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Anomalies, symmetries and strangeness content of the proton

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Abstract. The matrix elements of the operators of strange quark fields $\bar{s}\Gamma s$ where Γ is 1 or $\gamma_\mu\gamma^5$ between a proton state is calculated. The sigma term is found to be ≈41 MeV and the $SU(3)$ singlet axial matrix element is found to be ≈0.22, both in agreement with experiment. The sigma term is found using the trace anomaly, while the determination of the axial vector current matrix element is from QCD sum rules. These correspond to $\langle p|2\bar{s}s|p\rangle/\langle p|\bar{u}u+\bar{d}d|p\rangle \approx 0.12$ and for the axial current $\Delta s \approx -0.12$, respectively. The role of the anomalies in maintaining flavor symmetry in the presence of substantial differences in quark masses is pointed out. This suggests that there is no need to invoke an intrinsic strange quark component in the proton wave function.

Keywords. Strange quark; proton.

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1. Introduction

By strangeness content we mean the value of the matrix elements $\langle p|\bar{s}\Gamma s|p\rangle$, with $\Gamma = 1, \gamma_\mu, \gamma_\mu\gamma_5, \sigma_{\alpha\beta}, \gamma_5$. Here $|p\rangle$ refers to a proton state and s is the strange quark field. Naive application of OZI rule will suggest that all these matrix elements are zero. In this note I shall discuss only the scalar and the axial vector matrix elements, i.e. $\Gamma = 1$, $\gamma_\mu\gamma_5$. The scalar matrix element is closely connected with sigma term while the axial vector matrix element occurs in the Bjorken sum rule and is connected with the so-called proton spin puzzle.

2. Scalar matrix element

Several years ago Cheng [1] computed the ratio of strange quark to up- and down-quark matrix elements

$$y = \langle p|2\bar{s}s|p\rangle/\langle p|\bar{u}u+\bar{d}d)|p\rangle \qquad (1)$$

using the Gell–Mann–Okubo (GMO) mass formula and pion–nucleon scattering data as follows.

Define

$$\sigma \equiv \langle p|\hat{m}(\bar{u}u + \bar{d}d)|p\rangle, \quad \hat{m} \equiv (m_u + m_d)/2. \tag{2}$$

Write the strong interaction Hamiltonian as

$$H_{\text{strong}} = \hat{H}_0 + H_m, \tag{3}$$

where $\hat{H}_0$ is the QCD Hamiltonian obtained when all quark masses are set equal to zero and [2]

$$H_m = m_u\bar{u}u + m_d\bar{d}d + m_s\bar{s}s. \tag{4}$$

Let us ignore the up- and down-quark mass differences and set $m_u = m_d = \hat{m}$. We can then write the mass term as a singlet plus octet

$$H_m = \frac{1}{3}(m_s + 2\hat{m})(\bar{u}u + \bar{d}d + \bar{s}s) - \frac{1}{3}(m_s - \hat{m})(\bar{u}u + \bar{d}d - 2\bar{s}s). \tag{5}$$

From the Gell–Mann–Okubo mass formula then we derive

$$\langle p|\tfrac{1}{3}(m_s - \hat{m})(\bar{u}u + \bar{d}d - 2\bar{s}s)|p\rangle = m_\Xi - m_\Lambda. \tag{6}$$

Now use $m_s/\hat{m} = (2M_K^2/M_\pi^2) - 1 \approx 25$ to get

$$\sigma(1-y) = 26 \text{ MeV}. \tag{7}$$

To determine σ in eq. (2) one uses the pion–nucleon scattering data. Using low-energy theorems of current algebra, one has the relation

$$F_\pi^2 T^+\left(\nu = 0, t = 2m_\pi^2\right) \; = \; \sigma\left(t = 2m_\pi^2\right), \tag{8}$$

where T^+ is the isoscalar amplitude, ν is the energy and t is the momentum transfer. Using the phase shift data, sigma can be determined (see [3])

$$\sigma\left(t = 2m_\pi^2\right) \; \approx \; 60 \text{ MeV}. \tag{9}$$

If we neglect the t-dependence of sigma, then combining eq. (7) with eq. (9) we find a rather large value for $y \approx 0.57$ which more over would suggest that most of the nucleon's mass is contributed by the strange quark, a rather strange conclusion.

By now it is understood that there are important corrections to Cheng's discussion. First there are corrections to GMO mass formula which have been computed by Gasser and Leutwyler $(O(m_q^{3/2})$ [4] and by Borasoy and Meisner $(O(m_q^2))$ [5]. Sainio [6] has summarized these corrections as

$$\hat{\sigma} = \sigma(1-y) = 36 \pm 7 \text{ MeV}. \tag{10}$$

Cheng also ignored the fact that the scalar matrix element derived from the pion–nucleon data corresponds to a value of $t = 2m_\pi^2$ and should be continued to $t = 0$. This was done by Gasser *et al* [3], who obtained the value $\sigma = \sigma(t=0) = 45$ MeV.

Combining with eq. (10) one has $y \approx 0.2$ substantially smaller than Cheng's initial determination.

Here I shall give an update of an elementary determination [7] of σ and y using the trace anomaly. Let Θ_μ^ν be the energy–momentum tensor. Its trace has an anomaly [8]

$$\Theta_\alpha^\alpha = -\left(11 - \frac{2}{3}N_\mathrm{F}\right)\frac{\alpha_s}{8\pi}G^a_{\mu\nu}G^{a\mu\nu} + m_u\bar{u}u + m_d\bar{d}d + m_d\bar{d}d + m_s\bar{s}s + m_c\bar{c}c + m_b\bar{b}b + m_t\bar{t}t. \tag{11}$$

We have the equation for the mass

$$\langle p|\Theta_\alpha^\alpha|p\rangle = m_p, \tag{12}$$

which is an exact relation. (This follows from writing the most general matrix element of $\Theta_{\mu\nu}$ and using the fact that Θ_{00} is the Hamiltonian density.) Now by the decoupling theorems, one expects the proton mass to be unchanged if we remove a heavy quark from the QCD Lagrangian [9]. Writing

$$G^2 = \frac{\alpha_s}{8\pi}\langle p|G^a_{\mu\nu}G^{a\mu\nu}|p\rangle \tag{13}$$

decoupling of heavy quarks in eq. (11) leads to:

$$2/3G^2 + m_h\langle p|\bar{h}h|p\rangle = 0 \quad \text{for } h = t,b,c. \tag{14}$$

Suppose we assert that the strange quark also decouples in a similar manner, then we obtain

$$m_s\langle p|\bar{s}s|p\rangle = -2/3G^2 \tag{15}$$

from which using eq. (1) we get

$$\sigma\cdot y = -\frac{4}{3}\frac{\hat{m}}{m_s}\cdot G^2. \tag{16}$$

We can now write

$$\sigma(1-y) = \sigma + 4/75\cdot G^2 = 36\ \text{MeV}. \tag{17}$$

On the other hand, using eq. (11) we have from the nucleon mass

$$\sigma - 29/3G^2 = 938\ \text{MeV}. \tag{18}$$

Solving for G^2 we find

$$\sigma \approx 41\ \text{MeV} \tag{19}$$

and

$$G^2 \approx -93\ \text{MeV}.$$

We get a value of $y \approx 0.12$, which shows that strange quark matrix element is non-zero by virtue of its coupling to the gluon field. These values of σ and y are consistent with those of ref. [6] within errors. It would be surprising if strange quark decouples precisely according to eq. (15). More precise determination of y will decide whether strange quark in fact decouples exactly like heavy quarks or not.

3. Axial vector matrix element

The first point to note is that the Ellis–Jaffe sum rule [10] is internally inconsistent. On the one hand it assumes the validity of flavor $SU(3)$ symmetry and on the other it sets the matrix element of strange quark to zero by a naive application of OZI rule. As pointed out by Gross *et al* [11], the large differences in the quark masses would lead to large violations of flavor symmetry in the Bjorken sum rule if the anomaly is neglected.

Bearing this in mind, the matrix element of the isoscalar combination

$$\langle p,s|\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d|p,s\rangle \tag{20}$$

was evaluated by Gupta *et al* [12] by the QCD sum rule method. The details can be found in [12]. Define

$$\langle p,s|\bar{q}\gamma_\sigma\gamma_5 q|p,s\rangle \equiv \Delta q(\mu^2)s_\sigma \quad \text{with } q=u,d,s, \tag{21}$$

where s_σ is the covariant spin vector and μ is the renormalization scale.

$$a(I=0;\mu^2)\ s_\sigma = (\Delta u + \Delta d)s_\sigma \tag{22}$$

$$a_8 s_\sigma = (\Delta u + \Delta d - 2\Delta s)s_\sigma \tag{23}$$

and the $SU(3)$ singlet

$$\Sigma(\mu^2)s_\sigma = (\Delta u + \Delta d + \Delta s)s_\sigma. \tag{24}$$

Gupta *et al* [12] found the value of iso-singlet in eq. (22) to be

$$a(I=0;\mu^2) \approx 0.35, \tag{25}$$

substantially different from $a_8 \approx 0.58$. It corresponds to $\Delta s \approx -0.12$ and for the $SU(3)$ flavor singlet

$$\Sigma(\mu^2) \approx 0.22. \tag{26}$$

As in all old QCD sum rule calculations, the numbers in ref. [12] were obtained using a low value of $\Lambda = 100$ MeV and $\mu = 500$ MeV. We shall return elsewhere to a thorough re-analysis of ref. [12] but, for the present, note that simple change of Λ to 250 MeV and μ to 1 GeV marginally increases the value of a.

We also note that experimentally Σ is substantially lower than a_8 which is in conflict with Ellis and Jaffe [10]. Anthony *et al* [13] determined Σ to be 0.18 ± 0.09, while Abe *et al* [14] arrived at an average value of 0.30. The difference arises in the extrapolation procedures used by them to determine the low x distribution functions.

4. Anomalies and flavor symmetry

As mentioned earlier, it was pointed out by Gross *et al* [11], that if the anomaly is ignored, one should expect large violations of isospin in the Bjorken sum rule. In particular, setting the anomaly to zero, they derived the relation

$$\begin{aligned}&\langle N|\partial_\mu\left[\bar{u}\gamma_\mu\gamma_5 u+\bar{d}\gamma_\mu\gamma_5 d\right]|N\rangle_{I=1}\\&\quad=\frac{m_u-m_d}{m_u+m_d}\langle N|\partial_\mu\left[\bar{u}\gamma_\mu\gamma_5 u-\bar{d}\gamma_\mu\gamma_5 d\right]|N\rangle_{I=1},\end{aligned}\tag{27}$$

where the subscript $I=1$ on the nuclear matrix elements denotes the difference between proton and neutron matrix elements, $m_u\neq m_d$ and further

$$(m_u-m_d)/(m_u+m_d)=O(1).\tag{28}$$

Equation (27) implies large violation of isospin invariance in Bjorken sum rule. The left-hand side of eq. (27) should be zero or nearly so if isospin is a good symmetry while the right-hand side is far from zero. This erroneous conclusion disappears if anomaly is taken into account, as shown in ref. [11].

Turning now to the scalar matrix element, again if we set the anomaly to zero we have

$$\langle p|\Theta^\alpha_\alpha|p\rangle=\langle p|m_u\bar{u}u+m_d\bar{d}d+m_s\bar{s}s|p\rangle=m_p.\tag{29}$$

Inserting the phenomenological numbers found by Sainio [6], $\sigma=45$ MeV and $y=0.2$. The left-hand side is seen to be just 158 MeV far short of the nucleon mass.

Alternately, without using the value of σ one can write equations identical to eq. (29) for proton for Σ^+ by replacing d by s and for Ξ^- by replacing u by s and solving the resulting equations for $\bar{u}u,\bar{d}d,\bar{s}s$ elements. One finds the term $m_u\bar{u}u+m_d\bar{d}d$ contributes 94 MeV while $m_s\bar{s}s$ contributes 894 MeV for the proton mass!

5. Conclusion

Anomalies play a crucial role in maintaining flavor symmetry. Since strange quark mass is still small by the scale of strong interactions, a substantial coupling to the nucleon is possible via the gluon field anomaly both in the scalar and axial vector case.

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