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Horizon Problem and Inflation

T. Padmanabhan & T. R. Seshadri *Astrophysics Group, Tata Institute of Fundamental Research, Homi Bhabha Road, Bombay 400005*

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Abstract. We show that, the part of the universe that is observable today (in principle), could not have evolved out of a domain which was causally connected in the past. This and other issues related to horizon problem in inflationary models are discussed.

Key words: cosmology, inflationary models—universe, horizon problem

1. Introduction

Models for the inflationary universe have enjoyed immense popularity in the last few years. The original model for inflation proposed by Guth is quite elegant and simple. It was originally invoked to explain: (i) the homogeneity of the surface of last scattering (LSS) and the consequent isotropy of cosmic microwave background radiation (CMBR), (ii) the closeness of the rate of expansion of the universe to that of a spatially flat universe ('flatness problem') and (iii) the absence of GUT monopoles. Unfortunately the model turned out to be unworkable due to other reasons—it produced a universe very different from what we observe.

In the subsequent years several other models for inflation were proposed. It was also discovered that any inflationary model can produce density inhomogeneities out of the inherent quantum fluctuations of the field responsible for inflation. All these models are, however, unnatural in the sense that dimensionless parameters have to be fine-tuned to very low values for the model to produce an acceptable universe. (Thus, in order to avoid fine-tuning of initial conditions we have to invoke fine-tuning of theoretical parameters.) It is generally believed that, once this fine-tuning of the theoretical model is accepted, inflation would provide a natural explanation to (i) and (ii) of the previous paragraph.

In this paper, we examine the claim that 'inflation solves the horizon problem'. To motivate the discussion, consider a conventional inflationary scenario: The universe evolves from a singularity at $t = 0$, and is radiation dominated until $t = t_i$; it inflates for $t_i < t < t_f$ and is radiation dominated for $t_f < t$. The proper distance to the horizon —which is the linear extent of the causally connected domain—is given by

$$d_H(t) \equiv S(t) \int_0^t \frac{dx}{S(x)} \tag{1}$$

where $S(t)$ is the expansion factor. Thus $d_H(t)$ also represents the size of the observable universe. At any time t, an observer at the origin can receive signals from proper distances up to $d_H(t)$. The reader, at this stage, is invited to decide for himself the correctness (or otherwise) of the following statements:

1. In a model with sufficient inflation, the currently observable part of the universe has evolved out of a region which was causally connected in the past. In other words, the observable universe is causally connected (Turner 1983: p. 237, lines 6–9; Linde 1984: p. 946, lines 6–8).
2. Perturbations at some wavelengths can grow bigger than the horizon during the inflationary phase and reenter the horizon at a later stage.
3. Inflation can explain the isotropy of the CMBR in a natural manner.

In this paper we shall discuss the above claims and show: (a) The only way to make the whole of observable universe causally connected is to have a horizon-free model, *i.e.* choose a model with infinite $d_H(t)$ for all $t > 0$. (This can be realized if, for example, $S(t) \sim t^n$ with $n > 1$ near $t = 0$.) (b) The second statement is incorrect if 'horizon' is defined as $d_H(t)$. Unfortunately, the term 'horizon' is used in literature to denote two very different objects: $d_H(t)$ and the inverse Hubble distance $H^{-1}(t) = (\dot{S}/S)^{-1}$. For $S(t) \sim t^n$, with $n \sim 0$ (1) both $H^{-1}(t)$ and $d_H(t)$ are proportional to t and are of the same order of magnitude. But during an inflationary epoch, they behave very differently. The statement (2) is correct if horizon is interpreted as 'inverse Hubble distance'—which is the sense in which the term 'horizon' is used in literature dealing with perturbations. (c) The explanation of the isotropy of CMBR in inflationary models involves a surprising fine tuning.

2. Horizons in RW cosmology

Consider a $k = 0$, Robertson-Walker universe with an expansion factor $S(t)$. (Since we are not interested in 'flatness problem' we shall set $k = 0$; our results are independent of this assumption.) This proper distance to the horizon, $d_H(t)$, is defined by (1). Let t_0 denote the present moment (10^{18} s) and $\lambda(t_0) = \lambda_0$ be any proper length scale in the present day universe, which would correspond to the size

$$\lambda(t) = \lambda_0 \frac{S(t)}{S(t_0)} \tag{2}$$

at any other epoch t. We are interested in the ratio,

$$r(t) \equiv \frac{d_H(t)}{\lambda(t)} = \frac{S_0}{\lambda_0} \int_0^t \frac{dx}{S(x)} \qquad (t \leqslant t_0). \tag{3}$$

A particular scale λ_0 is 'within the horizon' at some time t if $r(t) > 1$ and is 'outside the horizon' if $r(t) < 1$. We can rewrite (3) as,

$$r(t) = \frac{S_0}{\lambda_0} \left\{ \int_0^{t_0} \frac{dx}{S(x)} - \int_t^{t_0} \frac{dx}{S(x)} \right\} = r(t_0) - \frac{S_0}{\lambda_0} \int_t^{t_0} \frac{dx}{S(x)}. \tag{4}$$

The complete, observable, region of the universe today corresponds to a length scale $\lambda_u(t_0) = d_H(t_0)$. In other words, for this scale $r(t_0) = 1$. From (4) it follows that,

$$\frac{d_H(t)}{\lambda_u(t)} = 1 - \frac{S_0}{\lambda_u(t_0)} \int_t^{t_0} \frac{dx}{S(x)} < 1 \quad (t \leqslant t_0). \tag{5}$$

Notice that $\lambda_u(t)$ represents the size of that region at time t, which expands to form the currently observable region of the universe. Equation (5) shows that the horizon size will always be smaller than the size of the region which evolves to form the currently observable universe. *In other words, it is simply impossible for the observable universe to have evolved out of a single causally connected domain in the past.* Statement 1 is false, in spite of repeated assertions to the contrary in the literature (Turner 1983: p. 237, lines 6–9; Linde 1984: p. 946, lines 6–8).

It is clear from the definition that, for $t_1 \leqslant t_2$

$$r(t_1) \leqslant r(t_2). \tag{6}$$

So if $r(t_2) < 1$ (the scale is outside the horizon at $t = t_2$) then $r(t)$ must be less than unity for all $0 \leqslant t_1 \leqslant t_2$ ('the scale must be outside the horizon in the past'). In other words no scale can grow bigger than the horizon and 'go outside the horizon' sometime in the past. (To do so, one would require $r(t_2) < 1$ and $r(t_1) > 1$ with $t_2 > t_1$; this is impossible.) Thus the claim in statement 2 is impossible if 'horizon' is taken to mean $d_H(t)$. We will come back to this point later.

The validity or otherwise of statement 3 depends on the specific form of $S(t)$ used. We shall take $S(t)$ to be,

$$S(t) = \begin{cases} S_i\left(\dfrac{t}{t_i}\right)^{1/2} & 0 < t \leqslant t_i & (7) \\ S_i \exp[H(t-t_i)] & t_i \leqslant t \leqq t_f & (8) \\ S_i Z\left(\dfrac{t}{t_f}\right)^{1/2} & t_f < t < t_0 & (9) \end{cases}$$

where $Z = \exp H(t_f - t_i)$. (We have neglected the matter-dominated epoch after recombination, but this hardly changes the results.) The coordinate length along the surface of last scattering (which is observable today) is,

$$l(t_0, t_{rec}) = \int_{t_{rec}}^{t_0} \frac{dx}{S(x)} \tag{10}$$

coordinate horizon distance at $t = t_{rec}$ is,

$$l(t_{rec}, 0) = \int_0^{t_{rec}} \frac{dx}{S(x)}. \tag{11}$$

Therefore, the number of causally disconnected volumes in CMBR is about $\sim N^3$ with,

$$N = \frac{l(t_0, t_{rec})}{l(t_{rec}, 0)} = \frac{l(t_0, 0)}{l(t_{rec}, 0)} - 1. \tag{12}$$

Using (7)–(9) we can easily compute $l(t,0)$. For $t > t_f$, this is given by,

$$l(t,0) = \frac{1}{S_i z}\{(2t_i + H^{-1})Z - H^{-1} + 2\sqrt{t_f}(\sqrt{t} - \sqrt{t_f})\} \tag{13}$$

$$\simeq \frac{1}{S_i z}\{4t_i z + 2\sqrt{t t_f}\}. \tag{14}$$

In arriving at (14) we have made the usual assumptions regarding inflation: $2t_i \simeq H^{-1}$

($\simeq 10^{10}$ GeV$)^{-1}$), $Z \gg 1$ and have taken $t \gg t_f$. Using (14) in (12) we get,

$$N = \frac{\sqrt{t_f}(\sqrt{t_0} - \sqrt{t_r})}{2t_i Z + \sqrt{t_f t_r}} \simeq \frac{\sqrt{t_f t_0}}{2t_i Z + \sqrt{t_f t_r}}. \tag{15}$$

If the whole of CMBR has to be within a causally connected patch, then $N \leqslant 1$. In other words,

$$\sqrt{t_f}(\sqrt{t_0} - \sqrt{t_{rec}}) \simeq \sqrt{t_f t_0} < 2t_i Z \tag{16}$$

or equivalently,

$$Z \geqslant \left(\frac{t_f}{t_i}\right)^{1/2} \left(\frac{t_0}{t_i}\right)^{1/2} \tag{17}$$

(Guth 1981). The value of right-hand side in conventional models is $\sim 3 \times 10^{27}$. So the usual inflation with $z \simeq 10^{29}$ will make CMBR homogeneous.

There is however, a surprise hidden in (17). Let us rewrite (17) in the form,

$$t_0 < t_i Z^2 (t_i/t_f) = \frac{t_i^2}{t_f} \exp[2H(t_f - t_i)]. \tag{18}$$

Now, all the parameters on the right-hand side of (18) are fixed by microscopic physics at a very early epoch. (All the parameters t_i, t_f and H are fixed in terms of the fundamental theory producing inflation.) *The isotropy of CMBR will hold true only as long as the age of the universe t_0 (the 'present' epoch) is smaller than the pre-decided timescale on the right-hand side!* CMBR may appear to be isotropic today; but may appear anisotropic sometime in future!

There is another way of presenting the same result: Given a microscopic theory , t_i, t_f are fixed. One can now violate (17) by simply taking a sufficiently large t_0. Thus, if we wait long enough, CMBR will appear to be anisotropic. (The same result can be restated in terms of T_0 the CMBR temperature at the present epoch. We also would like to stress the fact our results are independent of various approximations invoked in the discussion.)

3. Comparison and conclusions

The apparent contradiction between the Standard lore of inflation and our results above demands a comparison, which we shall now provide.

Let us begin with statement 2. The growth of perturbations is actually governed by the scale (inverse Hubble distance).

$$h(t) \equiv H^{-1}(t) = \left(\frac{\dot{S}}{S}\right)^{-1} \tag{19}$$

rather than by d_H t). Bounds like (6) are not applicable to the ratio $[d_H(t)/h(t)]$. There are, however, two aspects related to this issue that require comment: (i) It is sometimes stated in literature that only perturbations smaller than the 'horizon' (meaning $h(t)$) are physically relevant *because* they are inside *the causally connected region.* This is clearly untrue because $h(t)$ is much smaller than $d_H(t)$. Wavelengths in the range $h(t) < \lambda(t) < d_H(t)$ are well within the causally connected domain but outside the

Hubble distance. The validity of the statement 'microphysics cannot operate at scales $\lambda(t) > h(t)$, is not obvious in inflationary models with $d_H(t) \gg h(t)$. (Brandenberger 1985: p. 46; Turner 1985: p. 243). The issue is somewhat subtle, (ii) The behaviour of $\lambda(t)$, $h(t)$ and $d_H(t)$ are shown in Fig. 1, for an inflationary model described by (7)–(9). Note the exponential growth of $\lambda(t)$ during $t_i < t < t_f$.

In a simplified picture described by (7)–(9), $h(t)$ will be discontinuous at $t = t_f$. Proper discussion of reheating is necessary to smoothen this discontinuity.

Let us now consider statement 1. This statement is simply false. To understand how it is conventionally tackled, let us estimate $d_H(t_0)$—the size of the horizon today. It is easily computed to be,

$$d_H(t_0) = \left(\frac{t_0}{t_f}\right)^{1/2} [(2t_i + H^{-1})z - H^{-1} + 2\sqrt{t_f}(\sqrt{t_0} - \sqrt{t_f})] \tag{20}$$

$$\simeq (4t_i)Z\left(\frac{t_0}{t_f}\right)^{1/2} + 2t_0 \tag{21}$$

for $Z \gg 1$, $t \gg t_f$, $2t_i \simeq H^-$ Taking $t_i \simeq 10^-$ s, $t_f \simeq 10$ s, $Z \simeq 10^{29}$, and $t_0 \simeq 10^{18}$ s we get

$$d_H(t_0) \simeq 3 \times 10^{30} \text{ cm}. \tag{22}$$

In Standard inflationary lore, the size of observable universe is taken to be $\lambda(t_0) \simeq 10^{28}$ cm $\ll d_H(t_0)$. Such a scale would have been inside the causally connected domain all the way down to $t = t_i$. At the end of inflation, $(t = t_f)$ $d_H(t_f) \simeq 10^5$ cm while $\lambda(t_f) \simeq 3 \times 10^2$ cm; at the beginning of inflation $(t = t_i)$, $d_H(t_i) \simeq 10^-$ cm while $l(ti) \simeq 3 \times 10^{-27}$ cm. The arguments in (5)–(7), of course, are not applicable if $r(t_0) > 1$.

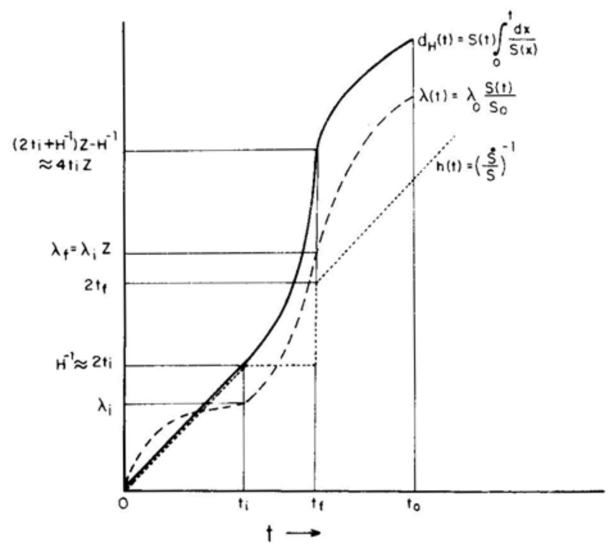

Figure 1. Evolution of various lengthscales with time. Thick line denotes the evolution of the particle horizon, dashed line denotes the growth of a comoving length scale and dotted line represents H^{-1}.

But what is *the* size of observable universe today? All the region in the universe that is causally connected to *us are observable in principle.* Thus the size of the region which is observable by us today (in principle) is $\lambda_u(t_0) = d_H(t_0)$. *This region would have been outside the horizon for all* $t < t_0$.

The claim that 'observable universe has a size of 10^{28} cm' arises from the existence of a last scattering surface (LSS) at $\simeq 10^3$. It is usual to say that universe at higher redshifts is 'opaque' and unobservable. Such a statement is misleading, to say the least. If cosmic neutrino background is discovered, one can proceed to redshifts $\gg 10^3$. Nothing—in principle—precludes the possible existence of one exotic relic particles which decoupled at very large . Therefore, it is more proper to consider $d_H(t_0)$ to be the size of observable universe.

If the above definition is accepted, the following conclusion is inescapable: Inflation can never produce the observable region of the universe from a single causally connected region. It can only be achieved in models which are strictly horizon-less. (This happens in many quantum gravitational models; for example, see Narlikar & Padmanabhan 1983, 1985.)

Lastly, we repeat that the conventional inflation does explain the homogeneity of CMBR. But only because our universe is still young, *i.e.,* t_0 is less than some predecided value.

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