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Baryon asymmetry from leptogenesis with four zero neutrino Yukawa textures

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Abstract

The generation of the right amount of baryon asymmetry η of the Universe from supersymmetric leptogenesis is studied within the type-I seesaw framework with three heavy singlet Majorana neutrinos N_i $(i = 1, 2, 3)$ and their superpartners. We assume the occurrence of four zeroes in the neutrino Yukawa coupling matrix Y_ν, taken to be $\mu\tau$ symmetric, in the weak basis where N_i (with real masses $M_i > 0$) and the charged leptons l_α $(\alpha = e, \mu, \tau)$ are mass diagonal. The quadrant of the single nontrivial phase, allowed in the corresponding light neutrino mass matrix m_ν, gets fixed and additional constraints ensue from the requirement of matching η with its observed value. Special attention is paid to flavor effects in the washout of the lepton asymmetry. We also comment on the role of small departures from high scale $\mu\tau$ symmetry due to RG evolution.

1 Introduction

Baryogenesis through leptogenesis [1, 2, 3] is a simple and attractive mechanism to explain the mysterious excess of matter over antimatter in the Universe. A lepton asymmetry is first

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generated at a relatively high scale ($> 10^9$ GeV). This then gets converted into a nonzero η, the difference between the baryonic and antibaryonic number densities normalized to the photon number density $(n_B - n_{\bar{B}})n_\gamma^{-1}$, at electroweak temperatures [4] due to $B + L$ violating but $B - L$ conserving sphaleron interactions of the Standard Model. Since the origin of the lepton asymmetry is from out of equilibrium decays of heavy unstable singlet Majorana neutrinos [5], the type-I seesaw framework [6, 7, 8, 9, 10], proposed for the generation of light neutrino masses, is ideal for this purpose. We study baryogenesis via supersymmetric leptogenesis [11] with a type-I seesaw driven by three heavy ($> 10^9$ GeV) right-chiral Majorana neutrinos N_i $(i = 1, 2, 3)$ with Yukawa couplings to the known left chiral neutrinos through the relevant Higgs doublet. There have been some recent investigations [12, 13, 14, 15] studying the interrelation between leptogenesis, heavy right-chiral neutrinos and neutrino flavor mixing. However, our angle is a little bit different in that we link supersymmetric leptogenesis to zeroes in the neutrino Yukawa coupling matrix. In fact, we take a $\mu\tau$ symmetric [16] neutrino Yukawa coupling matrix Y_ν with four zeroes [17] in the weak basis specified in the abstract.

There are several reasons for our choice. First, a seesaw with three heavy right chiral neutrinos is the simplest type-I scheme yielding a square Yukawa coupling matrix Y_ν on which symmetries can be imposed in a straightforward way. Second, $\mu\tau$ symmetry [18] - [46] in the neutrino sector provides a very natural way of understanding the observed maximal mixing of atmospheric neutrinos. Though it also predicts a vanishing value for the neutrino mixing angle θ_{13}, the latter is known from reactor experiments to be rather small. A tiny nonzero value of θ_{13} could arise at the 1-loop level via the charged lepton sector, where $\mu\tau$ symmetry is obviously broken, though RG effects if the said symmetry is imposed at a high scale [16]. Third, four has been shown [17] to be the maximum number of zeroes phenomenologically allowed in Y_ν within the type-I seesaw framework in the weak basis described earlier. Finally, four zero neutrino Yukawa textures provide [47] a very constrained and predictive theoretical scheme - particularly if $\mu\tau$ symmetry is imposed [16].

The beautiful thing about such four zero textures in Y_ν is that the high scale CP violation, required for leptogenesis, gets completely specified here [17] in terms of CP violation that is observable in the laboratory with neutrino and antineutrino beams. In our $\mu\tau$ symmetric scheme [16], which admits two categories A and B, the latter is given in terms of just one phase (for each category) which is already quite constrained by the extant neutrino oscillation data. Indeed, the quadrant in which this phase lies - which was earlier unspecified by the same data - gets fixed by the requirement of generating the right size and sign of the baryon asymmetry. Moreover, the magnitude of this phase is further constrained.

In computing the net lepton asymmetry generated at a high scale, one needs to consider not only the decays of heavy right-chiral neutrinos N_i into Higgs and left-chiral lepton doublets as well as their superpartner versions but also the washout caused by inverse decay processes in the thermal bath. The role of flavor [48, 49, 50, 51] can be crucial in the latter. In the Minimal Supersymmetric Standard Model (MSSM [52]), this has been studied [53] through flavor dependent Boltzmann equations. The solutions to those equations demonstrate that flavor effects show up differently in three distinct regimes depending on the mass of the lightest of the three heavy neutrinos and an MSSM parameter $\tan\beta$ which is the ratio v_u/v_d of the up-type and down-type Higgs VEVs. In each regime there are three N_i mass hierarchical cases : (a) normal, (b) inverted and (c) quasidegenerate. All these, considered in both categories A and B, make up eighteen different possibilities for each of which the lepton asymmetry is calculated here. That then is converted into the baryon asymmetry by standard sphaleronic conversion and compared with observation. These lead to the phase constraints mentioned above as well as a stronger restriction on the parameter $\tan\beta$ in some cases.

If $\mu\tau$ symmetry is posited at a high scale characterized by the masses of the heavy Majorana neutrinos, renormalization group evolution down to a laboratory energy λ breaks it radiatively. Consequently, a small nonzero θ_{13}^{λ}, crucially dependent on the magnitude of $\tan\beta$, gets induced. The said new restrictions on $\tan\beta$ coming from η in some cases therefore cause strong constraints on the nonzero value of θ_{13}^{λ} which we enumerate.

One possible problem with high scale supersymmetric thermal leptogenesis is that of the overabundance of gravitinos caused by the high reheating temperature. For a decaying gravitino, this can lead to a conflict with Big Bang Nucleosynthesis constraints, while for a stable gravitino (dark matter) this poses the danger of overclosing the Universe. The problem can be evaded by appropriate mass and lifetime restrictions on the concerned sparticles, cf. sec. 16.4 of ref [52]. Such is the case, for instance, with gauge mediated supersymmetry breaking with a gravitino as light as O(KeV) in mass. In gravity mediated supersymmetry breaking there are sparticle mass regions where the problem can be avoided – especially within an inflationary scenario. An illustration is a model [54], with a gluino and a neutralino that are close in mass, which satisfies the BBN constraints. Purely cosmological solutions within the supersymmetric inflationary scenario have also been proposed, e.g. [55]. We feel that, while the gravitino issue is one of concern, it can be resolved and therefore need not be addressed here any further.

The plan of the rest of the paper is as follows. In section 2 we recount the properties of the allowed $\mu\tau$ symmetric four zero Y_ν textures. Section 3 contains an outline of the basic steps in our calculation of η. In section 4, η is computed in our scheme for the three different heavy neutrino mass hierarchical cases in the regimes of unflavoured, fully flavored and τ-flavored leptogenesis for both categories A and B. Section 5 consists of our results on constraints emerging from η on the allowed $\mu\tau$ symmetric four zero Y_ν textures. In section 6 we discuss the departures - due to RG evolution down to laboratory energies - from $\mu\tau$ symmetry imposed at a high scale $\sim$ min $(M_1, M_2, M_3) \equiv M_{lowest}$. Section 7 summarizes our conclusions. Appendices A, B and C list the detailed expressions for η in each of the eighteen different possibilities.

2 Allowed $\mu\tau$ symmetric four zero textures of Y_ν

The complex symmetric light neutrino Majorana mass matrix m_ν is given in our basis by

$$m_\nu = -\frac{1}{2}v_u^2 Y_\nu \text{diag.}(M_1^{-1}, M_2^{-1}, M_3^{-1})Y_\nu^T = U\text{diag.}(m_1, m_2, m_3)U^T. \tag{2.1}$$

We work within the confines of the MSSM [52] so that $v_u = v\sin\beta$ and the W-mass equals $\frac{1}{2}gv$, g being the $SU(2)_L$ semiweak gauge coupling strength. The unitary PMNS mixing matrix U is parametrized as

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & -s_{23} \\ 0 & s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & -s_{13}e^{-i\delta_D} \\ 0 & 1 & 0 \\ s_{13}e^{i\delta_D} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\alpha_M} & 0 & 0 \\ 0 & e^{i\beta_M} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{2.2}$$

where $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$ and δ_D, α_M, β_M are the Dirac phase and two Majorana phases respectively.

The statement of $\mu\tau$ symmetry is that all couplings and masses in the pure neutrino part of the Lagrangian are invariant under the interchange of the flavor indices 2 and 3. Thus

$$(Y_\nu)_{12} = (Y_\nu)_{13}, \tag{2.3a}$$

$$(Y_\nu)_{21} = (Y_\nu)_{31}, \tag{2.3b}$$

$$(Y_\nu)_{23} = (Y_\nu)_{32}, \tag{2.3c}$$

$$(Y_\nu)_{22} = (Y_\nu)_{33} \tag{2.3d}$$

and

$$M_2 = M_3. \tag{2.4}$$

Eqs. (2.3) and (2.4), in conjunction with eq.(2.1), lead to a custodial $\mu\tau$ symmetry in m_ν :

$$(m_\nu)_{12} = (m_\nu)_{21} = (m_\nu)_{13} = (m_\nu)_{31}, \tag{2.5a}$$

$$(m_\nu)_{22} = (m_\nu)_{33}. \tag{2.5b}$$

Eqs. (2.5) immediately imply that $\theta_{23} = \pi/4$ and $\theta_{13} = 0$. With this $\mu\tau$ symmetry, it was shown in Ref. [16] that only four textures with four zeroes in Y_ν are allowed. These fall into two categories A and B - each category containing a pair of textures yielding an identical form of m_ν. These allowed textures may be written in the form of the Dirac mass matrix $m_D = Y_\nu v_u/\sqrt{2}$ in terms of complex parameters a_1, a_2, b_1, b_2.

$$\textit{Category } A : m_{DA}^{(1)} = \begin{pmatrix} a_1 & a_2 & a_2 \\ 0 & 0 & b_1 \\ 0 & b_1 & 0 \end{pmatrix}, m_{DA}^{(2)} = \begin{pmatrix} a_1 & a_2 & a_2 \\ 0 & b_1 & 0 \\ 0 & 0 & b_1 \end{pmatrix}, \tag{2.6a}$$

$$\textit{Category } B : m_{DB}^{(1)} = \begin{pmatrix} a_1 & 0 & 0 \\ b_1 & 0 & b_2 \\ b_1 & b_2 & 0 \end{pmatrix}, m_{DB}^{(2)} = \begin{pmatrix} a_1 & 0 & 0 \\ b_1 & b_2 & 0 \\ b_1 & 0 & b_2 \end{pmatrix}, \tag{2.6b}$$

The corresponding expressions for m_ν, obtained via eq.(2.1), are much simplified by a change of variables. We introduce overall mass scales $m_{A,B}$, real parameters k_1, k_2, l_1, l_2 and phases $\bar{\alpha}$ and $\bar{\beta}$ defined by

Category A :

$$m_A = -b_1^2/M_2, \qquad k_1 = \left|\frac{a_1}{b_1}\right|\sqrt{\frac{M_2}{M_1}}, \qquad k_2 = \left|\frac{a_2}{b_1}\right|, \qquad \bar{\alpha} = \arg\frac{a_1}{a_2}. \tag{2.7a}$$

Category B :

$$m_B = -b_2^2/M_2, \quad l_1 = \left|\frac{a_1}{b_2}\right|\sqrt{\frac{M_2}{M_1}}, \quad l_2 = \left|\frac{b_1}{b_2}\right|\sqrt{\frac{M_2}{M_1}}, \quad \bar{\beta} = \arg\frac{b_1}{b_2}. \tag{2.7b}$$

Then the light neutrino mass matrix for each category can be written as [53]

$$m_{\nu A} = m_A \begin{pmatrix} k_1^2 e^{2i\bar{\alpha}} + 2k_2^2 & k_2 & k_2 \\ k_2 & 1 & 0 \\ k_2 & 0 & 1 \end{pmatrix}, m_{\nu B} = m_B \begin{pmatrix} l_1^2 & l_1 l_2 e^{i\bar{\beta}} & l_1 l_2 e^{i\bar{\beta}} \\ l_1 l_2 e^{i\bar{\beta}} & l_2^2 e^{2i\bar{\beta}} + 1 & l_2^2 e^{2i\bar{\beta}} \\ l_1 l_2 e^{i\bar{\beta}} & l_2^2 e^{2i\bar{\beta}} & l_2^2 e^{2i\bar{\beta}} + 1 \end{pmatrix}. \tag{2.8}$$

We shall also employ the matrix

$$h = m_D^\dagger m_D \tag{2.9}$$

which is identical for the two textures of *Category A* as well as for the two textures of *Category B*. Indeed, it can be given separately for the two categories as

$$h_A = |m_A|\, M_1 \begin{pmatrix} k_1^2 & x^{1/4}k_1k_2e^{-i\bar{\alpha}} & x^{1/4}k_1k_2e^{-i\bar{\alpha}} \\ x^{1/4}k_1k_2e^{i\bar{\alpha}} & \sqrt{x}(1+k_2^2) & \sqrt{x}k_2^2 \\ x^{1/4}k_1k_2e^{i\bar{\alpha}} & \sqrt{x}k_2^2 & \sqrt{x}(1+k_2^2) \end{pmatrix}, \tag{2.10a}$$

$$h_B = |m_B|\, M_1 \begin{pmatrix} l_1^2+2l_2^2 & x^{1/4}l_2e^{-i\bar{\beta}} & x^{1/4}l_2e^{-i\bar{\beta}} \\ x^{1/4}l_2e^{i\bar{\beta}} & \sqrt{x} & 0 \\ x^{1/4}l_2e^{i\bar{\beta}} & 0 & \sqrt{x} \end{pmatrix}, \tag{2.10b}$$

where

$$x = \frac{M_{2=3}^2}{M_1^2}. \tag{2.11}$$

Restrictions on the parameters k_1, k_2, $\cos\bar{\alpha}$ and l_1, l_2, $\cos\bar{\beta}$ from neutrino oscillation data were worked out in ref. [16]. The relevant measured quantities are the ratio of the solar to atmospheric neutrino mass squared differences $R = \Delta m_{21}^2/\Delta m_{32}^2$ and the tangent of twice the solar mixing angle $\tan 2\theta_{12}$. One can write

$$R = 2(X_1^2+X_2^2)^{1/2}[X_3-(X_1^2+X_2^2)^{1/2}]^{-1}, \tag{2.12a}$$

$$\tan 2\theta_{12} = \frac{X_1}{X_2}. \tag{2.12b}$$

The quantities $X_{1,2,3}$ are given for the two categories as follows :

Category A :

$$X_{1A} = 2\sqrt{2}k_2[(1+2k_2^2)^2+k_1^4+2k_1^2(1+2k_2^2)\cos 2\bar{\alpha}]^{1/2}, \tag{2.13a}$$

$$X_{2A} = 1-k_1^4-4k_2^4-4k_1^2k_2^2\cos 2\bar{\alpha}, \tag{2.13b}$$

$$X_{3A} = 1-4k_2^4-k_1^4-4k_1^2k_2^2\cos 2\bar{\alpha}-4k_2^2. \tag{2.13c}$$

Category B :

$$X_{1B} = 2\sqrt{2}l_1l_2[(l_1^2+2l_2^2)^2+1+2(l_1^2+2l_2^2)\cos 2\bar{\beta}]^{1/2}, \tag{2.13d}$$

$$X_{2B} = 1+4l_2^2\cos 2\bar{\beta}+4l_2^4-l_1^4, \tag{2.13e}$$

$$X_{3B} = 1-(l_1^2+2l_2^2)^2-4l_2^2\cos 2\bar{\beta}. \tag{2.13f}$$

We also choose to define

$$X_{A,B} = (X^2_{1A,B} + X^2_{2A,B})^{1/2}. \tag{2.14}$$

At the 3σ level, $\tan 2\theta_{12}$ is presently known to be [56] between 1.83 and 4.90. For this range, only the inverted mass ordering for the light neutrinos, i.e. $\Delta m^2_{32} < 0$, is allowed for *Category A* with the allowed interval for R being $-4.13 \times 10^{-2}\ eV^2$ to $-2.53 \times 10^{-2}\ eV^2$. In contrast, the same range of $\tan 2\theta_{12}$ allows only the normal light neutrino mass ordering $\Delta m^2_{32} > 0$ for *Category B* with R restricted to be between $2.46 \times 10^{-2}\ eV^2$ and $3.92 \times 10^{-2}\ eV^2$. A thin sliver is allowed [16] in the $k_1 - k_2$ plane for *Category A*, while a substantial region with two branches is allowed [16] in the $l_1 - l_2$ plane for *Category B*. Finally, $\cos\bar{\alpha}$ is restricted to the interval bounded by 0 and 0.0175, while $\cos\bar{\beta}$ is restricted to the interval bounded by 0 and 0.0523. Thus, $\bar{\alpha}$, $\bar{\beta}$ could be either in the first or in the fourth quadrant. The interesting new point in the present work is that the baryogenesis constraint leads to restrictions on $\sin 2\bar{\alpha}$ and $\sin 2\bar{\beta}$ to the extent of removing the quadrant ambiguity in $\bar{\alpha}$ and $\bar{\beta}$.

3 Basic calculation of baryon asymmetry

Armed with $\mu\tau$ symmetry as well as eqs. (2.8) and (2.10), we can tackle leptogenesis at a scale $\sim M_{lowest}$. There are three possible mass hierarchical cases for N_i. Case (a) corresponds to a normal hierarchy of the heavy Majorana neutrinos (NHN), i.e. $M_{lowest} = M_1 << M_2 = M_3$. In case (b) one has an inverted hierarchy for N_i (IHN) with $M_{lowest} = M_2 = M_3 << M_1$. Case (c) refers to the quasidegenerate (QDN) situation with $M_1 \sim M_2 \sim M_3 \sim M_{lowest}$. Working within the MSSM [52] and completely neglecting possible scattering processes [53] which violate lepton number, we can take the asymmtries generated by N_i decaying into a doublet of leptons L_α and a Higgs doublet H_u as

$$\epsilon^\alpha_i = \frac{\Gamma(N_i \to L^C_\alpha H_u) - \Gamma(N_i \to L_\alpha H^C_u)}{\Gamma(N_i \to L^C_\alpha H_u) + \Gamma(N_i \to L_\alpha H^C_u)} \simeq \frac{1}{4\pi v_u^2 h_{ii}} \sum_{j \neq i} \left[\mathcal{I}^\alpha_{ij} f(x_{ij}) + \mathcal{J}^\alpha_{ij} \frac{1}{1 - x_{ij}} \right], \tag{3.1}$$

$$\mathcal{I}^\alpha_{ij} = \mathrm{Im}\ [(m^\dagger_D)_{i\alpha} (m_D)_{\alpha j} h_{ij}], \tag{3.2}$$

$$\mathcal{J}^\alpha_{ij} = \mathrm{Im}\ [(m^\dagger_D)_{i\alpha} (m_D)_{\alpha j} h_{ji}], \tag{3.3}$$

where

$$x_{ij} = M^2_j / M^2_i \tag{3.4}$$

and

$$f(x_{ij}) = \sqrt{x_{ij}} \left[\frac{2}{1 - x_{ij}} - \ln \frac{1 + x_{ij}}{x_{ij}} \right]. \tag{3.5}$$

We note here that the $\mathcal{J}_{ij}^\alpha$ term does not contribute to ϵ_i^α in our scheme since it vanishes [16] on account of $\mu\tau$ symmetry. Further, contributions to ϵ_i^α from N_i decaying into sleptons and higgsinos and from sneutrinos $\tilde{N}_i$ decaying into sleptons and Higgs as well as into leptons and higgsinos have been included by appropriately choosing the x_{ij}-dependence in the RHS of eq. (3.5). Observe also that $\mathcal{I}_{1j}^\alpha$ (and hence ϵ_1^α) gets an overall minus sign from $\mathrm{Im}(e^{-i\bar{\alpha}}, e^{-i\bar{\beta}})$, whereas $\mathcal{I}_{2j}^\alpha$, $\mathcal{I}_{3j}^\alpha$ (and hence $\epsilon_{2,3}^\alpha$) get an overall plus sign from $\mathrm{Im}(e^{i\bar{\alpha}}, e^{i\bar{\beta}})$. Except for being positive in the region $0.4 \leq x_{ij} < 1$, the function $f(x_{ij})$ of eq.(3.5) is negative for all other values of its argument. These signs are crucial in determining the sign of η and hence those of $\bar{\alpha}$, $\bar{\beta}$.

The decay asymmetries ϵ_i^α get converted into a lepton asymmetry $Y^\alpha = (n_l^\alpha - \bar{n}_l^\alpha)s^{-1}$, s being the entropy density and n_l^α ($\bar{n}_l^\alpha$) being the leptonic (antileptonic) number density (including superpartners) for flavor α via the washout relation [53]

$$Y^\alpha = \sum_i \epsilon_i^\alpha \mathcal{K}_i^\alpha g_{\star i}^{-1}. \tag{3.6}$$

In eq. (3.6), $g_{\star i}$ is the effective number of spin degrees of freedom of particles and antiparticles at a temperature equal to M_i. Furthermore, when all the flavors are active, the quantity $\mathcal{K}_i^\alpha$ is given by the approximate relation [12, 51], neglecting contributions from off-diagonal elements of A,

$$(\mathcal{K}_i^\alpha)^{-1} \simeq \frac{8.25}{|A^{\alpha\alpha}|K_i^\alpha} + \left(\frac{|A^{\alpha\alpha}|K_i^\alpha}{0.2} \right)^{1.16}. \tag{3.7}$$

In eq. (3.7), K_i^α is the flavor washout factor given by

$$K_i^\alpha = \frac{\Gamma\left(N \to L_\alpha H_u^C\right)}{H(M_i)} = \frac{|m_{D\alpha i}|^2}{M_i} \frac{M_{Pl}}{6.64\pi\sqrt{g_{\star i}} v_u^2}, \tag{3.8}$$

M_{Pl} being the Planck mass. This follows since the Hubble expansion parameter $H(M_i)$ at a temperature M_i is given by $1.66\sqrt{g_{\star i}} M_i^2 M_{Pl}^{-1}$. Moreover, to the lowest order, $\Gamma(N_i \to L_\alpha H_u^C)$ equals $|m_{D\alpha i}|^2 M_i (4\pi v_u^2)^{-1}$. An additional quantity, appearing in eq. (3.7), is $A^{\alpha\alpha}$, a diagonal element of the matrix $A^{\alpha\beta}$ defined by

$$Y_{\mathrm{L}}^\alpha = \sum_\beta A^{\alpha\beta} Y_\Delta^\beta. \tag{3.9}$$

Here $Y_{\rm L}^{\alpha} = s^{-1}(n_{\rm L}^{\alpha} - \bar{n}_{\rm L}^{\alpha})$, n_L^{α} being the number density of left-handed lepton and slepton doublets of flavor α and $Y_{\Delta}^{\alpha} = \frac{1}{3}Y_B - Y^{\alpha}$, Y_B being the baryonic number density (normalized to the entropy density s) including all superpartners. The precise forms for $A^{\alpha\beta}$ in different regimes of leptogenesis will be specified later.

One can now utilize the relation between $Y_B = (n_B - n_{\bar{B}})s^{-1}$ and $Y_l = \sum_{\alpha} Y^{\alpha}$, namely [57]

$$Y_B = -\frac{8n_F + 4n_H}{22n_F + 13n_H} Y_l, \tag{3.10}$$

where n_F (n_H) is the number of matter fermion (Higgs) $SU(2)_L$ doublets present in the theory at electroweak temperatures. For MSSM, $n_F = 3$ and $n_H = 2$ so that eq. (3.10) becomes

$$Y_B = -\frac{8}{23} Y_l. \tag{3.11}$$

The baryon asymmetry $\eta = (n_B - n_{\bar{B}})n_{\gamma}^{-1}$ can now be calculated, utilizing the result [58] that $sn_{\gamma}^{-1} \simeq 7.04$ at the present time, to be

$$\eta = \frac{s}{n_{\gamma}} Y_B \simeq 7.04 Y_B \simeq -2.45 Y_l. \tag{3.12}$$

Leptogenesis occurs at a temperature of the order of M_{lowest} and the effective values of $A^{\alpha\alpha}$ and $\mathcal{K}_i^{\alpha}$ depend on which flavors are active in the washout process. This is controlled [53] by the quantity $M_{lowest}(1 + \tan^2\beta)^{-1}$. There are three different regimes which we discuss separately.

(1) $\mathbf{M_{lowest}(1 + \tan^2\beta)^{-1} > 10^{12}}$ GeV.

In this case there is no flavor discrimination and unflavored leptogenesis takes place. Thus $A^{\alpha\beta} = -\delta^{\alpha\beta}$ and all flavors α can just be summed in eqs. (3.1). Thus $\epsilon_i = \sum \epsilon_i^{\alpha}$, $\sum_{\alpha} \mathcal{J}_{ij}^{\alpha} = 0$, $\mathcal{I}_{ij} \equiv \sum_{\alpha} \mathcal{I}_{ij}^{\alpha} = \mathrm{Im}\,(h_{ij})^2$ and $Y = \sum_i \epsilon_i g_{\star i}^{-1} \mathcal{K}_i$ with $\mathcal{K}_i^{-1} = 8.25 K_i^{-1} + (K_i/0.2)^{1.16}$ and $K_i = \sum_{\alpha} K_i^{\alpha} = h_{ii} M_{Pl} (6.64\pi\sqrt{g_{\star i}} M_i v_u^2)^{-1}$. For the normal hierarchical heavy neutrino (NHN) case (a), $M_{2=3}$ may be ignored and the index i can be restricted to just 1, taking $g_{\star 1} = 232.5$. For the corresponding inverted hierarchical (IHN) case (b) M_1 can be ignored and i made to run over 2 and 3 with $g_{\star 2=3} = 236.25$, all quantities involving the index 2 being identical to the corresponding ones involving 3. Coming to the quasidegenerate (QDN) heavy neutrino case (c), $g_{\star} = 240$ and the contributions from $i = 1$ must be separately added to identical contributions from $i = 2, 3$.

(2) $\mathbf{M_{lowest}(1 + \tan^2\beta)^{-1} < 10^9}$ GeV.

Here, all flavors are separately active and one has fully flavored leptogenesis. Now the A-matrix needs to be taken as [53]

$$A^{MSSM} = \begin{pmatrix} -93/110 & 6/55 & 6/55 \\ 3/40 & -19/30 & 1/30 \\ 3/40 & 1/30 & -19/30 \end{pmatrix} \tag{3.13}$$

and eqs. (3.6) – (3.8) used for each flavor α. Once again, we consider the different cases (a), (b) and (c) of heavy neutrino mass ordering. Ignoring $M_{2=3}$ for case (a) and with $g_{\star 1} = 232.5$, we have $\eta \simeq -1.05 \times 10^{-2} \sum_\alpha \epsilon_1^\alpha \mathcal{K}_1^\alpha$. Similarly, ignoring M_1 for case (b) and with $g_{\star 2=3} = 236.25$, one gets $\eta \simeq -1.04 \times 10^{-2} \sum_\alpha (\epsilon_2^\alpha \mathcal{K}_2^\alpha + \epsilon_3^\alpha \mathcal{K}_3^\alpha)$. For case (c), $g_\star = 240$ and $\eta \simeq -1.02 \times 10^{-2} \sum_\alpha (\epsilon_1^\alpha \mathcal{K}_1^\alpha + \epsilon_2^\alpha \mathcal{K}_2^\alpha + \epsilon_3^\alpha \mathcal{K}_3^\alpha)$.

(3) $\mathbf{10^9}$ GeV $< \mathbf{M_{lowest}(1 + \tan^2 \beta)^{-1}} < \mathbf{10^{12}}$ GeV.

In this regime the τ- flavor decouples first while the electron and muon flavors act indistinguishably. The latter, therefore, can be summed. Now effectively A becomes a 2×2 matrix $\tilde{A}$ given by [53]

$$\tilde{A} = \begin{pmatrix} -541/761 & 152/761 \\ 46/761 & -494/761 \end{pmatrix} \tag{3.14}$$

and acting in a space spanned by $e + \mu$ and τ. Indeed, we can define $\mathcal{K}_i^{e+\mu}$ and $\tilde{\mathcal{K}}_i^\tau$ by

$$(\mathcal{K}_i^{e+\mu})^{-1} = \frac{8.25}{|\tilde{A}^{11}|(K_i^e + K_i^\mu)} + \left(\frac{|\tilde{A}^{11}|(K_i^e + K_i^\mu)}{0.2}\right)^{1.16}, \tag{3.15a}$$

$$(\tilde{\mathcal{K}}_i^\tau)^{-1} = \frac{8.25}{|\tilde{A}^{22}|K_i^\tau} + \left(\frac{|\tilde{A}^{22}|(K_i^\tau)}{0.2}\right)^{1.16}. \tag{3.15b}$$

Now, for case (a) with $g_{\star 1} = 232.5$, $\eta \simeq -1.05 \times 10^{-2}[(\epsilon_1^e + \epsilon_1^\mu)\mathcal{K}_1^{e+\mu} + \epsilon_1^\tau \tilde{\mathcal{K}}_1^\tau]$. Case (b) has $g_{\star 2=3} = 236.25$ and $\eta \simeq -1.04 \times 10^{-2} \sum_{k=2,3}[(\epsilon_k^e + \epsilon_k^\mu)\mathcal{K}_k^{e+\mu} + \epsilon_k^\tau \tilde{\mathcal{K}}_k^\tau]$. Finally, case (c), with $g_\star = 240$, has $\eta \simeq -1.02 \times 10^{-2} \sum_i [(\epsilon_i^e + \epsilon_i^\mu)\mathcal{K}_i^{e+\mu} + \epsilon_i^\tau \tilde{\mathcal{K}}_i^\tau]$.

4 Baryon asymmetry in the present scheme

(1) Regime of unflavored leptogenesis

As explained in Sec. 3, there is no flavor discrimination if $M_{lowest}(1 + \tan^2 \beta)^{-1} > 10^{12}$ GeV. The lepton asymmetry parameters ϵ_i can now be given after summing over α. Additional simplifications can be made by taking $v_u = v\sin\beta$ with $v \simeq 246$ GeV and substituting

$$|m| = \left(\Delta m^2_{21}/X\right)^{1/2}. \tag{4.1}$$

The relevant expressions for the two categories then are the following

Category A :

$$\epsilon_{1A} \simeq -2.35\times 10^{-8}\frac{M_1}{10^9\ \text{GeV}}\frac{k_2^2\sqrt{x}f(x)\sin 2\bar{\alpha}}{X_A^{1/2}\sin^2\beta}, \tag{4.2a}$$

$$\begin{aligned}\epsilon_{2A} = \epsilon_{3A} &\simeq 1.18\times 10^{-8}\frac{M_{2=3}}{10^9\ \text{GeV}}\frac{k_1^2k_2^2\frac{1}{\sqrt{x}}f(\frac{1}{x})\sin 2\bar{\alpha}}{(1+k_2^2)X_A^{1/2}\sin^2\beta}\\ &= 1.18\times 10^{-8}\frac{M_1}{10^9\ \text{GeV}}\frac{k_1^2k_2^2f(\frac{1}{x})\sin 2\bar{\alpha}}{(1+k_2^2){X_A}^{1/2}\sin^2\beta}.\end{aligned} \tag{4.2b}$$

Category B:

$$\epsilon_{1B} \simeq -2.35\times 10^{-8}\frac{M_1}{10^9\ \text{GeV}}\frac{l_2^2\sqrt{x}f(x)\sin 2\bar{\beta}}{(l_1^2+2l_2^2)X_B^{1/2}\sin^2\beta}, \tag{4.3a}$$

$$\begin{aligned}\epsilon_{2B} = \epsilon_{3B} &\simeq 1.18\times 10^{-8}\frac{M_{2=3}}{10^9\ \text{GeV}}\frac{l_2^2f(\frac{1}{x})\sin 2\bar{\beta}}{X_B^{1/2}\sin^2\beta}\\ &= 1.18\times 10^{-8}\frac{M_1}{10^9\ \text{GeV}}\frac{l_2^2\sqrt{x}f(\frac{1}{x})\sin 2\bar{\beta}}{X_B^{1/2}\sin^2\beta}.\end{aligned} \tag{4.3b}$$

Note that x was defined in eq.(2.11). We are now in a position to discuss the three N_i mass hierarchical cases. For case (a), with the much heavier $M_2 = M_3$ ignored and only M_1 contributing, we can give the following expressions for the flavor-summed washout factors.

Category A :

$$K_{1A} \simeq \frac{86.36\ k_1^2}{\sqrt{g_\star}X_A^{1/2}\sin^2\beta}, \tag{4.4a}$$

$$K_{2A} = K_{3A} \simeq \frac{86.36\ (1+k_2^2)}{\sqrt{g_\star}X_A^{1/2}\sin^2\beta}. \tag{4.4b}$$

Category B :

$$K_{1B} \simeq \frac{86.36\ (l_1^2+2l_2^2)}{\sqrt{g_\star}X_B^{1/2}\sin^2\beta}, \tag{4.4c}$$

$$K_{2B} = K_{3B} \simeq \frac{86.36}{\sqrt{g_\star}X_B^{1/2}\sin^2\beta}. \tag{4.4d}$$

Consequently,

$$\eta_A^{NHN} \simeq -1.05 \times 10^{-2} \, (\epsilon_{1A}\mathcal{K}_{1A})_{g_\star=232.5}, \tag{4.5a}$$

$$\eta_B^{NHN} \simeq -1.05 \times 10^{-2} \, (\epsilon_{1B}\mathcal{K}_{1B})_{g_\star=232.5}, \tag{4.5b}$$

with the dependence on the category (A or B) coming both through ϵ_1 and K_1 occuring in $\mathcal{K}_1$. For case (b), one can ignore M_1 and hence ϵ_1 and $\mathcal{K}_1$. Thus we have

$$\eta_A^{IHN} \simeq -2.06 \, (\epsilon_{2A}\mathcal{K}_{2A})_{g_\star=236.25}, \tag{4.6a}$$

$$\eta_B^{IHN} \simeq -2.06 \, (\epsilon_{2B}\mathcal{K}_{2B})_{g_\star=236.25}, \tag{4.6b}$$

where once again the category dependence comes in through ϵ_2 and K_2 occuring in $\mathcal{K}_2$. Finally, for case (c) with all three $M's$ contributing,

$$\eta_{A,B}^{QDN} \simeq -1.02 \times 10^{-2} \, (\epsilon_{1A,B}\mathcal{K}_{1A,B} + 2\epsilon_{2A,B}\mathcal{K}_{2A,B})_{g_\star=240}. \tag{4.7}$$

The expressions for $\mathcal{K}_{1,2}$ in terms of $K_{1,2}$, have already been given in Sec. 3. Detailed expressions for the right hand sides of eqs. (4.5), (4.6) and (4.7) are given in appendix A.

(2) Regime of fully flavored leptogenesis

If $M_{lowest}(1+\tan^2\beta)^{-1} < 10^9$ GeV, all leptonic flavors become active causing fully flavored leptogenesis, cf. Sec. 3. We now need to resort to eqs. (3.1) – (3.8) to compute the lepton (flavor) asymmetry Y^α. However, $\mathcal{J}_{ij}^\alpha$ vanishes explicitly for all the four cases of four zero textures of m_D being considered by us. Thus we need be concerned only with the $\mathcal{I}_{ij}^\alpha$ term in eq. (3.1). Even some of the latter vanish on account of the zeroes in our textures. However, let us first draw some general conclusions about the two categories of textures before taking up the three N_i hierarchical cases separately.

Category A:

It is clear from eq. (2.6a) that the presence of two zeroes in rows 2 and 3 in both textures $m_{DA}^{(1)}$ and $m_{DA}^{(2)}$ implies the vanishing of ${(m_D)^\dagger}_{i\mu}(m_D)_{\mu j}$ and ${(m_D)^\dagger}_{i\tau}(m_D)_{\tau j}$ for $i \neq j$. As a result, $I_{ij}^\mu = I_{ij}^\tau = 0$ which imply that

$$\epsilon_{iA}^\mu = \epsilon_{iA}^\tau = 0. \tag{4.8}$$

Thus K_{iA}^μ and K_{iA}^τ do not contribute to η. The expressions for the pertinent nonvanishing quantities are given by

$$\epsilon_{1A}^e \simeq -2.35 \times 10^{-8} \frac{M_1}{10^9 \text{ GeV}} \frac{k_2^2 \sqrt{x} f(x) \sin 2\bar{\alpha}}{X_A^{1/2} \sin^2\beta}, \tag{4.9a}$$

$$\epsilon^e_{2A} = \epsilon^e_{3A} \simeq 1.18 \times 10^{-8} \frac{M_1}{10^9 \text{ GeV}} \frac{k_1^2 k_2^2 f(\frac{1}{x}) \sin 2\bar{\alpha}}{(1+k_2^2) X_A^{1/2} \sin^2\beta} , \tag{4.9b}$$

$$K^e_{1A} \simeq \frac{86.36 \ k_1^2}{\sqrt{g_\star} X_A^{1/2} \sin^2\beta} , \tag{4.9c}$$

$$K^e_{2A} = K^e_{3A} \simeq \frac{86.36 \ k_2^2}{\sqrt{g_\star} X_A^{1/2} \sin^2\beta} . \tag{4.9d}$$

Category B :

In this case, each allowed texture of m_D in eq.(2.6b) has two zeroes in the first row in consequence of which ${(m_D)^\dagger}_{ie} (m_D)_{ej}$ vanishes for $i \neq j$. Therefor, $I^e_{ij} = 0$ because of which

$$\epsilon^e_{iB} = 0. \tag{4.10}$$

in either case. Furthermore, with $(h_B)_{23}$ and $(h_B)_{32}$ being zero, $I^{\mu,\tau}_{23}$ and $I^{\mu,\tau}_{32}$ vanish here for both textures $m^{(1)}_{DB}$ and $m^{(2)}_{DB}$. An additional point is that, for the texture $m^{(1)}_{DB}$, $I^\mu_{12} = 0 = I^\tau_{13}$ but $I^\tau_{12} \neq 0 \neq I^\mu_{13}$ while, for $m^{(2)}_{DB}$, $I^\mu_{13} = 0 = I^\tau_{12}$ but $I^\mu_{12} \neq 0 \neq I^\tau_{13}$. Consequently, ϵ^μ_{1B} is the same for both allowed textures and so is ϵ^τ_{1B}. Moreover, for $m^{(1)}_{DB}$, ϵ^μ_{2B} and ϵ^τ_{3B} vanish but ϵ^τ_{2B} and ϵ^μ_{3B} do not while, for $m^{(2)}_{DB}$, ϵ^τ_{2B} and ϵ^μ_{3B} vanish but ϵ^μ_{2B} and ϵ^τ_{3B} do not. In fact, explicitly one has

$$\epsilon^{(1)\mu}_{2B} = \epsilon^{(1)\tau}_{3B} = \epsilon^{(2)\tau}_{2B} = \epsilon^{(2)\mu}_{3B} = 0, \tag{4.11a}$$

$$\epsilon^{(1)\mu}_{1B} = \epsilon^{(1)\tau}_{1B} = \epsilon^{(2)\mu}_{1B} = \epsilon^{(2)\tau}_{1B} \simeq -1.18 \times 10^{-8} \frac{M_1}{10^9 \text{ GeV}} \frac{l_2^2 \sqrt{x} f(x) \sin 2\bar{\beta}}{(l_1^2 + 2l_2^2) X_B^{1/2} \sin^2\beta}, \tag{4.11b}$$

$$\epsilon^{(1)\tau}_{2B} = \epsilon^{(1)\mu}_{3B} = \epsilon^{(2)\mu}_{2B} = \epsilon^{(2)\tau}_{3B} \simeq 1.18 \times 10^{-8} \frac{M_{2=3}}{10^9 \text{GeV}} \frac{l_2^2 \frac{1}{\sqrt{x}} f(x) \sin 2\bar{\beta}}{X_B^{1/2} \sin^2\beta}. \tag{4.11c}$$

In these equations and henceforth the superscripts (1),(2) refer to $m^{(1)}_D$, $m^{(2)}_D$ respectively. Coming to the washout factors, one sees a similar pattern. For $m^{(1)}_{DB}$, K^μ_{2B} and K^τ_{3B} vanish while for $m^{(2)}_{DB}$, K^τ_{2B} and K^μ_{3B} are zero. Explicitly,

$$K^{(1)\mu}_{1B} = K^{(1)\tau}_{1B} = K^{(2)\mu}_{1B} = K^{(2)\tau}_{1B} \simeq \frac{86.36 \ l_2^2}{\sqrt{g_\star} X_B^{1/2} \sin^2\beta}. \tag{4.12a}$$

$$K^{(1)e}_{2B} = K^{(2)e}_{2B} = K^{(1)\mu}_{2B} = K^{(1)\tau}_{3B} = K^{(2)\tau}_{2B} = K^{(2)\mu}_{3B} = K^{(1)e}_{3B} = K^{(2)e}_{3B} = 0, \tag{4.12b}$$

$$K^{(1)\tau}_{2B} = K^{(1)\mu}_{3B} = K^{(2)\mu}_{2B} = K^{(2)\tau}_{3B} \simeq \frac{86.36}{\sqrt{g_\star} X_B^{1/2} \sin^2\beta}. \tag{4.12c}$$

Let us finally draw attention to an important consequence of eqs. (4.11) and (4.12). Since $\mathcal{K}_i^\alpha$ is just a known function of $A^{\alpha\alpha}$ as well as K_i^α and since $A^{\mu\mu}$ equals $A^{\tau\tau}$, the combination

$$\epsilon^\mu_{2B}\mathcal{K}^\mu_{2B} + \epsilon^\mu_{3B}\mathcal{K}^\mu_{3B} + \epsilon^\tau_{2B}\mathcal{K}^\tau_{2B} + \epsilon^\tau_{3B}\mathcal{K}^\tau_{3B} \tag{4.13}$$

is identical for $m_{DB}^{(1)}$ and $m_{DB}^{(2)}$ and is a characteristic of just *Category B*.

Now, for the normal N_i-hierarchical case (a), with $M_{2,3}$ neglected, we have the following expression for the baryon asymmetry.

Category A :

$$\eta_A^{NHN} \simeq -1.05 \times 10^{-2}\ \epsilon^e_{1A}(\mathcal{K}^e_{1A})_{g_\star=232.5}. \tag{4.14a}$$

Category B:

$$\eta_B^{NHN} \simeq -1.05 \times 10^{-2}\ [(\epsilon^\mu_{1B}\mathcal{K}^\mu_{1B} + \epsilon^\tau_{1B}\mathcal{K}^\tau_{1B}) - 2.1 \times 10^{-2}(\epsilon^\mu_{1B}\mathcal{K}^\mu_{1B})]_{g_\star=232.5}, \tag{4.14b}$$

where $\mu\tau$ symmetry has been used in the last step. For the inverted N_i- hierarchical case (b), with M_1 neglected, the results are given below.

Category A :

$$\eta_A^{IHN} \simeq -1.03 \times 10^{-2}\ (\epsilon^e_{2A}\mathcal{K}^e_{2A} + \epsilon^e_{3A}\mathcal{K}^e_{3A})_{g_\star=236.25} \simeq -2.06 \times 10^{-2}\epsilon^e_{2A}(\mathcal{K}^e_{2A})_{g_\star=236.25}. \tag{4.15a}$$

Category B :

$$\begin{aligned} \eta_B^{IHN} &\simeq -1.03 \times 10^{-2}\ (\epsilon^\mu_{2B}\mathcal{K}^\mu_{2B} + \epsilon^\tau_{2B}\mathcal{K}^\tau_{2B} + \epsilon^\mu_{3B}\mathcal{K}^\mu_{3B} + \epsilon^\tau_{3B}\mathcal{K}^\tau_{3B})_{g_\star=236.25} \\ &\simeq -2.06 \times 10^{-2}\ (\epsilon^\mu_{2B}\mathcal{K}^\mu_{2B} + \epsilon^\mu_{3B}\mathcal{K}^\mu_{3B})_{g_\star=236.25}. \end{aligned} \tag{4.15b}$$

In eq. (4.15b), the first (second) term in the RHS bracket vanishes for $m_{DB}^{(1)}$ ($m_{DB}^{(2)}$); the non-vanishing terms have identical expressions for both textures. Lastly, for the quasidegenerate case (c), the expressions for the baryon asymmetry are as follows.

Category A :

$$\eta_A^{QDN} \simeq -1.02 \times 10^{-2}\ (\epsilon^e_{1A}\mathcal{K}^e_{1A} + 2\epsilon^e_{2A}\mathcal{K}^e_{2A})_{g_\star=240}. \tag{4.16a}$$

Category B :

$$\eta_B^{QDN} \simeq -1.02\times 10^{-2}\ (\epsilon^\mu_{1B}\mathcal{K}^\mu_{1B}+\epsilon^\tau_{1B}\mathcal{K}^\tau_{1B}+\epsilon^\mu_{2B}\mathcal{K}^\mu_{2B}+\epsilon^\tau_{2B}\mathcal{K}^\tau_{2B}+\epsilon^\mu_{3B}\mathcal{K}^\mu_{3B}+\epsilon^\tau_{3B}\mathcal{K}^\tau_{3B})_{g_\star=240}$$
$$\simeq -2.04\times 10^{-2}\ (\epsilon^\mu_{1B}\mathcal{K}^\mu_{1B}+\epsilon^\mu_{2B}\mathcal{K}^\mu_{2B}+\epsilon^\mu_{3B}\mathcal{K}^\mu_{3B})_{g_\star=240}. \qquad (4.16b)$$

The second (third) term within the RHS bracket vanishes for $m^{(1)}_{DB}$ ($m^{(2)}_{DB}$), while the remaining terms are identical for both textures of *Category B*. Detailed expressions for the right hand sides of eqs. (4.14), (4.15) and (4.16) appear in appendix B.

(3) Regime of τ-flavored leptogenesis

We have discussed in Sec. 3 that, with 10^9 GeV $< M_{lowest}(1+\tan^2\beta)^{-1} < 10^{12}$ GeV, there is flavor active leptogenesis in the τ-sector but the electron and muon flavors can be summed. Thus, use can be made here of the flavor dependent results of Regime (2), but there is a proviso : both the generation and washout of Y_L take place in a flavor subspace spanned by $e+\mu$ and τ, cf. eqs. (3.13) and (3.14). Using the notation of eq. (3.15), we can then write the consequent baryon asymmetry as

$$\eta \simeq -2.45\sum_i g^{-1}_{\star i}[(\epsilon^e_i+\epsilon^\mu_i)\mathcal{K}^{e+\mu}_i+\epsilon^\tau_i\tilde{\mathcal{K}}^\tau_i]. \qquad (4.17)$$

In discussing the lepton asymmetries and washout factors in detail here, it will be useful to consider the situation for each texture in either category by itself. We shall therefore separately enumerate the N_i-hierarchical cases (a), (b) and (c) for each of the four textures using the subscripts A, B for the category and subscripts $(1), (2)$ for the textures.

Category A, $m^{(1)}_{DA}$.

Now $\epsilon^{(1)\mu}_{iA}=0=\epsilon^{(1)\tau}_{iA}$, cf. eq. (4.8). But, in addition, we have

$$0 = K^{(1)\mu}_{1A} = K^{(1)\tau}_{1A} = K^{(1)\mu}_{2A} = K^{(1)\tau}_{3A}. \qquad (4.18)$$

Here the nonvanishing $\epsilon^{(1)e}_{1A}$, $\epsilon^{(1)e}_{2A}=\epsilon^{(1)e}_{3A}$, $K^{(1)e}_{1A}$ and $K^{(1)e}_{2A}=K^{(1)e}_{3A}$ are as given by eqs. (4.9a) – (4.9d). Additionally,

$$K^{(1)e}_{1A} = \frac{86.36}{\sqrt{g_\star}}\frac{k_1^2}{X_A^{1/2}\sin^2\beta}, \qquad (4.19a)$$

$$K^{(1)e}_{2A} = K^{(1)e}_{3A} = \frac{86.36}{\sqrt{g_\star}}\frac{k_2^2}{X_A^{1/2}\sin^2\beta}, \qquad (4.19b)$$

$$K^{(1)\mu}_{3A} = K^{(1)\tau}_{2A} = \frac{86.36}{\sqrt{g_\star}}\frac{1}{X_A^{1/2}\sin^2\beta}. \qquad (4.19c)$$

Now for the NHN case (a), we have

$$\eta_A^{(1)NHN} \simeq -1.05 \times 10^{-2} \epsilon_{1A}^{e} \left[\left(\mathcal{K}_{1A}^{e+\mu} \right)_{K_{1A}^{\mu}=0} \right]_{g_\star=232.5} \tag{4.20}$$

with $\mathcal{K}_{1A}^{e+\mu}$ calculated as per eq. (3.15a) but setting $K_{1A}^{\mu} = 0$. For the IHN case (b), we can write

$$\eta_A^{(1)IHN} \simeq -1.03 \times 10^{-2} \left[\epsilon_{2A}^{e} \left(\mathcal{K}_{2A}^{e+\mu} \right)_{K_{2A}^{\mu}=0} + \epsilon_{3A}^{e} \mathcal{K}_{3A}^{e+\mu} \right]_{g_\star=236.5}. \tag{4.21}$$

Here again $\mathcal{K}_{2A}^{e+\mu}$ is calculated by putting $K_{2A}^{\mu} = 0$.

For the QDN case (c), the expression is

$$\eta_A^{(1)QDN} \simeq -1.02 \times 10^{-2} \left[\epsilon_{1A}^{e} \left(\mathcal{K}_{1A}^{e+\mu} \right)_{K_{1A}^{\mu}=0} + \epsilon_{2A}^{e} \left(\mathcal{K}_{2A}^{e+\mu} \right)_{K_{2A}^{\mu}=0} + \epsilon_{3A}^{e} \mathcal{K}_{3A}^{e+\mu} \right]_{g_\star=240}. \tag{4.22}$$

Once more, appropriate washout factors have to be set at zero as shown earlier in the calculation of $\mathcal{K}_{iA}^{e+\mu}$.

Category A, $m_{DA}^{(2)}$.

Again, $\epsilon_{iA}^{(2)\mu} = 0 = \epsilon_{iA}^{(2)\tau}$, but the vanishing washout factors now are

$$0 = K_{1A}^{(2)\mu} = K_{1A}^{(2)\tau} = K_{3A}^{(2)\mu} = K_{2A}^{(2)\tau}. \tag{4.23}$$

The pertinent nonzero quantities namely, $\epsilon_{1A}^{(2)e}$, $\epsilon_{2A}^{(2)e} = \epsilon_{3A}^{(2)e}$, $K_{1A}^{(2)e}$, $K_{2A}^{(2)e}$ and $K_{3A}^{(2)e}$ are the same as for $m_{DA}^{(1)}$. In addition,

$$K_{2A}^{(2)\mu} = K_{3A}^{(2)\tau} = \frac{86.36}{\sqrt{g_\star}} \frac{1}{X_A^{1/2} \sin^2\beta}. \tag{4.24}$$

Thus, for the NHN case (a),

$$\eta_A^{(2)NHN} \simeq -1.05 \times 10^{-2}\ \epsilon_{1A}^{e} \left[\left(\mathcal{K}_{1A}^{e+\mu} \right)_{K_{1A}^{\mu}=0} \right]_{g_\star=232.5}, \tag{4.25}$$

i.e. the same as in eq. (4.20). Then, for the IHN case (b), we have

$$\eta_A^{(2)IHN} \simeq 1.03 \times 10^{-2} \left[\epsilon_{2A}^{e} \mathcal{K}_{2A}^{e+\mu} + \epsilon_{3A}^{e} \left(\mathcal{K}_{3A}^{e+\mu} \right)_{K_{3A}^{\mu}=0} \right]_{g_\star=236.25}, \tag{4.26}$$

i.e. $\mathcal{K}_{2A}^{e+\mu}$ is calculated fully but $\mathcal{K}_{3A}^{e+\mu}$ by setting $K_{3A}^{\mu} = 0$. This expression turns out to be the same as for $m_{DA}^{(1)}$.

Finally, the QDN case (c) has the baryon asymmetry as

$$\eta_A^{(2)QDN} \simeq -1.02 \times 10^{-2} \left[\epsilon_{1A}^e \left(\mathcal{K}_{1A}^{e+\mu}\right)_{K_{1A}^\mu=0} + \epsilon_{2A}^e \mathcal{K}_{2A}^{e+\mu} + \epsilon_{3A}^e \left(\mathcal{K}_{3A}^{e+\mu}\right)_{K_{3A}^\mu=0}\right]_{g_\star=240} \tag{4.27}$$

with appropriate washout factors set to zero in $\mathcal{K}_{iA}^{e+\mu}$, as shown. Again, this turns out to be equal to that for $m_{DA}^{(1)}$. Detailed expressions for the right hand side of eqs. (4.20) and (4.25), which are identical, appear in appendix C. We make the same statement for eqs. (4.21) and (4.26) as well as for eqs. (4.22) and (4.27).

Category B, $m_{DB}^{(1)}$.

Here, $\epsilon_{iB}^{(1)e} = 0 = \epsilon_{2B}^{(1)\mu} = \epsilon_{3B}^{(1)\tau}$ and $K_{2B}^{(1)e}$ cf. eqs. (4.10) and (4.11a), while the vanishing washout factors are $K_{2B}^{(1)\mu}$, $K_{3B}^{(1)\tau}$, $K_{3B}^{(1)e}$. The pertinent nonzero quantities, as given in eqs. (4.11b), (4.11c) and (4.12a), (4.12c), are $\epsilon_{1B}^{(1)\mu} = \epsilon_{1B}^{(1)\tau}$, $\epsilon_{2B}^{(1)\tau} = \epsilon_{3B}^{(1)\mu}$ and $K_{1B}^{(1)\mu} = K_{1B}^{(1)\tau}$, $K_{2B}^{(1)\tau} = K_{3B}^{(1)\mu}$. Additionally,

$$K_{1B}^{(1)e} = \frac{86.36}{\sqrt{g_\star}} \frac{l_1^2}{X_B^{1/2} \sin^2\beta}, \tag{4.28a}$$

$$K_{1B}^{(1)\mu} = K_{1B}^{(1)\tau} = \frac{86.36}{\sqrt{g_\star}} \frac{l_2^2}{X_B^{1/2} \sin^2\beta}. \tag{4.28b}$$

$$K_{2B}^{(1)\tau} = K_{3B}^{(1)\mu} = \frac{86.36}{\sqrt{g_\star}} \frac{1}{X_B^{1/2} \sin^2\beta}. \tag{4.28c}$$

Therefore, for the NHN case (a),

$$\eta_B^{(1)NHN} \simeq -1.05 \times 10^{-2} \left[\epsilon_{1B}^\mu \mathcal{K}_{1B}^{e+\mu} + \epsilon_{1B}^\tau \tilde{\mathcal{K}}_{1B}^\tau\right]_{g_\star=232.5}. \tag{4.29}$$

Coming to the IHN case (b), we have

$$\eta_B^{(1)IHN} \simeq -1.03 \times 10^{-2} \left[\epsilon_{3B}^\mu \left(\mathcal{K}_{3B}^{e+\mu}\right)_{K_{3B}^\mu=0} + \epsilon_{2B}^\tau \tilde{\mathcal{K}}_{2B}^\tau\right]_{g_\star=236.25}, \tag{4.30}$$

with the first term within the RHS bracket calculated by setting $K_{3B}^\mu = 0$. For the final QDN case (c), the expression is

$$\eta_B^{(1)QDN} \simeq -1.02 \times 10^{-2} \left[\epsilon_{1B}^\mu \mathcal{K}_{1B}^{e+\mu} + \epsilon_{3B}^\mu \left(\mathcal{K}_{3B}^{e+\mu}\right)_{K_{3B}^e=0} + \epsilon_{1B}^\tau \tilde{\mathcal{K}}_{1B}^\tau + \epsilon_{2B}^\tau \tilde{\mathcal{K}}_{2B}^\tau\right]_{g_\star=240}, \tag{4.31}$$

where $\mathcal{K}_{3B}^{e+\mu}$ is calculated with K_{3B}^e set to vanish.

Category B, $m_{DB}^{(2)}$.

Here we have $\epsilon_{iB}^{(2)e} = 0 = \epsilon_{2B}^{(2)\tau} = \epsilon_{3B}^{(2)\mu}$ from eqs. (4.10) and (4.11a), while the washout factors $K_{2B}^{(2)\tau}$, $K_{3B}^{(2)\mu}$, $K_{2B}^{(2)e}$, $K_{3B}^{(2)e}$ vanish. The remaining nonzero quantities of relevance, as appear in eqs. (4.11b), (4.11,c) and (4.12a) (4.12c), are $\epsilon_{1B}^{(2)\mu} = \epsilon_{1B}^{(2)\tau}$, $\epsilon_{2B}^{(2)\mu} = \epsilon_{3B}^{(2)\tau}$ and $K_{1B}^{(2)\mu} = K_{1B}^{(2)\tau}$, $K_{3B}^{(2)\tau} = K_{2B}^{(2)\mu}$. In addition, $K_{1B}^{(2)e}$ has the same expression as $K_{1B}^{(1)e}$ i.e.

$$K_{1B}^{(2)e} = \frac{86.36}{\sqrt{g_\star}} \frac{l_1^2}{X_B^{1/2} \sin^2\beta}. \tag{4.32}$$

The NHN case (a) now yields

$$\eta_B^{(2)NHN} \simeq -1.05 \times 10^{-2} \Big[\epsilon_{1B}^{\mu} \mathcal{K}_{1B}^{e+\mu} + \epsilon_{1B}^{\tau} \tilde{\mathcal{K}}_{1B}^{\tau} \Big]_{g_\star = 232.5}, \tag{4.33}$$

as with $m_{DB}^{(1)}$. For the IHN case (b), the baryon asymmetry reads

$$\eta_B^{(2)IHN} \simeq -1.03 \times 10^{-2} \left[\epsilon_{3B}^{\tau} \Big(\tilde{\mathcal{K}}_{3B}^{\tau} \Big) + \epsilon_{2B}^{\mu} \Big(\mathcal{K}_{2B}^{e+\mu} \Big)_{K_{2B}^{e}=0} \right]_{g\star = 236.25} \tag{4.34}$$

which happens to have the same expression as for $m_{DB}^{(1)}$. Finally, for the QDN case (c), the baryon asymmetry is

$$\eta_B^{(2)QDN} \simeq -1.02 \times 10^{-2} \left[\epsilon_{1B}^{\mu} \mathcal{K}_{1B}^{e+\mu} + \epsilon_{2B}^{\mu} \Big(\mathcal{K}_{2B}^{e+\mu} \Big)_{K_{2B}^{e}=0} + \epsilon_{1B}^{\tau} \tilde{\mathcal{K}}_{1B}^{\tau} + \epsilon_{3B}^{\tau} \tilde{\mathcal{K}}_{3B}^{\tau} \right]_{g\star = 240} \tag{4.35}$$

which also turns out to be the same as for $m_{DB}^{(1)}$. Thus the baryon asymmetry in each of the three N_i-hierarchical cases has the same expression for both $m_D^{(1)}$ and $m_D^{(2)}$ in *Category A* and the same statement holds for *Category B*. Detailed expressions of η in *Category B* for the NHN, IHN and QDN cases are given in appendix C.

5 Results and discussion

We had earlier deduced [16] from neutrino oscillation data with 3σ errors the constraints $0 \leq \cos\bar{\alpha} \leq 0.0175$ and $0 \leq \cos\bar{\beta} \leq 0.0523$ for the phases $\bar{\alpha}$ and $\bar{\beta}$ of *Categories A* and *B* respectively. Thus each phase could have been in either the first or the fourth quadrant with $89^o \leq |\bar{\alpha}| \leq 90^o$ and $87^o \leq |\bar{\beta}| \leq 90^o$. The new requirement of matching the generated baryon asymmetry η_A (η_B) for *Category A* (B) with its observed value in the 3σ range 5.5×10^{-10} to 7.0×10^{-10} [57]–[62] puts restrictions on $\sin 2\bar{\alpha}$ ($\sin 2\bar{\beta}$) which fix both the magnitude and the sign of $\bar{\alpha}$ ($\bar{\beta}$). To be specific in our numerical analysis, we choose $x = M_{2=3}^2/M_1^2$ for the different hierarchical cases as follows : (a) for NHN, $x \geq 10$, (b) for IHN, $x \leq 0.1$, (c) for QDN, $0.1 \leq x \leq 10$. So far, we did not dwell on the mass ordering (normal or inverted) of the right handed heavy neutrinos N_i in the QDN case. For the normal ordering (NON) case, we take $1.1 \leq x \leq 10$, while for an inverted ordering (ION), our choice is $0.1 \leq x \leq 0.9$. As mentioned earlier, the function $f(x)$ is positive for $0.4 \leq x < 1.0$ and negative elsewhere. We need to avoid the point $x = 1$ which corresponds to the complete degeneracy of the N_i, i.e. $M_1 = M_2 = M_3$ since $f(x)$ diverges at this point. The inclusion of finite width corrections to propagators of right handed neutrinos in the one loop decay diagrams avoids this problem. Now, both the previously divergent part of the modified $f(x)$ and the lepton asymmetry vanish there. We also avoid the near $x = 1$ region, $0.9 < x < 1.1$, to exclude the so called resonant leptogenesis [63] since that is not part of our scenario. Tables 1 – 3 enumerate the emergent constraints on $\bar{\alpha}$, $\bar{\beta}$ in consequence of matching η_A, η_B for each

Category A				
Parameters	NHN	IHN	QDN	
			NON	ION
$\bar{\alpha}$	$\bar{\alpha} < 0$ $89.0^o - 89.9^o$	$\bar{\alpha} > 0$ $89.95^o - 89.99^o$	$\bar{\alpha} < 0$ $89.1^o - 89.9^o$	$\bar{\alpha} > 0$ $89.10^o - 89.99^o$
x	$10 - 10^3$	$0.001 - 0.1$	$2.0 - 9.1$	$0.1 - 0.9$
$\tan\beta$	$2 - 60$	$2 - 5$	$2 - 60$	$2 - 60$
$\frac{M_{lowest}}{10^9 GeV}$	5.0×10^3 — 4.9×10^6	5.0×10^3 — 2.6×10^4	5×10^3 — 3.6×10^6	5.0×10^3 — 4.9×10^6
Category B				
Parameters	NHN	IHN	QDN	
			NON	ION
$\bar{\beta}$	$\bar{\beta} < 0$ $88.8^o - 89.9^o$	$\bar{\beta} > 0$ $89.48^o - 89.99^o$	$\bar{\beta} < 0$ $87.0^o - 89.9^o$	$\bar{\beta} > 0$ $89.84^o - 89.99^o$
x	$10 - 10^3$	$0.001 - 0.1$	8.3 - 9.5	$0.1 - 0.9$
$\tan\beta$	$2 - 8$	$2 - 12$	$2 - 60$	$2 - 10$
$\frac{M_{lowest}}{10^9 GeV}$	8.4×10^3 — 8.5×10^4	5.0×10^3 — 1.6×10^5	5.0×10^3 — 4.9×10^6	5.0×10^3 — 1.0×10^5

Table 1: Allowed $\bar{\alpha}$,$\bar{\beta}$ and other parameters for unflavored leptogenesis

of the eighteen different possibilities described earlier with corresponding restrictions on the

Category A				
Parameters	NHN	IHN	QDN	
			NON	ION
$\bar{\alpha}$	$\bar{\alpha} < 0$ $89.4^o - 89.9^o$	$\bar{\alpha} > 0$ $89.0^o - 89.8^o$	$\bar{\alpha} < 0$ $89.1^o - 89.9^o$	$\bar{\alpha} > 0$ $89.0^o - 89.9^o$
x	$10 - 10^3$	$0.001 - 0.1$	$1.1 - 10.0$	$0.1 - 0.9$
$\tan\beta$	$25 - 60$	$22 - 60$	$2 - 60$	$2 - 60$
$\frac{M_{lowest}}{10^9 GeV}$	67 — 3.6×10^3	4.9×10^2 — 3.6×10^3	23 — 3.60×10^3	10 — 3.6×10^3
Category B				
Parameters	NHN	IHN	QDN	
			NON	ION
$\bar{\beta}$	$\bar{\beta} < 0$ $87.0^o - 89.9^o$	$\bar{\beta} > 0$ $87.0^o - 89.9^o$	$\bar{\beta} < 0$ $87.0^o - 89.9^o$	$\bar{\beta} > 0$ $87.0^o - 89.9^o$
x	$10 - 10^3$	$0.001 - 0.1$	$1.1 - 10$	$0.3 - 0.9$
$\tan\beta$	$16 - 60$	$24 - 60$	$6 - 60$	$7 - 60$
$\frac{M_{lowest}}{10^9 GeV}$	2.4×10^2 — 3.6×10^3	5.7×10^2 — 3.6×10^3	0.35×10^2 — 3.6×10^3	0.49×10^2 — 3.6×10^3

Table 2: Allowed $\bar{\alpha}$, $\bar{\beta}$ and other parameters for fully flavored leptogenesis

parameters x, $\tan\beta$ and M_{lowest} as shown. We would like to make the following comments on the information contained in tables 1 – 3.

1. *Signs of phase angles* : We have a positive baryon asymmetry in our universe. From the formulae for all NHN cases in the Apendices, we can say that sign of $f(x)\sin 2(\bar{\alpha}, \bar{\beta})$ has to be positive in order to generate such a positive asymmetry. But $f(x)$ is negative in the NHN region of $x \geq 10$. So, $\bar{\alpha}$, $\bar{\beta}$ have to be negative for all NHN cases. On the contrary, for all IHN cases, there is an overall negative sign in the formulae for η since $\mathrm{Im}(h_{21}^2) = \mathrm{Im}(h_{31}^2)$ here is opposite in sign to $\mathrm{Im}(h_{12}^2) = \mathrm{Im}(h_{13}^2)$ that come in for the NHN case. So, for a positive η, a negative sign of $f(x)\sin 2(\bar{\alpha}, \bar{\beta})$ is needed in all IHN cases. Again, $f(x)$ is negative in the NHN region of $x \leq 0.1$. For this

Category A				
Parameters	NHN	IHN	QDN	
			NON	ION
$\bar{\alpha}$	$\bar{\alpha} < 0$ $89.0^o - 89.9^o$	$\bar{\alpha} > 0$ $89.0^o - 89.9^o$	$\bar{\alpha} < 0$ $89.0^o - 89.9^o$	$\bar{\alpha} > 0$ $89.0^o - 89.9^o$
x	$10 - 10^3$	$0.001 - 0.1$	$1.1 - 10.0$	$0.1 - 0.9$
$\tan\beta$	$2 - 60$	$2 - 60$	$2 - 60$	$2 - 60$
$\frac{M_{lowest}}{10^9 GeV}$	1.7×10^3 — 4.0×10^4	50 — 1.03×10^4	100 — 1.97×10^4	100 — 1.45×10^4
Category B				
Parameters	NHN	IHN	QDN	
			NON	ION
$\bar{\beta}$	$\bar{\beta} < 0$ $87.0^o - 89.9^o$	$\bar{\beta} > 0$ $87.0^o - 89.9^o$	$\bar{\beta} < 0$ $87.0^o - 89.9^o$	$\bar{\beta} > 0$ $87.0^o - 89.9^o$
x	$10 - 10^3$	$0.001 - 0.1$	$1.1 - 10.0$	$0.1 - 0.9$
$\tan\beta$	$2 - 60$	$2 - 60$	$2 - 60$	$2 - 60$
$\frac{M_{lowest}}{10^9 GeV}$	3.25×10^2 — 2.3×10^4	6.25×10^2 — 5.0×10^4	0.37×10^2 — 2.1×10^4	0.37×10^2 — 1.6×10^5

Table 3: Allowed $\bar{\alpha}$, $\bar{\beta}$ and other parameters for τ-flavored leptogenesis

reason, $\bar{\alpha}$, $\bar{\beta}$ are positive in all IHN cases. For QDN cases we need to discuss the possibilities of normal and inverted ordering of M_i separately. Here there are two terms with $f(x)$ and $-f(1/x)$ along with an overall factor $\sin 2(\bar{\alpha}, \bar{\beta})$. For the NON region $1.1 \leq x \leq 10.0$, $f(x)$ is negative while $-f(1/x)$ is negative for $1.1 \leq x \leq 2.5$. So, for the region $1.1 \leq x \leq 2.5$, $\bar{\alpha}$, $\bar{\beta}$ are required to be negative. For the remaining part of the NON region $2.5 \leq x \leq 10$, $f(x)$ is negative and $-f(1/x)$ positive but the $f(x)$ term dominates over the $-f(1/x)$ term. So, negative signs also are needed for $\bar{\alpha}$, $\bar{\beta}$, in the region $2.5 \leq x \leq 10$. Thus all QDN cases with NON require negative sign of $\bar{\alpha}$, $\bar{\beta}$. Again, for QDN with ION, both $f(x)$ and $-f(1/x)$ are positive in the region $0.4 \leq x \leq 0.9$. In the rest of the ION region $0.1 \leq x \leq 0.4$, $f(x)$ is negative and $-f(1/x)$ is positive. But, now the latter term dominates over the former one. So, positive $\bar{\alpha}$, $\bar{\beta}$ are needed in all QDN cases with ION. In fact, we see (tables 1 – 3) that for all normal (both hierarchical and quasidegenrate) mass ordering cases of M_i, the phases are negative whereas, for all inverted (both hierarchical and quasidegenrate) mass ordering cases, they are positive. One may also note that in all cases and regimes the size of the allowed range of $\tan\beta$ is correlated with that of the phase $\bar{\alpha}/\bar{\beta}$.

2. *Magnitudes of phase angles and other parameters* : Neither $\bar{\alpha}$ nor $\bar{\beta}$ can be strictly 90^o since η then vanishes. Therefore, a nonzero η is incompatible in *Category A* with tribimaximal mixing which requires [16] $\bar{\alpha} = \pi/2$. The numerical value of η is most sensitive to the values of $\sin 2(\bar{\alpha}, \bar{\beta})$, M_{lowest} (M_1 for normal mass ordering, M_2 for inverted mass ordering) and to some extent to the function f (and hence x) for acceptable ranges of k_1, k_2 (*Category* A) and l_1, l_2 (*Category B*). The latter are of course restricted [16] by the neutrino oscillation data. For unflavored leptogenesis with $M_{lowest} > (1 + \tan^2\beta)10^{12}$ GeV, the minimum value of M_{lowest} is 5×10^{12} GeV, while we cut the maximum value at 5×10^{15} GeV to avoid the GUT scale whereabouts all produced asymmetry gets washed out by inflation. Such a large value of M_{lowest} forces a small value of $\sin 2(\bar{\alpha}, \bar{\beta})$ in order to have the baryon asymmetry in the right range. In *Category A*, the range of $|\bar{\alpha}|$ is restricted to $89^o \leq |\bar{\alpha}| \leq 90^o$ so that $\sin 2\bar{\alpha}$ is small there. In the IHN case of *Category A*, other associated factors including $f(1/x)$ cause further restrictions on $\bar{\alpha}$, cf. Table 1. In *Category B* the range $87^o \leq |\bar{\beta}| \leq 90^o$ is curtailed to $|\bar{\beta}| > 88.8^o$ due to the large value of M_{lowest} in flavor independent leptogenesis except the QDN (NON) case where other factors are responsible for necessary suppression.

3. The quadrants of $\bar{\alpha}$, $\bar{\beta}$ do not change between unflavored, fully flavored and τ-flavored leptogenesis, nor is there any dependence of them on the value of $\tan\beta$. They only depend on whether N_i have a normal ($M_1 < M_{2=3}$) or inverted ($M_1 > M_{2=3}$) mass ordering. For the former, $\bar{\alpha}$ and $\bar{\beta}$ are always in the fourth quadrant (< 0) since ϵ_1 always has a minus sign in front, while the latter always forces them to be in the first quadrant (> 0) since $\epsilon_2 = \epsilon_3$ always has a plus sign in front.

4. The constraints on $\sin 2\bar{\alpha}$, $\sin 2\bar{\beta}$ - extracted from $\eta_{A,B}$ - restrict the allowed intervals for $|\bar{\alpha}|$, $|\bar{\beta}|$ more stringently than do constraints on $\cos\bar{\alpha}$, $\cos\bar{\beta}$ obtained [16] from neutrino oscillation phenomenology.

6 Effect of radiative $\mu\tau$ symmetry breaking

While explaining a maximal value for θ_{23}, exact $\mu\tau$ symmetry predicts a vanishing θ_{13}. The latter will make the CP violating Dirac phase δ_D unobservable in neutrino oscillation experiments, many of which are being planned to study CP violation in the neutrino sector. Thus it may be desirable to have a nonzero θ_{13}, however small.

Suppose $\mu\tau$ symmetry is exact at a high energy $\Lambda \sim 10^{12}$ GeV characterizing the heavy Majorana neutrino mass scale. Running down to a laboratory scale $\lambda \sim 10^3$ GeV, via one-loop renormalization group evolution, one picks up small factorizable departures from $\mu\tau$ symmetry, induced by charged lepton mass terms, in the elements of the light neutrino mass matrix m_ν. These cause small departures from 45^o in θ_{23}^λ and tiny nonzero values for θ_{13}^λ. Neglecting $m^2_{\mu,e}$ in comparison with m^2_τ, one obtains [16] that

$$m_\nu^\lambda \simeq \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1-\Delta_\tau \end{pmatrix} m_\nu^\Lambda \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1-\Delta_\tau \end{pmatrix}, \tag{6.1}$$

where m_ν^Λ is $\mu\tau$ symmetric and the deviation Δ_τ is given in MSSM by

$$\Delta_\tau \simeq \frac{m_\tau^2}{8\pi^2 v^2}(1+\tan^2\beta)\ln\frac{\Lambda}{\lambda} \simeq 6\times 10^{-6}\ (1+\tan^2\beta). \tag{6.2}$$

Working to the lowest nontrivial order in Δ_τ, the phenomenological consequences of eq. (6.1), derived from extant neutrino oscillation data, were worked out in ref.[15]. The allowed regions in the $k_1 - k_2$ ($l_1 - l_2$) plane for *Category A* (B) get slightly extended. Moreover, one finds that $\theta_{23}^\lambda \leq 45^o$ as well as $0 \leq \theta_{13}^\lambda \leq 2.7^o$ for *Category A* and $45^o \leq \theta_{23}^\lambda$ as well as $0 \leq \theta_{13}^\lambda \leq 0.85^o$ for *Category B*. The upper bounds on θ_{13}^λ in both categories correspond to $\tan\beta = 60$.

RG evolution from Λ to λ has no direct effect on the baryon asymmetry η. The lepton asymmetry Y_l, produced at the heavy Majorana neutrino mass scale, remains frozen till the temperature comes down to the weak scale where it is converted to η. The requirement of the latter being in the observed range leads to correlated constraints on x, M_{lowest} and $\tan\beta$, vide tables 1 – 3. While the constraints on x and M_{lowest} have some effects on the magnitude of Λ, they are numerically quite weak. Such is, however, not the case with the $\tan\beta$ constraints, owing to eq. (6.2). In particular, the bounds on θ_{13}^λ can be significantly affected by restrictions on $\tan\beta$.

Let us discuss the consequent effects on the said bounds in the three regimes.

(1) Flavor independent leptogenesis. Here $\tan\beta$ can go from 2 to 60, as taken in Ref.[15], for the NHN and QDN cases of *Category A* and the QDN (NON) case of *Category B*, cf. Table 1. Therefore the range of θ_{13}^{λ} remains unchanged for those cases. But the stronger restrictions on $\tan\beta$ given in Table 1 for the IHN case of *Category A* and the NHN, IHN and QDN (ION) cases of *Category B* force the corresponding θ_{13}^{λ} and θ_{23}^{λ} to be practically equal to 0^o and 45^o respectively for those two situations.

(2) Fully flavored leptogenesis. We can deduce from the information given in table 2 that the ranges of θ_{13}^{λ} are affected here for either category in each case. The results are given in table 4.

	Category A			Category B		
	NHN	IHN	QDN	NHN	IHN	QDN
$\tan\beta$	25-60	$22-60$	$2-60$	$16-60$	$24-60$	$6-60$ (NON) $7-60$ (ION)
θ_{13}^{λ}	$0.47^\circ-2.7^\circ$	$0.36^\circ-2.7^\circ$	$0^\circ-2.7^\circ$	$0.06^\circ-0.85^\circ$	$0.14^\circ-0.85^\circ$	$0^\circ-0.85^\circ$

Table 4: Effect on θ_{13}^{λ} of the more restricted range of $\tan\beta$ in fully flavored leptogenesis.

(3) τ-flavored leptogenesis. There is no additional restriction on $\tan\beta$ here as compared with unflavored leptogenesis, vide table 3. Hence the ranges of θ_{13}^{λ} stand unchanged in either category for the NHN, IHN and QDN cases.

Now that there is a nonzero θ_{13}^{λ}, one has CP violation in the neutrino sector which can be measured from the difference in oscillation probabilities $P(\nu_\mu \to \nu_e) - P(\bar{\nu_\mu} \to \bar{\nu_e})$ [64]. For the CKM CP phase δ^λ, we find the 3σ range of its value to be $1.0^o \leq \delta^\lambda \leq 70^o$ (*Category A*) and $1.5^o \leq \delta^\lambda \leq 90^o$ (*Category B*) for both flavored and unflavored leptogenesis in all regimes. The sign of δ^λ is opposite to the sign of $\bar{\alpha}/\bar{\beta}$ for *Category A/B* and hence it does change from one regime for $M_{lowest}(1+\tan^2\beta)^{-1}$ to another for a given mass ordering of N_i.

7 Conclusion

In this paper we have studied the generation of the observed amount of baryon asymmetry η in our scheme of $\mu\tau$ symmetric four zero neutrino Yukawa textures within the type-I seesaw. For each of the two categories A and B of our scheme, we have identified three regimes depending on the value of $M_{lowest}(1+\tan^2\beta)^{-1}$ and have studied the normal-hierarchical (NHN), inverted-hierarchical (IHN) and quasidegenerate (QDN) cases for the masses of the heavy Majorana neutrinos N_i. The requirement of matching the right value of η forces the phases $\bar{\alpha}$ (*Category A*) and $\bar{\beta}$ (*Category B*) to be in the fourth quadrant for the NHN and QDN cases and in the first quadrant for the IHN case in each regime. Restrictions on small but nonzero θ_{13}, arising from radiative $\mu\tau$ symmetry breaking, have also been worked out.

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Note added

A new paper on supersymmetric leptogenesis appeared [65] after this work was completed. The authors of ref. [65] have highlighted certain additional contributions to Y_Δ. These arise from soft supersymmetry breaking effects involving gauginos and higgsinos as well as anomalous global symmetries causing a different pattern of sphaleron induced lepton flavor mixing. While some of the numerical coeffcients – given in the various expressions for η in our analysis – are likely to change if these effects are included, their overall signs will not. Consequently, there will be no alteration in our conclusions on the quadrants of the phases $\bar{\alpha}$ and $\bar{\beta}$ which remain robust.

A Baryon Asymmetry in flavor independent leptogenesis

Category A

$$\begin{aligned}\eta_A^{NHN} &\simeq 2.47\times 10^{-10}\frac{M_1}{10^9\mathrm{GeV}}\frac{k_2^2\sin 2\bar{\alpha}}{X_A^{1/2}\sin^2\beta}\frac{M_{2=3}}{M_1}f\left(M_{2=3}^2/M_1^2\right)\times\\ &\left[\frac{1.46\sin^2\beta X_A^{1/2}}{k_1^2}+\left(\frac{28.3k_1^2}{X_A^{1/2}\sin^2\beta}\right)^{1.16}\right]^{-1}. \end{aligned} \tag{A.1}$$

$$\begin{aligned}\eta_A^{IHN} &\simeq -2.43\times 10^{-10}\frac{M_{2=3}}{10^9\mathrm{GeV}}\frac{k_1^2k_2^2\sin 2\bar{\alpha}}{(1+k_2^2)X_A^{1/2}\sin^2\beta}\frac{M_1}{M_{2=3}}f\left(M_1^2/M_{2=3}^2\right)\times\\ &\left[1.47(1+k_2^2)^{-1}X_A^{1/2}\sin^2\beta+\left(\frac{28\ (1+k_2^2)}{X_A^{1/2}\sin^2\beta}\right)^{1.16}\right]^{-1}. \end{aligned} \tag{A.2}$$

$$\begin{aligned}\eta_A^{QDN} \simeq 2.40\times 10^{-10}\frac{k_2^2\sin 2\bar{\alpha}}{X_A^{1/2}\sin^2\beta}\Bigg\{&\frac{M_1}{10^9\mathrm{GeV}}\frac{M_{2=3}}{M_1}f\left(M_{2=3}^2/M_1^2\right)\times\\ &\left[1.48(k_1^2)^{-1}X_A^{1/2}\sin^2\beta+\left(\frac{27.9\ k_1^2)}{X_A^{1/2}\sin^2\beta}\right)^{1.16}\right]^{-1}\\ -\frac{M_{2=3}}{10^9\mathrm{GeV}}\frac{k_1^2}{(1+k_2^2)}\frac{M_1}{M_{2=3}}f\left(M_1^2/M_{2=3}^2\right)\times&\left[1.48(1+k_1^2)^{-1}X_A^{1/2}\sin^2\beta+\left(\frac{27.9\ (1+k_2^2)}{X_A^{1/2}\sin^2\beta}\right)^{1.16}\right]^{-1}\Bigg\}. \end{aligned} \tag{A.3}$$

Category B

$$\begin{aligned}\eta_B^{NHN} \simeq 2.47\times 10^{-10}\frac{M_1}{10^9\mathrm{GeV}}\frac{l_2^2\sin 2\bar{\beta}}{(l_1^2+2l_2^2)X_B^{1/2}\sin^2\beta}\frac{M_{2=3}}{M_1}f\left(M_{2=3}^2/M_1^2\right)\times\\ \left[\frac{1.46X_B^{1/2}\sin^2\beta}{(l_1^2+2l_2^2)}+\left(\frac{28.3(l_1^2+2l_2^2)}{X_B^{1/2}\sin^2\beta}\right)^{1.16}\right]^{-1}. \end{aligned} \tag{A.4}$$

$$\eta_B^{IHN} \simeq -2.43 \times 10^{-10} \frac{M_{2=3}}{10^9\mathrm{GeV}} \frac{l_2^2 \sin 2\bar{\beta}}{X_B^{1/2} \sin^2\beta} f\left(M_1^2/M_{2=3}^2\right) \times$$

$$\left[1.47 X_B^{1/2} \sin^2\beta + \left(\frac{28.0}{X_B^{1/2} \sin^2\beta}\right)^{1.16}\right]^{-1}. \tag{A.5}$$

$$\eta_B^{QDN} \simeq 2.40 \times 10^{-10} \left\{ \frac{M_1}{10^9\mathrm{GeV}} \frac{l_2^2 \sin 2\bar{\beta}}{(l_1^2 + 2l_2^2) X_B^{1/2} \sin^2\beta} \frac{M_{2=3}}{M_1} f\left(M_{2=3}^2/M_1^2\right) \times \right.$$

$$\left[\frac{1.48 X_B^{1/2} \sin^2\beta}{(l_1^2 + 2l_2^2)} + \left(\frac{27.9(l_1^2 + 2l_2^2)}{X_B^{1/2} \sin^2\beta}\right)^{1.16}\right]^{-1}$$

$$- \frac{M_{2=3}}{10^9\mathrm{GeV}} \frac{l_2^2 \sin 2\bar{\beta}}{X_B^{1/2} \sin^2\beta} f\left(M_1^2/M_{2=3}^2\right) \times$$

$$\left. \left[1.48 X_B^{1/2} \sin^2\beta + \left(\frac{27.9}{X_B^{1/2} \sin^2\beta}\right)^{1.16}\right]^{-1} \right\}. \tag{A.6}$$

B Baryon Asymmetry in fully flavored leptogenesis

Category A

$$\eta_A^{NHN} \simeq 2.47 \times 10^{-10} \frac{M_1}{10^9\mathrm{GeV}} \frac{k_2^2 \sin 2\bar{\alpha}}{X_A^{1/2} \sin^2\beta} \frac{M_{2=3}}{M_1} f\left(M_{2=3}^2/M_1^2\right) \times$$

$$\left[\frac{1.72 X_A^{1/2} \sin^2\beta}{k_1^2} + \left(\frac{23.9 k_1^2}{X_A^{1/2} \sin^2\beta}\right)^{1.16}\right]^{-1}. \tag{B.1}$$

$$\eta_A^{IHN} \simeq -2.43 \times 10^{-10} \frac{M_{2=3}}{10^9\mathrm{GeV}} \frac{k_1^2 k_2^2 \sin 2\bar{\alpha}}{(1 + k_2^2) X_A^{1/2} \sin^2\beta} f\left(M_1^2/M_{2=3}^2\right) \times$$

$$\left[\frac{1.74 X_A^{1/2} \sin^2\beta}{k_2^2} + \left(\frac{23.8 k_2^2}{X_A^{1/2} \sin^2\beta}\right)^{1.16}\right]^{-1}. \tag{B.2}$$

$$\eta_A^{QDN} \simeq 2.40\times 10^{-10}\Bigg\{\frac{M_1}{10^9\mathrm{GeV}}\frac{k_2^2\sin 2\bar{\alpha}}{X_A^{1/2}\sin^2\beta}\frac{M_{2=3}}{M_1}f\left(M_{2=3}^2/M_1^2\right)\times$$

$$\left[\frac{1.75X_A^{1/2}\sin^2\beta}{k_1^2}+\left(\frac{23.6k_1^2}{X_A^{1/2}\sin^2\beta}\right)^{1.16}\right]^{-1}$$

$$-\frac{M_{2=3}}{10^9\mathrm{GeV}}\frac{k_1^2k_2^2\sin 2\bar{\alpha}}{(1+k_2^2)X_A^{1/2}\sin^2\beta}f\left(M_1^2/M_{2=3}^2\right)\times$$

$$\left[\frac{1.75X_A^{1/2}\sin^2\beta}{k_2^2}+\left(\frac{23.6k_2^2}{X_A^{1/2}\sin^2\beta}\right)^{1.16}\right]^{-1}\Bigg\}. \tag{B.3}$$

Category B

$$\eta_B^{NHN} \simeq 2.47\times 10^{-10}\frac{M_1}{10^9\mathrm{GeV}}\frac{l_2^2\sin 2\bar{\beta}}{(l_1^2+2l_2^2)X_B^{1/2}\sin^2\beta}\frac{M_{2=3}}{M_1}f\left(M_{2=3}^2/M_1^2\right)\times$$

$$\left[\frac{2.30X_B^{1/2}\sin^2\beta}{l_2^2}+\left(\frac{17.9l_2^2}{X_B^{1/2}\sin^2\beta}\right)^{1.16}\right]^{-1}. \tag{B.4}$$

$$\eta_B^{IHN} \simeq -2.43\times 10^{-10}\frac{M_{2=3}}{10^9\mathrm{GeV}}\frac{l_2^2\sin 2\bar{\beta}}{X_B^{1/2}\sin^2\beta}\frac{M_1}{M_{2=3}}f\left(M_1^2/M_{2=3}^2\right)\times$$

$$\left[2.32X_B^{1/2}\sin^2\beta+\left(\frac{17.8}{X_B^{1/2}\sin^2\beta}\right)^{1.16}\right]^{-1}. \tag{B.5}$$

$$\eta_B^{QDN} \simeq 2.40\times 10^{-10}\Bigg\{\frac{M_1}{10^9\mathrm{GeV}}\frac{l_2^2\sin 2\bar{\beta}}{(l_1^2+2l_2^2)X_B^{1/2}\sin^2\beta}\frac{M_{2=3}}{M_1}f\left(M_{2=3}^2/M_1^2\right)\times$$

$$\left[\frac{2.34X_B^{1/2}\sin^2\beta}{l_2^2}+\left(\frac{17.7l_2^2}{X_B^{1/2}\sin^2\beta}\right)^{1.16}\right]^{-1}-\frac{M_{2=3}}{10^9\mathrm{GeV}}\frac{l_2^2\sin 2\bar{\beta}}{X_B^{1/2}\sin^2\beta}\frac{M_1}{M_{2=3}}f\left(M_1^2/M_{2=3}^2\right)\times$$

$$\left[2.34X_B^{1/2}\sin^2\beta+\left(\frac{17.7}{X_B^{1/2}\sin^2\beta}\right)^{1.16}\right]^{-1}\Bigg\}. \tag{B.6}$$

C Baryon asymmetry in τ-flavored leptogenesis

Category A

$$\eta_A^{NHN} \simeq 2.47 \times 10^{-10} \frac{M_1}{10^9\mathrm{GeV}} \frac{k_2^2 \sin 2\bar{\alpha}}{X_A^{1/2} \sin^2\beta} \frac{M_{2=3}}{M_1} f\left(M_{2=3}^2/M_1^2\right) \times$$

$$\left[\frac{2.05 X_A^{1/2} \sin^2\beta}{k_1^2} + \left(\frac{20.1 k_1^2}{X_A^{1/2} \sin^2\beta}\right)^{1.16}\right]^{-1}. \tag{C.1}$$

$$\eta_A^{IHN} \simeq -1.22 \times 10^{-10} \frac{M_{2=3}}{10^9\mathrm{GeV}} \frac{k_1^2 k_2^2 \sin 2\bar{\alpha}}{(1+k_2^2) X_A^{1/2} \sin^2\beta} \frac{M_1}{M_{2=3}} f\left(M_1^2/M_{2=3}^2\right) \times$$

$$\left\{\left[\frac{2.07 X_A^{1/2} \sin^2\beta}{k_2^2} + \left(\frac{20.0 k_2^2}{X_A^{1/2} \sin^2\beta}\right)^{1.16}\right]^{-1} + \left[\frac{2.07 X_A^{1/2} \sin^2\beta}{(1+k_2^2)} + \left(\frac{20.0(1+k_2^2)}{X_A^{1/2} \sin^2\beta}\right)^{1.16}\right]^{-1}\right\}. \tag{C.2}$$

$$\eta_A^{QDN} \simeq 1.20 \times 10^{-10} \frac{k_2^2 \sin 2\bar{\alpha}}{X_A^{1/2} \sin^2\beta} \left\{ \frac{2M_1}{10^9\mathrm{GeV}} \frac{M_{2=3}}{M_1} f\left(M_{2=3}^2/M_1^2\right) \times\right.$$

$$\left[\frac{2.08 X_A^{1/2} \sin^2\beta}{k_1^2} + \left(\frac{19.8 k_1^2}{X_A^{1/2} \sin^2\beta}\right)^{1.16}\right]^{-1} - \frac{M_{2=3}}{10^9\mathrm{GeV}} \frac{k_1^2}{(1+k_2^2)} \frac{M_1}{M_{2=3}} f\left(M_1^2/M_{2=3}^2\right) \times$$

$$\left.\left\{\left[\frac{2.08 X_A^{1/2} \sin^2\beta}{k_2^2} + \left(\frac{19.8 k_2^2}{X_A^{1/2} \sin^2\beta}\right)^{1.16}\right]^{-1} + \left[\frac{2.08 \sin^2\beta X_A^{1/2}}{(1+k_2^2)} + \left(\frac{19.8(1+k_2^2)}{X_A^{1/2} \sin^2\beta}\right)^{1.16}\right]^{-1}\right\}\right\}. \tag{C.3}$$

Category B

$$\eta_B^{NHN} \simeq 1.23 \times 10^{-10} \times \frac{M_1}{10^9\mathrm{GeV}} \frac{l_2^2 \sin 2\bar{\beta}}{(l_1^2 + 2l_2^2) X_B^{1/2} \sin^2\beta} \frac{M_{2=3}}{M_1} f\left(M_{2=3}^2/M_1^2\right) \times$$

$$\left\{\left[\frac{2.05 X_B^{1/2} \sin^2\beta}{(l_1^2 + l_2^2)} + \left(\frac{20.1(l_1^2 + l_2^2)}{X_B^{1/2} \sin^2\beta}\right)^{1.16}\right]^{-1} + \left[\frac{2.24 X_B^{1/2} \sin^2\beta}{l_2^2} + \left(\frac{18.4 l_2^2}{X_B^{1/2} \sin^2\beta}\right)^{1.16}\right]^{-1}\right\}. \tag{C.4}$$

$$\eta_B^{IHN} \simeq -1.22 \times 10^{-10} \frac{M_{2=3}}{10^9\mathrm{GeV}} \frac{l_2^2 \sin 2\bar{\beta}}{X_B^{1/2} \sin^2\beta} \frac{M_1}{M_{2=3}} f\left(M_1^2/M_{2=3}^2\right) \times$$

$$\left\{\left[\frac{2.07 X_B^{1/2} \sin^2\beta}{l_2^2} + \left(\frac{20.0 l_2^2}{X_B^{1/2} \sin^2\beta}\right)^{1.16}\right]^{-1} + \left[2.26 X_B^{1/2} \sin^2\beta + \left(\frac{18.2}{X_B^{1/2} \sin^2\beta}\right)^{1.16}\right]^{-1}\right\}. \tag{C.5}$$

$$
\eta_B^{QDN} \simeq 1.20 \times 10^{-10} \frac{l_2^2 \sin 2\bar{\beta}}{X_B^{1/2} \sin^2\beta} \Bigg(\frac{M_1}{10^9 \mathrm{GeV}} \frac{1}{(l_1^2 + 2l_2^2)} \frac{M_{2=3}}{M_1} f\left(M_{2=3}^2/M_1^2\right) \times
$$

$$
\left\{ \left[\frac{2.16 X_B^{1/2} \sin^2\beta}{(l_1^2 + l_2^2)} + \left(\frac{19.1(l_1^2 + l_2^2)}{X_B^{1/2} \sin^2\beta} \right)^{1.16} \right]^{-1} + \left[\frac{2.28 X_B^{1/2} \sin^2\beta}{l_2^2} + \left(\frac{18.1 l_2^2}{X_B^{1/2} \sin^2\beta} \right)^{1.16} \right]^{-1} \right\}
$$

$$
- \frac{M_{2=3}}{10^9 \mathrm{GeV}} \frac{M_1}{M_{2=3}} f\left(M_1^2/M_{2=3}^2\right) \times
$$

$$
\left\{ \left[\frac{2.16 X_B^{1/2} \sin^2\beta}{l_2^2} + \left(\frac{19.1 l_2^2}{X_B^{1/2} \sin^2\beta} \right)^{1.16} \right]^{-1} + \left[2.28 X_B^{1/2} \sin^2\beta + \left(\frac{18.1}{X_B^{1/2} \sin^2\beta} \right)^{1.16} \right]^{-1} \right\} \Bigg). \tag{C.6}
$$

References

[1] M. Fukugita and T. Yanagida, *Baryogenesis Without Grand Unification, Phys. Lett.* B **174** (1986) 45.

[2] A. Riotto and M. Trodden, *Recent progress in baryogenesis, Ann. Rev. Nucl. Part. Sci.* **49** (1999) 35 [arXiv:hep-ph/9901362] [SPIRES].

[3] M. Yu. Khlopov, *Cosmoparticle Physics, World Scientific, Singapore* (1999).

[4] V. A. Kuzmin, V. A. Rubakov and M. E. Shaposhnikov, *On The Anomalous Electroweak Baryon Number Nonconservation In The Early Universe, Phys. Lett.* B **155** (1985) 36.

[5] S. Davidson, E. Nardi and Y. Nir, *Leptogenesis, Phys. Rept.* **466** (2008) 105 [arXiv:0802.2962] [SPIRES].

[6] P. Minkowski, *Mu → E Gamma At A Rate Of One Out Of 1-Billion Muon Decays*?, *Phys. Lett.* B **67** (1977) 421.

[7] M. Gell-Mann, P. Ramond and R. Slansky, *Complex spinors and unified theories* in proceedings of *Supergravity workshop,* Stony Brook, U.S.A. September 27-29, 1979, P. Van Nieuwenhuizen and D. Z. Freedman eds., North-Holland, Amsterdam The Netherlands (1979), pg.341.

[8] T. Yanagida, *Horizontal symmetry and masses of neutrinos* in proceedings of the *Workshop on the Unified baryon Number of the universe and unified theories*, Tsukuba Japan, 13-14 February 1979, O. Sawada and A. Sugamoto eds.

[9] R. N. Mohapatra and G. Senjanovic, *Neutrino mass and spontaneous parity nonconservation, Phys. Rev. Lett.* **44** (1980) 912.

[10] J. Schechter and J. W. F. Valle *Neutrino masses in $SU(2) \otimes U(1)$ theories*, *Phys. Rev.* D **22** (1980) 2227.

[11] G. F. Giudice, A. Notari, M. Raidal, A. Riotto and A. Strumia, *Towards a complete theory of thermal leptogenesis in the SM and MSSM, Nucl. Phys.* B **685** (2004) 89 [arXiv:0310123][SPIRES].

[12] Y. H. Ahn, S. K. Kang, C. S. Kim and T. P. Nguyen, *A direct link between neutrinoless double beta decay and leptogenesis in a seesaw model with S_4 symmetry* arXiv:1004.3469 [SPIRES].

[13] T. Asaka and H. Ishida, *Flavour Mixing of Neutrinos and Baryon Asymmetry of the Universe, Phys. Lett.* B **692** (2010) 105 [arXiv:1004.5491][SPIRES].

[14] L. Canetti and M. Shaposhnikov, *Baryon Asymmetry of the Universe in the NuMSM* arXiv:1006.0133 [SPIRES].

[15] B. Adhikary and A. Ghosal, *Nonzero U_{e3}, CP violation and leptogenesis in a see-saw type softly broken A_4 symmetric model, Phys. Rev.* D **78** (2008) 073007 [arXiv:0803.3582][SPIRES].

[16] B. Adhikary, A. Ghosal and P. Roy, *Mu-Tau symmetry, tribimaximal mixing and four zero neutrino Yukawa textures, JHEP* **0910** (2009) 040 [arXiv:0908.2686] [SPIRES].

[17] G. C. Branco, D. Emmanuel-Costa, M. N. Rebelo and P. Roy, *Four Zero Neutrino Yukawa Textures in the Minimal Seesaw Framework, Phys. Rev. D* **77** (2008) 053011 [arXiv:0712.0774] [SPIRES].

[18] T. Fukuyama and H. Nishiura, *Mass Matrix of Majorana neutrinos*, hep-ph/9702253 [SPIRES].

[19] W. Rodejohann, *Broken mu - tau symmetry and leptonic CP violation*, Prepared for 12th *International Workshop on Neutrinos Telescopes: Twenty Years after the Supernova 1987A Neutrino Bursts Discovery,* Venice, Italy, 6-9 Mar 2007[SPIRES].

[20] J. C. Gomez-Izquierdo and A. Perez-Lorenzana, *Softly broken $\mu \leftrightarrow \tau$ symmetry in the minimal see-saw model, Phys. Rev.* D **77** (2008) 113015 [arXiv:0711.0045] [SPIRES]].

[21] T. Baba, *What does mu - tau symmetry imply about leptonic CP violation?, Int. J. Mod. Phys.* E **16** (2007) 1373.

[22] N. Nimai Singh, H. Zeen Devi and M. Patgiri, *Phenomenology of neutrino mass matrices obeying μ-τ reflection symmetry* arXiv:0707.2713 [hep-ph].

[23] A. S. Joshipura and B. P. Kodrani, *Complex CKM matrix, spontaneous CP violation and generalized μ-τ symmetry, Phys. Lett.* B **670** (2009) 369 [arXiv:0706.0953][SPIRES].

[24] B. Adhikary, *Soft breaking of L(mu) - L(tau) symmetry: Light neutrino spectrum and leptogenesis, Phys. Rev.* D **74** (2006) 033002 [arXiv:0604009][SPIRES].

[25] T. Baba and M. Yasue, *Correlation between leptonic CP violation and mu - tau symmetry breaking Phys. Rev.* D **75** (2007) 055001 [arXiv:0612034][SPIRES].

[26] W. Grimus, *Realizations of mu - tau interchange symmetry*, [arXiv:hep-ph/0610158].

[27] Z. z. Xing, H. Zhang and S. Zhou, *Nearly tri-bimaximal neutrino mixing and CP violation from mu - tau symmetry breaking, Phys. Lett.* B **641** (2006) 189 [arXiv:0607091][SPIRES].

[28] N. Haba and W. Rodejohann, *A Supersymmetric Contribution to the Neutrino Mass Matrix and Breaking of mu-tau Symmetry, Phys. Rev.* D **74** (2006) 017701 [arXiv:0603206].

[29] R. N. Mohapatra, S. Nasri and H. B. Yu, *Grand unification of mu - tau symmetry, Phys. Lett.* B **636** (2006) 114 [arXiv:0603020][SPIRES].

[30] Y. H. Ahn, S. K. Kang, C. S. Kim and J. Lee, *Phased breaking of mu - tau symmetry and leptogenesis, Phys. Rev.* D **73** (2006) 093005 [arXiv:0602160][SPIRES].

[31] K. Fuki and M. Yasue, *What does mu - tau symmetry imply in neutrino mixings?, Phys. Rev.* D **73** (2006) 055014 [arXiv:0601118][SPIRES].

[32] S. Nasri, *Implications of mu $\leftrightarrow$ tau symmetry on neutrinos and leptogenesis, Int. J. Mod. Phys.* A **20** (2005) 6258.

[33] I. Aizawa and M. Yasue, *A new type of complex neutrino mass texture and mu - tau symmetry, Phys. Rev.* D **73** (2006) 015002 [arXiv:0510132][SPIRES].

[34] R. N. Mohapatra and W. Rodejohann, *Broken mu-tau Symmetry and Leptonic CP Violation, Phys. Rev.* D **72** (2005) 053001 [arXiv:0507312][SPIRES].

[35] I. Aizawa, M. Ishiguro, M. Yasue and T. Kitabayashi, *mu tau permutation symmetry and neutrino mixing for a two-loop radiative mechanism, J. Korean Phys. Soc.* **46** (2005) 597.

[36] T. Kitabayashi and M. Yasue, *mu - tau symmetry and maximal CP violation, Phys. Lett.* B **621** (2005) 133 [arXiv:0504212][SPIRES].

[37] R. N. Mohapatra, S. Nasri and H. B. Yu, *Leptogenesis, $\mu - \tau$ Symmetry and θ_{13}, Phys. Lett.* B **615** (2005) 231 [arXiv:0502026][SPIRES].

[38] R. N. Mohapatra and S. Nasri, *Leptogenesis and mu - tau symmetry, Phys. Rev.* D **71** (2005) 033001 [arXiv:0410369][SPIRES].

[39] R. N. Mohapatra, *theta(13) as a probe of mu $\leftrightarrow$ tau symmetry for leptons, JHEP* **0410** (2004) 027 [arXiv:0408187].

[40] I. Aizawa, M. Ishiguro, T. Kitabayashi and M. Yasue, *Bilarge neutrino mixing and mu - tau permutation symmetry for two-loop radiative mechanism, Phys. Rev.* D **70** (2004) 015011 [arXiv:0405201][SPIRES].

[41] I. de Medeiros Varzielas and G. G. Ross, *SU(3) family symmetry and neutrino bi-tri-maximal mixing, Nucl. Phys.* B **733** (2006) 31 [arXiv:0507176][SPIRES].

[42] P. F. Harrison and W. G. Scott, *mu - tau reflection symmetry in lepton mixing and neutrino oscillations, Phys. Lett.* B **547** (2002) 219 [arXiv:0210197][SPIRES].

[43] T. Kitabayashi and M. Yasue, *S(2L) permutation symmetry for left-handed mu and tau families and neutrino oscillations in an SU(3)L x U(1)N gauge model, Phys. Rev.* D **67** (2003) 015006 [arXiv:0209294][SPIRES].

[44] Zhi-zhong Xing, Ye-Ling Zhou, *A Generic Diagonalization of the 3X3 Neutrino Mass Matrix and Its Implications on the mu-tau Flavor Symmetry and Maximal CP Violation*, [arXiv:1008.4906][SPIRES].

[45] S. F. Ge, H. J. He and F. R. Yin, *Common Origin of Soft mu-tau and CP Breaking in Neutrino Seesaw and the Origin of Matter, JCAP* **1005** (2010) 017 [arXiv:1001.0940][SPIRES].

[46] A. Ghosal, *A neutrino mass model with reflection symmetry, Mod. Phys. Lett.* A **19** (2004) 2579.

[47] S. Choubey, W. Rodejohann and P. Roy, *Phenomenological consequences of four zero neutrino Yukawa textures, Nucl. Phys.* B **808** (2009) 272 [Erratum-ibid. **818** (2009) 136 (2009)] [arXiv:0807.4289][SPIRES].

[48] R. Barbieri, P. Creminelli, A. Strumia and N. Tetradis, *Baryogenesis through leptogenesis, Nucl. Phys.* B **575** (2000) 61 [arXiv:9911315][SPIRES].

[49] A. Abada, S. Davidson, F. X. Josse-Michaux, M. Losada and A. Riotto, *Flavour Issues in Leptogenesis, JCAP* **0604** (2006) 004 [arXiv:0601083][SPIRES].

[50] E. Nardi, Y. Nir, E. Roulet and J. Racker, *The importance of flavor in leptogenesis, JHEP* **0601** (2006) 164 [arXiv:0601084][SPIRES].

[51] A. Abada, S. Davidson, A. Ibarra, F. X. Josse-Michaux, M. Losada and A. Riotto, *Flavour matters in leptogenesis, JHEP* **0609** (2006) 010 [arXiv:hep-ph/0605281][SPIRES].

[52] M. Drees, R. Godbole and P. Roy, *Theory and phenomenology of sparticles: An account of four-dimensional N=1 supersymmetry in high energy physics, Hackensack, USA: World Scientific* (2004).

[53] S. Antusch, S. F. King and A. Riotto, *Flavour-dependent leptogenesis with sequential dominance, JCAP* **0611** (2006) 011 [arXiv:hep-ph/0609038][SPIRES].

[54] L. Covi, M. Olechowski, S. Pokorski, K. Turzynski and J. D. Wells, *Supersymmetric mass spectra for gravitino dark matter with a high reheating temperature*, [arXiv:1009.3801] [hep-ph] [SPIRES].

[55] J. C. B. Sanchez, M. Bastero-Gil, A. Berera, K. Dimopoulos and K. Kohri, *The gravitino problem in supersymmetric warm inflation*, [arXiv:1011.2398] [hep-ph] [SPIRES].

[56] G. L. Fogli, E. Lisi, A. Marrone, A. Palazzo and A. M. Rotunno, *Probing Theta(13) With Global Neutrino Data Analysis, J. Phys. Conf. Ser.* **203** (2010) 012103.

[57] J. A. Harvey and M. S. Turner, *Cosmological baryon and lepton number in the presence of electroweak fermion number violation, Phys. Rev. D* **42** (1990) 3344.

[58] E. W. Kolb and M. S. Turner, *The Early Universe (Addison-Wesley, Redwood City, CA, U.S.A,* (1990).

[59] D. N. Spergel *et al.* [WMAP Collaboration], *First Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Determination of Cosmological Parameters, Astrophys. J. Suppl.* **148** (2003) 175 [arXiv:0302209][SPIRES].

[60] M. Tegmark *et al.* [SDSS Collaboration], *Cosmological parameters from SDSS and WMAP, Phys. Rev. D* **69**, (2004) 103501 [arXiv:0310723][SPIRES].

[61] C. L. Bennett *et al.* [WMAP Collaboration], *First Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Preliminary Maps and Basic Results, Astrophys. J. Suppl.* **148** (2003) 1 [arXiv:0302207][SPIRES].

[62] M. Trodden, *Baryogenesis and leptogenesis, In the Proceedings of 32nd SLAC Summer Institute on Particle Physics (SSI 2004): Natures Greatest Puzzles, Menlo Park, California, 2-13 Aug 2004, pp L018* [arXiv:0411301][SPIRES].

[63] A. Pilaftsis and T. E. J. Underwood, *Resonant leptogenesis, Nucl. Phys. B* **692**, 303 (2004) [arXiv:hep-ph/0309342] [hep-ph] [SPIRES].

[64] H. Minakata, *Looking for Leptonic CP Violation with Neutrinos, Acta Phys. Polon. B* **39**, 283 (2008) [arXiv:0801.2427] [hep-ph] [SPIRES].

[65] C. S. Fong, M. C. Gonzalez-Garcia, E. Nardi and J. Racker, *Supersymmetric Leptogenesis*, arXiv:1009.0003 [hep-ph] [SPIRES].