Pump-noise-induced Hanle effect in forward four-wave mixing

G. S. Agarwal
School of Physics, University of Hyderabad, Hyderabad 500 134, Andra Pradesh, India

C. V. Kunasz and J. Cooper
Joint Institute for Laboratory Astrophysics, University of Colorado and National Bureau of Standards, P.O. Box 440, Boulder, Colorado 80309-0440

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This is an addendum to our previous publication [Phys. Rev. A **36**, 143 (1987)] dealing with cross-correlation–induced resonances in four-wave mixing. The temporal fluctuations of the source lead to well-resolved Hanle resonances in forward four-wave mixing which is in contrast to the dephasing-induced resonance in the same geometry.

In a previous paper,[1] which we shall refer to subsequently as AKC, we discussed the effects of cross correlation between the pump and probe fields on four-wave-mixing signals. We developed a consistent theory to show the existence of the cross-correlation–induced extra resonances which were first predicted by Prior *et al.*[2] There might be practical difficulties in producing complete cross correlation between the pump and probe fields especially for the case in which pump and probe frequencies differ greatly (by many many cm^{-1}). The case of Hanle resonances[3] is especially attractive since in this case both pump and probe fields are derived from the same source and hence they are automatically fully correlated. Thus fluctuation-induced resonances should be seen much more easily in experiments on Hanle systems.[4] In this paper we consider forward four-wave mixing on the optical transition $j=0$ to $j=1$ and we study the fluctuation-induced Hanle resonances. It turns out that for the geometry under consideration, it is difficult to see the collisional dephasing-induced Hanle resonances because of the large background produced by the dephasing collisions. The background on the other hand is negligible for the case of fluctuation-induced resonances. In this sense forward four-wave mixing is ideally suited for studying fluctuation-induced Hanle resonances.

The geometrical arrangement is shown in Fig. 1. The pump and probe are assumed to be derived from the same source and hence their frequencies are identical. These beams make a small angle ϕ with each other. We assume that the pump is linearly polarized and that its

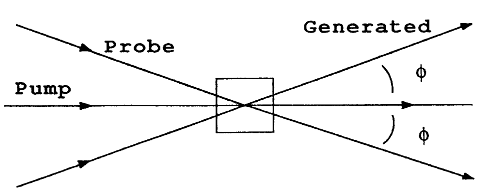

FIG. 1. Beam geometry.

σ_+ (σ_-) component acts on the transition $|0,0\rangle \leftrightarrow |1,-1\rangle$ [$|0,0\rangle \leftrightarrow |1,+1\rangle$]. The probe is assumed to be σ_+ polarized. Since the radiation generated by four-wave mixing is dominated by the σ_- component, we will focus only on this contribution. Any background σ_+ component of the generated radiation can be removed by an appropriate polarizer. Thus we will calculate the induced coherence given by the density matrix element ρ_{13} with states labeled as $|3\rangle \equiv |0,0\rangle$, $|1\rangle = |1,+1\rangle$, $|2\rangle = |1,-1\rangle$. The four-wave-mixing signal I is to be scanned as a function of the magnetic field or ω_{12}, the Zeeman splitting between the two excited states.

We first discuss briefly the analytic form of the signal produced by the dephasing collisions. For large values of the detuning Δ one finds the approximate value of the signal as [cf. AK (Ref. 5) Eq. (2.17)]

$$I = \frac{|g_1|^2 |g_2|^2 |\lambda_2|^2}{\Delta^6} \left| 1 + \frac{2\Gamma_2}{\gamma} + \frac{\Gamma_1 + \Gamma_2 - \Gamma_0}{\Gamma_0 + i\omega_{12}} \right|^2 . \tag{1}$$

Here $g_1[g_2]$ is the Rabi frequency associated with the $\sigma_-[\sigma_+]$ pump and λ_2 is the Rabi frequency of the σ_+ probe; γ^{-1} is the lifetime of the excited state and the Γ's give the decay rate of the off diagonal elements of ρ. In the special case $\Gamma_1 = \Gamma_2 = \gamma/2 + \Gamma^{ph}, \Gamma_0 = \gamma + \Gamma^{ph}$, Eq. (1) reduces to

$$I = \frac{|g_1|^2 |g_2|^2 |\lambda_2|^2}{\Delta^6} \times \left| 1 + \left[1 + \frac{2\Gamma^{ph}}{\gamma} \right] + \frac{\Gamma^{ph}}{\gamma + \Gamma^{ph} + i\omega_{12}} \right|^2 . \tag{2}$$

This result shows why it is that for the forward geometry the dephasing collisions not only lead to the presence of the Hanle resonance but also substantially increase the background,[6] thereby making impossible the observation of dephasing-induced Hanle resonance forward four-wave mixing.

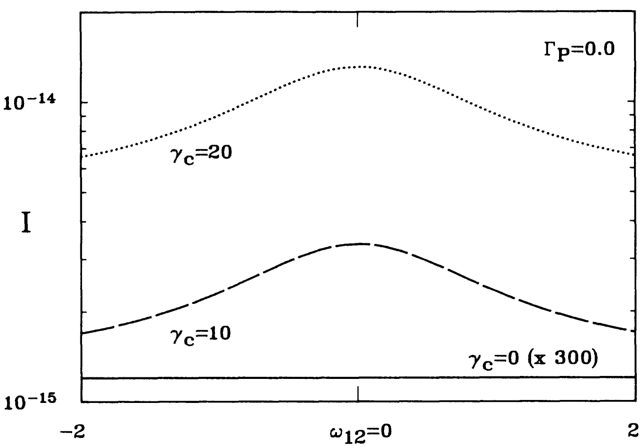

FIG. 2. The forward four-wave-mixing signal produced in the $j=0$ to $j=1$ optical transition, as a function of the Zeeman splitting ω_{12} (magnetic field) for various values of the fluctuation parameter γ_c. The medium has no dephasing collisions ($\Gamma^{ph}=\Gamma_p\gamma=0$) and the detuning Δ has been set equal to 1000. All parameters are in units of γ.

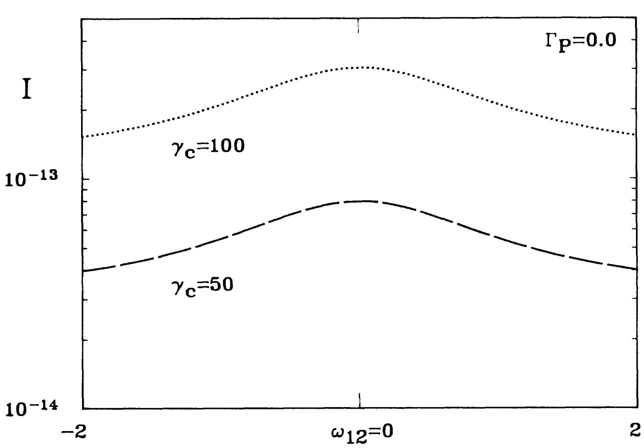

FIG. 3. Same as in Fig. 2 but for larger values of γ_c.

We next discuss how temporal fluctuations of the source can lead to well-defined and well-resolved Hanle resonance in forward four-wave mixing.[7] For this purpose we assume that no dephasing collisions are present and we use the theory developed in our previous paper. The calculations are based on Eq. (15) of AKC,[1] which gives the spectrum of the coherently generated radiation. It should be noted that in addition to the four-wave-mixing contribution to this spectrum there are also several other frequencies generated by the fluctuations of the source. We isolate the poles responsible for four-wave mixing and compute the residue at such poles. The spectrum has some poles of order two because the frequencies of the pump and probe are the same ($\delta=0$). The four-wave-mixing contribution is obtained by summing up the contributions from all the poles corresponding to the eigenvalues of M in the neighborhood of zero and $\pm\omega_{12}$. Note that ω_{12} is being scanned in the vicinity of zero, as is typical for Hanle resonance experiments.

Some typical results of our computations are shown in Figs. 2 and 3. There we see the existence of a well-resolved fluctuation-induced Hanle resonance[8] for which peak height scales as γ_c^2. The width of the resonance is essentially independent of γ_c and of the order of γ. The lines are far from Lorentzian. This should be contrasted with the usual[9] pressure-induced resonance whose peak height (width) is independent of (proportional to) pressure.

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[1]G. S. Agarwal, C. V. Kunasz, and J. Cooper, Phys. Rev. A **36**, 143 (1987).

[2]Y. Prior, I. Schek, and J. Jortner, Phys. Rev. A **31**, 3775 (1985).

[3]For the theory of the coherent Hanle effect, see A. Corney, B. P. Kibble, and G. W. Series, Proc. R. Soc. London, Ser. A **293**, 70 (1966); W. Gawlik and G. W. Series, in *Laser Spectroscopy IV*, edited by H. Walter and K. W. Rothe (Springer-Verlag, Berlin, 1979), p. 340.

[4]The effects of pump fluctuations on Hanle signals in *fluorescence* have been studied extensively [P. Avan and C. Cohen-Tannoudji, J. Phys. B. **10**, 171 (1977); G. S. Agarwal, Phys. Rev. A **18**, 1490 (1978); P. Ananthalakshmi and G. S. Agarwal, *ibid.* **23**, 2553 (1981); R. Saxena and G. S. Agarwal, *ibid.* **25**, 2123 (1982)].

[5]G. S. Agarwal and C. V. Kunasz, Phys. Rev. A **27**, 996 (1983); equations from this paper will be referred to as AK.

[6]Experiments on dephasing-induced Hanle resonances are done in phase-conjugate geometry [R. Scholz, J. Mlynek, and W. Lange, Phys. Rev. Lett. **51**, 1761 (1983); N. Bloembergen, Y. H. Zou, and L. J. Rothberg, *ibid.* **54**, 186 (1985)]. In a future publication we will treat the case of fluctuation-induced Hanle resonances in phase-conjugate geometry.

[7]Doppler broadening is negligible if the detuning Δ is large compared to the Dopper width γ_D and if the angle ϕ is small. Note that $\langle\delta\rangle\approx\langle(k-k')v\rangle\sim\langle k(1-\cos\phi)v\rangle\sim(\phi^2/2)\gamma_D\sim10^4-10^5$ Hz if $\phi\sim1-2°$, $\gamma_D\sim1$ GHz. Thus $\langle\delta\rangle\ll\gamma$ if $\gamma\sim10^6-10^7$ Hz.

[8]The ytterbium transition ${}^1S_0$-${}^3P_1$ may be a good one for the study of fluctuation-induced Hanle resonances.

[9]See, for example, Scholz *et al.*, Ref. 6.