Interferometric measurement of the degree of polarization and control of the contrast of intensity fluctuations

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We introduce a technique for determining the polarimetric characteristics of light by measuring the contrast of the intensity fluctuations in an interferometric setup. The method permits simultaneous measurement of the degree of polarization and of the second normalized Stokes component, which is related to the ellipsometric parameters azimuth and ellipticity, based on only two measurements. We also show that by using phase modulation we can increase the signal-to-noise ratio as much as 40% under certain conditions. © 2004 Optical Society of America

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The electric field components and therefore the total intensity of a partially polarized random electromagnetic field are generally fluctuating. Mandel and Wolf[1] gave the result $\langle(\Delta I)^2\rangle = (1 + P^2)\langle I\rangle^2/2$ for the variance of the intensity fluctuations for partially polarized light in a coordinate system in which $\langle I_x\rangle = \langle I_y\rangle$, where the angle brackets $\langle\ldots\rangle$ denote the ensemble average, P is the degree of polarization, and $\langle I\rangle$ is the average intensity. This result is obtained assuming Gaussian statistics for the fluctuations of electric field components E_x and E_y, which are partially correlated; the degree of correlation is related to the degree of polarization P. The contrast of the intensity fluctuations can be written as $C = [(1 + P^2)/2]^{1/2}$. This formula shows a simple relationship between the contrast C of the intensity fluctuations and the degree of polarization P of light. The degree of polarization, the intensity, and its variance, are all coordinate system invariant. Experimentally, one can determine the degree of polarization by simply measuring the contrast with a regular intensity detector that is also coordinate system invariant. However, if additional information about the polarized component of the light is required, more measurements are necessary with optical components that are polarization sensitive (polarizers, wave plates), and therefore a coordinate system has to be specified for the orientation of the optical components and of the state of polarization. The use of polarizers to select specific polarimetric components is energetically disadvantageous, and here we present a method of performing polarimetric measurements without using a polarizer. A Mach–Zehnder interferometer is used for simultaneous measurement of the degree of polarization and of the second component of the Stokes vector, based on only two measurements. In addition to obtaining polarimetric information from contrast measurement, our technique permits an increase of the signal-to-noise ratio up to 40% in certain circumstances. A similar experimental setup was previously[2] used for adjusting the degree of polarization and the spectrum of light based on tuning the correlations between parallel components of the electric field coming from the two arms of the interferometer. The degree of polarization rather than the full description of the state of polarization is of interest in multiple scattering[3] and free-space propagation,[4,5] where the statistical nature and not the deterministic component of light bears relevant information.

To describe the proposed technique we recall that the degree of polarization of the input optical field is given by[6] $P = [1 - 4\,\mathrm{Det}\,J/(\mathrm{Tr}J)^2]^{1/2}$, where Det and Tr are the determinant and the trace, respectively, of the coherence matrix[7] $J_{ij} = \langle E_i^* E_j\rangle (i,\ j = x,\ y)$.

For a Mach–Zehnder interferometer the total output field is represented by the ensemble $\{\mathbf{E}^{(T)}\} = \{E_x^{(1)} + E_x^{(2)}\}\hat{\mathbf{x}} + \{E_y^{(1)} + E_y^{(2)}\}\hat{\mathbf{y}}$, where $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ denote the unit vectors along the x and y directions, with 1 and 2 representing the arms of the interferometer, as shown in Fig. 1. If we consider that the field components in the two arms differ by only a phase factor $\exp(i\varphi_j)$, the total field can be expressed as a function of the initial field components as $\{\mathbf{E}^{(T)}\} = \{E_x\}f_x\hat{\mathbf{x}} + \{E_y\}f_y\hat{\mathbf{y}}$, where

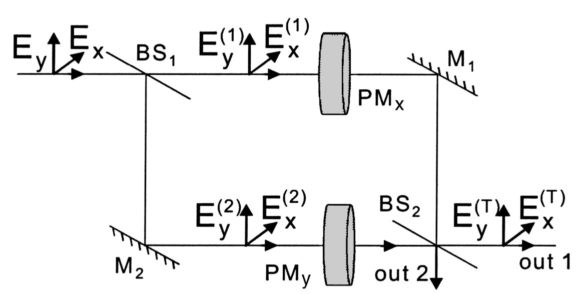

Fig. 1. Mach–Zehnder interferometer: BS_1, BS_2, nonpolarizing beam splitters; M_1, M_2, mirrors; PM_x and PM_y, phase modulators controlling the phase along the x and y directions, respectively.

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$f_j = [1 + \exp(i\varphi_j)]/2$. The total output average intensity is then $\langle I \rangle = |f_x|^2 \langle |E_x|^2 \rangle + |f_y|^2 \langle |E_y|^2 \rangle$.

To quantify the fluctuations of the output intensity we need the intensity–intensity correlation, which, for Gaussian statistics, is

$$\langle I^2 \rangle = 2|f_x|^4 J_{xx}^2 + 2|f_y|^4 J_{yy}^2 + 2|f_x|^2 |f_y|^2 (J_{xx} J_{yy} + |J_{xy}|^2). \quad (1)$$

The variance of the intensity fluctuations is given by $\langle (\Delta I)^2 \rangle = \langle I^2 \rangle - \langle I \rangle^2$, and therefore the contrast is obtained as

$$C = \frac{(|f_x|^4 J_{xx}^2 + |f_y|^4 J_{yy}^2 + 2|f_x|^2 |f_y|^2 J_{xy}|^2)^{1/2}}{|f_x|^2 J_{xx} + |f_y|^2 J_{yy}} \cdot \quad (2)$$

We further note that J_{xy} can be expressed as a function of the degree of polarization P and the intensity ratio $r = J_{xx}/J_{yy} = I_x/I_y$. Using this expression in Eq. (2), one can obtain the contrast of the output light fluctuations as a function of the degree of polarization of the input light and the input intensity ratio r. However, since r is not independent of P, we need a more meaningful representation of the contrast as a function of the state of polarization. Here we use the usual decomposition of the Stokes vector, $S = [I, Q, U, V]^T$, into the fully polarized and fully unpolarized components.[8] The intensity ratio r is then given by $r = (I + Q)/(I - Q) = (1 + Pq)/(1 - Pq)$, where q is the normalized second element of the Stokes vector that describes the pure polarized component. Using this representation of r, one obtains the contrast C as a function of P, q, and phase factors f_j. After simple algebraic manipulations, C is given by

$$C(f_x, f_y, q, P) = \left[1 - \frac{B(1 - P^2)}{(APq + 1)^2}\right]^{1/2}, \quad (3)$$

where the parameters $A = (|f_x|^2 - |f_y|^2)/(|f_x|^2 + |f_y|^2)$ and $B = 2|f_x|^2 |f_y|^2 (|f_x|^2 + |f_y|^2)^{-2}$ can be adjusted experimentally by tuning the phase factors f_j. This simple formula directly relates the contrast C of the output intensity fluctuations to the degree of polarization P and the normalized Stokes element q of the input light. Using CCD cameras as detectors, one can determine P and q in every point of the beam. In the following we analyze several practical consequences of this relationship. The relationship between the contrast C and the polarimetric characteristics P and q can be used both ways: one can determine P and q by measuring C, or one can modify C by either changing the input state of polarization or adjusting the phase factors f_j.

Until now, we have considered only one output of the Mach–Zehnder interferometer. The second output is complementary to the first, and there is an additional π phase shift between the parallel components of the electric field that overlap. The previous analysis is similar for the second output, the only required modification being the replacement of phase factors f_j by $1 - f_j$. Parameters A and B in Eq. (3) can be explicitly written as functions of the phases φ_j:

$$A_{1,2} = \frac{\pm\cos(\varphi_x) \mp \cos(\varphi_y)}{2 \pm \cos(\varphi_x) \mp \cos(\varphi_y)},$$

$$B_{1,2} = \frac{2[1 \pm \cos(\varphi_x)][1 \pm \cos(\varphi_y)]}{[2 \pm \cos(\varphi_x) \pm \cos(\varphi_y)]^2}, \quad (4)$$

where 1 and the top sign correspond to output 1, while 2 and the bottom sign correspond to output 2.

One can simultaneously measure the contrast of the intensity fluctuations for the two outputs of the Mach–Zehnder interferometer and solve the following equation:

$$C_{1,2} = \left[1 - \frac{B_{1,2}(1 - P^2)}{(1 + A_{1,2}Pq)^2}\right]^{1/2} \quad (5)$$

for the degree of polarization P and the normalized Stokes element q of the input light to finally obtain

$$P = \left[1 - \frac{1 - C_1^2}{B_1}\left(\frac{A_2 - A_1}{A_2 - MA_1}\right)^2\right]^{1/2}, \quad (6)$$

$$q = \frac{M - 1}{A_2 - MA_1}\left[1 - \frac{1 - C_1^2}{B_1}\left(\frac{A_2 - A_1}{A_2 - MA_1}\right)^2\right]^{-1/2}, \quad (7)$$

where

$$M = \left(\frac{B_2}{B_1}\frac{1 - C_1^2}{1 - C_2^2}\right)^{1/2}. \quad (8)$$

Since B_1 is always positive and the contrast C is always smaller than unity, the second term in Eq. (6) is also positive. Therefore the measured value of the degree of polarization will always be nonnegative and smaller than or equal to unity. We mention here that for $\varphi_j = 0$ and $q = 0$, Eq. (3) reduces to the one given by Mandel and Wolf[1] in the particular case of $\langle I_x \rangle = \langle I_y \rangle$.

Note that in this measurement the entire energy of the input light is used, since measurements are made on both outputs of the interferometer. Most of the standard polarimetric techniques that use polarizers waste a considerable amount of energy. We should also point out that this technique requires only two measurements for determining the degree of polarization. In Stokes polarimetry four measurements are necessary to completely determine the Stokes vector and, subsequently, the degree of polarization.

In Fig. 2 we show the contrast C of the intensity fluctuations of output 1 of the interferometer as a function of the phase φ_x for different values of q as indicated in each plot. As seen from Fig. 2, the contrast strongly depends on both P and q while the phase φ_x changes. However, $C = 1$ for $\varphi_x = \pi$, independent of P and q of the input light, as expected, since the output light is fully polarized (destructive interference for the x components of the electric field). Note that q is related to the ellipsometric parameters azimuth (α) and ellipticity [$\tan(\omega)$] through $q = \cos(2\omega)\cos(2\alpha)$.

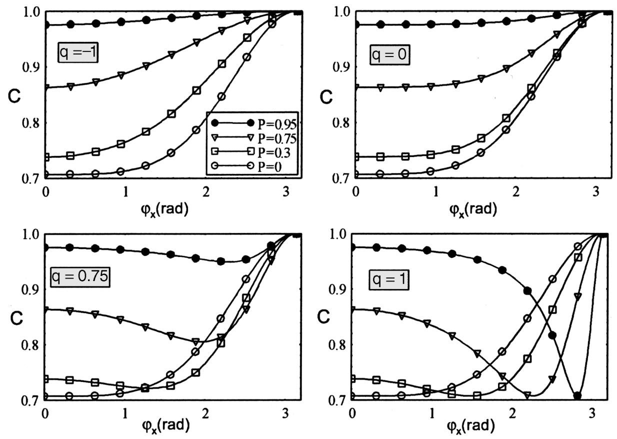

Fig. 2. Contrast of the intensity fluctuations of output 1 of the interferometer as a function of the phase φ_x for different values of q as indicated in each plot and different degrees of polarization as indicated in the legend.

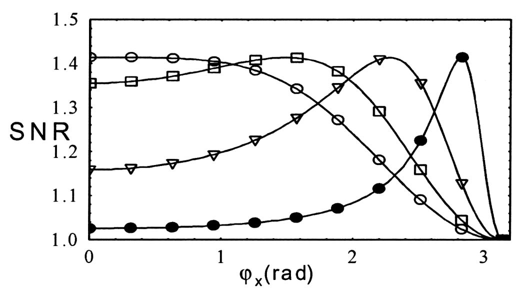

Fig. 3. SNR of output 1 of the interferometer as a function of the phase φ_x for $q = 1$ and different degrees of polarization as indicated in the legend of Fig. 2.

We point out that by using both outputs it is not necessary to tune the phases along the two arms of the interferometer. A fixed wave plate placed in one arm of the interferometer and oriented with its axis parallel to the $x-y$ coordinate system introduces different phases φ_x and φ_y along the x and y polarizations, sufficiently different such that $A_{1,2} \neq 0$. However, it is also possible to use only one output of the interferometer. Instead of $A_{1,2}$ and $B_{1,2}$ as given in Eqs. (4), one can use A_1 and B_1 for different values of φ_x while keeping $\varphi_y = 0$ and still obtain both P and q. In this case, sequential measurements are required, as opposed to simultaneous measurements when one is using both outputs.

Another important practical consequence of the relationship among C, P, and q shown in Eq. (3) is that the contrast can be modified by either changing the input state of polarization or adjusting the phase factors f_j. Modifying the state of polarization might not always be possible, whereas the phase factors can be easily adjusted using simple phase modulators. For simplicity let us assume that there is no phase introduced along the y direction and use only one phase modulator along the x axis in one arm of the interferometer. The signal-to-noise ratio (SNR), defined as the inverse of the contrast ($\text{SNR} = 1/C$), also depends on the input state of polarization and the phase factors. For Gaussian statistics of the fluctuations of an unpolarized input, SNR decreases from $\sqrt{2}$ to 1 while the phase φ_x changes from 0 to π. For a partially polarized input, however, SNR can be increased up to 40% while the phase φ_x is changed. The SNR or C, rather than the intensity fluctuation, is the relevant quantity, since both the variance and the average intensity are modified by φ_x.

In Fig. 3 we show the SNR for output 1 of the interferometer as a function of the phase φ_x for $q = 1$. As seen from Fig. 3, the SNR can be increased by changing the phase φ_x of partially polarized light.

In conclusion, we have introduced a technique for determining the polarimetric characteristics of light (governed by Gaussian statistics) by measuring the contrast of the intensity fluctuations in an interferometric setup. The method allows simultaneous measurement of the degree of polarization P and of the second normalized Stokes component q based on only two measurements. By measuring q one can determine the ellipticity if there is *a priori* knowledge of the orientation α, or vice versa, by knowing the ellipticity one can get the orientation. Another advantage of the method presented here is that, since both outputs of the interferometer are used for measurements, no input light is wasted, unlike the case when a polarizer is used. We have also shown that the SNR can be increased by phase modulation in certain conditions. Finally, in our analysis we have considered only a uniform phase applied across the beam. Using spatial light modulators it is possible to control the contrast and therefore the SNR in every point across the beam, a capability that might be of interest for certain applications involving random electromagnetic beams.

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