PHYSICAL REVIEW D VOLUME 12, NUMBER 3 1 AUGUST 1975

Hydrodynamical expansion with frame-independence symmetry in high-energy multiparticle production*

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(Received 23 December 1974; revised manuscript received 3 April 1975)

We describe the space-time development of the hardronic system formed immediately after the high-energy hadron collision with the following picture. Initially the system is highly compressed along the longitudinal direction. The sudden relaxation of this compression leads to a violent acceleration along this direction and perhaps a weak acceleration along the transverse direction. When these accelerations cease, we propose that the system acquires certain frame-independence symmetry with its further expansion governed by the hydrodynamic equation of motion. Within our scheme, this symmetry provides a natural mechanism which eventually leads to a flat inclusive longitudinal rapidity distribution and it also admits a sharp cutoff in the inclusive transverse momentum distribution. These features differ from those of Landau's model.

I. INTRODUCTION

For a space-time description of hadron multiparticle production at high energies, it is convenient to divide the process immediately after collision into three stages: the formation of a highly condensed matter system, its expansion, and the eventual decay. The formation of a hadronic system with a global thermal equilibrium was first suggested by Fermi.[1] Its subsequent expansion process was treated as a fluid shock by Heisenberg[2] and as a hydrodynamical problem by Landau.[3] Bhabha also treated this problem in a somewhat different context.[4] Recently there has been renewed interest in hydrodynamical models,[5-11] especially Landau's model. Within his model the expansion process begins with a one-dimensional longitudinal expansion and it is followed by a three-dimensional expansion. The one-dimensional part is solved analytically.[12-13] However, so far there is no completely satisfactory solution for the three-dimensional expansion.

We have also investigated hadron multiparticle production, making use of some ideas of hydrodynamics and mainly concerned with longitudinal expansion. However, we take a somewhat different point of view from that of Landau. We propose that the local elements of the condensed matter system first undergo a violent acceleration along the longitudinal direction. The system reaches a state of certain frame-independence symmetry. Only from this point onward is the system governed by the hydrodynamical equation of motion. This equation demands a specific rate of expansion. This rate in turn ensures the symmetry to prevail throughout the expansion. Our solution predicts a central plateau for the inclusive longitudinal rapidity distribution.[14] We show in the Appendix, even after we have taken into account the transverse motion of the matter system, that our solution remains compatible with having an energy-independent average transverse momentum for the inclusive pions. These features differ from those of Landau's model.

II. HADRONIC MATTER SYSTEM PRIOR TO ITS BREAKUP

A. The initial matter system

For simplicity we consider collisions between two identical hadrons such as nucleon-nucleon collisions. But we shall neglect their spins. Initially in the center-of-mass system, each incident nucleon is surrounded by a meson cloud with a characteristic transverse dimension $\pi\mu^{-2}$, where μ is the pion mass. Along the longitudinal direction there is Lorentz contraction, so the dimension is $\Delta \sim (\mu\gamma)^{-1}$ with γ being the usual Lorentz contraction factor $W/(2m_N)$ and W and m_N being the c.m. energy and the nucleon mass.

Let the collision take place at $t=0$. The two "bare" nucleons pass by each other and emerge as leading particles in the final state, which may also be accompanied by some fragmentation production. In the meantime, the meson clouds left behind form a condensed hadronic system. The matter system should be regarded as some effective system averaged over headon and nonheadon collisions.

We denote the initial c.m. energy and momentum of the nucleons by E^* and p^* and those of the two leading-particle systems by E_1^* and $\vec{p}_1^*$ and E_2^* and $\vec{p}_2^*$. Thus the energy and momentum of the remaining matter system are

$$E_M = 2E^* - (E_1^* + E_2^*),$$
$$\vec{p}_M = -(\vec{p}_1^* + \vec{p}_2^*),$$
and
$$M = (E_M{}^2 - \vec{p}_M{}^2)^{1/2}, \tag{1}$$

where M is the invariant mass of the matter system. Typically we expect $M \sim e_1 W$, where e_1 is the inelasticity parameter which is presumably constant at high energies. Our consideration below will be given in the c.m. frame of the matter system.

Due to the earlier longitudinal motion of the meson clouds, the matter system left behind continues to be compressed until, say, $t = t_1$, when the associated kinematic energy has transformed into matter and also into some sort of potential energy. Presumably, at this point the entire system is approximately at rest. We suggest $t_1 \sim O(\Delta)$, which corresponds to the characteristic time taken for the complete overlap of the two initial systems.

B. Violent expansion and the frame-independence symmetry

This stationary state is immediately followed by a violent acceleration along the longitudinal direction. We divide the entire matter system into cells with their coordinates specified by

$$x^\mu = (x^0, x^1, x^2, x^3) \equiv (\rho \cosh\alpha, x^1, x^2, \rho \sinh\alpha). \quad (2)$$

In general, different local cells may experience different accelerations. After a certain proper time has elapsed, the acceleration ceases. Probably the proper time interval involved is again of the order of the initial longitudinal dimension of the system. For definiteness, say this is at the local proper time $\rho = \rho_2 \sim O(\Delta)$. At this stage, we assume that the system has local thermal equilibrium. And it has also reached the state of a certain frame-independence symmetry. In particular, it appears to have lost its memory about the longitudinal direction except for the original space-time point of collision. In other words, the system appears to be identical in all frames related to the c.m. frame by a boost transformation along the longitudinal direction.[15]

We assume that the motion of the cells along the transverse direction is negligible (see Appendix A for justification on this assumption). With the neglect of the edge effect, the proper quantities of the cells in the interior of the system are expected to be independent of the transverse coordinates x_1 and x_2. It is convenient to describe this frame-independence symmetry in the two-dimensional subspace with the coordinates

$$\tilde{x}^\mu = (x^0, x^3) = (\rho \cosh\alpha, \rho \sinh\alpha). \quad (3)$$

At $\rho = \rho_2$, the only scalar and unit vectors at one's disposal are

$$\rho = (\tilde{x}_\mu \tilde{x}^\mu)^{1/2}$$

and (4)

$$\tilde{\eta}^\mu = \tilde{x}^\mu / \rho = (\cosh\alpha, \sinh\alpha).$$

Therefore, all proper quantities are functions of ρ only, and tensorial quantities are to be formed from $\tilde{\eta}_\mu$.

Returning to the four-dimensional space-time at $\rho = \rho_2$, for $r = (x_1^2 + x_2^2)^{1/2} < \mu^{-1}$, we have

$$\begin{aligned} u^\mu &= (\cosh\alpha, 0, 0, \sinh\alpha), \\ T^{\mu\nu}(x) &= [\epsilon(\rho) + p(\rho)] u^\mu u^\nu - g^{\mu\nu} p(\rho), \end{aligned} \quad (5)$$

where ϵ and p are the proper density and the proper pressure of the cells. We will see that these equations hold for $\rho > \rho_2$ also.

The quantity α may be identified as the rapidity of the cell. The parameter ρ is by definition the proper time of the frame which is related to the c.m. frame by a boost transformation along the x_3 axis with velocity x^3/x^0. For $\rho \gg \rho_2$, ρ is essentially the proper time of those cells to be located in the transverse plane which passes through the origin of the transformed frame.

For $\rho > \rho_2$, the hydrodynamical equations of motion are given by[16]

$$\partial_\mu T^{\mu\nu} = 0. \quad (6)$$

In terms of the variables α and ρ, Eqs. (6) are reduced to one nontrivial equation,

$$\frac{d\epsilon}{d\rho} + \frac{\epsilon + p}{\rho} = 0. \quad (7)$$

With the thermodynamical relations, $\epsilon + p = Ts$ and $d\epsilon = T\,ds$, one obtains

$$\frac{ds}{d\rho} + \frac{s}{\rho} = 0, \quad (8)$$

where s is the proper entropy density. This is equivalent to the continuity equation for the entropy,

$$\partial_\mu (s u^\mu) = 0. \quad (9)$$

The solution of the differential equation of Eq. (8) is

$$s\rho = \text{const or } s \propto 1/\rho. \quad (10)$$

We see that, once the status of the frame-independence symmetry is assumed at $\rho = \rho_2$, the hydrodynamical equations of motion predict an inverse law of expansion for the entropy density and maintain this status until the breakup of the matter system.

During this expansion interval, the cell velocity is given by $v = x_3/t$, so each cell appears to be expanding with a constant velocity. However, this does not imply that the pressure gradient at each cell is zero. We observe that for any given cell with $x_3 > 0$ ($x_3 < 0$), there is always a positive (negative) temperature gradient along the longitudinal direction, and in turn a net inward force on the cell toward the origin. This inward force is exactly compensated by the force due to the inward flow of the matter flux. This is reminiscent of the mechanism for a rocket traveling in outer space which maintains a constant velocity, despite the constant bombardment of the meteorites.

III. BREAKUP OF THE MATTER SYSTEM AND INCLUSIVE DISTRIBUTION

A. The breakup criterion and the decay temperature

While the expansion is going on, the proper matter density of local elements decreases rapidly. Similarly to the usual approach,[17] we assume that there is some critical matter density n_π below which the hadronic matter does not exist.[18] This density is presumably governed by the range of strong interaction or $n_\pi = (4/3\pi)\mu^{-3}$. Therefore, when the local element has reached this critical density, the matter inside it coalesces into hadrons, such as pion and other species. We make the usual assumption that the boson and fermion constituents within local elements satisfy the Bose-Einstein and the Fermi-Dirac distribution, respectively.

For the determination of the decay temperature, we shall neglect the contribution of other species. Then the matter density

$$n_\pi = \frac{3}{4\pi}\mu^3 = \frac{g_\pi}{(2\pi)^3}\int d^3p f(\vec{p}, \mu) \tag{11}$$

with

$$f(\vec{p}, \mu) = \{\exp[(\mu^2 + \vec{p}^2)^{1/2}/T_\pi] - 1\}^{-1}, \tag{12}$$

where $g_\pi = 3$ is the statistical weight of the pion isospin multiplet. From Eq. (11), the temperature at the breakup is found to be $T_\pi \approx \mu$.

We mention that at this temperature the ratio of the nucleon density to the pion density can be shown to be

$$n_N/n_\pi \sim g_N\left(\frac{T_\pi m_N}{2\pi\mu^2}\right)^{3/2} \exp(-m_N/T_\pi) \quad \text{with } g_N = 4, \tag{13}$$

which is much less than unity and is energy independent. This ratio of particle densities governs the ratio of the heights of the central plateau of the corresponding rapidity distributions at asymptotic energies.

B. The total entropy and the average multiplicity

The hydrodynamical equation provides the space-time development of the entropy density. However, to calculate the particle distribution, we need the matter density. We relate these two densities at breakup through the statistical distribution assumed. In particular,

$$s_\pi = \frac{g_\pi}{(2\pi)^3}\int d^3p \frac{1}{T_\pi}\left(E + \frac{p^2}{3E}\right) f(p, \mu) \equiv n_\pi/b, \tag{14}$$

where $E = (\mu^2 + \vec{p}^2)^{1/2}$. With $T_\pi \approx \mu$, one gets $b = 4.2$. Note for massless case ($\mu = 0$), one has $b = 4$. Summing over the contributions from the appropriate volume elements at the fixed decay temperature T_π, one finds that the total entropy S and the average multiplicity N are linearly related:

$$N = bS. \tag{15}$$

Hence, the particle distribution can be deduced from the entropy distribution.

For most discussions given in this work, further detailed specification of the average multiplicity N is not needed. However, for completeness, we suggest a possible lower bound for the energy dependence of N. We recall the total entropy is conserved for $\rho \geq \rho_2$. Denote the proper time at breakup by ρ_π. Then,

$$S(\rho_\pi) = S(\rho_2) \geq S(t = t_1). \tag{16}$$

If one were to assume that the thermal equilibrium had already been reached at $t = t_1$, and the equation of state is $\epsilon = p/3$, one gets the usual relation

$$S(t_1) \propto M^{1/2},$$

or

$$N(\rho_\pi) \gtrsim M^{1/2}. \tag{17}$$

C. The cell longitudinal rapidity distribution

At the breakup, the velocity distribution of the cells is given by

$$dN = b\,dS = b s_\pi u_\mu d\sigma^\mu, \tag{18}$$

with

$$d\sigma^\mu = (dx^1dx^2dx^3, dx^2dx^3dx^0, dx^3dx^0dx^1, dx^0dx^1dx^2). \tag{19}$$

The quantity dN is the number of pions inside the cell, which has a four-velocity u_μ. From Eqs. (5) and (18), one has at $\rho = \rho_\pi$,

$$dN = b s_\pi(u^0dz - u^3dt)dx_1dx_2 = b s_\pi \rho_\pi d\alpha\, dx_1\, dx_2. \tag{20}$$

We know that within $r < \mu^{-1}$, s_π does not depend on x_1 and x_2. With Eqs. (5) and (20), it follows that the cell velocity distribution is given by

$$\frac{dN}{d\alpha} = \pi b s_\pi \rho_\pi \mu^{-2} \text{ for } u^1 = u^2 = 0,$$
$$= 0 \text{ otherwise}. \tag{21}$$

D. The pion momentum inclusive distribution

The inclusive momentum distribution of decay pions can be obtained by folding the assumed statistical distribution of pion thermal motion into the cell velocity distribution. Denote the pion momenta in the c.m. frame of the matter system and in the rest frame of the cell, respectively, by

$$p_\mu = [(\mu^2 + \vec{p}_T{}^2)^{1/2}\cosh\alpha, \vec{p}_T, (\mu^2 + \vec{p}_T{}^2)^{1/2}\sinh\alpha] \tag{22}$$

and

$$p'_\mu = [(\mu^2 + \vec{p}'^2_T)^{1/2}\cosh\alpha', \vec{p}'_T, (\mu^2 + \vec{p}'^2_T)^{1/2}\sinh\alpha']. \tag{23}$$

Then, the pion momentum distribution is given by

$$dN = \frac{\rho_\pi g_\pi}{8\pi^2\mu^2}\int_{-\alpha_{\max}}^{\alpha_{\max}} d\alpha'' f(\vec{p}', \mu) d^3p'. \tag{24}$$

With the kinematic relations,

$$p'_T = p_T,$$
$$\alpha' = \alpha - \alpha'',$$
and
$$d\alpha' dp'^2_T = d\alpha dp_T{}^2, \tag{25}$$

one arrives at

$$\frac{dN}{d\alpha\, dp_T{}^2} = \frac{\rho_\pi g_\pi}{4\pi\mu^2}(\mu^2 + p_T{}^2)^{1/2}\int_{-\alpha_{\max}}^{\alpha_{\max}} d\alpha'' \frac{\cosh(\alpha - \alpha'')}{\exp[(\mu^2 + p_T{}^2)^{1/2}\cosh(\alpha - \alpha'')/T_\pi] - 1}, \tag{26}$$

where the limit of α'' is to be determined below. The pion momentum distribution of Eq. (26) is defined in the c.m. frame of the matter system. One may in a straightforward manner convert this distribution to that given in the initial c.m. system.[19] Furthermore, taking into account the leading-particle momentum distribution, one can eventually arrive at the single-pion inclusive distribution.[20]

We proceed now to determine $\alpha_{\max}$. We will only look at its asymptotic behavior and thus neglect the effect of the thermal motion. Making use of the inclusive sum rules one has

$$N = \int dN = 2\pi b \mu^{-2} s_\pi \rho_\pi \alpha_{\max}, \tag{27}$$

and

$$M = \int E\, dN$$
$$= \int \mu\cosh\alpha\, dN$$
$$= 2\pi\mu b \mu^{-2} s_\pi \rho_\pi \sinh\alpha_{\max}. \tag{28}$$

It turn the maximum rapidity and the decay proper time are given by

$$\alpha_{\max} \sim \ln\frac{2M}{N}$$
and
$$\rho_\pi \sim N/[2\pi\mu^{-2}s_\pi \ln(2M/N)]. \tag{29}$$

If we take the lower limit of Eq. (17), we find

$$\alpha_{\max} \sim \tfrac{1}{2}\ln M. \tag{30}$$

On the other hand, if we assume $N \sim \ln M$, which is also compatible with the data,

$$\alpha_{\max} \sim \ln M. \tag{31}$$

Our prediction on the inclusive transverse momentum distribution for $\alpha = 0$ (with $T_\pi = 140$ and 150 MeV), taking into account the cell longitudinal rapidity distribution, and the data[21] are illustrated in Fig. 1. Notice that the agreement with the low- and high-energy data are reasonable. In the small p_T region ($p_T < 0.2$ GeV), only the 24-GeV/c data are available. It will be interesting to see whether the predicted transverse momentum distribution in this region is in agreement with future data at higher energies.

IV. COMPARISON OF OUR WORK WITH LANDAU'S MODEL AND THE MULTIPERIPHERAL MODEL

The main difference between Landau's approach and ours is in the degree to which one incorporates hydrodynamics into the time development of the hadronic system. Within his work, Landau assumed that the collision phenomena can be accounted for solely by hydrodynamical considerations. Let us recall briefly here Landau's picture. At the collision, shock waves are generated at the surface of contact, which propagate longitudinally outward along both directions. In each direction, after the shock wave reaches the corresponding boundary of the resulting system, running waves begin to develop. At each side, there are two wave fronts, with one flowing outward into the vacuum and the other towards the center. The two inward-moving wave fronts from the two sides collide at the center and subsequently give rise to the general waves. These waves are then propa-

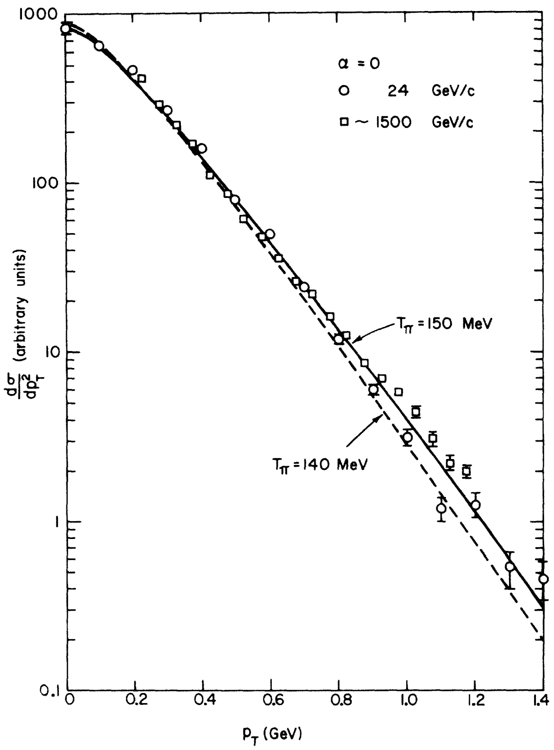

FIG. 1. Pion inclusive transverse momentum distribution at $\alpha = 0$. The theoretical curves are predictions of Eq. (26), with $T_\pi = 140$ and 150 MeV. For the data see Ref. 21.

gating outward. Features of the multiparticle production are completely governed by the properties of the general waves.

This is to be contrasted with our picture, where in a very short time after the collision there is a violent acceleration for the entire system which is followed by the achievement of the frame-independence symmetry. Our use of hydrodynamics is only confined to the application of its equation of motion after ρ_2.

Owing to this difference, the solutions to the hydrodynamical equations for these two models are quite different. For instance, in Landau's model for the running waves there are two vectors specifying the matter system: the space-time two-vector $\tilde{x}_\mu$, and the initial velocity of the entire system, which in the c.m. frame is given by $b_\mu = (1, 0)$ for the two-dimensional case. With these two independent scalars, $\rho^2 = \tilde{x}_\mu \tilde{x}^\mu$ and $\omega = \tilde{x}_\mu b^\mu$. So in general, proper quantities can be functions of both ρ and ω. Here the proper matter density is a function of the variable x^3/x^0, which is in fact $[1-(\rho/\omega)^2]^{1/2}$. Within our approach, due to the violent acceleration prior to $\rho \sim \rho_2$, the matter system becomes frame independent. Thus, for the description of the development of the longitudinal expansion, there is only one two-vector at our disposal.

The rapidity distributions predicted by the two models are also quite different. Consider the one-dimensional case for which there are exact solutions available. Landau's model gives an approximate Gaussian function for this distribution with its width given by $\sim(\ln M)^{1/2}$, while we get a plateau distribution with the magnitude of its limits extending to $\pm\frac{1}{2}\ln M$, for $N \propto M^{1/2}$ as in Landau's model.

We now turn to the multiperipheral model[22] which also predicts a flat longitudinal rapidity distribution. Recall that in the multiperipheral models, particles are emitted along the multiperipheral chain with sharp transverse momentum cutoffs. They are ordered in the rapidity space. Within this model, apart from the two particles on both ends of the chain, the probabilities of the production of all interior particles are comparable in strength. In other words, their production probabilities are insensitive to specific locations along the chain. This may be regarded as an example having the frame-independence symmetry.

In the multiperipheral model the transverse momentum cutoff is mainly due to the Reggeon-Reggeon-particle form factor. This is to be contrasted with our model, where the cutoff is due to the smallness of the transverse momentum of the local elements and the thermal motion within them. Furthermore, with a fixed breakup temperature our approach leads to a universal transverse momentum distribution, which is also in good agreement with the data.

ACKNOWLEDGMENT

After the completion of this work, we learned that Dr. F. Cooper had also discussed the solution similar to the one we presented in a somewhat different context; see the review talk on Landau's model given by F. Cooper at the Division of Particles and Fields meeting, Williamsburg, 1974.[23]

APPENDIX A: AN ESTIMATION ON THE TRANSVERSE HYDRODYNAMICAL EXPANSION

In the main text, the transverse motion of cells is assumed to be negligible. The dominance of the longitudinal motion is plausible since the original impact is along the longitudinal direction. In general, when one takes into account the transverse motion, the space-time development of the hydrodynamical system may be very complicated. To give an estimate on the average transverse cell velocity, we again make use of the notion of the frame-independence symmetry and propose a crude picture for the space-time development.

We suppose that the acceleration along the transverse direction similar to the longitudinal case also begins near $t = t_1$. Along with the longitudinal motion there is a weak acceleration in the transverse direction. Now we go to the longitudinal rest frame of some transverse slice of the system. The proper time taken for the propagation of any signal from its center to its edge is of the order of μ^{-1}. So it is plausible that the acceleration along the transverse direction should proceed at least for a proper time $\tau \equiv (x_0{}^2 - x_1{}^2 - x_2{}^2 - x_3{}^2)^{1/2} = \tau_3 \approx \mu^{-1}$. For definiteness we assume that the transverse acceleration ceases at $\tau = \tau_3$. Analogous to the longitudinal case, at this point we assume that the system may be described in an identical manner in all frames related by four-dimensional homogeneous Lorentz transformations except at the boundary. We recall that due to the earlier asymmetric accelerations along the longitudinal and the transverse directions, the boundary is very much elongated along the longitudinal direction. We introduce the quantity $u_T^{\max}$ to specify the maximum of the transverse component of the four-velocity of the cells.

Now, the space-time four-vector may be parameterized as

$$x_\mu = \tau(\cosh\beta\cosh\alpha,\ \sinh\beta\cos\phi,\ \sinh\beta\sin\phi,\ \cosh\beta\sinh\alpha) \equiv \tau\eta_\mu\,. \tag{A1}$$

The frame-independence symmetry at τ_3 leads to

$$u_\mu = x_\mu/\tau = \eta_\mu$$

and

$$T^{\mu\nu}(x) = [\epsilon(\tau) + p(\tau)]u^\mu u^\nu - g^{\mu\nu}p(\tau)\,. \tag{A2}$$

The observables for this case may be obtained in a similar manner as those for the two-dimensional symmetry case. We briefly state the results. The equations of motion are now reduced to

$$\frac{d\epsilon}{d\tau} + \frac{3}{\tau}(\epsilon + p) = 0 \quad \text{or} \quad \frac{ds}{d\tau} + \frac{3s}{\tau} = 0\,. \tag{A3}$$

The solution is

$$s\tau^3 = s_\pi\tau_\pi^3, \text{ or } s \propto \frac{1}{\tau^3}\,, \tag{A4}$$

where s_π and τ_π are the entropy density and the corresponding proper time at breakup. As expected, for this case the entropy density decreases much more rapidly as compared to that for a pure longitudinal expansion. The cell velocity distribution is given by

$$\begin{aligned}\frac{dN}{u_T du_T d\alpha} &= \frac{2\pi b s_\pi \tau_\pi^3}{\mu^2} \text{ for } u_T < u_T^{\max}\,,\\ &= 0 \text{ otherwise.}\end{aligned} \tag{A5}$$

If one assumes $N \sim M^{1/2}$, from the inclusive sum rules one finds that at breakup, the proper time is given by

$$\tau_\pi \propto M^{1/6}/(\ln M)^{1/3}\,. \tag{A6}$$

Hence, if $u_T^{\max}$ is nonzero, one has for large enough M, $\tau_3 \ll \tau_\pi$. Therefore, for this M, there is always the proper time interval where the three-dimensional expansion is significant.

Now we need to fold the thermal motion into the cell velocity distribution to obtain the pion inclusive momentum distribution. The resulting expression is rather complicated. However, we need not go into the details here except to point out the fact that because of the flat cell transverse velocity distribution of Eq. (A5), there should be a corresponding flat region in the pion transverse momentum distribution near $p_T = 0$. From the data shown in Fig. 1, by allowing an additional flat region near $p_T = 0$ in the p_T distribution, we estimated that $u_T^{\max} \leq 0.5$. We see that for large M, even though there is always a three-dimensional expansion, this expansion essentially does not affect the pion inclusive distributions concerned in the text.

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*Work supported in part by the U. S. Atomic Energy Commission under Contract No. (40-1)3992.

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[14]A plateau of the longitudinal rapidity distribution has also been deduced by Hwa in Ref. 9, based on his constituent hydrodynamical model.

[15]The intuitive picture prior to this symmetry described is by no means unique. For example, this state of symmetry may alternatively be achieved without going through an intermediate stationary state. For example, when the two incident nucleons pass through each other, local elements may be decelerated and some of their kinetic energies may be transferred into matter. At a certain stage the frame-independence symmetry may be reached directly. Within this latter picture, the average multiplicity of pions is not specified.

[16]Of course, these equations of motion are valid only in the interior of the matter. At the boundary, there is an outward pressure, as is the case for any hydrodynamical system. We assume this pressure is balanced by the uncompensated attractive force of strong interactions exerted by the interior of the matter.

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[18]We observe that this breakup criterion differs from that of Landau in Ref. 3. In particular, with the usual hydrodynamical consideration, he postulated that the breakup occurs when the ratio of the mean free path to the least linear dimension of the system is of the order of unity. With his criterion, the matter density at decay varies as a function of the incident energy.

[19]For the case $p_M = 0$, of course, there is no transformation involved. For the case $\vec{p}_M \neq 0$, owing to our assumption of the frame-independence symmetry, the longitudinal rapidity distribution of the cells remains the same. However, the boundary of the distribution will be distorted; this modified boundary can be determined through Lorentz transformations. For instance, for the case where the transverse part of $\vec{p}_M$ and the transverse momentum of the local elements p_T are negligible, the longitudinal inclusive distribution in the rapidity scale is shifted by the amount of $\tanh^{-1}|\vec{p}_M|/E_M$.

[20]Experimentally, the leading-particle momentum distributions at high energies are peaked near the two ends of the kinematic boundaries. We expect that the resulting pion distribution should still have the central plateau and be smeared out somewhat near the two boundaries.

[21]Pion inclusive transverse momentum distribution data: 24 GeV/c: V. Blobel *et al.*, Bonn-Hamburg-München Collaboration, Nucl. Phys. B69, 237 (1974); ISR data: $\sqrt{s} = 52.7$ GeV (or $P_{\rm Lab} \sim 1500$ GeV/c) by M. Banner *et al.*, Saclay-Strasbourg Collaboration, Phys. Lett. 41B, 547 (1972).

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