Proc. Indian Acad. Sci. (Earth Planet. Sci.), Vol. 89, Number 3, November 1980, pp. 369-382.
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Inversion of microearthquake arrival time data

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MS received 28 December 1979 ; revised 26 July 1980

Abstract. The paper presents a generalised inverse approach to determine the spatio-temporal coordinates of microearthquake events from wave kinematics data consisting of *P*-wave arrival times recorded by an array of seismographs. It is shown how a generalised inverse can be constructed for this problem so that iterations required to obtain a least squares solution become stabilised. The analysis further shows how well model parameters seduced from a given data set may be resolved and delineated their correspondence with data errors.

Keywords. Microearthquake: spatio-temporal coordinates: hypocentral location: inversion.

1. Introduction

Microearthquakes are extremely small seismic events which are recordable manifestations of the entire rock deterioration regime of a region preparing for a major earthquake. Laboratory investigations on rocks under stress have also revealed the occurrence of numerous tiny acoustic emissions generated throughout the sample due to micro-fracturing, leading to its eventual failure. The pattern of their spatial distribution and their progression in time, appear to anticipate the nature and location of the final rupture, whilst their slip mechanisms are consistent with the ambient stress field. Considerable interest has consequently centred on the monitoring of microearthquakes in seismically active regions, towards identifying their premonitory features for earthquake prediction.

The determination of the spatio-temporal coordinates of a seismic event, of its physical mechanism, as well as of the material properties of the medium, from ground displacements recorded at a number of points on the earth's surface, forms an inverse problem. The present paper is devoted to the construction of a generalised inverse for determining the first of these unknowns for an assumed velocity structure and to elucidating the nature of possible solutions when the data set is inadequate or inaccurate.

Hypocentral location of a microearthquake source from arrival time data collected in its vicinity, has been attempted by a number of workers (Bolt 1960; Flinn

1960; Nordquist 1962; Cisternas 1963; Eaten 1969; Lee and Lahr 1972; Peters and Crosson 1972; Rai 1977). Flinn (1960) analysed the problem as fitting the observed p_g and s_g arrival times with those predicted theoretically according to a least squares optimisation. Eaten (1969) incorporated p_g, s_g, p^*, s^*, p_n and s_n phases and used Geiger's method for optimising squared arrival time discrepancies. Lee and Lahr (1972) used stepwise multiple regression. Most of these authors modelled the earth's crust as a horizontally-layered structure. Rai (1977) in his model introduced a superficial sedimentary formation in which the velocity increased exponentially with depth. Rai (1977) also attempted a ray tracing technique to account for inhomogeneities of more complex nature in the medium. A usual perturbing factor is the rapidly changing relief particularly in mountainous terrain such as the Himalayan, which can introduce considerable discrepancies in the arrival times. For greater accuracy, however, the gross structure of the crust adopted for analysis of microearthquake data, must be derived from local data, keeping in view the lateral variations in the crust.

2. Statement of the problem

Arrival times of *P*-waves emanating from a microearthquake source are recorded at a number of stations on the surface of the earth by a network of microearthquake recorders. The problem is to deduce from these, the location of the event and its time of occurrence.

2.1. *Formulation of the problem*

Let a seismic event occur at coordinates (x_0, y_0, z_0, t_0) in a horizontally stratified earth, the depth-to-layer interfaces being $D(L)$ and velocities therein equal to $V(L)$. The travel time T_i taken by the *P*-wave from this source to arrive at the *i*th station (X_i, Y_i, Z_i) can be expressed as a functional of two parameter vectors.

$$\mathcal{T}_i = \mathcal{T}_i(P, X), \; i = 1, n, \tag{1}$$

where P is the vector of the unknown parameters to be determined, its components being,

$$P \equiv (x_0, y_0, z_0, t_0), \tag{2}$$

and X, of the layer thicknesses, velocities, and station coordinates, (i.e.)

$$\begin{aligned} &X \equiv [X_i, Y_i, Z_i, V(L), D(L)], \\ &i = 1 \ldots n, \\ &L = 1 \ldots N. \end{aligned} \tag{3}$$

Let P^0 be an estimated solution of P and $\triangle P$ the desired deviator to render coincidence of calculated and observed travel times. Expanding (1) in Taylor's series and retaining only the first term,

$$\mathcal{T}_i(P, X^0) = \mathcal{T}_i(P^0, X^0) + \sum_{j=1}^{m} (p_j - p_j^0) \frac{\partial \mathcal{T}_i(P,X)}{\partial p_j} \Bigg| \begin{matrix} P = P^0 \\ X = X^0 \end{matrix} \tag{4}$$

which can be written in a matrix form as follows:

$$\triangle \mathcal{T} = A \triangle P, \tag{5}$$

where, $\triangle \mathcal{T} = \mathcal{T}_i (P, X^0) - \mathcal{T}_i (P^0, X^0)$ is the travel time anomaly vector $(n \times 1)$,

$\triangle P = p_j - p_j^0$, is the parameter correction vector $(m \times 1)$,

and $A = \dfrac{\partial \mathcal{T}_i (P, X)}{\partial p_j} \Big| \begin{matrix} P = P^0 \\ X = X^0 \end{matrix}$ is the $(n \times m)$ system matrix of influence coefficients.

$i = 1, n \; ; \; j = 1, m.$

The problem as posed above has been linearised in $\triangle P$ to exploit the facilities of a matrix formulation, although it means that the final solution would be obtained iteratively by updating the vector P^0 after very iteration. In a given iteration, however, $\triangle P$ is obtained simply from the relation $\triangle P = G \triangle \mathcal{T}$ where G will be identified as an inverse of the matrix A.

When the number of data elements n is equal to the number m of parameters to be resolved and the determinant of A is non-zero, the matrix A has a unique inverse. In all other cases one must choose from a set of its possible generalised inverses, using criteria most suitable to the physical nature of a given problem. For example, in an overdetermined problem $(n > m)$ where A is a full rank matrix and $A^T A$ therefore nonsingular, one obtains (Morrison 1969) for the inverse of A

$$A_s^{-1} = (A^T A)^{-1} A^T \tag{6}$$

which turns out to be a least squares solution; for a weighted least squares solution given by

$$A_w^{-1} = (A^T Q^{-1} A)^{-1} A^T Q^{-1}, \tag{7}$$

where Q is a diagonal weighting matrix such as the covariance matrix of data errors if they happen to be estimable. On the other hand, for an underdetermined system $(n < m)$, one obtains an inverse

$$A_E^{-1} = A^T (A A^T)^{-1}, \tag{8}$$

which proves to be a solution giving a least Euclidean length.

Indeed, there are a number of generalised inverses, but one making use of a decomposition theorem by Penrose is particularly instructive. For, it enables one to investigate on the one hand, the ' goodness ' of the inverse solution yielded by a given data set, and on the other to analyse the relative influence of various data elements in illuminating the solution space thereby delineating desirable trade-offs between the quality of acceptable solution and the expense of augmenting the data set. Accordingly (Penrose 1955)

$$\underset{(n \times m)}{A} = \underset{(n \times n)}{U} \; \underset{(n \times m)}{\Lambda} \; \underset{(m \times m)}{V^T} \tag{9}$$

where Λ is the diagonal matrix comprising the eigenvalues of the matrix $A^T A$ or AA^T arranged in descending order of magnitude, and V and U are respectively the orthogonal matrices comprising the eigenvectors of $A^T A$ and AA^T. For an overdetermined system the least squares inverse of A can be written following (6).

$$A_p^{-1} = (V \Lambda U^T U \Lambda V^T)^{-1} V \Lambda U^T = V \Lambda^{-1} U^T. \tag{10}$$

It will be seen that the vectors of the parameter or solution space would be proportional to the reciprocal of the eigenvalues. In case some of these happen to be near singular, the parameter vectors will not only reflect the data errors in a highly magnified space, but would also suffer wide excursions in the parameter space upon every iteration, thereby prevetning convergence. A modified inverse popularised by Lanczos (1961) which circumvents these problems, is defined as

$$\underset{(m \times n)}{A_L^{-1}} = \underset{(m \times p)}{V_p} \quad \underset{(p \times p)}{\Lambda_p^{-1}} \quad \underset{(p \times n)}{U_p^T} \tag{11}$$

where Λ_p is a shortened version of Λ in (10), comprising of only p significant eigenvalues of the latter, and V_p and U_p are the corresponding new eigenvectors of $A^T A$ and AA^T.

To obtain P, one first begins by constructing the Lanczos inverse A_L^{-1} choosing a considerably shortened order of the eigenvalue matrix, comprising only the q significant eigenvalues. After a few iterations which incidentally progress in small steps upholding the assumption of linearity, this leads to some stationary point in the parameter space fairly close to the final solution. The largest of the rejected eigenvalues is then incorporated to form a new eigenvalue matrix and the above process repeated, whereafter the next largest of the rejected eigenvalues is also incorporated and so on, till the objective function is reduced to the same order of magnitude as the mean square error of observations or lower.

Whilst inclusion of as many eigenvalues as possible is desirable to exploit maximum information contained in the data, the iterations are often found to diverge upon inclusion of the small eigenvalues. This can be forestalled by using a modification of Marquordt* (1963) algorithm as discussed by Johanson (1975) which uses an estimator $[(A^T A + KI)^{-1} A^T)]$ in place of the usual $[(A^T A)^{-1} A^T]$. It consists in stepping back as soon as the iteration procedure is found to diverge, say, upon the inclusion of the $(q + 1)$th eigenvalue, and adding the value λ_q of the qth eigenvalue to all q eigenvalues retained, thereby defining a new $(q \times q)$ eigenvalue matrix. Thus the next progression will be reduced by half in the 'bad' direction V_q in the parameter space and marginally so in others. If this fails to reverse divergence, the process is repeated by adding λ_{q-1} to all eigenvalues to define yet another eigenvalue matrix and so on, till convergence is achieved. In rare cases, it may of course become impossible to minimise the objective function any further. Then it serves no useful purpose to add additional

* Marquordt algorithm is an optimum algorithm which resembles the gradient method ($k \to \infty$) when the estimator is far from the minimum and the Newton-Raphson method ($K \to 0$) when the estimator is near the minimum, the optimum value of K being 1·0 and 0·01—Johanson (1977) modified this to permit an adaptive choice for the value of K in successive iteration.

small eigenvalues since the corresponding minimisation of the objective function would be insignificant.

The Lanczos inverse can be further modified as shown by Jackson (1972) so as to incorporate additional information such as statistics of data errors and limits to variations of model parameters estimated on geological grounds or from results of some independent geo-investigations. This can be accomplished through the use of appropriate weighting matrices.

2.2. *Resolution of model parameters*

Whilst the errors in estimated model parameters can be prevented from blowing up by excluding the smaller eigenvalues in the construction of the estimator A_L^{-1} the corresponding forfeiture of information leads to a certain degradation in the resolution of parameters. For, if $\triangle P_q$ be the estimated deviator vector of model parameters obtained through the Lanczos inverse $= A_{Lq}^{-1}$ constructed from only q significant eigenvalues, then, from (5)

$$\triangle P_q = A_{Lq}^{-1} \triangle \mathcal{F} = [A_{Lq}^{-1} A] \triangle P. \tag{12}$$

The matrix $[A_{Lq}^{-1} A] = R$ is obviously a measure of the resolution of the model parameters as $\triangle P_q$ appears as a result of convolution of the true solution $\triangle P$ with the rows of R. Ideally, the resolution matrix R is desired to be an identity matrix whose diagonal elements are unity and off-diagonal elements zero. But as we suspend small eigenvalues from consideration, its rows tend to get smeared, thereby exhibiting a trade-off between resolution of model parameters and their errors as in the case of the linear inverse theory of Backus and Gilbert (1970).

For underdetermined systems, this trade-off may not be too significant since the solution can never be unique and in practice drastic reduction of errors in model parameters can be achieved by sacrificing a small loss in resolution.

2.3. *Informative density*

The generalised inverse formulation also enables one to explore the information value of the data set and thereby design a data set for desired quality of information by drawing up appropriate data collection plans. If $\triangle \mathcal{F}_L$ be the data set corresponding to an estimate $\triangle P_q$ of the model parameters obtained through the Lanczos inverse A_{Lq}^{-1} then from (5) and (12) we have

$$\triangle \mathcal{F}_L = A \triangle P_q = [A\, A_{Lp}^{-1}] \triangle \mathcal{F} = S \triangle \mathcal{F}, \tag{13}$$

where $$S = [A\, A_{Lq}^{-1}], \tag{14}$$

is obviously a measure of the information density and is therefore termed as information density matrix. S will be an identity matrix only if $n \leqslant m$, when there would be a one-to-one correspondence between the transformed data $\triangle \mathcal{F}_L$ and the actual data set $\triangle \mathcal{F}$. Otherwise the two will deviate according to the deviation of S from an identity matrix. Once, therefore a geophysical problem has been posed one can analyse the nature of S and so design it as to achieve an optimum trade-off between the cost of data collection and information gained.

2.4. *The forward problem*

As the purpose of the present exercise is to invert the arrival time data for the determination of the spacio-temporal co-ordinates of the source, a specific earth model must be *a priori* selected based on information regarding the velocity structure of the crust in the area. However, for the present-layered earth model in which the velocity increases in successively deeper layers, has been chosen to illuminate the general nature of the problem.

Let $V(L)$, $D(L)$, $L = 1, N$ be an N-layered velocity model where $V(L)$ and $D(L)$ are the wave velocities in successive deeper layers and depths to the interfaces respectively. The focus lies in the Jth layer at a depth H below the surface (figure 1).

Following Eaton (1969) and Katiyar (1978) expressions for the travel time of a critically refracted wave issuing from a focus in the Jth layer at a depth H below the surface and refracted from the top of the Mth layer, to an epicentral distance Δ is given by

$$T = \frac{\Delta}{V(M)} + \sum_{L=J}^{M-1} \frac{\mathrm{THK}(L)\,[V(M)^2 - V(L)^2]^{1/2}}{V(M)\,V(L)} + \sum_{L=1}^{M-1} \frac{\mathrm{THK}(L)\,[V(M)^2 - V(L)^2]^{1/2}}{V(M)\,V(L)} - \frac{\mathrm{TKJ}\,[V(M)^2 - V(J)^2]^{1/2}}{V(M)\,V(J)}. \tag{15}$$

Also, the travel time of the direct wave to an epicentral distance Δ, from a focus in the first layer at a depth H below the surface is given by

$$T = (H^2 + \Delta^2)^{1/2}/V(1). \tag{16}$$

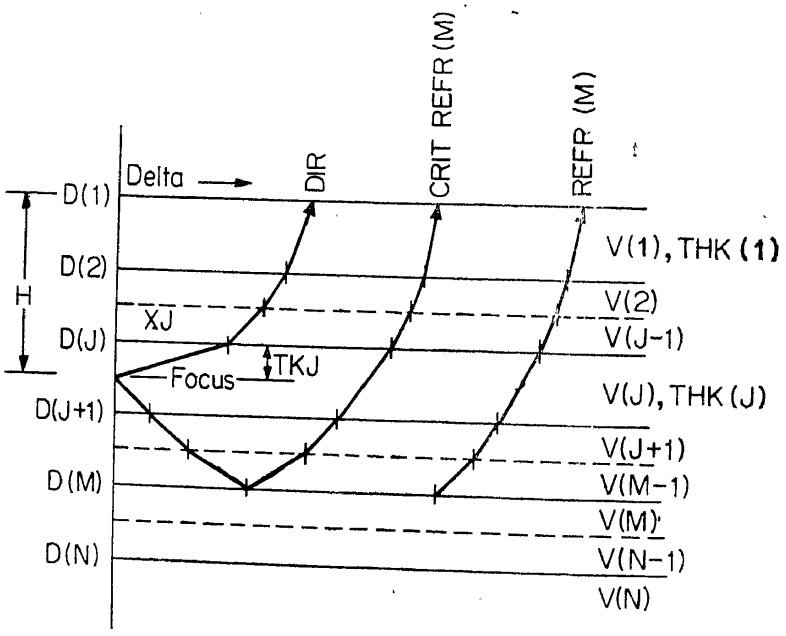

Figure 1. Notation used to specify the crustal velocity structures.

Expressions for T as a function of $\triangle$ are quite complex when the focus lies in the Jth layer ($J = 2, N$). However, both T and $\triangle$ are relatively simple functions of sin θ_i where θ_i is the angle of incidence of the ray at the focus. Denoting sin $\theta_i = u$ and $h = TKJ = H - D(J)$, T and $\triangle$ can be expressed as functions of u.

$$T = \frac{h}{V(J)(1-u^2)^{1/2}} + \sum_{L=1}^{J-1} \frac{THK(L)\,V(J)}{V(L)^2\left(\frac{V(J)^2}{V(L)^2} - u^2\right)^{1/2}}, \tag{17}$$

and

$$\triangle = \frac{hu}{(1-u^2)^{1/2}} + \sum_{L=1}^{J-1} \frac{THK(L)\,u}{\left(\frac{V(J)^2}{V(L)^2} - u^2\right)^{1/2}}. \tag{18}$$

u is determined following Fermat's principle which requires it to be so adjusted as to cause the function T to become stationary.

The system matrix A which characterises the nature of the problem, maps the unknown parameter-space onto the observation space. This matrix is a scheme of 'variational parameters' or the 'influence coefficients' which are derivatives of the arrival times with respect to each element of the unknown parameter set x, y, z, t, individual elements being arranged columnwise, whilst observations at different stations correspond to the individual rows of the matrix. In other words, all the derivates of arrival times at different stations with respect to a single unknown parameter are arranged to form one column of the matrix. These derivatives can be obtained from equations (15), (16), (17) and (18) by noting that

$$\triangle = [(x_i - x_0)^2 + (y_i - y_0)^2]^{1/2}, \tag{19}$$

and

$$\mathcal{T} = t_0 + T.$$

It is to be observed at this point that the three partial derivatives $\partial T/\partial x_0$, $\partial T/\partial y_0$ and $\partial T/\partial z_0$ are in fact respectively equal to $\partial f/\partial x_0$, $\partial f/\partial y_0$ and $\partial f/\partial z_0$ because travel times are invariants of the origin time of the event.

3. Results and discussion

In order to test the performance of the generalised inverse formulation developed here for inverting the micro-earthquake arrival time data and to examine its capabilities in illuminating the nature of this inverse problem, theoretical data for a given event at a depth of 20 km was generated. The velocity model chosen for this exercise, consisted of three homogeneous horizontal layers, admitting velocities $V(1) = 5{\cdot}0$ km/sec, $V(2) = 6{\cdot}5$ km/sec, $V(3) = 8{\cdot}0$ km/sec, depths to interfaces being $D(1) = 0{\cdot}0$ km, $D(2) = 30$ km and $D(3) = 80$ km. The focus was assumed to be located in the topmost lowest velocity layer. The data points were evenly distributed around the source upto distances of 202 km. The origin time was chosen arbitrarily.

The selected station locations and the first arrival times at different stations are given in table 1. The system matrix A for the initial point in the parameter space, representing the partial derivatives of arrival times with respect to the unknown parameters at each station can be written as follows:

	$\partial\mathcal{T}/\partial x_0$	$\partial\mathcal{T}/\partial y_0$	$\partial\mathcal{T}/\partial z_0$	$\partial\mathcal{T}/\partial t_0$
Station 1	−0·124	−0·152	0·040	1·000
Station 2	−0·012	−0·155	0·126	1·000
Station 3	−0·141	−0·061	−0·128	1·000
Station 4	−0·097	−0·119	−0·128	1·000
Station 5	−0·126	−0·088	−0·128	1·000

The condition number $\eta_i = \lambda_1/\lambda_i$ and eigenvalues λ_i^2 of the matrices AA^T and A^TA arranged in decreasing order of magnitude are given in table 2.

Initial estimates of the parameters and final inverted solution are given in table 3. The epicentral distances for various stations corresponding to the initial and final values of parameters are given in table 4.

The problem as posed above is an overdetermined one using 5 data elements to solve for 4 unknowns. The parameter in this case can be uniquely obtained but the model data fits the observed data only in least squares sense. The solution

Table 1. The selected station locations and the first arrival times at different stations.

Station name	Station latitude	Station longitude	Time of the first arrival					
			Year	Month	Day	Hour	Min.	Sec.
STN 1	35° 38·24′	120° 46·47′	78	05	12	8	5	20·40
STN 2	34″ 46·84′	119° 57·15′	,,	,,	,,	,,	,,	6·40
STN 3	34° 32·25′	119° 27·36′	,,	,,	,,	,,	,,	30·27
STN 4	36° 26·05′	121° 44·57′	,,	,,	,,	,,	,,	42·42
STN 5	35° 47·32′	121° 39·61′	,,	,,	,,	,,	,,	34·77

Table 2. The eigenvalues and condition numbers arranged in decreasing order of magnitudes.

$\lambda_1 = 2·264$	$\lambda_2 = 0·261$	$\lambda_3 = 0·066$	$\lambda_4 = 0·042$
$\eta_1 = 1·000$	$\eta_2 = 8·674$	$\eta_3 = 34·303$	$\eta_4 = 53·905$

has a 4 degree of freedom but retention of all of them, whilst rendering the resolution matrix to be an identity matrix, causes the inversion scheme to diverge owing to the presence of large condition number η_4, which in turn results in large parameter corrections.

Upon exclusion of the corresponding eigenvalue $\lambda 4$, however, the solution is found to converge after 9 iterations, when further change in the objective functions falls below 0·005. The courses of corrections in individual parameter and of the objective functions through the entire iteration procedure are shown in table 5.

The inversion of arrival time data also neatly delineates those data points which contain significant information. Figure 2 (b) shows a profile of the rows of the information density matrix which has four peaks of information corresponding to diagonal elements. These four points are the most vital data points for the resolution of parameters, information being greatest for the nearest data point and falling monotonically with increasing epicentral distance. Nearer data points thus appear to exercise greater control in determination of parameter as expected. However, the matrix in figure 2 (b) is in respect of the initial estimate of the model, and the peaks of information appearing here may not be the same as for the actual model that best fits the data. In many cases there will be a corresponding progression of the peaks of information during the iterative sequence

Table 3. Initial estimate of the parameters and final inverted solution.

Origin time of the event	Latitude of the epicentre	Longitude of the epicentre	Depth of the focus
Initial : 780512854·40	35° 4·41′	120° 6·75′	18·0
Final : 780512854·10	35° 0·71′	120° 0·87′	19·6

Table 4. The epicentral distance for various stations corresponding to the initial and final values of parameters.

Epicentral distances in km for initial estimate	Epicentral distance in km for the inverted solution
22·18	23·14
88·35	89·53
123·44	124·53
148·04	149·75
202·04	202·24

Table 5. The courses of corrections in individual parameters and of the objective functions through the entire iteration procedure.

Iteration number	dx_0 (Km)	dy_0 (Km)	dz_0 (Km)	dt_0 (Sec)	Objective function ϵ (sec)
1.	—0·266	—0·305	—0·118	2·636	2·633
2.	—0·177	—0·203	—0·078	1·758	1·920
3.	—0·118	—0·135	—0·052	1·172	1·498
4.	—0·078	—0·090	—0·035	0·781	1·266
5.	—0·052	—0·060	—0·023	0·521	1·128
6.	—0·035	—0·040	—0·015	0·347	1·091
7.	—0·023	—0·027	—0·010	0·232	1·064
8.	—0·015	—0·018	—0·007	0·151	1·053
9.	—0·010	—0·012	—0·005	0·103	1·049

as evident from figure 2 (c) which represents the information density matrix for the final model. These peaks of information are rather broad suggesting that two or more additional data points, if included at intermediate points should greatly improve the definition of peaks by enhancing information thereat. We also observe from figure 2 that significant information is contributed by a point 148 km away from the source. This is in fact near the first critical distance (166·12 km). The fact that information is contributed substantially by data points near the critical distance, is also corroborated by second and third row profiles.

The information density matrix proves to be instructive in analysing noisy data. A high level of noise at data points corresponding to the information peaks may have substantial influence on the estimation of the final model whereas noise at points between peaks will not produce as great an effect. However, it is advantageous to include intervening data points. For example, by incorporating a data point at 202 km (*vide* figure 2), the information content would be substantially enhanced at points 123 km and 146 km distant from the source. Furthermore, redundancy in data increases the magnitude of the eigenvalues, causing them to become better behaved. An appropriate degree of redundancy improves noise elimination from the solution, since the least squares fitting tends to smoothen out data errors. This is the argument for not restricting the number of data points to four for the determination of the four unknown parameters. For, in such a case, whilst the information density matrix would be an identity matrix (figure 2a), large premiums given to each of the data points corresponding to the information peaks would tend to inflate the associated errors at these points. Incorporating additional data points increases information content without giving undue weightage to any single data element which contribute information at a particular point.

The eigenvectors of AA^T and A^TA respectively may be called data and parameter eigenvectors of the system matrix A and are plotted in figure 3. The parameter eigenvectors represent a combination of the initially chosen parameters which may be uniquely determined while the data eigenvectors represent a combination of

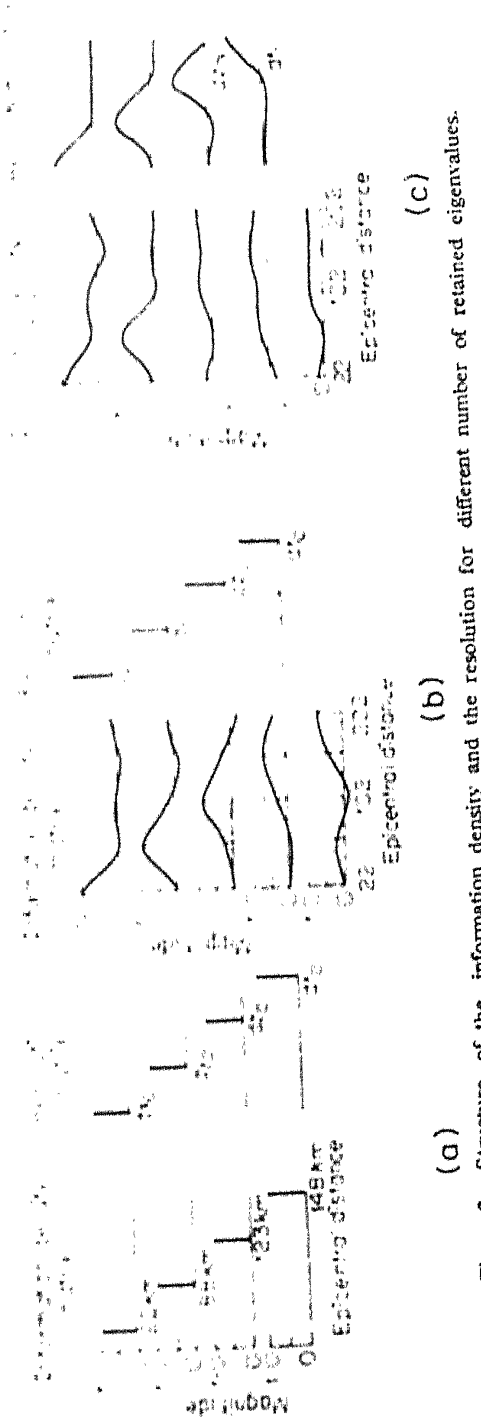

Figure 2. Structure of the information density and the resolution for different number of retained eigenvalues.

the original data elements. The relative magnitudes of the eigenvalues are important in determining the error propagation in the system (Inman *et al* 1973). If the data points at which the data eigenvectors are large contain a certain amount of error and if the associated eigenvalues are small, the error will be greatly magnified in the direction of the associated parameter eigenvector (Lanczos 1961).

Consider the eigenvectors associated with the smallest eigenvalue $\lambda_4 = 0{\cdot}042$. If any or all the data elements at 88, 128 and 202 km are assumed to contain some errors, the resulting error in dy_0 would become rather large while their effect in the determination of dt_0 or dz_0 which are chiefly associated with larger eigenvalues $\lambda_1 = 2{\cdot}264$ and $\lambda_2 = 0{\cdot}261$ would not be so serious. On the other hand any error in data at 88 and 202 km would have a greater effect on dx_0 because of the associated smaller eigenvalue $\lambda_3 = 0{\cdot}066$.

The parameters dt_0 is associated chiefly with the larger eigenvalue $\lambda_1 = 2{\cdot}264$ (figure 3) and all data points contribute almost uniformly to its resolution. The parameter eigenvector associated with the eigenvalue $\lambda_2 = 0{\cdot}261$ points chiefly along dz_0; and points which receive a direct wave as well as those near the first critical distance, are crucial for its determination. We also observe that dx_0 is resolved mainly by a direct wave data point and the farthest point whilst the resolution of dy_0 is mainly determined by information from the first critical distance point and that from the farthest point. If convergence requires the rejection of

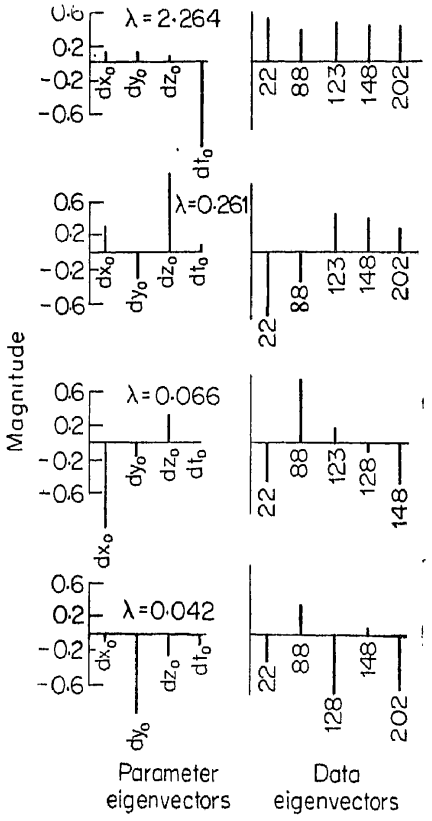

Figure 3. Data and the parameter eigenvectors. Also indicated are data points important for resolution of individual parameters.

the two smaller eigenvalues $\lambda_3 = 0{\cdot}066$ and $\lambda_4 = 0{\cdot}042$, the resolution of dx_0 and dy_0 would suffer most because of their preferred association with these eigenvalues. The parameters associated with the largest eigenvalues are found most accurately and are most quickly determined by the inversion technique, whereas those associated with the smaller eigenvalues are the least accurately determined and converge slowly.

The effect of random noise will be to cause oscillations of dx_0 and dy_0 about some mean value but will not be as conspicuous in the case of dt_0 and dz_0.

The least squared error in arrival times fit is 1·049 sec (table 5) which is very nearly equal to the accuracy of arrival time data. Figure 2 (c) shows the information density and the resolution matrices when three eigenvalues are retained. We notice that the diagonal dominance of both the matrices and the peak information is reduced. Also, the resolution matrix is no longer an identity matrix. Incorporating data at intermediate points between the peaks and beyond 202 km, near the second critical distance point should greatly amplify the information. This would increase the magnitudes of the eigenvalues, thereby improving both resolution and the rate of convergence. Whereas there is always a trade-off between the errors and resolution of the solution, a proper control of redundancy can prove helpful in striking a balance between the two.

The generalised inverse technique can prove to be a powerful tool in the planning of microearthquake investigations. This automatic method requires minimum human interaction for solving a geophysical inverse problem, and delineates the significance of various data points in achieving a desired resolution and accuracy.

Acknowledgements

The authors thank Praveen Gupta and Dr R N Singh of the National Geophysical Research Institute, Hyderabad, for many useful discussions and assistance in computations. Thanks are also due to Prof. B B S Singhal for providing facilities for this work.

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