods

| $j$ \ $t$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1041 | 521 | 22 | 44 | 17 | 83 | 439 | 521 | 603 | 192 | 83 | 384 |
| 2 | | 1164 | 521 | 49 | 47 | 84 | 446 | 681 | 797 | 633 | 209 | 393 |
| 3 | | | 1164 | 523 | 52 | 95 | 447 | 686 | 909 | 819 | 638 | 437 |

The optimisation provides the policy given in Table 3. Orders occur in periods 1, 2, 4, 7, 9, 10 and 12, seven times. The order-up-to level of period 2 is equal to 1511. This is the amount to fulfil demand of periods 2 and 3 and the safety stock at $t = 3$ and $j = 2$ according to Table 2. The actual amount ordered is equal to the order-up-to level minus the inventory at the end of period 1. To fulfil demand in period 2, the one-period-old items of period 1 are used. After fulfilling demand there are still 91 items left. These 91 items are two periods old at the end of period 2. The fresh produced items are not used. They are one period old at the end of period 2. In period 3 the demand for 40 items is fulfilled from the 91 two-periods-old items of period 2, resulting in 51 items of waste at the end of period 3. In period 4 a new production run takes place, to fulfil demand of periods 4, 5 and 6 and the safety stock for these periods. So 80 + 30 + 150 + 95 = 355 items are required, and one would expect an order-up-to level of 355. Instead, $E(S_4) = 745$ in Table 3. Note that the demand of period 4 is fulfilled by the two-periods-old items of period 3. Afterwards, there are 390 items waste, which cannot be used in periods 5 and 6. The order-up-to level of period 4 is corrected for the amount of waste: 355 + 390 = 745, so the order-up-to level is waste-compensating. As illustrated, the MILP model determines order-up-to levels by taking into account the expected age-distribution of the inventory rather than only the actual inventory level.

**Table 3** Order policy and model output for the example problem, CV = 0.333

| $t$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $E(S_t)$ | 2941 | 1511 | 561 | 745 | 275 | 245 | 2431 | 1631 | 1703 | 709 | 359 | 1084 |
| $E(Q_t)$ | 2941 | 470 | 0 | 275 | 0 | 0 | 2431 | 0 | 1022 | 106 | 0 | 978 |
| $E(d_t)$ | 1900 | 950 | 40 | 80 | 30 | 150 | 800 | 950 | 1100 | 350 | 150 | 700 |
| $\Sigma_b E(I_{bt})$ | 1041 | 561 | 521 | 665 | 245 | 95 | 1631 | 681 | 603 | 359 | 209 | 384 |
| $E(I_{1t})$ | 1041 | 470 | 0 | 275 | 0 | 0 | 1631 | 0 | 603 | 106 | 0 | 384 |
| $E(I_{2t})$ | 0 | 91 | 470 | 0 | 245 | 0 | 0 | 681 | 0 | 253 | 106 | 0 |
| $E(waste_t)$ | 0 | 0 | 51 | 390 | 0 | 95 | 0 | 0 | 0 | 0 | 103 | 0 |

## 6.2 Relaxation of FIFO

In the practical decision problem a FIFO issuing policy is used. A different approach to determine a production plan is to use no predetermined issuing policy. Therefore, FIFO constraints (43) to (47) are replaced by the inventory balance constraints (54) to (56).

$$\sum_{b=1}^{M-1} E(I_{b,t-1}) - E(d_t) + E(Q_t) = \sum_{b=1}^{M} E(I_{b,t}) \qquad t = 1,..,T \qquad (54)$$

$$E(I_{b,t-1}) \geq E(I_{b+1,t}) \qquad t = 1,..,T;\ b = 1,..,M-1 \qquad (55)$$

$$E(Q_t) \geq E(I_{1,t}) \qquad t = 1,..,T \qquad (56)$$

Application of these constraints to the numerical example of Section 6.1, results into the policy shown in Table 4. Notice that in period 2, 390 fresh items are used, while older items are still available. Due to the relaxation of the FIFO constraints, the expected total costs of this production plan are reduced to 45968 compared to those of the FIFO production plan of 46358. For this specific instance, the amount of waste is the same, but the timing is different, resulting in lower holding cost. This example shows that, although FIFO seems intuitively the logical issuing rule to provide low costs, it is not necessarily the optimal way to issue the items.

**Table 4** Order policy and model output for the example problem, CV = 0.333, no issuing policy

| $t$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $E(S_t)$ | 2941 | 1511 | 561 | 355 | 275 | 245 | 2431 | 1631 | 1703 | 709 | 359 | 1084 |
| $E(Q_t)$ | 2941 | 470 | 0 | 275 | 0 | 0 | 2431 | 0 | 1022 | 106 | 0 | 978 |
| $E(d_t)$ | 1900 | 950 | 40 | 80 | 30 | 150 | 800 | 950 | 1100 | 350 | 150 | 700 |
| $\Sigma_b E(I_{bt})$ | 1041 | 561 | 521 | 275 | 245 | 95 | 1631 | 681 | 603 | 359 | 209 | 384 |
| $E(I_{1t})$ | 1041 | 80 | 0 | 275 | 0 | 0 | 1631 | 0 | 603 | 106 | 0 | 384 |
| $E(I_{2t})$ | 0 | 481 | 80 | 0 | 245 | 0 | 0 | 681 | 0 | 253 | 106 | 0 |
| $E(waste_t)$ | 0 | 0 | 441 | 0 | 0 | 95 | 0 | 0 | 0 | 0 | 103 | 0 |

## 7 Results of the MILP model

This section, we  investigate the behaviour of the model for different parameter values and different demand patterns. Section 7.1 describes the design of experiments. In Section 7.2, we show by Monte Carlo simulation to what extend the MILP policies meet the service level requirements. In Section 7.3, we study the influence of the service level, the cost of waste and the coefficient of variation for different values of the setup cost and different demand patterns, on expected total costs, percentage of waste and the production plan.

### 7.1 Design of experiments

The model is tested with a shelf life of $M = 3$, initially assuming a demand pattern that is erratic due to promotions in weeks 1, 2, 4, 5, 8 and 9, as depicted in Fig. 1. Table 5 reports the design of experiments: in total 84 experiments are done. Systematically we vary fixed setup cost $k$ (1500, 500 and 2000), cost of waste $w$ (-0.5, 0 and 0.5), α-service levels (90%, 95% and 98%), and the CV (0.1, 0.25 and 0.333). The underlined values are our base parameter values. The other cost values are constant: unit production cost $c = 2$ and unit holding cost $h = 0.5$. Note, negative cost of waste means the product has a salvage value, which is usually much less than the unit production cost $c$, zero cost of waste means that only the unit production cost are lost in case of waste, and positive cost of waste means that there is a cost to discard the wasted items.

For the base parameter values we tested also a variant of the erratic demand pattern, with different mean demands per period but with the same overall mean and standard deviation (experiment 82). In the erratic variant a clustering of promotions can be observed in weeks 3 and 10. In experiments 83 and 84, the effect of the two other demand patterns depicted in Fig. 1 are studied: a highly erratic demand pattern and a stationary demand pattern. The total expected demand is 7200 for all patterns.

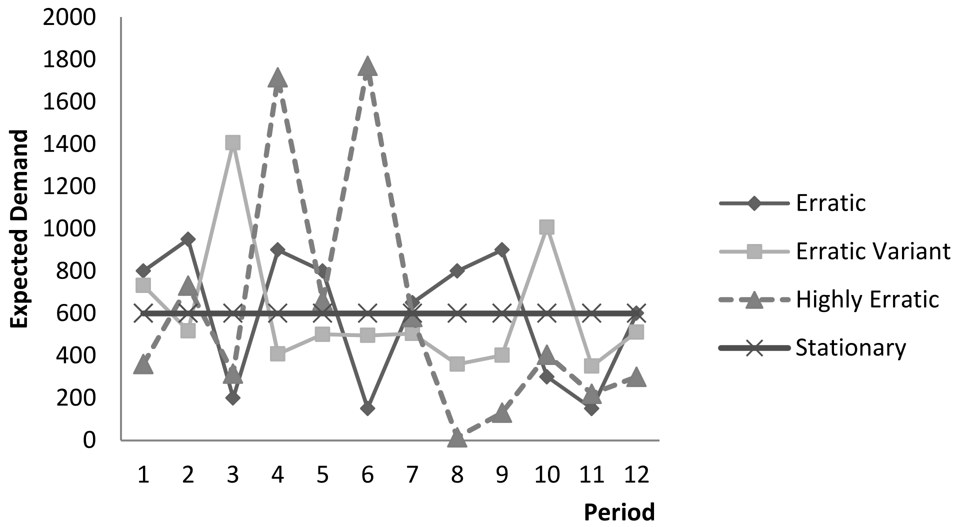


**Fig. 1** Demand patterns

**Table 5** Design of Experiments

| Scenario | Demand | k | w | α-service (%) | CV |
|---|---|---|---|---|---|
| Base | Erratic | 1500 | 0 | 95 | 0.25 |
| 1 – 9 | Erratic | 1500 | -0.5, 0, 0.5 | 90, 95, 98 | 0.10 |
| *10 – 18 | Erratic | 1500 | -0.5, 0, 0.5 | 90, 95, 98 | 0.25 |
| 19 – 27 | Erratic | 1500 | -0.5, 0, 0.5 | 90, 95, 98 | 0.33 |
| 28 – 36 | Erratic | 500 | -0.5, 0, 0.5 | 90, 95, 98 | 0.10 |
| 37 – 45 | Erratic | 500 | -0.5, 0, 0.5 | 90, 95, 98 | 0.25 |
| 46 – 54 | Erratic | 500 | -0.5, 0, 0.5 | 90, 95, 98 | 0.33 |
| 55 – 63 | Erratic | 2000 | -0.5, 0, 0.5 | 90, 95, 98 | 0.10 |
| 64 – 72 | Erratic | 2000 | -0.5, 0, 0.5 | 90, 95, 98 | 0.25 |
| 73 – 81 | Erratic | 2000 | -0.5, 0, 0.5 | 90, 95, 98 | 0.33 |
| 82 | Err Variant | 1500 | 0 | 95 | 0.25 |
| 83 | Highly Err | 1500 | 0 | 95 | 0.25 |
| 84 | Stationary | 1500 | 0 | 95 | 0.25 |

* including the base case

## 7.2 Quality of the MILP model approach

The MILP model provides approximate solutions in terms of the service level. We investigate how well the MILP solutions approximate the service level requirements. The solutions corresponding to the experiments listed in Table 5 were evaluated by simulating the inventory system using the same (pseudo) random number series of 5000 runs, what implies that the costs are within 0.1% of the true costs with 95% confidence. Each simulation run starts with no inventory in stock and lasts $T = 12$ periods. For every period $t$ the α-service level is computed by calculating the fraction of runs in which the inventory at the end of period $t$ is nonnegative. The resulting α-service levels are presented for the base case in the next-to-last row of Table 6.

**Table 6** Base case: MILP production plan and simulation results of measured service level and fill rate

| $T$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $E(S_t)$ | 1129 | 1550 | 600 | 2350 | 1450 | 650 | 1874 | 1224 | 1271 | 1333 | 1033 | 883 |
| $E(Q_t)$ | 1129 | 1221 | 0 | 1950 | 0 | 0 | 1874 | 0 | 847 | 962 | 0 | 0 |
| $E(d_t)$ | 800 | 950 | 200 | 900 | 800 | 150 | 650 | 800 | 900 | 300 | 150 | 600 |
| $\Sigma_b E(I_{bt})$ | 329 | 600 | 400 | 1450 | 650 | 500 | 1224 | 424 | 371 | 1033 | 883 | 283 |
| $E(I_{1t})$ | 329 | 600 | 0 | 1450 | 0 | 0 | 1224 | 0 | 371 | 962 | 0 | 0 |
| $E(I_{2t})$ | 0 | 0 | 400 | 0 | 650 | 0 | 0 | 424 | 0 | 71 | 883 | 0 |
| $E(waste_t)$ | 0 | 0 | 0 | 0 | 0 | 500 | 0 | 0 | 0 | 0 | 0 | 283 |
| α-service level | 94.9 | 99.5 | 95.4 | 100 | 98.6 | 95.3 | 100 | 95.1 | 94.8 | 100 | 100 | 89.0 |
| fill rate | 100 | 100 | 97.8 | 100 | 99.8 | 96.0 | 100 | 99.4 | 99.5 | 100 | 100 | 98.2 |

For the base case, the MILP order policy prescribes to order in periods 1, 2, 4, 7, 9 and 10. We observe that the α-service level of 95% is met in 9 periods. In periods 1 and 9, the service level is almost met: the difference may not be significant from a statistical point of view and is acceptable from a practical point of view. In period 12 the service level is only 89.0%, this may be well accepted in practice as production plans are usually updated every week. Reasons for not meeting the service level in period 12 are the combination of ordering in period 10, a low demand in periods 10 and 11 and a higher demand in period 12. In period 10 there is an expected starting inventory of 371. If the total demand of periods 10 and 11 is less than the starting inventory, there will be waste at the end of period 11 and if the demand in period 12 is above the expected demand, there will be out-of-stock. However, consider the fill rate, which measures how well demand is met and consequently, how much shortage takes place. One can observe from the measurements that in period 12, the fill rate is 98.2%, so there was limited shortage.

There are two main reasons why service levels are not always met. First, the real inventory levels are fluctuating around the expected values, while the model is compensating for the expected waste. As explained in Section 5.3, the expected waste in the MILP model is underestimated, while the expected inventory level of the freshest items is overestimated. Second, the combination of demand pattern and parameter values may result in inconvenient production moments. In some cases, the replenishment cycle consists of multiple periods. When the expected demand increases and the starting inventory level is relatively high or consists of items of different ages, the service level requirement might not be met.

In Appendix B, the service levels for all periods of every experiment is listed. The last column reports the Sum of Squared Errors of the α-service level: SSE(α) = $\Sigma_t$ (max{0, α – realised service level in period $t$})$^2$. A service level above the requirements is considered as good, so this is not considered an error. The lower the SSE(α), the better the service levels are met. For a quick overview of the quality of the MILP approximation, we also report in Appendix C the SSE(α) for every experiment.

In the experiments, the setup cost of $k$ = 500 gives, for a CV of 0.25 and 0.33, service levels close to the requirements. There are production runs in 9 of the 12 periods: only when demand is low, a production run is skipped. For a CV of 0.10, there are less production runs, resulting in no production in period 12, where the demand is high. The service level in period 12 is 80.4%. A setup cost of $k$ = 2000 gives in most instances an order policy of ordering in period 1, 4, 7 and 10. Considering the maximum shelf life, this is the minimum amount of production runs that is needed. That means that the inventory level before production starts is 0. There is no waste during the replenishment cycle due to inventory on hand. The service levels are close to the requirements with this order policy.

In the experiments, the setup cost of $k$ = 1500 gives mixed results with respect to meeting the service level requirements, they are reasonable, with some ups and downs. The order policies prescribe more than the minimum amount of 4 production runs and less than 9, which results in a considerable amount of older items in stock. Table 7 shows the experiment with highly erratic demand and the base parameter values. In period 9 the service level is 73.0%. The last production run before this period takes place in period 7, when there was a relatively high inventory level followed by relatively small amounts of demand. When the realised waste in period 8 is more than expected, the chance of out-of-stock in period 9 is higher than allowed by the service level requirement.

**Table 7** Highly Erratic demand: MILP production plan and simulation results of measured service level and fill rate

| $t$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $E(S_t)$ | 1764 | 1404 | 674 | 3131 | 1416 | 2498 | 1104 | 522 | 376 | 1150 | 746 | 526 |
| $E(Q_t)$ | 1764 | 0 | 0 | 3131 | 0 | 1742 | 376 | 0 | 0 | 1150 | 0 | 0 |
| $E(d_t)$ | 360 | 730 | 315 | 1715 | 660 | 1770 | 582 | 14 | 130 | 404 | 220 | 300 |
| $\Sigma_b E(I_{bt})$ | 1404 | 674 | 359 | 1416 | 756 | 728 | 522 | 508 | 246 | 746 | 526 | 226 |
| $E(I_{1t})$ | 1404 | 0 | 0 | 1416 | 0 | 728 | 376 | 0 | 0 | 746 | 0 | 0 |
| $E(I_{2t})$ | 0 | 674 | 0 | 0 | 756 | 0 | 146 | 376 | 0 | 0 | 526 | 0 |
| $E(waste_t)$ | 0 | 0 | 359 | 0 | 0 | 0 | 0 | 132 | 246 | 0 | 0 | 226 |
| α-service level | 100 | 99.9 | 95.1 | 99.9 | 95.6 | 95.2 | 100 | 100 | 73.0 | 100 | 100 | 94.6 |
| fill rate | 100 | 100 | 98.7 | 100 | 98.7 | 99.6 | 100 | 100 | 77.6 | 100 | 100 | 98.97 |

Over all the performed experiments, in 26.9% of the periods the service level requirement is not met. However, in 22.7% of the periods the realised service level is within 1% lower than the required service level. The MILP approximation provides a practical solution to the SP problem. As production plans are updated frequently, not meeting the service level in the last period(s) is less relevant. The performance of the MILP solutions with respect to the required service level becomes more complicated when there are many items (of different ages) in stock. The simulation determines the order quantity as the order-up-to level of the MILP model minus the on-hand inventory level of ages one and two. The results show that in the application of the model, it is important to investigate the need of considering the age-distribution of the items in stock for the determination of the order quantity.

### 7.3 Sensitivity analysis

Appendix C shows the results of the MILP model for the experiments listed in Table 5. For every experiment we report the expected total costs, the total expected production quantity, waste as a percentage of the total expected production quantity and the order policy denoting the periods in which there is a production run. The SSE(α) gives an indication of the quality of the MILP approximation with respect to meeting the service level requirements, as explained in Section 7.2.

*Summary of the sensitivity analysis: insights*

Table 8 shows a summary of the sensitivity analysis. The effect of varying the α-service level or the CV on the expected total costs follows the intuition: the higher the service level or the CV, the higher the expected total costs $E(TC)$. However, in the experiments, the effect on the total expected order quantity $E(Q)$ and the percentage of waste is not unidirectional, because the number of production runs as well as the timing of production runs may change. More production runs lead to a lower total expected order quantity and less waste. A higher cost of waste or a lower setup cost may increase the number of production runs.

**Table 8** Summary of the sensitivity analysis

| Vary: | Observed effect on: | | | |
|---|---|---|---|---|
| | *E*(*TC*) | Total *E*(*Q*) | % Waste | # production runs |
| α-service level ↑ | ↑ | ↑ or ↓ | ↑ or ↓ | ↑ or = |
| CV ↑ | ↑ | ↑ or ↓ | ↑ or ↓ | ↑ or = |
| Cost of waste ↑ | ↑ or = | ↓ or = | ↓ or = | ↑ or = |
| Setup cost ↓ | ↓ | ↓ or = | ↓ or = | ↑ or = |

*Influence of α-service level and cost of waste*

In Fig. 2 the influence of the α-service level and the cost of waste on the number of production runs en percentage of waste is depicted. In the base case an increase in service level from 95% to 98% leads to a 2.5% increase of the expected total costs and a decrease of percentage of waste from 9.8% to 4.8%, due to a change in the production plan, from 6 to 7 production runs. A decrease in service level from 95% to 90% leads to a 3.2% decrease of the expected total costs and the percentage of waste increases up to 12.3%. When the cost of waste is -0.5, an increase in service level results in higher order-up-to levels but not in a different timing or a different number of production runs. Consequently, the percentage of waste increases. When the cost of waste is increased to 0.5, an increase in service level from 90% to 95% leads to a change in production plan and the percentage of waste decreases. A further increase in service level from 95% to 98% results only in higher order-up-to levels but not in a different timing or a different number of production runs. Consequently, the percentage of waste increases. Clearly, the model can be used to manage the amount of waste, while maintaining a certain service level. For example, increasing the cost of waste from 0 to 0.5 reduces waste from 9.8% to 3.8% of the total expected production quantity.

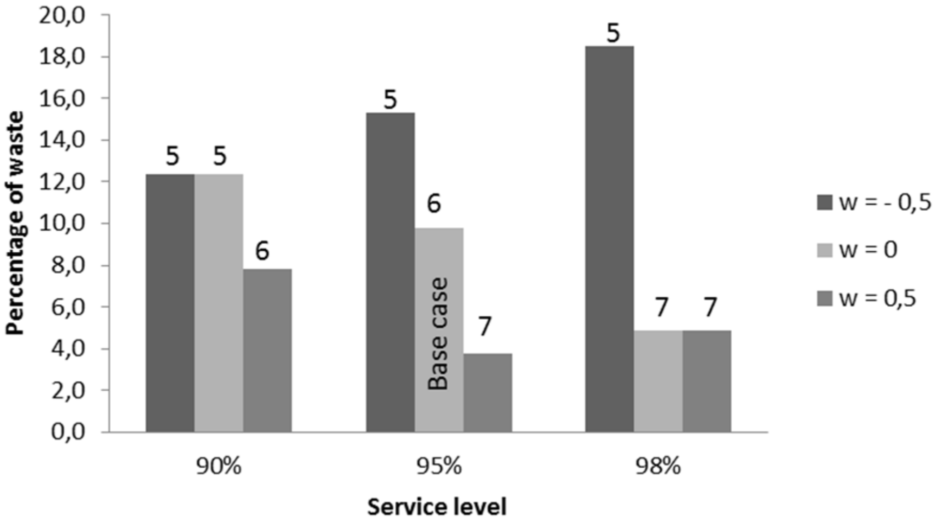


**Fig. 2** The case of erratic demand and CV = 0.25: influence of service level on number of production runs and percentage of waste; influence of the cost of waste on percentage of waste at different service levels. The numbers on top of the bars denote the number of production runs

*Influence of setup cost*

Motivated by the practical problem, the setup cost in the base case is $k$ = 1500. Varying the other parameter values we observe frequent changes in the order policies. A decrease of setup cost to 500 results in more frequent production runs and low percentages of waste in all 27 experiments considered. In some experiments with $w$=-0.5 or $w$=0 the expected waste is reduced to 0. For experiments at which the expected waste is zero, increasing the cost of waste does not change the production plan. An increase of the setup cost to 2000 shows order policies with a minimum number of production runs.

*Comparing the consequence of demand patterns*
The base case parameter values have also been applied to other demand patterns. The solutions of the MILP model are in line with the production plans for the erratic demand pattern. The order policy for the stationary demand is a regular production plan, with production in every other period.

## 8 Conclusions

We studied the practical production planning problem of a food producer facing a non-stationary erratic demand for a perishable product with a fixed life time, under a service-level constraint. The case includes a fixed setup cost for every production run, zero lead time and a First In First Out issuing policy. In case of out-of-stock, demand is backlogged. A theoretical Stochastic Programming model for this problem has been presented, that considers the age-distribution of the items in stock. The question is how to generate a waste-compensating replenishment cycle policy by applying commercial MILP solvers. Therefore, an MILP model has been formulated to generate approximate solutions. A solution provides a plan specifying simultaneously the periods to produce and the corresponding order-up-to-levels. To meet a certain α-service level, the model considers and corrects for the expected age-distribution of the items in stock. The model can be solved in a fraction of a second. This makes it interesting for practical application.

The MILP solutions provide approximations of the required service level. Simulation shows that in 95.8% of the periods the service level requirements are met, with an error tolerance of 1%. The performance of the MILP model with respect to service level requires attention when there are many items of different ages in stock. In the application of the model, it is important to investigate the need of considering the age-distribution of the items in stock for the determination of the order quantity.

Currently we are investigating the applicability of this model in a food company under a rolling horizon setting and a fixed lead time. The company faces an erratic stochastic demand and has service levels to meet to its customers. Future research aims to adapt the model using fill rates instead of service levels, and to incorporate lost sales instead of backlogging. Another interesting option would be to investigate the incorporation of a decision variable to get rid of excessive inventory before it becomes waste, to reduce holding cost. Also a model for multiple products, taking production capacities into account, is an interesting extension for practice. Finally, the model can be modified to accommodate a Last In First Out issuing policy, or even a mixed FIFO/LIFO policy, to be able to use the model in a situation (e.g. retail) in which the customer selects the items from the shelf.

**Acknowledgement**
This research has been supported and partly funded by (1) TI Food and Nutrition (project RE002), The Netherlands, (2) the European Community's Seventh Framework Programme (FP7) under grant agreement no 244994 (project VEGi-TRADE), and (3) The Spanish Ministry of Science and Innovation (project TIN2008-01117) and Junta de Andalucía (P11-TIC-7176), in part financed by the European Regional Development Fund (ERDF).

## References

Bakker, M., Riezebos, J., Teunter, R.H., 2012. Review of inventory systems with deterioration since 2001. Eur. J. Oper. Res. 221, 275-284.

Bookbinder, J.H., Tan, J.Y., 1988. Strategies for the probabilistic lot-sizing problem with service-level constraints. Manage. Sci. 34, 1096-1108.
Broekmeulen, R., van Donselaar, K.H., 2009. A heuristic to manage perishable inventory with batch ordering, positive lead-times, and time-varying demand. Comput. Oper. Res. 36, 3013-3018.
Chopra, S., Meindl, P., 2010. Supply Chain Management: Strategy, Planning, and Operation, Fourth Edition ed. Pearson Education, Inc., New Yersey.
Fries, B.E., 1975. Optimal Ordering Policy for a Perishable Commodity with Fixed Lifetime. Operations Research 23, 46-61.
Goyal, S.K., Giri, B.C., 2001. Recent trends in modeling of deteriorating inventory. Eur. J. Oper. Res. 134, 1-16.
Graves, S.C., Willems, S.P., 2000. Optimizing Strategic Safety Stock Placement in Supply Chains. Manufacturing & Service Operations Management 2, 68-83.
Graves, S.C., Willems, S.P., 2008. Strategic inventory placement in supply chains: Nonstationary demand. Manufacturing & Service Operations Management 10, 278-287.
Haijema, R., 2011. A new class of stock-level dependent ordering policies for perishables with a short maximum shelf life. Int. J. Prod. Econ. doi:10.1016/j.ijpe.2011.05.021.
Haijema, R., van der Wal, J., van Dijk, N.M., 2007. Blood platelet production: Optimization by dynamic programming and simulation. Comput. Oper. Res. 34, 760-779.
Haijema, R., van Dijk, N., van der Wal, J., Sibinga, C.S., 2009. Blood platelet production with breaks: optimization by SDP and simulation. Int. J. Prod. Econ. 121, 464-473.
Karaesmen, I.Z., Scheller-Wolf, A., Deniz, B., 2011. Managing Perishable and Aging Inventories: Review and Future Research Directions, in: Kempf, K.G., Keskinocak, P., Uzsoy, R. (Eds.), Planning Production and Inventories in the Extended Enterprise Springer, pp. 393-436.
Minner, S., Transchel, S., 2010. Periodic review inventory-control for perishable products under service-level constraints. Or Spectrum 32, 979-996.
Mood, A.M., Graybill, F.A., Boes, D.C., 1974. Introduction to the theory of statistics Third Edition ed. McGraw-Hill.
Nahmias, S., 1975. Optimal Ordering Policies for Perishable Inventory-II. Operations Research 23, 735-749.
Nahmias, S., 1982. Perishable inventory-theory - a review. Operations Research 30, 680-708.
Neale, J.J., Willems, S.P., 2009. Managing Inventory in Supply Chains with Nonstationary Demand. Interfaces 39, 388-399.
Pujawan, I.N., Silver, E.A., 2008. Augmenting the lot sizing order quantity when demand is probabilistic. Eur. J. Oper. Res. 188, 705-722.
Rossi, R., Tarim, S.A., Bollapragada, R., 2011a. Constraint-Based Local Search for Inventory Control Under Stochastic Demand and Lead Time. INFORMS Journal on Computing.
Rossi, R., Tarim, S.A., Hnich, B., Prestwich, S., 2010. Computing the non-stationary replenishment cycle inventory policy under stochastic supplier lead-times. Int. J. Prod. Econ. 127, 180-189.
Rossi, R., Tarim, S.A., Hnich, B., Prestwich, S., 2011b. A state space augmentation algorithm for the replenishment cycle inventory policy. Int. J. Prod. Econ. 133, 377-384.
Silver, E.A., Pyke, D.F., Peterson, R., 1998. Inventory Management and Production Planning and Scheduling, Third Edition ed. John Wiley & Sons.
Tarim, S.A., Dogru, M.K., Özen, U., Rossi, R., 2011. An efficient computational method for a stochastic dynamic lot-sizing problem under service-level constraints. Eur. J. Oper. Res. 215, 563-571.
Tarim, S.A., Kingsman, B.G., 2004. The stochastic dynamic production/inventory lot-sizing problem with service-level constraints. Int. J. Prod. Econ. 88, 105-119.

Tekin, E., Gurler, U., Berk, E., 2001. Age-based vs. stock level control policies for a perishable inventory system. Eur. J. Oper. Res. 134, 309-329.
Tempelmeier, H., 2007. On the stochastic uncapacitated dynamic single-item lotsizing problem with service level constraints. Eur. J. Oper. Res. 181, 184-194.
Tempelmeier, H., 2011. A column generation heuristic for dynamic capacitated lot sizing with random demand under a fill rate constraint. Omega 39, 627-633.
Tunc, H., Kilic, O.A., Tarim, S.A., Eksioglu, B., 2011. The cost of using stationary inventory policies when demand is non-stationary. Omega 39, 410-415.
van der Vorst, J.G.A.J., Beulens, A.J.M., van Beek, P., 2000. Modelling and simulating multi-echelon food systems. Eur. J. Oper. Res. 122, 354-366.

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## Appendix A

**Table A.1** Used symbols, where applicable: E(·) = expected value of…

| SP | MILP | |
|---|---|---|
| $T$ | $T$ | maximum number of periods of the finite time horizon |
| $t$ | $t$ | index denoting the period |
| $M$ | $M$ | fixed maximum (internal) shelf life |
| $b$ | $b$ | index denoting the age of the item |
| $k$ | $k$ | fixed setup cost for every production run |
| $c$ | $c$ | variable production cost per item produced |
| $h$ | $h$ | holding cost per item, for items that are carried over from one period to the next |
| $w$ | $w$ | disposal cost per item of waste |
| α | α | service level |
| | $\boldsymbol{M}$ | big number |
| $d_t$ | $E(d_t)$ | non-stationary stochastic demand |
| $g_t(\cdot)$ | $g_t(\cdot)$ | probability density function of demand $d_t$ |
| $G_t(\cdot)$ | $G_t(\cdot)$ | cumulative distribution function of demand $d_t$ |
| $I_{bt}$ | $E(I_{bt})$ | the inventory level of items with age $b$ at the end of period $t$ |
| $I_{Mt}$ | $E(I_{Mt})$ | inventory of age $M$ at the end of period $t$ is considered waste |
| $Q_t$ | $E(Q_t)$ | production quantity in period $t$ |
| $S_t$ | $E(S_t)$ | the order-up-to level or starting inventory level at the beginning of period $t$ |
| $E(TC)$ | $E(TC)$ | expected total costs over the time horizon |
| $Y_t$ | $Y_t$ | binary variable takes the value of 1 if there is a production run in period $t$, and 0 otherwise |
| | $Z_{tj}$ | binary variable takes the value of 1 if the most recent order prior to period $t$ was in period $t - j+1$, and 0 otherwise |
| | $E(X_{bt})$ | auxiliary variable denotes the residual demand for items of age $b$ with $b = 1,..,M-1$ in period $t$ |
| | $BX_{bt}$ | binary variable takes the value of 1 if $E(X_{bt}) > 0$, and 0 otherwise |

# Appendix B

**Table B.1** Results of the simulation: service levels for periods 1 to 12.

| No.\ t | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | Alpha | SSE(α) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 100.00 | 99.82 | 89.84 | 100.00 | 99.44 | 90.68 | 100.00 | 100.00 | 89.38 | 100.00 | 100.00 | 90.10 | 90 | 0.4 |
| 2 | 100.00 | 99.82 | 89.84 | 100.00 | 99.44 | 90.68 | 100.00 | 100.00 | 89.38 | 100.00 | 100.00 | 90.10 | 90 | 0.4 |
| 3 | 100.00 | 99.82 | 89.84 | 100.00 | 99.44 | 90.68 | 100.00 | 90.02 | 100.00 | 89.50 | 100.00 | 77.98 | 90 | 144.8 |
| 4 | 100.00 | 99.94 | 94.74 | 100.00 | 99.76 | 95.36 | 100.00 | 100.00 | 94.88 | 100.00 | 100.00 | 95.18 | 95 | 0.1 |
| 5 | 100.00 | 99.94 | 94.74 | 100.00 | 99.76 | 95.36 | 100.00 | 100.00 | 94.88 | 100.00 | 100.00 | 95.18 | 95 | 0.1 |
| 6 | 100.00 | 99.94 | 94.74 | 100.00 | 99.76 | 95.36 | 100.00 | 95.08 | 100.00 | 94.60 | 100.00 | 80.42 | 95 | 212.8 |
| 7 | 100.00 | 100.00 | 97.94 | 100.00 | 99.96 | 98.06 | 100.00 | 100.00 | 98.18 | 100.00 | 100.00 | 98.06 | 98 | 0.0 |
| 8 | 100.00 | 100.00 | 97.94 | 100.00 | 99.96 | 98.06 | 100.00 | 98.16 | 100.00 | 97.84 | 100.00 | 87.44 | 98 | 111.5 |
| 9 | 100.00 | 100.00 | 97.94 | 100.00 | 99.96 | 98.06 | 100.00 | 98.16 | 100.00 | 97.84 | 100.00 | 87.44 | 98 | 111.5 |
| 10 | 100.00 | 97.28 | 89.76 | 100.00 | 96.48 | 90.62 | 100.00 | 89.94 | 89.52 | 100.00 | 100.00 | 85.40 | 90 | 21.5 |
| 11 | 100.00 | 97.28 | 89.76 | 100.00 | 96.48 | 90.62 | 100.00 | 89.94 | 89.52 | 100.00 | 100.00 | 85.40 | 90 | 21.5 |
| 12 | 90.20 | 98.54 | 90.70 | 100.00 | 96.46 | 90.60 | 100.00 | 89.94 | 89.52 | 100.00 | 100.00 | 85.40 | 90 | 21.4 |
| 13 | 100.00 | 98.92 | 94.78 | 100.00 | 98.64 | 95.36 | 100.00 | 95.06 | 94.78 | 100.00 | 100.00 | 89.00 | 95 | 36.1 |
| 14 | 94.86 | 99.54 | 95.42 | 100.00 | 98.62 | 95.32 | 100.00 | 95.06 | 94.78 | 100.00 | 100.00 | 89.00 | 95 | 36.1 |
| 15 | 94.86 | 99.54 | 95.42 | 95.56 | 99.22 | 94.86 | 100.00 | 94.86 | 94.78 | 100.00 | 100.00 | 88.88 | 95 | 37.6 |
| 16 | 100.00 | 99.72 | 97.94 | 100.00 | 99.46 | 98.06 | 100.00 | 98.16 | 97.88 | 100.00 | 100.00 | 92.10 | 98 | 34.8 |
| 17 | 98.10 | 99.80 | 98.44 | 98.28 | 99.80 | 97.98 | 100.00 | 97.94 | 97.88 | 100.00 | 100.00 | 91.78 | 98 | 38.7 |
| 18 | 98.10 | 99.80 | 98.44 | 98.28 | 99.80 | 97.98 | 100.00 | 97.94 | 97.88 | 100.00 | 100.00 | 91.78 | 98 | 38.7 |
| 19 | 100.00 | 96.26 | 89.76 | 100.00 | 95.64 | 90.64 | 100.00 | 89.96 | 89.58 | 100.00 | 100.00 | 81.36 | 90 | 74.9 |
| 20 | 90.20 | 97.88 | 90.66 | 100.00 | 95.56 | 90.28 | 100.00 | 89.96 | 89.58 | 100.00 | 100.00 | 81.36 | 90 | 74.8 |
| 21 | 90.20 | 97.88 | 90.66 | 90.52 | 96.78 | 89.82 | 100.00 | 88.92 | 89.52 | 100.00 | 100.00 | 81.04 | 90 | 81.7 |
| 22 | 94.88 | 99.06 | 95.42 | 100.00 | 98.04 | 95.06 | 100.00 | 95.06 | 94.82 | 100.00 | 100.00 | 88.36 | 95 | 44.1 |
| 23 | 94.88 | 99.06 | 95.42 | 95.56 | 98.64 | 94.84 | 100.00 | 93.98 | 94.80 | 100.00 | 100.00 | 87.58 | 95 | 56.2 |
| 24 | 94.88 | 99.06 | 95.42 | 95.56 | 98.64 | 94.84 | 100.00 | 93.98 | 94.80 | 100.00 | 100.00 | 87.58 | 95 | 56.2 |
| 25 | 98.10 | 99.74 | 98.44 | 98.30 | 99.72 | 97.98 | 100.00 | 97.32 | 97.88 | 100.00 | 100.00 | 94.88 | 98 | 10.2 |
| 26 | 98.10 | 99.74 | 98.44 | 98.30 | 99.72 | 97.98 | 100.00 | 97.32 | 97.88 | 100.00 | 100.00 | 94.88 | 98 | 10.2 |
| 27 | 98.10 | 99.74 | 98.44 | 98.30 | 99.72 | 97.98 | 100.00 | 97.32 | 97.88 | 100.00 | 100.00 | 94.88 | 98 | 10.2 |
| 28 | 90.22 | 99.98 | 90.76 | 90.70 | 99.90 | 89.90 | 100.00 | 90.02 | 100.00 | 99.76 | 89.58 | 90.44 | 90 | 0.2 |
| 29 | 90.22 | 99.98 | 90.76 | 90.70 | 99.90 | 89.90 | 100.00 | 90.02 | 100.00 | 89.50 | 100.00 | 77.98 | 90 | 144.7 |
| 30 | 90.22 | 99.98 | 90.76 | 90.70 | 99.90 | 89.90 | 100.00 | 90.02 | 100.00 | 89.50 | 100.00 | 77.98 | 90 | 144.7 |
| 31 | 94.88 | 99.98 | 95.42 | 95.72 | 99.96 | 94.86 | 100.00 | 95.08 | 100.00 | 94.60 | 100.00 | 80.42 | 95 | 212.8 |
| 32 | 94.88 | 99.98 | 95.42 | 95.72 | 99.96 | 94.86 | 100.00 | 95.08 | 100.00 | 94.60 | 100.00 | 80.42 | 95 | 212.8 |
| 33 | 94.88 | 99.98 | 95.42 | 95.72 | 99.96 | 94.86 | 100.00 | 95.08 | 100.00 | 94.60 | 100.00 | 80.42 | 95 | 212.8 |
| 34 | 98.20 | 100.00 | 98.44 | 98.28 | 100.00 | 98.06 | 100.00 | 98.16 | 97.88 | 100.00 | 97.82 | 98.20 | 98 | 0.0 |
| 35 | 98.20 | 100.00 | 98.44 | 98.28 | 100.00 | 98.06 | 100.00 | 98.16 | 97.88 | 100.00 | 97.82 | 98.20 | 98 | 0.0 |
| 36 | 98.20 | 100.00 | 98.44 | 98.28 | 100.00 | 98.06 | 100.00 | 98.16 | 97.88 | 100.00 | 97.82 | 98.20 | 98 | 0.0 |
| 37 | 90.20 | 98.54 | 90.70 | 90.60 | 97.92 | 89.78 | 100.00 | 89.68 | 89.52 | 99.96 | 90.58 | 90.56 | 90 | 0.4 |
| 38 | 90.20 | 98.54 | 90.70 | 90.60 | 97.92 | 89.78 | 100.00 | 89.68 | 89.52 | 99.96 | 90.58 | 90.56 | 90 | 0.4 |
| 39 | 90.20 | 98.54 | 90.70 | 90.60 | 97.92 | 89.78 | 100.00 | 89.68 | 89.52 | 99.96 | 90.58 | 90.56 | 90 | 0.4 |
| 40 | 94.86 | 99.54 | 95.42 | 95.56 | 99.22 | 94.86 | 94.80 | 94.96 | 94.78 | 99.98 | 95.48 | 95.52 | 95 | 0.1 |
| 41 | 94.86 | 99.54 | 95.42 | 95.56 | 99.22 | 94.86 | 94.80 | 94.96 | 94.78 | 99.98 | 95.48 | 95.52 | 95 | 0.1 |
| 42 | 94.86 | 99.54 | 95.42 | 95.56 | 99.22 | 94.86 | 94.80 | 94.96 | 94.78 | 99.98 | 95.48 | 95.52 | 95 | 0.1 |

| No.\ t | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | Alpha | SSE(α) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 43 | 98.10 | 99.80 | 98.44 | 98.28 | 99.80 | 97.98 | 97.70 | 97.84 | 97.88 | 100.00 | 98.26 | 98.14 | 98 | 0.1 |
| 44 | 98.10 | 99.80 | 98.44 | 98.28 | 99.80 | 97.98 | 97.70 | 97.84 | 97.88 | 100.00 | 98.26 | 98.14 | 98 | 0.1 |
| 45 | 98.10 | 99.80 | 98.44 | 98.28 | 99.80 | 97.98 | 97.70 | 97.84 | 97.88 | 100.00 | 98.26 | 98.14 | 98 | 0.1 |
| 46 | 90.20 | 97.88 | 90.66 | 90.52 | 96.78 | 89.82 | 90.00 | 89.34 | 89.46 | 99.80 | 91.62 | 90.44 | 90 | 0.8 |
| 47 | 90.20 | 97.88 | 90.66 | 90.52 | 96.78 | 89.82 | 90.00 | 89.34 | 89.46 | 99.80 | 91.62 | 90.44 | 90 | 0.8 |
| 48 | 90.20 | 97.88 | 90.66 | 90.52 | 96.78 | 89.82 | 90.00 | 89.34 | 89.46 | 99.80 | 91.62 | 90.44 | 90 | 0.8 |
| 49 | 94.88 | 99.06 | 95.42 | 95.56 | 98.64 | 94.84 | 94.80 | 94.96 | 94.78 | 99.98 | 96.08 | 95.52 | 95 | 0.1 |
| 50 | 94.88 | 99.06 | 95.42 | 95.56 | 98.64 | 94.84 | 94.80 | 94.96 | 94.78 | 99.98 | 96.08 | 95.52 | 95 | 0.1 |
| 51 | 94.88 | 99.06 | 95.42 | 95.56 | 98.64 | 94.84 | 94.80 | 94.96 | 94.78 | 99.98 | 96.08 | 95.52 | 95 | 0.1 |
| 52 | 98.10 | 99.74 | 98.44 | 98.30 | 99.72 | 97.98 | 97.68 | 97.86 | 97.88 | 100.00 | 98.60 | 98.12 | 98 | 0.1 |
| 53 | 98.10 | 99.74 | 98.44 | 98.30 | 99.72 | 97.98 | 97.68 | 97.86 | 97.88 | 100.00 | 98.60 | 98.12 | 98 | 0.1 |
| 54 | 98.10 | 99.74 | 98.44 | 98.30 | 99.72 | 97.98 | 97.68 | 97.86 | 97.88 | 100.00 | 98.60 | 98.12 | 98 | 0.1 |
| 55 | 100.00 | 99.82 | 89.84 | 100.00 | 99.44 | 90.68 | 100.00 | 100.00 | 89.38 | 100.00 | 100.00 | 90.10 | 90 | 0.4 |
| 56 | 100.00 | 99.82 | 89.84 | 100.00 | 99.44 | 90.68 | 100.00 | 100.00 | 89.38 | 100.00 | 100.00 | 90.10 | 90 | 0.4 |
| 57 | 100.00 | 99.82 | 89.84 | 100.00 | 99.44 | 90.68 | 100.00 | 100.00 | 89.38 | 100.00 | 100.00 | 90.10 | 90 | 0.4 |
| 58 | 100.00 | 99.94 | 94.74 | 100.00 | 99.76 | 95.36 | 100.00 | 100.00 | 94.88 | 100.00 | 100.00 | 95.18 | 95 | 0.1 |
| 59 | 100.00 | 99.94 | 94.74 | 100.00 | 99.76 | 95.36 | 100.00 | 100.00 | 94.88 | 100.00 | 100.00 | 95.18 | 95 | 0.1 |
| 60 | 100.00 | 99.94 | 94.74 | 100.00 | 99.76 | 95.36 | 100.00 | 100.00 | 94.88 | 100.00 | 100.00 | 95.18 | 95 | 0.1 |
| 61 | 100.00 | 100.00 | 97.94 | 100.00 | 99.96 | 98.06 | 100.00 | 100.00 | 98.18 | 100.00 | 100.00 | 98.06 | 98 | 0.0 |
| 62 | 100.00 | 100.00 | 97.94 | 100.00 | 99.96 | 98.06 | 100.00 | 100.00 | 98.18 | 100.00 | 100.00 | 98.06 | 98 | 0.0 |
| 63 | 100.00 | 100.00 | 97.94 | 100.00 | 99.96 | 98.06 | 100.00 | 100.00 | 98.18 | 100.00 | 100.00 | 98.06 | 98 | 0.0 |
| 64 | 100.00 | 97.28 | 89.76 | 100.00 | 96.48 | 90.62 | 100.00 | 100.00 | 89.28 | 100.00 | 100.00 | 89.98 | 90 | 0.6 |
| 65 | 100.00 | 97.28 | 89.76 | 100.00 | 96.48 | 90.62 | 100.00 | 100.00 | 89.28 | 100.00 | 100.00 | 89.98 | 90 | 0.6 |
| 66 | 100.00 | 97.28 | 89.76 | 100.00 | 96.48 | 90.62 | 100.00 | 100.00 | 89.28 | 100.00 | 100.00 | 89.98 | 90 | 0.6 |
| 67 | 100.00 | 98.92 | 94.78 | 100.00 | 98.64 | 95.36 | 100.00 | 100.00 | 94.82 | 100.00 | 100.00 | 95.04 | 95 | 0.1 |
| 68 | 100.00 | 98.92 | 94.78 | 100.00 | 98.64 | 95.36 | 100.00 | 100.00 | 94.82 | 100.00 | 100.00 | 95.04 | 95 | 0.1 |
| 69 | 100.00 | 98.92 | 94.78 | 100.00 | 98.64 | 95.36 | 100.00 | 95.06 | 94.78 | 100.00 | 100.00 | 89.00 | 95 | 36.1 |
| 70 | 100.00 | 99.72 | 97.94 | 100.00 | 99.46 | 98.06 | 100.00 | 100.00 | 98.12 | 100.00 | 100.00 | 97.98 | 98 | 0.0 |
| 71 | 100.00 | 99.72 | 97.94 | 100.00 | 99.46 | 98.06 | 100.00 | 98.16 | 97.88 | 100.00 | 100.00 | 92.10 | 98 | 34.8 |
| 72 | 98.10 | 99.80 | 98.44 | 98.28 | 99.80 | 97.98 | 100.00 | 97.94 | 97.88 | 100.00 | 100.00 | 91.78 | 98 | 38.7 |
| 73 | 100.00 | 96.26 | 89.76 | 100.00 | 95.64 | 90.64 | 100.00 | 100.00 | 89.26 | 100.00 | 100.00 | 89.92 | 90 | 0.6 |
| 74 | 100.00 | 96.26 | 89.76 | 100.00 | 95.64 | 90.64 | 100.00 | 89.96 | 89.58 | 100.00 | 100.00 | 81.36 | 90 | 74.9 |
| 75 | 100.00 | 96.26 | 89.76 | 100.00 | 95.64 | 90.64 | 100.00 | 89.96 | 89.58 | 100.00 | 100.00 | 81.36 | 90 | 74.9 |
| 76 | 100.00 | 98.42 | 94.74 | 100.00 | 98.10 | 95.32 | 100.00 | 100.00 | 94.82 | 100.00 | 100.00 | 95.08 | 95 | 0.1 |
| 77 | 100.00 | 98.42 | 94.74 | 100.00 | 98.10 | 95.32 | 100.00 | 95.06 | 94.82 | 100.00 | 100.00 | 88.36 | 95 | 44.2 |
| 78 | 94.88 | 99.06 | 95.42 | 95.56 | 98.64 | 94.84 | 100.00 | 93.98 | 94.80 | 100.00 | 100.00 | 87.58 | 95 | 56.2 |
| 79 | 100.00 | 99.66 | 97.94 | 100.00 | 99.34 | 98.06 | 100.00 | 100.00 | 98.12 | 100.00 | 100.00 | 97.98 | 98 | 0.0 |
| 80 | 98.10 | 99.74 | 98.44 | 98.30 | 99.72 | 97.98 | 100.00 | 97.32 | 97.88 | 100.00 | 100.00 | 94.88 | 98 | 10.2 |
| 81 | 98.10 | 99.74 | 98.44 | 98.30 | 99.72 | 97.98 | 100.00 | 97.32 | 97.88 | 100.00 | 100.00 | 94.88 | 98 | 10.2 |
| EV | 100.00 | 94.72 | 99.92 | 95.20 | 100.00 | 77.16 | 100.00 | 100.00 | 94.04 | 100.00 | 99.98 | 94.54 | 95 | 319.5 |
| HE | 100.00 | 99.92 | 95.14 | 99.94 | 95.60 | 95.22 | 100.00 | 100.00 | 72.98 | 100.00 | 100.00 | 94.64 | 95 | 485.0 |
| ST | 100.00 | 94.80 | 100.00 | 94.88 | 100.00 | 94.46 | 100.00 | 94.86 | 100.00 | 94.06 | 100.00 | 94.26 | 95 | 1.8 |

# Appendix C

**Table C.1** Results of the MILP model with a total demand of 7200 over 12 periods. The base case is marked grey.

Erratic demand pattern

| | CV = 0.10 | | | | | | CV = 0.25 | | | | | | CV = 0.33 | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| k=1500 | | E(TC) | Total E(Q) | % Waste | Order Policy | SSE(α) | | E(TC) | Total E(Q) | % Waste | Order Policy | SSE(α) | | E(TC) | Total E(Q) | % Waste | Order Policy | SSE(α) |
| Service level 90% | | | | | | | | | | | | | | | | | | |
| w=-0.5 | 1 | 25057 | 7783 | 7.5 | 1.4.7.10 | 0.4 | 10 | 27210.5 | 8214 | 12.3 | 1.4.7.9.10 | 21.5 | 19 | 28172.5 | 8538 | 15.7 | 1.4.7.9.10 | 74.9 |
| w=0 | 2 | 25349 | 7783 | 7.5 | 1.4.7.10 | 0.4 | 11 | 27717.5 | 8214 | 12.3 | 1.4.7.9.10 | 21.5 | 20 | 28748 | 8005 | 10.1 | 1.2.4.7.9.10 | 74.8 |
| w=0.5 | 3 | 25583 | 7598 | 4.2 | 1.4.7.9.11 | 144.8 | 12 | 28176 | 7810 | 7.8 | 1.2.4.7.9.10 | 21.4 | 21 | 28912 | 7491 | 3.9 | 1.2.4.5.7.9.10 | 81.7 |
| Service level 95% | | | | | | | | | | | | | | | | | | |
| w=-0.5 | 4 | 25467 | 7947 | 9.4 | 1.4.7.10 | 0.1 | 13 | 28062 | 8501 | 15.3 | 1.4.7.9.10 | 36.1 | 22 | 29335.5 | 8272 | 13.0 | 1.2.4.7.9.10 | 44.1 |
| w=0 | 5 | 25841 | 7947 | 9.4 | 1.4.7.10 | 0.1 | 14 | 28648 | 7983 | 9.8 | 1.2.4.7.9.10 | 36.1 | 23 | 29606 | 7613 | 5.4 | 1.2.4.5.7.9.10 | 56.2 |
| w=0.5 | 6 | 26050 | 7716 | 5.4 | 1.4.7.9.11 | 212.8 | 15 | 28835 | 7483 | 3.8 | 1.2.4.5.7.9.10 | 37.6 | 24 | 29812.5 | 7613 | 5.4 | 1.2.4.5.7.9.10 | 56.2 |
| Service level 98% | | | | | | | | | | | | | | | | | | |
| w=-0.5 | 7 | 25932 | 8133 | 11.5 | 1.4.7.10 | 0.0 | 16 | 29045 | 8836 | 18.5 | 1.4.7.9.10 | 34.8 | 25 | 30429.5 | 7827 | 8.0 | 1.2.4.5.7.9.10 | 10.2 |
| w=0 | 8 | 26383 | 7882 | 7.0 | 1.4.7.9.11 | 111.5 | 17 | 29357 | 7566 | 4.8 | 1.2.4.5.7.9.10 | 38.7 | 26 | 30743 | 7827 | 8.0 | 1.2.4.5.7.9.10 | 10.2 |
| w=0.5 | 9 | 26660 | 7882 | 7.0 | 1.4.7.9.11 | 111.5 | 18 | 29540 | 7566 | 4.8 | 1.2.4.5.7.9.10 | 38.7 | 27 | 31056.5 | 7827 | 8.0 | 1.2.4.5.7.9.10 | 10.2 |
| k=500 | | E(TC) | Total E(Q) | % Waste | Order Policy | SSE(α) | | E(TC) | Total E(Q) | % Waste | Order Policy | SSE(α) | | E(TC) | Total E(Q) | % Waste | Order Policy | SSE(α) |
| Service level 90% | | | | | | | | | | | | | | | | | | |
| w=-0.5 | 28 | 19750 | 7401 | 1.7 | 1.2.4.5.7.9.12 | 0.2 | 37 | 20962 | 7393 | 0.0 | 1.2.4.5.7.9.10.12 | 0.4 | 46 | 21540.5 | 7454 | 0.0 | 1.2.4.5.7.8.9.10.12 | 0.8 |
| w=0 | 29 | 19759 | 7280 | 0.0 | 1.2.4.5.7.9.11 | 144.7 | 38 | 20962 | 7393 | 0.0 | 1.2.4.5.7.9.10.12 | 0.4 | 47 | 21540.5 | 7454 | 0.0 | 1.2.4.5.7.8.9.10.12 | 0.8 |
| w=0.5 | 30 | 19759 | 7280 | 0.0 | 1.2.4.5.7.9.11 | 144.7 | 39 | 20962 | 7393 | 0.0 | 1.2.4.5.7.9.10.12 | 0.4 | 48 | 21540.5 | 7454 | 0.0 | 1.2.4.5.7.8.9.10.12 | 0.8 |
| Service level 95% | | | | | | | | | | | | | | | | | | |
| w=-0.5 | 31 | 20003 | 7309 | 0.1 | 1.2.4.5.7.9.11 | 212.8 | 40 | 21474.5 | 7447 | 0.0 | 1.2.4.5.7.8.9.10.12 | 0.1 | 49 | 22257.5 | 7565 | 0.5 | 1.2.4.5.7.8.9.10.12 | 0.1 |
| w=0 | 32 | 20007 | 7309 | 0.1 | 1.2.4.5.7.9.11 | 212.8 | 41 | 21474.5 | 7447 | 0.0 | 1.2.4.5.7.8.9.10.12 | 0.1 | 50 | 22277 | 7565 | 0.5 | 1.2.4.5.7.8.9.10.12 | 0.1 |
| w=0.5 | 33 | 20010 | 7309 | 0.1 | 1.2.4.5.7.9.11 | 212.8 | 42 | 21474.5 | 7447 | 0.0 | 1.2.4.5.7.8.9.10.12 | 0.1 | 51 | 22296.5 | 7565 | 0.5 | 1.2.4.5.7.8.9.10.12 | 0.1 |
| Service level 98% | | | | | | | | | | | | | | | | | | |
| w=-0.5 | 34 | 20276 | 7324 | 0.0 | 1.2.4.5.7.9.10.12 | 0.0 | 43 | 22066.5 | 7522 | 0.2 | 1.2.4.5.7.8.9.10.12 | 0.1 | 52 | 23142 | 7768 | 2.1 | 1.2.4.5.7.8.9.10.12 | 0.1 |
| w=0 | 35 | 20276 | 7324 | 0.0 | 1.2.4.5.7.9.10.12 | 0.0 | 44 | 22073 | 7522 | 0.2 | 1.2.4.5.7.8.9.10.12 | 0.1 | 53 | 23222.5 | 7768 | 2.1 | 1.2.4.5.7.8.9.10.12 | 0.1 |
| w=0.5 | 36 | 20276 | 7324 | 0.0 | 1.2.4.5.7.9.10.12 | 0.0 | 45 | 22079.5 | 7522 | 0.2 | 1.2.4.5.7.8.9.10.12 | 0.1 | 54 | 23303 | 7768 | 2.1 | 1.2.4.5.7.8.9.10.12 | 0.1 |
| k=2000 | | E(TC) | Total E(Q) | % Waste | Order Policy | SSE(α) | | E(TC) | Total E(Q) | % Waste | Order Policy | SSE(α) | | E(TC) | Total E(Q) | % Waste | Order Policy | SSE(α) |
| Service level 90% | | | | | | | | | | | | | | | | | | |
| w=-0.5 | 55 | 27057 | 7783 | 7.5 | 1.4.7.10 | 0.4 | 64 | 29232.5 | 8653 | 16.8 | 1.4.7.10 | 0.6 | 73 | 30392.5 | 9117 | 21.0 | 1.4.7.10 | 0.6 |
| w=0 | 56 | 27349 | 7783 | 7.5 | 1.4.7.10 | 0.4 | 65 | 29959 | 8653 | 16.8 | 1.4.7.10 | 0.6 | 74 | 31341.5 | 8538 | 15.7 | 1.4.7.9.10 | 74.9 |
| w=0.5 | 57 | 27641 | 7783 | 7.5 | 1.4.7.10 | 0.4 | 66 | 30685.5 | 8653 | 16.8 | 1.4.7.10 | 0.6 | 75 | 32010.5 | 8538 | 15.7 | 1.4.7.9.10 | 74.9 |
| Service level 95% | | | | | | | | | | | | | | | | | | |
| w=-0.5 | 58 | 27468 | 7947 | 9.4 | 1.4.7.10 | 0.1 | 67 | 30260 | 9064 | 20.6 | 1.4.7.10 | 0.1 | 76 | 31747.5 | 9659 | 25.5 | 1.4.7.10 | 0.1 |
| w=0 | 59 | 27841 | 7947 | 9.4 | 1.4.7.10 | 0.1 | 68 | 31192 | 9064 | 20.6 | 1.4.7.10 | 0.1 | 77 | 32750 | 8955 | 19.6 | 1.4.7.9.10 | 44.2 |
| w=0.5 | 60 | 28215 | 7947 | 9.4 | 1.4.7.10 | 0.1 | 69 | 31863 | 8501 | 15.3 | 1.4.7.9.10 | 36.1 | 78 | 33312.5 | 7613 | 5.4 | 1.2.4.5.7.9.10 | 56.2 |
| Service level 98% | | | | | | | | | | | | | | | | | | |
| w=-0.5 | 61 | 27933 | 8133 | 11.5 | 1.4.7.10 | 0.0 | 70 | 31415 | 9526 | 24.4 | 1.4.7.10 | 0.0 | 79 | 33275 | 10270 | 29.9 | 1.4.7.10 | 0.0 |
| w=0 | 62 | 28399 | 8133 | 11.5 | 1.4.7.10 | 0.0 | 71 | 32363 | 8836 | 18.5 | 1.4.7.9.10 | 34.8 | 80 | 34243 | 7827 | 8.0 | 1.2.4.5.7.9.10 | 10.2 |
| w=0.5 | 63 | 28866 | 8133 | 11.5 | 1.4.7.10 | 0.0 | 72 | 33040 | 7566 | 4.8 | 1.2.4.5.7.9.10 | 38.7 | 81 | 34556.5 | 7827 | 8.0 | 1.2.4.5.7.9.10 | 10.2 |

| k=1500 | w=0 | Service level 95% | | | |
|---|---|---|---|---|---|
| | E(TC) | Total E(Q) | % Waste | Order Policy | SSE(α) |
| EV | 27719.5 | 8093 | 11.0 | 1.3.5.7.10 | 319.5 |
| HE | 27400 | 8163 | 11.8 | 1.4.6.7.10 | 485.0 |
| ST | 27992 | 7549 | 0.0 | 1.3.5.7.9.11 | 1.8 |