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A study of the empirical functions of the height fields over India and their relation with the rainfall

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Abstract. The empirical orthogonal functions have been obtained for the individual summer monsoon (June through September) months using the grid point values of monthly 700 mb geopotential heights over Indian region. The data for 21 summer monsoon months for the years 1958 to 1978 have been used in the present computation.

The major variance reduction is due to the first three dominant functions accounting over 80% of the total variance in each month. The variance reduction only due to the first function ranges from 45 to 65%.

The first function has in-phase oscillation throughout the area indicating that the area under study is homogeneous and the centre of the oscillation lies over northwest India. The amplitudes of the first function also show generally quasipers stence in their sign within a season. The second function has two centres of action over the region of monsoon trough which are in phase. The third function has also two centres oriented in the east-west direction but they are in the opposite phase.

Fairly large values of correlation coefficients between the patterns of the different monsoon months suggest that the patterns for these months corresponding to the first and the second functions respectively are quite similar. The patterns for these months also evolve with time in a related way. The spectrum analysis to the time series of amplitudes indicates the presence of the quasi-periodicity of 3 years during these monsoon months. The amplitudes corresponding to the dominant functions are found to be significantly related with the rainfall of central and western parts of India.

Keywords. Empirical orthogonal functions analysis applied to 700 mb heights over India; relationship of EOF with monsoon rainfall.

1. Introduction

Most of the meteorological variables are correlated with each other and it is useful to transform them into a few number of orthogonal functions according to interrelationships they possess. This will eliminate the multi-collinearity problem arising in the statistical prediction scheme. The other advantages of the scheme can be the convenience in analysis and interpretation of the data and in understanding the external sources of the variation in the system which the data sets

represent. Thus, the empirical orthogonal functions analysis provides the most efficient and convenient method of transforming the intercorrelated variables into a small number of empirical orthogonal functions. The functions are derived directly from the data without having any pre-determined form, unlike other parametric descriptions of the data.

Recently, we find increasing number of applications of the technique in analysis of the different meteorological variables. Kutzback (1967, 1970) used this technique for the analysis of monthly pressure, temperature and precipitation data over North America and sea level pressure data over northern hemisphere. Craddock and Flood (1969) used this technique for examining 500 mb daily geopotential heights over northern hemisphere. Kidson (1975 a, b) examined monthly surface pressure, temperature and precipitation data over northern and southern hemisphere and also over tropical region. Trenberth (1975) used this technique for the study of monthly sea surface pressure patterns over Australia and sea surface temperature over Tasman sea. Weare (1977) used this technique for examining sea-surface temperature patterns over Atlantic ocean.

Bedi and Bindra (1980) used this technique for analysis of Indian monsoon rainfall. Sikka and Prasad (1981) used this technique for the analysis of height fields over Asiatic regions.

In the present study, the technique has been used for the study of the dominant patterns of the monthly anomalies of geopotential heights over Indian region for the monsoon months, June to September, and to examine their relation with the rainfall.

2. Method

The monthly anomaly of 700 mb height over the network of M grid points and for the periods of N years is represented by the $M \times N$ matrix Z. The column of the matrix represents M grid point values corresponding to a year. The anomaly over a grid point is considered as the departure of an observed value from the N years mean value over that point. The functions (EOF's) have been obtained from the symmetric covariance matrix (A) defined as:

$$A = (ZZ')/(N-1) \tag{1}$$

where prime denotes the transposed matrix. This covariance matrix takes into account all the possible spatial covariance relationship within the network of grid points. The functions thus obtained from the covariance matrix will describe the relationship between the regions of maximum variance. The anomaly of 700 mb height in terms of functions can be expressed as

$$Z_{it} = \sum_{n=1}^{M} C_{nt} f_{ni} \tag{2}$$

where *i* refers to grid points, *t* refers to time, f_n refers to the *n*th function and C_n refers to amplitude associated with the *n*th function. The function and the amplitude satisfy the following orthogonal conditions.

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$$\sum_{i=1}^{M} f_{ri} f_{si} = \delta_{re} \tag{3}$$

$$\sum_{t=1}^{N} C_{rt}C_{st} = \lambda_{r}\delta_{rs} \tag{4}$$

where $\delta_{rs}=0$ for $r\neq s$ and $\delta_{rs}=1$ for r=s. λ is the eigen value of the function.

Based on such orthogonality conditions, the expression for the amplitude in terms of the functions can be written as:

$$C_{nt} = \sum_{i=1}^{M} f_{ni}' Z_{it} \tag{5}$$

The eigen value satisfies the relation

$$Af_n = \lambda f_n \tag{6}$$

The relationship between the eigen value and the amplitude can be expressed as:

$$\lambda_n = \sum_{t=1}^N C^2_{nt} \tag{7}$$

The *n*th eigen value λ_n gives the measure of the variance explained by the *n*th function. The functions f_n were obtained by iterating the equation (6) by Jacobi method.

3. Data

Daily mean (average of 00 GMT and 12 GMT) 700 mb geopotential heights at 98 grid points over the Indian region (figure 1) were obtained from analysed weather maps from 1st June to 30th September for the years 1958 to 1978. The data were obtained for the region between 5° N to 35° N and 60° E to 100° E and refer to the corners and centre of 5° square grid. The monthly mean grid point values were obtained by averaging the daily grid point values for the month. The anomaly was computed as the departure of the monthly mean value from the long term (21 years) mean. The anomaly fields at these 98 grid points for the years 1958 to 1978 were treated separately for each of the months June to September in the computation of the functions.

4. Variance reductions

the cumulative variance reductions expressed as percent of the total variance by the first 10 functions for the months June through September are shown in

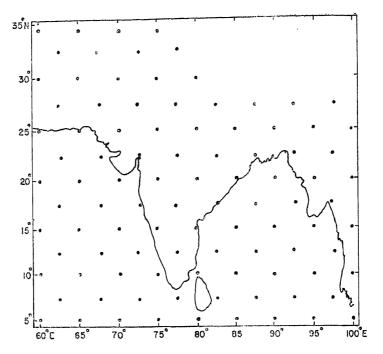


Figure 1. Area of analysis showing 98 grid points used in the study.

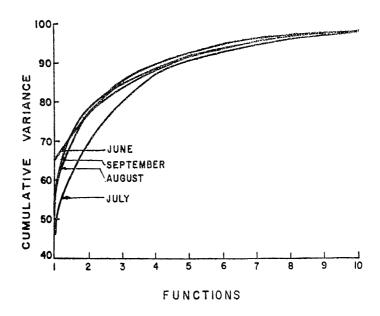


Figure 2. Cumulative variance in percent explained by the first 10 functions.

figure 2. It can be seen that the major variance reductions in the different months come from the first function accounting to 46 to 65% of the total variance. The contribution to the total variance by the higher order function decreases sharply and the contribution made by the 10th function is merely 0.5%.

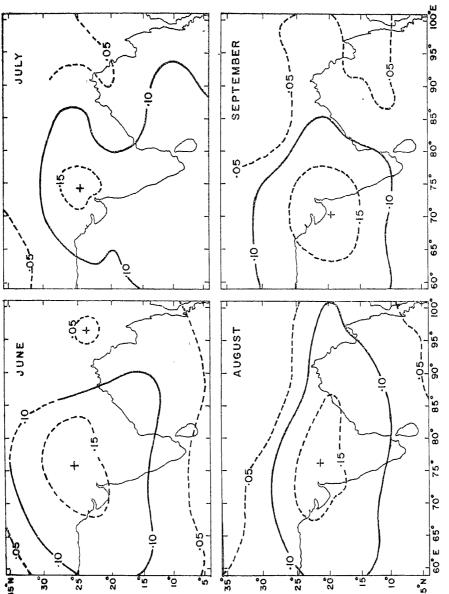


Figure 3. Spatial pattern corresponding to the first function. The isolines drawn at 0.05 interval represent non-dimensional configuration of 700 mb height anomaly fields. The functions for June to September explain 64.9, 46.4, 57.4 and 51.2% respectively of the total variance.

The contribution to the total variance made by the third function alone is 6% or more and the first three functions combined explain 80% of the total variance in each of the months. Since each higher order function explains a decreasing amount of the total variance and incorporates more random noise of the original data analysis, detailed discussion is restricted to the first three dominant functions. These three functions are more stable and represent the important modes of oscillations present at 700 mb geopotential height over the area.

5. Discussion

5.1. The first function

The patterns corresponding to the first dominant function for the months June through September are presented in figure 3. The pattern contributes a major portion of the total variance which ranges from 46 to 65% in different months.

The patterns for all the months are similar in appearance. They depict in phase oscillation over the entire area, the centre of which is located over the western/central India near 20-25° N and 70-75° E. The orientation of the isolines suggests that the oscillation even extends beyond the area considered in the present study.

The pattern of the function has been compared with the pattern of total variance. The two patterns are in good agreement in each of these months. For comparison the pattern of variance for the typical month September is shown in figure 4.

The variations in amplitudes with time corresponding to the first function for the months June through September are shown in figure 5. The amplitudes maintain the same sign in all the four months of the season in about 60% of the

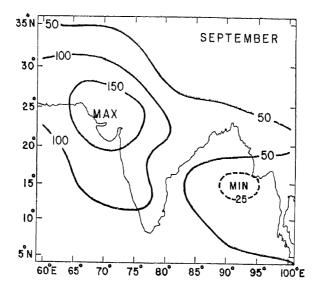


Figure 4. Spatial patterns of the variance of 700 mb height fields (September).

seasons studied. This quasi-persistent nature within a season of the most dominant function reflects the seasonal behaviour. This aspect is further discussed in § 5.4. The amplitude varies over a wide range and exhibits oscillatory characteristics. The large positive values have been obtained in the following years for the months: June 1958, 1959, 1965, 1966; July 1958, 1965, 1966; August 1958, 1965, 1966; September 1958, 1965, 1966, 1972. The large negative peaks have been obtained in the following years of the months: June 1971, 1975; July 1974,

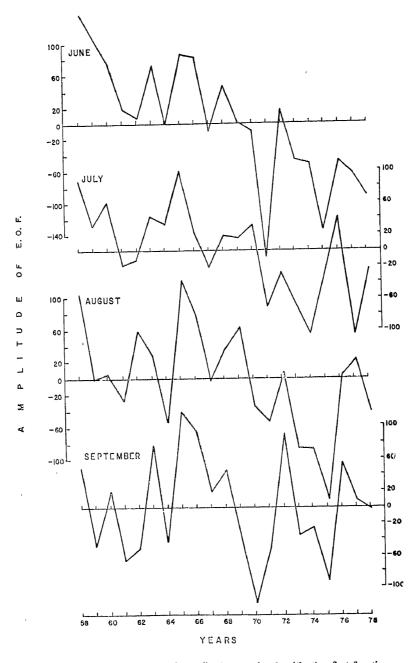


Figure 5. Time variation of amplitude associated with the first function.

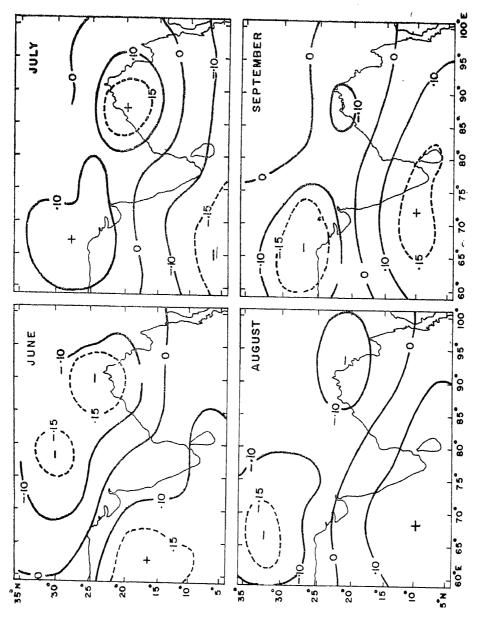


Figure 6. As in figure 3 except for the second function. The functions for June to September explain 11.7, 23.6, 21.6 and 26.8% respectively of the total variance.

1977; August 1973, 1974, 1975; September 1970, 1975. However, the highest negative peak is obtained corresponding to year 1971 during June.

5.2. The second function

The second dominant function (figure 6) contributes 12% of the total variance during June and 22 to 27% of the total variance during July to September. The patterns indicate two oscillation centres in phase to each other in all the months and laying along the seasonal monsoon trough. The centre of one oscillation lies over the Head Bay of Bengal and the adjoining area and the centre of the other lies over northwest India and the adjoining area. The circulation associated with these two centres extends over almost the entire area north of 15° N. During June the circulation occupies a relatively northward position and the north-south gradient in the anomaly fields over the area is weaker during the month than either during July, August or September. The north-south gradient over the Head Bay of Bengal is maximum during July. The maximum gradient occurs over northwest India during August and September.

The pattern has a third weak centre of oscillation which remains, throughout the monsoon months, in opposite phase to the centres of oscillation located over the monsoon trough regime. It is centred over the Arabian sea near 16° N and 62° E during June. During the subsequent month it shifts southeastward and lies centred near 10° N and 72° E during September. The gradient associated with it is quite weak throughout the monsoon months.

In phase oscillation of both the centres over the monsoon trough region and their out of phase behaviour with respect to the centre in the lower latitude would affect the north-south contour gradient consequently affecting the zonal flow over the region of their influence.

The time variation of amplitudes corresponding to the functions for different months are presented in figure 7. The positive (negative) value of amplitude translates into below (above) normal 700 mb geopotential heights over the monsoon trough regime and above (below) normal 700 mb geopotential heights over the area south of the monsoon trough (south of 15° N) regime, except during July in which the reverse is the case. The pattern of function changes its sign accordingly during the different years of a month depending upon the sign of the amplitude. During June and September the negative and positive high values of amplitudes are comparable. High positive values have been obtained during June 1968, September 1958 and high negative values during June 1972 and September 1974. Large positive values have been obtained during July 1960, 1964 and 1975. During August, the amplitudes have generally negative values or small positive values except the large positive values obtained in 1967, 1968 and 1978.

5.3. The third function

The patterns corresponding to the third functions for the months June through September are displayed in figure 8. They contribute 6 to 10% of the total variance during these months. Except for June the patterns for the other months

consist of three centres of oscillation. The pattern for June consists of two centres of oscillation one over northwest India, Pakistan and adjoining area centred between 25–30° N and 60–65° E and the second one, in the opposite phase to the first one, over the south Bay of Bengal centred between 5–10° N and 80–85° E. The isopleths in the pattern are separated wide apart and the centres are not well marked.

The patterns for the months July to September have three centres of oscillation. One of them lies over northwest India, Pakistan and the adjoining area centred near 24°N and 62°E during July. During August and September it lies centred

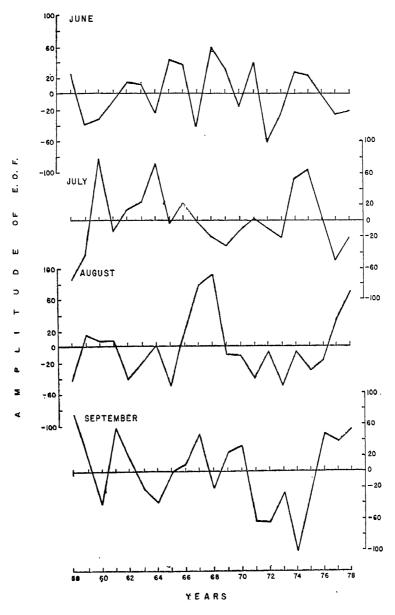


Figure 7. As in figure 5 except for the amplitude associated with the second function.

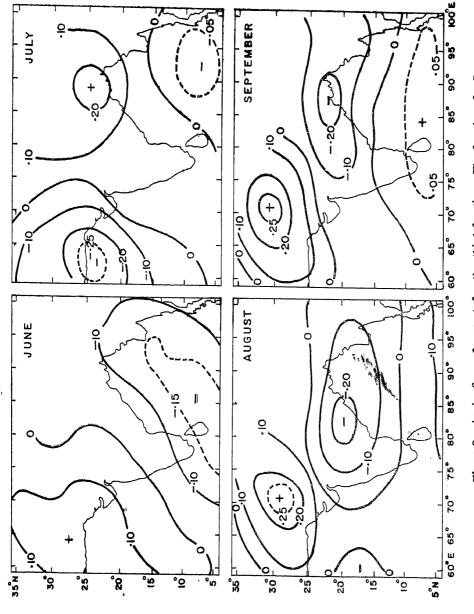


Figure 8. As in figure 3 except for the third function. The functions for June to September explain 9.0, 10.4, 6.2 and 5.9% respectively of the total variance.

between 29-31° N and 70-72° E. The second centre of oscillation, in the opposite phase to the first one, lies over Bengal and the adjoining area centred near 25° N and 88° E during July, near 19° N and 83° E during August and near 22° N and 88° E during September. The gradient surrounding it is weaker during July than during August or September. The presence of these two centres in opposite phase to

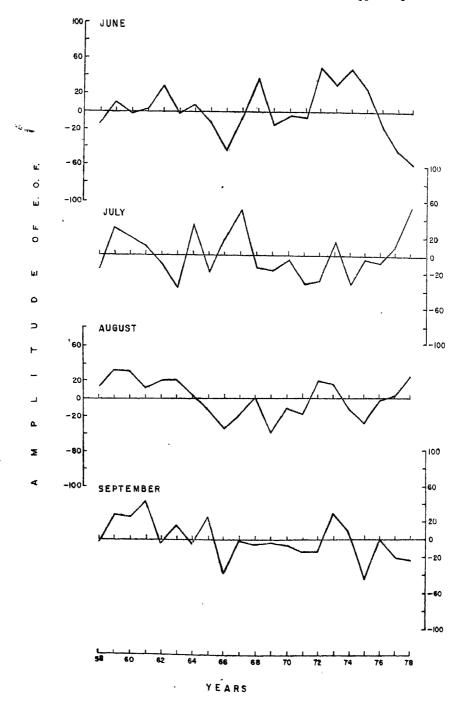


Figure 9. As in figure 5 except for the amplitude associated with the third function

each other would affect the meridional circulation. For example, during June and August, if the amplitude of the third function is negative (positive), this function would contribute to southerly (northerly) flow over the Peninsular India, Central India and most of the Bay of Bengal.

The third centre of oscillation in the pattern for July lies over south Bay of Bengal centred near 8°N and 92°E which is weaker than the other centres of oscillation during the month. During August it is still less defined and has shifted southward. During September it is centred between 5–10°N and 85–90°E and is still not well marked. The oscillations over northwest India and south Bay of Bengal are in the opposite phase during June but they are in the same phase during July and September.

The time variation of amplitudes corresponding to the third functions from June to September are shown in figure 9. The amplitude variations are smaller than those associated with the first and second functions. However, higher positive values of amplitudes have been obtained during the following years for months: June 1972, 1974; July 1967, 1978; September 1961 and high negative value during June 1976, 1978; September 1975.

5.4. Similarity between the patterns of different months and their evolution with time

A quantitative measure of spatial similarity between the patterns of different months can be given by the correlation coefficient between them. Similarly, the similarity between the evolution of the functions with time during different months can be examined by finding the correlation coefficient between the associated time series of amplitudes. The correlation coefficients between the spatial patterns of different months and between the time series of amplitudes associated with these functions are presented in tables 1 and 2 respectively.

It can be seen that the patterns corresponding to the first functions for June to September are highly similar (C. C. ≥ 0.92) among each other. The amplitude associated with the function of June is also significantly related with the corresponding amplitudes of the other months (table 2). This means that the spatial pattern corresponding to the first function of June to September are similar to each other and they evolve every year during these months in a more or less similar way.

The patterns corresponding to the second functions also have high (C.C. ≥ 0.62) spatial similarity during these months but the amplitudes associated with them are not significantly correlated. This indicates that, though the patterns of the second function are similar, they evolve every year during these months in a different way. Therefore, in some years/months the contribution to the height anomaly due to the first and the second components may superpose each other in the same sense whereas in others they may annul each other.

In case of the pattern corresponding to the third function, a good similarity is noticed between the patterns of August and September and the changes in the patterns with time during these two months are also significantly (table 2) related.

6. The spectrum analysis of the amplitude time series

The time series of amplitudes corresponding to the first three functions were examined for the presence of cyclic behaviour. The spectral density was computed

Table 1. Correlation coefficients between the functions of different months, only the first three functions are considered for the purpose.

September	3 1 2 3	.17 .94** —.10 .03	56**	September	Functions 1 2 3	1 .95** .0113	•
August	2				3 Fun	90.	
	1	.95**	- 14	September	2.	.23*	
	33	. 63.		Š	1	.93**	
July	2	.35**	.31**		т т	.24*	
	1	.92**	. 2.3. 18	August	2	.16	
		н с	7 6		1	• 93 **	
	Functions	1	June		Functions	-	

Single and double asterisks indicate correlation coefficient significant at 95 and 99% levels respectively.

Table 2. Same as in table 1 except for the amplitudes associated with the functions,

	3	· 40 —· 18 · 24		3	· 09 19 · 44*
September	7	· 23 —· 11 —· 62**	September	2	.39
	_	.50* .111		-	.65** .11 .06
	(en	. 28 48* . 15		Amplitudes	August 2
August	2	· 01 —· 21 —· 22		3	· 18 — 09 — 03
	1	.72** .27 —.32	September	বে	.32 60** .43*
	3	.00 54** 43*			.37
July	2	20 - 02 - 26		3	· 09 —· 10 · 11
	1	· 71** · 17 18	August	7	15 10 55**
		3 2 1		-	.57** 43* 15
					- N 6
	Amplitudes	June		Amplitudes	July

following Blackman and Tuckey method after removing persistence from the series. The smooth spectral estimate was obtained from Hamming method of 3 term weighted average.

Figure 10 shows that a spectral peak significant at 95% confidence limit corresponding to a 3 year periodicity exists in the following amplitude series:

Month	Amplitude corresponding to the function shown below
June July August September	Second First and third First Second

The variance accounted by these peaks were 28 to 32% of the total variance of their amplitudes.

7. The cluster analysis

The time amplitudes $C_1(t)$ and $C_2(t)$ corresponding to the first and the second dominant functions respectively have been used to determine similarity of behaviour (analogy) of the anomaly fields between different years. For this purpose, the amplitudes corresponding to different years of a particular month, have been plotted in $C_1(t)/C_2(t)$ space. The nearness of the euclidean distance in each quadrant has been taken to give a cluster of years of similar behaviour. Typical cluster corresponding to July are presented in figure 11.

The contribution of a function to the actual anomaly field for a particular year is obtained from the product of each element of the corresponding

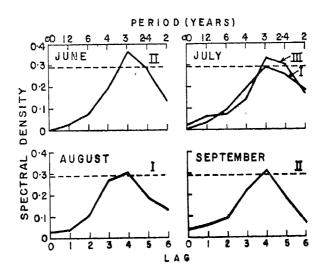


Figure 10. Spectral density estimates of the time series of amplitude associated with (I) the first function, (II) the second function, (III) the third function. The horizontal broken line indicates 95% confidence limit.

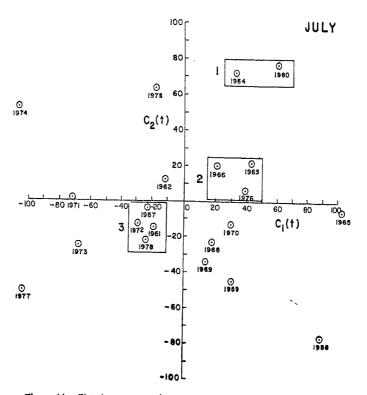


Figure 11. Plot between amplitude-space C_1 (t)—Amplitude corresponding to the first function, C_2 (t)—Amplitude corresponding to the second function. The three sets of analogues found during the period 1958 to 1978 are shown enclosed within the three rectangles.

function and the amplitude (see equation 2). The behaviour of the anomaly field resulting from the combination of the first and the second dominant function over central parts of India (17.5–25° N and 72.5–85° E) can be tested with the observed anomaly fields for different months in each year under study.

The variance in the height fields over this belt of India is found to be large during the monsoon months. The years in which the sign of the resulting anomaly fields are similar can be compared against the actual observed anomaly fields. Table 3 shows the results of such comparison. The last column of the table summarises the analogy of the anomaly fields over central parts of India for a particular month of different years. It can be seen that the analogous behaviour is uncertain only in case of July 1967 and August 1972, 1976. In these years of the months combination of the first two functions does not satisfactorily represent the observed anomaly field and higher order functions are needed to represent satisfactorily the observed field.

8. The relation between the amplitude and the rainfall

8.1. Relationship with failure of seasonal monsoon rainfall over India as a whole

During the years 1958-78, there have been four years, viz. 1965, 1966, 1972 and 1974 during which the monsoon seasonal rainfall on all India basis showed large nega-

Month	Cluster No.	Years which appeared in the same claster	Sign of the anomaly field which is similar for the first and second function over central parts of India	Sign of the observed anomely field over central parts of India	Analogous behaviour or otherwise
June	7 7	1959, 1950 1971, 1975	Pojtive Negative	Ponitive Negative	Analogous do.
July	3 2 1	1969, 1964 1953, 1966, 1976 1961, 1967, 1972, 1978	Positive Positive Nogative	Positive do. Positive do. Nosative for 1961, 1961, 1972, 1978 1972, 1978; Positive for analogous; 1967 1957	Analogous do. 1961, 1972, 1978 x analogous; 1967 uncartain
August	-	1963, 1972, 19 76	Positive	Weak positive for 1953; nogutive for	1963, analogous; 1972, 1976 unostain
	7	1951, 1964	Nogative	1972, 1976 Negative	Analogous
September	1 2	1960, 1968 1939, 1932, 1969	Positive Nagative	Positive Negative	Analogous Analogous

tive departure (< 10% of the normal). We attempt a possible relationship between the failure of rain in the above four seasons with the persistence of anomalous monthly height patterns resulting from the combination of the first two functions. It is expected that the persistence of above normal height fields throughout the season would signify the persistence of the anomalous downward motion in the middle troposphere over most of the Indian region. For this purpose figures 3, 5, 6 and 7 are examined in conjunction with each other to determine the contribution of each of these functions for the above normal height field over central and northern India. The first function would contribute to large positive 700 mb height anomaly during June 1958, 1959, 1965, 1966; July 1958, 1965, 1966; August 1958, 1965, 1966 and September 1958, 1965, 1966, 1972. Note that during 1965 and 1966 large positive amplitude values persisted throughout the monsoon season from June to September which would contribute to the above normal height values over India. It is possible that the persistence of positive departures of height fields through the first component is related to the large seasonal rainfall deficit during these two years as the contributions due to the second component are not significant in these years owing to much smaller values of the corresponding amplitudes.

The failure of the monsoon rains during 1972, however, cannot be accounted through the persistence of above normal height fields as during July the hight anomaly fields due to the first two functions are below normal and during August they are above normal. During 1974, the contribution of the first function to the 700 mb height anomaly is negative throughout June to September. However, the contribution from the second function is opposite to that of the first during July and September 1974 and hence the height anomaly due to the first and the second functions would annul each other in these two months. Therefore, the dominant cause of the monsoon failure in 1974 also may not be the persistence of above normal height fields during the entire season.

The above discussion suggests that different mechanisms may be operating for the failure of monsoon rain and one of them could be the persistence of the above normal height fields in the lower middle troposphere throughout the season as it occurred in 1965 and 1966.

8.2. The relationship with the monthly rainfall of different meteorological subdivisions over India

The amplitudes corresponding to three dominant functions have been correlated with the rainfall of different meteorological subdivisions for the same months. The plot of the significant correlation at 95% level in the map form revealed that the significant correlation existed over a large contiguous area generally in the case of the amplitude of the first function for all the months. However, this contiguous area covered western and central India for June, August and September (correlation coefficient—ve). When this relationship is examined in combination of the pattern of the first function it is found that, in the months of June, August and September, below normal 700 mb height fields over the western and central India is associated with above normal rainfall. The significant correlation coefficient did not exist for a contiguous large area in case of the amplitudes associated with the second and the third functions, except for the month of September.

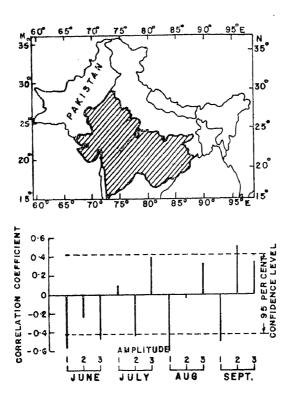


Figure 12. Correlation coefficients between the average rainfall of the area which are hatched in the upper diagram and time series of amplitudes corresponding to the first three functions.

Even though the meteorological subdivisions are more or less homogeneous, it is rather difficult to expect that the rainfall of a subdivision (size 10⁵ sq. km) is related with the pressure patterns which have been obtained from the 2·5° latitude/longitude grid point data. As most of the rainfall during the season occurs under the influence of synoptic scale disturbances, it is appropriate to seek relationship on the monthly basis for a larger part of India consisting of a few contiguous subdivisions which are influenced by transient monsoon disturbances. Figure 12 shows the area which covers 43% of the total area of India and accounts for 42% of the seasonal rainfall of India as a whole. The rainfall over this area is not affected by the orographic features and is mostly associated with the large scale fluctuations of the monsoon trough and the synoptic scale perturbations (lows and depressions) which move over the region during their life history. Therefore, we thought it proper to relate the average rainfall over this region with the amplitudes associated with the first three functions.

The relation is presented at the lower end of figure 12. It can be seen that the average rainfall of the area indicates significant negative correlation with the amplitude of the first function for June, August and September. This implies that below (above) normal anomaly (figure 3) pattern is responsible for the above

(below) normal rainfall for the area. For July and September, the amplitude of the second function also indicates the similar results. Though the amplitude of the third function also indicates higher correlation coefficient with the rainfall of the area, the significant relation at 95% level exists only for June. However, only two correlation coefficients, viz., the first function of June and August, are significant at 99% (C.C. ≥ 0.54) level.

The above analyses suggest that below normal (above normal) height fields over the central and western India associated with the first function for June, August and September are associated with above normal (below normal) rainfall over the area. Similar result is obtained for July and September in case of the pattern corresponding to the second function.

9. Conclusion

The first dominant function of 700 mb heights over Indian region contributes 45 to 65% of the total variance during each of the months June through September. The major variance reduction is due to the first three functions explaining over 80% of the total variance.

The first function has in phase oscillation throughout the area indicating that the area is homogeneous. The amplitudes of this function also show quasi-persistence in their sign in different months within a season which may imply that the function depicts the characteristics of a season. The centre of oscillation lies over northwest India where the variance in the 700 mb height fields is maximum.

The second function describes the relation between the regions of maximum variance lying over the monsoon trough zone. As the two dominant centres of this function show in phase oscillation, the contribution to the anomaly field due to this function is expected to be in phase over the entire monsoon trough region.

The centres of oscillation due to the third function also lie over the monsoon trough zone. However, they are in opposite phase to each other in contrast to the similar centres due to the second function. The presence of these two centres in opposite phase to each other would affect the meridional flow. It is also observed that in about 50% of the months the contribution due to the second and the third function strengthen each other and in other months they annul each other.

The spatial patterns of the monsoon months corresponding to the first and the second function respectively are quite similar. The patterns corresponding to the first function also evolve from year to year during these months in a significantly related way.

The spectrum analysis of the amplitude time series corresponding to the dominant functions indicates the presence of a quasi-periodicity of 3 years during the monsoon months.

The amplitudes corresponding to the dominant functions indicate significant relation with the rainfall of central and western parts of India.

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