

String theory: a framework for quantum gravity and various applications

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In this semi-technical review we discuss string theory (and all that goes by that name) as a framework for a quantum theory of gravity. This is a new paradigm in theoretical physics that goes beyond relativistic quantum field theory. We provide concrete evidence for this proposal. It leads to the resolution of the ultra-violet catastrophe of Einstein's theory of general relativity and an explanation of the Bekenstein–Hawking entropy (of a class of black holes) in terms of Boltzmann's formula for entropy in statistical mechanics. We discuss 'the holographic principle' and its precise and consequential formulation in the AdS/CFT correspondence of Maldacena. One consequence of this correspondence is the ability to do strong coupling calculations in $SU(N)$ gauge theories in terms of semi-classical gravity. In particular, we indicate a connection between dissipative fluid dynamics and the dynamics of black hole horizons. We end with a discussion of elementary particle physics and cosmology in the framework of string theory. We do not cover all aspects of string theory and its applications to diverse areas of physics and mathematics, but follow a few paths in a vast landscape of ideas.

Keywords: Cosmology, elementary particles, quantum gravity, string theory.

Introduction

In this article, we discuss the need for a quantum theory of gravity to address some of the important questions of physics related to the very early universe and the physics of blackholes. We will posit the case for string theory as a framework to address these questions. The bonus of string theory is that it has the tenets of a unified theory of all interactions, electro-magnetism, weak and strong interactions, and gravitation. Given this, string theory provides a framework to address some fundamental issues in cosmology and elementary particle physics. Examples are dark matter, supersymmetric particles, dark energy, unification for all interactions, etc.

Perhaps the most important success of string theory, in recent times, is in providing a microscopic basis of black-

hole thermodynamics. The discovery of the AdS/CFT correspondence, as a precise realization of the holographic principle of black hole physics, has shed new light on the solution of large N gauge theories and other field theories at strong coupling. Given the diversity of concepts and techniques, string theory has a healthy interface with various branches of mathematics and statistical mechanics. More recently, there have appeared connections with fluid mechanics and strongly coupled condensed matter systems.

We begin with a brief review of the current theories of physics and their limitations.

Quantum mechanics and general relativity

Quantum mechanics

Quantum mechanics is the established framework to describe the world of molecules, atoms, nuclei and their constituents. Its validity has been tested to very short distances like 10^{-18} m in high-energy collision experiments at CERN and Fermi Lab. In quantum mechanics the scale of quantum effects is set by Planck's constant $\hbar = 1.05 \times 10^{-27}$ erg s, and a new mathematical formulation is required. Position and momentum do not commute $xp - px = i\hbar$, and the Heisenberg uncertainty relation $\Delta x \Delta p \geq \hbar$ implies a limit to which we can localize the position of a point particle. This fuzziness that characterizes quantum mechanics resolves, e.g. the fundamental problem of the stability of atoms.

Relativistic quantum field theory

The application of quantum mechanics to describe relativistic particles, requires the framework of quantum field theory (QFT), which is characterized by \hbar and the speed of light c . The successful theories of elementary particle physics, viz. electro-weak theory and the theory of strong interactions are formulated in this framework. The inherent incompatibility between a 'continuous field' and the notion of a quantum fluctuation at a space–time point is resolved in the framework of renormalization theory and the concept of the renormalization group that was developed by Wilson¹.

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General relativity

General relativity was conceived by Einstein to resolve an apparent contradiction between special relativity and Newton's theory of gravitation. In special relativity, interactions take a finite time to propagate due to the finiteness of the speed of light. But in Newton's theory the gravitational interaction is instantaneous! The resolution in general relativity is that space–time is not static and responds to matter by changing its geometry (the metric of space–time) in accordance with Einstein's equations. General relativity is a successful theory (by success we mean it is experimentally well-tested) for distances R and masses M characterized by $R \gtrsim \ell_{\text{Pl}}$, where $\ell_{\text{Pl}} = 2G_{\text{N}}M/c^2$ (G_{N} is Newton's constant) is the Planck length. This includes a large range of phenomena in relativistic astrophysics. General relativity also provides the framework of the standard inflationary model of cosmology within which one successfully interprets the cosmic microwave background (CMB) data.

Now let us indicate the problem one encounters when one tries to quantize general relativity.

Divergent quantum theory: Just like in Maxwell's theory, where electro-magnetic waves exist in the absence of sources, Einstein's equations predict 'gravity' waves. The 'graviton' is the analogue of the 'photon'. The graviton can be considered as a 'particle' in the quantum theory with mass $M = 0$ and spin $S = 2$. It is a fluctuation of the geometry around flat Minkowski space–time.

Given this, one does the obvious like in any quantum field theory. One discusses emission and absorption processes of gravitons. The non-linearity of Einstein's theory implies that besides the emission and absorption of gravitons by matter, gravitons can be emitted and absorbed by gravitons. These processes are characterized by a dimensionless 'coupling constant': $E = E_{\text{Pl}}$, where E is the energy of the process and $E_{\text{Pl}} = (\hbar c^5/G_{\text{N}})^{1/2} \simeq 10^{19}$ GeV. As long as we are discussing processes in which $E/E_{\text{Pl}} \ll 1$, the effects of quantum fluctuations are suppressed and negligible. When $E = E_{\text{Pl}} \gtrsim 1$ gravity is strongly coupled and graviton fluctuations are large. It is basically this fact that renders the standard quantum theory of the metric fluctuations around a given classical space–time badly divergent and meaningless. Note that in discussing quantum gravity effects, we introduced the Planck energy which involves all the three fundamental constants of nature: \hbar , c and G_{N} .

Big Bang singularity: If we consider the Friedmann–Robertson–Walker (FRW) expanding universe solution and extrapolate it backward in time, the universe would be packed in a smaller and smaller volume in the far past. Once again when we reach the Planck volume ℓ_{Pl}^3 , we would expect a breakdown of the quantized general relativity, due to large uncontrolled fluctuations. In the absence

of a theory of this epoch of space–time, it would be impossible to understand in a fundamental way the evolution of the universe after the 'Big Bang'. In fact the very notion of a 'initial time' may lose meaning. Perhaps a new framework may provide a new language in terms of which we may address such questions about the very early universe.

Black hole information paradox: Quantized general relativity also runs into difficulties with quantum mechanics in the description of phenomena in the vicinity of the horizon of a black hole. A black hole is formed when a large mass is packed in a small volume, characterized by the radius $r_{\text{h}} = 2G_{\text{N}}M/c^2$. If so, then even light cannot escape from its interior and hence the name black hole. The surface with radius r_{h} is called the horizon of a black hole and it divides the space–time into two distinct regions. The horizon of the black hole is a one way gate: If you check in you cannot get out! However, Hawking in 1974 realized that in quantum mechanics black holes radiate. He calculated the temperature of a black hole of mass M : $T = \hbar c^3/8\pi G_{\text{N}}M$. Using the first law of thermodynamics and Bekenstein's heuristic proposal that the entropy of a black hole is proportional to the area of its horizon, he arrived at one of the most important facts of quantum gravity. The entropy of a large black hole is given by

$$S_{\text{bh}} = \frac{A_{\text{h}}c^3}{4\hbar G_{\text{N}}}, \quad (1)$$

where A_{h} is the area of the horizon of the black hole. In general, the black hole is characterized by its mass, charge and angular momentum, and the Bekenstein–Hawking formula (1) is valid for all of them. Note that it involves all the three fundamental constants \hbar , c and G_{N} . Defining $A_{\text{Pl}} = \hbar G_{\text{N}}/c^3$ (Planck area) we can write it suggestively as

$$S_{\text{bh}} = \frac{A}{4A_{\text{Pl}}}. \quad (2)$$

A/A_{Pl} represents the number of degrees of freedom of the horizon. In 3 + 1 dim, $A_{\text{Pl}} \approx 2.6 \times 10^{-70}$ m². *This formula is a benchmark for any theory of quantum gravity to reproduce.*

The fact that the entropy is proportional to the area of the horizon and not the volume it encloses, gives a clue that even though the degrees of freedom of the black hole are apparently behind the horizon, they seem to leave an imprint (hologram) on the horizon.

Once the black hole is formed it will emit Hawking radiation. It is here that we run into a problem with quantum mechanics. The quantum states that make up the black hole cannot be reconstructed from the emitted radiation, even in principle, within the framework of general relativity because the emitted radiation is thermal.

This is the celebrated ‘information paradox’ of black hole physics. It is clear that the resolution of this paradox is intimately connected with an understanding of the Bekenstein–Hawking entropy formula and the degrees of freedom that constitute the black hole.

Why string theory?

Now that we have spelt out (in three important instances) why quantizing general relativity does not produce a theory of quantum gravity, we would like to posit the view that the correct framework to address the issues we have raised is string theory. *Unlike the development of general relativity which had the principle of equivalence as a guide from the very beginning, string theory has no such recognizable guiding principle. But perhaps it may just turn out that the ‘holographic principle’, which we will discuss later on, is one such, guiding principle.*

String theory primer (see note 1)

Perturbative string theory

The laws of nature in classical and quantum mechanics are usually formulated in terms of point particles. When many particles are involved, the mechanical laws are described in terms of fields. A good example is the Navier–Stokes equation which is Newton’s laws for a fluid. Even in quantum field theory we essentially deal with the ‘particle’ concept because in an approximate sense a field at a point in space creates a particle at that point from the vacuum. *The main paradigm shift in string theory is that the formulation of the dynamical laws is not restricted to point particles.*

Historically, the first example beyond point particles is the one-dimensional string. Strings can be open or closed. The open string sweeps out a world sheet with end points moving at the speed of light (Figure 1). The closed string sweeps out a cylindrical surface in space–time. The dynamical

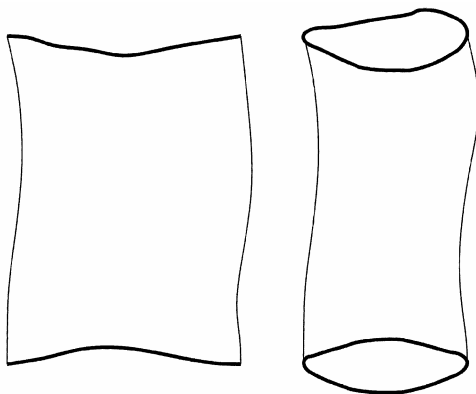


Figure 1. Propagating open and closed strings.

is determined by an action principle which states that the area swept out between initial and final configurations is minimum.

String theory comes with an intrinsic length scale ℓ_s which is related to the string tension $T = \ell_s^{-2}$. One of the great discoveries of string theory is that a string can carry bosonic as well as fermionic coordinates: $X^\mu(\sigma, t)$, $\psi^\mu(\sigma, t)$, $\bar{\psi}^\mu(\sigma, t)$. X^μ is a space–time coordinate and ψ^μ , $\bar{\psi}^\mu$ are additional ‘anti-commuting’ coordinates. This is a radical extension of our usual notion of space–time. The action describing the free string dynamics is

$$S = \frac{1}{\ell_s^2} \int d\sigma dt (\partial_\alpha X^\mu \partial^\alpha X_\mu - i \bar{\psi}^\mu \rho^\alpha \partial_\alpha \psi_\mu). \tag{3}$$

σ parametrizes the length of the string and t is time. The index ‘ μ ’ is a space-time index, ‘ α ’ is a 2-dim world sheet index and ρ^α are 2-dim Dirac matrices. Besides the standard conformal symmetry, this action is invariant under a supersymmetry transformation (see note 2) which transforms fermions into bosons and vice versa:

$$\delta X^\mu = \bar{\epsilon} \psi^\mu, \quad \delta \psi^\mu = -i \rho^\alpha \partial_\alpha X^\mu \epsilon. \tag{4}$$

The symmetry transformation parameter ϵ is a constant anti-commuting spinor. The string described by eq. (3) is called a superstring.

String spectrum

The various vibrational modes of the superstring correspond to an infinite tower of particle states of spin J and mass M , satisfying a linear relation $M^2 = 1/\ell_s^2 J + \text{const}$. It turns out that the spectrum of the superstring can be organized into space–time supersymmetry multiplets. *The most important aspect of the spectrum of the string is that in 10 space–time dimensions (and none other), its spectrum has a graviton and a gluon. The graviton ($J = 2, M = 0$) is the massless particle of the closed string, while the gluon ($J = 1, M = 0$) is the massless particle of the open string. Their space–time supersymmetric partners are the gravitino ($J = 3/2; M = 0$) and the gluino ($J = 1/2, M = 0$).*

String interactions

String interactions involve the splitting and joining of strings, characterized by a coupling constant denoted by g_s , the string coupling (Figure 2a).

These interactions generate 2-dim surfaces with non-trivial topology. For example the splitting and rejoining of a closed string creates a handle on the world sheet. A similar process for the open string creates a hole in the world sheet. There are also diagrams describing the inter-

actions of closed and open strings (Figure 2 *b*). These interactions can be consistently described only in a 10-dim space–time.

Now the rules of perturbative string theory which we have briefly stated enable us to calculate scattering of the particle states of the free string theory. In particular, graviton scattering amplitudes in string theory turn out (to leading order in the $E/M_{\text{Pl}}c^2$) to be identical to those calculated from general relativity (actually supergravity) with the identification of the 10-dim Newton’s constant

$$G_{\text{N}}^{(10)} = 8\pi^6 \ell_s^8 g_s^2. \quad (5)$$

The deep result here is that unlike general relativity, string theory has no short distance divergences. This is because in general relativity point-like gravitons with large energy come arbitrarily close together in accordance with the uncertainty principle $\Delta x \geq \hbar/\Delta p$. In string theory due to the existence of a length scale ℓ_s , high energy gravitons do not come arbitrarily close together (see Figure 2 *a*), but the energy gets distributed in the higher modes of the string, so that instead of probing short distances the size of the string grows. This point can be summarized in a plausible generalization of the Heisenberg uncertainty principle

$$\Delta x \geq \frac{\hbar}{\Delta p} + \ell_s^2 \frac{\Delta p}{\hbar}. \quad (6)$$

Hence string theory by its construction, includes an additional length scale ℓ_s , and gives rise to a ultra-violet finite theory. This result is radically different from standard quantum field theory. In this way string theory passes its

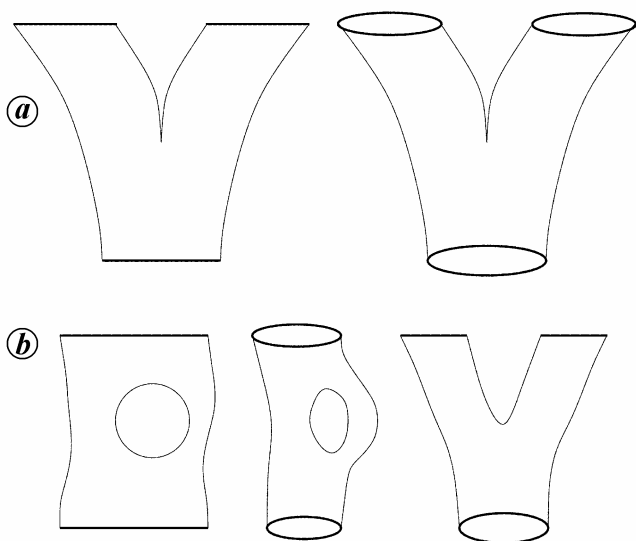


Figure 2. Strings interact by splitting and joining.

first test towards a theory of quantum gravity: it is perturbatively finite and calculable.

Types of string theories in 10-dim

Various perturbatively consistent superstrings have been constructed in 10 space–time dimensions. There are five such theories:

- (i) Type I strings are open and can have non-abelian charges at their ends. These open strings can also evolve into closed Type I strings. (Open Type I strings are analogous to QCD strings with quarks at their end points. Closed Type I strings are analogous to closed QCD strings which describe gluons.)
- (ii) Type II A and Type II B in which the strings are closed and oriented.
- (iii) The 2 heterotic string theories corresponding to the gauge groups $SO(32)$ and $E_8 \times E_8$.

Non-perturbative string theory

(i) *Duality and M-theory.* Going beyond a perturbative description of string theory is rendered difficult by the fact that unlike in standard quantum field theory, string theory is defined by a set of rules that enable a perturbative definition. However, progress was made towards understanding non-perturbative effects in string theory by the discovery of duality symmetries.

The duality symmetries of string theory are of two-kinds. One called *S*-duality, has a correspondence in standard quantum field theory. It says that two theories with coupling constants e and g are actually the same theory when $eg = c$, a constant. This implies that the theory with coupling constant e , can be studied at strong coupling ($e \gg 1$), by the equivalent version with coupling constant $g = c/e \ll 1$. Examples are the Sine-Gordon and Thirring model, Z_2 gauge theory and the Ising model in $2 + 1$ dim and Maxwell’s theory in $3 + 1$ dim. In statistical physics, this duality is called the Kramers–Wannier duality. In the case of Maxwell’s theory it is called ‘electric–magnetic’ duality, where the electric and magnetic charges are related by $eg = \hbar/2$.

The other duality symmetry called *T*-duality has no analogue in quantum field theory and is stringy in origin⁴. An example is a string moving in a space–time, one of whose dimensions is a circle of radius R . It is easy to see that the energy levels are given by (assuming motion is only along the circle)

$$E_n = \frac{c\hbar|n|}{R} + \frac{c\hbar|m|R}{\ell_s^2}, \quad (7)$$

where $\hbar n/R$ is the momentum of the state characterized by the integer n and $c\hbar|m|R/\ell_s^2$ is the energy due to the wind-

ing modes of the string. The energy formula is symmetric under the interchange of momentum and winding modes ($|n\rangle \rightarrow |m\rangle$) and $R \rightarrow \ell_s^2/R$. For $R \gg \ell_s$ the momentum mode description is appropriate while for $R \ll \ell_s$ the winding mode description is more appropriate. At $R = \ell_s$ both descriptions are valid. This simple example also generalizes to higher dimensional objects called ‘branes’.

As Polchinski observed duality has a profound consequence for string theory. T -duality led to the inference of D -branes, which are solitonic domain walls on which open strings end⁵ (that explains the D , because the open string has a Dirichlet boundary condition on the brane). A combination of S and T dualities led to the realization that the five perturbatively defined string theories are different phases of a meta-theory called M -theory which lives in 11-dim. The radius of the 11th dimension $R_{11} = \ell_s g_s$, so that as $g_s \rightarrow \infty$, $R_{11} \rightarrow \infty$. One of the great challenges of string theory is to discover ‘new principles’ that would lead to a construction of M -theory.

(ii) D -branes: We now give a brief introduction to D -branes as they, among other things, play a crucial role in resolving some of the conundrums of black hole physics. As we have mentioned a $D-p$ brane (in the simplest geometrical configuration) is a domain wall of dimension p , where $0 \leq p \leq 9$. It is characterized by a charge and it couples to a $(p+1)$ form abelian gauge field $A^{(p+1)}$ (see note 3). The $D-p$ brane has a brane tension T_p which is its mass per unit volume. The crucial point is that $T_p \propto 1/g_s$. This dependence on the coupling constant (instead of g_s^2) is peculiar to string theory. It has a very important consequence. A quick estimate of the gravitational field of a $D-p$ brane gives, $G_N^{(10)} T_p \sim g_s^2/g_s \sim g_s$. Hence as $g_s \rightarrow 0$, the gravitational field goes to zero! If we stack N $D-p$ branes on top of each other then the gravitational field of the stack $\sim Ng_s$. A useful limit to study is to hold $g_s N = \lambda$ fixed, as $g_s \rightarrow 0$ and $N \rightarrow \infty$. In this limit when $\lambda \gg 1$ the stack of branes can source a solution of supergravity. On the other hand, when $\lambda \ll 1$ there is also a description of the stack of D -branes in terms of open strings. A stack of D -branes interacts by the exchange of open strings very

much like quarks interact by the exchange of gluons. Figure 3a illustrates the self-interaction of a $D2$ -brane by the emission and absorption of an open string and Figure 3b illustrates the interaction of two $D2$ -branes by the exchange of an open string. In the infra-red limit only the lowest mode of the open string contributes and hence the stack of N D -branes can be equivalently described as a familiar $SU(N)$ non-abelian gauge theory in $p+1$ dim.

A precise formulation of the dual description of a large stack of D -branes in terms of gauge fields (or in general a field theory) and gravity, gives a realization of the holographic principle of black hole physics. More on this later.

Black hole micro-states (see note 4)

In order to discuss the question of the micro-states of a black hole it will be best to ‘construct’ a black hole whose states one can count. This is exactly what Strominger and Vafa⁸ did. They considered a system of Q_1 $D1$ branes and Q_5 $D5$ branes and placed them on five circles. The radius $R_5 \gg \ell_s$ and $R_i \sim \ell_s$, $i = 1, 2, 3, 4$. This system of branes interacts by the exchange of open strings and interacts with gravitons around the flat 10-dim space-time. If we are interested in the long wave length dynamics (compared to ℓ_s) of the $D1-D5$ system, one can ignore the gravitons and also the massive modes of the open strings with masses $\propto 1/\ell_s$. What one is left with is a non-abelian gauge theory of the lowest lying modes of the open strings exchanged between the $D1$ and $D5$ branes.

These open strings are described by a super-conformal field theory (SCFT) on a 2-dim cylinder of radius R_5 . Time flows along the length of the cylinder (Figure 4). The central charge of the SCFT is $C = 6Q_1 Q_5$ which basically is the number of open strings (both bosonic and fermionic) that are exchanged between the $D1$ and $D5$ branes. The SCFT can have excitations moving around the circle. These can be either left moving or right moving waves.

Now consider a state in the SCFT consisting of these left/right moving waves with energy E and momentum P (note that momentum on the circle is quantized and $P = \hbar n/R_5$, where n is an integer). There is a well-known formula due to Cardy which enables us to calculate the degeneracy of states $\Omega(E, P)$ for fixed values of E and P , for large values of the central charge,

$$\ln \Omega(E, P) = 2\pi \sqrt{Q_1 Q_5} \left(\sqrt{\frac{(E + cP)R_5}{2\hbar c}} + \left(\sqrt{\frac{(E - cP)R_5}{2\hbar c}} + o(\sqrt{Q_1 Q_5}) \right) \right) \tag{8}$$

(the ‘ c ’ in the above formula is the speed of light).

From here we can calculate the entropy and temperature using Boltzmann’s formulas $S = \ln \Omega$ and $T^{-1} = (\partial S/\partial E)$.

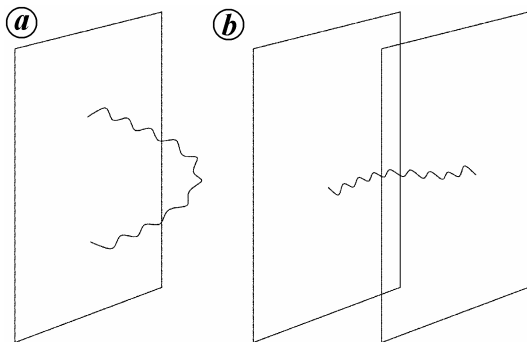


Figure 3. Emission and absorption of open strings by $D2$ -branes.

In particular, in the case $E = cP$ we get the famous formula

$$S = 2\pi\sqrt{Q_1 Q_5 n}, \tag{9}$$

where $E = cP = n = R_5$. Now one can find the supergravity solution corresponding to this system of branes. It is a black hole in 4 + 1 dim which is asymptotically, a 4 + 1 dim space–time. The brane sources living in the higher dimensions appear as point sources in 4 + 1 dim. The entropy and temperature of the black hole are calculated using the Bekenstein–Hawking formula,

$$S = \frac{A_h c^3}{4G_N \hbar}, \tag{10}$$

with

$$A_h = 2\pi^2 r_h^3, \quad r_h^2 = \left(g Q_1 g Q_5 \frac{g^2 n}{R_5^2} \right)^{1/3} \tag{11}$$

and

$$G_N^{-1} = \frac{2\pi R_5 (2\pi)^4}{8\pi^6 g_s^2} \tag{12}$$

is the 5-dim Newton constant. We see an exact matching of the entropy calculated in string theory and general relativity.

The impressive agreement of eq. (10) and the microscopic counting of black hole entropy continues to hold when we include higher order corrections (in ℓ_s) to gen-

eral relativity. In this case eq. (10) was generalized by Wald to be consistent with the first law of thermodynamics. Here too the string theory answer, exactly agrees with Wald’s formula. This agreement is a ‘precision test’ of string theory as microscopic theory underlying general relativity⁹.

Hawking radiation

The next important question is whether the microscopic theory can describe Hawking radiation and predict the decay rate of a black hole. For the D1–D5 system the answer to this question is in the affirmative. Modeling the emission of Hawking radiation is similar to the way we describe the emission of a photon from an atom which is a bound state of electrons and the nucleus.

In the case of the D1–D5 system, the left and right moving waves can collide to form a closed string mode (Figure 4). One can calculate an ‘S-matrix’ from an initial state of open strings (left/right moving waves in the SCFT) to a closed string mode. The absorption and decay probabilities are calculated using standard formulae of quantum statistical mechanics.

In a nutshell, consider a microcanonical ensemble \mathcal{S} specified by the energy and a set of charges. Consider the process of absorption of some particles by the brane system which changes the energy and charges corresponding to another microcanonical ensemble \mathcal{S}' . The ensembles \mathcal{S} and \mathcal{S}' have total number of states Ω and Ω' respectively. The absorption probability from a state $|i\rangle \in \mathcal{S}$ to a state $|f\rangle \in \mathcal{S}'$ is given by

$$P_{\text{abs}}(i \rightarrow f) = \frac{1}{\Omega} \sum_{i,f} |\langle f | \mathcal{S} | i \rangle|^2. \tag{13}$$

The sum in eq. (13) is over all final states and there is an average over all initial states. Similarly, the decay probability is given by

$$P_{\text{decay}}(i \rightarrow f) = \frac{1}{\Omega'} \sum_{i,f} |\langle f | \mathcal{S} | i \rangle|^2. \tag{14}$$

These formulae then enable us to calculate the decay rate of Hawking radiation which is given by the formula

$$\Gamma_H = \frac{\sigma_{\text{abs}}(\omega)}{e^{\omega/T_H} - 1} \frac{d^4 k}{(2\pi)^4}, \tag{15}$$

where $\omega = |k|$, T_H is the Hawking temperature and $\sigma_{\text{abs}}(\omega)$ is the absorption cross section of a given species of particles at frequency ω . For spherically symmetric waves $\sigma_{\text{abs}}(\omega \rightarrow 0) = A_h$ the area of the horizon of the black hole.

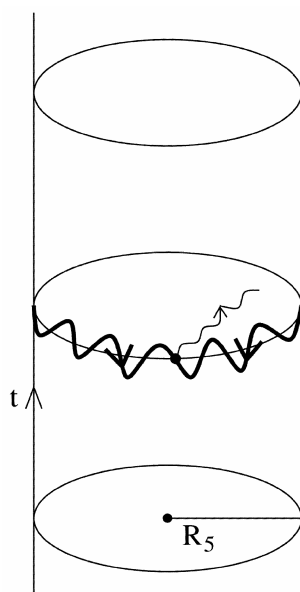


Figure 4. Colliding collective modes of open strings give rise to Hawking radiation.

The formulas calculated from D -branes and gravity match exactly with those calculated from semi-classical gravity. The important point is that S_{fi} is a standard S -matrix and thermodynamics emerges as an averaging process over microstates of a micro-canonical ensemble.

In this way, the microstate model does indeed explain how Hawking radiation emerges from a unitary theory. However the ‘information paradox’ remains, because its resolution would need us to explain why there is loss of unitarity in semi-classical general relativity. More precisely, even though semi-classical general relativity leads to the correct formulae for black hole entropy and Hawking radiation rates, consistent with the laws of thermodynamics, it seems to lack the ingredients to obtain an in-principle unitary answer without being embedded in the larger framework of string theory.

Lessons from the D1–D5 system: Holography and the AdS/CFT correspondence (see note 5)

We have briefly explained in the preceding section that thermodynamical properties of black holes and Hawking radiation are exactly derivable from the dynamics of a stack of D1–D5 branes which then constitute the microstates of the black hole. On closer examination it turns out that these results (on the black hole side) are determined entirely by the near horizon region of the black hole. *This suggests a general fact that the infrared dynamics of the brane system is equivalent to gravitational physics in the near horizon region of the black hole*¹⁰.

These suggestive facts about the D1–D5 system, combined with the ‘holographic principle’ led Maldacena to precisely formulate the AdS/CFT conjecture. This is a duality of a non-gravitational theory with a theory of superstrings (which has supergravity as its low energy limit). It gives a precise formulation of the ‘holographic principle’ (we shall explain this a bit later). It is simplest to explain this set of ideas in the context of a stack of N D3 branes to which we now turn.

D3 branes and the AdS/CFT correspondence^{2,7}

A D3 brane is a 3 + 1 dim object. A stack of N D3 branes interacts by the exchange of open strings (Figure 5). In the long wavelength limit ($\ell_s \rightarrow 0$), only the massless modes of the open string are relevant. These correspond to 4 gauge fields A_μ , six scalar fields ϕ^I ($I = 1, \dots, 6$) (corresponding to the fact that the brane extends in six transverse dimensions) and their supersymmetric partners. These massless degrees of freedom are described by $\mathcal{N} = 4$, $SU(N)$ Yang–Mills theory in 3 + 1 dim. This is a maximally supersymmetric, conformally invariant SCFT in 3 + 1 dim. The coupling constant of this gauge theory g_{YM} , is simply related to the string coupling $g_s = g_{YM}^2$. The ‘tHooft coupling is $\lambda = g_s N$ and the theory admits a sys-

tematic expansion in $1/N$, for fixed λ . Further as $\ell_s \rightarrow 0$ the coupling of the D3 branes to gravitons also vanishes, and hence we are left with the $\mathcal{N} = 4$ SYM theory and free gravitons.

On the other hand when $\lambda \gg 1$, N D3 branes (for large N) source a supergravity solution in 10-dim. The supergravity fields include the metric, two scalars, two 2-form potentials, and a 4-form potential whose field strength F_5 is self-dual and proportional to the volume form of S^5 . The fact that there are N D3 branes is expressed as $\int_{S^5} F_5 = N$. There are also fermionic fields required by supersymmetry. It is instructive to write down the supergravity metric:

$$ds^2 = H^{-1/2}(-dt^2 + d\vec{x} \cdot d\vec{x}) + H^{1/2}(dr^2 + r^2 d\Omega_5^2)$$

$$H = \left(1 + \frac{R^4}{r^4}\right), \quad \left(\frac{R}{\ell_s}\right)^4 = 4\pi g_s N. \tag{15}$$

Since $|g_{00}| = H^{-1/2}$ the energy depends on the 5th coordinate r . In fact the energy at r is related to the energy at $r = \infty$ (where $g_{00} = 1$) by $E_\infty = \sqrt{|g_{00}|} E_r$. As $r \rightarrow 0$ (the near horizon limit), $E_\infty = (r/R) E_r$ and this says that E_∞ is red-shifted as $r \rightarrow 0$. We can allow for an arbitrary excitation energy in string units (i.e. arbitrary $E_r \ell_s$) as $r \rightarrow 0$ and $\ell_s \rightarrow 0$, by holding a mass scale ‘ U ’ fixed:

$$\frac{E_\infty}{\ell_s E_r} \cong \frac{r}{\ell_s^2} = U. \tag{16}$$

Note that in this limit the bulk gravitons also decouple from the near horizon region. This is the famous near

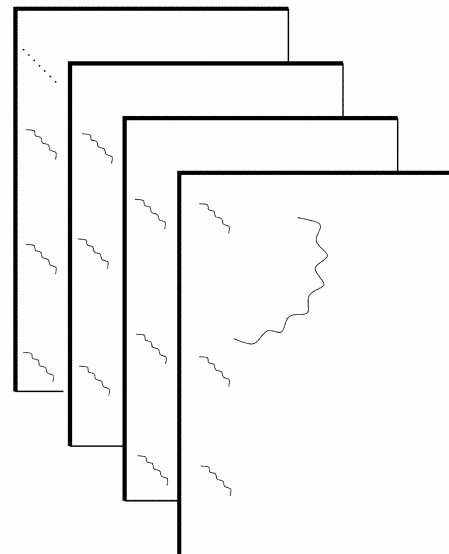


Figure 5. A stack of N D3 branes interacting via open strings.

horizon limit of Maldacena and in this limit the metric eq. (16) becomes

$$ds^2 = \ell_s^2 \left[\frac{U^2}{\sqrt{4\pi\lambda}} (-dt^2 + d\vec{x} + d\vec{x} \cdot d\vec{x}) + 4\sqrt{4\pi\lambda} \frac{dU^2}{U^2} + \sqrt{4\pi\lambda} d\Omega_5^2 \right]. \quad (17)$$

This is the metric of $AdS_5 \times S^5$. AdS_5 is the anti-de Sitter space in 5 dim. This space has a boundary at $U \rightarrow \infty$, which is conformally equivalent to 3 + 1 dim Minkowski space-time.

The AdS/CFT conjecture^{7,10,11}

The conjecture of Maldacena is that $\mathcal{N}=4$, $SU(N)$ super Yang–Mills theory in 3 + 1 dim is dual to type IIB string theory with $AdS_5 \times S^5$ boundary conditions.

The gauge/gravity parameters are related as $g_{YM}^2 = g_s$ and $R/\ell_s = (4\pi g_{YM}^2 N)^{1/4}$. It is natural to consider the $SU(N)$ gauge theory living on the boundary of AdS_5 . The gauge theory is conformally invariant and its global exact symmetry $SO(2, 4) \times SO(6)$, is also an isometry of $AdS_5 \times S^5$. In order to have a common definition of time in the gauge theory and AdS, the gauge theory is defined on $S^3 \times R^1$ which is the conformal boundary of AdS_5 and which is conformally equivalent to $R^3 \times R^1$. Since S^3 is compact the gauge theory has no infrared divergences and hence it is well defined.

The AdS/CFT conjecture is difficult to test because at $\lambda \ll 1$ the gauge theory is perturbatively calculable but the string theory is defined in $AdS_5 \times S^5$ with $R \ll \ell_s$. On the other hand for $\lambda \gg 1$, the gauge theory is strongly coupled and hard to calculate. In this regime $R \gg \ell_s$ the string theory can be approximated by supergravity in a derivative expansion in ℓ_s/R . The region $\lambda \sim 1$ is most intractable as we can study neither the gauge theory nor the string theory in a reliable way.

Interpretation of the radial direction of AdS

Before we discuss the duality further we would like to explain the significance of the extra dimension ‘ r ’. Let us recast the AdS_5 metric by a redefinition: $u = (R^2/r)$

$$ds^2 = (R^2/u^2)(-dt^2 + d\vec{x} \cdot d\vec{x} + du^2) + R^2 d\Omega_5^2. \quad (18)$$

The boundary in these coordinates is situated at $u = 0$. Now this metric has a scaling symmetry. For $\alpha > 0$, $u \rightarrow \alpha u$, $t \rightarrow \alpha t$ and $\vec{x} \rightarrow \alpha \vec{x}$, leaves the metric invariant. From this it is clear that the additional dimension ‘ u ’ represents a length scale in the boundary space–time: $u \rightarrow 0$ corresponds to a localization or short distances in the

boundary coordinates (\vec{x}, t) , while $u \rightarrow \infty$ represents long distances on the boundary.

Another indicator that the fifth-dimension represents a scale in the gauge theory on the boundary is provided by the fact that the $r/\ell_s^2 = U$, is the energy of an open string connecting the stack of N D3 branes and a single D3 brane placed at a distance ‘ r ’ from it. In the gauge theory this represents an expectation value of the scalar field ϕ^I which corresponds to the symmetry breaking $U(N + 1) \rightarrow U(N) \times U(1)$.

Holography

We now indicate why the AdS/CFT correspondence gives a holographic description of physics in AdS. In order to see this (following Susskind and Witten) we recast the metric eq. (17) to another form by a coordinate transformation

$$ds^2 = R^2 \left[-\left(\frac{1+r^2}{1-r^2} \right)^2 dt^2 + \frac{4}{(1-r^2)^2} (dr^2 + r^2 d\Omega^2) \right], \quad (19)$$

so that the boundary of AdS is at $r = 1$. If we calculate the entropy of AdS using the Bekenstein–Hawking formula we will get infinity. Hence we stay near the boundary and set $r = 1 - \delta$, δ is a small and $\delta > 0$. The entropy can now be computed. An elementary calculation gives

$$S = \frac{\text{Area}}{4G_N} \cong \frac{R^8 \delta^{-3}}{4G_N} \sim \frac{R^8 \delta^{-3}}{g_s^2 \ell_s^8} \sim N^2 \delta^{-3}. \quad (20)$$

When $r = 1 - \delta$, δ is an ultra-violet cut-off of the boundary theory because as $\delta \rightarrow 0$, the induced metric on the boundary is

$$ds^2 = (R^2/\delta^2)(-dt^2 + d\vec{x} \cdot d\vec{x}). \quad (21)$$

Now we can easily estimate the degrees of freedom of the $SU(N)$ gauge theory on $S^3 \times R^1$. Since S^3 is compact, the number of cells into which we can divide it is δ^{-3} and hence $S \sim N^2 \delta^{-3}$. Hence the estimate of the number of degrees of freedom in the gauge theory and AdS matches. This is in accordance with the holographic principle which states that in a quantum theory of gravity the degrees of freedom and their interactions can be described in terms of a non-gravitational theory living on the boundary of the space–time.

This principle due to ‘tHooft and elaborated by Susskind was motivated by Bekenstein’s entropy bound. Bekenstein argued that the maximum entropy of a system in a region of space-time is $S = (A/4G_N)$, where A is the area of the boundary bounding the region. To show this assume that this is not true and that it is possible to have

a state in the region with $\bar{S} > (A/4G_N)$. By pumping enough energy into the region one can create a black hole of area $A_{\text{bh}} \leq A$, which means $S_{\text{bh}} \leq S < \bar{S}$. On the other hand, the second law of thermodynamics requires $S_{\text{bh}} > \bar{S}$. The only solution is that there is no such state with entropy \bar{S} and the maximum entropy possible is precisely that of a black hole with horizon area = A .

In summary we see that the AdS/CFT conjecture is a precise and explicit realization of the Holographic Principle and that the radial dimension of AdS corresponds to a length scale in the theory on the boundary of AdS.

AdS/CFT correspondence rules

If the AdS/CFT conjecture is to serve a useful purpose then we need to give a precise dictionary that relates processes in AdS to those in the gauge theory. Firstly, we would expect that to a local gauge invariant operator in the gauge theory there corresponds a field propagating in AdS. The boundary value of the field acts as a source of the operator. If we denote the field in AdS by $\phi(\vec{x}, t, z)$ and the corresponding operator by $\mathcal{O}(\vec{x}, t)$, then to first order the operator-source coupling is given by the interaction

$$S_1 = \int d\vec{x} dt \phi(\vec{x}, t, z = \delta) \mathcal{O}(\vec{x}, t) \delta^{\Delta-4}. \tag{22}$$

$\lim_{\delta \rightarrow 0} \phi(\vec{x}, t, \delta) = \phi_0(\vec{x}, t)$ is the boundary value of the AdS field $\phi(\vec{x}, t, z)$, and Δ is the scaling dim of $\mathcal{O}(\vec{x}, t)$ and the correspondence of the two theories is stated as:

$$\begin{aligned} & \left\langle \exp i \int d\vec{x} dt \delta^{\Delta-4} \mathcal{O}(\vec{x}, t) \phi_0(\vec{x}, t) \right\rangle_{\text{SCFT}} \\ & = \mathcal{Z}_{\text{string}}(\phi(\vec{x}, t, \delta \rightarrow 0) = \phi_0(\vec{x}, t)). \end{aligned} \tag{23}$$

The lhs is very precisely defined while the rhs involves the full type IIB string theory partition function, which is (at present) defined only in certain limits. There is a whole class of operators with special super symmetry properties for which we can compute Δ in terms of the mass and spin of the field $\phi(\vec{x}, t, z)$.

Tests of the AdS/CFT conjecture

To test eq. (23) we should be able to calculate both sides and compare. The strong form of the conjecture is that eq. (23) is valid for finite N and g_s . The crucial ingredient that enables us to test the conjecture are the identical symmetries of the gauge theory and the string theory. The $\mathcal{N} = 4$ gauge theory is invariant under the super-conformal group $SU(2, 2|4)$ whose bosonic sub-group is $SO(4, 2) \times SU(4)$. $SO(4, 2)$ is the conformal group in 4-dim and $SU(4)$ is the R -symmetry group corresponding to $\mathcal{N} = 4$

supersymmetry. $SU(2, 2|4)$ is also a symmetry of the IIB string theory with $AdS_5 \times S^5$ boundary conditions. We expect these symmetries to be valid for all values of g_s and N . We can make further progress if we organize the gauge theory in the 'tHooft $1/N$ expansion for fixed values of $\lambda = g_s N$. In this case, the basic relation $R/\ell_s = (4\pi\lambda)^{1/4}$ implies that $\lambda \rightarrow \infty$ corresponds to $R/\ell_s \rightarrow \infty$. In this limit and as $N \rightarrow \infty$ the string theory can be approximated by supergravity as an expansion in powers of ℓ_s/R and ℓ_p/R . ℓ_p is the Planck length defined by $\ell_p^8 = \ell_s^8 g_s^2$.

In supergravity one can perform a Kaluza–Klein reduction from $AdS_5 \times S^5$ to AdS_5 . For example, a scalar field $\Phi(x, y)$ can be expanded as

$$\Phi(x, y) = \sum_{k=0}^{\infty} \phi^k(x) Y_k(y),$$

where x is a coordinate in AdS_5 and y is a coordinate in S^5 . $Y_k(y)$ are the scalar spherical harmonics on S^5 and are given by the symmetric tensors $Y_k(y) \sim y^{l_1} y^{l_2} \dots y^{l_k}$, y^l are the coordinates of a unit vector on S^5 . From the wave equation for $\phi(x, y)$ we can infer the mass of the field $\phi^k(x)$ in AdS_5 . It turns out to be $R^2 m_k^2 = k(k+4)$ where $k(k+4)$ is the value of the Casimir in the $(0, k, 0)$ representation of $SO(6) \sim SU(4)$. On the Yang–Mills side the fields $\phi^k(x)$ correspond to traceless symmetric tensors of $SO(6)$ formed out of the operator $\mathcal{O}^{l_1 \dots l_k} = \text{Tr}(\phi^{l_1} \dots \phi^{l_k})$. Those operators also transform in the $(0, k, 0)$ representation of $SO(6)$ and have dimension k . Hence the formula $R^2 m_k^2 = k(k+4)$ implies $k = \Delta = 2 + (4 + R^2 m_k^2)^{1/2}$, a relation between the dimension Δ of an operator in the gauge theory and the mass of the corresponding field in string theory.

In a similar way one can achieve a complete correspondence of *all* supergravity short multiplets in AdS_5 and chiral primary operators in the gauge theory. A crucial point about the correspondence is that the formulae that relate dimensions of the gauge theory operators and masses of the supergravity fields are independent of λ , and hence are valid at both strong and weak couplings.

The AdS/CFT correspondence can also be tested by an exact (λ independent) matching of anomalous terms: the non-abelian anomaly of the $SU(4)$ R -symmetry currents is exactly reproduced by a corresponding $SU(4)$ Chern–Simons term in the string theory. It also turns out that the 3-point functions of chiral primary operators in the gauge theory exactly match the calculation from the string theory side.

Uses of the AdS/CFT conjecture^{2,4}

Given the physical basis, the maximal superconformal symmetry, and the spectral correspondence we sketched in the previous section, we believe that the conjecture is on a firm footing and can be used to derive complementary results and insights on both sides of the correspondence.

Heavy quark potential at strong coupling

Let us first discuss an example of a strong coupling calculation in the gauge theory using the correspondence. We wish to calculate the energy $E(L)$ of two heavy quarks, in the fundamental representation of $SU(N)$, separated by a distance L in the gauge theory. The correct operator whose expectation value evaluates this energy is a generalization of the standard Wilson loop because the ‘heavy quarks’ couple both to the gauge fields and the scalar fields Φ^I .

$$W(C) = \text{Tr} \left[P \exp \left(\oint \left(iA_\mu \frac{dx^\mu}{d\tau} + y_I \Phi^I \sqrt{\dot{x}^2} \right) d\tau \right) \right], \quad (24)$$

$x^\mu(\tau)$ defines the loop in space-time. y_I is a unit vector on S^5 and $\dot{x}^2 = \dot{x}_\mu \dot{x}^\mu$. In the limit where $R/\ell_s \rightarrow \infty$, $\langle W(C) \rangle$ can be represented as a first quantized string path integral in $\text{AdS}_5 \times S^5$, with boundary conditions at the curve C . It can be evaluated semi-classically for $\lambda \gg 1$, by a surface that minimizes the path integral. For a loop C specified by the distance L and time T , we have $\langle W \rangle \approx e^{-TE(L)}$ and this gives

$$E(L) = -\frac{4\pi^2 \sqrt{2\lambda}}{\Gamma\left(\frac{1}{4}\right)^4 L}. \quad (25)$$

The fact that $E(L) \propto 1/L$ can be inferred from conformal invariance. It is the $\sqrt{\lambda}$ dependence that is the prediction of AdS/CFT and it differs from the perturbative estimate of $E(\lambda) \propto \lambda/L$.

Thermal gauge theories at strong coupling and black holes¹²

The $\mathcal{N} = 4$, super Yang–Mills theory defined on $S^3 \times R^1$ can be considered at finite temperature if we work with euclidean time and compactify it to be a circle of radius $\beta = 1/T$, where T is the temperature of the gauge theory. We have to supply boundary conditions which are periodic for bosonic fields and are anti-periodic for fermions. These boundary conditions break the $\mathcal{N} = 4$ supersymmetry, and the conformal symmetry. However, the AdS/CFT conjecture continues to hold and we will discuss the relationship of the thermal gauge theory with the physics of black holes in AdS.

As we have mentioned in the limit of large N (i.e. $G_N \ll 1$) and large λ (i.e. $R \gg \ell_s$), the string theory is well approximated by supergravity, and we can imagine considering the Euclidean string theory partition function as a path integral over all metrics which are asymptotic to AdS_5 space-time (for the moment we ignore S^5).

The saddle points are given by the solutions to Einstein’s equations in 5-dim with a $-ve$ cosmological constant

$$R_{ij} + (4/R^2)g_{ij} = 0. \quad (26)$$

As was found by Hawking and Page, a long time ago, there are only two spherically symmetric metrics which satisfy these equations with AdS_5 boundary conditions: AdS_5 itself and a black hole solution. The metric for both solutions can be written as

$$ds^2 = -V(r)dt^2 + V^{-1}(r)dr^2 + r^2 d\Omega_3^2$$

$$V(r) = 1 + \frac{r^2}{R^2} - \frac{\mu}{r^2}, \quad (27)$$

$\mu = 0$ corresponds to AdS_5 and $\mu > 0$ leads to a horizon radius r_+ given by $V(r_+) = 0$. The temperature of the black hole is

$$\frac{1}{TR} = \frac{2\pi R r_+}{2r_+^2 + R^2}. \quad (28)$$

Let us denote the $\mu = 0$ solution by X_1 and the $\mu > 0$ solution by X_2 . These two spaces are topologically distinct because in X_2 the boundary circle can be shrunk to zero while in X_1 that is not possible. In fact this property of X_2 defines a Euclidean black hole. As a function of temperature eq. (28) has in fact two roots and we will choose the largest root as it corresponds to a black hole with positive specific heat. Note that for $r_+ \gg R$, $TR \sim (r_+/R)$.

We can also calculate the euclidean Einstein–Hilbert action of X_1 and X_2 . Since both actions are infinite we can calculate both of them using an appropriate cut off and then evaluating the difference

$$I(X_2) - I(X_1) = \frac{\pi^2 r_+^3 (R^2 - r_+^2)}{4G_5 (2r_+^2 + R^2)}. \quad (29)$$

The important point is that there occurs a change of dominance from X_1 to X_2 at $r_+ = R$ and for $r_+ > R$, $I(X_2) < I(X_1)$. The temperature at $r_+ = R$ is $TR = (3/2\pi) \sim o(1)$. Hawking and Page interpreted this as a first order phase transition from a phase consisting of thermal gravitons to a large black hole. For $r_+ \gg R$, eq. (29) becomes

$$I(X_2) - I(X_1) = -\frac{\pi^5}{8} (TR)^3 \frac{1}{R^{-3}G_5} = -\frac{\pi^5}{8} N^2 (RT)^3, \quad (30)$$

where we have used $R^5 G_5 = G_{10} = g_s^2 \ell_s^8$ and $(R/\ell_s) = (g_s N)^{1/4}$.

Now let us discuss the above phenomenon in dual gauge theory at finite temperature. The $\mathcal{N} = 4$ Super Yang–

Mills theory at finite temperature involves a continuation to periodic Euclidean time $\tau \sim \tau + 2\pi\beta$. Hence the theory is defined on $S^3 \times S^1$. A well-known order parameter at finite temperature is the Polyakov line defined as

$$P(\bar{x}, \beta) = \text{Tr} \left[P \exp \left(\int_0^\beta d\tau A_0(\bar{x}, \tau) \right) \right]. \tag{31}$$

Now $SU(N)$ has a non-trivial centre given by $Z_N \{ e^{i2\pi k/N}, k = 1, \dots, N \}$.

All local gauge invariant operators are invariant under Z_N . However, the order parameter eq. (31) transforms as

$$P'(\bar{x}, \beta) = gP(\bar{x}, \beta), \quad g \in Z_N. \tag{32}$$

The phase in which $\langle P \rangle = 0$, Z_N symmetry is intact. It is called the ‘confinement phase’ because the dominant excitations in this phase correspond to colour singlet single trace matrix products of the fields of the gauge theory. Denoting a typical $SU(N)$ matrix valued field by M these excitations correspond to $\text{Tr}(M_{i_1} M_{i_2} \dots M_{i_n})$, where the length of the ‘string’ $n < N^2$. The free energy in this phase $F \sim N^0$.

In the phase $\langle P \rangle \neq 0$, Z_N symmetry is broken. This is called the ‘deconfinement phase’, because the N^2 ‘colour’ degrees of freedom are deconfined and the free energy $F \sim o(N^2)$. In fact by using the fact that the underlying theory is conformally invariant, the free energy in this ‘deconfinement phase’ is given by

$$F \sim N^2(RT)^3. \tag{33}$$

This answer matches eq. (30). Since the gauge theory is strongly coupled it is not possible to compute the numerical coefficient in eq. (32), however on the gravity side it can be computed!

The conclusion we can draw from the agreement of eqs (30) and (32) is that the ‘deconfinement’ phase of the gauge theory corresponds to the presence of a large black hole in AdS. Besides qualitative predictions like eq. (32) it is difficult to make precise quantitative statements about the gauge theory at strong coupling ($\lambda \gg 1$). However, on the AdS side the calculation in gravity is semi-classical and hence there are precise quantitative answers for

- (i) the temperature at which the first order confinement–deconfinement transition occurs:

$$T_c = \frac{3}{2\pi R}$$

- (ii) the latent heat at $T = T_c$

$$F(T_c) = N^2 \frac{9\pi^2}{64}$$

- (iii) the free energy for $T > T_c$

$$F(T) = N^2 \frac{\pi^5}{8} (RT)^3.$$

In the preceding two examples we saw how calculations in the strongly coupled gauge theory can be done using the AdS correspondence by using semiclassical gravity in the limit of $G_N \sim (1/N^2) \ll 1$ and $(R/\ell_s) \sim \lambda^{1/4} \gg 1$.

We will now indicate how the correspondence can be used to make some non-trivial statements about the string theory (supergravity) in AdS.

- (i) Just like in the case of the D1–D5 system, since the $\mathcal{N} = 4SU(N)$ gauge theory is unitary, we can assert that there cannot be information loss in any process in AdS, including the process of the formation and evaporation of black holes. However these processes need to be identified and worked out in this gauge theory.
- (ii) Another direction involved a study of the dynamics of small Schwarzschild black holes in AdS. Here when the horizon of the black hole is of order the string scale, $r_h \sim \ell_s$, the supergravity (space–time) description breaks down. The string theory in this region is not (yet) defined except by its correspondence with the gauge theory. The string size black hole of temp $T \sim \ell_s^{-1}$ is studied using a double scaling limit of the GWW (Gross–Witten–Wadia) large N phase transition in the gauge theory. In the scaling region $T - T_c \sim N^{-2/3}$ a non-perturbative description of a small string length size black hole was obtained¹³.

Conformal fluid dynamics and the AdS/CFT correspondence

We have seen that the thermodynamics of the strongly coupled gauge theory in the limit of large N and large λ is calculable, in the AdS/CFT correspondence, using the thermodynamic properties of a large ($r_+ \gg R$) black hole in AdS₅. We now discuss how this correspondence can be generalized to real time dynamics in this gauge theory when both N and λ are large. We will discuss a remarkable connection between the (relativistic) Navier–Stokes equations of fluid dynamics and the long wavelength oscillations of the horizon of a black brane which is described by Einstein’s equations of general relativity in AdS₅ space–time.

On general physical grounds a local quantum field theory at very high density can be approximated by fluid dynamics. In a conformal field theory in 3 + 1 dim we expect the density $\rho \propto T^4$, where T is the local temperature of the fluid. Hence fluid dynamics is a good approximation for length scales $L \gg 1/T$. The dynamical variables of relativistic fluid dynamics are the four

velocities: $u_\mu(x)$ ($u_\mu u^\mu = -1$), and the densities of local conserved currents. The conserved currents are expressed as local functions of the velocities, charge densities and their derivatives. The equations of motion are given by the conservation laws. An example is the conserved energy–momentum tensor of a charge neutral conformal fluid:

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu + P\eta^{\mu\nu} - \eta \left(P^{\mu\alpha} P^{\nu\beta} (\partial_\alpha u_\beta + \partial_\beta u_\alpha) - \frac{1}{3} P^{\mu\nu} \partial_\alpha u^\alpha \right) + \dots \quad (34)$$

where ϵ is the energy density, P the pressure, η the shear viscosity and $P^{\mu\nu} = u^\mu u^\nu + \eta^{\mu\nu}$. These are functions of the local temperature. Since the fluid dynamics is conformally invariant (inheriting this property from the parent field theory) we have $\eta_{\mu\nu} T^{\mu\nu} = 0$ which implies $\epsilon = 3P$. The pressure and the viscosity are then determined in terms of temperature from the microscopic theory. In this case conformal symmetry and the dimensionality of space–time tells us that $P \sim T^4$ and $\eta \sim T^3$. However the numerical coefficients need a microscopic calculation.

The Navier–Stokes equations are given by eq. (34) and

$$\partial_\mu T^{\mu\nu} = 0. \quad (35)$$

The conformal field theory of interest to us is a gauge theory and a gauge theory expressed in a fixed gauge or in terms of manifestly gauge invariant variables is not a local theory. In spite of this eq. (34) seems to be a reasonable assumption and the local derivative expansion in eq. (34) can be justified using the AdS/CFT correspondence.

We now briefly indicate that the eqs (34) and (35) can be deduced systematically from black brane dynamics. Einstein’s eq. (26) admits a boosted black-brane solution

$$ds^2 = -2u_\mu dx^\mu dv - r^2 f(br) u_\mu u_\nu dr^\mu dx^\nu + r^2 P_{\mu\nu} dx^\mu dx^\nu, \quad (36)$$

where v, r, x^μ are in-going Eddington–Finkelstein coordinates and

$$f(r) = 1 - \frac{1}{r^4}$$

$$u^\nu = \frac{1}{\sqrt{1 - \beta_i^2}}, \quad u^i = \frac{\beta^i}{\sqrt{1 - \beta_i^2}}, \quad (37)$$

where the temperature $T = 1/\pi b$ and the velocities β_i are all constants. This 4-parameter solution can be obtained from the solution with $\beta^i = 0$ and $b = 1$ by a boost and a scale transformation. The key idea is to make b and β^i

slowly varying functions of the brane volume, i.e. of the co-ordinates x^μ . One can then develop a perturbative non-singular solution of eq. (26) as an expansion in powers of $1/LT$. Einstein’s equations are satisfied provided the velocities and pressure that characterize eq. (36) satisfy the Navier–Stokes equation¹⁴. The pressure P and viscosity η can be exactly calculated to be^{14,15}

$$P = (\pi T)^4 \text{ and } \eta = 2(\pi T)^3. \quad (38)$$

Using the thermodynamic relation $dP = s dT$ we get the entropy density to be $s = 4\pi^4 T^3$ and hence obtain the famous equation of Policastro, Son and Starinets¹⁶,

$$\frac{\eta}{s} = \frac{1}{4\pi}, \quad (39)$$

which is a relation between viscosity of the fluid and the entropy density.

Systematic higher order corrections to eq. (34) can also be worked out.

In summary we have a truly remarkable relationship between two famous equations of physics, viz. Einstein’s equations of general relativity and the relativistic Navier–Stokes equations. This relationship is firmly established for a 3 + 1 dim conformal fluid dynamics which is dual to gravity in AdS₅ space-time. A similar connection holds for 2 + 1 dim fluids and AdS₄ space–time.

Finally, it is hoped that the AdS/CFT correspondence lends new insights to the problem of turbulence in fluids. Towards this goal the AdS/CFT correspondence has also been established for forced fluids, where the ‘stirring’ term is provided by an external metric and dilaton field¹⁷.

In summary: (i) The AdS/CFT correspondence allows us to discuss a strongly coupled ($\lambda \gg 1$) gauge theory at large density ($T \gg 1$) as a fluid dynamics problem whose equations are the relativistic Navier–Stokes equations. The various transport coefficients and thermodynamic functions can be exactly calculated! These results have encouraging implications for experiments involving the collisions of heavy ions in spite of the fact that the relevant gauge theory is QCD rather than $\mathcal{N} = 4$ Yang–Mills theory. The experiments at RHIC seem to support rapid thermalization and a strongly coupled quark–gluon plasma with very low viscosity coefficient. There exists a window of temperatures where the plasma behaves like a conformal fluid. The AdS/CFT correspondence also provides a calculational scheme for propagation of heavy quarks and jet quenching in a strongly coupled plasma.

(ii) The Einstein equations enable a systematic determination of higher derivative (in the velocities) terms of the Navier–Stokes equations.

(iii) The relationship between dissipative fluid dynamics and black hole horizons, known as the membrane

paradigm, has found a precise formulation within the AdS/CFT correspondence.

(iv) The Navier–Stokes eqs (34) and (35) imply dissipation and violates time reversal invariance. The scale of this violation is set by η/ρ which has the dim of length (in units where the speed of light $c = 1$). There is no paradox here with the fact that the underlying theory is non-dissipative and time reversal invariant, because we know that the Navier–Stokes equations are not a valid description of the system for length scales $\ll \eta/\rho$, where the micro-states should be taken into account.

QCD type theories and the AdS/CFT correspondence

QCD is not a conformally invariant theory. As is well known at weak coupling and at a length scale L , we have asymptotic freedom

$$Ng^2(L\Lambda) = -\frac{1}{\beta_0 \ln L\Lambda}, \quad \beta_0 = \frac{11}{24\pi^2},$$

Λ^{-1} is a fixed length that characterizes the theory. In the real world $\Lambda \simeq 200$ meV, which corresponds to a length scale $\simeq 10^{-13}$ cm. For $L\Lambda \gtrsim 1$, the theory is strongly coupled and difficult to calculate.

Presently the best we can do is to study modifications of the $\mathcal{N} = 4$ super-Yang–Mills theory, which we briefly mention.

(a) Deforming the $\mathcal{N} = 4$ theory by ‘relevant’ operators can lead to new massive fixed points which are characterized by a scale. This can be established at strong coupling by seeking a new supergravity solution that, in the interior of the 5-dim space–time, differs from AdS_5 . The new solution breaks the supersymmetry from $\mathcal{N} = 4$ to $\mathcal{N} = 1$ (ref. 18).

(b) One can wrap branes on cycles of space–time geometry that break the supersymmetry. The simplest example of this, is the wrapping of D4 branes on the thermal circle¹². The dual supergravity background is quite simple as compared to a model in which D5-branes are wrapped on a collapsed 2-cycle at a conifold singularity¹⁹.

Apart from reproducing qualitative features of non-perturbative phenomena such as confinement, chiral symmetry breaking and the low-energy spectrum of QCD, perhaps the most interesting qualitative results that have been obtained in this approach are the QCD-like behaviour in high energy and fixed angle (hard) scattering and the qualitative properties of pomeron exchange. These studies have also yielded exciting connections of the Froissart bound in high energy scattering with black hole physics²⁰.

Finally, we mention a novel application of the AdS/CFT correspondence to gluon scattering amplitudes that employs a momentum space dual of AdS_5 space–time²¹.

String theory, elementary particles and cosmology

In the preceding sections, we have tried to present a case for string theory as a framework to formulate a quantum theory of gravity. String theory has passed many tests in this regard because it leads to a perturbatively finite theory of gravity, and enables a calculation of black hole entropy for a class of black holes. This has been possible primarily due to two important ingredients of string theory: (i) supersymmetry and (ii) new degrees of freedom ‘ p -branes’ and in particular D -branes. At weak string coupling these are the solitons of string theory.

The idea of supersymmetry requires that space–time has additional fermionic co-ordinates besides the familiar bosonic co-ordinates, and supersymmetric transformations mix these bosonic and fermionic co-ordinates. The existence of D -branes is also intimately connected with the fact that the underlying theory is supersymmetric. As we saw, in the discussion of black hole entropy, they are essential for a unitary description of quantum gravity.

The microscopic understanding of black hole entropy and Hawking radiation led to a precise formulation of holography, which may very well be one of the guiding principles of string theory. The Maldacena conjecture gives a precise and calculable formulation of this idea. Presently there are various such dualities (AdS/CFT correspondence) known in various space–time dimensions. The most recent addition to this list is a correspondence of a Chern–Simons gauge theory in 3-dim and M2 branes which are objects of 11-dim M-theory²².

The AdS/CFT correspondence seems to be useful for doing strong coupling calculations not only in gauge theories but also for several strongly coupled condensed matter systems, see e.g. refs 23 and 24. Besides this, it also led to a duality between long wavelength motions on a black brane horizon and dissipative fluid dynamics at high temperature and density.

Besides, the unearthing of deep theoretical and mathematical structures, we must ask whether the string theory framework enables a description of elementary particle physics and cosmology. We briefly comment on this point.

Elementary particle physics (see note 6)

Our present understanding of elementary particle physics is based on the standard model (SM) based on the gauge groups $SU(2) \times U(1) \times SU(3)$. $SU(2) \times U(1)$ describes the electroweak sector and $SU(3)$ the strong interactions. All particles of the SM, except the Higgs, have been experimentally discovered and the SM gives a precise description of elementary particle physics and the strong interactions up to energies $\lesssim 100$ GeV. However, the Higgs is not yet discovered (see note 7) and further there are 23 phenomenological parameters that are not calculable

within the SM. These parameters account for the masses of the quarks and leptons, the sum of the Higgs, the masses of the neutrinos and the small parameters that account for CP violation in the weak and strong interactions.

It is well known that while a non-zero expectation value of the Higgs field accounts for electro-weak symmetry breaking and gives non-zero masses to the quarks and leptons, its fluctuations led to a quadratic dependence on the ultra-violet cut-off Λ , which make it difficult to explain the hierarchy of the electro-weak scale (~ 100 GeV) to the Planck scale ($\sim 10^{19}$ GeV): $M_{EW}/M_{Pl} \simeq 10^{-17}$. One of the proposed and robust mechanisms that preserves this hierarchy, is supersymmetry. The presence of ‘fermionic loops’ cancels the quadratic divergence!

The minimal supersymmetric standard model (MSSM) predicts a unification of weak, electromagnetic and strong interactions at $\sim 10^{16}$ GeV, which is quite close to the Planck scale $\sim 10^{19}$ GeV. This fact hints at a possible unification of the weak, electromagnetic and strong interactions with gravity. It is important to note that this unification fails to happen in the absence of supersymmetry. Besides, unification, MSSM also provides a ‘dark matter’ candidate, viz. a neutralino, which is the lightest, stable, neutral supersymmetric fermionic particle. It is ‘dark matter’ because it has only weak and gravitational interactions. One of the great expectations of the LHC (large Hadron collider), besides the discovery of the Higgs particle, is the discovery of supersymmetry, and a possible dark matter candidate! Such a discovery would indeed have profound consequences for both elementary particles and cosmology²⁶. Another proposed dark matter candidate is the axion.

Besides the MSSM there are other ‘effective field theory’ proposals involving higher dimensions, low scale supersymmetry, and warped Randall–Sundrum type compactifications, that are being explored. In these scenarios the hierarchy problem disappears because the string scale (or equivalently the 10-dim Planck scale) is brought down to 1 TeV, so that quantum gravity effects can become experimentally accessible at accelerators. In the Randall–Sundrum scenario the SM lives on a 3-brane while only gravity extends in the additional six dimensions. The hierarchy of scales, or the smallness of the weak scale as compared to the Planck scale, is explained in terms of a gravitational red shift that occurs from a Planck brane situated in the additional dimension.

We will not discuss the details of the various ‘beyond the standard model’ proposals. Fortunately the LHC, which goes into operation a few months from now, will in the next few years give (hopefully!) a verdict about nature’s choice. For the sake of argument let us optimistically assume that supersymmetry is discovered at the LHC. It is likely that most of the physics below 1 TeV can be described in terms of an effective 4-dim lagrangian, with a partial list of parameters fitted from experiment. It

would be difficult to find direct evidence of string theory in a ‘low energy’ effective lagrangian unless we can calculate its parameters from an underlying theory. The issue is similar to asking whether we can infer the details of the atomic and molecular composition of a fluid knowing the fluid dynamics equations and the various coefficients like viscosity that enter the equations.

In spite of the difficulty in identifying a signature of string theory at low energies, the discovery of supersymmetry at the LHC would be an encouraging sign about the string theory framework. The difficult theoretical problem is the exact emergence of the MSSM (or its variants) from string theory. Brane constructions, compactification on manifolds with singularities do come close to delivering the MSSM, within the framework of a theory which also includes a consistent quantum theory of gravity. This point may have a consequence for one of the most puzzling facts of nature, viz. the cosmological constant is a small but non-zero number which is equivalent to a vacuum energy density of 10^{-8} erg/cm³.

Cosmology^{27–29}

The so called ‘SM of cosmology’ is not as well developed as the SM of particle physics. It is an effective theory with fewer parameters and its basic equations are Einstein’s equations of general relativity, where the stress tensor depends upon the matter composition of the universe. Even though there is no concrete dynamical model of inflation (there are proposals), it is a key idea that reconciles the Big Bang hypothesis with a large, homogeneous and isotropic universe with fluctuations that eventually led to the formation of matter and galaxies over a period of 14 billion years.

The large volume of experimental data from the cosmic micro-wave background (CMB) radiation seems to be consistent with a ‘flat’ homogeneous and accelerating universe. Another important fact is that only 4% of the energy density of the universe is the matter that we are familiar with, viz. the matter content of the standard model of particle physics. The next abundant 22% energy density source comes from ‘dark matter’, which is electrically neutral and hence optically dark. The rest of the 76% is ‘vacuum energy’ parametrized in the Einstein theory by the cosmological constant.

There are several very basic questions we do not have answers to and which are subjects of active research. We briefly comment on these.

(i) Inflation is usually modelled by a single scalar field called the inflaton. String theory which seems to be a compelling ultra-violet completion of general relativity does not lead to a single scalar field but to a whole host of scalar fields (moduli corresponding to shapes and sizes of the internal compact newfold) which naturally occur in string compactifications. The moduli stabilization problem is the same as giving a mass to the corresponding scalar

fields. One possibility is to turn on discrete fluxes in the six compact directions of string theory. This leads to, in models that have been studied, to a lifting of all but the ‘size’ or volume modulus, and the potential as a function of this modulus can be calculated. The minimum turns out to be an AdS space–time. In order to break the supersymmetry and raise the ground state energy by a small amount, a probe anti-D3 brane is introduced. In this way a meta-stable vacuum with positive vacuum energy is achieved. This leads to a de Sitter space–time with a non-zero and positive cosmological constant. The above construction is called the Kachru, Kallosh, Linde and Trivedi (KKLT) scenario³⁰. This and various other D -brane inflation scenarios are a subject of active investigations. It is fair to say that presently, the ‘slow roll’ or prolonged inflation seems to be difficult to achieve. Perhaps progress in locating the SM vacuum in string theory may contribute to a solution of this important and difficult problem.

(ii) The SM of cosmology has the inevitable space like ‘cosmological’ singularity, as one scales the size of the universe to zero. Here general relativity breaks down, and string theory should certainly be relevant for a resolution of this singularity. There are various attempts in this direction within effective field theory, perturbative string theory and matrix models. What would be desirable is to find a model cosmology in AdS₅ and study its hologram in the gauge theory on the boundary of AdS₅. A resolution of the space-like singularity of a black hole is bound to shed light on this fundamental question.

(iii) As we mentioned in (i), the turning on fluxes in the compact manifold leads to a large number ($\sim 10^{500}$) of acceptable ground states of string theory. The question arises whether there is a drastic reduction of these consistent ground states in the full non-perturbative theory. We do not know the answer to this question, because we do not know non-perturbative string theory well enough except in the case of AdS space–times, where it is dual to a $SU(N)$ gauge theory. However one cannot preclude the possibility that our universe, which presently has a vacuum energy density $\rho_{\text{rac}} \simeq 10^{-8}$ erg/cm³, is not special! As Weinberg³¹ noted, if $\rho_{\text{rac}} > 10^{-8}$ erg/cm³, we would have an universe in which galaxies could not have formed. The key question is whether such an uninteresting universe is also a consistent solution of ‘string theory’?

In summary, it is fair to say that we presently do not know string theory well enough both conceptually and technically to provide answers to questions and issues we have discussed above. It seems certain that we will need to explore models in which both cosmology and particle physics are tied up in a dynamical way.

Epilogue

In this review, we have made the case for string theory as a framework for a finite theory of quantum gravity that goes beyond quantum field theory. Along the way to realize

this, a host of new and intricate structures have been discovered. Prominent among these are supersymmetry and the holographic principle. The *raison d’être* of string theory is elementary particle physics and cosmology, and the quest to answer basic questions related to the fundamental structure of matter and the laws of the cosmos. However like all fertile ideas in science, string theory and its methods make a connection with other areas of physics and mathematics. Its influence on geometry and topology is well known (see note 8)³². Its ability to solve outstanding strong coupling problems in condensed matter physics is being realized in the AdS/CFT correspondence. The connection with fluid dynamics is also tantalizing and may perhaps shed light on the problem of turbulence.

A more popular exposition of the topics we have discussed here can be found in ref. 33. Especially relevant are the articles by D. Gross, M. F. Atiyah, A. Sen, A. Dabholkar and S. Sarkar.

Notes

1. A concise modern reference on string theory is the book by E. Kiritsis².
2. Supersymmetry was discovered first in string theory by Ramond, Neveu, Schwarz, and Gervais and Sakita. This later inspired the construction of supersymmetric field theories³ in $3 + 1$ dim.
3. e.g. a D0 brane couples to a 1-form gauge field $A_{\mu}^{(1)}$, a D1 brane couples to a 2-form gauge field $A_{\mu\nu}^{(2)}$ etc.
4. For a review see refs 2, 6 and 7.
5. For a review see refs 2 and 4.
6. A modern reference is the book by M. Dine²⁵.
7. The SM precision tests put a bound on the mass of the Higgs: $M_{\text{H}} \gtrsim 114$ GeV.
8. Also see M. F. Atiyah’s article on ‘Einstein and Geometry’ in ref. 33.

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