Vacuum-field-induced filamentation in laser-beam propagation

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We show that filamentation initiated by quantum fluctuations is a process that limits the intensity of a laser beam that can propagate stably through a nonlinear optical medium. We also describe the experimental signatures of this process, which allow it to be distinguished from classical processes such as filamentation induced by wave-front irregularities. [S1050-2947(97)00702-6]

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As a laser beam passes through a material medium, it is often observed to break up into a large number of filaments [1]. This filamentation process is initiated by the presence of weak perturbations on the laser wave front [2,3], which can grow by means of four-wave-mixing processes [4] and become large enough to disturb the overall propagation of the beam. Laser-beam filamentation can be suppressed either by reducing the intensity of the incident laser beam or by using a beam with extremely uniform wave fronts. The maximum intensity that can be transmitted through a given medium is thus determined by the extent to which the wave-front perturbations of the incident beam can be reduced. In this paper we show that vacuum fluctuations of the electromagnetic field [5] constitute a fundamental perturbation to the incident laser field and that filamentation initiated by these quantum fluctuations places a realistic upper limit on the laser intensity that can be transmitted through a given nonlinear optical material without the occurrence of beam breakup.

Our theoretical formalism follows closely that of Bespalov and Talanov [6], which treats the filamentation process classically by considering the gain experienced by a wave-front perturbation on a strong, monochromatic pump beam propagating through a Kerr material. Our treatment differs from theirs in that we consider the optical field to be a quantum-mechanical quantity. The quantum fluctuations of such a field (i.e., vacuum fluctuations) are necessarily spectrally broadband. Our model thus predicts that quantum-initiated filamentation differs from its classical counterpart in that it is accompanied by a spectral broadening of the transmitted laser field. Our model also differs from its classical counterpart in that it leads to explicit predictions regarding the strength of the fluctuations that initiate the filamentation process.

Let us consider the propagation of a laser beam through a Kerr material. We express the positive-frequency part of the total field as

$$\hat{E}^{(+)}(\mathbf{r},t)=[\mathcal{E}_0+\hat{\mathcal{E}}_1(\mathbf{r},t)]e^{i\gamma_0 z}e^{ik_0 z-i\omega_0 t}. \tag{1}$$

Here $\mathcal{E}_0$ denotes the amplitude of the strong pump field, which we treat classically and assume to have frequency ω_0 and wave vector $k_0+\gamma_0$, where $k_0=n_0\omega_0/c$ is its linear contribution and $\gamma_0=n_2 I_0\omega_0/c$ is its nonlinear contribution, with $n_2=(12\pi^2/n_0^2 c)\chi^{(3)}$ and $I_0=(n_0 c/2\pi)|\mathcal{E}_0|^2$. The perturbation, which we treat as a quantum-mechanical operator, is denoted $\hat{\mathcal{E}}_1$ and is conveniently decomposed in terms of its frequency components ω and transverse wave-vector components $\mathbf{q}$ as

$$\hat{\mathcal{E}}_1(\mathbf{r},t)=\int d^2q\int_0^\infty d\omega\; N(\omega)\hat{a}(\mathbf{q},\omega;z)\times e^{i\mathbf{q}\cdot\mathbf{r}}e^{i[k_z(\omega)-k_0]z-i(\omega-\omega_0)t}. \tag{2}$$

The mode amplitudes are denoted by $\hat{a}(\mathbf{q},\omega;z)$ and satisfy the usual commutation relation $[\hat{a}(\mathbf{q},\omega;z),\hat{a}^\dagger(\mathbf{q}',\omega';z)]=\delta^2(\mathbf{q}-\mathbf{q}')\delta(\omega-\omega')$. We have also introduced the mode normalization factor $N(\omega)=\sqrt{\hbar\omega^2 n^2(\omega)/4\pi^2 k_z(\omega)c^2}$. Note that we are assuming that the nonlinear response can be modeled adequately by a dispersionless third-order susceptibility. We are thus ignoring effects such as those of population trapping, which under certain conditions can prevent the occurrence of self-focusing [7]. We assume that n_0 is frequency independent; the validity of this assumption is discussed below. We further assume that $n_2 I_0 \ll n_0$. Note that we have taken the input pump beam to be a perfect plane wave. We have done so to make clear that the onset of filamentation stems from the presence of vacuum fluctuations and not from other perturbations to the input profile. While the introduction of a finite beam width would bring new transverse components into play, the evolution of these components leads simply to diffraction and to whole beam self-focusing and does not alter the conclusions of this paper in any significant way.

We insert the field $\hat{E}^{(+)}(\mathbf{r},t)$ into the wave equation

$$\nabla^2\hat{E}^{(+)}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\hat{E}^{(+)}=\frac{4\pi}{c^2}\frac{\partial^2}{\partial t^2}\hat{P}^{(+)}, \tag{3}$$

where $\hat{P}^{(+)}$ is the sum of the linear and nonlinear (i.e., $3\chi^{(3)}\hat{E}^{(-)}\hat{E}^{(+)}\hat{E}^{(+)}$) contributions of the material polarization. We linearize this equation in the perturbation and we make the paraxial and slowly varying amplitude approximations. The paraxial approximation can break down under conditions of catastrophic self-focusing, but the present calculation deals only with the initiation of the filamentation

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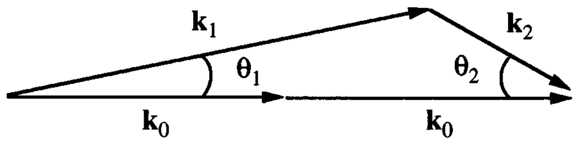

FIG. 1. Wave-vector matching diagram describing the four-wave-mixing process that leads to filamentation.

process, which can be treated adequately within the framework of the paraxial approximation [8]. We thereby obtain the coupled equations that describe the spatial evolution of the mode amplitudes

$$\frac{d}{dz}\,\hat{a}(\mathbf{q},\omega;z)=C(q,\omega)\hat{a}(\mathbf{q},\omega;z)+D(q,\omega)[e^{-i\Delta kz}\hat{a}^{\dagger}(-\mathbf{q},2\omega_0-\omega;z)], \tag{4a}$$

$$\frac{d}{dz}\,[e^{-i\Delta kz}\hat{a}^{\dagger}(-\mathbf{q},2\omega_0-\omega;z)]=F(q,\omega)\hat{a}(\mathbf{q},\omega;z)+E(q,\omega)\times[e^{-i\Delta kz}\hat{a}^{\dagger}(-\mathbf{q},2\omega_0-\omega;z)]. \tag{4b}$$

The coefficients that appear in these equations are given by

$$C(q,\omega)=-\frac{i}{2[k_z(\omega)+\gamma_0]}\left[q^2+[k_z(\omega)+\gamma_0]^2-k^2(\omega)-4k_0\gamma_0\left(\frac{\omega}{\omega_0}\right)^2\right], \tag{5a}$$

$$D(q,\omega)=\frac{i\gamma_0 k_0}{k_z(\omega)+\gamma_0}\left(\frac{\omega}{\omega_0}\right)^2, \tag{5b}$$

$$E(q,\omega)=-i\Delta k(\omega)-C(q,2\omega_0-\omega), \tag{5c}$$

$$F(q,\omega)=-D(q,2\omega_0-\omega), \tag{5d}$$

where

$$\Delta k(\omega)\equiv k_z(\omega)+k_z(2\omega_0-\omega)-2k_0. \tag{5e}$$

We solve Eqs. (4) to obtain

$$\hat{a}(\mathbf{q},\omega;z)=\hat{a}(\mathbf{q},\omega;0)e^{g_0z}\left\{\cosh g_1z+\frac{C-E}{2g_1}\sinh g_1z\right\}+e^{-i\Delta kz}\hat{a}^{\dagger}(-q,2\omega_0-\omega;0)e^{g_0z}\left\{D\,\frac{\sinh g_1z}{g_1}\right\}, \tag{6}$$

where $g_0=(C+E)/2$ and $g_1=\sqrt{(C-E)^2+4DF}/2$. Note that we have not assumed that the angle between the wave vectors of the pump and the perturbation is necessarily small. We can thus treat interactions, such as that illustrated in Fig. 1, which for $|\mathbf{k}_2|\ll|\mathbf{k}_1|$ can be nearly phase matched even for large angles θ_2. We are required to include interactions of this sort because the quantum fluctuations that initiate the four-wave-mixing process are spectrally broadband. Inspection of the form of the coefficients $C(q,\omega)$ and $E(q,\omega)$ shows that the gain eigenvalue $g_0(q,\omega)$ is always imaginary, but that the other eigenvalue $g_1(q,\omega)$ can be either real or imaginary depending on the values of q and ω. When $g_1(q,\omega)$ is real, the mode amplitude undergoes nearly exponential growth.

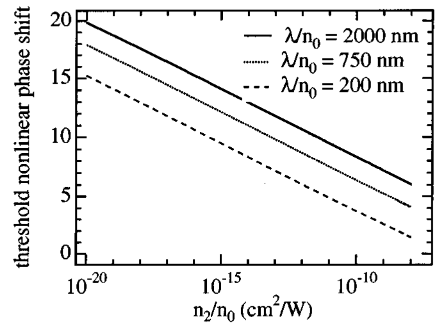

FIG. 2. Normalized laser intensity at the threshold for filamentation (i.e., the nonlinear phase shift B_{th}) vs the nonlinearity parameter n_2/n_0 for various laser wavelengths.

We next consider the effects of the growth of the mode amplitudes on the overall propagation of the optical beam. We calculate the total intensity of the perturbation as

$$I_{\mathrm{filament}}=\frac{nc}{2\pi}\,\langle\hat{\mathcal{E}}_1^{\dagger}(\mathbf{r},t)\hat{\mathcal{E}}_1(\mathbf{r},t)\rangle=\frac{nc}{2\pi}\int_0^{2\omega_0}d\omega\int_R dq\;2\pi q|N|^2|D|^2\,\frac{\sinh^2 g_1z}{g_1^2}, \tag{7}$$

where we have assumed that at the input to the nonlinear medium the perturbation is in the vacuum state and R denotes integration only over those transverse wave vectors for which $g_1(q,\omega)$ is real. We have evaluated this integral and find that for $z\gg\gamma_0^{-1}$ the result can be approximated by

$$I_{\mathrm{filament}}(z)=0.051\,\frac{n_0c}{2\pi}\left[\frac{\hbar\omega_0k_0^2}{4\pi}\,\gamma_0\right]e^{2\gamma_0z}, \tag{8}$$

which indicates that vacuum fluctuations supply an effective input intensity of

$$I_0^{\mathrm{vac}}=0.051\,\frac{n_0c}{2\pi}\,\frac{\hbar\omega_0k_0^2}{4\pi}\,\gamma_0. \tag{9}$$

Using Eq. (8), we readily determine the filamentation threshold distance z_f, which we arbitrarily take to be the distance at which the filament intensity reaches one-tenth of the input pump intensity I_0 and is given by

$$z_f=\frac{1}{2\gamma_0}\ln\frac{8\pi^2}{0.51\hbar\omega_0k_0^3n_2c}. \tag{10}$$

This result is illustrated in Fig. 2, which shows how the dimensionless parameter $B_{\mathrm{th}}=\gamma_0z_f=n_2I_0z_f\omega_0/c$, which can be interpreted as the nonlinear phase shift experienced by the pump wave at the threshold for filamentation, depends on the nonlinearity parameter n_2/n_0 for a variety of laser wave-

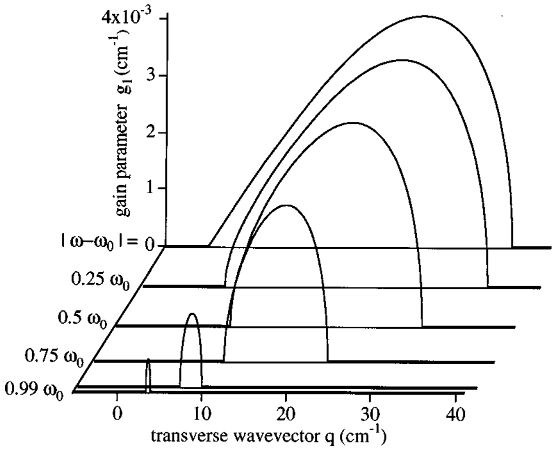

FIG. 3. Gain parameter as a function of the transverse wave vector for various detunings for a 775-nm beam with intensity 10^{11} W/cm^2 propagating through air.

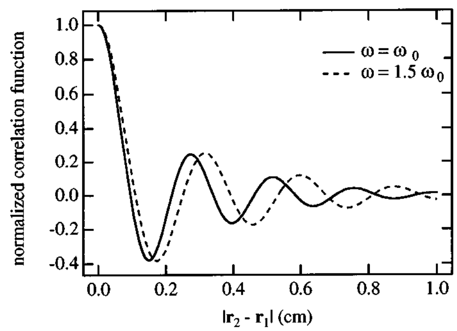

FIG. 4. Field correlation function at $z=4000$ cm for two frequencies.

lengths. Note that typical threshold phase shifts are in the range 5–15, comparable to those of stimulated scattering processes.

As a specific example of the application of these results, let us consider the case of a laser beam propagating through air, for which $n_2=5\times10^{-19}$ cm^2/W [9]. For the case of an input beam at a wavelength of 775 nm with intensity 10^{11} W/cm^2, we find that the effective vacuum input intensity is 0.128 mW/cm^2 and that the filament intensity reaches one-tenth of the input intensity at a distance of 40 m. This distance is comparable to those that have been used in high-intensity laser-beam propagation experiments [10]. We note, for comparison, that a beam with a diameter of 3 cm has a whole beam self-focusing [11] distance of about 67 m and a Rayleigh range of about 2 km. Filamentation induced by vacuum fluctuations is thus seen to impose a realistic limitation on the intensities and distances over which a high-power beam can propagate.

There are many observable differences between the properties of the quantum-initiated process and the classical process. The origin of these differences lies primarily in the large range of frequencies present in vacuum fluctuations. The quantum filamentation process is thereby accompanied by a significant broadening of the optical spectrum. Under certain conditions, such as those that pertain before the catastrophic filamentation has occurred, these new frequency components are emitted in the form of a frequency-dependent cone.

The nature of this cone-emission process is determined by phase-matching considerations. For the classical situation in which filamentation is seeded by a narrow-band perturbation at the pump frequency (i.e., $\delta=0$ so that $|\mathbf{k}_1|=|\mathbf{k}_2|$ in Fig. 1), a simple calculation [4] shows that the cone angle is given by $\theta\approx\sqrt{2n_2I_0/n_0}$. However, for seeding by broadband perturbations such as vacuum fluctuations, different frequencies experience maximum gain at different angles. This behavior can be seen in Fig. 3, which shows the gain parameter $g_1(q,\omega)$ plotted as a function of the magnitude of the transverse wave vector for various values of the emission frequency. We see that the transverse wave vector that experiences maximum gain decreases with increasing detuning from ω_0. The peak value of the gain decreases with detuning as well. Nonetheless, the filamentation is accompanied by a significant broadening of the overall optical spectrum, leading to a width (full width at half maximum) of the order of $0.4\omega_0$. The model we have developed in this paper thus may prove useful in the formulation of a more complete theory of supercontinuum generation [12]. We have plotted Fig. 3 for the situation considered above (a 775-nm laser beam in air with intensity 10^{11} W/cm^2), but we have found that the behavior is qualitatively similar for other values of n_2 and I_0.

Another difference between the classical and quantum treatments is that for quantum-initiated filamentation the statistical properties of the fluctuations that initiate the process are known. We can thus obtain an explicit prediction for the field correlation function, which provides a measure of the nature of the transverse, spatial pattern created by the filamentation process [13] and is given by

$$\begin{aligned}\langle\hat{\mathcal{E}}_1^\dagger(\mathbf{r}_1,t)\hat{\mathcal{E}}_1(\mathbf{r}_2,t)\rangle &= \int_0^{2\omega_0} d\omega\int_R dq\; 2\pi q|N|^2|D|^2\,\frac{\sinh^2 g_1 z}{g_1^2}\\ &\quad\times J_0(q|\mathbf{r}_2-\mathbf{r}_1|)\\ &= \int_0^{2\omega_0} d\omega\; S(\omega,z_0;|\mathbf{r}_2-\mathbf{r}_1|). \qquad (11)\end{aligned}$$

This correlation function is frequency dependent, as shown in Fig. 4, which displays $S(\omega,z_0;|\mathbf{r}_2-\mathbf{r}_1|)$ versus $|\mathbf{r}_2-\mathbf{r}_1|$ at $z_0=4000$ cm for $\omega=\omega_0$ and $\omega=1.5\omega_0$.

For simplicity and definiteness, we have neglected the dispersion of the refractive index in performing the calculations presented above. For several special cases, we have included dispersion of the refractive index and we find that most of the predictions of the calculation remain qualitatively similar. For example, we have modeled dispersion by assuming that the refractive index changes linearly with frequency at a rate appropriate to that of air at visible wavelengths. We find that the predictions for the dependence of g_1 on $\mathbf{q}$ and on ω are then somewhat different from before, because the phase-matching condition is now modified both by nonlinearity and by dispersion. However, we find that the predictions for the overall growth of the filament intensity with distance z is essentially unchanged.

In conclusion, we have shown that a laser beam propagating through a nonlinear optical medium is subject to breakup through the process of filamentation initiated by quantum-mechanical vacuum fluctuations. We have presented explicit predictions for the limitations that this process places on the distance that an intense laser beam can propagate without breakup and have described how this process can be distinguished experimentally from its classical counterpart by means of its unique spectral signature.

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