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MASS LOSS, LONG-PERIOD VARIABLES, AND THE FORMATION OF CIRCUMNEBULAR SHELLS

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ABSTRACT

We have found that the rate of mass loss $\dot{M}$ increases with an increase in the period of pulsation for Mira-type variables. This result suggests that the rate of mass loss is accelerated with time until a maximum value is reached before the ejection of the outer envelope. The matter from the continuous mass loss during the evolution of the star produces supersonic shock waves that sweep up the interstellar gas upon encountering the interstellar medium, so that a shell is formed. This phenomenon may account for the observations of extended regions of emission that surround planetary nebulae.

Subject headings: nebulae: planetary — stars: long-period variables — stars: mass loss

I. INTRODUCTION

Mass loss has been observed in early- as well as late-type stars. The rate of mass loss may depend on the luminosity L, the mass M, and the radius R as well as on chemical composition. As many late-type red giants are also known to be variable, the rate of mass loss may also depend on the period of pulsation Π or vice versa. The purpose of this paper is to examine the coupling of mass loss and pulsation as well as to investigate some of its consequences on the evolution of red giants and on the formation of circumnebular shells.

II. SECULAR PERIOD VARIATIONS AND MASS LOSS

Reimers (1975) recently found that the rate of mass loss $\dot{M}$ in late-type stars follows an empirical relation

$$\dot{M} = -A \frac{L_*}{g_* R_*} = -A \frac{L_* R_*}{M_*}, \qquad (1)$$

where $A = 4 \times 10^{-13}\, M_\odot\, \text{yr}^{-1}$, the asterisked quantities are in solar units, and g is the surface gravity.

As many cool stars are also variables, the possibility exists that the rate of mass loss may also depend on the period of pulsation. In Figure 1 we have plotted the rate of mass loss, derived from the observations of Gehrz and Woolf (1971), as a function of period for semiregular and Mira-type variables.

The data suggest that the rate of mass loss has a tendency to increase with the period of pulsation. The scatter of the data points in this figure is not surprising due to the uncertainty in the value of $\dot{M}$, where errors can be as large as a factor of 2 or 3. This follows because, first, relation (1) requires an accurate knowledge of both the stellar radius and mass; second, as these stars are known to show large-amplitude variations over a period, $\dot{M}$ probably depends on the phase of the observations; third, a multiplicity of stellar masses may exist in the sample observed by Gehrz and Woolf (1971); and last, differences in chemical

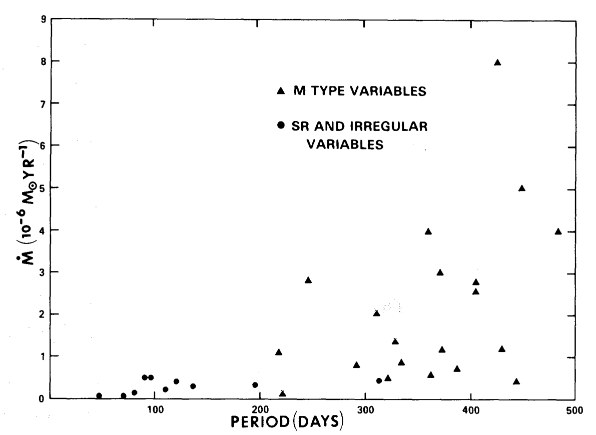

FIG. 1.—Rate of mass loss $\dot{M}$ from Gehrz and Woolf (1971) as a function of period. The periods adopted are from Kukarkin *et al.* (1969).

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1977ApJ...216..526K

composition may exist for stars with the same pulsation period. However, in spite of this scatter, which is probably produced by a combination of the above factors, a tendency for $\dot{M}$ to increase with period is evident.

A precise relationship between the visual period and mass-loss rate is difficult to formulate owing to the complex structure of an extended cool envelope and to the uncertainty in the nature of the driving force and mechanism of pulsation. However, we will assume that the period-density formula (Ritter's relation) is valid in order to obtain the pulsation period Π, so that

$$\Pi(\bar{\rho}/\bar{\rho}_{\odot})^{1/2} = Q = 0.03\text{–}0.08 \text{ days}, \quad (2)$$

where we have adopted the lower limit 0.03 for a highly condensed core appropriate to red giants (Cox and Giuli 1968). Here $\bar{\rho}$ and $\bar{\rho}_{\odot}$ denote the mean density of the star and the Sun, respectively. For $M_* \approx 1$ and $R_* \approx 500$, the period $\Pi \approx 330$ days (see discussion after relation [5]).

Equation (2) can also be written as

$$\Pi = Q(R_*^3/M_*)^{1/2}. \quad (3)$$

Differentiating the above equation with respect to time, we obtain

$$\dot{\Pi} = \tfrac{1}{2}\Pi\left(3\frac{\dot{R}}{R} - \frac{\dot{M}}{M}\right). \quad (4)$$

As the rate of mass loss $\dot{M} < 0$, $\dot{\Pi} > 0$ if $\dot{R} \geq 0$. To assume that $\dot{R}$ is zero over the evolutionary time scale of the star in the giant phase is a crude assumption, as $\dot{R}$ is most likely a positive quantity in the evolutionary phase under consideration. Hence, the assumption that $\dot{R} = 0$ in equation (4) provides a lower limit to the change in the period of pulsation.

Combining equations (1) and (3), and setting $L_* = R_*^2(T_e/T_{e,\odot})^4$, we can write

$$\dot{M} = -BT_e^4\Pi^2, \quad (5)$$

where

$$B = \frac{A}{T_{e,\odot}^4 Q^2},$$

and T_e and $T_{e,\odot}$ are the effective temperatures of the star and the Sun, respectively. As the evolution of the stars in the giant phase is basically parallel to the Hayashi track (cf. Iben 1974), with the effective temperature remaining essentially constant, the implication is that the rate of mass loss depends on the period of pulsation. The increase in the rate of mass loss with period may be somewhat reduced if the temperature is not constant but decreases slightly with evolution.

We can justify our use of Ritter's formula (3) for Π by using in equation (5) the values of $\dot{M}$ from Gehrz and Woolf (1971) to find the period of pulsation of Mira variables. Taking T_e appropriate to the spectral class of the star in relation (5) yields values of Π that are within a factor of 2 of the observed visual periods. Accordingly, our confidence that Ritter's simple relation is applicable to Mira variables is enhanced.

a) The Period Distribution of Mira Variables

The systematic variation of $\dot{M}$ with pulsation period supports the notion that red giants continuously evolve until a critical value of mass-loss rate is achieved. This notion suggests that the number distribution of Mira variables should decrease as the rate of mass loss and the corresponding period of pulsation increase. This statement is supported in part by Figure 2, which shows the distribution of pulsation period for 1030 Mira-type variables from Kukarkin *et al.* (1969). The median period lies between 250 and 300 days. The number of Miras declines with increase in visual period, and very few Miras are observed with periods in excess of 700 days. One should note that the period distribution above 700 days may be somewhat affected by observational selection. The decrease in the number of Miras as we go to periods shorter than 250 days is perhaps related to the definition of Miras vis-à-vis semiregular (SR) and short-period variables.

If $\dot{M}$ is an increasing function of time such that $\dot{M}$ approaches a critical value 10^{-5} to $10^{-4}\ M_{\odot}\ \text{yr}^{-1}$ (Sparks and Kutter 1972), then the ejection rate cannot persist at these high values for long periods of time. Härm and Schwarzschild (1975) have found that the ejection phase for stellar envelopes to form planetary nebulae lasts about 6000 years at mass-loss rates $\sim 7 \times 10^{-4}\ M_{\odot}\ \text{yr}^{-1}$.

That the period Π and its derivative $\dot{\Pi}$ are related to the rate of mass loss $\dot{M}$ in equations (4) and (5) suggests an increase in $\dot{M}$ during the evolution of red giants, as is indicated by Figure 1. Therefore, if $\dot{M}$ increases substantially for periods of pulsation greater than 300 days, the time spent at a particular period of pulsation will decrease progressively.

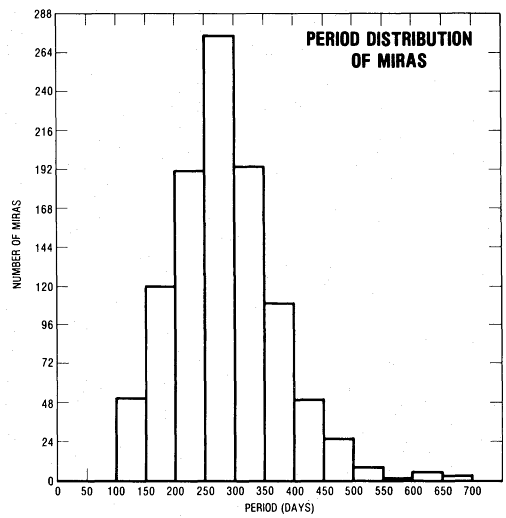

FIG. 2.—Distribution of Mira variables with period. This distribution consists of 1030 Mira-type variables sampled from Kukarkin *et al.* (1969).

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1977ApJ...216..526K

b) *Red-Giant Evolutionary Time Scales and the Formation of Circumnebular Shells*

It is of interest to compute the evolutionary time scale of red giants in the mass-loss phase. We can write equation (5) in the form

$$\dot{M} = -|\dot{M}_0|\left(\frac{\Pi}{\Pi_0}\right)^2 , \quad (6)$$

where $|\dot{M}_0|$ is a constant given by

$$|\dot{M}_0| = BT_{e,0}{}^4\Pi_0{}^2 . \quad (7)$$

Here $|\dot{M}_0|$ is the rate of mass loss when the period of pulsation is Π_0 and the effective temperature T_e is assumed equal to $T_{e,0}$. For $\Pi_0 \approx 275$ days (Fig. 1) and $T_{e,0} \approx 3000$ K, $|\dot{M}_0| = 2.4 \times 10^{-6}\ M_\odot\ \text{yr}^{-1}$; this value of $\dot{M}$ agrees remarkably well with the value suggested by Figure 1.

Assuming $\dot{R} = 0$, we obtain from equations (3), (4), and (6)

$$\ddot{M} = -|\dot{M}_0|M_0\left(\frac{\dot{M}}{M^2}\right) , \quad (8)$$

where M_0 is the initial mass of the star. Integrating equation (8), one finds

$$M^2(t) = M_0{}^2 - 2|\dot{M}_0|M_0 t . \quad (9)$$

Therefore, the time scale required to eject one-half of the stellar mass $t_{1/2} = 2 \times 10^5$ yr, if $\dot{M}_0 = 2 \times 10^{-6}\ M_\odot\ \text{yr}^{-1}$.

This value is actually an upper limit, as we have neglected the change in the stellar radius. It is difficult to estimate $\dot{R}$ precisely, owing to the numerous factors that complicate M-giant envelope models. However, if $\dot{R}/R$ is comparable to or greater than $\dot{M}/M$ in equation (4), the time scale will be reduced by a factor of 4 or more.

Therefore, if $t_{1/2}$ is reduced by an order of magnitude, we approach time scales required for the rapid ejection of planetary nebulae (cf. Abell and Goldreich 1966). Accordingly, we anticipate that as the mass-loss rate $\dot{M}$ accelerates to higher values toward later stages of evolution, we should approach the critical value $\dot{M} \approx 10^{-5}$ to $10^{-4}\ M_\odot\ \text{yr}^{-1}$ (Sparks and Kutter 1972) beyond which a star cannot exist at such high levels of mass loss for more than $\sim 10^4$ yr.

For an extended star with $R_* \approx 500$ and $\dot{M} \approx 2 \times 10^{-6}\ M_\odot\ \text{yr}^{-1}$, the luminosity (from eq. [1]) $L_* \approx 10^4$. However, current evolutionary models cannot accurately predict the maximum value of luminosity for red giants just prior to ejection of the outer envelope. Some bright supergiants in the double cluster h and χ Persei have been observed to have $L_* \approx 3 \times 10^5$ and $T_e \approx 2500$ K, values which would give $R_* \approx 3000$. Equation (1) implies a mass-loss rate of $4 \times 10^{-4}/M_*$ ($M_\odot\ \text{yr}^{-1}$). Accordingly, these stars may represent the phase just prior to ejection of the outer envelope by the star.

III. MASS LOSS AND THE INTERSTELLAR MEDIUM

It has been suggested for early-type stars that a high rate of mass loss, accompanied by high velocities, produces supersonic shock waves when the ejected stellar matter encounters the interstellar medium (Castor, McCray, and Weaver 1975). As many late-type stars are losing mass at comparable rates, with observed velocities ~ 10 km s^{-1} (Pottash 1970), such an effect should occur for M giants as well. Accordingly, we propose that in the vicinity of long-period variables or around planetary nebulae one should expect to find a region of compressed interstellar matter that has been created by the continuous mass loss from the star during its evolution in the giant phase prior to final ejection of the stellar envelope. In what follows we will discuss this idea without going into the details of the mechanism(s) of mass loss.

We can assume that the stellar wind velocity near the star is essentially equal to the escape velocity of the star, i.e., $V_{\text{wind}} \approx V_{\text{esc}} = 617(M_*/R_*)^{1/2}$ km s^{-1}, which is $\sim$10–100 km s^{-1} for red giants. If $V_{100} = V_{\text{esc}}/100$ km s^{-1}, $\dot{M}_5 = \dot{M}/10^5\ M_\odot\ \text{yr}^{-1}$, and the time scale $t_5 = t/10^5$ yr, the equations governing the motion of the expanding shell (cf. Avedisova 1972; Steigman, Strittmatter, and Williams 1975) can be written as

$$r_s = 3.5\left(\frac{\dot{M}_5 V_{100}}{n_0}\right)^{1/4} t_5{}^{1/2}\ \text{pc} , \quad (10)$$

$$V_s \approx 17\left(\frac{\dot{M}_5 V_{100}}{n_0}\right)^{1/4} t_5{}^{-1/2}\ \text{km s}^{-1} , \quad (11)$$

and

$$M_s \approx 6n_0{}^{1/4}(\dot{M}_5 V_{100})^{3/4} t_5{}^{3/2}\ M_\odot . \quad (12)$$

Here n_0 is the ambient number density of the interstellar medium and M_s is the mass of the swept-up moving shell. The shock front created when the mass expelled from the star encounters the interstellar medium is characterized by an expanding shell that gradually sweeps up the ambient interstellar material. Equations (10)–(12) are applicable only when the mass swept up from the interstellar medium exceeds the mass $\dot{M}t$ expelled from the star, i.e., when

$$t \geq 3 \times 10^3\left(\frac{\dot{M}_5}{n_0 V_{100}{}^3}\right)^{1/2}\ \text{yr} . \quad (13)$$

The shell will eventually come to rest when its velocity, given by relation (11), approaches the speed of sound (or turbulent speed) of the H I ambient medium. If, for example, $V_{100} = 0.3$, $\dot{M}_5 = 1$, $n_0 = 1$ cm^{-3}, and $V_s = 3$ km s^{-1}, then the time of dissipation of the shell is about 2×10^6 yr.

If we consider the thermal properties of this shell, the amount of radiation ionizing the shell depends on the amount of radiation absorbed by the ejected planetary nebula. If most of the energy from the star is absorbed within the planetary nebula, we expect a shell of neutral and/or molecular hydrogen to surround the planetary nebula. The number density of

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1977ApJ...216..526K

the shell is given by Castor, McCray, and Weaver (1975) as

$$n_s = \frac{\mu m_{\rm H} V_s^2 n_0}{kT_s} \ {\rm cm}^{-3} , \tag{14}$$

where μ is the mean molecular weight and T_s is the mean temperature of the shell. Eliminating V_s from the above equation by means of relation (11) and taking $T_s = 80$ K, one finds a density for H I or for H I and H_2,

$$n_s \approx 10^3 (n_0 \dot{M}_5 V_{100})^{1/2} t_5^{-1} \ {\rm cm}^{-3} . \tag{15}$$

The column density $N_s = n_0 r_s/3$ is then given by

$$N_s \approx 3.6 \times 10^{18} (n_0^3 \dot{M}_5 V_{100})^{1/4} t_5^{1/2} \ {\rm cm}^{-2} . \tag{16}$$

Equations (10)–(16) apply to those phases of giant evolution in which $\dot{M}$ does not vary appreciably. As planetary nebulae are observed to have typical masses of a few tenths of a solar mass, one is tempted to speculate on whether the swept-up interstellar gas can appear as a planetary nebula. With $\dot{M}_5 \approx 1$, $t_5 \approx 1$, $V_{100} \approx 0.3$, and $n_0 \approx 1$, we derive from equations (10)–(12) values of $r_s \approx 2.5$ pc, $V_s \approx 13$ km s^{-1}, and $M_s \approx 2.5 \ M_\odot$. The radius and mass of the shell are far larger than those *normally* obtained for planetary nebulae. Therefore, it is very likely that the mass of a planetary nebula arises from the dynamic instability of the star at the end of the long-period-pulsation phase rather than from the swept-up interstellar matter. Note that if the mass of the shell is predominantly ejected rather than swept-up matter, $\dot{M}_5$ is rapidly increasing on time scales shorter than those given by equation (13); accordingly, relations (10)–(12) and (14)–(16) do not apply.

A compressed shell formed by continuous mass loss from a giant star well ahead of the planetary nebula may be observable in H_2 and other molecular lines. Recently, Mufson, Lyon, and Marionni (1975) observed CO emission in three planetary nebulae: NGC 7027, IC 418, and NGC 6543. Treffers *et al.* (1976) observed three lines—2.12 μm, 2.4 μm, and 2.42 μm—of the 1–0 quadrupole spectrum of molecular hydrogen in NGC 7027. According to Mufson *et al.*, the CO emitting region in NGC 7027, with an associated mass of 1.4 $M_\odot$, is 4–6 times larger than the visual planetary nebula, whose mass is probably 0.2 $M_\odot$ (Allen 1973). These results give credence to our picture in which continuous mass loss leads finally to the rapid ejection of the planetary nebula. NGC 7027 may be unusual, however. Even though it is about as large, as dense, and as distant as IC 418 and NGC 6543 (Allen 1973), its CO emission is more pronounced by a factor of 20. In fact, the ^{13}CO column density of NGC 7027 implies a molecular hydrogen column density greater than $\sim 5 \times 10^{20}$ cm^{-2}. This nebula is also observed to have high extinction around 1 μm. On the other hand, the CO observations of IC 418 and NGC 6543 suggest column densities of hydrogen that are more in agreement with our formula (16).

In order that extended regions of emission outside the nebula show molecular lines including H_2, molecules will have to be shielded in the dense circumnebular shell from the background UV radiation field as well as from the UV photons that escape the planetary nebula. The normal dust that is found in the ambient interstellar gas cannot provide the necessary extinction from the ambient UV starlight in the column of material to protect the molecules. Appreciably more dust must be present in the form of very small grains. At least half of the observable planetary nebulae show an infrared excess, i.e., evidence for dust; but it is not known for certain what the properties of the dust grains are or where they are located. If the grains are indeed formed in the atmospheres of cool giants (cf. Aannestad and Purcell 1973), we should not be surprised to find a large amount of dust inside as well as outside the planetary nebula. Hence, grain formation in cool-giant atmospheres should be fairly efficient. The mean size of the dust grains in the atmospheres of red giants may be smaller than that in the interstellar medium and, therefore, more efficient at providing UV extinction.

Attempts should be made to observe extended regions around long-period variables as well as regions surrounding visual planetary nebulae in the radio and infrared in order to detect molecular emission. Such detection would strengthen our contention that the rapid ejection phase of planetary nebulae is preceded by a comparatively low rate of continuous mass loss and by the formation of a large amount of dust in the atmospheres of cool giants.

Several planetary nebulae have been observed with expansion velocities as large as 100 km s^{-1}, which perhaps is due to their lower-than-average values of density, a consequence of the sweeping effect of the interstellar medium. The large expansion velocities and the observations of Millikan (1974), which indicate that regions of very large angular dimensions surround several planetary nebulae, are best explained in terms of our picture.

Finally, even for optically thin planetary nebulae, the radiation capable of escaping the nebula will ionize the surrounding shell. Hence, estimates of the radiative flux that is available for ionizing the interstellar medium would be significantly less than have been made previously.

IV. CONCLUSIONS

A correlation appears to exist between the rates of mass loss and the period of pulsation for Mira-type variables. Undoubtedly, an exact formulation would require an in-depth treatment of the dynamics associated with extended stellar envelopes, including convection. The question of whether the photon flux drives the stellar wind through radiation pressure directly on grains (Wickramasinghe, Donn, and Stecher 1966; Kwok 1975), or whether the luminosity of the star is first converted to acoustic waves in the convective zone that subsequently propagate outward and expand the envelope (Fussi-Pecci and

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1977ApJ...216..526K

Renzini 1975), must be resolved before the dependence of pulsation on $\dot{M}$ can be understood in detail.

Mindful of the assumptions implicit in Ritter's formula, we have found that the period of pulsation increases with increasing rate of mass loss during the evolution of Mira variables. The precise behavior of $\dot{M}$ as a function of time, however, is unknown. Nevertheless, it is probably correct to assume that as the star becomes larger, its pulsation period increases for two reasons: (*a*) The characteristic travel time $2R/C_S$, where C_S is the speed of sound, increases as R increases. (*b*) The value of Q in Ritter's formula (2) appears to have a lower bound (Cox and Giuli 1968), and for long-period variables we are operating close to this lower limit. Hence, as R increases, $\bar{\rho}$ decreases, forcing Π to increase. Furthermore, the consequent decrease in surface gravity will make the outer envelope more susceptible to dynamic expansion, and the rate of mass loss will increase.

We feel that our picture of continuously accelerated mass loss during the evolutionary course of long-period variables is implied in the observations of red giants. Surveys of the molecular lines of planetary nebulae, infrared observations of extended regions of emission associated with planetary nebulae, and observations of large "halos" around visual planetary nebulae should further strengthen our contention that extended circumnebular regions of emission, and possibly planetary nebulae, originate from the mass loss associated with long-period variables.

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