The Challenge of Fluid Flow

1. The Diversity of Flow Phenomena

Roddam Narasimha

You look up at the sky, and see a lovely cloud; you look down, and may see lovely ripples on a rivulet (or river). On a hot summer afternoon you see dancing dust devils; on a cold winter evening you can see smoke rising lazily from a *chulah*, and hang up there as if it has given up.

You peer at a telescope, and see intense supersonic jets, or vast whirling galaxies; you measure in a wind tunnel, and sense powerful tornadoes behind an aircraft wing. The universe is full of fluid that flows in crazy, beautiful or fearsome ways.

In our machines and in the lab, as in terrestrial nature, one sees this amazing diversity in the flow of such a simple liquid like water or a simple gas like air. What is it that makes fluid flows so rich, so complex – some times so highly ordered that their patterns can adorn a saree border, sometimes so chaotic as to defy analysis? Do the *same* laws govern *all* that extraordinary variety?

We begin with a picture gallery of a number of visible or visualized flows, and consider which ones we understand and which ones we do not, which ones we can compute and which ones we cannot; and it will be argued that behind those all-too-common but lovely flows lie deep problems in physics and mathematics that still remain mysteries.

The Diversity of Flows

Almost anywhere we look we see a fluid flow: air, water, smoke, steam, oil or something. And the diversity of the flows we see, even with our unaided eyes, is astonishing. You look up at the

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sky, and may see striking clouds – white, pretty and fluffy against a perfectly blue sky, or black and threatening against a gray one. You look down at the ground, and may see a wonderful medley of ripples, whirlpools, jumps or bores on a tiny rivulet of water – or on a mighty river. On a hot summer afternoon you can see dust devils – suddenly twisting themselves into shape picking up all kinds of loose dirt, and darting wildly around for some seconds before they flop out. On a cool winter evening you can see warm smoke rising lazily from a *chulah* out in the open, and hang up there a few metres above ground with no energy left to push into the cooler, heavier air around. You can go to Jog and keep gazing at the four waterfalls there – majestic, awesome, graceful, and so different from each other although they are all making about the same leap.

With the power of modern instrumentation the range of what we can see goes up enormously. If you peer at a telescope, or the pictures it takes, we can see swirling nebulae whose sizes are measured in hundreds of thousands of light years. If you have a microscope you can see minute organisms (like spermatozoa, for example) darting here and there in the surrounding medium. If you can inject smoke and shine laser beams you can see powerful vortices that would otherwise be invisible in air, such as those that trail behind aircraft. From satellites we can see cyclones that can wreak havoc. And there can be terrible tsunamis of the kind we experienced in south and south-east Asia on 26 December 2004. I have put together a collection of some flow pictures of this kind in *Figure* 1 – I hope you will agree that they can be crazy, beautiful and fearsome in turn. (A wonderful album compiled by Van Dyke has many others.)

One can go on endlessly like this. This diversity of fluid motion has fascinated man for thousands of years. The *Yoga-Vāsiṣṭha* (written perhaps around +6 or 7 c.) uses fluid flow metaphors extensively, and compares the complexity of the human mind with that of flowing water:
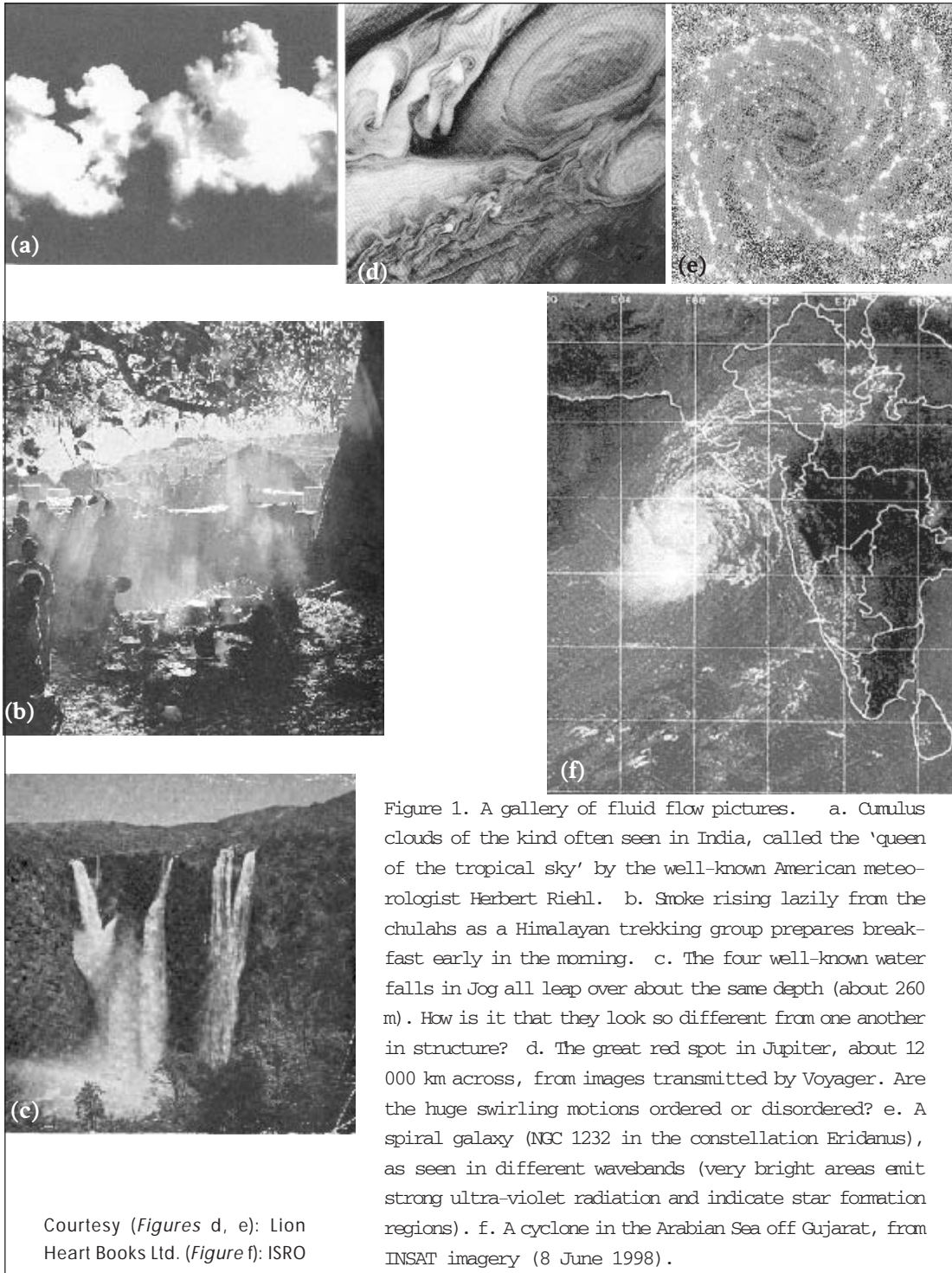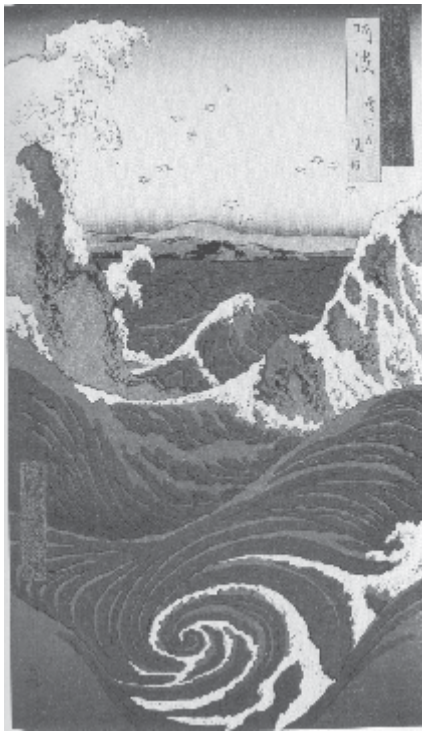
Figure 1. A gallery of fluid flow pictures.   a. Cumulus clouds of the kind often seen in India, called the 'queen of the tropical sky' by the well-known American meteorologist Herbert Riehl. b. Smoke rising lazily from the chulahs as a Himalayan trekking group prepares breakfast early in the morning. c. The four well-known water falls in Jog all leap over about the same depth (about 260 m). How is it that they look so different from one another in structure? d. The great red spot in Jupiter, about 12 000 km across, from images transmitted by Voyager. Are the huge swirling motions ordered or disordered? e. A spiral galaxy (NGC 1232 in the constellation Eridanus), as seen in different wavebands (very bright areas emit strong ultra-violet radiation and indicate star formation regions). f. A cyclone in the Arabian Sea off Gujarat, from INSAT imagery (8 June 1998).

Courtesy (*Figures* d, e): Lion Heart Books Ltd. (*Figure* f): ISRO

As water displays itself richly

　　In current, wave, foam and spray,

So does the mind exhibit

　　A strange, splendid diversity.

dhārā-kaṇ' -ōrmi-phēṇa-śrīr.
yathā samlakṣyatē.' mbhasaḥ , |
tathā vicitra-vibhavā
nānāt''. ēyam hi cētasaḥ . ||

(3:110.48)

Elsewhere the author sings of the beauty of a whirlpool in swaying water *(āhola-salil'-āvarta-sundarī)*. And, as if in illustration of Vāsiṣṭha poetry, the Japanese painter Hiroshige Utagawa (1797-1858) catches its essential spirit in a lovely print of *Wind and Wave* (*Figure* 2). In renaissance Europe Leonardo da Vinci (1452-1519), who combined within himself the hands and eyes of an engineer as well as an artist, drew wonderfully faithful pictures of vortices in turbulent water flow.

Do we *understand* these diverse flows? How many of them can be described by a convincing theory? How many can be computed? After all, isn't the flow of fluids a 'classical' subject, whose governing laws were set down in the first half of the 19th century? It may be interesting to put together an album of pretty flow pictures, but are there new things to discover? (Answer: yes.) Why do people (like me) spend a life-time investigating fluid flow problems, pursuing the rather esoteric discipline of 'fluid dynamics'?

It is my intention here to try and answer these questions. But before offering answers, we need to look at those flows and grasp their nature a little better.

Order and Chaos

One of the most striking features of many fluid flows that has fascinated all observers – ancient and modern – is the strange mix of order and chaos that is their characteristic. At one extreme, a flow can be incredibly well-ordered. For example, consider the case of what is called Rayleigh-Benard convection between two large horizontal plates, the lower one of which is warmer
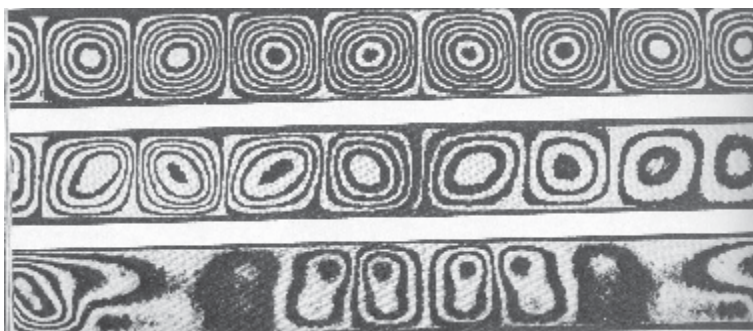

Figure 2. A Japanese print entitled 'Wind and wave at Awa-Nruto', by Hiroshige Utagawa (1797–1858).

than the upper. If the temperature difference between the plates is sufficiently small there is no flow at all, and heat is transported from the lower to the upper plate through thermal conduction in the enclosed fluid – which might just as well have been a solid, it would have made no difference. But as the temperature difference exceeds a critical value the fluid suddenly overturns into motion, in a row of highly organized rolls (*Figure* 3) (or a pattern of hexagonal cells as seen from the top if the upper surface is free). Those rolls can be so steady in time and so repeatable in space that they could well decorate a saree border. As the temperature difference increases the rolls get distorted and begin to sway and list, and eventually break down into chaotic motion. We see one example of such breakdown in the bottom panels of *Figure* 3. (But, as we shall see shortly, such apparently chaotic motion may well contain hidden order.)

Saree-border type organization may also be seen in a variety of other situations – most famously in the flow in the wake of a circular cylinder. (By the principle of Galilean invariance, the relative flow is the same whether the cylinder moves in still fluid or the fluid flows past a cylinder at rest; so we shall use either frame interchangeably.) In this case, as a critical velocity is exceeded the flow in the wake spontaneously breaks into two parallel rows of staggered vortices – clockwise in one row, anti-clockwise in the other (*Figure* 4), forming what is called a Karman vortex street. (See article in current issue.) (We often 'hear' those vortices when power lines sing in wind; the Greeks even had an 'aeolian' harp, whose strings were resonantly

Suggested Reading

[1] J H Arakeri, Bernoulli's Equation, *Resonance*, Vol.5, No.8, pp.54-71, August 2000.

[2] J H Arakeri and P N Shankar, Ludwig Prandtl and Boundary Layers in Fluid Flow, *Resonance*, Vol.5, No.12, pp. 48-63, December 2000.

[3] R Govindarajan , Turbulence and Flying Machines, *Resonance*, Vol.4, No. 11, pp.54-62, November 1999.

[4] R Narasimha, Order and chaos in fluid flows, *Curr. Sci.*, Vol. 56, pp.629-645, 1987.

[5] R Narasimha, Turbulence on computers, *Curr. Sci.*, Vol. 64, pp. 8-32, 1993.

[6] R Narasimha, Divide, conquer and unify, *Nature*, Vol. 432, p. 807, 2004.

[7] M Van Dyke, *An album of fluid motion*, Parabolic Press, Stanford, CA, 1982.

Figure 4. Another ordered motion: the Karman vortex street behind a circular cylinder. This staggered arrangement of vortices in two parallel rows is 'stable'. (See Box 2 on Reynolds number.)


'plucked' by natural wind.) As the velocity is increased further, the Karman vortex street breaks down into apparent chaos, but again, it turns out, with hidden order in it.

Instability and Transition to Turbulence

In these and other cases, the appearance of order actually marks the onset of an instability (*Box* 1). The critical condition that sparks this onset is given not so much by a temperature difference or flow velocity, which may be specific to a particular apparatus and fluid, as (much more generally) by an appropriate non-dimensional number like the Rayleigh number in convection and the Reynolds number in flow past bodies. (*Boxes* 2 and 3).

Figure 5. An idealized, schematic picture of how transition from laminar to turbulent flow occurs in the boundary layer on a flat plate, from two-dimensional instability through turbulent spots to fully developed turbulent flow.
(Reproduced from Frank M White, Viscous Fluid Flow, McGraw-Hill International Editions, 1991).
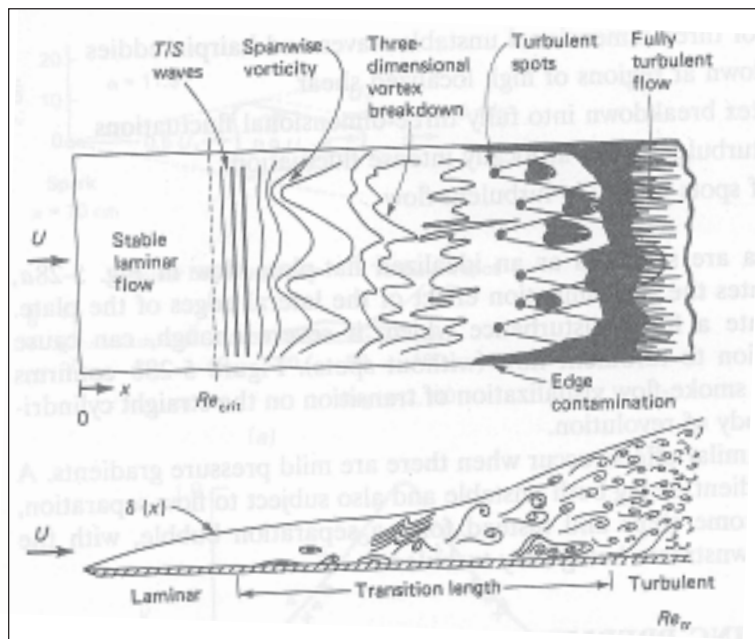
Box 1. Flow Instability

The words stability and instability are used in so many different senses that it is not possible to go into all of them here. Briefly, the idea in linear stability theory is that a system in a certain state is stable if a small perturbation made on it decays, unstable if it grows. In some flow systems, a small perturbation may grow exponentially under certain conditions – e.g. if the relevant non-dimensional number (see *Box* 2 or 3) exceeds a critical value. This growth may however soon reach saturation through nonlinear effects. Consider the Karman vortex street as an example. The arrangement of *Figure* 4 is stable in the sense that if any vortex in the system is displaced, the effect of the rest of the vortices is such as to take the perturbed vortex back to its original position. However, the birth of the vortex street itself is a result of an instability that occurs at a critical cylinder Reynolds number (see *Box* 2) of about 48.5. In fact, the Karman vortex street is best seen as an example of a global fluid-dynamical oscillator, which makes its appearance at the critical Reynolds number of 48.5. (At this critical value the flow encounters what is known as a bifurcation: this is a point of structural instability, at which the solution can exhibit a significant qualitative change in its structure - e.g. from a steady state to an oscillatory state.)

In the boundary layer on a flat plate, as *Figure* 5 suggests, the flow is laminar and stable to small perturbations till a critical Reynolds number of about $\mathrm{Re} = U x / \nu = 10^5$ is reached (here x is distance from the leading edge). Beyond this point sinusoidal disturbances of certain frequencies can grow as they travel downstream in space, till they reach a point where the boundary layer has grown enough to be no longer unstable to those frequencies. From then on the disturbance decays. However, the flow is now unstable to other frequencies, which again can grow in amplitude before they also eventually decay again. On the whole, disturbance of some frequency or other will keep growing beyond the critical point. All these disturbances travel downstream, so if the original disturbance is a pulse at some point, the disturbance at that point vanishes eventually but will be present in some form at some downstream location at some later time. Such an instability is called convective.

In contrast to the Karman vortex street, the boundary layer is best seen as a selective amplifier, in the sense that the instability waves appear only because there is a forcing somewhere - almost always in the environment. If the environmental disturbance levels are reduced transition to turbulence is delayed; the best reading of the experimental evidence suggests that the critical Reynolds number is inversely proportional to the environmental disturbance intensity, and so can increase without limit as the environment becomes increasingly 'quiet'. There is in this view no such thing as 'natural' transition on a flat plate, although that phrase is unfortunately still widely used in text books and in engineering practice.

I myself have long been interested in flow past surfaces, such as e.g. an aircraft wing. What happens here in a simple and rather idealized situation is illustrated in *Figure* 5. At a critical distance from the leading edge (more precisely a Reynolds number based on that distance) instability waves can appear in the flow, and grow downstream; eventually they break down into islands of

Box 2. The Reynolds Number

Fluid flows are determined by a variety of forces – pressure, viscous stresses, gravity (including buoyancy), surface tension, electro- and ferro-magnetic etc. As it is impossible (and rarely desirable) to take all of them into account in analysing specific flows, fluid dynamicists identify various non-dimensional groups that give some indication of which among these many forces may be dominant in any given situation. The grand-father of all such non-dimensional groups is the Reynolds number, named after the British engineer Osborne Reynolds who first showed the value of the concept in his studies of pipe flow, treated elsewhere in another box (see *Box* 4).

The Reynolds number is commonly thought of as a measure of the ratio of inertial to viscous forces. The inertial force is another name for the effective acceleration of the fluid. Because a fluid is a continuous medium, all these forces are most conveniently estimated for unit volume of the fluid. It must be remembered that only estimates of orders of magnitude are involved in forming non-dimensional groups like the Reynolds number.

As an example, consider the flow of an incompressible fluid of density ρ and viscosity μ at a velocity U past a circular cylinder of diameter D. As velocity changes of order U may be expected to occur over distances of order D, a characteristic flow time is of order D/U, and a characteristic acceleration of a fluid element is of order $U/(D/U) = U^2/D$. So the inertial force is of order $\rho U^2/D$ per unit volume. The viscous stress is typically of order $\mu U/D$, and its spatial rate of change $\mu U/D^2$ gives a measure of the viscous force per unit volume. The ratio of the two is the Reynolds number

$$Re = \frac{\rho U^2}{D} \cdot \frac{D^2}{\mu U} = \frac{UD}{\nu}$$

where it is convenient to introduce $\nu = \mu/\rho$ as the 'kinematic' viscosity (so called because its units, m^2/s in the metric system, do not involve mass).

Note that we could just as well have used the radius of the cylinder (rather than the diameter) in defining the Reynolds number. In that case the numerical value of the Reynolds number would be half of what is given above. This only shows that there can be arbitrary numerical constants in defining the Reynolds number, and when quoting numerical values for it we must clearly state the specific choices for the quantities that go into it, and stick to those choices. All the more reason why the Reynolds number in one class of flow (flow past cylinders, e.g.) cannot be directly compared with values in a different class (e.g. wings). Once a choice is made, however, we can say that a doubling of the Reynolds number will double the inertial force in comparison with viscous forces.

Reynolds numbers vary widely for different flow types – from something like 10^{-2} for the mote in the sunbeam to nearly 10^{12} for a tropical cyclone.

The Reynolds number is a non-dimensional group in the sense that its value will be the same in any internally consistent system of units – SI, British, etc.

chaos called turbulent spots, which themselves grow and propagate downstream till the flow becomes 'fully turbulent'.

What I have described above is the phenomenon of flow transition – from laminar flow (no disorder) to turbulent flow (apparently complete disorder). Such a transition occurs in virtually every known flow type, often through one or more intermediate stages of instability of one kind or another. In some flows transition is abrupt or 'hard', in others it can be gradual and slow ('soft'). It can even be both: spots can appear suddenly, but full-time turbulence emerges slowly as spots need time to grow before they cover the surface. If the non-dimensional parameter governing the flow, like a local flow Reynolds number, varies in space (as in *Figure* 5) or in time (as it does in the upper reaches of our lungs especially if we are breathing hard, say after jogging), the transitional sequence from laminar to turbulent flow may occur in space or in time (or in both).

Box 3. The Rayleigh Number

The Rayleigh number does for free convection what the Reynolds number does for incompressible flows. Consider the situation illustrated in *Figure* 3: two horizontal plates separated by a vertical distance h, the gap being filled with a fluid of characteristic density ρ_o, viscosity μ_o and temperature T_o. The flow is now driven by a temperature difference ΔT. As no characteristic flow velocity is explicitly prescribed in the statement of the problem, it has to be estimated from the other parameters. We do it this way. Thermal conduction diffuses temperature, the corresponding fluid property being the 'kinematic' diffusivity κ_o (= $k_o/\rho_o\, c$, where k_o is the thermal conductivity and c the specific heat). Now a characteristic diffusion time to smear out a temperature difference of ΔT over a distance h is h^2/κ_o (easily checked dimensionally). The effective acceleration due to buoyancy force per unit mass is $g\,\Delta\rho/\rho_o$, $\sim g\,\Delta T/T_o$, where $\Delta\rho$ and ΔT are the characteristic density and temperature differences. So the velocity that develops due to this 'acceleration' over the diffusion time is $g\,(\Delta T/T_o)\,(h^2/\kappa_o)$. The corresponding Reynolds number works out to what is called the Rayleigh number

$$\text{Ra} = \frac{gh^2 \Delta T}{\nu_o T_o}\frac{h}{\kappa_o} = \frac{gh^3 \Delta T}{T_o \nu_o \kappa_o}.$$

Another non-dimensional group we will run across is the well-known Mach number, the ratio of flow velocity to the acoustic speed. This ratio provides a measure of the thermodynamic elastic forces operating in a compressible flow. If the Mach number is small the 'elastic' forces leading to sound propagation are also relatively small, so the flow behaves as if it is incompressible.

Reverse Transition –Turbulent Flow Becoming Laminar

Under certain conditions (e.g. if the local Reynolds number *decreases* downstream) a turbulent flow can revert to a laminar state. This indeed happens in our lungs, as the air flows down from the wind pipe into numerous tiny bronchioles. This *reverse* transition from disorder to order does not violate the second law of thermodynamics, because the flows we are discussing are open systems. Some instances of such relaminarization are illustrated in *Figure* 6. You can sometimes see it in the evening sky, when a cloud, bubbling up from the ground, seems all of a sudden to lose its rough edges and bulges out into a smooth bump. This happens when the cloud hits what is called an 'inversion' – a situation where the temperature locally *increases* with height, rather than decreasing as it most commonly does. The rising air then becomes heavier than the air above the inversion and subsides, having converted its kinetic energy to gravitational. This is similar to the *chulah* smoke mentioned earlier, and one of the pictures in *Figure* 6 is a laboratory simulation of this effect.

Figure 6. 'Reverse' transition from turbulent to laminar flow. a (left). Flow comes into coiled tube from top left in a turbulent state (notice how tube is filled with red dye), but leaves in a laminar state (green filament injected at bottom does not spread). b (right). Jet of dyed water flow up in a tank. On the left, the jet starts out laminar, transitions to turbulence. On the right, the turbulent jet so formed reverts to a laminar state as it hits a stabilizing density gradient created by heating the water at the top and making it lighter.
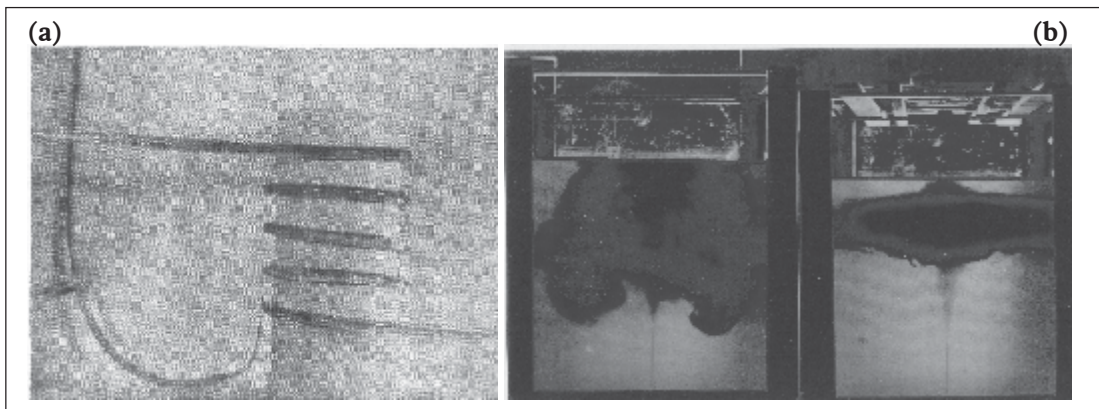
Box 4. Laminar-turbulent Transition

It is a matter of common observation that fluid flow can be in one of two fundamentally different states. One of them, called laminar, is regular, smooth or ordered, and the other, called turbulent, is irregular, chaotic, disordered. In steady laminar flow the velocity at any point is either a constant in time, or has orderly (periodic or related) fluctuations. In turbulent flow the velocity is fundamentally time-dependent, exhibits no apparent order, has contributions from all frequencies, and has vorticity (.i.e. fluid elements have local spin). Some typical turbulent traces are shown in *Figure* 7.

The first scientific study of the transition from laminar to turbulent flow was conducted by Reynolds (*Figure* a) in a pipe. He detected turbulent flow by injecting a filament of dye at a point on the centre-line near the entry to the pipe. If the flow was laminar the dye remained a straight filament along the centre line. But if the flow were turbulent the dye spread across the pipe colouring the fluid in the whole pipe. Laminar and turbulent flow can be made out easily in *Figure* 6.

By experimenting with different fluids in pipes of different diameter, Reynolds discovered that transition occurred at more or less the same value of what later came to be called the Reynolds number (see *Box* 2). (The name was given by the physicist Arnold Sommerfeld.) In Reynolds's own experiments the critical value of Re, defined as UD/ν, where U is the average velocity across the pipe of diameter D, turned out to be around 2300.

We now know that the situation is not so simple; if great care is taken in reducing environmental disturbances, flow perturbations, asymmetries, pipe roughness etc., very high transition Reynolds numbers (multiple of 10^5) can in fact be obtained. Between fully laminar and fully turbulent states appear travelling 'slugs' of turbulent flow – preceded and followed by laminar flow – called 'flashes' by Reynolds.

As we show elsewhere, there are also conditions when a turbulent flow can revert to a laminar state – a transition in the reverse direction, often called relaminarization, is therefore also possible.
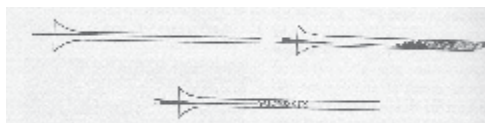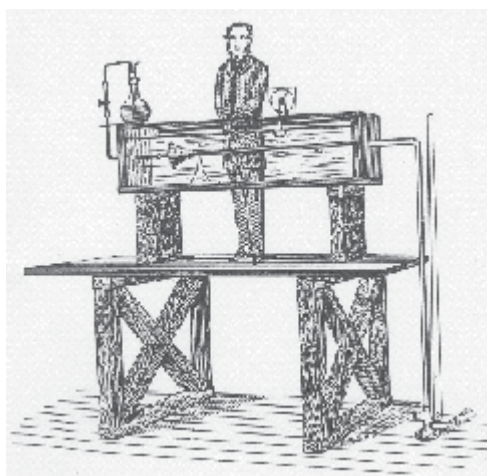

Figure: Osborne Reynolds demonstrating transition in pipe flow. Inset shows sketches of the nature of the flow in the pipe, as visualized by a filament of dye injected along the centre-line of the pipe. The sketches illustrate laminar flow, turbulent flow, and turbulent 'flashes'. (Reynolds 1883 Phil. Trans. Roy. Soc.174:935–982. Interestingly, the paper starts with the sentence, 'The results of this investigation have both a practical and philosophical aspect' – a thought that captures the enduring appeal of fluid dynamics.)
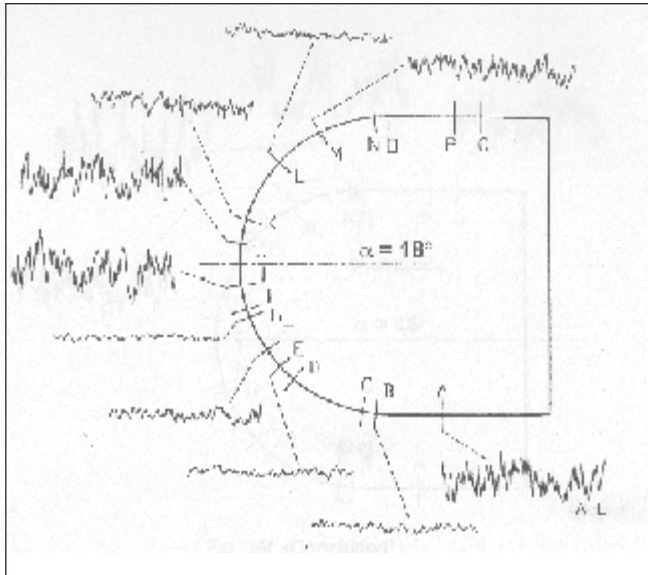
We recently discovered, to our astonishment, that as many as six such transitions – back and forth between laminar and turbulent flow – can occur in a narrow strip around the leading edge of the kind of swept wing that is typical of modern civilian passenger aircraft (such as Boeing and Airbus make). *Figure* 7 shows how wildly the output from hot-film sensors stuck on the surface of such a wing can vary from one sensor to the next, under conditions typical of take-off or landing.

Figure 7. Flow past an idealized leading edge of a swept wing, of the kind commonly used in transport aircraft. Conditions correspond to those at high angles of attack, as aircraft is landing or taking off. The flow appears to go through several transitions, as seen from the traces of the wall stress. The most striking characteristic of these traces is the dramatic variations in fluctuation intensity; to clearly identify transitions between laminar and turbulent periods requires some sophisticated signal processing, involving estimates of intermittency or skewness in the signals.

Is Turbulence 'Totally' Random?

I have talked loosely of transition from order to disorder or the other way round, but this is slightly misleading because (as I have been hinting) that fully turbulent flow is often *not* completely disordered. Instantaneous pictures of simple turbulent mixing layers – i.e. the flow at the interface between two streams moving at different velocities – showed dramatic evidence of unsuspected spatial organization into large-scale vortices (*Figure* 8), of which there had been hardly any hint in the numerous local measurements that had earlier been made in the same flows. These Brown-Roshko vortices (so called after their discoverers) are no longer strictly periodic, nor are they identical to each other. (So they are okay for modernist saree borders.) The

Figure 8. Ordered structure in a 'turbulent' mixing layer, as seen in a wavelet transform of a picture due to Brown and Roshko.
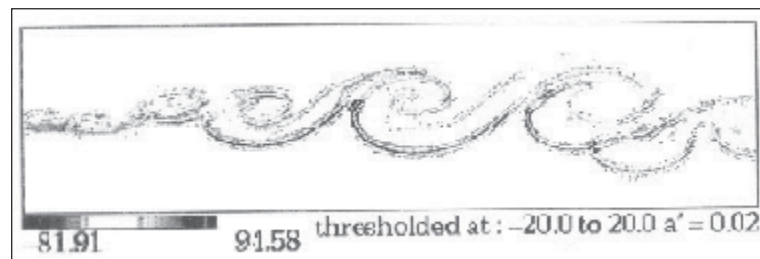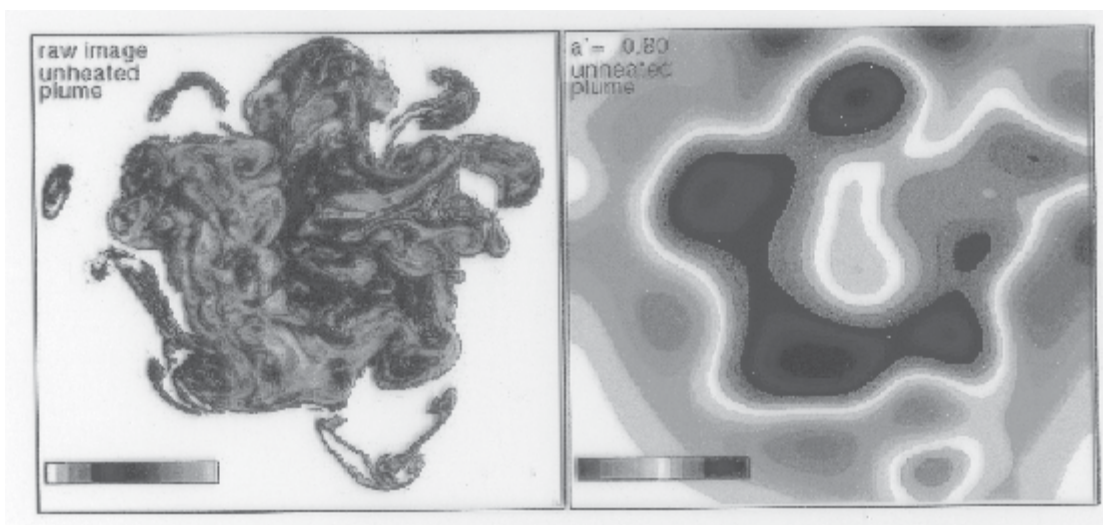
turbulence reveals itself in the jitter and variability that characterize these vortices, and in the disordered motion at smaller scales. So even what is called fully turbulent flow often contains coherent structures of organized motion within itself.

Let us look at a less obvious example. *Figure* 9 shows the cross-section of a jet illuminated by a sheet of laser light. (We can think of it as a slice of flow issuing towards this sheet of paper from a circular orifice well below.) The fluid coming out of the orifice is dyed, so the picture shows how this jet fluid is mingling and mixing with the ambient fluid drawn into the jet. The raw image on the left – with false colouring indicating dye concentration – is what we normally associate with fully turbulent flow. (Notice by the way all those thin structures in the picture.) The image on the right is a wavelet transform – a spatial filter that eliminates both smoother and sharper variations than a specified scale. At the scale chosen, we found that a lobed ring-like structure pops out, presumably representing a wobbly, azimuthally unstable vortex ring. At much smaller scales the wavelet transform looks random.

In Part 2, we will consider how these phenomena have been handled, what we can compute and what we can measure, and where the hard, grand challenges to our understanding lie.

Figure 9. On the left is a false-colour image of the diametral cross section of a dyed jet (like the one in Figure 6b), taken by laser-induced fluorescence techniques. The flow seems disordered, but a wavelet transform at a suitable scale reveals a fluted ring-like structure – perhaps a lobed vortex ring exhibiting the well-known Widnall instability, all hidden in the picture on the left.

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