# Is the quark- mixing matrix moduli symmetric?

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**Abstract**

If the unitary quark- mixing matrix, $V$, is moduli symmetric then it depends on three real parameters. This means that there is a relation between the four parameters needed to parametrize a general $V$. It is shown that there exists a very simple relation involving $|V_{11}|^2$, $|V_{33}|^2, \overline{\rho}$ and $\overline{\eta}$. This relation is compared with the present experimental data. It is concluded that a moduli symmetric $V$ is not ruled out.

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# 1 Introduction

It is well known that for three generations, the general parametrization [1],[2] of the Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing matrix, $V$, depends on four parameters, namely, three angles and a phase. Experimental data gives the values of the moduli $|V_{ij}|$ and a particular parametrization of $V$ is needed to determine the complex matrix elements of the unitary matrix $V$. Four moduli (obtained from data) are needed to determine the four parameters in $V$.

The present data [2], gives us the ranges of $|V_{ij}|$. These are

$$V_{EXP} = \begin{pmatrix} 0.9741 - 0.9756 & 0.219 - 0.226 & 0.0025 - 0.0048 \\ 0.219 - 0.226 & 0.9732 - 0.9748 & 0.038 - 0.044 \\ 0.004 - 0.014 & 0.037 - 0.044 & 0.9990 - 0.9993 \end{pmatrix}. \tag{1}$$

It is clear from these values that there is a possibility that $V$ might turn out to be moduli symmetric. The ranges suggest that $|V_{ij}| = |V_{ji}|$ for $(i,j) = (1,2)$ and $(2,3)$, but it seems that $|V_{13}| \neq |V_{31}|$. However, the latter matrix elements are difficult to measure and may change in future. Since $V$ is unitary, it follows that

$$\Delta \equiv |V_{12}|^2 - |V_{21}|^2 = |V_{23}|^2 - |V_{32}|^2| = V_{31}|^2 - |V_{13}|^2. \tag{2}$$

So either $V$ is completely moduli symmetric ( $\Delta = 0$ ) or it is fully asymmetric ($\Delta \neq 0$). Recently an attempt to understand the smallness of this asymmetry (i.e. smallness of $\Delta$) has been made [3].

In this note, we explore the experimental consequences of a moduli symmetric $V$, denoted by $V_{MS}$. Since, $\Delta = 0$ for $V_{MS}$, this gives an extra condition[1]and consequently a general parametrization of $V_{MS}$ contains only three real parameters [3], [4].

The important point is that if $V = V_{MS}$ then there will be a relation between four measurables, which for a general $V$ would be independent. In this note we obtain a general relation and confront it with available data.

# 2 The relation

There is a lot of interest in measuring the quantities connected with the unitarity relation or triangle, viz.,

$$V_{11}V_{13}^* + V_{21}V_{23}^* + V_{31}V_{33}^* = 0, \tag{3}$$

[1]An explicit parametrization for $V_{MS}$ was considered inreferences 5 and 6. A relation involving $|V_{12}|, |V_{23}|$ and theparameters $\overline{\rho}$ and $\overline{\eta}$ of the unitarity triangle was pointed out.

Define $z_i$=$V_{i1}V_{i3}^*$ ; $i = 1,\ 2,\ 3$ then Eq.(3) can be written as

$$-\, z_1/z_2 - z_3/z_2 = 1. \tag{4}$$

This defines a triangle. Define the complex numbers[2]

$$-\, z_1/z_2 = \overline{\rho} + i\overline{\eta}, \tag{5}$$

so using Eq.(4),

$$-\, z_3/z_2 = (1 - \overline{\rho}) - i\overline{\eta} \tag{6}$$

This notation like that for the angles of the triangle has become standard. The angles $\alpha = arg(-z_3/z_1)$, $\beta = arg(-z_2/z_3)$, and $\gamma = arg(-z_1/z_2)$ of the triangle satisfy

$$\sin\alpha = \frac{\sin\beta}{\sqrt{\overline{\rho}^2 + \overline{\eta}^2}} = \frac{\sin\gamma}{\sqrt{(1-\overline{\rho})^2 + \overline{\eta}^2}}, \tag{7}$$

and

$$\tan\gamma = \overline{\eta}/\overline{\rho}. \tag{8}$$

To obtain the desired relation we note that from Eqs.(5, 6)

$$\frac{(1-\overline{\rho})^2 + \overline{\eta}^2}{\overline{\rho}^2 + \overline{\eta}^2} = \frac{|V_{33}V_{31}|^2}{|V_{11}V_{13}|^2} = \frac{|V_{33}|^2}{|V_{11}|^2} \equiv r \tag{9}$$

The last equality follows if $V$ is moduli symmetric since then $|V_{13}|^2 = |V_{31}|^2$. Thus, for $V_{MS}$, the four independent quantities $\overline{\rho}, \overline{\eta}$, $|V_{11}|$ and $|V_{33}|$ are related.

To compute the ratio $r$, we convert the ranges for $|V_{ij}|$ given in Eq.(1) into a central value with errors. This procedure gives, $|V_{11}| = 0.97485 \pm 0.00075$, $|V_{33}| = 0.99915 \pm 0.00015$ $|V_{13}| = 0.00365 \pm 0.00115$ and $|V_{31}| = 0.009 \pm 0.005$. Using these we find for $V_{MS}$, $r_{MS} = |V_{33}|^2/|V_{11}|^2 = 1.05048 \pm 0.00165$, otherwise $r = |V_{31}V_{33}|^2/|V_{13}V_{11}|^2 = 6.38683 \pm 8.15826$. The extremely large error in $r$ reflects the large errors in $|V_{13}|$ and $|V_{31}|$.

According to the relation in Eq.(9), $\overline{\rho}$ and $\overline{\eta}$ lie on the circle

$$(\overline{\rho} + 1/(r-1))^2 + \overline{\eta}^2 = (\sqrt{r}/(r-1))^2 \tag{10}$$

The circles for $r_{MS}$ are plotted in Fig 1. The relevant portion in the first quadrant is shown since $\overline{\rho}$ and $\overline{\eta}$ are both positive. It should be noted for $r = 1$ the circle degenerates into a straight line $\overline{\rho} = 1/2$. As $r$ increases, the radius increases and the centre approaches the origin along negative $\overline{\rho}$-axis. For $r = 6.38683$ the centre of the circle is at $(-0.185638, 0)$ and the radius is 0.469148. Since there is a large error in $r$ (for the asymmetric case), it is clear that the range of values for $r$ contain those for $r_{MS}$. Given the present data it seems that the possibility that $V$ is moduli symmetric is not ruled out. Our point here is that Eq.(10) provides a very simple and direct way to check if $V$ is moduli symmetric or not. One has to await more accurate data for $|V_{13}|$ and $|V_{31}|$ to come to a definitive conclusion in this regard.

From Eqs,(7, 8), we can determine

$$\sin 2\beta = \frac{2\overline{\eta}(1-\overline{\rho})}{(1-\overline{\rho})^2+\overline{\eta}^2}. \tag{11}$$

The curve in Eq.(11) represents the product of straight lines given by

$$\overline{\eta} = \tan\beta(1-\overline{\rho}), \tag{12}$$
$$\overline{\eta} = \cot\beta(1-\overline{\rho}). \tag{13}$$

Experimentally, different groups and different decay modes give a wide range of values for $\sin 2\beta$. Using the average of all modes and groups [7], $\sin 2\beta = 0.699 \pm 0.054$, the straight lines in Eq.(12, 13) are also plotted in Fig.1. It is interesting to note that the circles for $r = r_{MS} = 1.05048 \pm 0.00165$ and the line in Eq(13) with $\cot\beta = 2.45368 \pm 0.265066$ have a small region of intersection around $\overline{\rho} = 0.447577$, $\overline{\eta} = 1.36391$. However, this is excluded by constraints from other data [2]. For the general case, taking 1/2 the error into account, that is $r = 6.38683 \pm 4.07913$, one finds that there is a large region of intersection region with the lines in Eq.(12 with $\tan\beta = 0.407551 \pm 0.044027$, though there is no intersection with Eq.(13). As one can see the lines corresponding to Eq.(12) with $\tan\beta = 0.407551 \pm 0.044027$ have a small region of intersection with the circles for $r = r_{MS} = 1.05048 \pm 0.00165$ around the point $\overline{\rho} = 0.492779$ and $\overline{\eta} = 0.206728$ keeping open the possibility that $V$ be moduli symmetric.

Further, we note that $\sin^2\gamma / \sin^2\beta = r$ so that knowledge of $\beta$ from $\sin 2\beta$ enables one to obtain angles $\alpha$ and $\gamma$. The values of the angles for $r = r_{MS}$ and the general $r$ are given in Table I. The values in the two columns, as expected, are fairly different. The point to note is that for the moduli symmetric case, since $r_{MS} \approx 1$, one expects $\beta \approx \gamma$, unlike the general or asymmetric $V$ where the two angles can be quite different (viz. Table I). It is very interesting to note the value of $\sin 2\alpha$ (which is in the process of being measured) in the two cases. From Table I, for the central values, one expects $\sin 2\alpha = -0.9974$ for the moduli symmetric case in contarast to a value of 0.163 for a asymmetric $V$. An experimental value of $\sin 2\alpha$ near $-1$ would favour a moduli symmetric $V$.

In conclusion, we have pointed out that a simple, model independent relation between $\overline{\rho}, \overline{\eta}, |V_{11}|$ and $|V_{33}|$ provides a direct test of the moduli symmetry of $V$. The present data does not rule out such a symmetry. For a conclusive answer we must await future data.

In our view a moduli symmetric quark-mixing matrix would be far more elegant and physically interesting than one with a tiny, difficult to explain, asymmetry.

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[7] For details see http://www.slac.stanford.edu/xorg/hfag/ triangle/winter2003/index.shtml.

| ANGLES | $r = r_{MS} = 1.05048 \pm 00165$ | $r = 6.38683 \pm (8.15826)/2$ |
| --- | --- | --- |
| $\beta$ | $22.1734 \pm 2.16325^\circ$ | $22.1734 \pm 2.16325^\circ$ |
| $\gamma$ | $22.7568 \pm 2.17246^\circ$ | $72.5159 \pm 58.4675^\circ$ |
| $\alpha$ | $137.07 \pm 3.06581^\circ$ | $85.3107 \pm 58.5075^\circ$ |

Table I Numerical values of the unitarity triangle angles with errors corresponding to $r = r_{MS}$ and for general $r$. Note that in the latter case we have taken the error in $r$ to be half of its actual value.

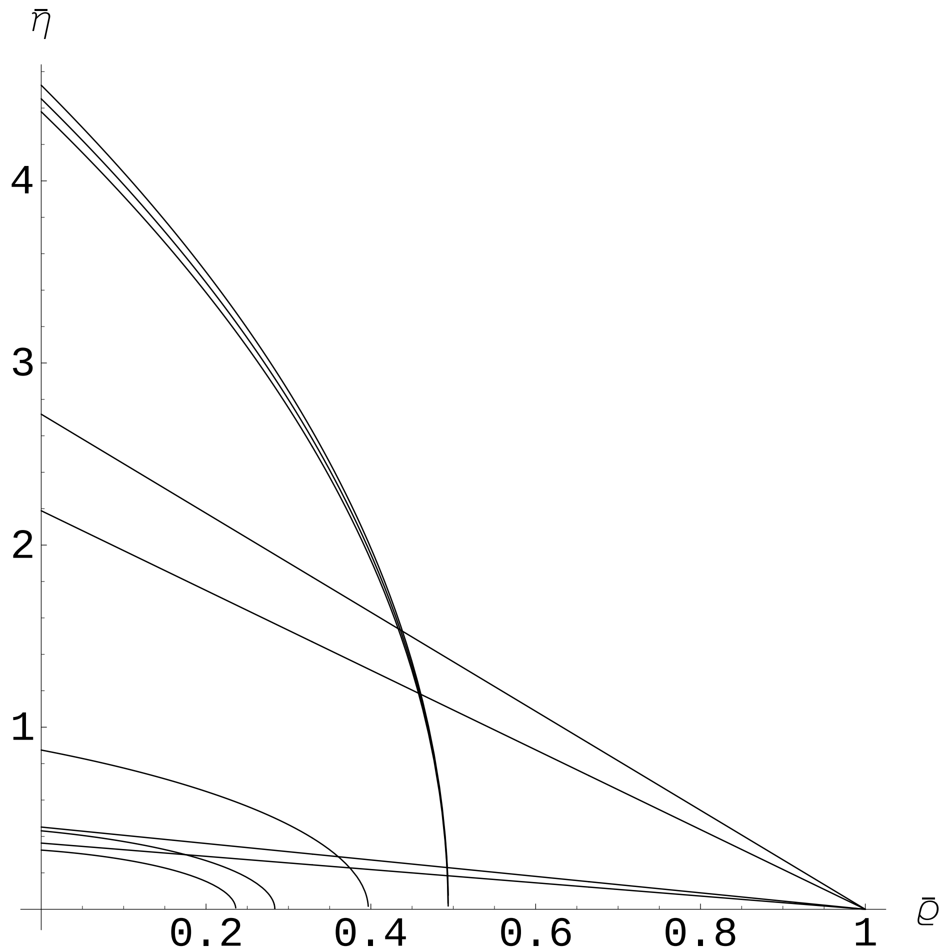


Figure 1: Plots of $\overline{\eta}$ versus $\overline{\rho}$ : (a) General Case: The lower three curves represent Eq(10) for $r = 6.38683 + (8.15826)/2, 6.38683$ and $6.38683 - (8.15826)/2$. They are parts of circles of radii $0.341763, 0.469148, 1.1617$ with centres at $(-0.105643, 0), (-0.185638, 0), (-0.764701, 0)$ respectively. (b) Moduli Symmetric Case: The upper three curves again represent Eq(10) for $r = 1.05048 + 0.00165, 1.05048$ and $1.05048 - 0.00165$. The radii of these circles are $19.6765, 20.3037, 20.9733$ with centres at $(-19.1828, 0), (-19.8098, 0), (-20.4792, 0)$ respectively. (c) The lower pair of straight lines corresponds to Eq.(12) with $\tan\beta = 0.407551 \pm 0.044027$ while the upper pair of straight lines corresponds to Eq.(13) with $\cot\beta = 2.45368 \pm 0.265066$.