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An Application of Martingale Convergence Theorem to Analyse Company Data

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Abstract

Martingale Convergence Theorem, in a slightly general form, when applied to the process of investor's fortune tells that an investor with a limited aim of enhancing her fortune k-fold will, in due course, either attain her aim or lose the entire initial investment. We verify and apply this theorem to analyse certain company data.

AMS (2000) *subject classification.* Primary 62-07, 62G99, 62M99.
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1 Introduction

We will discuss some basic ideas of financial mathematics in a simple manner and draw certain conclusions rarely pointed out in the books on the subject. We will discuss some real life data to corroborate our conclusions. The Martingale Convergence Theorem applied to a rather simple process of investor's fortune at once shows that an investor with a limited aim of enhancing her initial investment k-folds will, with probability one, either achieve her aim or lose the entire initial investment, unless she winds up the business in the middle. The probabilities of achieving the aim or losing the entire initial investment are very much dependent on the initial investment. We verify this theoretical conclusions by analysing certain company data obtained from Reserve Bank of India, and observe that, in Indian setting, the proportion of companies whose performance is at one of the extremes is high. There are also other related observations made at appropriate places in the paper.

We shall use the word investor to mean any individual, institution or a body, which has to take probability related risk in doing its work, business or transaction. Thus an investor may be an entrepreneur, a farmer, a financial institution such as a bank or an insurance company, etc. The word also includes a gambler whose risk taking is of a perverse kind, can be fraudulent in nature, and not a risk taking dictated by need. Mathematics of risk taking is, however, impersonal and independent of who is taking risk, of what kind or for what purpose.

In Section 2 we set up the model for the progress of investor's fortune while in Section 3 we prove the convergence theorem we have in mind for this process. It is proved under a condition more general than the martingale requirement, since the real life probabilities need not be martingalian, and this is the form we need for application. The case when the process is a martingale is discussed in Section 4 (see Athreya and Nadkarni, 2009). In Section 5 we explain the term 'net worth' and our reasons for choosing this parameter and the related data for analysis. In Section 6 we discuss certain data and related ratios which we use for comparison in later discussions and also to show that these ratios are good indicators of the state of the economy during the years under consideration. In Section 7 we discuss the main data, with a verification of Theorem 1, and summarize our findings and its implications for the economy in Section 8.

2 A Model for the Progress of Investor's Fortune

We will consider a simple probability model for the progress of an investor's fortune. We will call it an investor's walk. It is more general than the usual random walk where one takes only one step in either direction.

The walk is confined to first $N+1$ non-negative integers $\{0,1,2,,N\}$. At time 0 the investor invests an amount $a, 0 < a < N$, with the hope of receiving, at time 1, an amount $x_1 > a, x_1 \leq N$. Howevever this is not certain and the investor may end up receiving an amount, $0 \leq x_1 < a$. More precisely, for some $x_1, a < x_1 \leq N$, the probability $p_a(x_1)$ of the investor receiving the amount x_1 at time 1 is positive, and for some other $x_1, 0 \leq x_1 < a$, the probability $p_a(x_1)$ of the investor receiving the amount x_1 at time 1 is positive.

If the investor is lucky enough to receive the full fortune N at time 1 he is contented and he does not invest any more and the game stops. The barrier N may also be interpreted to mean that there are regulations which

do not permit unlimited gain. If the investor is unlucky enough to receive the amount 0 at time 1, then he can not invest any more and the game stops. If x_1 is the amount the investor receives at time 1 and if it is strictly between 0 and N then the investor invests the amount x_1 again, hoping to receive, at time 2, an amount $x_2 \leq N$ bigger than x_1 but she may end up receiving an amount $x_2 \geq 0$ smaller than x_1. In other words the probabilities $p_a(x_1 < x_2 \leq N)$, $p_a(0 \leq x_2 < x_1)$ are both positive. As before, if $x_2 = N$ or if $x_2 = 0$ then the investor becomes inactive and the game stops. In case $0 < x_2 < N$ the amount x_2 is invested again and the game proceeds as before.

Let us write

$$p_a(x_n|x_1, x_2, \cdots, x_{n-1})$$

to denote the conditional probability that the investor receives an amount x_n at time n, given that she has received amounts $x_1, x_2, \cdots, x_{n-1}$ at times $1, 2, \cdots, n-1$, respectively. Then

$$p_a(x_1, x_2, \cdots, x_n) = p_a(x_n|x_1, x_2, \cdots, x_{n-1}).p_a(x_1, x_2, \cdots, x_{n-1}),$$

whence, by iteration

$$p_a(x_1, x_2, \cdots, x_n) = \prod_{i=1}^{n} p_a(x_i|x_1, x_2, \cdots, x_{i-1}) \tag{2.1}$$

where x_0 stands for a, the starting capital. By our assumption, from any position x_i at time i satisfying $0 < x_i < N$, there is a positive probability that the investor receives an amount $x_{i+1} > x_i$ at time $i+1$. Hence for some $k \leq N - a$ there is a path $a < x_1 < x_2 \cdots < x_k = N$ which has positive probability so that $p_a(x_k = N) > 0$. This may be interpreted to mean:

No matter how investor-unfriendly the economy and how small the starting capital a, there is a positive probability that the investor will reach the top in finite time (e.g., Infosys, Google).

Similarly, our assumptions also imply:

No matter how investor-friendly the economy is and how large the starting capital a, there is a positive probability that the investor will hit the bottom in finite time (e.g., Enron, Satyam (a major Indian company)).

It is important to emphasize here that our 'risk taking' allows gambler-like risk taking where one bluffs even when one's cards are bad and the risk that one takes is to hope that the bluff will not be called. Most books on

basic financial mathematics do not classify types of risk or mention ethical or unethical risks, but are mainly concerned with deriving the Black-Scholes formula which is usually the center-piece (Delbaen and Schachermayer, 2006; Duffie, 2001; Medina and Merino, 2003; Ross, 1999).

3 A Convergence Theorem

Let us now assume (assumption A) that there is a constant $c > 0, c < 1$, such that for all a and i, whenever $p_a(x_i|x_1, x_2, \cdots, x_{i-1}) > 0$,

$$p_a(x_i|x_1, x_2, \cdots, x_{i-1}) > c > 0.$$

Under this assumption, from (2.1) we conclude that

$$p_a(x_1, x_2, \cdots, x_n) > c^n, \text{ whenever } p_a(x_1, x_2, \cdots, x_n) > 0.$$

Our assumptions imply that there is a path

$$a > x_1 > x_2 > \cdots > x_k = 0, k \leq a$$

whose probability is bigger than $c^k > c^N$. Similarly there is a path

$$a < x_1 < x_2 < \cdots < x_l = N, l \leq N - a$$

whose probability is $> c^l > c^{N-a} > c^N$.

Let A_n denote the set of paths $(x_1, x_2, \cdots, x_n)$ with $x_n = 0$ or $x_n = N$, and let B_n denote the remaining set of paths, namely, those paths $(x_1, x_2, \cdots, x_n)$ of length n for which $0 < x_n < N$. Note that for any path $(x_1, x_2, \cdots, x_n)$ in B_n, each x_i is strictly between 0 and N. Clearly,

$$p_a(A_N) > 2c^N,$$

whence

$$p_a(B_N) = 1 - p_a(A_N) < 1 - 2c^N.$$

Consider now a path $(x_1, x_2, \cdots, x_N)$ of length N with $0 < x_N < N$. Then the probability of the set of paths starting at x_N and not hitting 0 and N during the time points $N+1, N+2, \cdots, 2N$ is again $< 1-2c^N$. This implies that

$$p_a(B_{2N}|B_N) < 1 - 2c^N.$$

Therefore,

$$p_a(B_{2N}) = p_a(B_{2N}|B_N).p_a(B_N) < (1-2c^N)^2.$$

In general we have,

$$p_a(B_{kN}) < (1-2c^N)^k, k = 1, 2, \cdots$$

Hence

$$p_a(B_{kN}) \to 0, \quad \text{as} \quad k \to \infty.$$

Since the sequence $(B_n)_{n=1}^{\infty}$ is a decreasing sequence of sets, we see that their intersection $C = \cap_{n=1}^{\infty} B_n$ has probability zero. Thus, we have proved:

THEOREM 1. *Under assumption A, the event that the investor reaches N or 0 in finite time has probability one.*

Let us recast this theorem slightly differently: Let

$$Y_n((x_i)_{i=1}^{\infty}) = 1 \quad \text{if} \quad (x_1, x_2, \cdots, x_n) \in A_n, Y_n((x_i)_{i=1}^{\infty}) = 0 \quad \text{otherwise,}$$

Then Theorem 1 states that under assumption A, $Y_n \to 1$ a.e. as $n \to \infty$.

If we have L investors, with the stochastic process of their fortunes satisfying assumption A, and if $Y_n^l, 1 \leq l \leq L$ are the associated random variables Y_n as above, then clearly

$$\frac{1}{L}\sum_{l=1}^{L} Y_n^l \to 1 \quad \text{a.e.} \quad \text{as} \quad n \to \infty$$

In our application (Section 7) we will be verifying Theorem 1 in this form. Note that Y_n^l's are not assumed to be independent and it is not L but n that tends to infinity, so it is not the case of a standard Law of Large Numbers.

4 Special Case of Martingale Processes

Let X_n denote the investor's fortune at time n. It is a random variable taking values in the set $\{0, 1, 2, \cdots, N\}$. The sequence $(X_n)_{n=1}^{\infty}$ is the stochastic process of investor's fortune. It is said to be a martingale if for all n

$$E(X_n|X_1, X_2, \cdots, X_{n-1}) = E(X_{n-1}).$$

Equivalently, for all n and for all $x_1, x_2, \cdots, x_n$

$$\sum_{i=1}^{N} i p_a(i|x_1, x_2, \cdots, x_n) = x_n$$

If $(X_n)_{n=1}^{\infty}$ is a martingale, then

$$E(X_n) = E(X_{n-1}) = \cdots = E(X_0) = a$$

and since each X_n is non-negative, by Martingale Convergence Theorem $(X_n)_{n=1}^{\infty}$ converges almost surely (Doob, 1953). If this limit is denoted by X_∞, then $E(X_\infty) = a$. In addition, if X_∞ assumes only two values 0 and N, as is the case in Theorem 1, we have

$$0 \cdot p_a(X_\infty = 0) + N p_a(X_\infty = N) = a,$$

which yields

$$p_a(X_\infty = N) = \frac{a}{N}, \quad p_a(X_\infty = 0) = 1 - \frac{a}{N}.$$

Thus we have

Theorem 2. *If in Theorem 1 we assume in addition that the investor's walk is a martingale, then the probability that the investor reaches N in finite time is $\frac{a}{N}$ and the probability that she reaches 0 in finite time is $1 - \frac{a}{N}$.*

Thus, in Theorem 1, if the process is a martingale then the probability of reaching N is proportional to the initial investment. If $N = 2a$, then the probability of reaching N is $\frac{1}{2}$ which is same as the probability of reaching 0.

In case $N = \infty$, then $1 - \frac{a}{N} = 1$. This may be interpreted to mean that *if the investor is greedy or if there are no regulations, then, in the long run, the investor's fortune hits the bottom with probability one.*

Although Theorem 1 is more general than Theorem 2 which is a special case of Martingale Convergence Theorem, we will refer to Theorem 1 also as a Martingale Convergence Theorem, and this is the theorem we have in mind in the discussions below.

Let A denote the set of investor's paths which end up reaching N. Let B denote the set of investor's paths which end up reaching 0. The function $a \to P_a(A)$, $a \to p_a(B)$ seem to be good indicators of the state of the economy in the sense that if $P_a(A)$ is 'large', equivalently, if $p_a(B)$ is 'small', especially for small a, then an investor with small initial capital has better chances of doing well. In the next part we apply these concepts and results to actual data.

5 Net Worth of a Company

In the rest of this paper we will analyze and discuss certain data on movement of net worth of companies over a period of time. First we explain the term 'net worth' and our reasons for choosing this parameter for analysis and the source of data.

The Reserve Bank of India (RBI) regularly conducts studies on financial performance of private corporate sector in India. Each year, the RBI brings out studies on 'Finances of public limited companies' based on analysis of audited annual accounts of select non-Government non-financial public limited companies. The RBI study provides detailed analysis of the performance of private corporate sector at aggregate level and also an overview of performance of companies in various size classes, classified according to size of paid-up capital and sales. While the aggregated data of companies provides useful analysis of the performance of corporate sector, it does not reveal the extent of divergence in performance of individual companies. The RBI, however, maintains detailed firm-level data on financial performance of companies selected for their studies. We have used the firm-level data, obtained from RBI, for our analysis.

The RBI studies analyse financial performance of corporate sector using a large number of performance indicators. For the purpose of simplicity, we have chosen the variable 'net worth' for our analysis, which is a proxy of overall performance of a company. The term 'net worth' is defined as the sum of 'paid-up capital' and 'reserves and surplus'. Thus net worth includes changes in the paid-up capital, changes in reserves and also changes in surplus. Companies often raise money from shareholders through issue of capital at a premium. The premium part goes to share premium account under the head reserves. The reserve of a company also changes due to allocation of funds from profits of the company or withdrawal of funds for specific uses. The surplus of profits (or deficit in case of losses) after allocation of funds is credited to reserves. Hence, net worth reflects the overall financial performance of companies both under capital account and revenue account.

In economic terms the net worth is the excess of a company's assets over its liabilities, i.e., net worth = assets − liabilities. In the annual accounts of companies, assets and liabilities are given at book value; hence net worth is also at book value. Net worth is also referred to as shareholder's equity. As the name suggests, it is the money which belongs to the shareholders

(investors) of the company. In normal situations companies tries to maximize the shareholder's equity, as its market valuation, capacity to raise fresh capital, credit worthiness, etc are directly linked to the net worth. Hence, the net worth could be taken as a single measure for the evaluation of a company's performance. The changes in net worth of a company over the years indicate gains or losses of the investors in the company.

A company's net worth could be completely eroded due to losses, and in the worst case scenario may become negative, indicating that its liabilities are more than its assets and it has become insolvent. However, such companies may also continue their business for some more time. Some of these companies may manage to turn around and achieve positive net worth. This may happen due to various reasons like improvement in performance, infusion of fresh capital, profits from sale of assets (e.g., sale of surplus land), reduction in liabilities (e.g., debt restructure), etc. In our analysis, the performance of the company is measured by the change in net worth as at the end of the study period compared with the beginning of the period.

6 Data Description and Analysis

In this section we explain certain tables of data on net worth, discuss certain ratios which correspond to the quantities $p_a(A)$ and $p_a(B)$, mentioned in Section 4. These ratios seem to be good indicators for the state of the economy for the years under consideration. We analyse the performance of companies with reference to these ratios for the periods 1999-2002 and 2004-2007. In the former period the Indian economy grew moderately at an average rate of 5.5 per cent per annum (growth in gross domestic product (GDP) at current prices)[1], while in the later period it grew at a higher average rate of 8.8 per cent per annum. As per the RBI studies, the overall performance of private corporate sector in terms of growth and profitability improved remarkably during 2004-2007 after witnessing a slow down during the period 1999-2002. Hence, the ratios observed for these two periods are used as benchmark for comparison.

In the tables below we have used the following abbreviations: +ve for positive, -ve for negative, Rs. for rupees (Indian currency), G. Total for Grand Total, Rs. cr. to mean 'in rupees crore '.

[1]National Accounts Statistics, Central Statistical Organisation, Government of India

Table A gives certain data on net worth of 1844 companies for the three year period 1999-2002. The companies were classified according to their net worth during the base year 1999-2000. Column 1 in the tables below gives the class intervals of this classification. There are in all nine class intervals, starting with companies with net worth less than Rs.1 crore[2] to companies with net worth greater than Rs.200 crore. We will call these class intervals ranges. For each range the companies in that range were further divided into companies which record growth or decline in the net worth within a certain percentage of their original net worth. Columns 2, 3, 4, 5 give the classification of companies experiencing positive growth and columns 6, 7, 8, 9 give the breakup of companies experiencing negative growth. Column 10 gives the total of companies experiencing positive growth in the given range and column 11 gives similarly total of companies experiencing negative growth rate in the specified range. Column 12 gives the total number of companies in each range. Last row sums the numbers in the column above it.

TABLE A: CHANGE IN NET WORTH - FREQ. DIST., 1999-00 TO 2001-02

Number of companies

NW SIZE IN 1999-00	Positive growth in NW in the 3rd year				Negative Growth in NW in the 3rd year				+ve growth total	-ve growth total	Total
	<20%	20 to 50%	50 to 100%	>100%	-20 to 0%	-50 to -20%	-100 to -50%	< -100%			
Range (Rs. cr.)				p				q	k	l	n
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
< 1	19	14	7	12	16	13	8	22	52	59	111
1 - 2	21	8	5	9	15	8	7	9	43	39	82
2 - 5	71	40	20	18	62	23	22	17	149	124	273
5 - 10	73	35	21	11	76	29	11	26	140	142	282
10 - 20	83	39	17	13	77	32	20	18	152	147	299
20 - 50	86	60	22	9	96	40	14	14	177	164	341
50 -100	60	33	9	4	47	20	9	8	106	84	190
100 - 200	35	20	10	5	39	17	5	1	70	62	132
> 200	46	17	7	5	35	15	8	1	75	59	134
G. Total	494	266	118	86	463	197	104	116	964	880	1844

Companies with net worth (NW) > 0 in the base year 1999-2000

[2] 1 crore = ten million

Table B gives similar data for 2728 companies for the years 2004-2007, which were generally considered to be good years for the Indian economy. Comparing the last rows of the two tables makes it clear that 2004-2007 were better years for the corporate sector than the years 1999-2002.

Table B: Change in Net worth-freq dist., 2004-05 to 2006-07

Number of companies

NW SIZE IN 2004-05	Positive Growth in NW in the 3rd year				Negative Growth in NW in the 3rd year				+ve growth total	-ve growth total	Total
	<20%	20 to 50%	50 to 100%	>100%	-20 to 0%	-50 to -20%	-100 to -50%	<-100%			
Range (Rs. cr.)				p				q	k	l	n
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
< 1	9	13	5	21	1	3	3	6	48	13	61
1 - 2	19	6	5	21	7	4		3	51	14	65
2 - 5	69	45	40	48	31	11	6	4	202	52	254
5 - 10	144	93	50	97	32	7	3	10	384	52	436
10 - 20	131	107	73	97	35	24	5	3	408	67	475
20 - 50	132	140	102	133	33	12	7	6	507	58	565
50 -100	78	81	52	76	19	9	4	1	287	33	320
100 - 200	46	79	55	48	11	9	2	1	228	23	251
> 200	53	91	78	53	20	6			275	26	301
G. Total	681	655	460	594	189	85	30	34	2390	338	2728

Companies with net worth (NW) > 0 in the base year 2004-05

We would like to make some further analysis. For each range, let
p = the number of companies experiencing more than 100 percent growth
q = the number of companies experiencing less than -100 percent growth
n = total number of companies
k = total number of companies experiencing positive growth
l = total number of companies experiencing negative growth
Note that $l + k = n$.

Tables C and D record numbers p and q for each range and gives certain ratios for the years 1999-2002 and 2004-2007, respectively. The ratios $\frac{p}{n}, \frac{q}{n}$ correspond to probabilities $p_a(A), p_a(B)$ for a belonging to the appropriate range for which p, q and n are considered. The relative state of the economy for companies is clearly reflected in these ratios.

TABLE C: CHANGE IN NET WORTH - FREQ. DIST., 1999-00 TO 2001-02

Number of companies

NW SIZE IN 1999-00	+ve Growth in NW in 3rd yr.		-ve Growth in NW in 3rd yr.							
	+ve growth total	of which: >100%	-ve growth total	of which: <-100%	Total	**Ratios**				
Range (Rs. cr.)	k	p	l	q	n	$\frac{p}{k}$	$\frac{q}{l}$	$\frac{p}{n}$	$\frac{q}{n}$	$\frac{p+q}{n}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
<1	52	12	59	22	111	0.23	0.37	0.11	0.20	0.31
1 - 2	43	9	39	9	82	0.21	0.23	0.11	0.11	0.22
2 - 5	149	18	124	17	273	0.12	0.14	0.07	0.06	0.13
5 - 10	140	11	142	26	282	0.08	0.18	0.04	0.09	0.13
10 - 20	152	13	147	18	299	0.09	0.12	0.04	0.06	0.10
20 - 50	177	9	164	14	341	0.05	0.09	0.03	0.04	0.07
50 -100	106	4	84	8	190	0.04	0.10	0.02	0.04	0.06
100 - 200	70	5	62	1	132	0.07	0.02	0.04	0.01	0.05
> 200	75	5	59	1	134	0.07	0.02	0.04	0.01	0.04
G. Total	964	86	880	116	1844	0.09	0.13	0.05	0.06	0.11

The ratios in columns 9 & 10 may not add up to the entries in column 11 due to rounding

TABLE D: CHANGE IN NET WORTH - FREQ. DIST., 2004-05 TO 2006-07

Number of companies

NW SIZE IN 2004-05	+ve Growth in NW in 3rd yr.		-ve Growth in NW in 3rd yr.							
	+ve growth total	of which: >100%	-ve growth total	of which: <-100%	Total	**Ratios**				
Range (Rs. cr.)	k	p	l	q	n	$\frac{p}{k}$	$\frac{q}{l}$	$\frac{p}{n}$	$\frac{q}{n}$	$\frac{p+q}{n}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
< 1	48	21	13	6	61	0.44	0.46	0.34	0.10	0.44
1 - 2	51	21	14	3	65	0.41	0.21	0.32	0.05	0.37
2 - 5	202	48	52	4	254	0.24	0.08	0.19	0.02	0.20
5 - 10	384	97	52	10	436	0.25	0.19	0.22	0.02	0.25
10 - 20	408	97	67	3	475	0.24	0.04	0.20	0.01	0.21
20 - 50	507	133	58	6	565	0.26	0.10	0.24	0.01	0.25
50 -100	287	76	33	1	320	0.26	0.03	0.24	0.00	0.24
100 - 200	228	48	23	1	251	0.21	0.04	0.19	0.00	0.20
> 200	275	53	26		301	0.19	0.00	0.18	0.00	0.18
G. Total	2390	594	338	34	2728	0.25	0.10	0.22	0.01	0.23

The ratios in columns 9 & 10 may not add up to the entries in column 11 due to rounding

For the sake of clarity we juxtapose the ratios $\frac{p}{n}$ and $\frac{q}{n}$ from Tables C and D against each period in Table E and Figure 1. It is clear from this that

the period 2004-2007 was obviously better for the corporate sector compared to the period 1999-2002.

Table E: Ratios $\frac{p}{n}$ and $\frac{q}{n}$ from tables C and D

Range (Rs. cr.)	$\frac{p}{n}$ from Table C	$\frac{p}{n}$ from Table D	$\frac{q}{n}$ from Table C	$\frac{q}{n}$ from Table D
(1)	(2	3)	(4)	(5)
< 1	0.11	0.34	0.20	0.10
1 - 2	0.11	0.32	0.11	0.05
2 - 5	0.07	0.19	0.06	0.02
5 - 10	0.04	0.22	0.09	0.02
10 - 20	0.04	0.20	0.06	0.01
20 - 50	0.03	0.24	0.04	0.01
50 -100	0.02	0.24	0.04	0.00
100 - 200	0.04	0.19	0.01	0.00
> 200	0.04	0.18	0.01	0.00
All Companies	0.05	0.22	0.06	0.01

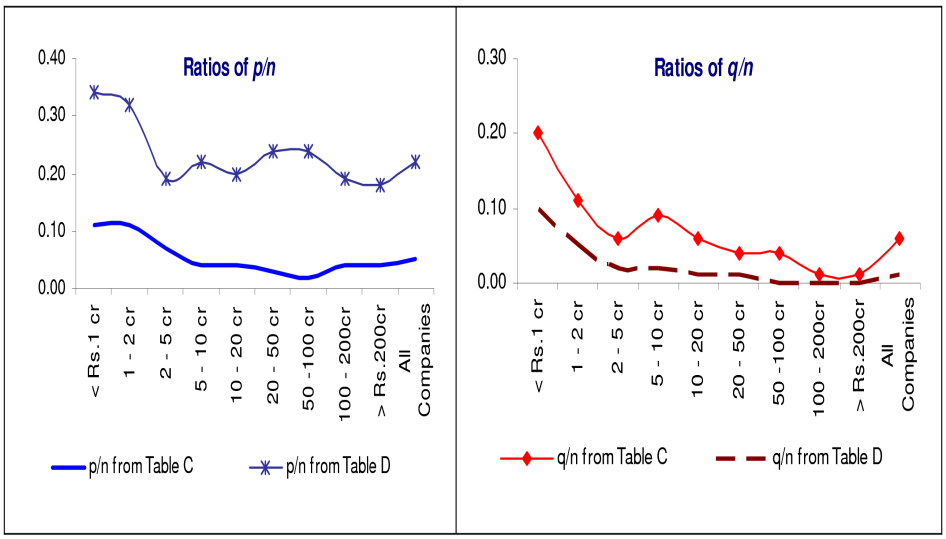

Figure 1: Ratios $\frac{p}{n}$ and $\frac{q}{n}$ from Tables C and D

7 Verification of Theorem 1

In this section we analyse and discuss certain data on movement of net worth of companies over the periods 1994-1995 to 1998-1999, 1994-1995 to 2002-2003 and 1994-1995 to 2006-2007 so that in all a period of 13 years from 1994 to 2007 is covered. A set of 2222 companies, having positive net worth in the base year 1994-95, were considered. They were classified according to their net worth during the year 1994-1995. As before, Tables 1, 2, 3 below give this classification, except that column 12 is for the number of 'missing' companies in each range and column 13 is for the totals. This is explained below. A company is 'missing' if information on its net worth is not on record at the end of the period of the table under consideration. Each entry in column on missing companies counts the number of missing companies for the end year of the table and for the range under consideration. With regard to entries in this column, a company may have ceased to exist in the intervening period for want of profits or may have merged with another company, or possibly simply not provided the required information for that particular year. In a separate section we will discuss the data taking into consideration the available information about the missing companies. A non-missing company is called robust. There are 2222-1242= 980 such companies.

Table 1: Change in Net worth - freq. dist., 1994-95 to 1998-99

NW SIZE IN 1994-95	+ve Growth in NW in 5th year				-ve Growth in NW in 5th year							
	$< 20\%$	20 to 50%	50 to 100%	$> 100\%$	-20 to 0%	-50 to -20 %	-100 to -50%	$< -100\%$	+ve growth total	-ve growth total	Missing*	G. Total
Range (Rs. cr.)				p				q	k	l	m	n
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
<1	18	17	18	51	8	6	3	20	104	37	65	206
1 - 2	16	18	21	42	7	6	7	13	97	33	50	180
2 - 5	33	48	59	78	14	14	20	29	218	77	106	401
5 - 10	40	37	39	69	23	21	10	19	185	73	90	348
10 - 20	26	48	56	64	20	15	14	18	194	67	78	339
20 - 50	40	52	71	91	15	19	11	18	254	63	62	379
50 -100	15	30	33	32	8	6	7	1	110	22	29	161
100 - 200	8	9	21	20	10	9	6	3	58	28	14	100
>200	11	24	31	17	7	5	1	2	83	15	10	108
G.Total	207	283	349	464	112	101	79	123	1303	415	504	2222
Companies with NW> 0 in the base year 1994-95												

NW : Net Worth (=Paid-up Capital + Reserves & Surpluses)

* Missing: Information on NW is not available at the end of the reference period

Table 2: Change in Net worth - freq. dist., 1994-95 to 2002-03

Number of companies

NW SIZE IN 1994-95	+ve Growth in NW in the 9th year				-ve Growth in NW in the 9th year							
	$< 20\%$	20 to 50%	50 to 100%	$> 100\%$	-20 to 0%	-50 to -20 %	-100 to -50%	$< -100\%$	+ve growth total	-ve growth total	Missing*	G. Total
Range (Rs. cr.)				p				q	k	l	m	n
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
<1	6	5	12	40	8	2	5	20	63	35	108	206
1 - 2	4	11	7	41	5	5	2	9	63	21	96	180
2 - 5	14	13	32	83	14	11	12	29	142	66	193	401
5 - 10	19	16	14	62	16	14	16	22	111	68	169	348
10 - 20	17	23	30	77	15	10	6	28	147	59	133	339
20 - 50	22	25	38	98	12	20	20	30	183	82	114	379
50 -100	8	19	22	37	8	5	10	12	86	35	40	161
100 - 200	8	3	14	26	5	8	6	6	51	25	24	100
>Rs.200	9	14	11	25	11	3	4	6	59	24	25	108
G. Total	107	129	180	489	94	78	81	162	905	415	902	2222

Table 3: Change in Net worth - freq. dist., 1994-95 to 2006-07

Number of companies

NW SIZE IN 1994-95	+ve Growth in NW in the 13th year				-ve Growth in NW in the 13th year							
	$< 20\%$	20 to 50%	50 to 100%	$> 100\%$	-20 to 0%	-50 to -20 %	-100 to -50%	$< -100\%$	+ve growth total	-ve growth total	Missing*	G. Total
Range (Rs. cr.)				p				q	k	l	m	n
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
<1		2	4	34		1	1	10	40	12	154	206
1 - 2		1	5	31	2	1	3	2	37	8	135	180
2 - 5		8	15	77	2	6	4	16	100	28	273	401
5 - 10	4	9	22	66	8	7	6	13	101	34	213	348
10 - 20	2	6	15	112	5	5	5	18	135	33	171	339
20 - 50	9	8	20	126	6	8	11	27	163	52	164	379
50 -100	4	8	11	55	5	6	3	7	78	21	62	161
100 - 200	2	3	7	42	1	1	1	8	54	11	35	100
>200	3	7	11	38	2	3	3	6	59	14	35	108
G.Total	24	52	110	581	31	38	37	107	767	213	1242	2222

Having set forth the tables, we make some immediate observations on them: (1) The grand totals of the number of companies which record more than 100 per cent growth in net worth increase as we go from Table 1 to

Table 2 to Table 3. This is also true for the bigger companies, but not so for the smaller companies. (2) The grand totals of the number of companies which record less than -100 per cent growth increase from first to second table and then decreases in the third table, these numbers being 123, 162 and 107. (3) The grand totals of number of missing companies increase as we move from first to second to third table, these numbers being 504, 902, 1242. Thus, at the end of the period under consideration more than half the companies go missing. If large proportion of them cease to exist for want of profits, then it does not look like an acceptable situation. This will be discussed separately. (4) The grand totals of number of companies in columns 2, 3, 4, 6 and 7 decrease as we move from first table to second to third table. The corresponding entries in the 8th column are 79, 81 and 37. Thus, we surmise that, in general, the tendency of a company in the robust category is to move to column 5, i.e., to double its net worth, or, to move to column 9, i.e., to do very badly. In Tables 4, 5, 6 we give the ratios for Tables 1, 2, 3 respectively.

TABLE 4: CHANGE IN NET WORTH - FREQ. DIST., 1994-95 TO 1998-99

NW SIZE IN 1994-95	+ve Growth in NW in **5**th yr.		-ve Growth in NW in **5**th yr.				**Ratios**				
	+ve growth total	of which: $> 100\%$	-ve growth total	of which: $< -100\%$	Missing*	Total	$\frac{p}{k}$	$\frac{q}{l}$	$\frac{p}{k+l}$	$\frac{q}{k+l}$	$\frac{p+q}{k+l}$
Range (Rs. cr.)	k	p	l	q	m	n					
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
<1	104	51	37	20	65	206	0.49	0.54	0.36	0.14	0.50
1 - 2	97	42	33	13	50	180	0.43	0.39	0.32	0.10	0.42
2 - 5	218	78	77	29	106	401	0.36	0.38	0.26	0.10	0.36
5 - 10	185	69	73	19	90	348	0.37	0.26	0.27	0.07	0.34
10 - 20	194	64	67	18	78	339	0.33	0.27	0.25	0.07	0.31
20 - 50	254	91	63	18	62	379	0.36	0.29	0.29	0.06	0.34
50 -100	110	32	22	1	29	161	0.29	0.05	0.24	0.01	0.25
100 - 200	58	20	28	3	14	100	0.34	0.11	0.23	0.03	0.27
> 200	83	17	15	2	10	108	0.20	0.13	0.17	0.02	0.19
G. Total	1303	464	415	123	504	2222	0.36	0.30	0.27	0.07	0.34

The ratios in col.10 &11 may not add up to the totals in col.12 due to rounding off
* Missing: Information on NW is not available at the end of the reference period

Table 5: Change in Net worth - freq. dist., 1994-95 to 2002-03

NW SIZE IN 1994-95	+ve Growth in NW in 9th yr		-ve Growth in NW in 9th yr				**Ratios**				
	+ve growth total	of which: $> 100\%$	-ve growth total	of which: $< -100\%$	Missing*	Total	$\frac{p}{k}$	$\frac{q}{l}$	$\frac{p}{k+l}$	$\frac{q}{k+l}$	$\frac{p+q}{k+l}$
Range (Rs. cr.)	k	p	l	q	m	n					
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
<1	63	40	35	20	108	206	0.63	0.57	0.41	0.20	0.61
1 - 2	63	41	21	9	96	180	0.65	0.43	0.49	0.11	0.60
2 - 5	142	83	66	29	193	401	0.58	0.44	0.40	0.14	0.54
5 - 10	111	62	68	22	169	348	0.56	0.32	0.35	0.12	0.47
10 - 20	147	77	59	28	133	339	0.52	0.47	0.37	0.14	0.51
20 - 50	183	98	82	30	114	379	0.54	0.37	0.37	0.11	0.48
50 -100	86	37	35	12	40	161	0.43	0.34	0.31	0.10	0.40
100 - 200	51	26	25	6	24	100	0.51	0.24	0.34	0.08	0.42
> 200	59	25	24	6	25	108	0.42	0.25	0.30	0.07	0.37
G. Total	905	489	415	162	902	2222	0.54	0.39	0.37	0.12	0.49

Table 6: Change in Net worth - freq. dist., 1994-95 to 2006-07

NW SIZE IN 1994-95	+ve Growth in NW in 13th yr		-ve Growth in NW in 13th yr				**Ratios**				
	+ve growth total	of which: $> 100\%$	-ve growth total	of which: $< -100\%$	Missing*	Total	$\frac{p}{k}$	$\frac{q}{l}$	$\frac{p}{k+l}$	$\frac{q}{k+l}$	$\frac{p+q}{k+l}$
Range (Rs. cr.)	k	p	l	q	m	n					
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
<1	40	34	12	10	154	206	0.85	0.83	0.65	0.19	0.85
1 - 2	37	31	8	2	135	180	0.84	0.25	0.69	0.04	0.73
2 - 5	100	77	28	16	273	401	0.77	0.57	0.60	0.13	0.73
5 – 10	101	66	34	13	213	348	0.65	0.38	0.49	0.10	0.59
10 - 20	135	112	33	18	171	339	0.83	0.55	0.67	0.11	0.77
20 - 50	163	126	52	27	164	379	0.77	0.52	0.59	0.13	0.71
50 -100	78	55	21	7	62	161	0.71	0.33	0.56	0.07	0.63
100 - 200	54	42	11	8	35	100	0.78	0.73	0.65	0.12	0.77
> 200	59	38	14	6	35	108	0.64	0.43	0.52	0.08	0.60
G. Total	767	581	213	107	1242	2222	0.76	0.50	0.59	0.11	0.70

We observe the following:

(1) In each range the ratios $\frac{p}{k}$ increase as we pass from Table 4 to 5 to 6 and are high for smaller companies in relation to bigger companies. Thus,

among the robust companies that record positive growth, high proportion of such companies record more than 100 percent growth. In addition, among the robust companies that record positive growth in net worth, the smaller companies double their net worth with higher proportion than bigger companies. (2) In general, in each range, the ratios $\frac{q}{l}$ increase as we pass from Table 4 to 5 to 6 and the ratio is high for smaller companies in relation to bigger companies. Thus, among the companies which suffer loss, higher proportion of small companies do very badly compared to larger companies. (3) The ratios $\frac{p}{k+l}$ show that they increase with time and eventually exceeds $\frac{1}{2}$ in all ranges and substantially high for smaller companies. (4) The ratios $\frac{q}{k+l}$ remain rather small although higher for smaller companies. (5) The ratios $\frac{p+q}{k+l}$ increase in each range and overall and eventually exceeds 1/2 in all ranges and rather big for small companies.

Thus, we see that among the robust companies the performance in the long run is either 'very good' or 'very bad' with high probability. This is in conformity with the theory in the third section. We juxtapose in Table 7 and Figure 2 the ratios $\frac{p}{k+l}$ and $\frac{q}{k+l}$ against the corresponding ratios from Tables D and C.

Table 7: Comparison of the ratios $\frac{p}{k+l}$ AND $\frac{q}{k+l}$ from Table 6 with the corresponding ratios from Tables D and C

Period	2004-2007	1994-2007	1999-2002	1994-2007
Range (Rs. cr.)	$\frac{p}{n}$	$\frac{p}{k+l}$	$\frac{q}{n}$	$\frac{q}{k+l}$
	from Table D	from Table 6	from Table C	from Table 6
(1)	(2)	(3)	(4)	(5)
< 1	0.34	0.65	0.20	0.19
1 - 2	0.32	0.69	0.11	0.04
2 - 5	0.19	0.60	0.06	0.13
5 - 10	0.22	0.49	0.09	0.10
10 - 20	0.20	0.67	0.06	0.11
20 - 50	0.24	0.59	0.04	0.13
50 -100	0.24	0.56	0.04	0.07
100 - 200	0.19	0.65	0.01	0.12
> 200	0.18	0.52	0.01	0.08
All Companies	0.22	0.59	0.06	0.11

It is clear from this that if we ignore the 'missing companies' the statistics gives a rather rosy picture of the performance of the companies in the

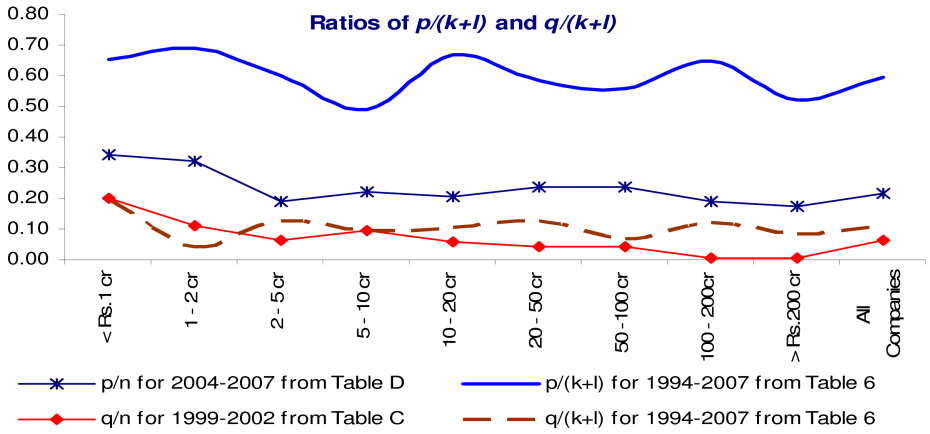

Figure 2: The ratios $\frac{p}{k+l}$ and $\frac{q}{k+l}$ from Table 6 plotted against the corresponding ratios from Tables D and C

economy as a whole. However relatively accurate picture will be given by the analysis of the whole data, and not just part of the data. Before we do this we analyse the set of missing companies.

7.1. Case of 'missing' or non-robust companies. We now consider the set of 'missing' or non-robust companies. First thing is to observe that high proportion of smaller companies go missing than the bigger companies. The initial reaction is to presume that a company goes missing from the data because it was performing very badly and so decided to wind up. A careful look at the data, however, shows a somewhat different situation. A non-trivial number of companies go missing even if they are performing well. The situation is in a way dual or mirror image of the situation for the set of robust companies, namely there are more companies in the negative category than the number of companies which double their net worth. Table 8 gives the frequency distribution and the ratios for the set of 1242 missing companies from Table 3. The classification is done according to the company's net worth in the year just before it went missing.

TABLE 8. CHANGE IN NET WORTH (NW): FREQ. DIST., 1994-95 TO 2006-07

NW SIZE IN 1994-95 Range (Rs. cr.)	Positive growth in NW in 13th year: +ve growth total k	of which >100% p	Negative growth in NW in 13th year: -ve growth total l	of which <-100% q	Total $n = k + l$	**Ratios** $\frac{p}{k}$	$\frac{q}{l}$	$\frac{p}{n}$	$\frac{q}{n}$	$\frac{p+q}{n}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
<1	85	48	69	27	154	0.56	0.39	0.31	0.18	0.49
1 - 2	82	36	53	20	135	0.44	0.38	0.27	0.15	0.41
2 - 5	162	79	111	42	273	0.49	0.38	0.29	0.15	0.44
5 - 10	120	60	93	29	213	0.50	0.31	0.28	0.14	0.42
10 - 20	104	42	67	28	171	0.40	0.42	0.25	0.16	0.41
20 - 50	108	50	56	22	164	0.46	0.39	0.30	0.13	0.44
50 -100	44	21	18	5	62	0.48	0.28	0.34	0.08	0.42
100 - 200	17	8	18	3	35	0.47	0.17	0.23	0.09	0.31
> 200	27	10	8	4	35	0.37	0.50	0.29	0.11	0.40
Grand total	749	354	493	180	1242	0.47	0.37	0.29	0.14	0.43

TABLE 9. A COMPARISON OF THE RATIOS $\frac{p}{n}$ AND $\frac{q}{n}$ FROM TABLE 8 WITH CORRESPONDING RATIOS FROM TABLES D AND C

Period	2004-2007	1994-2007	1999-2002	1994-2007
Range (Rs. cr.)	$\frac{p}{n}$ from Table D	$\frac{p}{n}$ from Table 8	$\frac{q}{n}$ from Table C	$\frac{q}{n}$ from Table 8
(1)	(2)	(3)	(4)	(5)
< 1	0.34	0.31	0.20	0.18
1 - 2	0.32	0.27	0.11	0.15
2 - 5	0.19	0.29	0.06	0.15
5 - 10	0.22	0.28	0.09	0.14
10 - 20	0.20	0.25	0.06	0.16
20 - 50	0.24	0.30	0.04	0.13
50 -100	0.24	0.34	0.04	0.08
100 - 200	0.19	0.23	0.01	0.09
> 200	0.18	0.29	0.01	0.11
All Companies	0.22	0.29	0.06	0.14

As before, a comparison of the ratios $\frac{p}{n}$ and $\frac{q}{n}$ from (Table 8) with corresponding ratios from Tables D and C, respectively, given in Table 9 and Figure 3, clearly shows that the collective performance of these companies over the years is not satisfactory, and the years under consideration certainly not good for the small companies among these.

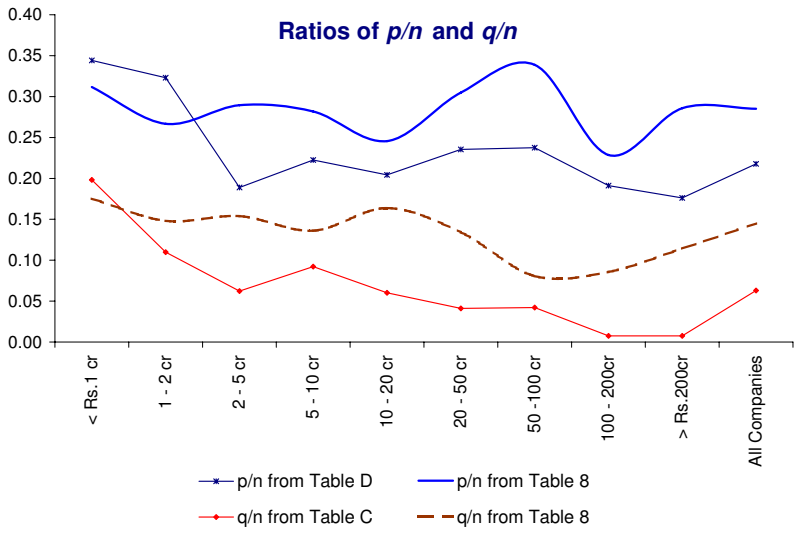

Figure 3: Ratios $\frac{p}{n}$ and $\frac{q}{n}$ from Table 8, plotted against values from Tables C & D

Let us finally consider the full set of 2222 companies, the missing companies being classified by their net worth in the year preceding the year they go missing. Table 10 below gives the numbers k, p, l, q etc. and the associated ratios.

Juxtaposing the ratios $\frac{p}{n}$, $\frac{q}{n}$ from this table against the corresponding numbers from Tables D and C in Table 11, we see immediately, considering the long period for which the data is being considered, that these ratios are not satisfactory, especially for the smaller companies. The ratios $\frac{p}{n}$ should exceed $\frac{1}{2}$ and the ratios $\frac{q}{n}$ should be smaller. Also, the ratios $\frac{p}{k}$ are no more

superior for small companies as is the case in the case of robust companies (in Table 6).

Table 10: Change in Net worth – freq. dist., 1994-95 to 2006-07

NW SIZE IN 1994-95	+ve Growth in NW in **13**th yr		-ve Growth in NW in **13**th yr			Ratios				
	+ve growth total	of which: $> 100\%$	-ve growth total	of which: $< -100\%$	Total	$\frac{p}{k}$	$\frac{q}{l}$	$\frac{p}{n}$	$\frac{q}{n}$	$\frac{p+q}{n}$
Range (Rs. cr.)	k	p	l	q	$n = k+l$					
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
< 1	125	82	81	37	206	0.66	0.46	0.40	0.18	0.58
1 - 2	119	67	61	22	180	0.56	0.36	0.37	0.12	0.49
2 - 5	262	156	139	58	401	0.60	0.42	0.39	0.14	0.53
5 - 10	221	126	127	42	348	0.57	0.33	0.36	0.12	0.48
10 - 20	239	154	100	46	339	0.64	0.46	0.45	0.14	0.59
20 - 50	271	176	108	49	379	0.65	0.45	0.46	0.13	0.59
50 -100	122	76	39	12	161	0.62	0.31	0.47	0.07	0.55
100 - 200	71	50	29	11	100	0.70	0.38	0.50	0.11	0.61
> 200	86	48	22	10	108	0.56	0.45	0.44	0.09	0.54
G. Total	1516	935	706	287	2222	0.62	0.41	0.42	0.13	0.55

Table 11: A comparison of the ratios $\frac{p}{n}$ & $\frac{q}{n}$ from Table 10 corresponding ratios from Tables D & C

Period	2004-2007	1994-2007	1999-2002	1994-2007
Range (Rs. cr.)	$\frac{p}{n}$ from Table D	$\frac{p}{n}$ from Table 10	$\frac{q}{n}$ from Table C	$\frac{q}{n}$ Table 10
(1)	(2)	(3)	(4)	(5)
< 1	0.34	0.40	0.20	0.18
1-2	0.32	0.37	0.11	0.12
2 - 5	0.19	0.39	0.06	0.14
5 - 10	0.22	0.36	0.09	0.12
10 - 20	0.20	0.45	0.06	0.14
20 - 50	0.24	0.46	0.04	0.13
50 -100	0.24	0.47	0.04	0.07
100 - 200	0.19	0.50	0.01	0.11
> 200	0.18	0.44	0.01	0.09
All Companies	0.22	0.42	0.06	0.13

If we write $t = l(non - robust) + q(robust)$, then for any given range, the number t may be treated as the number of companies that become insolvent. (We treat a loss making company which goes 'missing' or a company which records negative net worth as insolvent).

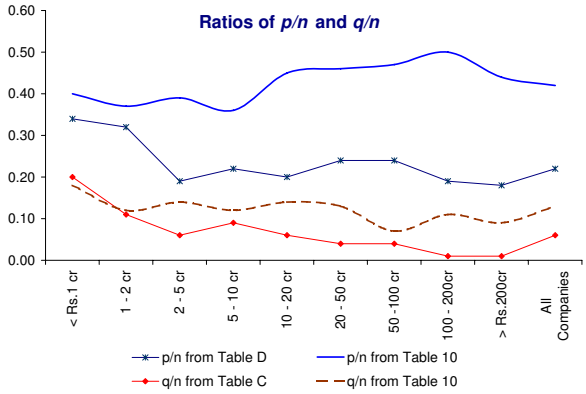

Figure 4: Ratios $\frac{p}{n}$ & $\frac{q}{n}$ from Table 10, plotted against values from Tables C & D

The ratios $\frac{t}{n}$ do not seem satisfactory, whether for big or for small companies. Finally the ratios $\frac{p+t}{n}$ are in the range of 0.57 to 0.78, and the ratio for all companies is 0.69, confirming thus the tendency towards extremities.

Table 12. Proportions Showing Tendency towards Extremes

Range (Rs. cr.)	t	$p+t$	n	t/n	$(p+t)/n$
< 1	79	161	206	0.38	0.78
1-2	55	122	180	0.31	0.68
2 - 5	127	283	401	0.32	0.71
5 - 10	106	232	348	0.30	0.67
10 - 20	85	239	339	0.25	0.71
20 - 50	83	259	379	0.22	0.68
50 -100	25	101	161	0.16	0.63
100 - 200	26	76	100	0.26	0.76
> 200	14	62	108	0.13	0.57
All Companies	600	1535	2222	0.27	0.69

We confirm the long term tendency of the performance of the company to the extremities by giving similar tables (I-V) for a set of 2564 companies considered over a nine-year period 1998-1999 to 2006-2007.

TABLE I: CHANGE IN NET WORTH - FREQ. DIST., 1998-99 TO 2002-03

NW SIZE IN 1998-99	+ve Growth in NW in 5th yr		-ve Growth in NW in 5th yr				**Ratios**				
	+ve growth total	of which: $> 100\%$	-ve growth total	of which: $< -100\%$	Missing*	Total	$\frac{p}{k}$	$\frac{q}{l}$	$\frac{p}{k+l}$	$\frac{q}{k+l}$	$\frac{p+q}{k+l}$
Range (Rs. cr.)	k	p	l	q	m	n					
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
< 1	45	16	39	17	53	137	0.36	0.44	0.19	0.20	0.39
1 - 2	38	13	41	13	55	134	0.34	0.32	0.16	0.16	0.33
2 - 5	141	34	100	26	153	394	0.24	0.26	0.14	0.11	0.25
5 - 10	137	36	118	24	148	403	0.26	0.20	0.14	0.09	0.24
10 - 20	154	32	137	31	120	411	0.21	0.23	0.11	0.11	0.22
20 - 50	214	46	144	14	118	476	0.21	0.10	0.13	0.04	0.17
50 -100	124	17	81	13	55	260	0.14	0.16	0.08	0.06	0.15
100 - 200	89	11	54	2	35	178	0.12	0.04	0.08	0.01	0.09
$>$ Rs.200	77	12	59	3	35	171	0.16	0.05	0.09	0.02	0.11
G. Total	1019	217	773	143	772	2564	0.21	0.18	0.12	0.08	0.20

Note: The ratios in col.10 &11 may not add up to the totals in col.12 due to rounding off
**Missing*: Information on NW is not available at the end of the reference period

TABLE II: CHANGE IN NET WORTH - FREQ. DIST., 1998-99 TO 2004-05

NW SIZE IN 1998-99	+ve Growth in NW in 7th yr		-ve Growth in NW in 7th yr				**Ratios**				
	+ve growth total	of which: $>100\%$	-ve growth total	of which: $< -100\%$	Missing*	Total	$\frac{p}{k}$	$\frac{q}{l}$	$\frac{p}{k+l}$	$\frac{q}{k+l}$	$\frac{p+q}{k+l}$
Range (Rs. cr.)	k	p	l	q	m	n					
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
< 1	39	18	22	13	76	137	0.46	0.59	0.30	0.21	0.51
1 - 2	25	9	26	8	83	134	0.36	0.31	0.18	0.16	0.33
2 - 5	120	58	69	23	205	394	0.48	0.33	0.31	0.12	0.43
5 - 10	147	46	80	22	176	403	0.31	0.28	0.20	0.10	0.30
10 - 20	175	62	101	33	135	411	0.35	0.33	0.22	0.12	0.34
20 - 50	234	78	103	25	139	476	0.33	0.24	0.23	0.07	0.31
50 -100	134	53	71	15	55	260	0.40	0.21	0.26	0.07	0.33
100 - 200	97	30	46	5	35	178	0.31	0.11	0.21	0.03	0.24
$>$ 200	94	22	47	8	30	171	0.23	0.17	0.16	0.06	0.21
G. Total	1065	376	565	152	934	2564	0.35	0.27	0.23	0.09	0.32

Table III: Change in Net worth - freq. dist., 1998-99 to 2006-07

NW SIZE IN 1998-99	+ve Growth in NW in 9th yr		-ve Growth in NW in 9th yr				**Ratios**				
	+ve growth total	of which: >100%	-ve growth total	of which: <-100%	Missing*	Total	$\frac{p}{k}$	$\frac{q}{l}$	$\frac{p}{k+l}$	$\frac{q}{k+l}$	$\frac{p+q}{k+l}$
Range (Rs. cr.)	k	p	l	q	m	n					
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
< 1	26	17	10	6	101	137	0.65	0.60	0.47	0.17	0.64
1 - 2	18	13	16	8	100	134	0.72	0.50	0.38	0.24	0.62
2 - 5	100	48	31	12	263	394	0.48	0.39	0.37	0.09	0.46
5 - 10	137	71	40	10	226	403	0.52	0.25	0.40	0.06	0.46
10 - 20	163	85	56	14	192	411	0.52	0.25	0.39	0.06	0.45
20 - 50	219	129	53	16	204	476	0.59	0.30	0.47	0.06	0.53
50 -100	120	69	38	9	102	260	0.58	0.24	0.44	0.06	0.49
100 - 200	97	49	22	5	59	178	0.51	0.23	0.41	0.04	0.45
> Rs.200	93	52	20	4	58	171	0.56	0.20	0.46	0.04	0.50
G. Total	973	533	286	84	1305	2564	0.55	0.29	0.42	0.07	0.49

Table IV: Change in Net worth - freq. dist., 1998-99 to 2006-07

NW SIZE IN 1998-99	+ve Growth in NW in 9th year		-ve Growth in NW in 9th year			**Ratios**				
	+ve growth total	of which: >100%	-ve growth total	of which: <-100%	Total	$\frac{p}{k}$	$\frac{q}{l}$	$\frac{p}{n}$	$\frac{q}{n}$	$\frac{p+q}{n}$
Range (Rs. cr.)	k	p	l	q	n					
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
< 1	49	16	52	21	101	0.33	0.40	0.16	0.21	0.37
1 - 2	49	13	51	15	100	0.27	0.29	0.13	0.15	0.28
2 - 5	136	39	127	32	263	0.29	0.25	0.15	0.12	0.27
5 - 10	112	33	114	29	226	0.29	0.25	0.15	0.13	0.27
10 - 20	109	27	83	19	192	0.25	0.23	0.14	0.10	0.24
20 - 50	111	25	93	18	204	0.23	0.19	0.12	0.09	0.21
50 -100	56	15	46	9	102	0.27	0.20	0.15	0.09	0.24
100 - 200	36	11	23	3	59	0.31	0.13	0.19	0.05	0.24
> 200	38	6	20	4	58	0.16	0.20	0.10	0.07	0.17
G. Total	696	185	609	150	1305	0.27	0.25	0.14	0.11	0.26

Table IV gives the frequency distribution and the ratios for the set of 1305 missing companies from Table III. The classification is done according to the company's net worth in the year just before it went missing.

TABLE V: CHANGE IN NET WORTH - FREQ. DIST., 1998-99 TO 2006-07

NW SIZE IN 1998-99	+ve Growth in NW in 9th year		-ve Growth in NW in 9th year			**Ratios**				
	+ve growth total	of which: >100%	-ve growth total	of which: <-100%	Total	$\frac{p}{k}$	$\frac{q}{l}$	$\frac{p}{n}$	$\frac{q}{n}$	$\frac{p+q}{n}$
Range (Rs. cr.)	k	p	l	q	n					
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
< 1	75	33	62	27	137	0.44	0.44	0.24	0.20	0.44
1 - 2	67	26	67	23	134	0.39	0.34	0.19	0.17	0.37
2 - 5	236	87	158	44	394	0.37	0.28	0.22	0.11	0.33
5 - 10	249	104	154	39	403	0.42	0.25	0.26	0.10	0.35
10 - 20	272	112	139	33	411	0.41	0.24	0.27	0.08	0.35
20 - 50	330	154	146	34	476	0.47	0.23	0.32	0.07	0.39
50 -100	176	84	84	18	260	0.48	0.21	0.32	0.07	0.39
100 - 200	133	60	45	8	178	0.45	0.18	0.34	0.04	0.38
> 200	131	58	40	8	171	0.44	0.20	0.34	0.05	0.39
G. Total	1669	718	895	234	2564	0.43	0.26	0.28	0.09	0.37

Table V gives the numbers k, p, l, q etc. and the associated ratios for the full set of 2564 companies, where the missing companies are classified as per Table IV.

The results confirm the observations made in Tables 1 to 11. Other scenario is similar, except that the ratios are different.

A reader may observe that in table 8 the ratio $\frac{p+q}{n} < \frac{1}{2}$ and thus conclude that this does not confirm the theory above. However this is not so. For the theory assumes that the investor remains active until she hits N or 0, and does not wind up her activity in the middle. On the other hand, in the absence of any information, 'missing companies' must be assumed to have stopped from the year they go missing. Indeed this part of the data is a good 'practical' illustration of the concept of stopping time of martingale theory.

8 Conclusion

The main conclusions of our study are as follows:

(1) In the long run, there is a strong tendency for a company to either do very well (double its net worth) or go insolvent, as predicted by the Martingale Convergence Theorem.

(2) The ratios $\frac{p}{n}$, $\frac{q}{n}$, $\frac{t}{n}$ are good indicators of the performance of the industry. There can be a program to compute them on yearly basis to be compared against benchmark values. In absence of known bench mark values, one may recommend the values corresponding to the robust data as the bench mark values. Decisions which influence the economy should be such that for small companies the ratios $\frac{p}{n}$ should be large, while the ratios $\frac{q}{n}$, $\frac{t}{n}$ should be small. Like the old adage, 'take care of pennies, Pounds will take care of themselves' one may say here, "pay attention to the small enterprises, large ones will take care of themselves".

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