QCD predictions for total cross-sections and the Froissart Bound

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1 Introduction

We discuss a mechanism to explain the increase in total hadronic cross-sections with energy and examine the dynamics beneath the Froissart bound for the asymptotic behavior of total cross-sections. We present the ansätz that in QCD mini-jet driven models for the total cross-section, soft gluon emission resummed down to the infrared region leads to the Froissart bound. We also show predictions for the survival probability of Large Rapidity Gaps in hadronic collisions.

Recently, we have shown predictions for the total pp, $p\bar{p}$ cross-sections [1] from a model [2] which has the following ingredients:

1. A hard component of scattering responsible for the rise of the total cross-section,

2. Eikonal transformation which includes multiple scattering and requires impact parameter (b-)distributions inside scattering particles, along with the basic scattering cross-sections,

3. Soft gluon emission from scattering particles giving rise to an s-dependent impact parameter distribution and softening the rate of rise of cross-sections with energy.

2 QCD in minimum bias processes: Minijets and Soft Gluon emission

According to perturbative QCD, processes occurring in hadronic collisions, involving parton-parton scattering down to $p_t \approx 1 \div 2\ GeV$ can be described in the p(erturbative)QCD improved parton model by:

$$\sigma_{\rm jet}^{AB}(s, p_{tmin}) = \int_{p_{tmin}}^{\sqrt{s/2}} dp_t \int_{4p_t^2/s}^{1} dx_1 \int_{4p_t^2/(x_1 s)}^{1} dx_2 \times \sum_{i,j,k,l} f_{i|A}(x_1, p_t^2) f_{j|B}(x_2, p_t^2) \frac{d\hat{\sigma}_{ij}^{kl}(\hat{s})}{dp_t}. \quad (1)$$

Here the scale dependent parton densities in the hadrons $A, B = p, \overline{p}$ (GRV, MRST, CTEQ [3]) are obtained by analysing the Deep Inelastic Scattering (DIS) data as well as the large p_t processes at the colliders in the framework of pQCD; the subprocess cross-sections $\frac{d\hat{\sigma}_{ij}^{kl}(\hat{s})}{dp_t}$ are also given by pQCD. In mini-jet models, this hard component of scattering is considered responsible for the rise of the total cross-section. This mini-jet cross-section is strongly dependent upon p_{tmin}, the minimum transverse momentum allowed to the scattered partons in the final state and has a power law growth with energy, s^{ϵ}. In particular :

$$\sigma_{jet}^{GRV} \approx s^{0.4} \qquad \sigma_{jet}^{MRST} \approx s^{0.3} \qquad \sigma_{jet}^{CTEQ} \approx s^{0.3}$$

Such behaviour follows from the low-x behaviour of the gluon densities in QCD, which is an infinite range theory. By contrast we notice that such is not the behaviour of the hadronic total cross-section, which for proton-proton scattering say, must obey the Froissart bound, namely $\sigma_{tot} \leq log^2(s)$.

According to our model, soft gluon emission down to zero momentum modes is responsible for the initial decrease in the pp cross-section with energy, as well as for transforming the subsequent sharp rise, due to gluon-gluon interactions, into a more smooth behaviour. This observation is based upon the realization that initial state emission of zero mass quanta such as photons in charged QED processes or gluons in QCD, decreases the observed cross-section. This is due to the fact that initially collinear particles, upon emission of radiation, acquire a non-zero relative transverse momentum, thereby reducing the scattering cross-section.

The probability of observing a total transverse momentum $\mathbf{K_t}$ due to soft gluon emission from initially collinear quarks is a well known function, given by

$$d^2P(\mathbf{K}_\perp) = d^2\mathbf{K}_\perp \frac{1}{(2\pi)^2} \int d^2\mathbf{b}\ e^{i\mathbf{K}_\perp \cdot \mathbf{b} - h(b, q_{max})} \quad (2)$$

with

$$h(b, q_{max}) = \int_0^{q_{max}} d^3\overline{n}(k)[1 - e^{-i\mathbf{k_t}\cdot\mathbf{b}}] \quad (3)$$

In QED $d^3\overline{n}(k) \propto \alpha \log(\frac{2q_{max}}{m_{electron}})$ and resummation in transverse momentum variable is well approximated by first order expansion in α. In QCD, the situation is completely different because α_s is (i) not a constant and (ii) can become very large as the gluon transverse momentum goes to zero. In phenomenological applications, resummation is typically exploited by splitting the integral so that

$$h(b, q_{max}) = c_0 b^2 + \int_{\mu}^{q_{max}} d^3\overline{n}(k)[1 - e^{-i\mathbf{k_t}\cdot\mathbf{b}}] \approx c_0 b^2 + c_1 \int_{\mu}^{q_{max}} \frac{dk_t^2}{k_t^2} \alpha_s(k_t^2) \log(\frac{2q_{max}}{k_t}) \tag{4}$$

with $c_1 = 8/(3\pi)$, i.e. separating the infrared region from the one where one can apply the asymptotic freedom expression for the strong coupling constant. In this energy range, k_t is small but non-zero and the exponential term $e^{i\mathbf{k_t}\cdot\mathbf{b}}$ is neglected. In the above $d^3\overline{n}$ is evaluated in LO, but the resulting Sudakov form factor [4] is often implemented beyond LO.

Our approach is different and focuses on the infrared (IR) part of the above integral, where resummation plays a more fundamental role. Two major observations apply in this region : (i) in the IR region one cannot count the number of gluons and so soft gluons have to be resummed; (ii) resummation results in exponentiation (summing of all the very soft k-distributions) and an integration over the low gluon momentum. The exponentiated integral must include the zero momentum values *and* requires the spectrum to be integrable. The asymptotic freedom expression for $\alpha_s(k_t^2) \propto \frac{1}{\log(k_t^2/\Lambda^2)}$ does not satisfy the second condition. Phenomenological choices for α_s in the infrared region include freezing it at or around 1 GeV [5]. We make an altogether different ansätz, namely that in the infrared region the dynamics of zero momentum gluon coupling to a quark source is described by a singular, but integrable, power law behaviour, namely

$$\alpha_s(k_t^2) \approx (\frac{\Lambda^2}{k_t^2})^p \qquad k_t \to 0 \quad with\ p < 1 \tag{5}$$

We then introduce a phenomenological expression to interpolate between the infrared and the asymptotic freedom regime as in [2].

Everything is now in place for using soft gluon resummation to describe the impact parameter distribution of partons in hadron-hadron scattering. We propose the following expression

$$A_{BN}(b, s) = N \int d^2\mathbf{K}_\perp\ e^{-i\mathbf{K}_\perp\cdot\mathbf{b}} \frac{d^2P(\mathbf{K}_\perp)}{d^2\mathbf{K}_\perp} = \frac{e^{-h(b,q_{max})}}{\int d^2\mathbf{b}\ e^{-h(b,q_{max})}} \tag{6}$$

with

$$h(b, q_{max}) = \frac{16}{3} \int_0^{q_{max}} \frac{\alpha_s(k_t^2)}{\pi} \frac{dk_t}{k_t} \log \frac{2q_{\max}}{k_t} [1 - J_0(k_t b)] \tag{7}$$

where N is a normalization constant such that $\int A(b)d^2\mathbf{b} = 1$ and $\alpha_s = \frac{12\pi}{33-2N_f}\frac{p}{\ln[1+p(\frac{k_t}{\Lambda})^{2p}]}$. This distribution is our input to the eikonal representation of the total cross-section, which, upon neglecting the real part of the eikonal function, reads

$$\sigma_{tot} = 2\int d^2\mathbf{b}[1 - e^{-n(b,s)/2}] \tag{8}$$

with $n(b,s) = n_{soft}(b,s) + n_{hard}^{PDF}(b,s) = n_{soft}(b,s) + A_{BN}(b,s)\sigma_{jet}^{PDF}(s,p_{tmin})$. The division between *soft* and *hard* corresponds to including in the average number of hard collisions all those processes for which $p_t^{partons} > p_{tmin}$.

3 Restoration of Froissart bound through soft gluon emission

The soft-gluon (positive definite) spectral function $h(b,s)$ goes to zero as b^2 for small b and for large b it goes as a power law b^{2p}. For algebraic simplicity, we shall illustrate below the case for $p = 1$. Thus, we let

$$h(b, q_{max}) \sim b^2 c(s) \qquad as \quad \sqrt{s} \text{ increases.}$$

Hence, $A(b,s) \simeq e^{-b^2c(s)}$. The number of collisions $n(b,s)$ for $b \to 0$ and large s gets huge due to the steep rise of the jet cross-section.(See for instance, the sum rule [6]). For large b on the other hand, we have a Gaussian cutoff in n. Thus, the function $(1 - e^{-n/2})$ is practically unity for small b and is zero for large b, so that we may assume $1 - e^{-n(b,s)/2} \sim \theta(b_o^2 - b^2)$, i.e., an ideal Fermi function. It then follows

$$\sigma_{tot} = 2\int d^2\mathbf{b}\ \theta(b_0 - b) = 2\pi b_0^2$$

where b_0 is that value of b for which $[1 - e^{-n(b_0,s)/2}] = 1/2$, or, equivalently

$$e^{-n(b_0,s)/2} = 1/2 \quad or\ n(b_0,s) = 2\ln 2$$

We estimate b_0 by including only the jet part for n

$$n(b,s) \simeq n_{hard}^{PDF}(b,s) \approx A_{hard}(b,s)\sigma_{jet}^{PDF}(s,p_{tmin}) \qquad as \quad \sqrt{s} \text{ rises}$$

so that

$$n_{hard}^{PDF}(b,s) \approx \frac{c(s)}{\pi}e^{-b^2c(s)}\sigma_{jet}^{PDF}(s,p_{tmin})$$

and at very large s

$$\sigma_{total} \approx 2\pi b_0^2 = \frac{2\pi}{c(s)}\ln\frac{c(s)\sigma_{jet}^{PDF}(s,p_{tmin})}{2\pi\ln 2}$$

$\sigma_{jet}^{PDF}(s, p_{tmin}) \sim s^{\epsilon}$ as expected from an infinite range theory such as QCD, but if $c(s) \sim constant$ or rises with $\sqrt{s}$, we finally obtain

$$\sigma_{total} \approx \log s$$

with increasing $\sqrt{s}$. We note that for the allowed values of p between 1/2 and 1, the total cross-section remains bounded between $[log(s)]^2$ and $[log(s)]$ always obeying the Froissart bound.

This is confirmed also from a phenomenological point of view [1], when we apply the model described in [2] to predict the total cross-section and survival probabilities at LHC as shown in Figure 1.

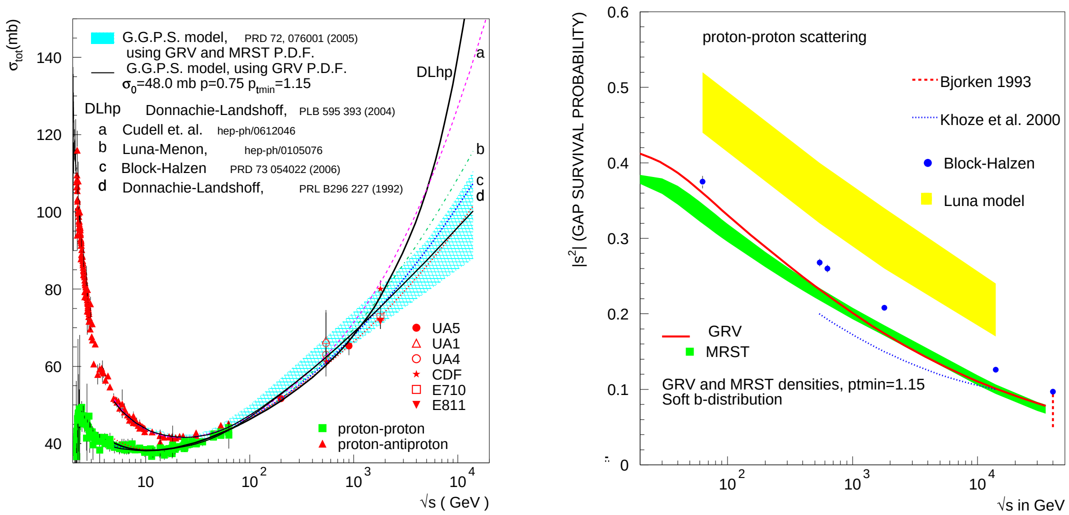

Figure 1: Total cross-section models [7] compared with data [8] (left) and survival probability for Large Rapidity Gaps [9] (right) for different input parton densities and different models from [1].

4 Conclusions

We have discussed the restoration of the Froissart bound on total cross-section models with rise driven by QCD mini-jets. We have shown that soft gluon emission down to zero momentum modes can be responsible for transforming the sudden fast rise due to low-x gluon gluon collisions into the gentler observed logarithmic rise. No violation of the Froissart bound implies that indeed the finite range of the interaction is restored through soft gluon emission.

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