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Return current instability and its effects on beam-plasma system

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Abstract. The return current induced in a plasma by a relativisitc electron beam generates a new electron-ion two-stream instability (return current instability). Although the effect of these currents on the beam-plasma e–e instability is negligible, there exists a range of wave numbers which is unstable only to return current (RC) instability and not to e–e instability. The electromagnetic waves propagating along the direction of the external magnetic field, in which the plasma is immersed, are stabilized by these currents but the e.m. waves with frequencies, $\omega^2 \ll \Omega_e^2 \ll \omega_{pe}^2$ (Ω_e and ω_{pe} being cyclotron and plasma frequency for the electrons of the plasma respectively) propagating transverse to the magnetic field get destabilized. Heuristic estimates of plasma heating, due to RC instability and due to decay of ion-acoustic turbulence generated by the return current, are made. The fastest time scale on which the return current delivers energy to the plasma due to the scattering of ion-sound waves by the electrons can be $\sim \omega_{pi}^{-1}$ (ω_{pi} being the plasma frequency for the ions).

Keywords. Return current instability; beam-plasma interactions; plasma heating.

1. Introduction

It is well known that under certain conditions the electrons and the ions of a plasma can be heated to high temperatures by passing an energetic electron beam through the plasma. Such beam-plasma interactions have been studied extensively both theoretically and experimentally during the last two decades (Bunemann 1959, Briggs 1964, Nezlin 1971). Most of the work in the field is confined to the case of beams moving with nonrelativistic velocities. Lately the interest is being shifted towards the use of relativistic beams because they offer a convenient mechanism for transporting high energy into a small area and also due to their possible application as a tool to initiate a thermonuclear reaction (Roberts and Bennett 1968, Andrews *et al* 1970, Watson *et al* 1960, Hammer and Rostoker 1970, Cox and Bennett 1970, Lee and Sudan 1971, Finkelstein and Sturrock 1961, Buti 1972, Winterberg 1972). However, the passage of a relativistic beam through a plasma inherently gives rise to the phenomenon of ‘return current’ which ultimately limits the beam current passing through the plasma. The presence of return currents is expected to change the stability characteristics of the waves in the beam-plasma system. Recently Nebenzhal (1972) has examined the influence of these currents on the transverse waves for the cold plasma in the absence of any external magnetic field. He finds that only in the extreme relativistic case when $(\gamma_0 n_b/n_p) > 1$ (n_b and n_p being the densities of electrons in the beam and in the plasma, respectively, and $\gamma_0 = (1 - U^2/c^2)^{-1/2}$, where U and c are the velocity of the beam and that of light, respectively), the return current effects are important.

In this paper we have attempted to ascertain the role of return currents on the stability of longitudinal as well as transverse waves which are excited when a relativistic beam of electrons impinges on the hot plasma immersed in a uniform magnetic field.

We find that, besides the usual beam-plasma electron-electron (e-e) instability, the electrostatic waves can support an instability which arises because of the presence of return currents. This return current instability can be excited by beam currents which are less than I_{cr}, the critical currents required to excite the e-e instability. Moreover, for $I > I_{cr}$, there exists a range of wave numbers ($k_{cr} < k < k_{cr}^{R}$, where k_{cr} and k_{cr}^{R} are the largest wave numbers which are unstable to e-e instability and to return current instability, respectively) which is unstable to return current instability and not to e-e instability. The electromagnetic waves propagating in the direction of the magnetic field are stabilized whereas the one propagating in the transverse direction are destabilized by the return currents.

The possibility of heating a plasma to thermonuclear temperatures (Babykin *et al* 1971, Winterberg 1972) has recently drawn considerable attention of both theoretical and experimental physicists. Optimistic estimates of final heating due to return current instability (Guillory and Benford 1972), e-e two-stream instability (Thode and Sudan 1973) and due to decay of return currents because of ion-acoustic turbulence (Lovelace and Sudan 1971) have recently been made. A number of experiments (Altyntsev *et al* 1971, Miller and Kuswa 1972, Stallings *et al* 1972, Goldenbaum *et al* 1972, Kapetanakos *et al* 1972, Miller and Kuswa 1973, Korn *et al* 1973) for heating plasma electrons, using intense pulsed relativistic electron beams, have also been reported recently. The important questions that have to be answered are the rate at which the beam delivers energy to the plasma and the partitioning of the energy between the plasma particles and the waves. In section 3.1c we have considered the quasilinear development of the return current instability and have shown that the rate at which the electrons are heated is larger than the rate at which the ions are heated. Hence, the electrons will be preferentially heated by this instability and when the return current is such that it satisfies the condition $V_r > (c_s/2)^{1/2} \alpha_{\parallel e}$ (where V_r is the velocity of the return current electrons, c_s is the speed of sound and $\alpha_{\parallel e}$ is the thermal velocity of the plasma electrons), the ion-acoustic instability will set in. The rate at which the return current delivers energy to the plasma due to scattering of the ion-acoustic waves by the electrons is also reported in this section.

2. Dispersion relation

Let us consider a hot plasma imbedded in a uniform magnetic field $\boldsymbol{B}_0$ which we take along the Z-axis. A relativistic beam of electrons of radius a is streaming with velocity $\boldsymbol{U}$ through the plasma along the direction of $\boldsymbol{B}_0$. The beam density, n_b, is taken to be much smaller than the background plasma density, n_p, through which the beam is moving. We further assume that the plasma is hot but nonrelativistic; this restricts the analysis to temperatures such that $k_B T \ll mc^2$ (m being the electron rest mass).

As the beam propagates through the plasma the changing self-magnetic field towards the head of the beam induces a back current in the plasma which eventually neutralizes the beam current. The effect of such a return current can be taken into account in an indirect way through the inclusion of a return velocity V_r, similar to the usual drift velocity, in the equilibrium distribution function of the plasma electrons. This return velocity is related to the beam velocity U through the current neutralization relation, *viz.* $J_p^0 + J_b^0 = 0$, where J_p^0 and J_b^0 are the unperturbed plasma and beam current densities, respectively. On assuming that the return current is mainly due to the electrons, the return velocity is simply given by

$$V_r = -(n_b/n_p)\, U \tag{1}$$

Consequently for the plasma particles we can take the following equilibrium distribution function:

$$f_{0j}^{p} = \frac{1}{\pi^{3/2}\, \alpha_{\parallel j} \alpha_{\perp j}^2} \exp\left[-\, v_\perp^2/\alpha_{\perp j}^2 - (v_\parallel - V_{rj})^2/\alpha_{\parallel j}^2\right] \tag{2}$$

where $\alpha_{\parallel,\perp j} = (2k_B T_{\parallel,\perp}/m)_j^{1/2}$ are the parallel (perpendicular) thermal velocities for the jth species. The subscript labels the plasma species, *i.e.* $j = \mathrm{e}$ for electrons and i for ions. Moreover $V_{ri} = 0$ and $V_{re} \equiv V_r$, as given by eq. (1). For the beam, we choose a delta-type distribution function, namely

$$f_{0e}^{b} = \frac{1}{\pi p_\perp}\, \delta(p_\perp)\, \delta(p_\parallel - p_0) \tag{3}$$

The superscripts p and b are used to distinguish the plasma and the beam parameters.

For small perturbations the motion of the charged particles moving with relativistic speeds is governed by the linearized Vlasov equation. Following the procedure outlined by Buti (1963) and Montgomery and Tidman (1964), and on using the full set of Maxwell equations, we arrive at the following dispersion relation

$$|\mathbf{R}| = 0, \tag{4}$$

where

$$\mathbf{R} = (c^2 k^2 - \omega^2)\, \mathsf{I} - c^2\, \boldsymbol{kk} + \boldsymbol{\sigma} \tag{5}$$

with

$$\sigma = \mp \sum_{\alpha = \mathrm{p,\, b}} \sum_j \omega_{\alpha j}^2 \frac{\mathrm{i}\omega}{\Omega_j} \int \mathrm{d}\boldsymbol{p}\, \boldsymbol{p} \int_{\mp\infty}^{\phi} \mathrm{d}\phi'\, G(\phi') \times \left[\frac{\partial f_{0j}^{\alpha}}{\partial \boldsymbol{p}} + \frac{\boldsymbol{k} \times \boldsymbol{p} \times (\partial f_{0j}^{\alpha}/\partial \boldsymbol{p})}{m_j \omega \gamma_j}\right] \tag{6}$$

$$\ln G(\phi') = \pm \frac{\mathrm{i}}{m_j \Omega_j} \left[(k_\parallel p_\parallel - m_j \omega \gamma_j)(\phi - \phi') + k_\perp p_\perp (\sin\phi - \sin\phi')\right] \tag{7}$$

$\omega_{\alpha j} = (4\pi n_j e_j^2/m_j)^{1/2}$ and $\Omega_j = (e_j B_0/m_j c \gamma_j)$ are the plasma and cyclotron frequencies of the jth species. The label α appearing in eq. (6) implies that the summation is over the beam as well as the plasma parameters. Moreover, the upper sign in eq. (6) correponds to the ions and the lower one corresponds to the electrons. The rest of the symbols have their usual meaning and are defined in the above mentioned references (Buti 1963, Montgomery and Tidman 1964).

The various components of $\boldsymbol{\sigma}$ for the plasma and the beam, characterized by the distributions given by eq. (2) and eq. (3), respectively, are given in Appendix A. From eqs (4), (5) and (A–1) to (A–14) we observe that for a general $\boldsymbol{k} = (k_\perp, 0, k_\parallel)$, all the components of $\mathbf{R}$ are nonvanishing, and to analyse the dispersion relation in this case is a formidable job. Instead we shall restrict ourselves to the special cases $k_\perp = 0$, *i.e.*, parallel propagation or $k_\parallel = 0$, *i.e.*, perpendicular propagation only.

3. Parallel propagation

In this case, on putting $k_\perp = 0$ in eqs (5), (6) and (A–1)–(A–14), we find that the elements R_{xz}, R_{zx}, R_{yz} and R_{zy} vanish and the dispersion relation reduces to

$$R_{zz}\,(R_{xx} \pm \mathrm{i}R_{xy}) = 0 \tag{8}$$

The mode corresponding to $R_{zz} = 0$ is a purely electrostatic mode, whereas $(R_{xx} \pm \mathrm{i}R_{xy}) = 0$ correspond to the right handed and left handed circularly polarized electromagnetic modes, respectively. We shall investigate first the electrostatic mode.

3.1. *Electrostatic mode and return current instability*

For $k_\perp = 0$, the mode $R_{zz} = 0$ leads to the following relation:

$$\frac{\omega_{be}^{2}}{\gamma_0^{3}\,(\omega - kU)^2} + \frac{\omega_{pe}^{2}}{k^2\,a_{\parallel e}^{2}}\,Z'(\mu_e) + \frac{\omega_{pi}^{2}}{k^2\,a_{\parallel i}^{2}}\,Z'(\mu_i) = 1 \tag{9}$$

where $\mu_e = (\omega - kV_r)/ka_{\parallel e}$, $\mu_i = \omega/ka_{\parallel i}$ and $\gamma_0 = (1 - U^2/c^2)^{\frac{1}{2}}$. In the absence of return currents, *i.e.* for $V_r = 0$ eq. (9) represents the usual dispersion relation for the electrostatic wave in a cold-beam-hot plasma system. On the other hand if we treat V_r as some sort of relative velocity between the plasma electrons and the ions and neglect the beam term, then eq. (9) reduces to the dispersion relation for the current carrying plasmas (Stringer 1964). Hence eq. (9), as one would have expected, has the characteristics of the beam plasma system or the current carrying plasmas under appropriate conditions. Since for arbitrary values of μ_e and μ_i the above equation cannot be solved analytically, we shall now consider a few cases where it is possible to extract some information regarding the stability of the electrostatic waves.

3.1.a. Cold plasma—Under this approximation, eq. (9) simplifies to

$$F(\omega, k) \equiv \frac{\omega_{be}^{2}}{\gamma_0^{3}\,(\omega - kU)^2} + \frac{\omega_{re}^{2}}{(\omega + k\,|\,V_r\,|)^2} + \frac{\omega_{pi}^{2}}{\omega^2} = 1 \tag{10}$$

The instability occurs whenever minimum of function F becomes greater than one, *i.e.* for $F_{\min} > 1$. The case $F_{\min} = 1$ defines the boundary between the stable and the unstable regions (*see* figure 1).

From the schematic plot of eq. (10) shown in figure (1), we observe that in the absence of return current the instability can occur for $\omega_r > 0$ which is the usual e-e type instability (Briggs 1964, Nezlin 1971). However, when $V_r \neq 0$ even the frequencies $\omega_r < 0$ can support instability. The latter instability arises because of return currents and is essentially an e-i type instability which can occur in current carrying plasmas (Nezlin 1971, Stringer 1964). We shall now discuss the conditions under which the return current instability is important.

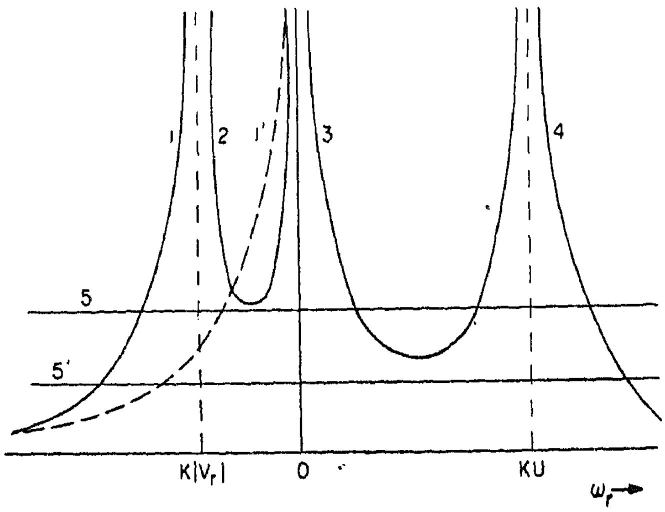

Figure 1. Schematic plot of eq. (10). Curves 1–4 are for $F(\omega, k)$ and curves 5 and 5′ are for $F(\omega, k) = 1$. In the absence of return currents the curves 1 and 2 merge together and form the curve 1′. Curves 5 and 2 show instability.

From eq. (10), we observe that for $\omega < 0$ and for $|\omega| \lesssim k|V_r|$, the beam term is approximately $\omega_{b0}^2/k^2 U^2 \gamma_0^3$ which for relativistic velocities is $\ll 1$ and hence can be neglected for the frequencies of interest. In such a case, F_{min} occurs at

$$\omega = -k|V_r|(1 + \epsilon^{-1/3})^{-1} \tag{11}$$

where $\epsilon = (m/M)$ is the electron to ion mass ratio. On substituting eq. (11) in eq. (10), we find that the critical plasma density is given by

$$n_{pcr} = \left(\frac{m}{4\pi e^2}\right) k^2 V_r^2 (1 + \epsilon^{1/3})^{-3} \tag{12 a}$$

and hence for the critical return current and consequentlyf or critical beam current, we have

$$\mathrm{I}_{cr}^{b} \equiv \mathrm{I}_{cr}^{R} = \pi a^2 n_{pcr} V_r$$

$$= \left(\frac{ma^2}{4e}\right) k^2 U^3 \left(\frac{n_b}{n_p}\right)^3 (1 + \epsilon^{1/3})^{-3} \tag{12 b}$$

The superscript R stands for return current.

In writing eq. (12 *a*) we have made use of eq. (1). From eqs (10) and (11) we find that the instability exists only in the range of wave numbers $0 < k < k_{cr}^{R}$; where

$$k_{cr}^{R} = \left(\frac{n_p}{n_b}\right)^{3/2} \frac{\omega_{bc}}{U} (1 + \epsilon^{1/3})^{3/2} \tag{12 c}$$

is the critical value of k. The maximum growth rate of this instability is given by

$$\gamma^{R}_{max} \sim 2^{-4/3}\, 3^{1/2}\, \epsilon^{1/3} k |V_r| \tag{12 d}$$

the wave number and the frequency corresponding to the maximally growing wave are given by $k^{R}_{max} \sim \omega_{p0}/|V_r|$ and $\omega_r^{R}(\max) \sim 2^{-4/3} \epsilon^{1/3} \omega_{i0}$.

For $\omega_r > 0$ and $\omega \sim kU$ we can again start with eq. (10) and find out critical condition for instability (beam plasma e-e instability) from the relation $F_{min} = 1$. The critical beam current necessary to excite this instability is given by

$$I_{cr} = \left(\frac{ma^2}{4e}\right) k^2 U^3 \gamma_0^3 [1 + (n_b/n_p)^{-1/3} \gamma_0]^{-3} (1 + |V_r|/U)^2 \tag{13 a}$$

The range of wave number unstable to this instability is $0 < k < k_{cr}$, where

$$k_{cr} = \frac{\omega_{be}}{U\gamma_0^{3/2}} \frac{[1+(n_p\gamma_0^3/n_b)^{\frac{1}{3}}]^{3/2}}{(1+|V_r|/U)} \tag{13 b}$$

The maximum growth rate of this instability is given by

$$\gamma_{max} \sim 2^{-4/3}\, 3^{1/2}\, (n_b/n_p)^{1/3}\, \gamma_0^{-1}\, kU\,(1+|V_r|/U) \tag{13 c}$$

which will occur for frequency and wave number corresponding to ω_r (max) $\sim 2^{-4/3}\,\gamma_0^{-1}\,(n_b/n_p)^{1/3}\,kU\,(1+|V_r|/U)$ and $k_{max} \sim \omega_{pe}/U$ respectively. From eqs (13 *a*), (13 *b*) and (13 *c*), it is clear that the effect of return currents on I_{cr}, k_{cr} or γ_{max} is of the order of $|V_r|/U$ and hence negligible.

In order to ascertain the condition under which the return current instability plays a dominant role, let us compare the critical currents, critical wave numbers and growth rates of this instability with the corresponding quantities for e-e instability. From eqs (12 *a*)–(12 *d*) and eqs (13 *a*)–(13 *c*) we find that for $U \sim c$

$$I_{cr}^{R}/I_{cr} \sim (n_b/n_p)^2 \tag{14 a}$$

$$k_{cr}^{R}/k_{cr} \sim (n_p/n_b) \tag{14 b}$$

and

$$\gamma^{R}_{max}/\gamma_{max} \sim [\epsilon\gamma_0^3\,(n_b/n_p)]^{1/3}. \tag{14 c}$$

From eqs (14 *a*)–(14 *c*), it is obvious that the return current instability will be important when the beam-plasma e-e instability is absent, *i.e.* either for $I < I_{cr}$ or for wave numbers lying in the range $k_{cr} < k < k_{cr}^{R}$. The effect of finite temperature on the RC instability is considered in the next section.

3.1.b. Hot electrons and cold ions—Under this approximation, *i.e.* for $\mu_e \ll 1$ and $\mu_i \gg 1$ the dispersion relation (9) simplifies to

$$\frac{\omega_{pi}^2}{\omega^2} - 2i\pi^{\frac{1}{2}} \frac{\omega_{pe}^2}{k^3 a_{\parallel e}^3}(\omega + k|V_r|) = 1 + \frac{2\omega_{pe}^2}{k^2 a_{\parallel e}^2} - \frac{\omega_{be}^2}{\gamma_0^3 k^2 U^2} \tag{15}$$

Let us write $\omega = \omega_r + i\gamma$ ($\gamma > 0$ for instability), and *a priori* assume that $|\gamma^2| \ll |\omega_r^2|$. Then on separating the real and the imaginary parts of eq. (15) and on solving for ω_r and γ, we get

$$\omega_r = \pm (\epsilon/2)^{1/2}\, k a_{\parallel e} X \tag{16}$$

and

$$\gamma = (\pi\epsilon/8)^{1/2}\, k\,(|V_r| - \sqrt{\epsilon/2}\, a_{\parallel e} X \tag{17}$$

with

$$X = \left[1 + \frac{2k^2 a_{\parallel e}^2}{\omega_{pe}^2} - \frac{2n_b}{n_p}\frac{a_{\parallel e}^2}{\gamma_0^3 U^2}\right]^{-\frac{1}{2}} \approx 1. \tag{18}$$

From eq. (17), we notice that γ will be positive only when the return velocity exceeds a critical value, *i.e.* if

$$|V_r| > |V_c| \approx a_{\parallel e}\,(\epsilon/2)^{1/2} \tag{19}$$

So the critical return current or the beam current is given by

$$I_{cr}^{hot} = \pi a^2 e n_p V_c = \left(\frac{ma^2}{4e}\right)(\epsilon/2)^{1/2}\,\omega^2_{pe}\, a_{\parallel e} \tag{20}$$

On comparing eq. (20) with eq. (13 *a*), we find that

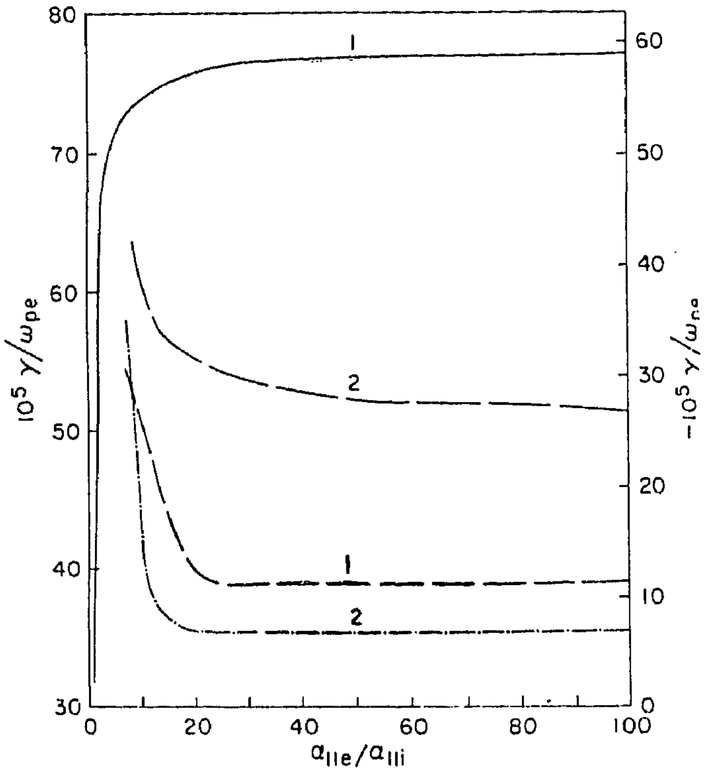

Figure 2. Variation of growth rate γ/ω_{pe})——— and real frequency ω_r/ω_{pe} (— — —) in the presence (curve 1) and absence (curve 2) of return currents *versus* $a_{\parallel e}/a_{\parallel i}$ for $n_b/n_p = 0{\cdot}05$, $U/c = 0{\cdot}95$, $\omega_{pe}/ka_{\parallel e} = 10$, and for $a_{\parallel e}/c = 10^{-2}$. The broken curve 2 (—·—·—·) denotes damping and follows the scale given on the right hand side of the figure.

$$\frac{I_{cr}^{\,hot}}{I_{cr}} = \frac{\omega_{pe}^{\;2}}{k^2\,U^2}\left(\frac{n_p}{n_b}\frac{a_{\parallel e}}{U}\right)\left(\frac{\epsilon}{2}\right)^{1/2} \approx O\left(\epsilon^{\frac{1}{2}}\right) \tag{21}$$

The general dispersion relation (9) has been solved numerically; the results are shown in figures 2 and 3. From these figures we observe that the presence of return current forces the damped waves to grow instead. The effects of increasing (n_b/n_p) is to increase the growth rates. As $a_{\parallel e}/a_{\parallel i}$ is increased, the growth rates first increase with $a_{\parallel e}/a_{\parallel i}$ but then get saturated for larger values.

3.1.c. Estimates of plasma heating—It is well known that growth of the beam plasma instability is limited by the trapping of the beam electrons in the potential of the growing wave. Following the simple model given by Drummond *et al* (1970) we can calculate the final saturation level of the return current instability which according to eq. (12 *d*) is given by

$$W_R = \frac{E^2\,(t_f)}{16\pi} = \frac{1}{2}\,n_p\,mV_r^{\;2}\,(1 - 2^{-4/3}\,\epsilon^{1/3}) \tag{22}$$

In order to have an order-of-magnitude estimate of how much heating can be achieved by the return current instability, we define an average kinetic energy per particle relative to the mean for the *j*th species as

$$K_j\,(t) = \tfrac{1}{2}\,m_j \int \mathrm{d}v\,[v - V_j\,(t)]^2\, f_j\,(v, t)$$

where $V_j\,(t)$ is the mean velocity defined by

$$V_j\,(t) = \int \mathrm{d}v\,v\,f_j\,(v, t)$$

and $f_j\,(v, t)$ is the distribution function for the *j*th species. The quasilinear evolution of the distribution function is governed by the equation (Drummond and Pines 1962, Vedenov *et al* 1961)

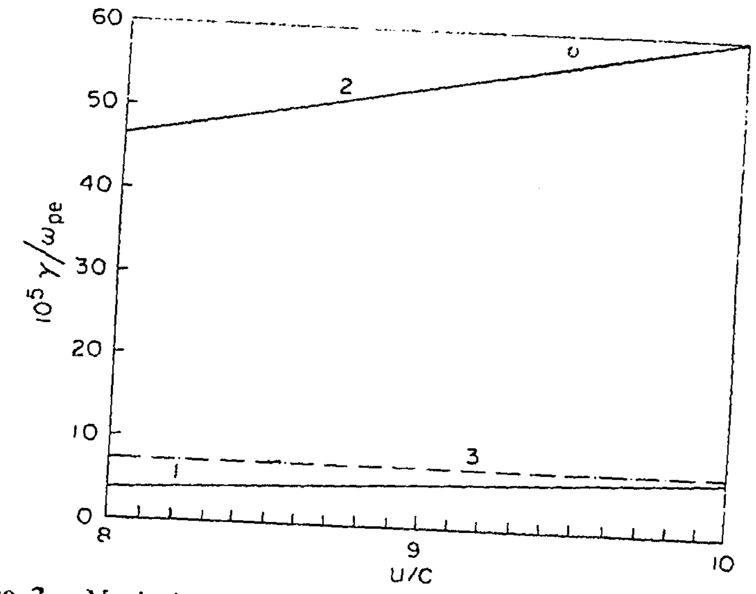

Figure 3. Variation of growth rate γ/ω_{ie} versus U/c for $a_{\|e}/a_{\|i} = 100$, $\omega_{pe}/ka_{\|e} = 10$, and $a_{\|e}/c = 0\cdot 1$ and for $n_b/n_p = 0\cdot 01$ and $0\cdot 05$ for the curves 1 and 2 respectively. The broken curve 3, which is for the case when return currents are neglected but the other parameters are same as above, denotes damping. The damping remains unaffected by the increase of n_b/n_p. The value of the real frequency is $\omega/\omega_{re} = -5\cdot 17 \times 10^{-3}$ for all the curves.

$$\frac{\partial}{\partial t} f_j(v, t) = \frac{\partial}{\partial v}\left[D_j(v, t)\frac{\partial}{\partial v} f_j(v, t)\right] \tag{23}$$

where the diffusion coefficient is defined by

$$D_j(v, t) = \frac{8\pi e^2}{m_j^2}\int dk\mathscr{E}_k(t)\frac{\gamma_k}{(\omega_k - kv)^2 + \gamma_k^2} \tag{24}$$

In eq. (24) $\mathscr{E}_k(t)$ is the spectral energy density of the wave and γ_k is the linear growth rate corresponding to the kth mode. Taking appropriate velocity moment of eq. (23), it can be easily shown that

$$n_j\frac{d}{dt}K_j(t) = \omega_{pj}^2\int dk 2\gamma_k\mathscr{E}_k(t)\int dv f_j(v, t) \times \frac{[(\omega_k - kV_j)^2 + \gamma_k^2 - k^2(v - V_j)^2]}{[(\omega_k - kv)^2 + \gamma_k^2]^2} \tag{25}$$

In obtaining eq. (25), eq. (24) has been used. From an order-of-magnitude estimate of the integrand in eq. (25), it can be shown (Davidson 1972) that the electron and the ion heating rates are approximately given by,

$$n_e\frac{d}{dt}K_e(t) \simeq \frac{d}{dt}\mathscr{E}_f(t) \tag{26}$$

and

$$n_i\frac{d}{dt}K_i(t) \simeq \epsilon^{1/3}\frac{d}{dt}\mathscr{E}_f(t) \tag{27}$$

where $\mathscr{E}_f(t) = \int dk\mathscr{E}_k(t)$ is the total field energy density in the unstable modes. From eqs (26) and (27) it is clear that it is the electrons that get preferentially heated. This fact is important in the sense that after the initial development of the return current instability, the electrons will be sufficiently heated so as to make ion-acoustic instability to take over.

Another important conclusion that can be drawn from the order-of-magnitude estimate of the integrand of eq. (25) is that K_j will continue to increase until (Davidson 1972)

$$\frac{K_j}{m_j} \simeq (\omega_k/k - V_j)^2 + (\gamma_k^{\mathrm{R}}/k^2) \tag{28}$$

Thus the maximum temperature of the electrons that can be achieved by the return current instability is given by

$$T_e \simeq \tfrac{1}{2} m \left[(\omega_{k_{\max}}/k_{\max} - V_r)^2 + (\gamma^{\mathrm{R}}{}_{\max}/k_{\max})^2\right] \tag{29}$$

For $n_p \sim 10^{14}$ cm^{-3}, $n_b/n_p \simeq 0 \cdot 05$ and $U/c \sim 0 \cdot 99$, which corresponds to an intense beam with peak current density ~ 20 kA/cm^2, the maximum temperatures to which the electrons will be heated by the return current instability is ~ 5 keV. This is in good agreement with the estimate made by Guillory and Benford (1972).

As was mentioned earlier, the electrons will be preferentially heated in the early stages of the development of the return current instability. When the electrons are sufficiently heated so that $T_e \gg T_i$ and the return current satisfies the condition given by eq. (19), the return current induced ion acoustic instability sets in. The turbulence generated by this instability will further heat the plasma.

We shall make a rough estimate of the electron heating on the assumption that the level of turbulence (*i.e.* the energy in the growing wave) is limited solely due to scattering of the ion sound wave by the electrons. Following Sisonenko and Stepanov (1969) and Krall and Book (1969), we can immediately show that the result of the above mentioned process is to limit the final level of turbulent energy W, to a value given by

$$\frac{W}{n_p T_e} \sim \frac{m}{M} \frac{V_r}{c_s}, \tag{30}$$

where $c_s = \sqrt{T_e/M}$ is the ion sound speed.

In order to make an estimate of the electron heating, we shall first calculate the effective collision frequency ν_{eff}, for the electrons. In the resonant region of velocity space, the space-averaged distribution function for the electrons, $f_e(v, t)$ evolves according to

$$\frac{\partial f_e(\boldsymbol{v}, t)}{\partial t} = \frac{e^2}{m^2} \pi \frac{\partial}{\partial \boldsymbol{v}} \int \frac{\mathrm{d}^3 k}{(2\pi)^3} kk \mid Ek \mid^2 \delta(\omega k - k \cdot v) \cdot \frac{\partial}{\partial v} f_e(v, t) \tag{31}$$

On taking the first moment of eq. (31) we obtain,

$$\frac{\mathrm{d}}{\mathrm{d}t}(nV_e) = -n_p V_e \nu_{\mathrm{eff}}$$

$$= -\pi \frac{|e^2}{m^2} \int \mathrm{d}^3 \boldsymbol{v} \int \frac{\mathrm{d}^3 k}{(2\pi)^3} kk \mid \mathrm{E}k \mid^2 \delta(\omega k - k \cdot v) \cdot \frac{\partial}{\partial v} f_e(v, t)$$

$$= -\int \frac{\mathrm{d}^3 k}{(2\pi)^3} k \frac{\mid Ek \mid^2}{4\pi m} 2\gamma_k \frac{\omega_{pi}{}^2}{\omega k^2} \tag{32}$$

On substituting eqs (16) and (17) in eq. (32) we get,

$$\nu_{\text{eff}} = \frac{2\pi^{1/2}\,\omega_{\text{pi}}^2}{4\pi m n_{\text{p}}\epsilon} \cdot \frac{|E_k|^2}{\langle k \rangle\, \alpha_{\|e}^3}\left[1 - \frac{c_s \langle X \rangle}{|V_r|}\right] \tag{33}$$

On treating these 'effective collisions' as an isotropic joule-heating mechanism, we can obtain the rate of heating of the electron as

$$\frac{3}{2} n_{\text{p}} \frac{dT_e}{dt} \approx \eta J_{\text{p}}^2 \tag{34}$$

where η is the resistivity defined by $\eta = 4\pi\nu_{\text{eff}}/\omega_{\text{pe}}^2$. Equations (33) and (34) immediately give

$$\frac{dT_e}{dt} \approx \frac{2^{5/2}\,\pi^{1/2}}{3\langle k \rangle \lambda_{\text{D}}}\left(\frac{W}{nT_e}\right)\omega_{\text{pe}}\left(\frac{1}{2} m\,|V_r|^2\right)\left[1 - \frac{c_s\langle X \rangle}{|V_r|}\right]$$

which can be rewritten as

$$\frac{d}{dt}(n_{\text{p}} T_e) \approx A\left(\frac{1}{2} m n_{\text{p}}\,|V_r|^2\right) \tag{35}$$

where

$$A = \frac{2^{5/2}\,\pi^{1/2}}{3\langle k \rangle \lambda_{\text{D}}}\left(\frac{m}{M}\frac{|V_r|}{c_s}\right)\omega_{\text{pe}}\left[1 - \frac{c_s\langle X \rangle}{|V_r|}\right] \tag{36}$$

Equation (35) tells us that the rate at which the return current is delivering energy to the plasma electrons is given by the quantity A. Since $X \sim 1$ and for the unstable modes under consideration $c_s < V_r$, the time scale of delivery of energy can be $\sim \omega_{\text{pi}}^{-1}$ when $V_r \sim (M/m)^{1/2} c_s$ and $\langle k \rangle \lambda_{\text{D}} \sim 1$.

3.2. *Electromagnetic modes*

As mentioned earlier, two electromagnetic modes of right handed and left handed circular polarization can propagate along the direction of the magnetic field. Here we shall discuss only the right handed mode which is given by $R_{xx} + iR_{xy} = 0$. A similar treatment can be applied to the left handed mode in a straight forward manner. On using the results of Appendix A in eq. (5), the dispersion relation for the right handed mode can be written as

$$-\omega^2 + c^2 k^2 + \frac{\omega_{\text{b0}}^2(\omega - kU)}{\gamma_0(\omega - kU + \Omega_e^{\text{r}})} + \sum_j \omega_{\text{pj}}^2\left[1 + \frac{\Omega_j}{k\alpha_{\|j}} Z(\eta_j) + \frac{\alpha_{\perp j}^2}{2\alpha_{\|j}^2} Z'(\eta_j)\right] = 0 \tag{37}$$

where

$$\Omega_e^{\text{r}} = \Omega_e/\gamma_0, \quad \eta_e = (\omega - \Omega_e - kV_r)/k\alpha_{\|e}$$

and

$$\eta_i = (\omega + \Omega_i)/k\alpha_{\|i}$$

For frequencies, $\omega \ll \Omega_i$ on solving eq. (37), we find that the return current effects are negligible. We shall therefore leave out this uninteresting case and discuss here the frequency range, $\Omega_i \ll \omega \lessapprox \Omega_e$, where the return current effects are expected to be important. Let us consider the following case:

3.2.*a*. $\eta_e \ll 1$ and $\eta_i \approx \omega/k\alpha_{\|i} \gg 1$

Under these restrictions eq. (37) reduces to

$$- \omega^2 + c^2 k^2 + \frac{\omega_{be}{}^2}{\gamma_0} - (\Gamma - 1)\, \omega_{pe}{}^2 + 2\omega_{pe}{}^2 X(\omega - \Omega_e - kV_r)/k\alpha_{\parallel e}$$
$$- i\pi^{1/2} X\omega_{pe}{}^2 + \omega_{pi}{}^2 = 0 \tag{38}$$

where

$$\Gamma = \alpha_{\perp e}{}^2/\alpha_{\parallel e}{}^2$$

and

$$X = [\Gamma(\omega - kV_r) - (\Gamma - 1)\, \Omega_e]/k\alpha_{\parallel e} \tag{39}$$

On separating eq. (38) into real and imaginary parts by writing, $\omega = \omega_r + i\gamma$, and assuming that $|\gamma| \ll |\omega_r|$, we obtain

$$\omega_r = \left[(\Gamma - 1) - \frac{2\gamma}{\pi^{1/2} k\alpha_{\parallel e}}\right] \frac{\Omega_e}{\Gamma} - k\,|V_r| \tag{40}$$

and

$$\gamma \sim \frac{k\alpha_{\parallel e}}{\pi^{1/2}\Gamma}\left[(\Gamma - 1)\left(1 - \frac{4\Omega_e{}^2}{\pi k^2 \alpha_{\perp e}{}^2 \Gamma}\right) + \left(\frac{\Gamma - 1}{\Gamma}\right)^2 \frac{\Omega_e{}^2}{\omega_{pe}{}^2}\left(1 - \frac{4}{\pi\Gamma}\right)\right.$$
$$\left. - \frac{2(\Gamma - 1)\, k\,|V_r|\, \Omega_e{}^2}{\Gamma\Omega_e \omega_{pe}{}^2}\left(1 - \frac{2}{\pi\Gamma}\right) - \frac{c^2 k^2}{\omega_{pe}{}^2}\right] \tag{41}$$

We may note that the assumptions $\omega \lesssim \Omega_e$ and $\eta_e = (\omega - \Omega_e + k\,|V_r|)/k\alpha_{\parallel e} \ll 1$ imply that $|V_r| \ll \alpha_{\parallel e}$. Therefore, eq. (41) simply shows the tendency of these modes towards stabilization due to the presence of the return currents. In fact, the influence of these currents is expected to become appreciable when $|V_r| \gtrsim \alpha_{\parallel e}$, *i.e.* for $\eta_e \sim 1$, a case which is difficult to handle analytically. However, if we consider the case when

$$\eta_e = (\omega - \Omega_e + k\,|V_r|)/k\alpha_{\parallel e} \approx \frac{|V_r|}{\alpha_{\parallel e}} \gg 1$$

and $\eta_i \gg 1$ as before; from eq. (38) we obtain the following dispersion relation:

$$\omega^2 = c^2 k^2 + \frac{\omega_{be}{}^2}{\gamma_0} + \omega_{pe}{}^2\left(1 + \frac{\Omega_e}{k\,|V_r|} + \frac{\alpha_{\perp e}{}^2}{|V_r|^2}\right) + \omega_{pi}{}^2\left(1 + \frac{k^2 \alpha_{\perp i}{}^2}{\omega^2}\right), \tag{42}$$

which shows no instability. Hence we can conclude that the influence of the return currents on this mode is to suppress the growth rates.

4. Transverse propagation ($k_{\parallel} = 0$)

In this case we have $\boldsymbol{k} = k\hat{\mathbf{e}}_x$ and on putting $k_{\parallel} = 0$ in eqs (5), (6) and (A–1)–(A–14) we notice that all the elements of $\mathbf{R}$ are nonvanishing and involve infinite summation over Bessel functions $I_n(\lambda)$. For any arbitrary value of λ it is impossible to obtain some analytical results. We shall, therefore, restrict ourselves to the case where $\lambda = k^2 \alpha_{\perp e}{}^2/\Omega_e{}^2 \to \infty$, *i.e.* no magnetic field case and when $\lambda \ll 1$, *i.e.* a strong magnetic field case.

4.1. *Zero magnetic field* ($\boldsymbol{B}_0 = 0$)

In the limit of $\Omega \to 0$ the various elements of $\mathbf{R}$ are given by

$$R_{xx} = -\omega^2 + \frac{\omega_{be}^2}{\gamma_0} - \omega^2 \sum_j \frac{2\omega_{Pj}^2}{k^2 \alpha_{\perp j}^2} [1 + \xi_j Z(\xi_j)] \quad (43)$$

$$R_{yy} = -\omega^2 + c^2 k^2 + \frac{\omega_{be}^2}{\gamma_0} - \sum_j \omega_{Pj}^2 \xi_j Z(\xi_j) \quad (44)$$

$$R_{xz} = R_{zx} = \frac{kU\omega_{be}^2}{\gamma_0 \omega} - \omega \sum_j \frac{2\omega_{Pj}^2 V_r}{k\alpha^2_{\perp j}} [1 + \xi_j Z(\xi_j)] \quad (45)$$

$$R_{zz} = -\omega^2 + c^2 k^2 + \frac{k^2 U^2 \omega_{be}^2}{\gamma_0 \omega^2} + \sum_j \omega_{Pj}^2 \left\{1 - \frac{\alpha_{\parallel j}^2}{\alpha_{\perp j}^2}\left(1 + \frac{2V_r^2}{\alpha_{\parallel j}^2}\right) \times [1 + \xi_j Z(\xi_j)]\right\} \quad (46)$$

and

$$R_{xy} = R_{yx} = R_{yz} = R_{zy} = 0$$

with

$$\xi_j = \omega / k\alpha_{\perp j}.$$

The dispersion relation in this case is simply given by

$$R_{yy}(R_{xx}R_{zz} - R_{xz}^2) = 0 \quad (47)$$

The mode $R_{yy} = 0$ is independent of V_r and hence we will not discuss it here. The other mode, $R_{xx}R_{zz} - R_{xz}^2 = 0$, is affected by the streaming velocity and we will study this in detail. The dispersion relation for this mode, on using eqs (43)–(46) and on neglecting ion contribution, can be written (after dropping the subscript e which is now not needed) as

$$G_1\omega^4 - \left(\left\{c^2 k^2 + \omega_P^2 - \frac{\alpha_\parallel^2}{\alpha_\perp^2}\omega_P^2 [1 + \xi Z(\xi)]\right\} G_1 - 2\omega_P^2 \frac{V_r^2}{\alpha_\perp^2} \times [1 + \xi Z(\xi)]\right)\omega^2 - \frac{\omega_b^2 \omega_P^2}{\gamma_0}\left\{\frac{2\tilde{U}^2}{\alpha_\perp^2}[1 + \xi Z(\xi)] - 1\right\} = 0, \quad (48)$$

where

$$G_1 = 1 + 2\frac{\omega_P^2}{k^2 \alpha_\perp^2}[1 + \xi Z(\xi)] \quad (49)$$

$$\tilde{U}^2 = (U + |V_r|)^2 + \alpha_\parallel^2/2 \quad (50)$$

For the case, $\xi \gg 1$ and $\omega_P^2/\omega^2 \gg 1$ eq. (48) reduces to

$$\omega^2 = \omega_P^2 + c^2 k^2\left(1 - \frac{\alpha_\parallel^2 + 2V_r^2}{2c^2}\right) + \frac{\omega_P^2 k^2}{\omega^2}\left[\left(1 + \frac{3}{2}\frac{k^2\alpha_\perp^2}{\omega^2}\right) \times \frac{\alpha_\parallel^2}{2} + \frac{n_b \tilde{U}^2}{n_P \gamma_0}\right]. \quad (51)$$

As $k^2 \alpha_\perp^2/\omega^2 \ll 1$, we can solve eq. (51) by an iterative procedure. To zeroth order eq. (51) reduces to a bi-quadratic equation which yeilds the solution

$$\omega_0^2 \approx \frac{k^2[\alpha_\parallel^2/2 + (n_b/n_P\gamma_0)\tilde{U}^2]}{1 + (c^2 k^2/\omega_P^2)[1 - (\alpha_\parallel^2 + 2V_r^2)/2c^2]} \quad (52)$$

The other root of the bi-quadratic equation is not consistent with our assumption $\omega_p^2 \gg \omega^2$.

On using eq. (52), the first order solution of eq. (51) is simply given by

$$\omega^2 = -\frac{k^2\{[1+(3/2)(k^2 a_\perp^2/\omega_0^2)](a_\parallel^2/2)+(n_b/n_p\gamma_0)\tilde{U}^2\}}{1+(c^2 k^2/\omega_p^2)[1-(a_\parallel^2+2V_r^2)/2c^2]} \tag{53}$$

The effect of return currents, as seen from eq. (53), is to increase the growth rate slightly.

We have solved the dispersion relation (48) numerically and the results are given in table 1. From this table, we observe that the return currents do affect the growth rates but very slightly and that the effect of increasing $a_\perp/a_\parallel$ is to stabilize the system; this is in agreement with eq. (53). From table 1 we also conclude that for low beam velocity the growth rate increases with U, attains a maximum value and then decreases as $U \to c$. This decrease of growth rate when $U \to c$ is due to the increase of relativistic mass, $m = m_0/(1 - U^2/c^2)^{1/2}$. This is in agreement with the results of Lakhina and Buti (1972).

Table 1. Variation of growth rate of the transverse e.m. waves in the absence of magnetic field for $c^2k^2/\omega_{pe}^2 = 0 \cdot 1$ and $\omega_b/n_p = 0 \cdot 01$. The parameter δ decides whether values correspond to the case of 'no return current' ($\delta = 0$) or 'with return currents' ($\delta = 1$) respectively. The waves are purely growing.
Growth rates γ/ω_{pe} (in units of 10^{-3})

U/c \ $a_\perp/a_\parallel$	0·1		1·0	
	$\delta=1$	$\delta=0$	$\delta=1$	$\delta=0$
0·1	21·20	21·19	5·04	4·99
0·2	21·84	21·82	8·86	8·79
0·3	22·83	22·79	12·16	12·06
0·4	24·04	23·99	15·11	14·99
0·5	25·36	25·29	17·73	17·57
0·6	26·65	26·55	19·96	19·79
0·7	27·71	27·59	21·66	21·48
0·8	28·28	28·16	22·54	22·35
0·9	27·78	27·66	21·80	21·62
0·91	27·61	27·50	21·56	21·37
0·92	27·42	27·31	21·26	21·08
0·93	27·18	27·07	20·89	20·72
0·94	26·90	26·80	20·44	20·28
0·95	26·56	26·46	19·89	19·73
0·96	26·14	26·04	19·19	19·04
0·97	25·61	25·52	18·28	18·13
0·98	24·91	24·84	17·00	16·86
0·99	23·93	23·83	14·94	14·82
0·995	23·10	23·06	13·08	12·98

4.2. *Strong magnetic field* ($\lambda = k^2 \alpha_\perp^2/\Omega^2 \ll 1$)

Under the approximation, $\lambda \ll 1$ the elements of $\mathbf{R}$ get simplified but all are non-vanishing (*see* Appendix B) and we have to use the full dispersion relation

$$R_{xx}\left(R_{yy}R_{zz} + R_{yz}^2\right) + R_{xy}\left(R_{xy}R_{zz} + R_{yz}R_{xz}\right) + R_{xz}\left(R_{xy}R_{yz} - R_{yy}R_{xz}\right) = 0 \tag{54}$$

where R_{ij} are as given in Appendix B. However, if we assume $\omega^2/\Omega_e^2 \approx 1/\gamma_0 \ll 1$, the terms $R_{xy}R_{yz}R_{xz}$ and $R_{yy}R_{xz}^2$ become much smaller than $R_{xy}^2R_{zz}$ and hence can be neglected. When we do this, the dispersion relation (54) reduces to (neglecting ion contribution and dropping the subscript once again)

$$\begin{aligned}
&\omega^6 - \omega^4 c^2 k^2\left[\left(1+\omega_p^2/\Omega^2\right)\left(\tau^2-\Phi\right) + 1 + \frac{\left(\tau^2-1\right)\omega_p^2/\Omega^2}{\left(1+\omega_p^2/\Omega^2\right)}\right] \\
&\quad + \omega^2 c^2 k^2\left[c^2k^2\left(\tau^2-\Phi\right) - \frac{\omega_b^2 U^2\left(1+\omega_p^2/\Omega^2\right)}{\gamma_0 c^2} - \frac{V_r^2\omega_p^4}{c^2\Omega^2}\right. \\
&\quad \left. + \frac{\left(\tau^2-\Phi\right)\omega_p^4/\Omega^2}{\left(1+\omega_p^2/\Omega^2\right)}\right] + \left[1 + \frac{\left(\tau^2-1\right)\omega_p^2/\Omega^2}{\left(1+\omega_p^2/\Omega^2\right)}\right]\frac{c^2U^2k^4\omega_b^2}{\gamma_0} = 0
\end{aligned} \tag{55}$$

where

$$\tau^2 = \left(1 + \omega_p^2/c^2k^2\right) \text{ and } \Phi = \frac{1}{2}\frac{\alpha_\parallel^2}{c^2}\frac{\omega_p^2}{\Omega^2}\left(1 + \frac{2V_r^2}{\alpha_\parallel^2}\right) \tag{56}$$

Since eq. (55) is a cubic equation in ω^2 with coefficient of constant term positive, it will always give one negative root for ω^2. However, for $\omega_p^2/\Omega^2 \gtrsim 1$ we find that ω^6 term in eq. (55) is O (ω^2/Ω^2 or ω^2/ω_p^2) as compared to other terms. So neglecting this term (which amounts to suppressing the root $\omega \sim \Omega$ or ω_p which in any case is an invalid root) in eq. (55), the resulting quadratic equation in ω^2, for the growth rate, yields

$$\gamma^2 \equiv -\omega^2 \approx \frac{\left(n_b/n_p\gamma_0\right)k^2U^2}{1 + \dfrac{c^2k^2}{\omega_p^2} - \dfrac{k^2\left(\alpha_\parallel^2 + 2V_r^2\right)}{2\Omega^2} - \dfrac{V_r^2\omega_p^2}{c^2\Omega^2}\left[1 + \dfrac{\left(\tau^2-1\right)\omega_p^2/\Omega^2}{1+\omega_p^2/\Omega^2}\right]^{-1}} \tag{57}$$

From eq. (57), we can immediately conclude that the growth rates will increase with V_r.

Once again we have solved the general dispersion relation (54) numerically. Some of the results obtained are shown in table 2, which shows that the effect of return current is important when $\omega \sim \Omega_e$. For ω deviating too much from Ω_e, the effect of return current on the growth rate is very small.

A brief comment on the heating produced by these transverse e.m. instabilities is in order here. Computer simulation experiments (Davidson *et al* 1971, Davidson *et al* 1972) have shown that the bulk response of the plasma, such as heating, is in very good agreement with the predictions of space averaged quasilinear theory in the initial stages of the instability. The computer simulation experiments also show that the magnetic field fluctuations get saturated *via* magnetic trapping governed by the equation

Table 2. Variation of growth rate and real frequency in the presence of a magnetic field for $c^2k^2/\omega^2_{pe} = 0 \cdot 1$, $n_b/n_p = 0 \cdot 1$, and $a_{\parallel}/c = 10^{-2}$. The parameter δ distinguishes the 'no return currents' ($\delta = 0$) case from the 'with return current' ($\delta = 1$) case.

Ω/ω_p	1·2				1·6				5·0			
	γ/ω_p (10^{-6})		ω_r/ω_p		γ/ω_p (10^{-5})		ω_r/ω_p		γ/ω_p (10^{-4})		ω_r/ω_p	
U/c	$\delta=1$	$\delta=0$	$\delta=1$	$\delta=0$	$\delta=1$	$\delta=0$	$\delta=1$	$\delta=0$	$\delta=1$	$\delta=0$	$\delta=1$	$\delta=0$
0·92	1·19	0·87	1·08	1·07	4·02	4·03	1·07	1·07	3·27	3·27	1·03	1·03
0·93	1·36	0·99	1·08	1·07	4·30	4·31	1·07	1·07	3·28	3·28	1·03	1·03
0·94	1·52	1·12	1·07	1·07	4·55	4·56	1·07	1·06	3·28	3·28	1·03	1·03
0·95	1·69	1·26	1·07	1·06	4·78	4·79	1·06	1·06	3·27	3·27	1·03	1·03
0·96	1·87	1·42	1·07	1·06	4·99	5·00	1·06	1·06	3·24	3·24	1·02	1·02
0·97	2·07	1·58	1·07	1·06	5·19	5·19	1·06	1·06	2·99	2·99	1·02	1·02
0·98	2·29	1·77	1·06	1·06	5·38	5·38	1·05	1·05	2·77	2·73	0·95	0·95
0·99	2·55	2·00	1·06	1·06	5·57	5·57	1·05	1·05	2·90	2·90	1·04	1·04
0·995	2·72	2·15	1·06	1·05	5·68	5·68	1·05	1·05	3·08	3·08	1·04	1·04
0·999	2·93	2·35	1·06	1·05	5·80	5·80	1·05	1·05	3·16	3·15	1·03	1·03

where $\omega_B = | ekV_{\perp}B_k/mc |^{\frac{1}{2}}$ (B_k being the magnetic field amplitude and $V_{\perp}$ is the characteristic particle velocity perpendicular to the direction of propagation) is the bounce frequency of the electrons in the potential of the magnetic fluctuations. A rough estimate of the rate of heating due to such an instability when $\omega \sim \Omega_0 \sim \omega_{pe}$ (the region, where the effect of return currents according to table 2 is most significant, was made by using quasilinear equations governing the rate of change of kinetic energy (Davidson *et al* 1972). It is found that the rate of heating achieved due to such an electromagnetic instability is extremely small compared to the rates obtained due to e.s instabilities (*see* section 3.1 *c*).

5. Conclusion

The return currents, arising because of the motion of a relativistic beam of electrons through a nonrelativistic plasma affect the stability of the waves excited by the beam-plasma interaction in a number of ways. These currents destabilize the electrostatic waves by exciting a return current instability which requires smaller beam currents than the one required for exciting the usual beam plasma e-e instability. The electromagnetic waves propagating along the direction of the magnetic field are stabilized by these currents. The growth rates of e.m. waves propagating in the transverse direction are, however, slightly increased by the presence of these currents. The return current instability can heat the plasma to keV temperatures. Moreover, when the ion-sound turbulence generated by the return current decays *via* scattering of ion-sound waves by electrons, the return current delivers energy to the plasma at the rate of $\sim (m/M)^{\frac{1}{2}} (V_r/C_s)\, \omega_{pi}$.

In this study the return velocities were taken to be nonrelativistic as the analysis was restricted to beam densities much smaller than the plasma density. It would be of interest to extend the analysis for relativistic return velocities and also for the case where the plasma temperatures are high, *i.e.* $k_B T \sim mc^2$. This work will be reported in a forthcoming paper.

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Appendix A

The elements of σ defined by eq. (6) can be evaluated by doing the integrations as indicated by Montgomery and Tidman (1964) and Bernstein (1958). For the case of plasma characterized by the distribution function as given by eq. (2) the various σ_{ij} elements turn out to have the following form:

$$\sigma_{xx}^{\mathrm{P}} = \sum_j \omega_{\mathrm{P}j}^2 \sum_{n=-\infty}^{\infty} \frac{n^2 I_n(\lambda_j)\, e^{-\lambda_j}}{\lambda_j} \left[1 - \frac{n\Omega_j}{k_\parallel \alpha_{\parallel j}} Z(\mu_{nj}) + \frac{\alpha_{\perp j}^2}{2\alpha_{\parallel j}^2} Z'(\mu_{nj})\right] \tag{A-1}$$

$$\sigma_{xy}^{\mathrm{P}} = -\sigma_{yx}^{\mathrm{P}} = \sum_j \omega_{\mathrm{P}j}^2 \sum_{n=-\infty}^{\infty} \frac{inF_1(\lambda_j)}{\lambda_j} \left[1 - \frac{n\Omega_j}{k_\parallel \alpha_{\parallel j}} Z(\mu_{nj}) + \frac{\alpha_{\perp j}^2}{2\alpha_{\parallel j}^2} Z'(\mu_{nj})\right] \tag{A-2}$$

$$\sigma_{xz}^{\mathrm{P}} = \sigma_{zx}^{\mathrm{P}} = \sum_j \omega_{\mathrm{P}j}^2 \sum_{n=-\infty}^{\infty} \frac{n^2 I_n(\lambda_j)\, e^{-\lambda_j}}{2\lambda_j} \frac{k_\perp}{k_\parallel} \times \left[\left(\frac{\alpha_{\parallel j}^2}{\alpha_{\perp j}^2} + \frac{\omega}{n\Omega_j} - 1\right) \frac{\alpha_{\perp j}^2}{\alpha_{\parallel j}^2} Z'(\mu_{nj}) - \frac{2V_r}{\alpha_{\parallel j}} Z(\mu_{nj})\right] \tag{A-3}$$

$$\sigma_{yy}^{\mathrm{P}} = \sum_j \omega_{\mathrm{P}j}^2 \sum_{n=-\infty}^{\infty} \Phi_1(\lambda_j) \left[1 - \frac{n\Omega_j}{k_\parallel \alpha_{\parallel j}} Z(\mu_{nj}) + \frac{\alpha_{\perp j}^2}{2\alpha_{\parallel j}^2} Z'(\mu_{nj})\right] \tag{A-4}$$

$$\sigma_{yz}^{\mathrm{P}} = -\sigma_{zy}^{\mathrm{P}} = \sum_j \omega_{\mathrm{P}j}^2 \sum_{n=-\infty}^{\infty} \frac{\Omega_j F_1(\lambda_j)}{ik_\perp k_\parallel \alpha_{\perp j}^2} \left[\left(\omega - n\Omega_j \frac{\alpha_{\perp j}^2}{\alpha_{\parallel j}^2} + n\Omega_j\right) Z'(\mu_{nj}) - \frac{2n\Omega_j V_r}{\alpha_{\parallel j}} Z(\mu_{nj})\right] \tag{A-5}$$

$$\begin{aligned}\sigma_{zz}^{\mathrm{P}} = -\sum_j \omega_{\mathrm{P}j}^2 \sum_{n=-\infty}^{\infty} \frac{I_n(\lambda_j)\, e^{-\lambda_j}}{k_\parallel \alpha_{\parallel j}} \Bigg(&\omega\left[\frac{1}{2} Z''(\mu_{nj}) - \frac{V_r}{\alpha_{\parallel j}} Z'(\mu_{nj}) + Z(\mu_{nj})\right] \\ &+ n\Omega_j \left\{\frac{1}{2}\left(\frac{\alpha_{\parallel j}^2}{\alpha_{\perp j}^2} + 1\right) Z''(\mu_{nj}) - \frac{V_r}{\alpha_{\parallel j}}\left(1 + \frac{\alpha_{\parallel j}^2}{\alpha_{\perp j}^2}\right) Z'(\mu_{nj}) \right. \\ &\left. + \left[\frac{\alpha_{\parallel j}^2}{\alpha_{\perp j}^2}\left(1 + \frac{2V_r^2}{\alpha_{\parallel j}^2}\right) + 1\right] Z(\mu_{nj})\right\}\Bigg)\end{aligned} \tag{A-6}$$

where

$$\lambda_j = (k^2 \alpha_{\perp j}^2)/(2\Omega_j^2), \qquad \mu_{nj} = \frac{\omega + k_\parallel |V_r|}{k_\parallel \alpha_{\parallel j}} - \frac{n\Omega_j}{k_\parallel \alpha_{\parallel j}}$$

$$F_1(\lambda) = \frac{\lambda}{2}\frac{\mathrm{d}}{\mathrm{d}\lambda}\{\mathrm{e}^{-\lambda}[I_n(\lambda) - I_{n+1}(\lambda)]\} + \frac{n}{2}\mathrm{e}^{-\lambda}[I_n(\lambda) - I_{n+1}(\lambda)] - \tfrac{1}{2} I_{n+1}(\lambda)\,\mathrm{e}^{-\lambda} \tag{A-7}$$

$$\Phi_1(\lambda) = \lambda\frac{\mathrm{d}}{\mathrm{d}\lambda}\{\mathrm{e}^{-\lambda}[I_n'(\lambda) - I_n(\lambda)]\} + \mathrm{e}^{-\lambda} I_n'(\lambda)$$

and

$$Z(\mu) = \pi^{-1/2}\int_{-\infty}^{\infty} \mathrm{d}x\, \mathrm{e}^{-x^2}/(x-\mu)$$

is the plasma dispersion function (Fried and Conte 1961) and $I_n(\lambda)$ is the modified Bessel function of nth order.

The elements of $\boldsymbol{a}$ for the beam characterized by the distribution function as given by eq. (3) have been evaluated by Montgomery and Tidman (1964). These are given below for the sake of completeness:

$$\sigma_{xx}^{\mathrm{b}} = -\omega_{\mathrm{be}}^2(-\omega + k_\parallel U)^2 A \tag{A-8}$$

$$\sigma_{xy}^{\mathrm{b}} = -\sigma_{yx}^{\mathrm{b}} = -\mathrm{i}\Omega_{\mathrm{e}}^{\mathrm{r}}\,\omega_{\mathrm{be}}^2(-\omega + k_\parallel U) A \tag{A-9}$$

$$\sigma_{xz}^{\mathrm{b}} = \sigma_{zx}^{\mathrm{b}} = k_\parallel U\omega_{\mathrm{be}}^2(-\omega + k_\parallel U) A \tag{A-10}$$

$$\sigma_{yy}^{\mathrm{b}} = -\omega_{\mathrm{be}}^2(-\omega + k_\parallel U)^2 A \tag{A-11}$$

$$\sigma_{yz}^{\mathrm{b}} = -\sigma_{zy}^{\mathrm{b}} = -\mathrm{i}k_\perp U\omega_{\mathrm{be}}^2\Omega_{\mathrm{e}}^{\mathrm{r}} A \tag{A-12}$$

$$\sigma_{zz}^{\mathrm{b}} = -\frac{\omega\omega_{\mathrm{be}}^2}{\Omega_{\mathrm{e}}}\frac{\partial}{\partial P}\left[\frac{\Omega_{\mathrm{e}} P}{\gamma_0(-\omega + k_\parallel U)}\right] - \omega_{\mathrm{be}}^2 k_\perp^2 U^2 A \tag{A-13}$$

where

$$\Omega_{\mathrm{e}}^{\mathrm{r}} = \Omega_{\mathrm{e}}/\gamma_0$$

and

$$A = \gamma_0^{-1}[\Omega_{\mathrm{e}}^{\mathrm{r}2} - (-\omega + k_\parallel U)^2]^{-1} \tag{A-14}$$

Appendix B

Elements of $\mathbf{R}$ for transverse propagation in the limit $\lambda_{\mathrm{j}} = k^2 a_{\perp\mathrm{j}}^2/2\Omega_{\mathrm{j}}^2 \ll 1$:

$$R_{xx} = -\omega^2 - \frac{\omega_{\mathrm{be}}^2\omega^2}{\gamma_0(\Omega_{\mathrm{e}}^{\mathrm{r}2} - \omega^2)} + \sum_j \frac{\omega_{\mathrm{pj}}^2\omega^2}{\omega^2 - \Omega_j^2} \tag{B-1}$$

$$R_{xy} = -R_{yx} = -\frac{\mathrm{i}\omega\Omega_{\mathrm{e}}\omega_{\mathrm{be}}^2}{\gamma_0^2(\Omega_{\mathrm{e}}^{\mathrm{r}2} - \omega^2)} + \sum_j \frac{\mathrm{i}\omega\Omega_j\omega_{\mathrm{pj}}^2}{\omega^2 - \Omega_j^2} \tag{B-2}$$

$$R_{xz} = R_{zx} = -\frac{\omega_{\mathrm{be}}^2 kU\omega}{\gamma_0(\Omega_{\mathrm{e}}^{\mathrm{r}2} - \omega^2)} + \sum_j \frac{kV_r\omega_{\mathrm{pj}}^2\omega}{\omega^2 - \Omega_j^2} \tag{B-3}$$

$$R_{yy} = -\omega^2 + c^2k^2 - \frac{\omega_{\mathrm{be}}^2\omega^2}{\gamma_0(\Omega_{\mathrm{e}}^{\mathrm{r}2} - \omega^2)} + \sum_j \frac{\omega^2\omega_{\mathrm{pj}}^2}{\omega^2 - \Omega_j^2} \tag{B-4}$$

$$R_{yz} = -R_{zy} = -\frac{\mathrm{i}kU\omega_{\mathrm{be}}^2\Omega_{\mathrm{e}}}{\gamma_0^2(\Omega_{\mathrm{e}}^{\mathrm{r}2} - \omega^2} - \sum_j \frac{\mathrm{i}kV_r\omega_{\mathrm{pj}}^2\Omega_{\mathrm{j}}}{\omega^2 - \Omega_j^2} \tag{B-5}$$

$$R_{zz} = -\omega^2 + c^2 k^2 + \frac{\omega_{be}{}^2}{\gamma_0{}^3} - \frac{k^2 U^2 \omega_{be}{}^2}{\gamma_0 (\Omega_e{}'^2 - \omega^2)} + \sum_j \omega_{pj}{}^2$$

$$+ \frac{1}{2} \sum_j \omega_{pj}{}^2 \left(1 + \frac{2V_r{}^2}{\alpha_{\|j}{}^2}\right) \frac{k^2 \alpha_{\|j}{}^2}{\omega^2 - \Omega_j{}^2} \tag{B–6}$$

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