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Unraveling unparticles through violation of atomic parity and rare beauty

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Abstract

We put constraints on unparticle physics, specifically on the scale $\Lambda_{\mathcal{U}}$ and the scale dimension $d_{\mathcal{U}}$ of unparticle operators, using (i) measurements of atomic parity violation as well as (ii) branching ratio and CP asymmetry measurements in some rare non-leptonic B decay channels.

I Introduction

The notion of 'unparticles', recently introduced by Georgi [1], constitutes a new window of physics beyond the Standard Model (SM), conceptually different from supersymmetry and/or extra dimensions. The latter scenarios are all about new particles more massive than what we have already encountered in the SM. This new idea stems from the assumption that there exists a scale invariant sector which couples to the SM. The origin of unparticles can be traced to the degrees of freedom of this hidden sector. We shall elaborate a bit on it later. For direct detection of unparticles one has to rely on missing energy signals or on distortions in momentum distributions. But even as propagators, unparticles exhibit some properties characteristically different from the usual expectations arising from new physics [2, 3]. As a result, interesting phenomena arise when an unparticle mediated tree graph interferes with a SM amplitude. In particular, a peculiar phase appears, only for time-like unparticle propagator, which depends on the scale dimension of the relevant unparticle operator.

In this Letter we study the novel aspects emerging from such unparticle exchange in two different sectors: (*i*) modification of parity-violating transitions in atoms (APV), specifically, Cesium and (*ii*) interference of flavor-violating unparticle mediated tree graph with the leading SM penguin diagrams in $B^\pm \to \pi^\pm K$ (quark level $b \to sd\bar{d}$) and $B_d \to \phi K_S$ (quark level $b \to ss\bar{s}$). While APV is mediated by space-like unparticle propagator, the unparticle propagator for B decays is time-like which gives rise to a CP-even (strong) phase. We have chosen to focus on these two B decay modes as they do not receive any tree level SM contributions, and hence provide interesting playgrounds for the competition between one-loop SM and tree level new physics contributions. The above mentioned strong phase associated with the unparticle propagator provides a potential source of large CP violation that may arise out of the above interferences in B decays. Conversely, a zeroish CP asymmetry, in addition to branching ratio measurements, would put strong constraints on unparticle parameters.

The main idea of unparticles is the following. Let us assume that at a very high energy scale there exists a non-trivial scale invariant sector with an infrared fixed point, whose fields will be called the BZ fields (as first studied by Banks and Zaks [4]). The SM sector interacts with the BZ sector by the exchange of heavy messenger particles (of mass scale $M_{\mathcal{U}}$) leading to non-renormalizable operators $\mathcal{O}_{\rm SM}\mathcal{O}_{\rm BZ}/M_{\mathcal{U}}^k$. The renormalizable couplings of the BZ fields then induce dimensional transmutation [5] leading to a scale $\Lambda_{\mathcal{U}}$. Below this scale the BZ operators with scale dimension $d_{\rm BZ}$ match to the so-called unparticle operator $O_{\mathcal{U}}$ of scale dimension $d_{\mathcal{U}}$ yielding an effective operator $\lambda\mathcal{O}_{\rm SM}\mathcal{O}_{\mathcal{U}}$ with a dimensionless coefficient $\lambda \sim \Lambda_{\mathcal{U}}^{d_{\rm BZ}-d_{\mathcal{U}}}/M_{\mathcal{U}}^k$. Since the hypothetical conformal BZ sector is self-interacting, $d_{\mathcal{U}}$ can be fractional as well. Now, scale invariance in that sector implies that $O_{\mathcal{U}}$ does not correspond to conventional particles, but rather describes unparticles symbolizing a continuous mass spectrum[1]. A sizeable body of literature covering various aspects of unparticle dynamics already exists [7].

The unparticles could, in principle, have any spin structure. However, with the mass spectrum being continuous and extending to zero, one would presume that any $\mathcal{O}_{\mathcal{U}}$ would be a singlet under the SM gauge group so as to avoid profuse production of unparticles whose detection would have been unavoidable. In this Letter, we restrict ourselves only to a discussion of *vector* unparticles. Their couplings to the fermion currents be parametrized as

$$\mathcal{L} = \Lambda_{\mathcal{U}}^{1-d_{\mathcal{U}}}\ \bar{f}'\,\gamma_\mu \left[a_L^{ff'}\ (1-\gamma_5) + a_R^{ff'}\ (1+\gamma_5)\right]\ f\ \mathcal{O}_{\mathcal{U}}^\mu\ , \tag{1}$$

where $\mathcal{O}_{\mathcal{U}}^\mu$ is a transverse and Hermitian operator of dimension $d_{\mathcal{U}} > 1$. Using scale invariance, the $\mathcal{U}$-propagator for time-like P^2 can be determined to be [2, 3]

$$\begin{aligned}\int e^{iPx}\ \langle 0|\,T(\mathcal{O}_{\mathcal{U}}^\mu(x)\,\mathcal{O}_{\mathcal{U}}^\nu(0))\,|0\rangle\ d^4x &= \frac{i}{2}\,A_{d_{\mathcal{U}}}\,\frac{-g^{\mu\nu}+P^\mu\,P^\nu/P^2}{\sin(d_{\mathcal{U}}\,\pi)}\,\frac{1}{(P^2)^{2-d_{\mathcal{U}}}}\,e^{i\theta_{\mathcal{U}}}\ ,\\ \text{where}\ \ A_{d_{\mathcal{U}}} &\equiv \frac{16\,\pi^{5/2}}{(2\,\pi)^{2\,d_{\mathcal{U}}}}\,\frac{\Gamma(d_{\mathcal{U}}+\frac{1}{2})}{\Gamma(d_{\mathcal{U}}-1)\,\Gamma(2\,d_{\mathcal{U}})}\ ,\quad \theta_{\mathcal{U}} = -d_{\mathcal{U}}\pi.\end{aligned} \tag{2}$$

For space-like P^2, the propagator is real, i.e. there is no such phase $\theta_{\mathcal{U}}$. The expression of $A_{d_{\mathcal{U}}}$ highlights that unparticle matter corresponds to a stream of $d_{\mathcal{U}}$ number of massless particles, where the peculiarity is that $d_{\mathcal{U}}$ can even be a fraction. Eq. (1), then, represents the effective interaction Lagrangian, and, along with Eq. (2), defines the new physics beyond the SM. In the spirit of effective theories, we shall consider the coefficients $a_{L,R}^{ff'}$ to be typically order unity. Note that, in general, both flavor diagonal and nondiagonal unparticle couplings to fermions may exist. However, a crucial observation is that flavor nonconserving vertices can lead to decay processes such as $f' \to f + \mathcal{U}$ and, for such couplings, one has to satisfy $d_{\mathcal{U}} > 2$ to avoid divergence of the decay width arising from enhanced density of states in the low P^2 regime[2]. Clearly, for the calculation of APV we need flavor diagonal unparticle current, while for $B^\pm \to \pi^\pm K$ and $B_d \to \phi K_S$ to be induced at the tree level we have to employ flavor non-diagonal unparticle vertices.

II Atomic Parity Violation (APV)

Improved measurements of APV, mainly in Cesium ($^{133}_{55}$Cs), have led to bounds on different incarnations of physics beyond the SM (e.g. leptoquark, R-parity violating supersymmetry, additional Z',

[1]It has been demonstrated [6] that the unparticle can be deconstructed as the limiting case of an infinite tower of particles of different masses with a regular mass spacing. This demonstration implies a conceptual connection between unparticles and extra dimension.

[2]Choudhury, Ghosh amd Mamta in [7].

extra dimensions, etc [8]). Such bounds are comparable in size to the ones obtained from high energy collider experiments. In this section we wish to explore how the exchange of vector unparticles leads to parity-violating effects in Cesium in addition to those already present within the SM.

For a given atom, the parity-violating electron-nucleus effects largely accrue from the combination of the Z-boson's axial coupling to the electron and vector couplings to the quarks within the nucleus. It is conventionally parametrized in terms of the weak charge of the nucleus $Q_W(Z,N)$. At the atomic energy scale (~ 1 MeV) the quarks inside the nucleus act coherently and Q_W is expressed as the coherent sum of the neutral current charges of the $(2\,Z+N)$ up-quarks and the $(2\,N+Z)$ down-quarks in the nucleus under question. While the derivation of the effect within the SM can be found in the literature [9], we briefly review it here, both for the sake of completeness as well as to establish the framework for the corresponding derivation for the case of the unparticle.

The SM and vector unparticle mediated electron-quark interaction can be expressed in the current-current form as

$$\begin{aligned}\mathcal{L}_{eeqq} &= \frac{e^2\,e_q}{P^2}\,[\bar e\,\gamma_\mu\,e]\;[\bar q\,\gamma^\mu\,q] + \frac{g^2}{4\,c_W^2}\,[\bar e\,\gamma_\mu\,(v_e + a_e\,\gamma_5)\,e]\;[\bar q\,\gamma^\mu\,(v_q + a_q\,\gamma_5)\,q]\;(P^2 - M_Z^2)^{-1} \\ &+ \;[\bar e\,\gamma_\mu\,(\mathcal{V}_e + \mathcal{A}_e\,\gamma_5)\,e\,]\;[\bar q\,\gamma^\mu\,(\mathcal{V}_q + \mathcal{A}_q\,\gamma_5)\,q\,]\;\frac{A_{d_\mathcal{U}}}{2}\;\Lambda_\mathcal{U}^{2-2\,d_\mathcal{U}}\,\frac{(P^2)^{d_\mathcal{U}-2}}{\sin\,(d_\mathcal{U}\,\pi)}\,,\end{aligned} \tag{3}$$

where

$$\begin{aligned} v_f &\equiv T_{3f} - 2\,s_W^2\,e_f, & a_f &\equiv -T_{3f}, \\ \mathcal{V}_f &\equiv a_R^{ff} + a_L^{ff}, & \mathcal{A}_f &\equiv a_R^{ff} - a_L^{ff}. \end{aligned} \tag{4}$$

Notice that flavor diagonal unparticle mediation with quarks at one end and electrons at the other involves space-like momentum transfer, and hence the propagator does not involve any strong phase ($\theta_\mathcal{U} = 0$).

The parity-violating part of the potential in the non-relativistic limit arising purely from the Z-boson exchange is given by

$$V_{\rm PV}^{(q)}\;({\rm SM}) \;=\; \frac{g^2}{4\,M_W^2}\;a_e\,v_q\;\left[2\pi^2 M_Z^2\,\frac{e^{(-M_Z\,r)}}{r}\right]\;[\vec\sigma_e\cdot\vec{\mathbf{v}}_e]\;\approx\;\sqrt{2}\,G_F\;a_e\,v_q\;\delta^3(\vec{\mathbf{r}})\;[\vec\sigma_e\cdot\vec{\mathbf{v}}_e]\,, \tag{5}$$

where the approximation in the last step is well justified as M_Z^{-1} is infinitesimal in comparison to the atomic length scale. Summing coherently the effects of all quarks in the nucleus, one can parametrize the APV effect in terms of the weak charge Q_W of the nucleon which appears in the parity-violating part of the potential of the whole nucleus as

$$V_{\rm PV}\;({\rm SM}) \;=\; \frac{G_F}{2\sqrt{2}}\,Q_W^{\rm SM}\,\delta^3(\vec{\mathbf{r}})\;[\vec\sigma_e\cdot\vec{\mathbf{v}}_e]\,, \tag{6}$$

and, to the leading order in electroweak theory[3], reads

$$Q_W^{\rm SM} = 2\;[(2\,Z+N)\,v_u + (Z+2\,N)\,v_d] = -N + (1-4\,s_W^2)\,Z\,. \tag{7}$$

[3]Radiative corrections to $Q_W^{\rm SM}$ have been calculated [10].

We now estimate the unparticle contribution to Q_W. A straightforward computation yields the following term for the parity-violating potential

$$
\begin{aligned}
V_{\rm PV}(\mathcal{U}) &= \frac{1}{\pi\,\Lambda_{\mathcal{U}}^2}\,\mathcal{A}_e\,\mathcal{V}_q\,A_{d_{\mathcal{U}}}\,\Gamma(2\,d_{\mathcal{U}}-2)\,\left\{2\pi^2\Lambda_{\mathcal{U}}^2\,(\Lambda_{\mathcal{U}}r)^{2-2\,d_{\mathcal{U}}}\,\frac{1}{r}\right\}\,[\vec{\sigma}_e\cdot\vec{\mathbf{v}}_e] \\
&\approx \frac{1}{\pi\,\Lambda_{\mathcal{U}}^2}\,\mathcal{A}_e\,\mathcal{V}_q\,A_{d_{\mathcal{U}}}\,\Gamma(2\,d_{\mathcal{U}}-2)\,\delta^3(\vec{r})\,[\vec{\sigma}_e\cdot\vec{\mathbf{v}}_e]\ .
\end{aligned}
\tag{8}
$$

With the cutoff $\Lambda_{\mathcal{U}} > M_Z$, the delta function approximation above has been done analogously to (but not *exactly* as) the SM derivation in Eq. (5). The above derivation is valid as long as $1 \leq d_{\mathcal{U}} \leq \frac{3}{2}$. Outside this range, the calculation of $V_{\rm PV}(\mathcal{U})$ needs the introduction of a regulator, which may bring in additional model and scheme dependence[4]. As regards the lower limit, we recall that the very description of the unparticle physics already restricts us to $d_{\mathcal{U}} > 1$. Note that, unlike in the case of many collider based observables, the factor $\sin(d_{\mathcal{U}}\,\pi)$ in the unparticle propagator cancels exactly in the expression of the potential, thus eliminating any sharp behavior as $d_{\mathcal{U}}$ approaches integral values. Expressing the total (SM + $\mathcal{U}$) contribution to APV in the form of Eq. (6) by replacing $Q_W^{\rm SM}$ with $Q_W^{\rm tot} = Q_W^{\rm SM} + \delta Q_W(\mathcal{U})$, we have

$$
\delta Q_W(\mathcal{U}) = \frac{8}{\Lambda_{\mathcal{U}}^2\,G_F}\,\frac{(2\pi)^{3/2-2\,d_{\mathcal{U}}}\,\Gamma(d_{\mathcal{U}}+\frac{1}{2})}{\Gamma(d_{\mathcal{U}})\,(2\,d_{\mathcal{U}}-1)}\quad\mathcal{A}_e\,\left[(2\,Z+N)\,\mathcal{V}_u+(2\,N+Z)\,\mathcal{V}_d\right]\ . \tag{9}
$$

where the last factor in Eq. (9) is but a manifestation of the coherent superposition (constructive or destructive as the case may be).

As per the latest compilation in the Review of Particle Properties [11], the experimental constraint on Q_W of Cesium ($^{133}_{55}$Cs) and its SM prediction[5] are given by

$$
\begin{aligned}
Q_W(\text{Expt}) &= -72.62 \pm 0.46 \\
Q_W(\text{SM}) &= -73.17 \pm 0.03,
\end{aligned}
\tag{10}
$$

thus admitting a small room for new physics:

$$
\delta Q_W \equiv Q_W(\text{Expt}) - Q_W(\text{SM}) = 0.55 \pm 0.46\ . \tag{11}
$$

Using Eqs. (9) and (11), we may now derive numerical constraints on the unparticle physics parameter space. In Fig. 1, we display the lower bound on $\Lambda_{\mathcal{U}}$ as a function of $d_{\mathcal{U}}$. Since $\delta Q_W(\mathcal{U})$ depends on the products $\mathcal{A}_e\,\mathcal{V}_d$ and $\mathcal{A}_e\,\mathcal{V}_u$, we may, without any loss of generality, fix the value of one of these and we have chosen to normalize to $\mathcal{A}_e = 1$. Different combinations of $(\mathcal{V}_d, \mathcal{V}_u)$ then lead to differing constraints. For ease of presentation, we restrict ourselves to $\mathcal{V}_{u,d} = 0, \pm 1$. Thus, for our choice of $\mathcal{A}_e = 1$, a positive (negative) value of the the combination $[(2\,Z+N)\,\mathcal{V}_u + (2\,N+Z)\,\mathcal{V}_d]$ leads to a positive (negative) δQ_W. As the magnitude (and sign) of this combination crucially depends on those of $\mathcal{V}_{d,u}$, this is manifested in the relative differences in the bounds for the various choices in Fig. 1. Since the only dependence (approximately exponential) on $d_{\mathcal{U}}$ is in the pre-factor in Eq. (9), the shape

[4]A direct translation of APV limit from an effective contact interaction scale to the unparticle parameter space, as inferred in Cheung, Keung and Yuan in [7], is not so straightforward, since the implicit Fourier transformation with massless unparticles yields an additional $\Gamma(2d_{\mathcal{U}}-2)$ factor, see Eq. (8).

[5]While experimental measurements as well as theoretical analyses have been made for other atoms [11], the current uncertainties are too large to compete in sensitivity.

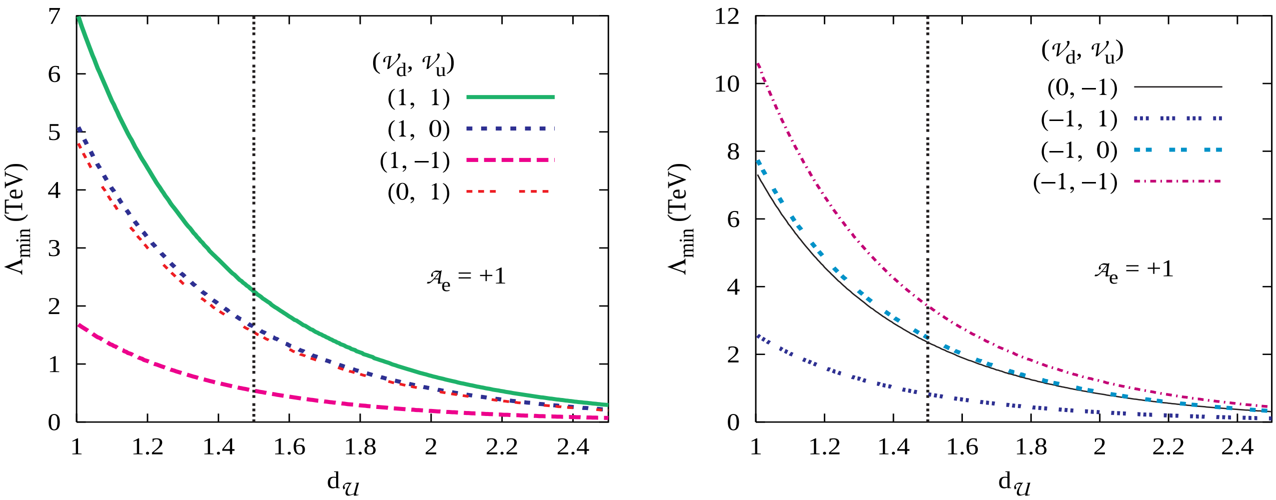

Figure 1: *The 3σ lower bound on $\Lambda_{\mathcal{U}}(\equiv \Lambda_{\rm min})$ as a function of $d_{\mathcal{U}}$ for various combinations of the unparticle's vector couplings to the u- and d-quark. The axial coupling to the electron has been held to unity. For the curves in the left (right) panel, the unparticle contribution δQ_W is positive (negative). The bounds to the right of the dotted vertical line corresponds to analytic continuation of Eq. (8) (see text and footnote).*

of the curves are easily understood. The extrapolation of curves in Fig. 1 beyond $d_{\mathcal{U}} = 1.5$ has been achieved by analytic continuation[6].

It should be noted that the current data prefers a small positive δQ_W. This, obviously, can be reproduced only for certain combinations of the couplings $\mathcal{A}_e$ and $\mathcal{V}_{u,d}$, e.g., those in the left panel of Fig. 1. Finally, we mention in passing that the unparticle contribution proportional to $\mathcal{V}_e\,\mathcal{A}_q$ probes nuclear spin, and hence the anapole moment, which we refrain from investigating here.

III Rare non-leptonic B decays

III.1 $B^\pm \to \pi^\pm K$

Thanks to the wealth of B-factory data, the $B^\pm \to \pi^\pm K$ decay mode could provide important clues to unparticle parameters, in particular, the CP-even strong phase $\theta_{\mathcal{U}}$ which is proportional to the scale dimension $d_{\mathcal{U}}$. We consider here flavor violating vector unparticle current to facilitate tree level unparticle mediated $b \to sq\bar{q}$ operator[7]. We specifically choose only those unparticle couplings which would contribute to $b \to sd\bar{d}$ for which there is no tree diagram in the SM. This corresponds to $B^\pm \to$

[6]By analytic continuation, we imply the existence of a regulator that makes Eq. (8) to be still useable beyond $d_{\mathcal{U}} = 3/2$ by effecting a smooth transition across the above boundary value.

[7]A correlated study of unparticle effects in $B \to \pi\pi$ and $B \to \pi K$ systems has been performed in Chen and Geng (arXiv:0706.0850 [hep-ph]) in [7], where flavor diagonal vector unparticle couplings are assumed. As a result, the leading new physics operators considered there are still penguins, with vector unparticle replacing the SM gauge boson. Obviously, such penguins would contribute both to $b \to sd\bar{d}$ and $b \to su\bar{u}$, leading to both neutral and charged B decays in all πK modes. On the contrary, we switch on unparticle flavor off-diagonal couplings as well which trigger just $b \to sd\bar{d}$ interaction leading to only $B^\pm \to \pi^\pm K$. A comparison of their results with ours is not so straightforward as Chen and

$\pi^{\pm}K$, the leading SM contribution coming from color-suppressed penguin operators. These penguins will interfere with tree level unparticle mediated graphs with appropriate couplings. Branching ratio and CP asymmetry measurements in this channel would enable us to constrain $\Lambda_{\mathcal{U}}$ and $d_{\mathcal{U}}$.

The SM effective Hamiltonian for $B^+ \to \pi^+ K$ is given by

$$H_{\rm eff}^{\rm SM}(\pi K) = \frac{G_F}{\sqrt{2}} |V_{tb}V_{ts}^*| \, C_{\pi K}^{\rm SM} \, f_K \, (m_B^2 - m_\pi^2) \, F_{B\pi} \left[1 + \rho \, e^{i\theta} \, e^{i\gamma} \right] , \tag{12}$$

where

$$\rho = \left| \frac{V_{ub}V_{us}^*}{V_{tb}V_{ts}^*} \right| , \qquad P \equiv \frac{m_{K^0}^2}{(m_s + m_d)(m_b - m_d)} , \qquad \gamma = {\rm Arg}(V_{ub}) \ .$$

The SM penguin operators are captured in the combination

$$C_{\pi K}^{\rm SM} = \frac{C_3}{3} + C_4 + P \left(\frac{2}{3} C_5 + 2\, C_6 - \frac{C_7}{3} - C_8 \right) - \frac{1}{2} \left(\frac{C_9}{3} + C_{10} \right) ,$$

where the Wilson coefficients C_3–C_{10} as well as the decay constant f_K and the form factor $F_{B\pi}$ may be found in Refs. [12,13]. The strong phase θ arises from rescattering, and conservatively, is expected to be small due to α_s suppression [14]. We shall assume $\theta = 0$ for simplicity.

To be specific, we consider only vector unparticles, and switch on only the coefficients $a^{bs}_{L,R}$ and $a^{bd}_{L,R}$. These couplings will induce tree level unparticle mediated $b \to s d \bar{d}$. The total (SM + $\mathcal{U}$) effective Hamiltonian can now be written as

$$H_{\rm eff}^{\rm tot}(\pi K) = \frac{G_F}{\sqrt{2}} |V_{tb}V_{ts}^*| \, C_{\pi K}^{\rm SM} \, f_K \, (m_B^2 - m_\pi^2) \, F_{B\pi} \left[1 + \rho \, e^{i\theta} \, e^{i\gamma} + \rho_{\mathcal{U}}^{\pi K} \, e^{i\theta_{\mathcal{U}}} \, e^{i\gamma_{\mathcal{U}}} \right] , \tag{13}$$

where

$$\begin{aligned} \rho_{\mathcal{U}}^{\pi K} &= \frac{-\mathcal{C}}{|V_{tb}V_{ts}^*| C_{\pi K}^{\rm SM}} \left(\frac{m_b^2}{\Lambda_{\mathcal{U}}^2} \right)^{d_{\mathcal{U}}-1} \frac{A_{d_{\mathcal{U}}}}{2 \sin(\pi d_{\mathcal{U}})} \left(\frac{\sqrt{2}}{G_F m_b^2} \right) , \\ \text{and} \quad \mathcal{C} &= \left[\left(a_L^{bd} a_L^{sd} - a_R^{bd} a_R^{sd} \right) + 2P \left(a_L^{bd} a_R^{sd} - a_R^{bd} a_L^{sd} \right) \right] \\ &+ N_c^{-1} \left[\left(a_L^{bs} a_L^{dd} - a_R^{bs} a_R^{dd} \right) + 2P \left(a_L^{bs} a_R^{dd} - a_R^{bs} a_L^{dd} \right) \right] . \end{aligned} \tag{14}$$

Above, $\gamma_{\mathcal{U}}$ is a possible CP-odd weak phase associated with the combination $\mathcal{C}$ of the unparticle couplings. The unparticle propagator is time-like in this case and gives rise to a CP-even strong phase $\theta_{\mathcal{U}} = -d_{\mathcal{U}}\pi$. It is worth recalling that the generation of a CP asymmetry requires both a weak phase difference and a strong phase difference between the two interfering amplitudes. Within our assumption of $\theta = 0$, the SM amplitude contributes only to the weak phase (γ). The unparticle amplitude, on the other hand, provides not only a strong phase ($\theta_{\mathcal{U}}$), but also has the potential of contributing to the weak phase ($\gamma_{\mathcal{U}}$). In this respect, unparticle physics scores over several other forms of new physics in the sense that it not only generates a tree level amplitude for $b \to s d \bar{d}$ (which R-parity violation also does) but also provides a sizable strong phase of a *different* origin. The expression for CP asymmetry ($\equiv$ [Br $(B^+ \to f) -$ Br $(B^- \to \bar{f})$]/[Br $(B^+ \to f) +$ Br $(B^- \to \bar{f})$], with $f \equiv \pi^+ K$)

Geng (arXiv:0706.0850 [hep-ph]) in [7] display results for $d_{\mathcal{U}} < 2$, while we are constrained to take $d_{\mathcal{U}} > 2$ since we deal with flavor non-diagonal unparticle couplings (see Introduction).

now reads ($\rho_\mathcal{U} \equiv \rho_\mathcal{U}^{\pi K}$)

$$A_{\rm CP}^{\rm dir} = \frac{2\rho\sin\theta\,\sin\gamma + 2\rho_\mathcal{U}\sin\theta_\mathcal{U}\,\sin\gamma_\mathcal{U} + 2\rho\rho_\mathcal{U}\sin(\theta-\theta_\mathcal{U})\,\sin(\gamma-\gamma_\mathcal{U})}{1+\rho^2+\rho_\mathcal{U}^2+2\rho\cos\theta\,\cos\gamma+2\rho_\mathcal{U}\cos\theta_\mathcal{U}\,\cos\gamma_\mathcal{U}+2\rho\rho_\mathcal{U}\cos(\theta-\theta_\mathcal{U})\,\cos(\gamma-\gamma_\mathcal{U})}$$
$$\approx \frac{2\rho_\mathcal{U}\sin\theta_\mathcal{U}\,[\sin\gamma_\mathcal{U}-\rho\,\sin(\gamma-\gamma_\mathcal{U})]}{1+\rho^2+\rho_\mathcal{U}^2+2\rho\,\cos\gamma+2\rho_\mathcal{U}\cos\theta_\mathcal{U}\,[\cos\gamma_\mathcal{U}+\rho\,\cos(\gamma-\gamma_\mathcal{U})]}\,. \quad (15)$$

where the approximate equality follows from the (excellent) approximation of $\theta = 0$. It should be noted that the CP asymmetry is sizable when (i) the magnitudes of the interfering amplitudes are roughly of the same size ($\rho_\mathcal{U} \sim 1$, we checked, just below $d_\mathcal{U} = 2.1$), and (ii) the weak and strong phase differences between the interfering amplitudes are large.

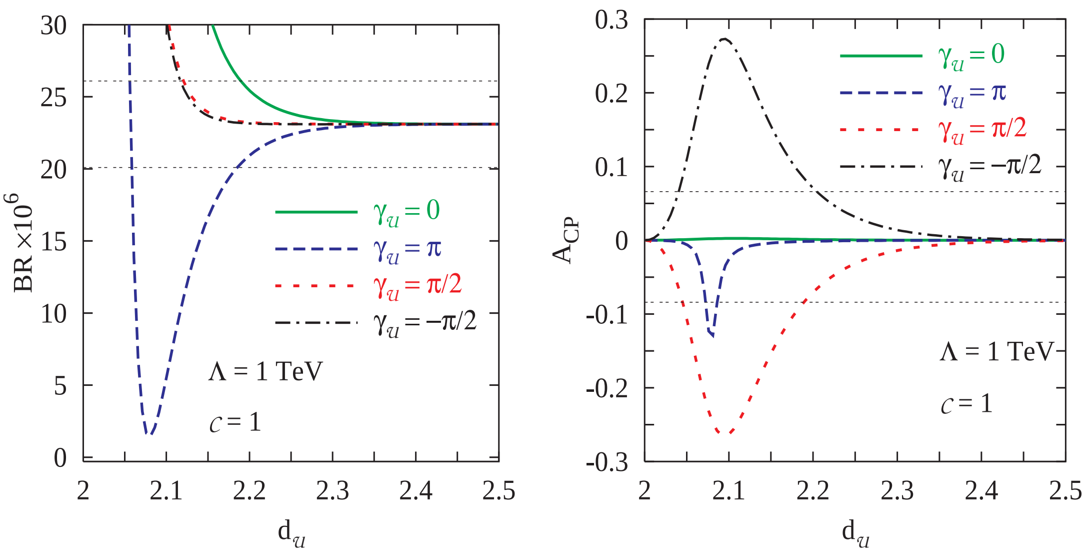

Figure 2: *The branching ratio for $B^+ \to \pi^+ K$ decays (left panel) and the direct CP asymmetry (right panel) as a function of $d_\mathcal{U}$ for $\Lambda_\mathcal{U} \equiv \Lambda = 1$ TeV and $\mathcal{C} = 1$ (see Eq. (14)). Also shown are the 3σ experimental bounds [13].*

In Fig. 2(left panel), we display the branching ratio Br ($B^+ \to \pi^+ K$) as a function of the scaling dimension $d_\mathcal{U}$. For definiteness, we set the combination $\mathcal{C}$ of the unparticle coupling constants—see Eq. (14)— to unity, the scale of the operator to 1 TeV, and the SM weak phase $\gamma = 63°$. The dependence on $\mathcal{C}$ and $\Lambda_\mathcal{U}$ is rather trivial. As the left panel of Fig. 2 shows, for $\gamma_\mathcal{U} = 0$, and $\pm\pi/2$, the branching fraction is a monotonic function of $d_\mathcal{U}$ and, for $\Lambda_\mathcal{U} = 1$ TeV, becomes indistinguishable from the SM value when $d_\mathcal{U} \gtrsim 2.3$ and $d_\mathcal{U} \gtrsim 2.2$ respectively. While the exact value of the SM expectations has a considerable dependence on the hadronic matrix elements, we assume here the central value as given in Ref. [12] which is quite close to the observed central value [13]:

$$\mathrm{Br}(B^\pm \to \pi^\pm K) = (23.1 \pm 1.0) \times 10^{-6}\,. \quad (16)$$

For the assumed benchmark value of $\Lambda_\mathcal{U}$ and for $\gamma_\mathcal{U} = 0$, consistency with the observed branching ratio would rule out $d_\mathcal{U} < 2.2$ at the 3σ level. For larger (smaller) $\Lambda_\mathcal{U}$, the curve moves to the left (right). For example, $\Lambda_\mathcal{U} = 10$ TeV is consistent with observations down to $d_\mathcal{U} = 2.025$.

If we consider $\gamma_\mathcal{U} = \pi$ (which still leaves the unparticle amplitude bereft of a weak phase), an interesting feature develops. Owing to the destructive interference between the SM and the unparticle amplitudes, the partial width now develops a minimum, and consequently, two disjoint ranges of $d_{\mathcal{U}}$ are now consistent with the data for a given value of $\Lambda_{\mathcal{U}}$ and the coupling combination $\mathcal{C}$. As far as the partial width is concerned, the two extreme cases $\gamma_{\mathcal{U}} = 0$ and $\gamma_{\mathcal{U}} = \pi$ constitute the envelopes of the effect of unparticle exchange, with those for any other choice of $\gamma_{\mathcal{U}}$ falling in between (see Fig. 2(left panel)). And, as in the case for $\gamma_{\mathcal{U}} = 0$, all such curves move to the left as $\Lambda_{\mathcal{U}}$ is increased or $|\mathcal{C}|$ is decreased.

The introduction of a weak phase in the unparticle couplings has a much more dramatic effect in the expectations for the direct CP asymmetry $A_{\rm CP}^{\rm dir}$. The measured value is the following [13]:

$$A_{\rm CP}^{\rm dir} = -0.009 \pm 0.025 \; . \tag{17}$$

Again, we assume the experimental central value as the SM expectation. As Fig. 2(right panel) shows, for $\gamma_{\mathcal{U}} = 0$, the CP-asymmetry is almost indistinguishable from the SM value. A sharp peak in $A_{\rm CP}^{\rm dir}$ shows up for $\gamma_{\mathcal{U}} = \pi$ which corresponds to a region of the parameter space that would lead to too large a value for the branching ratio. A non-trivial value for $\gamma_{\mathcal{U}}$, on the other hand, could lead to a significant enhancement in $A_{\rm CP}^{\rm dir}$ while maintaining consistency with the observed partial width. As expected, the cases $\gamma_{\mathcal{U}} = \pi/2$ and $\gamma_{\mathcal{U}} = -\pi/2$ provide the envelope for $A_{CP}^{\rm dir}$. With an increase in $\Lambda_{\mathcal{U}}$ (or, equivalently, a decrease in $\mathcal{C}$), the deviation from the SM value decreases, while the loci of the extrema move to the left. For example, $\Lambda_{\mathcal{U}} = 10$ TeV results in the maximum magnitude of $A_{CP}^{\rm dir}$ being reduced to $-0.01\,(+0.014)$ for $\gamma_{\mathcal{U}} = \pm\pi/2$ respectively.

III.2 $B \to \phi K_S$

The quark level process for this channel is $b \to s s \bar{s}$. This is dominated by a single amplitude in the SM and the leading contribution comes again from penguin operators. The SM effective Hamiltonian is given by [12]

$$\begin{aligned} H_{\rm eff}^{\rm SM}(\phi K) &= \frac{G_F}{\sqrt{2}} \, |V_{tb} V_{ts}^*| \, C_{\phi K}^{\rm SM} \, f_\phi \, F_{BK} \, \lambda(m_B^2, m_\phi^2, m_K^2), \quad \text{where} \\ C_{\phi K}^{\rm SM} &= (C_3 + C_4)\left(1 + \frac{1}{N_c}\right) + C_5 + \frac{C_6}{N_c} - \frac{1}{2}\left[C_7 + \frac{C_8}{N_c} + (C_9 + C_{10})\left(1 + \frac{1}{N_c}\right)\right], \end{aligned} \tag{18}$$

with $\lambda(x,y,z) = \sqrt{x^2 + y^2 + z^2 - 2xy - 2yz - 2zx}$. Once again, we consider only a vector unparticle following the structure of Eq. (1), but this time we switch on only the $a_{L,R}^{bs}$ and $a_{L,R}^{ss}$ couplings and assume them to be real. The total (SM + $\mathcal{U}$) Hamiltonian is then given by

$$\begin{aligned} H_{\rm eff}^{\rm tot} &= H_{\rm eff}^{\rm SM}(1 + \rho_{\mathcal{U}}^{\phi K} \, e^{i\theta_{\mathcal{U}}}) \; , \quad \text{where} \\ \rho_{\mathcal{U}}^{\phi K} &\equiv -\left(1 + \frac{1}{N_c}\right) \frac{\mathcal{D}}{|V_{tb} \, V_{ts}^*| \, C_{\phi K}^{\rm SM}} \, \frac{A_{d_{\mathcal{U}}}}{2\sin(\pi d_{\mathcal{U}})} \left(\frac{m_b^2}{\Lambda_{\mathcal{U}}^2}\right)^{d_{\mathcal{U}}-1} \left(\frac{\sqrt{2}}{G_F m_b^2}\right) , \\ \text{and} \;\; \mathcal{D} &\equiv a_L^{bs} a_L^{ss} + a_L^{bs} a_R^{ss} + a_R^{bs} a_L^{ss} + a_R^{bs} a_R^{ss} \; . \end{aligned} \tag{19}$$

In Fig. 3, we display the effect of unparticle exchange on this partial width. Once again, we use hadronic matrix elements, as given in [12], and compare with the experimental measurement [13], namely

$$\text{Br}\,(B_d^0 \to \phi K_S) = \left(8.3^{+1.2}_{-1.0}\right) \times 10^{-6} \; . \tag{20}$$

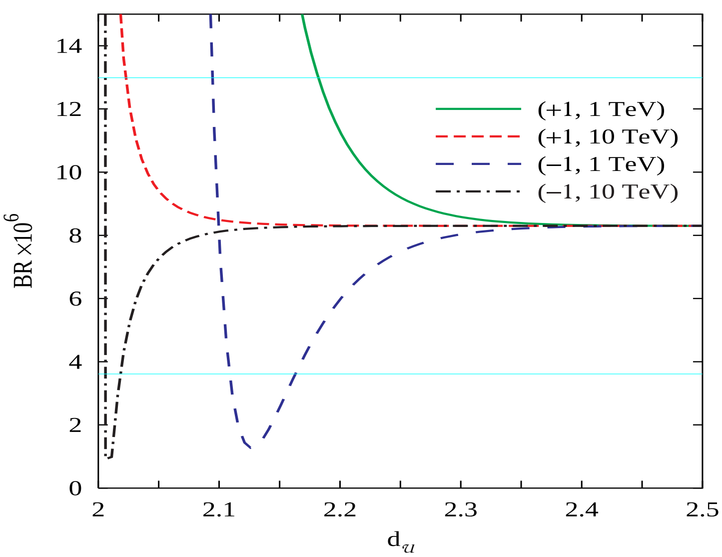

Figure 3: *The branching ratio for* $B_d^0 \to \phi K_S$ *decay as a function of* $d_{\mathcal{U}}$ *for different combinations of* $(\mathcal{D}, \Lambda_{\mathcal{U}})$ *(see Eq. (19)). Also shown are the* 3σ *experimental bounds [13].*

For a positive value of the coupling combination $\mathcal{D}$, the branching fraction decreases monotonically with $d_{\mathcal{U}}$. As expected, the deviation from the SM decreases with an increase in the scale $\Lambda_{\mathcal{U}}$ (a similar behavior would be seen if the magnitude of $\mathcal{D}$ were to decrease). A reversal of the sign of $\mathcal{D}$ renders the interference between the SM and the unparticle amplitudes destructive leading to the existence of a minimum. While we have restricted ourselves to real-valued unparticle couplings, it is easy to see that, for a complex unimodular $\mathcal{D}$, the corresponding branching fraction would lie in between the curves for $\mathcal{D} = \pm 1$ acting as envelope. A CP asymmetry in this channel provides an independent (from $B \to J/\Psi K_S$ mode) measurement of $\sin 2\beta$, but its experimental error is still too crude [15] to make it worthy of new physics probe. We note in passing that by selecting appropriate couplings and phases one can explain the current 2σ discrepancy between the values of $\sin 2\beta$ measured from these two modes.

IV Conclusions

In this note, we have explored the effect of vector unparticles as propagators in the atomic parity violating process as well as two rare non-leptonic B decay modes, namely, $B^{\pm} \to \pi^{\pm} K$ and $B \to \phi K_S$. In the APV process, the virtual unparticle propagator is space-like, while for B decays it is time-like, leading to an additional source of CP even strong phase which has a crucial impact on the phenomenology of B decays. Moreover, while APV offers a probe to the sensitivity of TeV scale physics through measurements at the ~ 1 MeV scale, B decays could provide clues to the TeV dynamics from measurements at a few GeV scale, so in a sense they provide complementary hunting grounds for new physics.

With the most precise information on APV coming from the Cesium ($^{133}_{55}$Cs) analysis (on account of accuracy in both experimental measurements as well as theoretical estimates), it can be used to obtain rather stringent conditions on the unparticle parameter space. While the derivation of our results strictly hold only for $d_{\mathcal{U}} \leq \frac{3}{2}$, they can be smoothly extended to larger $d_{\mathcal{U}}$ values as well. Thus, a measurement at around 1 MeV scale is shown to be sensitive to the unparticle scale all the way up to a few TeV. In particular, the discrepancy (admittedely small) between the experimental value and the SM expectations can be explained by turning an unparticle coupling to right-handed currents.

A flavor non-diagonal coupling of a vector unparticle with quarks provides additional tree-level contributions to both $B^\pm \to \pi^\pm K$ and $B_d \to \phi K_S$ decays. For either process, the time-like unparticle propagator gives rise to a CP even strong phase leading to a non-trivial contribution to the corresponding CP asymmetry. In addition to this strong phase, a possible weak phase $\gamma_{\mathcal{U}}$ may arise too if quark−unparticle couplings are complex. Thus, the experimental data for both the branching ratio of as well as CP asymmetry in the $B^\pm \to \pi^\pm K$ decays may be used to impose stringent constraints on the unparticle parameter space. As can be expected, such limits have a strong dependence on the value of the new weak phase $\gamma_{\mathcal{U}}$. Similar results obtain for the the $B_d \to \phi K_S$ decay process as well (although here the CP asymmetry is not a sensitive probe as of date).

We conclude by emphasizing two points that emerge from our analyses, and which are also shared by other authors studying other processes: (i) besides $\Lambda_{\mathcal{U}}$, the unparticle contribution to physical observables has a very strong dependence on $d_{\mathcal{U}}$, and (ii) flavor off-diagonal unparticle couplings, as expected, are more strongly constrained than the flavor diagonal ones.

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