Proc. Indian Acad. Sci., Vol. 85 A, No. 3, 1977, pp. 144–152.

Lubrication of porous solids in reference to human joints

P. N. TANDON AND S. JAGGI

Department of Mathematics, Harcourt Butler Technological Institute, Kanpur 208002

MS received 9 August 1976

ABSTRACT

A simple mechanical model which has some features in common with load bearing human joints is described. The normal approach of two plane surfaces, one of which is covered with porous material is analysed. The gap between the two surfaces is filled with micropolar fluid to represent a particulate suspension (*i.e*, synovial fluid) as lubricant. The pore-size diameter is so small that only the suspending medium, *i.e*, the viscous fluid enters into the porous matrix due to the filtration action. The problem has been solved separately in two regions; flow of viscous fluid in the porous matrix and the squeeze film lubrication with micropolar fluid as lubricant in between the two approaching surfaces along with suitable matching conditions at the porous boundary. Several interesting results have been brought out. Agreement with available experimental results and the computational results presented, herein, is quite good.

1. INTRODUCTION

THIS paper is concerned with a straightforward physical situation, the normal approach of a porous solid and a rigid impervious solid, separated by a particulate suspension whose concentration increases as the gap between the approaching surfaces decreases.[1] The suspending medium (viscous fluid) enters into the porous matrix of the solid as the surfaces approach each other. The work has been inspired by the variety of more complicated situations occurring in living human load bearing joints.[2] The non-newtonian thixotropic synovial fluid lubricates the two layers of porous articular cartilage which have non linear stress/strain rate/liquid content relations.

According to the theories of joint lubrication given by Dowson *et al.*[3] the human joint experiences fluid (including elastohydrodynamic) mixed and boundary situations[4] in its varied operating conditions[5] and a full appreciation of the exceptional characteristics of human bearing requires a recognition of the role of each mode of lubrication. These authors have also proposed a mechanism of " Boosted Lubrication " in the extreme situation of high normal loads and low or zero sliding velocities. This mechanism depends on an increase in the viscosity of synovial fluid as the gap between the approaching Cartilage surfaces decreases. Caused by the increasing concentration of hyaluronic acid which is largely responsible for the viscosity of the synovial fluid, it has been argued that the hyaluronic acid molecules are too big to pass through the pores in the Cartilage whereas the base fluid (the suspending medium, *i.e.*, water) is driven away through the pores.

Mc-Cutchen[6,7] has earlier stated that the porosity of the Cartilage, coupled with the impervious nature of the bone backing, is responsible for the very low coefficient of friction by a mechanism " weeping lubrication "[8] which is quite different from Dowson *et al.*[3]

From all that has been written about the functioning of human joints, no reliable description of their lubrication has emerged and one probable conclusion is that no single mechanism operates throughout the range of practical conditions as has been noted by Dowson *et al.*[3] It would therefore be adequate to study a straightforward phase of operations, with no need to enlist various complications. The work described here is devoted to those conditions where entraining velocity is small or zero, as such points in the walking cycle, the load is high, *viz.*, heel strike and toe off.[9] At these points the relative velocity is normal in approach and hydrodynamic lubrication must relay on the squeeze film effect[10] together with the flow of viscous fluid in the porous matrix and the micropolar fluid in the fluid film region to represent synovial fluid and the corresponding matching conditions at the porous boundary. As may be seen the model used in the investigation is very much simpler than the real thing, while clearly inspired by the human joints, it lays no claim to describe them adequately. The calculation also include the effect of concentration on the viscosity and thus the effect of viscosity on the pressure distributions and load carrying capacity of the fluid film.

2. FORMULATION OF PROBLEM

The configuration of bearing as shown in figure consists of two rectangular plates of infinite length (not shown in figure) in $-x_3$ direction and of length L in $-x_1$ direction. The upper surface is of porous pad of thickness H with fixed porosity K fitted over a rigid backing and the lower surface is of rigid impervious solid. These surfaces are kept apart by a micropolar

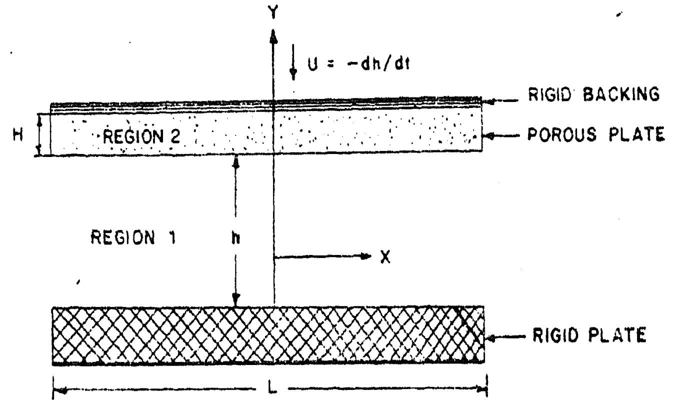

Figure 1. Bearing configuration.

fluid film of thickness h. The upper plate moves with velocity U in negative direction of $-x_2$ axis.

As the problem is two-dimensional, the velocity, spin and pressure fields are given as:

$$\bar{V} = i_1 v_1 (x_1, x_2) + i_2 v_2 (x_1, x_2)$$

$$\bar{\nu} = i_3 \nu_3 (x_1, x_2)$$

$$\bar{p} = p (x_1)$$

where i_1, i_2, i_3 are the unit vectors along x_1, x_2, x_3 axis.

GOVERNING EQUATIONS IN REGION 1 (FLUID FILM REGION)

For the present case in film region, the field equations for micropolar fluid given in detail[11] may be reduced in following form:

$$(1 + R) \frac{\partial^2 u}{\partial y^2} + R \frac{\partial \nu}{\partial y} - \frac{dP}{dx} = 0 \tag{1}$$

$$A \frac{\partial^2 \nu}{\partial y^2} - \frac{\partial u}{\partial y} - 2\nu = 0 \tag{2}$$

when the following non-dimensional quantities have been used

$$x = \frac{x_1}{L}, \quad y = \frac{x_2}{L}, \quad u = \frac{v_1}{U_0}, \quad v = \frac{v_2}{U_0}, \quad \nu = \frac{L\nu_3}{U_0}, \quad P = \frac{L}{\mu U_0} p$$

where R in eq. (1) is an index to measure the concentration of the suspended molecules (*e.g.*, hyaluronic acid molecules in synovial fluid) and A in eq. (2) measures the intensity of the influence on the action of the lubricant by the change in size and shape of macromolecules. Both the parameters are positive. U_0 the velocity at $t = 0$ when the gap is h_0, μ the classical viscosity coefficients and k, γ are additional viscosity coefficients for micropolar fluid.

GOVERNING EQUATION IN REGION 2 (POROUS MATRIX)

In the porous region, the flow is governed by the following equations in non-dimensional form

$$\bar{u} = -\frac{\phi}{\alpha}\frac{\partial \bar{p}'}{\partial x} \tag{3}$$

$$\bar{v} = -\frac{\phi}{\alpha}\frac{\partial \bar{p}'}{\partial x} \tag{4}$$

where $\bar{p}'$ is non-dimensional pressure in region 2 and $\phi = KH/L^3$ (design parameter), $\alpha = H/L$ and the equation of continuity

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} = 0. \tag{5}$$

Substitution of (3) and (4) in (5) gives a two-dimensional Laplace equation for pressure distribution as

$$\frac{\partial^2 \bar{p}'}{\partial x^2} + \frac{\partial^2 \bar{p}'}{\partial y^2} = 0 \tag{6}$$

TRANSFORMED BOUNDARY CONDITIONS IN NON-DIEMENSIONAL FORM

In fluid film region:

$$\begin{aligned} &u = 0,\ \ v = 0 \ \text{ at } \ y = 0 \\ &u = 0,\ \ v = 0 \ \text{ at } \ y = h/L = \beta,\ 0 < \beta < 1 \\ &P = 0 \ \text{ at } \ x = \pm \tfrac{1}{2}, \end{aligned} \tag{7}$$

In porous region:

$$\begin{aligned} &\bar{p}' = 0 \ \text{ at } \ x = \pm \tfrac{1}{2} \\ &\left.\frac{\partial \bar{p}}{\partial y}\right|_{y=\alpha+\beta} = 0 \end{aligned} \tag{8}$$

and the matching condition:

$$p\,(x) = \bar{p}'\,(x, \beta) \tag{9}$$

3. SOLUTION OF THE PROBLEM

Analytical solutions for velocity and spin field determined from eqs (1) and (2) using boundary conditions 7 are

$$u = T_1 \left[y^2 - \beta y + \left(\frac{T_2 \beta}{2}\right) \{C(1 - \cosh my) - \sin h\, my\} \right] \tag{10}$$

$$v = \left(\frac{T_1 \beta}{2}\right) [1 - \cosh my - C \sinh my] - T_1 y \tag{11}$$

where

$$T_1 = \frac{dP/dx}{R+2},\ T_2 = Am - \frac{2}{m},\ m = \left[\frac{R+2}{A(R+1)}\right]^{\frac{1}{2}}$$

and

$$C = [\beta(1 + \cosh m\beta) + T_2 \sinh m\beta] \times [(1 - \cosh m\beta) T_2 - \beta \sinh m\beta]^{-1}.$$

Equation of continuity in fluid film region gives

$$V|_{y=\beta} = \frac{-M}{R+2} \frac{\partial}{\partial x} \left[\frac{dP}{dx}\right] \tag{12}$$

where

$$M = \left[\frac{-\beta^3}{6} + \left(\frac{T_2 \beta}{2}\right) \left\{\frac{C(\beta m - \sinh m\beta)}{m} - \frac{(\cosh m\beta - 1)}{m}\right\}\right]$$

and

$$V|_{y=\beta} = \bar{v}|_{y=\beta} + \frac{dh}{dt}. \tag{13}$$

Solving eq. (6) with boundary conditions (8), we get

$$\bar{p}' = \sum_{n=1}^{\infty} A_n \exp(-\beta_n y) \{\exp[-2\beta_n(\alpha + \beta - y)] + 1\} \cosh \beta_n x \tag{14}$$

where

$$\beta_n = (2n-1)\pi.$$

With the help of eqs (13) and (14), eq. (12) becomes

$$-\frac{1}{U_0}\left(\frac{dh}{dt}\right) + \frac{\phi}{a} \sum_{n=1}^{\infty} A_n \beta_n \cos \beta_n x = \frac{M}{R+2} \frac{\partial}{\partial x} \left(\frac{dP}{dx}\right) \tag{15}$$

where

$$E = \exp(-\beta_n\beta)(\exp(-2\beta_n\alpha) - 1).$$

Now using $P = 0$ at $x = \pm\frac{1}{2}$, we obtain the pressure field for fluid film region from eq. (15)

$$P = \frac{R+2}{M}\left[-\frac{1}{U_0}\left(\frac{dh}{dt}\right)\left(\frac{x^2}{2} - \frac{1}{8}\right) - \frac{\phi}{\alpha}\sum_{n=1}^{\infty}\frac{A_n}{\beta_n}E\cos\beta_n x\right]. \quad (16)$$

Using matching condition at porous boundary

i.e., $P(x) = \bar{p}'(x, \beta)$

we obtain the following system of equations

$$\sum_{n=1}^{\infty} A_n F\cos\beta_n x = \frac{R+2}{M}\left[-\frac{1}{U_0}\left(\frac{dh}{dt}\right)\left(\frac{x^2}{2} - \frac{1}{8}\right)\right.$$

$$\left. - \frac{\phi}{\alpha}\sum_{n=1}^{\infty}\frac{A_n}{\beta_n}E\cos\beta_n x\right] \quad (17)$$

where

$$F = \exp(-\beta_n\beta)(\exp(-\beta_n\alpha) + 1)$$

$$A_n = \frac{\frac{4}{U_0}\left(\frac{dh}{dt}\right)(-1)^{n+1}}{\frac{\phi}{\alpha}E\left(1 + \frac{F}{E}\cdot\frac{M}{R+2}\frac{\alpha}{\phi}\beta_n\right)}. \quad (18)$$

Thus the dimensionless pressure distribution in region 1 is given by

$$\bar{P} = \frac{U_0 p'}{dh/dt} = \frac{R+2}{M}\left[-\left(\frac{x^2}{2} - \frac{1}{8}\right)\right.$$

$$\left. - \sum_{n=1}^{\infty}\frac{4(-1)^{n+1}\cos\beta_n x}{\beta_n^3\left(1 + \frac{F}{E}\cdot\frac{M}{R+2}\cdot\frac{\alpha}{\phi}\cdot\beta_n\right)}\right]. \quad (19)$$

The expression for load carrying capacity $\bar{W}$ in the squeeze film in non-dimensional form

$$\bar{W} = \frac{WU_0}{dh/dt} = \frac{R+2}{M}\left[\frac{1}{24} - \sum_{n=1}^{\infty}\frac{4(-1)^{2n+1}}{\beta_n^4\left(1 + \frac{F}{H}\cdot\frac{M}{R+2}\cdot\frac{\alpha}{\phi}\cdot\beta_n\right)}\right]$$

(20)

and the non-dimensional thickness and time relationship is

$$T = \int_1^\beta W(\xi)\, d\xi. \tag{21}$$

FRICTIONAL-DRAG AND COEFFICIENT OF FRICTION

The shear stress along the surface[11]

$$t_{21} = (1 + R)\frac{\partial u}{\partial y} - R\nu.$$

Since $\nu = 0$ at $y = 0$, therefore shear stress T_0 at surface is given by

$$T_0 = \left(\frac{1+R}{R+2}\right)\frac{dP}{dx}\left[\beta + \left(\frac{T_2\beta}{2}\right)\{Cm \sinh m\beta - m\cosh m\beta\}\right]$$

and the frictional drag F per unit length

$$F = \int_0^{1/2} F dx = \left(\frac{1+R}{R+2}\right)\int_0^{1/2}\frac{dP}{dx}$$

$$\times \left[\beta + \frac{T_2\beta}{2}\{Cm \sinh m\beta - m\cosh m\beta\}\right] dx.$$

The coefficient of friction can be obtained dividing frictional drag by load capacity which lies within the range $0{\cdot}005 - 0{\cdot}025$.

Tables 1 and 2 have been prepared from eqs. (20) and (21).

Table 1. Dimensionless load capacity ($\overline{W}$) for the different value of R (index to measure the concentration of suspended particles). A (influence of size and shape of suspended particles) β (dimensionless film thickness).

(a) ($A = 5$, $\beta = {\cdot}5$, $\alpha = {\cdot}1$, $\phi = {\cdot}01$)

R	1	2	3	4	5
$\overline{W}$	·019203	·023694	·035183	·138233	·190888

(b) ($R = 5$, $\beta = {\cdot}5$, $\alpha = {\cdot}1$, $\phi = {\cdot}01$)

A	1	2	3	4	5
$\overline{W}$	·070643	·884108	·100514	·132200	·192735

(c) ($R = 4$, $A = 6$, $\alpha = 0{\cdot}1$, $\phi = {\cdot}01$)

β	1	2	3	4	5
$\overline{W}$	·252142	·315634	·427029	·569575	·705270

(d) ($R = 0$, $A = 0$, $\alpha = {\cdot}1$, $\phi = {\cdot}01$)

β	·1	·2	·3	·4	·5
$\overline{W}$	2·201021	2·111081	2·102102	2·093315	2·069474

Table 2. Time of approach—height Relationship for $\phi = 0 \cdot 1$, $\alpha = 0 \cdot 1$, A = 5.

βR	3	5	7
·1	·654501	·250195	·250101
·2	·601090	·201019	·193461
·3	·590180	·179801	·169213
·4	·510290	·150690	·150418
·5	·492890	·123659	·115080

RESULTS AND DISCUSSION

Idealising a simple model for human joints, an attempt has been made in this paper to bring out the effects of changes of size and shape of macromolecules and then concentration on bearing characteristics using micropolar fluid as lubricant (Synovial fluid) including the effect of the porous nature of the cartilage. This simple model brings out many interesting observations which support the observations made by various authors reported earlier. Numerical results are shown in tables 1 and 2. It may be observed that:

1. Load carrying capacity increases with R, *i.e.*, concentration of macromolecules (hyluronic acid molecules). It may also be observed that as the surfaces approach each other, the gap decreases and the concentration increases.[7] In the ab ence of macrostructure and the macroscopic motions (*i.e.*, when $R = 0$, $A = 0$), the load capacity increases when the gap decreases 1 (*d*).

2. The variation of A in the micropolar fluid under consideration measures the change in the shape and size of the macromolecules in the suspension. It may be observed from table 1 (*b*) that the load capacity increases with A which measures the orientation of the molecules in suspension.

3. Referring to table 2 showing the variation of gap for different values of R with time. It may be observed that time of approach decreases when R increases, and for particular value of R, time of approach increases when h decreases. It shows that higher the concentration of the macromolecules, the greater the time of approach similar experimental observations have been made by Walker and Dowson.[12]

5. CONCLUSIONS

Although the values of the parameters employed in the proposed model are very much different from those of human joints, but the object of this paper is to confirm the approximate validity of the theory. As the gap between the two approaching surfaces decreases which, in turn, due to the filtration action increasing the concentration of the suspended particles (*i.e.*, macromolecules), the load carrying capacity increases. It, therefore, follows that the filtration action through the surface aggregates as suggested earlier by Unsworth is very much effective. The calculated closure time increases as the gap decreases and the concentration of the suspended particles is inversely proportional to the film thickness. These results are, in general, accord with the existing experimental evidence and provide support for squeeze film action as an important aspect in human joints.

ACKNOWLEDGEMENT

We are grateful to State Council of Science and Technology, U.P., for the award of the research scheme.

REFERENCES

1. Higginson, G. R. and Norman, R., *J. Mech. Eng. Sci.* 16 250 (1974).
2. Greenwald, A. S. and Haynes, D. W., *J. Bone Jt. Surg.* **B54** 157 (1972).
3. Dawson, D., Unsworth, A. and Wright, V., *J. Mech. Eng. Sci.* **12** 364 (1970).
4. Maroudas, A., *Lubrication and Wear in Joints*, Ed. Wright, V., Chap. 14 (1969).
5. Mac-Conail, M. A., *J. Bone Jt. Surg.* **32** 2 (1950).
6. Mc-Cutchen, C. W., *Nature* **184** 1284 (1950).
7. Mc-Cutchen, C. W., *Wear* **52** 5 (1962).
8. Mow, V. C. and Linn, F. F., *J. Appl. Math. Phys.* **20** 156 (1969).
9. Tanner, R. I., *Phys. Med. Biol.* **2** 1 (1966).
10. Fein, R. S., Rep. 3; *Proc. Inst. Mech. Eng.* **181** 122 (1966-67).
11. Eringen, A. C., *J. Math. Mech.* 16 1 (1968).
12. Walker, P. S., Dowson, D., Longfield, M. D. and Wright, V., *Ann. Rhem. Dis.* 30 3 (1971).