IC/72/12
INTERNAL REPORT
(Limited distribution)

International Atomic Energy Agency
and
United Nations Educational Scientific and Cultural Organization

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

MAGNETIC FIELD DUE TO THE SELF-GRAVITY-INDUCED ELECTRIC POLARIZATION OF A ROTATING MASSIVE BODY *

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ABSTRACT

It is shown that the gravitationally self-induced electric polarization of an otherwise neutral massive body, taken in conjunction with the latter's rotation, generates a magnetic field of the right type and order of magnitude for certain astrophysical objects.

MIRAMARE - TRIESTE
March 1972

* To be submitted for publication.

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Gravity-induced electric field and the concomitant electric polarization of an earth-based laboratory sample, arrested from free fall, has been studied theoretically (Schiff and Barnhill, 1966; Dessler et al., 1968; Rieger, 1970; and Leung, 1972) and experimentally (Witteborn and Fairbank, 1967) by several workers in the recent past. It has, however, not been realized, to the authors' knowledge, that such an electric polarization, when applied self-consistently to an entire massive body in rotation can give rise to a poloidal magnetic field of the right type and order of magnitude for certain astrophysical objects. In this preliminary communication we report the results of a simple-minded calculation of this effect for the rather unphysical case of an infinitely long, uniform cylindrical conductor spinning about its axis. The electric polarization is calculated in the ion-continuum Thomas-Fermi approximation while the electrodynamics of the continuous medium is treated in the non-relativistic approximation. The essential points of the physics and the calculation involved are sketched below.

The physical idea behind the gravity-induced electric polarization can be fixed most readily by considering the equilibrium of an isolated atom in a gravitational field ($\underline{g}$) when the point-massive ion core (nucleus plus the core electrons) must "fall" a certain distance relative to the extended (and hence externally supportable) essentially massless electron cloud such that the resulting electrostatic restoring force exerted on the ion by the negative electronic charge $(-|e|)$ "uncovered" balances the ionic weight. Thus, for the semiclassical atom of ionic mass M, electron cloud radius a_0 and valency ζ (number of outer electrons forming the extended charge cloud), elementary electrostatics gives, for the gravity-induced atomic electric dipole moment: $\mu^e_{\text{atomic}} = \underline{g}\, M\, a_0^3/\zeta|e|$, and for the bulk polarization: $\underline{p}^e = \underline{g}\, n\, M\, a_0^3/\zeta|e|$, where n is the number of atoms per unit volume. While the above simple argument is fairly accurate for an insulator, it needs to be modified in the case of a conductor. Here the gravitational compression of the massive ionic lattice through the relatively massless and incompressible neutralizing electron gas is sensed by the latter via the electron-phonon coupling. In the ion-continuum Thomas-Fermi approximation, the situation is well described by the Hamiltonian

$$
H = \sum_{\underline{q}} \hbar c_s q \left(b^{+}_{\underline{q}} b_{\underline{q}} + \tfrac{1}{2} \right) + \sum_{\underline{q}} (-i) T_{\underline{q}} \left(b^{+}_{\underline{q}} - b_{-\underline{q}} \right) \rho_{\underline{q}} +
$$

$$
+ \sum_{\underline{q}} \int_V d^3r \, (-i) \, g \, \rho_m \left(\frac{\hbar}{2 \rho_m V c_s q} \right)^{1/2} \left(b^{+}_{\underline{q}} - b_{-\underline{q}} \right) e^{i(q_x x + q_y y)} \cdot \sin(z q_z) \cdot \left(\frac{q_z}{q} \right)
$$

(1)

with the screened electron-phonon coupling matrix element given by

$$
T_{\underline{q}} = \frac{n \zeta |e| q}{\varepsilon_0 (\chi_s^2 + q^2)} \cdot \left(\frac{\hbar}{2 \rho_m V c_s q} \right)^{1/2} \tag{2}
$$

Here $b^{+}_{\underline{q}}$, $b_{\underline{q}}$ are the phonon creation, annihilation operators for the longitudinal (acoustic) phonon mode of wave-vector $\underline{q}$; c_s is the sound speed; $1/\chi_s$ is the Thomas-Fermi screening length; $\rho_{\underline{q}}$ is the $\underline{q}$ - Fourier component of the electron density fluctuation, and $V(=L_x L_y L_z)$ is the volume of the sample, supported at the bottom plane $z = 0$ while the top plane $z = L_z$ is free. The acceleration due to gravity $\underline{g}$ is along the negative z-axis and the zero of the gravitational potential is chosen so as to coincide with the equilibrium configuration of the lattice in the absence of any gravity. The boundary condition that the end $z = L_z$ is free can now be taken into account mathematically by setting $\cos q_z L_z = 0$ for all q_z .

The lattice part of the Hamiltonian can now be diagonalized by a simple canonical transformation that physically amounts to the displacement of the normal modes (i.e. compression of the lattice). This displacement produces an extra static term $(H^{\text{static}}_{e-p})$ in the electron-phonon part of the Hamiltonian given by

$$
H^{\text{static}}_{e-p} = \sum_{\underline{q}} \frac{n \zeta |e| g}{\chi_s^2 c_s^2} \frac{1}{L_z} \left(\frac{1}{q_z^2} - \frac{1}{q_z^2 + \chi_s^2} \right) \rho_{\underline{q}} \tag{3}
$$

Thus one can identify a gravity-induced electrostatic potential $V^{g}_{\underline{q}}$ of the form

$$
V^{g}_{q} = \left(\frac{n \zeta |e| g}{\chi_s^2 c_s^2} \right) \frac{1}{L_z q_z^2} - \frac{n \zeta |e| g}{\chi_s^2 (\chi_s^2 + q_z^2) c_s^2} \cdot \frac{1}{L_z} \tag{4}
$$

The first term is readily seen to correspond to a uniform gravity-induced electrostatic field E^g given by

$$\underline{E}^g = -\left(\frac{n\,\rho\,|e|\,\underline{g}}{\chi_s^2\,c_s^2}\right) , \tag{5}$$

while the second term in Eq.(4) corresponds to a fluctuating short-range effect and is of no consequence in the present context. It may be noted in passing that the continuum treatment, together with the boundary condition $\cos q_z L_z = 0$, avoids having to evaluate complicated sums arising in the discrete treatment because of the problem of proper counting of the degrees of freedom (modes), and there is no loss of physics (cf. Rieger, 1970). One can rewrite Eq.(5) in the form

$$\underline{E}^g = \frac{-n|e|\rho\,\underline{g}}{\chi_s^2\,c_s^2} = \frac{-\underline{g}}{12\pi}\left(\frac{\rho_{mass}}{\rho_{charge}}\right)\left(\frac{\epsilon_{Fermi}}{\frac{1}{2}Mc_s^2}\right) \tag{6}$$

which is seen to be similar to that obtained earlier semiclassically and depends essentially on the ratio of the ionic mass density ρ_{mass} to the electronic charge density ρ_{charge}.

In order to apply the above expressions to describe the macroscopic polarization induced by the self-gravity of a massive body we must replace the variables by their local values and, in particular, $\underline{g}$ is replaced by $\underline{g}(\underline{r})$. Thus for the case of the uniform cylindrical body in question we have

$$\underline{g}(\underline{r}: r,\phi,z) = (g(r),0,0), \quad \text{with} \quad g(r) = -(2\pi\rho_m G)\,r . \tag{7}$$

Here $\underline{r}$ is the position vector, and (r,ϕ,z) the cylindrical polar co-ordinates. The z-axis is along the cylinder axis. The gravity-induced electric charge density ρ^g is given by combining Eqs.(6) and (7),

$$\rho^g = \frac{\epsilon_0}{4\pi}\,\underline{\nabla}\cdot\underline{E}^g = \frac{n\,\rho\,|e|\,\rho_m G}{\chi_s^2\,c_s^2} \equiv \text{constant}. \tag{8}$$

Electrical neutrality is of course maintained by the surface charges. Now, let the cylinder rotate about the z-axis with an angular velocity $\underline{\Omega}(=0,0,\Omega)$. The electrodynamics referred to the space-fixed inertial co-ordinate system is described by Maxwell's equations:

$$\nabla \times \underline{H}^{ind} = 4\pi(\underline{j}_{conv} + \underline{j}_{cond}) + \epsilon_0 \frac{\partial \underline{E}}{\partial t} \tag{9}$$

$$\nabla \times \underline{E}^{ind} = -\frac{\mu_0}{c} \frac{\partial \underline{H}}{\partial t} \tag{10}$$

with $\underline{j}_{conv}$ ≡ convection current density = $\frac{1}{c}(\rho^{g} + \rho^{ind})\, \underline{\Omega} \times \underline{r}$, (11)

and $\underline{j}_{cond}$ ≡ conduction current density = $\frac{\sigma}{c}(\underline{E}^{ind} + \frac{\mu_0}{c}(\underline{\Omega} \times \underline{r}) \times \underline{H}^{ind})$, (12)

satisfying the continuity equation

$$\nabla \cdot (\underline{j}_{conv} + \underline{j}_{cond}) + \frac{1}{c}\frac{\partial \rho^{ind}}{\partial t} = 0 \quad . \tag{13}$$

Here the superscripts "g" and "ind" stand for the gravitationally-induced and the rotationally-induced effects, respectively. σ is the electrical conductivity. These equations must be solved subject to the initial-boundary value condition

$$H(r, \phi, z; t=0) = (0, 0, H_0(r)), \text{ with } H_0(r) = -\left(\frac{4\pi \rho^{g} \Omega}{c}\right) r^2 \tag{14}$$

Here c is the speed of light.

Operating with $\nabla \times$ on both sides of Eq.(9) and substituting from Eq.(10) and Laplace transforming the resulting equation from time (t)- to frequency (s)-domain and combining with Eq.(13), we get, on neglecting the displacement currents and for $(\Omega r)^2/c^2 \ll 1$,

$$\left(\frac{d^2 y}{dx^2}\right) + \frac{1}{x}\left(\frac{dy}{dx}\right) + y = x^2 \tag{15}$$

with

$$x^2 = -\alpha(s)\, r^2 \quad , \quad y = \frac{H^{ind}(r;s)\, \alpha^2(s)}{\gamma} + \frac{\alpha(s)\, \beta(s)}{\gamma} \quad ,$$

where

$$\alpha(s) = \left(\frac{4\pi\sigma\mu_0}{c^2}\right) s + \frac{16\pi\sigma\mu_0 \Omega^2}{c^2\left(\frac{4\pi\sigma}{\epsilon_0} + s\right)} \quad ,$$

$$\beta(s) = \left(\frac{8\pi \rho^{g} \Omega}{c}\right)\frac{1}{s}$$

$$\gamma = \left(\frac{4\pi\sigma\mu_0}{c^2}\right)\left(\frac{4\pi \rho^{g} \Omega}{c}\right) \quad ,$$

and $\underline{H}^{ind}(r,\phi,z;s) = (0,0,H^{ind}(r;s))$.

The inhomogeneous Bessel Eq.(15) has the well-known general integral

$$y(x) = c_1 J_0(x) + c_2 Y_0(x) + S_{3,0}(x) , \tag{16}$$

where $J_n(x)$, $Y_n(x)$ and $S_{\mu,\nu}(x)$ are, respectively, the Bessel, Neumann and Lommel functions of respective orders. Imposing the appropriate boundary conditions we get, after some reduction,

$$H(r;s) = -\frac{\beta(s)}{\gamma} + \frac{\gamma}{\alpha^2(s)}\left[4 - \alpha(s)\, r^2\right] \tag{17}$$

Using the initial value theorem, one can at once verify that Eq.(17) reduces to (14) for $t \to 0$. Eq.(17) can readily be inverted to the time domain and we quote the following significant result for the saturation value of the field at the axis:

$$H(r=0;t) \simeq -\left(\frac{2\pi \rho^{\theta}/c}{\mu_0 \epsilon_0 \Omega}\right) \equiv -H_0 \tag{18}$$

for $$t \gg \left(\frac{4\pi\sigma}{\epsilon_0 \mu_0 \Omega^2}\right) .$$

The above result, at first sight, appears somewhat queer, for it predicts $H_0 \to \infty$ for $\Omega \to 0$. This is not so, really, because the condition $t \gg (4\pi\sigma/\epsilon_0\mu_0\Omega^2)$ will require $t \to \infty^2$. In fact, on inverting Eq.(17) to time domain and by proper ordering of the limits $\Omega \to 0$ and $t \to \infty$, one finds indeed that $H_0 \to 0$ for $\Omega \to 0$.

To estimate the above effect we set $n \sim 10^{23}$ cm^{-3} , $\zeta \sim 1$, $1/\chi_s \sim 5 \times 10^8$ cm^{-1} , $c_s \sim 10^4$ cm sec^{-1} and $\rho_m \sim 5$ gm cm^{-3} , $\Omega \sim 10^{-4}$sec^{-1} and get $H_0 \sim 1$ Gauss. We assume that $4\pi\sigma/\mu_0\epsilon_0\Omega^2$ is small compared with the age of the object. The field is, of course, antiparallel to the rotation vector. The choice of parameters above is made to correspond to the relatively dense and hot interior of the astrophysical objects.

Finally, we should like to point out certain important conceptual points of interest implicit in the above treatment. The above effect has been calculated in the space-fixed co-ordinate system. The relativistic corrections in going over to the co-rotating body-fixed co-ordinate system are small for $(\Omega\gamma/c)^2 \ll 1$. Further, we are basically talking about an induced polarization of an otherwise neutral body and hence the effect is intrinsically temperature-independent, unlike the case of orientational polarization of permanent dipoles. Also, we are looking for the cumulative effect of large number (N, say) of small contributions coming from the different parts of an extended body and one wonders if the effect would be washed away by fluctuations. Indeed, on the contrary, in the sense of the central limit theorem for large N the fluctuational spread goes as $N^{-\frac{1}{2}}$. Finally, we have considered a special geometry. Work is in progress to consider the same problem for a sphere with due cognizance of the observed obliquity and reported reversals of the planetary magnetic fields, and will be reported elsewhere.

In conclusion, we have shown that gravity-induced electric field in conjuction with rotation can provide a mechanism for magnetic fields of astrophysical objects.

ACKNOWLEDGMENTS

One of us, (Miss R.N.) would like to thank the C.S.I.R., New Delhi, for financial support while her part of the work was performed.

The other (N.K.) thanks Professor Abdus Salam, the International Atomic Energy Agency and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste, where the work was completed.

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