

# Radiatively broken symmetries of nonhierarchical neutrinos

Amol Dighe,<sup>1</sup> Srubabati Goswami,<sup>2</sup> and Probir Roy<sup>1</sup>

<sup>1</sup> *Tata Institute of Fundamental Research,  
Homi Bhabha Road, Mumbai 400 005, India*

<sup>2</sup> *Harish-Chandra Research Institute,  
Chhatnag Road, Jhusi, Allahabad 211 019, India*

## Abstract

Symmetry-based ideas, such as the quark-lepton complementarity (QLC) principle and the tribimaximal mixing (TBM) scheme, have been proposed to explain the observed mixing pattern of neutrinos. We argue that such symmetry relations need to be imposed at a high scale  $\Lambda \sim 10^{12}$  GeV characterizing the large masses of right-handed neutrinos required to implement the seesaw mechanism. For nonhierarchical neutrinos, renormalisation group evolution down to a laboratory energy scale  $\lambda \sim 10^3$  GeV tends to radiatively break these symmetries at a significant level and spoil the mixing pattern predicted by them. However, for Majorana neutrinos, suitable constraints on the extra phases  $\alpha_{2,3}$  enable the retention of those high scale mixing patterns at laboratory energies. We examine this issue within the Minimal Supersymmetric Standard Model (MSSM) and demonstrate the fact posited above for two versions of QLC and two versions of TBM. The appropriate constraints are worked out for all these four cases. Specifically, a preference for  $\alpha_2 \approx \pi$  (i.e.  $m_1 \approx -m_2$ ) emerges in each case. We also show how a future accurate measurement of  $\theta_{13}$  may enable some discrimination among these four cases in spite of renormalization group evolution.

PACS numbers: 11.10.Hi, 12.15.Ff, 14.60.Pq

Keywords: neutrino masses and mixing, renormalisation group running, quark-lepton complementarity, tribimaximal mixing

arXiv:0704.3735v2 [hep-ph] 17 Sep 2007

## I. INTRODUCTION

Outstanding recent experiments have increased our knowledge [1] of neutrino masses and mixing angles enormously. We are already certain that at least two of the three known neutrinos are massive, the heavier and the lighter of them being respectively  $\gtrsim 0.05$  eV and  $\gtrsim 0.009$  eV in mass. We also know that two of the three neutrino mixing angles are large:  $\theta_{23} \approx 45^\circ$  and  $\theta_{12} \approx 34^\circ$ , while the third is significantly smaller:  $\theta_{13} < 12^\circ$ . The total sum of the neutrino masses is also cosmologically bounded from above by  $\mathcal{O}(1)$  eV. Much remains to be known, though. The values of  $\theta_{13}$  and the leptonic CP violating Dirac phase  $\delta_\ell$ , are still unknown. So is the ordering of the neutrino masses  $m_i$  ( $i = 1, 2, 3$ ) – whether it is normal ( $|m_3| > |m_{1,2}|$ ) or inverted ( $|m_3| < |m_{1,2}|$ ). We also do not know if the three neutrinos are hierarchically spaced in mass like charged fermions or if they are nonhierarchical. Our term nonhierarchical here includes both the inverted hierarchical (IH) case, i.e.  $|m_3| \ll |m_1| \sim |m_2| \sim 0.05$  eV, and the quasi-degenerate (QD) situation [3], i.e.  $|m_1| \sim |m_2| \sim |m_3| \gg 0.05$  eV, the latter with either a normal or an inverted mass ordering. Neither of these scenarios is observationally excluded as yet and we focus on them. As per our present knowledge, the average neutrino mass could still in fact be anywhere between half of the atmospheric oscillation mass scale, i.e.  $\approx 0.025$  eV and a third of the cosmological upper bound, i.e.  $\approx 0.3$  eV. Finally, most theoretical ideas expect the three neutrinos to be Majorana particles whose masses  $m_i$  can be complex. In that case, since one of their phases can be rotated away, there are two additional, possibly nonzero, phases [2] on which we do not have any direct information at present. This is because no convincing evidence exists as yet of neutrinoless nuclear double beta decay which is the only known direct probe [4] on these phases. Indirectly, of course, some constraints on these phases may also arise from considerations of leptogenesis [5]. It is nevertheless worthwhile to try to constrain these phases in some other way. That is one of the aims of the present work, which is an elaboration of our earlier shorter communication [6] with many additional results. In particular, we demonstrate here that, given the constraints on these Majorana phases, a measurement of  $\theta_{13}$  can make some discrimination among four scenarios considered by us despite renormalization group (RG) running.

The observed bilarge pattern of neutrino mixing has led to the idea of some kind of a

symmetry at work. Several symmetry-based relations<sup>1</sup> have in fact been proposed, which give rise to specific neutrino mixing patterns. Two of the most promising mixing patterns, that we will be concerned with here, are (i) quark-lepton complementarity (QLC) [7, 8, 9, 10, 11] and (ii) tribimaximal mixing (TBM) [12]. QLC involves bimaximal mixing [13] followed by the unitary transformation of quark mixing. A bimaximal mixing can in turn be generated by a  $\mu$ - $\tau$  exchange symmetry [14], an  $L_\mu - L_\tau$  gauge symmetry [15], or an  $S_3$  permutation symmetry [16]. The second step is inspired by SU(5) or SO(10) GUT, as discussed later. Similarly, a tribimaximal mixing pattern may be obtained from an  $A_4$  [17] or  $S_3$  [18] family symmetry. However, a major issue in connection with such symmetries is the scale at which they are to be implemented. Neutrino masses and mixing angles are related directly to the corresponding Yukawa coupling strengths which run with the energy scale. There is as yet no universally accepted explanation of the origin of neutrino masses, but the seesaw mechanism [19] is the most believable candidate so far. The form of the light neutrino mass matrix in family space in that case is  $\mathcal{M}_\nu = -(m_\nu^D)^T M_R^{-1} m_\nu^D$ , where  $m_\nu^D$  is the Dirac neutrino mass matrix (analogous to the charged fermion ones) and  $M_R$  the mass matrix for very heavy right chiral singlet neutrinos. If the Dirac mass of the heaviest neutrino is taken to be 1 – 100 GeV, the atmospheric neutrino data require typical eigenvalues of  $M_R$  to be in the  $10^{11}$ – $10^{15}$  GeV range [20]. This is also the desirable magnitude for  $M_R$  from the standpoint of a successful leptogenesis [21]. From these considerations, we choose to implement the above mentioned symmetries at the scale  $\Lambda \sim 10^{12}$  GeV. One can take issue with the particular value chosen for  $\Lambda$ . However, our conclusions are only logarithmically sensitive to the precise value of this scale.

A question arises immediately on the application of such a high scale symmetry on the elements of the neutrino mass matrix. It concerns their radiative breaking via RG evolution down to a laboratory energy scale  $\lambda \sim 10^3$  GeV. The actual evolution [22, 23, 24] needs to be worked out in a specific theory which we choose to be the minimal supersymmetric standard model (MSSM [25]). That is why we have taken  $\lambda$  to be of the order of the explicit supersymmetry breaking or the intra-supermultiplet splitting scale  $\mathcal{O}(\text{TeV})$ . Once again, our calculations are only logarithmically sensitive to this exact choice. The point, however, is

---

<sup>1</sup> Here one should perhaps make a distinction between a symmetry of the Lagrangian and just a special relation among coupling strengths or masses. Nevertheless, the relations of concern to us can be implemented through specific symmetries of the Lagrangian.

that – for nonhierarchical neutrinos – symmetry relations formulated at  $\Lambda$  will in general get spoiled on evolution down to  $\lambda$ .

The full RG equations for the evolution of neutrino masses and mixing angles in the MSSM have been worked out [23, 24] in detail. In particular, the evolution effects on the mixing angles are found to be controlled by the quantities [6]  $\Delta_\tau |m_i + m_j|^2 / (|m_i|^2 - |m_j|^2)$  where  $\Delta_\tau$ , to be specified later, is a small fraction  $\lesssim 10^{-2}$ , while  $i, j$  refer to the concerned neutrino mass eigenstates. Consequently, these effects are negligible for a normal hierarchical mass pattern with  $|m_3| \gg |m_2| \gg |m_1|$ . RG effects can become significantly large only when neutrinos are nonhierarchical. There is another important characteristic of the above-mentioned ratios. While their denominators involve only the absolute masses  $|m_i|$ , the numerators involve the combinations  $|m_i + m_j|^2$ . Therefore, with appropriate constraints on the neutrino Majorana phases, the desired symmetry relations can be approximately preserved at the laboratory scale  $\lambda$  even for nonhierarchical neutrinos – in agreement with the mixing pattern that has emerged from the oscillation data. The constraints on the majorana phases and the consequent discrimination among the scenarios by a measurement of  $\theta_{13}$  constitute our main results. Our work is somewhat complementary to that of Ref. [26] in the QLC sector and Ref. [27] in the TBM sector.

In this paper we work out in detail the last-mentioned constraints on the neutrino Majorana phases in the (i) bimaximal mixing + QLC and (ii) TBM scenarios respectively. Each of these comes in two variations. So we have in all four cases at hand. Thus, the scope of the present work is much larger than our earlier shorter communication [6] which addressed only one version of QLC and did not consider the implications for  $\theta_{13}$ . A major technical observation utilized by us is the following. Suppose  $\theta_{13}^\Lambda$ , the high scale value of the angle  $\theta_{13}$ , is sufficiently small (as is the case for the situations considered here) such that  $\mathcal{O}(\theta_{13})$  terms can be neglected in comparison with other  $\mathcal{O}(1)$  terms in the RG equations [24]. Then the neutrino mass matrix  $\mathcal{M}_\nu^\lambda$ , at the laboratory scale  $\lambda$ , becomes analytically tractable in terms of its high scale form  $\mathcal{M}_\nu^\Lambda$ . In fact, the relation obtained looks quite simple and transparent. The step from there to explicit constraints on the neutrino Majorana phases is then shown to be quite straightforward. The rest of this paper is organised as follows. Sec. II contains a description of the parametrisation that we find convenient to adopt for nonhierarchical neutrino masses. In Sec. III, we introduce two versions each of the QLC and TBM scenarios to be implemented at the high scale. In sec. IV, we discuss the energywise downward evolution

of the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix in general, and its effects on the predictions of the scenarios under consideration. In sec. V we study the constraints on neutrino and MSSM parameters in order for the scenarios to be valid and explore if these scenarios may be distinguished by means of more accurate measurements of the neutrino mixing angles. The concluding sec. VI consists of a summary and the discussion of our main results.

## II. PARAMETRISATION OF NONHIERARCHICAL NEUTRINO MASSES

We work in the convention [28] in which the neutrino mass eigenstates  $|\nu_1\rangle, |\nu_2\rangle, |\nu_3\rangle$  are related to the flavour eigenstates  $|\nu_e\rangle, |\nu_\mu\rangle, |\nu_\tau\rangle$  with the unitary mixing matrix  $U_\nu$ :

$$|\nu_\alpha\rangle = U_{\alpha i} |\nu_i\rangle, \quad (1)$$

$\alpha$  and  $i$  being flavour and mass indices respectively. We take the neutrino mass term in the Lagrangian to be

$$\mathcal{L}_{\text{mass}}^\nu = -\frac{1}{2} \overline{\nu_{L\alpha}^C} \mathcal{M}_{\nu\alpha\beta} \nu_{L\beta} + h.c. \quad (2)$$

Thus,

$$U_\nu^\dagger \mathcal{M}_\nu U_\nu^* = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}, \quad (3)$$

where  $m_i$  are in general complex. However, one of the three phases of  $m_{1,2,3}$  can be absorbed in the overall phase choice of  $\nu_L$  in (2). We can therefore choose

$$m_1 = |m_1|, \quad m_2 = |m_2| e^{i\alpha_2}, \quad m_3 = |m_3| e^{i\alpha_3}, \quad (4)$$

where  $\alpha_{2,3}$  are real. Experiments with atmospheric neutrinos tell us that [28, 29]

$$|\delta m_A^2| \equiv ||m_3|^2 - |m_{2,1}|^2| = (2.4 \pm 0.3) \times 10^{-3} \text{ eV}^2, \quad (5)$$

while experiments with solar electron neutrinos and reactor electron antineutrinos yield [28, 29]

$$\delta m_S^2 \equiv |m_2|^2 - |m_1|^2 = (7.9 \pm 0.4) \times 10^{-5} \text{ eV}^2. \quad (6)$$

For charged fermions ( $f = u, d, l$ ), the mass term is

$$\mathcal{L}_{\text{mass}}^f = -\frac{1}{2} \overline{f_{R\alpha}} m_{f\alpha\beta} f_{L\beta} + h.c. \quad (7)$$

The corresponding mass matrix  $m_f$  is put into a diagonal form by

$$U_f^\dagger m_f^\dagger m_f U_f = |m_f^{(D)}|^2 . \quad (8)$$

Now the unitary Cabbibo-Kobayashi-Maskawa (CKM) and the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrices, whose elements contribute to the observed quark and neutrino processes respectively, are given by

$$\begin{aligned} V_{\text{CKM}} &= U_u^\dagger U_d , \\ U_{\text{PMNS}} &= U_\ell^\dagger U_\nu . \end{aligned} \quad (9)$$

One can write  $U_{\text{PMNS}}$  in the standard basis [28] in terms of the angles  $\theta_{12}, \theta_{23}, \theta_{13}$  and the CP violating phase  $\delta_\ell$ . With  $s_{ij} \equiv \sin \theta_{ij}$  and  $c_{ij} \equiv \cos \theta_{ij}$ ,

$$U_{\text{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_\ell} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_\ell} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_\ell} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_\ell} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_\ell} & c_{23}c_{13} \end{pmatrix} . \quad (10)$$

The experiments mentioned earlier then also tell us that [28, 29]

$$\begin{aligned} \theta_{12} &= 33.9^\circ \pm 1.6^\circ , \\ \theta_{23} &= 43.3^\circ \pm 8.2^\circ , \\ \theta_{13} &< 12^\circ . \end{aligned} \quad (11)$$

We find it convenient to parametrise the absolute masses  $|m_i|$  for nonhierarchical neutrinos in terms of three real parameters  $m_0, \rho_A$  and  $\epsilon_S$  as follows:

$$\begin{aligned} |m_1| &= m_0(1 - \rho_A)(1 - \epsilon_S) , \\ |m_2| &= m_0(1 - \rho_A)(1 + \epsilon_S) , \\ |m_3| &= m_0(1 + \rho_A) . \end{aligned} \quad (12)$$

In eqs. (12),  $m_0$  defines the overall mass scale of the neutrinos, whereas  $\rho_A$  and  $\epsilon_S$  are dimensionless fractions with  $-1 \leq \rho_A \ll 1$  and  $0 < \epsilon_S < |\rho_A|$  for nonhierarchical neutrinos. The sign of  $\rho_A$  is positive (negative) for a normal (inverted) ordering of neutrino masses. Moreover,  $\rho_A \approx -1$  ( $|\rho_A| \ll 1$ ) for the IH (QD) case; in either case  $\epsilon_S \ll 1$ . For comparison,

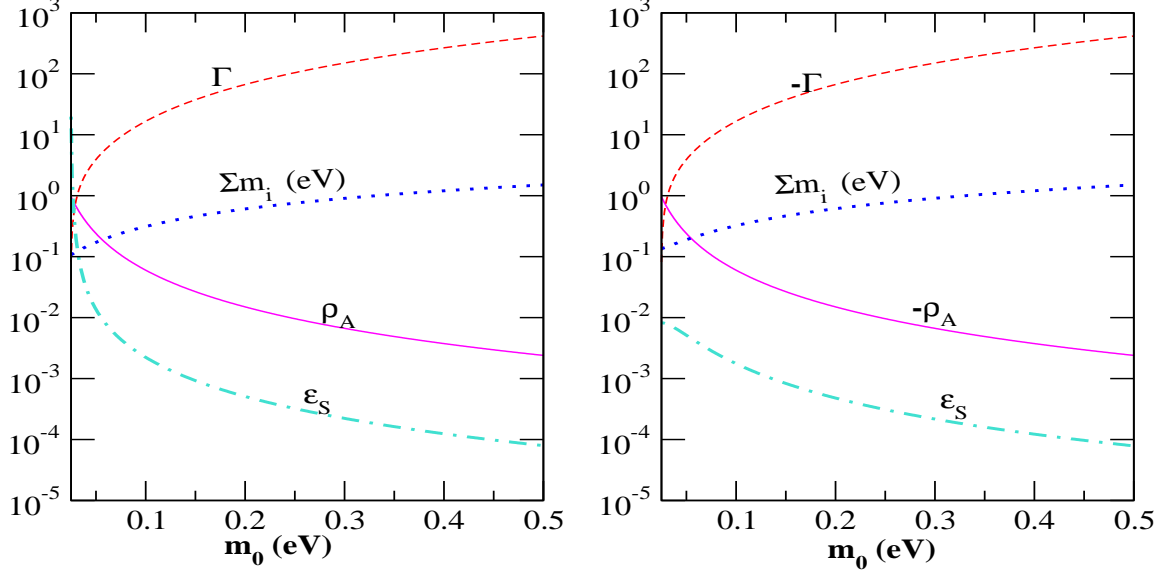


FIG. 1: The parameters  $\rho_A$ ,  $\epsilon_S$  and  $\Gamma$  as well as the value of  $\sum |m_i|$  (eV) as functions of  $m_0$ , for a normal (left panel) and an inverted (right panel) mass ordering of neutrinos. Note that  $\rho_A$  and  $\Gamma$  are negative for inverted mass ordering. For normal ordering,  $\epsilon_S$  goes to near unity with low  $m_0$ .

it may be noted that for normally hierarchical neutrinos,  $\rho_A \sim \epsilon_S \sim 1$ . We can further write the solar and atmospheric neutrino mass squared differences as

$$\begin{aligned} \delta m_S^2 &= |m_2|^2 - |m_1|^2 \approx 4m_0^2(1 - \rho_A)^2\epsilon_S, \\ |\delta m_A^2| &= ||m_3|^2 - (|m_1|/2 + |m_2|/2)^2| = 4m_0^2|\rho_A|. \end{aligned} \quad (13)$$

Utilizing (6), (5) and (13), we see that

$$m_0 > 0.024 \text{ eV}. \quad (14)$$

Also, the cosmologically bounded sum of neutrino absolute masses is given by

$$\sum_i |m_i| = 3m_0(1 - \rho_A/3) \lesssim 1 \text{ eV}. \quad (15)$$

From (13) and (15), it follows that

$$\frac{4(\sum |m_i|)^2}{9|\delta m_A^2|} = \frac{(1 - \rho_A/3)^2}{|\rho_A|}. \quad (16)$$

Utilising (5), (16) and the cosmological upper bound (15), we get  $|\rho_A| \gtrsim 5.5 \times 10^{-3}$ .

In Fig.1, we show how  $\rho_A$ ,  $\epsilon_S$  and  $\sum |m_i|$  behave as functions of  $m_0$ . We also find it convenient to define the derived dimensionless parameter

$$\Gamma \equiv \frac{1}{\rho_A} - \rho_A, \quad (17)$$

whose behaviour is included in the figure. Since  $|\rho_A| < 1$ , the sign of  $\Gamma$  is positive ( $\rho_A > 0$ ) for a normal ordering and negative ( $\rho_A < 0$ ) for an inverted ordering of the neutrino masses. The inequality  $5.5 \times 10^{-3} \lesssim |\rho_A| < 1$  translates to  $|\Gamma| \lesssim 180$ . We can generate typical numbers for nonhierarchical neutrinos: In the IH scenario, for instance, if  $m_0$  is chosen as 0.025 eV, we get  $\rho_A \approx -1$ ,  $\Gamma \approx 0^-$ , and  $\epsilon_S \approx 8 \times 10^{-3}$ . On the other hand, for QD neutrinos, choosing  $m_0 = 0.2$  eV, we have  $|\rho_A| \approx 1.5 \times 10^{-2}$ ,  $|\Gamma| \approx 65$ , and  $\epsilon_S \approx 5 \times 10^{-4}$ .

### III. SYMMETRIES AND MIXING ANGLES AT THE HIGH SCALE

The neutrino mass matrix at the high scale  $\Lambda$  originates from a dimension-5 operator

$$\mathcal{O} = c_{\alpha\beta} \frac{(\ell_\alpha \cdot H)(\ell_\beta \cdot H)}{\Lambda}. \quad (18)$$

In (18),  $\ell_\alpha$  and  $H$  are the SU(2) doublet lepton and Higgs fields respectively and  $c_{\alpha\beta}$  are dimensionless coefficients that run with the energy scale. Then

$$\mathcal{M}_{\nu\alpha\beta}^\Lambda \sim c_{\alpha\beta} \frac{v^2}{\Lambda}, \quad (19)$$

where  $v = 246$  GeV and  $\Lambda \sim M_{\text{MAJ}}$ , the Majorana mass characterising the heavy SM-singlet Majorana neutrino N. The symmetries at the high scale  $\Lambda$  give rise to specific structures for the matrix  $\mathcal{M}_\nu^\Lambda$ , and hence predict the values of the mixing angles  $\theta_{ij}^\Lambda$  at the scale  $\Lambda$ . There is an issue about a consistent definition of  $\nu_{1,2}$  at all scales. Solar neutrino experiments tell us that  $|m_2^\lambda| > |m_1^\lambda|$ , where  $\lambda$  is the laboratory scale. We *define*  $\nu_1$  and  $\nu_2$  at higher scales in a way such that  $|m_2| \geq |m_1|$  at *all* scales, and in particular,  $|m_2^\Lambda| > |m_1^\Lambda|$ .

In this section, we introduce four different symmetries and the corresponding predictions on the neutrino mixing angles at this high scale:

#### A. QLC1

Quark-lepton complementarity [7, 8, 9, 10, 11] links the difference between the measured and the maximal (i.e.  $45^\circ$ ) values of the neutrino mixing angle  $\theta_{12}$  to the Cabbibo angle  $\theta_c = 12.6^\circ \pm 0.1^\circ$  [28]. We first follow a particular basis independent formulation ‘‘QLC1’’



[9] of this principle <sup>2</sup>:

$$U_{\text{PMNS}} = V_{\text{CKM}}^\dagger U_{\nu, \text{bm}} , \quad (20)$$

where  $U_{\nu, \text{bm}}$  is the specific bimaximal form [13] for the unitary neutrino mixing matrix. Eq. (20) gives rise to the ‘‘QLC1’’ relation <sup>3</sup>

$$\theta_{12}^\Lambda + \frac{\theta_c}{\sqrt{2}} = \frac{\pi}{4} + \mathcal{O}(\theta_c^3) . \quad (21)$$

The identification of (20) as a statement of QLC becomes more transparent in the basis with  $U_u = I$ , i.e. where the matrix  $Y_u^\dagger Y_u$  is diagonal. It follows from (9) that  $V_{\text{CKM}} = U_d$  in this basis. Now a comparison of (9) and (20), together with the assumption of  $U_\nu$  being  $U_{\nu, \text{bm}}$ , yields the SU(5) GUT-inspired quark-lepton symmetry relation  $U_d = U_l$ . Eq. (20), as it stands, is basis independent, however.

Eq. (20) yields the neutrino mixing angles at the high scale  $\Lambda$  to be

$$\begin{aligned} \theta_{12}^\Lambda &= \frac{\pi}{4} - \frac{\theta_c}{\sqrt{2}} + \mathcal{O}(\theta_c^3) \approx 35.4^\circ \\ \theta_{23}^\Lambda &= \frac{\pi}{4} - |V_{cb}| - \frac{\theta_c^2}{4} + \mathcal{O}(\theta_c^3) \approx 42.1^\circ , \\ \theta_{13}^\Lambda &= \frac{\theta_c}{\sqrt{2}} + \mathcal{O}(\theta_c^3) \approx 8.9^\circ . \end{aligned} \quad (22)$$

Thus, QLC1 predicts a value of  $\theta_{13}^\Lambda$  that is close to the current experimental bound.

## B. QLC2

In a second version of quark-lepton complementarity, ‘‘QLC2’’ [8, 9] , one assumes a bimaximal structure for the charged lepton mixing matrix,  $U_\ell = U_{\ell, \text{bm}}$  and the form

$$U_{\text{PMNS}} = U_{\ell, \text{bm}} V_{\text{CKM}}^\dagger , \quad (23)$$

for the PMNS matrix. Eq. (23) yields in a straightforward way the relation

$$\theta_{12}^\Lambda + \theta_c = \frac{\pi}{4} + \mathcal{O}(\theta_c^3) . \quad (24)$$

---

<sup>2</sup> There could be a more general statement of QLC1 with an additional diagonal phase matrix  $\Gamma_\delta$  between  $V_{\text{CKM}}^\dagger$  and  $U_{\nu, \text{bm}}$ . But for consistency and simplicity, we choose eq. (20).

<sup>3</sup> We do not distinguish between  $\theta_c^\Lambda$  and  $\theta_c^\lambda$  since the running of  $\theta_c$  is negligible on account of the hierarchical nature of quarks belonging to different generations.

One may note that, in the basis with  $U_d = I$ , i.e. where  $Y_d^\dagger Y_d$  is diagonal, (23) yields the SO(10) GUT-inspired relation  $U_u = U_\nu$ .

Eq. (23) leads to the following values for the neutrino mixing angles at the high scale:

$$\begin{aligned}\theta_{12}^\Lambda &= \frac{\pi}{4} - \theta_c + \mathcal{O}(\theta_c^3) \approx 32.4^\circ, \\ \theta_{23}^\Lambda &= \frac{\pi}{4} - \frac{|V_{cb}|}{\sqrt{2}} + \mathcal{O}(\theta_c^3) \approx 43.4^\circ, \\ \theta_{13}^\Lambda &= \frac{|V_{cb}|}{\sqrt{2}} + \mathcal{O}(\theta_c^3) \approx 1.6^\circ.\end{aligned}\tag{25}$$

The value of  $\theta_{13}^\Lambda$  predicted in QLC2 is beyond the measuring capacity of the neutrino experiments planned during the next decade.

### C. TBM1

The tribimaximal form of the neutrino mixing matrix is given by

$$U_{\nu,\text{tbm}}^\Lambda = \frac{1}{\sqrt{6}} \begin{pmatrix} 2 & \sqrt{2} & 0 \\ -1 & \sqrt{2} & \sqrt{3} \\ 1 & -\sqrt{2} & \sqrt{3} \end{pmatrix} = R_{23}\left(\frac{\pi}{4}\right)R_{13}(0)R_{12}\left(\sin^{-1}\frac{1}{\sqrt{3}}\right).\tag{26}$$

In the standard TBM scenario [12], which we refer to as TBM1, one has  $U_{\text{PMNS}}^\Lambda = U_\nu^\Lambda$  since the charged lepton mass matrix at the high scale is already flavour diagonal. Then we have

$$\theta_{12}^\Lambda \approx 35.3^\circ, \theta_{23}^\Lambda = 45^\circ, \theta_{13}^\Lambda = 0^\circ.\tag{27}$$

### D. TBM2

Small deviations from the tribimaximal scenario TBM1 above have been considered in the literature [34, 35], where the deviation originates from the mixing in the charged lepton sector. Here we consider the version in Ref. [34], and call it TBM2. Here  $U_{\text{PMNS}} = V_{\ell L}^\dagger U_{\nu,\text{tbm}}$ , where  $V_{\ell L}$  has the form [11]

$$V_{\ell L} = \begin{pmatrix} 1 & \theta_c/3 & 0 \\ \theta_c/3 & 1 & -|V_{cb}| \\ 0 & |V_{cb}| & 1 \end{pmatrix} + \mathcal{O}(\theta_c^3),\tag{28}$$

with the factor of 1/3 coming from the Georgi-Jarlskog relation [36]  $m_\mu/m_s = 3$  at the GUT scale. As a result, we have at the high scale

$$\begin{aligned}\theta_{12}^\Lambda &= \sin^{-1} \frac{1}{\sqrt{3}} - \frac{\theta_c}{3\sqrt{2}} + \mathcal{O}(\theta_c^3) \approx 32.3^\circ, \\ \theta_{23}^\Lambda &= \frac{\pi}{4} - |V_{cb}| + \mathcal{O}(\theta_c^3) \approx 42.7^\circ, \\ \theta_{13}^\Lambda &= \frac{\theta_c}{3\sqrt{2}} + \mathcal{O}(\theta_c^3) \approx 3.1^\circ.\end{aligned}\tag{29}$$

#### IV. HIGH SCALE STRUCTURE AND DOWNWARD EVOLUTION

As explained in the Introduction, our idea is to start with a specific structure of the neutrino mass matrix  $\mathcal{M}_\nu^\Lambda$  that is dictated by some symmetry at a high scale  $\Lambda \sim 10^{12}$  GeV. We would then like to evolve the elements of  $\mathcal{M}_\nu$  down to a laboratory energy scale  $\lambda \sim 10^3$  GeV. This involves studying the (one-loop) RG evolution of the coefficient functions  $c_{\alpha\beta}$  in (18) between  $\Lambda \sim 10^{12}$  GeV and  $\lambda \sim 10^3$  GeV. In case the considered high scale neutrino symmetries are consequences of grand unification, we need to assume that the threshold effects [30] between the GUT scale  $\sim 2 \times 10^{16}$  GeV and  $\Lambda$  are flavor blind so that they do not spoil the assumed symmetry relations in the downward evolution from  $M_{\text{GUT}}$  to  $\Lambda$ . We also note that effects of evolution on the masses and mixing angles of charged fermions are known [31] to be negligibly small<sup>4</sup> on account of the hierarchical nature of their mass values.

At one loop, the neutrino mass matrices at the scales  $\Lambda$  and  $\lambda$  are homogeneously related [32, 33]:

$$\mathcal{M}_\nu^\lambda = I_K \mathcal{I}_\kappa^T \mathcal{M}_\nu^\Lambda \mathcal{I}_\kappa,\tag{30}$$

where

$$I_K \equiv \exp \left[ - \int_{t(\Lambda)}^{t(\lambda)} K(t) dt \right]\tag{31}$$

is a scalar factor common to all elements of  $\mathcal{M}_\nu^\lambda$ . In eq. (31),  $t(Q) \equiv (16\pi^2)^{-1} \ln(Q/Q_0)$  with  $Q$  ( $Q_0$ ) being a running (fixed) scale, and the integrand is given by

$$K(t) = -6g_2^2(t) - 2g_Y^2(t) + 6\text{Tr} (Y_u^\dagger Y_u)(t)\tag{32}$$

---

<sup>4</sup> The value of  $|V_{cb}|$  does run by about 0.01 in the MSSM due to the top quark U(1) coupling. However, at the level of accuracy that we are concerned with, this is inconsequential.

in a transparent notation,  $g_{2,Y}$  being the  $SU(2)_L, U(1)_Y$  gauge coupling strength and  $Y_u$  the up-type Yukawa coupling matrix. Finally, the matrix  $I_\kappa$  has the form

$$\mathcal{I}_\kappa \equiv \exp \left[ - \int_{t(\Lambda)}^{t(\lambda)} (Y_l^\dagger Y_l)(t) dt \right], \quad (33)$$

$Y_\ell$  being the Yukawa coupling matrix for charged leptons.

Although some of the neutrino mixing matrices in various scenarios in Sec. III have been motivated in terms of grand unification in bases where the symmetries involved may be clearly observed, for the RG evolution of all scenarios we choose to work in the basis where the charged lepton mass matrix is diagonal. In this basis,

$$Y_\ell^\dagger Y_\ell = \text{Diag} (y_e^2, y_\mu^2, y_\tau^2). \quad (34)$$

We can neglect  $y_{e,\mu}^2$  in comparison with  $y_\tau^2$  in (34) to get the result

$$\mathcal{I}_\kappa \approx \text{Diag}(1, 1, e^{-\Delta_\tau}) = \text{Diag}(1, 1, 1 - \Delta_\tau) + \mathcal{O}(\Delta_\tau^2), \quad (35)$$

where [32]

$$\Delta_\tau = \int_{t(\Lambda)}^{t(\lambda)} |y_\tau(t)|^2 = m_\tau^2 (\tan^2 \beta + 1) (8\pi^2 v^2)^{-1} \ln(\Lambda/\lambda). \quad (36)$$

Here  $v \equiv \sqrt{v_u^2 + v_d^2}$  and  $\tan \beta = v_u/v_d$ , where  $v_u$  ( $v_d$ ) is  $\sqrt{2}$  times the vev of the up (down) type neutral Higgs scalars. Evidently,  $\Delta_\tau$  is a small number for the allowed range of  $\tan \beta$ : e.g.  $\Delta_\tau \approx 6 \times 10^{-3}$  for  $\tan \beta = 30$ , justifying our neglect of the  $\mathcal{O}(\Delta_\tau^2)$  terms<sup>5</sup>.

The substitution of (35) into (30) leads us to

$$\mathcal{M}_\nu^\lambda \propto \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 - \Delta_\tau \end{pmatrix} \mathcal{M}_\nu^\Lambda \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 - \Delta_\tau \end{pmatrix} + \mathcal{O}(\Delta_\tau^2), \quad (37)$$

where the proportionality is through the scalar factor  $I_K$  given in (31). It must be emphasised that (37) is valid only when  $\mathcal{M}_\nu^{\Lambda(\lambda)}$  is written in the basis where the charged lepton mass matrix is diagonal.

The matrix  $\mathcal{M}_\nu^\Lambda$  is complex symmetric. Writing it in the general form

$$\mathcal{M}_\nu^\Lambda = \begin{pmatrix} A & B & C \\ B & D & E \\ C & E & F \end{pmatrix}, \quad (38)$$

---

<sup>5</sup> Note that a mistake of a factor of 2 in eq. 9 of [6] has been corrected here.

we have

$$\mathcal{M}_\nu^\lambda = \begin{pmatrix} A & B & C(1 - \Delta_\tau) \\ B & D & E(1 - \Delta_\tau) \\ C(1 - \Delta_\tau) & E(1 - \Delta_\tau) & F(1 - 2\Delta_\tau) \end{pmatrix} + \mathcal{O}(\Delta_\tau^2). \quad (39)$$

Since both  $M_\nu^{\Lambda, \lambda}$  are complex symmetric matrices, they can be diagonalized as

$$\begin{aligned} (U^\Lambda)^\dagger M_\nu^\Lambda (U^\Lambda)^* &= \text{Diag}(|m_1^\Lambda|, |m_2^\Lambda|, |m_3^\Lambda|), \\ (U^\lambda)^\dagger M_\nu^\lambda (U^\lambda)^* &= \text{Diag}(|m_1^\lambda|, |m_2^\lambda|, |m_3^\lambda|). \end{aligned} \quad (40)$$

Note that, since we are working in a basis where the charged lepton mass matrix is diagonal,  $U^{\Lambda(\lambda)}$  is the same as the net leptonic mixing matrix  $U_{\text{PMNS}}$  (9) at the scale  $\Lambda$  ( $\lambda$ ).

The unitary matrix  $U_{\text{PMNS}}$  may be parametrised in its most general form as

$$\begin{aligned} U_{\text{PMNS}} \equiv & \text{Diag}(e^{i\phi_e}, e^{i\phi_\mu}, e^{i\phi_\tau}) R_{23}(\theta_{23}) \text{Diag}(1, 1, e^{i\delta_\ell}) R_{13}(\theta_{13}) \times \\ & \text{Diag}(1, 1, e^{-i\delta_\ell}) R_{12}(\theta_{12}) \text{Diag}(1, e^{-i\alpha_2/2}, e^{-i\alpha_3/2}), \end{aligned} \quad (41)$$

where  $R_{ij}(\theta_{ij})$  is the matrix for rotation through the angle  $\theta_{ij}$  in the  $i - j$  plane,  $\delta_\ell$  is the CP violating Dirac phase,  $\alpha_{2,3}$  the Majorana phases and  $\phi_{e,\mu,\tau}$  the so-called ‘‘unphysical’’ additional phases required to diagonalise the neutrino mass matrix. Here we have already used the freedom of choosing  $\alpha_1 = 0$ .

If  $\theta_{13}^\Lambda$  vanishes on account of the symmetry requirement at the scale  $\Lambda$ , the evolution of the mixing angles can be computed analytically in a simple manner. The matrix  $U^\Lambda$  can then be written as

$$U^\Lambda = R_{23}(\theta_{23}^\Lambda) R_{12}(\theta_{12}^\Lambda) \text{Diag}(1, e^{-i\alpha_2^\Lambda/2}, e^{-i\alpha_3^\Lambda/2}), \quad (42)$$

since the Dirac phase contribution vanishes and the phases  $\phi_{e,\mu,\tau}$  can anyway be absorbed in the charged lepton phases. RG evolution will modify the angles  $\theta_{23}, \theta_{12}$  as well as the phases  $\alpha_{2,3}$ , at the same time generating nonzero values for the mixing angle  $\theta_{13}$  and the Dirac phase  $\delta_\ell$ . The phases  $\phi_{e,\mu,\tau}$  that may get generated can always be absorbed in the phases of the charged lepton flavour eigenstates.

If we now approximate the deviation of  $U_\nu^\lambda$  from  $U_\nu^\Lambda$  by the retention of only terms that are linear in  $\Delta_\tau$ , we can write the modified mixing angles as

$$\theta_{12}^\lambda = \theta_{12}^\Lambda + k_{12}\Delta_\tau + \mathcal{O}(\Delta_\tau^2), \quad \theta_{23}^\lambda = \theta_{23}^\Lambda + k_{23}\Delta_\tau + \mathcal{O}(\Delta_\tau^2), \quad \theta_{13}^\lambda = k_{13}\Delta_\tau + \mathcal{O}(\Delta_\tau^2). \quad (43)$$

The modified phases may similarly be written as

$$\alpha_{2,3}^\lambda = \alpha_{2,3}^\Lambda + a_{2,3}\Delta_\tau + \mathcal{O}(\Delta_\tau^2), \quad \delta_\ell^\lambda = d_\ell\Delta_\tau + \mathcal{O}(\Delta_\tau^2), \quad (44)$$

where we expect  $a_{2,3}$  and  $d_\ell$  to be  $\mathcal{O}(1)$  quantities. In this paper we shall only be concerned about the deviation  $k_{ij}\Delta_\tau$  of the mixing angles  $\theta_{ij}$  from their high scale values. We shall see a posteriori that  $|k_{ij}\Delta_\tau|$  is always much less than unity so that we can ignore its quadratic and higher powers.

The values of  $k_{ij}$  are found to be

$$\begin{aligned} k_{12} &= \frac{1}{2} \sin 2\theta_{12}^\Lambda \sin^2 \theta_{23}^\Lambda \frac{|m_1^\Lambda + m_2^\Lambda|^2}{|m_2^\Lambda|^2 - |m_1^\Lambda|^2}, \\ k_{23} &= \frac{1}{2} \sin 2\theta_{23}^\Lambda \left( \cos^2 \theta_{12}^\Lambda \frac{|m_2^\Lambda + m_3^\Lambda|^2}{|m_3^\Lambda|^2 - |m_2^\Lambda|^2} + \sin^2 \theta_{12}^\Lambda \frac{|m_1^\Lambda + m_3^\Lambda|^2}{|m_3^\Lambda|^2 - |m_1^\Lambda|^2} \right), \\ k_{13} &= \frac{1}{4} \sin 2\theta_{12}^\Lambda \sin 2\theta_{23}^\Lambda \left( \frac{|m_2^\Lambda + m_3^\Lambda|^2}{|m_3^\Lambda|^2 - |m_2^\Lambda|^2} - \frac{|m_1^\Lambda + m_3^\Lambda|^2}{|m_3^\Lambda|^2 - |m_1^\Lambda|^2} \right), \end{aligned} \quad (45)$$

where  $m_i^\Lambda$  are the masses  $|m_i^\Lambda|e^{i\alpha_i^\Lambda}$  at the high scale. The RG evolution of  $|m_i|$  may be parametrised by

$$|m_i^\lambda| = I_K |m_i^\Lambda| (1 + \mu_i \Delta_\tau + \mathcal{O}(\Delta_\tau^2)) \quad (46)$$

where  $I_K$  is the scalar factor given in (31) and  $\mu_i$  are  $\mathcal{O}(1)$  numbers [24]. Then, taking  $m_i$  to be the masses  $|m_i^\lambda|e^{i\alpha_i^\lambda}$  at the low scale introduces an error of  $\mathcal{O}(\Delta_\tau)$  in  $k_{ij}$ , and hence of  $\mathcal{O}(\Delta_\tau^2)$  in  $\theta_{ij}$ . Eqs. (45) are therefore valid even with the low scale values of the masses and Majorana phases. The same argument is true even for the mixing angles  $\theta_{ij}$ . Since all observations in the neutrino experiments are made at laboratory energies, we henceforth drop the superscript  $\lambda$  for all the quantities at the low scale.

The expressions (45) have been derived starting with  $\theta_{13}^\Lambda = 0$ , and they agree with the general expressions in [24] in the limit  $\theta_{13}^\Lambda \rightarrow 0$ . For finite  $\theta_{13}^\Lambda$ , the  $k_{ij}$  given in (45) have an error of  $\mathcal{O}(\theta_{13}^\Lambda)$ , i.e. the mixing angles at the low scale have an error of  $\mathcal{O}(\theta_{13}^\Lambda \Delta_\tau)$ , which we neglect in our analytical approximations. Since for all the scenarios considered here,  $\theta_{13}^\Lambda \Delta_\tau \lesssim 0.1^\circ$ , this is a justified assumption. Thus, even with small nonzero values of  $\theta_{13}^\Lambda$ , as is the case with QLC1, QLC2 and TBM2 scenarios, we can justifiably use the result  $\theta_{ij} \approx \theta_{ij}^\Lambda + k_{ij}\Delta_\tau$  with  $k_{ij}$  given by (45). Another requirement for the validity of eqs. (45) is that the values of  $m_i$  and  $|m_i|^2 - |m_j|^2$  should be described accurately by the  $\mathcal{O}(\Delta_\tau)$  terms in their RG evolution. This condition does not impose any restriction on  $m_i^\Lambda$ . However,

for  $\Delta_\tau \gtrsim (|m_2^\Lambda|^2 - |m_1^\Lambda|^2)/(m_0^\Lambda)^2$ , the  $\mathcal{O}(\Delta_\tau^2)$  terms dominate over the  $\mathcal{O}(\Delta_\tau)$  terms in the  $|m_2|^2 - |m_1|^2$  evolution [24], which results in eqs. (45) breaking down. Thus for the validity of these equations, we require

$$(m_0^\Lambda)^2 \cdot \Delta_\tau \lesssim |m_2^\Lambda|^2 - |m_1^\Lambda|^2. \quad (47)$$

Eq. (47) may be violated if  $|m_2^\Lambda|^2 - |m_1^\Lambda|^2$  is indeed very small. However, even in such a situation, many of the qualitative features following from our analytical treatment will continue to remain valid. This will be confirmed by the exact numerical analysis given later.

One can make some further observations on eqs. (45). Since  $k_{12}$  is always positive, the measured value of  $\theta_{12}$  can never be smaller than  $\theta_{12}^\Lambda$ . Moreover, in order to have any significant effect of RG evolution, some  $|k_{ij}|$  are required to be  $\gtrsim \mathcal{O}(10)$  since  $\Delta_\tau \lesssim \mathcal{O}(10^{-2})$ . This requirement is not met in the case of a normal hierarchy which has all  $|k_{ij}| \sim \mathcal{O}(1)$ , and hence there is no significant RG effect there. Specifically, for an enhancement of  $|k_{12}|$ ,  $|m_0^\Lambda|^2$  must necessarily be  $\gg |m_2^\Lambda|^2 - |m_1^\Lambda|^2$ . This inequality is obeyed whenever neutrinos are nonhierarchical, i.e. either in the IH or the QD case. On the other hand, the enhancement of both  $|k_{23}|$  and  $|k_{13}|$  requires the inequality  $|\rho_A^\Lambda| \ll 1$ , satisfied only by QD neutrinos. However, these are necessary conditions. The occurrence (or not) of an actual enhancement depends on the Majorana phases  $\alpha_{2,3}$  too. In order to see this dependence explicitly, let us rewrite (45) in terms of these phases and our parameters  $\rho_A^\Lambda$  and  $\epsilon_S^\Lambda$  of (12), defined at the high scale. We then have, with finite  $\theta_{13}^\Lambda$ ,

$$\begin{aligned} k_{12} &= \frac{1}{4\epsilon_S^\Lambda} \sin 2\theta_{12}^\Lambda \sin^2 \theta_{23}^\Lambda [1 + \cos \alpha_2^\Lambda + (\epsilon_S^\Lambda)^2(1 - \cos \alpha_2^\Lambda)] + \mathcal{O}(\theta_{13}^\Lambda), \\ k_{23} &= \frac{\Gamma^\Lambda}{4} \sin 2\theta_{23}^\Lambda [1 + \cos^2 \theta_{12}^\Lambda \cos(\alpha_2^\Lambda - \alpha_3^\Lambda) + \sin^2 \theta_{12}^\Lambda \cos \alpha_3^\Lambda] \\ &\quad + \frac{\rho_A^\Lambda}{2} \sin 2\theta_{12}^\Lambda \sin 2\theta_{23}^\Lambda + \mathcal{O}(\epsilon_S^\Lambda, \theta_{13}^\Lambda), \\ k_{13} &= \frac{\Gamma^\Lambda}{8} \sin 2\theta_{12}^\Lambda \sin 2\theta_{23}^\Lambda [\cos(\alpha_2^\Lambda - \alpha_3^\Lambda) - \cos \alpha_3^\Lambda] + \mathcal{O}(\epsilon_S^\Lambda, \theta_{13}^\Lambda). \end{aligned} \quad (48)$$

It is clear from (48) that the values of the Majorana phases  $\alpha_{2,3}^\Lambda$  are crucial in controlling whether the evolution terms  $k_{ij}\Delta_\tau$  are dangerously large or not. For instance, as  $\alpha_2^\Lambda \rightarrow 0$ , we have  $k_{12} \approx (4\epsilon_S^\Lambda)^{-1}$ . For nonhierarchical neutrinos, the latter is likely to destroy any high scale symmetry statement on  $\theta_{12}$ . When  $\alpha_2^\Lambda$  increases from zero,  $|k_{12}|$  decreases rapidly, becoming as small as  $|\epsilon_S^\Lambda|/4$  when  $\alpha_2^\Lambda = \pi$ , i.e.  $m_1^\Lambda \approx -m_2^\Lambda$ . Moreover, (48) also indicates

that both  $|k_{23}|$  and  $|k_{13}|$  are enhanced when either  $\alpha_3^\Lambda = 0$ , or  $\alpha_2^\Lambda = \alpha_3^\Lambda$  for a nonzero  $\alpha_2^\Lambda$ , though  $|k_{13}|$  gets highly suppressed when  $\alpha_2^\Lambda = 0$ .

As noted in Sec. II, the sign of  $\Gamma^\Lambda$  is positive for a normal ordering and negative for an inverted ordering of the neutrino masses. Thus  $k_{23}$  is always positive (negative) for a normal (inverted) ordering. However, the sign of  $k_{13}$  is controlled not only by  $\text{sgn}(\Gamma^\Lambda)$  but also by a combination of Majorana phases.

In the remaining part of this section, we enumerate the predictions for the four scenarios considered in this paper. On evolution down to the scale  $\lambda$ , the mixing angles of the corresponding mixing matrix  $U_{\text{PMNS}}$  are given in all the four cases by  $\theta_{ij} = \theta_{ij}^\Lambda + k_{ij}\Delta_\tau + \mathcal{O}(\theta_{13}^\Lambda\Delta_\tau, \Delta_\tau^2)$  where the values of  $k_{ij}$  are given by (48).

### A. QLC1

On evolving (22) to the laboratory scale, the net leptonic mixing angles are found to be

$$\begin{aligned}\theta_{12} &= \frac{\pi}{4} - \frac{\theta_c}{\sqrt{2}} + k_{12}^{\text{QLC1}}\Delta_\tau + \mathcal{O}(\theta_c^3, \theta_{13}^\Lambda\Delta_\tau, \Delta_\tau^2), \\ \theta_{23} &= \frac{\pi}{4} - |V_{cb}| - \frac{\theta_c^2}{4} + k_{23}^{\text{QLC1}}\Delta_\tau + \mathcal{O}(\theta_c^3, \theta_{13}^\Lambda\Delta_\tau, \Delta_\tau^2), \\ \theta_{13} &= \left| \frac{\theta_c}{\sqrt{2}} - k_{13}^{\text{QLC1}}\Delta_\tau \right| + \mathcal{O}(\theta_c^3, \theta_{13}^\Lambda\Delta_\tau, \Delta_\tau^2),\end{aligned}\tag{49}$$

where the neglected terms are  $\lesssim 0.1^\circ$ . The absolute value taken for the RHS of  $\theta_{13}$  is in order to keep to the convention of defining  $\theta_{ij} > 0$  [28].

Since  $\pi/4 - \theta_c/\sqrt{2} \approx 35.4^\circ$  and  $k_{12}^{\text{QLC1}}\Delta_\tau > 0$ , it follows from (49) that  $\theta_{12}$  in this scenario cannot be less than  $35.4^\circ$ . Further, since  $\pi/4 - |V_{cb}| - \theta_c^2/4 \approx 42.1^\circ$ , and  $k_{23}^{\text{QLC1}}\Delta_\tau > 0$  ( $< 0$ ) for a normal (inverted) ordering of neutrino masses, a consequence of (49) is that  $\theta_{23}$  is greater than (less than)  $42.1^\circ$  for a normal (inverted) ordering. Finally, the predicted value of  $\theta_{13}$  is  $\theta_c/\sqrt{2} \approx 8.9^\circ$  in the absence of RG running, but it can be greater or less than  $8.9^\circ$  depending on the values of the Majorana phases. The detailed numerical analysis will be presented in Sec. V.



## B. QLC2

After RG evolution, the high scale angles (25) in this scenario evolve to

$$\begin{aligned}
\theta_{12} &= \frac{\pi}{4} - \theta_c + k_{12}^{\text{QLC2}} \Delta_\tau + \mathcal{O}(\theta_c^3, \theta_{13}^\Lambda \Delta_\tau, \Delta_\tau^2), \\
\theta_{23} &= \frac{\pi}{4} - \frac{|V_{cb}|}{\sqrt{2}} + k_{23}^{\text{QLC2}} \Delta_\tau + \mathcal{O}(\theta_c^3, \theta_{13}^\Lambda \Delta_\tau, \Delta_\tau^2), \\
\theta_{13} &= \left| \frac{|V_{cb}|}{\sqrt{2}} - k_{13}^{\text{QLC2}} \Delta_\tau \right| + \mathcal{O}(\theta_c^3, \theta_{13}^\Lambda \Delta_\tau, \Delta_\tau^2),
\end{aligned} \tag{50}$$

where the neglected terms are  $\lesssim 0.1^\circ$  as before.

The lower bound on  $\theta_{12}$  in this scenario being  $\pi/4 - \theta_c \approx 32.4^\circ$ , significantly lower values of  $\theta_{12}$  than QLC1 are allowed. Also, here  $\theta_{23}$  is greater than (less than)  $\pi/4 - |V_{cb}|/\sqrt{2} \approx 43.4^\circ$  for normal (inverted) neutrino mass ordering. The major difference from QLC1 is in  $\theta_{13}$ : the value  $\theta_{13}$  in QLC2 is only  $|V_{cb}|/\sqrt{2} \approx 1.6^\circ$  in the absence of RG running. It can increase with RG running, but the extent of this increase is restricted from the observed  $\theta_{12}$  values which restrict the values of the Majorana phases in turn. The detailed numerical analysis will again be presented in Sec. V.

## C. TBM1

The mixing angles at the low scale here are simply given by

$$\begin{aligned}
\theta_{12} &= \sin^{-1} \frac{1}{\sqrt{3}} + k_{12}^{\text{TBM1}} \Delta_\tau + \mathcal{O}(\theta_c^3, \theta_{13}^\Lambda \Delta_\tau, \Delta_\tau^2), \\
\theta_{23} &= \frac{\pi}{4} + k_{23}^{\text{TBM1}} \Delta_\tau + \mathcal{O}(\theta_c^3, \theta_{13}^\Lambda \Delta_\tau, \Delta_\tau^2), \\
\theta_{13} &= k_{13}^{\text{TBM1}} \Delta_\tau + \mathcal{O}(\theta_c^3, \theta_{13}^\Lambda \Delta_\tau, \Delta_\tau^2).
\end{aligned} \tag{51}$$

On similar lines to the arguments given for the QLC scenarios, here (i) the minimum value of  $\theta_{12}$  is  $\sin^{-1}(1/\sqrt{3}) \approx 35.3^\circ$ , (ii) the value of  $\theta_{23}$  is greater than (less than)  $45^\circ$  for normal (inverted) hierarchy, and the value of  $\theta_{13}$  vanishes in the absence of RG running. Since (48) shows that  $\theta_{13}$  does not run if the Majorana phases vanish, *any observed deviation of  $\theta_{13}$  from zero in this scheme will indicate nonvanishing Majorana phases.* The detailed numerical analysis appears in Sec. V.

## D. TBM2

The mixing angles (29) for this scenario evolve to the laboratory scale and become

$$\begin{aligned}
\theta_{12} &= \sin^{-1} \frac{1}{\sqrt{3}} - \frac{\theta_c}{3\sqrt{2}} + k_{12}^{\text{TBM2}} \Delta_\tau + \mathcal{O}(\theta_c^3, \theta_{13}^\Lambda \Delta_\tau, \Delta_\tau^2), \\
\theta_{23} &= \frac{\pi}{4} - |V_{cb}| + k_{23}^{\text{TBM2}} \Delta_\tau + \mathcal{O}(\theta_c^3, \theta_{13}^\Lambda \Delta_\tau, \Delta_\tau^2), \\
\theta_{13} &= \left| \frac{\theta_c}{3\sqrt{2}} - k_{13}^{\text{TBM2}} \Delta_\tau \right| + \mathcal{O}(\theta_c^3, \theta_{13}^\Lambda \Delta_\tau, \Delta_\tau^2).
\end{aligned} \tag{52}$$

The minimum allowed value of  $\theta_{12}$  in this scenario is  $\sin^{-1}(1/\sqrt{3}) - \theta_c/(3\sqrt{2}) \approx 32.3^\circ$ , since  $k_{12}^{\text{TBM2}} > 0$ . The value of  $\theta_{23}$  is greater than (less than)  $\pi/4 - |V_{cb}| \approx 42.7^\circ$  for normal (inverted) hierarchy. Finally, the value of  $\theta_{13}$  is  $\theta_c/(3\sqrt{2}) \approx 3.1^\circ$  in the absence of RG running. Again, the detailed numerical analysis with RG running is presented in Sec. V.

## V. CONSTRAINTS ON $m_0$ , $\tan\beta$ AND MAJORANA PHASES

In this section, we explore the current limits on the parameters of the four scenarios considered above. The main parameters governing RG running are  $m_0$ ,  $\tan\beta$ , the Majorana phases  $\alpha_{2,3}$ , and (to a smaller extent) the Dirac phase  $\delta_\ell$ . Our aim is to find the range of values of these parameters allowed by the current data. The four scenarios also lead to slightly different predictions for the mixing angles; and accurate measurements of these angles should distinguish among them in the absence of RG running. However, since the latter spoils high scale symmetries in general, and since the values of all the relevant parameters are still not known, the low scale predictions of the mixing angles are expected to overlap. We explore in detail whether this still allows one to discriminate among the scenarios considered in this paper. Moreover, we study the correlations between the deviations of the mixing angles from their high scale values.

We use the  $3\sigma$  ranges for the neutrino mass and mixing parameters

$$7 \times 10^{-5} \text{ eV}^2 < \delta m_S^2 < 9.1 \times 10^{-5} \text{ eV}^2, \quad 1.7 \times 10^{-3} \text{ eV}^2 < |\delta m_A^2| < 3.3 \times 10^{-3} \text{ eV}^2, \tag{53}$$

$$30^\circ < \theta_{12} < 39.2^\circ, \quad 35.5^\circ < \theta_{23} < 55.5^\circ, \quad \theta_{13} < 12^\circ \tag{54}$$

at the low scale. At the high scale, we start with the values of  $\theta_{ij}^\Lambda$  dictated by the scenario under consideration, and a range of  $\delta m_{S/A}^2$  values that are consistent, after RG evolution,

with the low scale measurements (53) and (54). We show our results for a normal mass ordering of neutrinos. However, the constraints in the case of an inverted mass ordering are almost identical.

We give our constraints in terms of the values of  $m_0^\Lambda$  and  $\alpha_2^\Lambda$  at the high scale. These are related to the low scale values of  $m_0$  and  $\alpha_2$  as indicated in (46) and (44) respectively. In particular, the evolution of  $m_0$  is controlled mainly by  $I_K \approx 0.71$ , which makes  $m_0^\lambda \approx 0.71 m_0^\Lambda$ . The bounds on  $m_0^\Lambda$  shown in the figures in this section can then be easily translated into bounds on the low scale value of  $m_0$ . The RG evolution of  $\alpha_2$  is given by  $\alpha_2^\lambda \approx \alpha_2^\Lambda + a_2 \Delta_\tau$  with [24]

$$a_2 \approx \frac{-4|m_1^\Lambda m_2^\Lambda|}{|m_2^\Lambda|^2 - |m_1^\Lambda|^2} \cos 2\theta_{12}^\Lambda \sin^2 \theta_{23}^\Lambda \sin \alpha_2^\Lambda, \quad (55)$$

which may be used to translate the  $\alpha_2^\Lambda$  constraints to low scale values of  $\alpha_2$ .

We have neglected possible Planck scale effects [37] which may change the value of  $\theta_{12}$  by a few degrees for quasidegenerate neutrinos, leaving the other two angles virtually unaffected. Inclusion of these effects would relax [38] the constraints in the  $m_0^\Lambda - \alpha_2^\Lambda$  plane by a small amount.

### A. Limits on $m_0^\Lambda$ , $\alpha_2^\Lambda$ and $\tan \beta$

Out of the three leptonic mixing angles,  $\theta_{12}$  is the one measured with the greatest accuracy currently. Since all the scenarios have specific predictions for  $\theta_{12}$  in the absence of RG evolution, the measured value of  $\theta_{12}$  can put the strongest constraints on the running parameters. From (43) and (48), the running of  $\theta_{12}$  is expected to be independent of  $\alpha_3^\Lambda$ .

We show in Fig. 2 the  $3\sigma$  allowed regions in the  $m_0^\Lambda - \alpha_2^\Lambda$  plane for two  $\tan \beta$  values. This figure has been obtained from numerical solutions of the RG equations. The figure agrees very well with our expectations from the analytic expressions (48). On account of the occurrence of the small quantity  $\epsilon_S$  in the denominator of  $k_{12}$  in eq. (48), strong constraints ensue on  $m_0^\Lambda$  and  $\alpha_2^\Lambda$ . At large  $\tan \beta$  (equivalent to a relatively large  $\Delta_\tau$ ), the value of  $\alpha_2^\Lambda$  has to be near  $\pi$  (i.e.  $m_1^\Lambda \approx -m_2^\Lambda$ ) underscoring the necessity for a nontrivial Majorana phase. However, the requirement is less severe for a smaller  $\tan \beta$ . The coefficient  $a_2$  in (55) characterising the evolution of  $\alpha_2$  vanishes when  $\alpha_2^\Lambda = \pi$  [24]. As a result, the preferred value of  $\alpha_2$ , viz.  $\alpha_2^\Lambda = \pi$ , is equivalent to  $\alpha_2 = \pi$  at all scales.

The constraints displayed in the figure are calculated for two fixed  $\tan \beta$  values. The

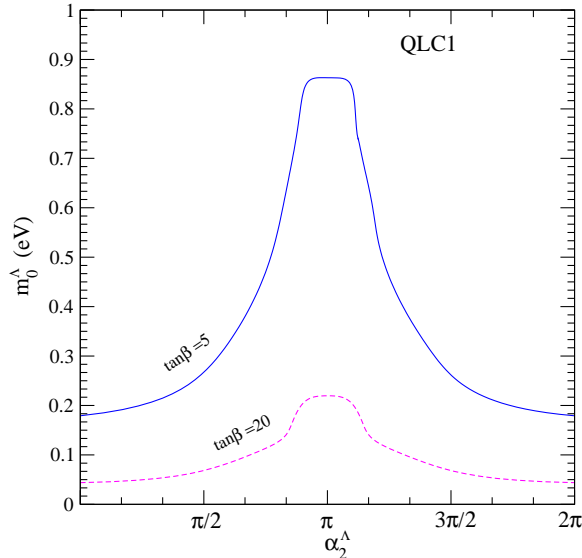


FIG. 2: Constraints from the  $3\sigma$  allowed range of  $\theta_{12}$  on the  $m_0$  (eV) –  $\alpha_2$  (radians) parameter space for  $\tan\beta = 5$  and  $20$  in the QLC1 scenario. The regions above the contours are excluded by data for that particular value of  $\tan\beta$ . The peak at  $\alpha_2 \approx \pi$  is noteworthy.

results for other values of  $\tan\beta$  may be extrapolated from the figure. However, we observe numerically that the  $\theta_{12}$  constraints on the  $m_0^\Lambda - \alpha_2^\Lambda$  plane (Fig. 2) depend essentially on the combination  $m_0^\Lambda \tan\beta$ . Therefore, in Fig. 3, we show the allowed region in the  $m_0 \tan\beta - \alpha_2$  parameter space for all four scenarios. The differences among the four scenarios arise primarily from the differences in the values of  $\theta_{12}^\Lambda$ . These are  $35.4^\circ$  (QLC1),  $32.4^\circ$  (QLC2),  $35.3^\circ$  (TBM1) and  $32.3^\circ$  (TBM2) in the four scenarios. The deviation of  $\theta_{12}$  from this value,  $\Delta\theta_{12} \equiv \theta_{12} - \theta_{12}^\Lambda$  differs in all the scenarios only through a factor of  $\sin 2\theta_{12}^\Lambda \sin^2 \theta_{23}^\Lambda$  [see eq. (48)], which is equal to unity within 5% for all the four scenarios. As a result, the allowed regions for QLC2 and TBM2 are nearly identical, and larger than those for QLC1 and TBM1, the last two regions being also almost identical to each other.

Note that, for all the scenarios, the larger the value of  $m_0^\Lambda \tan\beta$ , the closer the value of  $\alpha_2^\Lambda$  needs to be to  $\pi$ . This statement is true even if we use the laboratory values of  $m_0$  and  $\alpha_2$ . Moreover, the region with  $m_0^\Lambda \tan\beta \gtrsim 4.4$  eV (i.e.  $m_0 \tan\beta \gtrsim 3.1$  eV) is disallowed for all values of  $\alpha_2$  and for all scenarios.

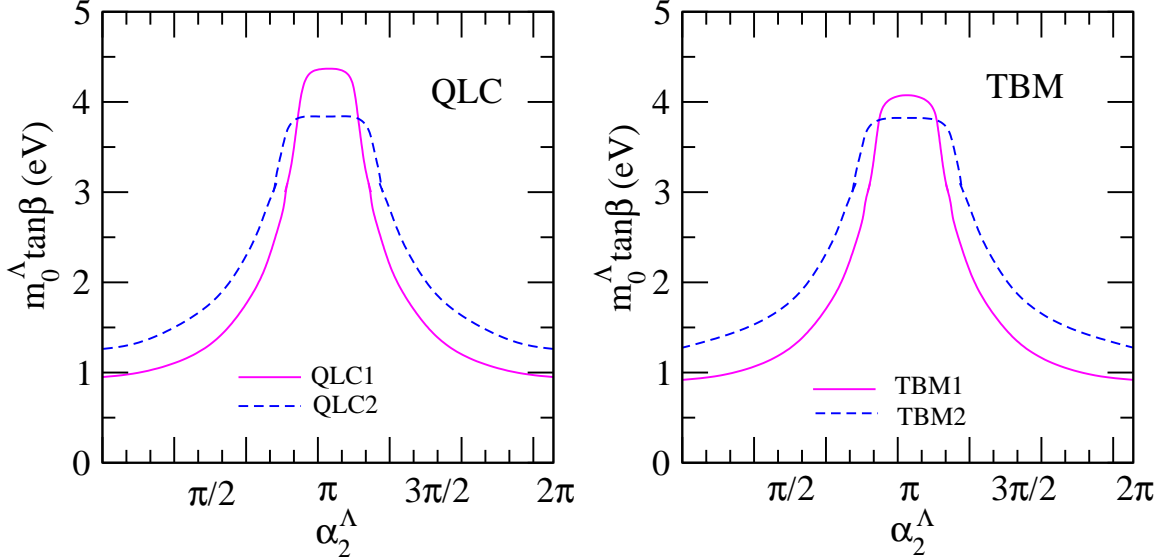


FIG. 3: Constraints from the  $3\sigma$  allowed range of  $\theta_{12}$  on the  $m_0 \tan \beta$  (eV) –  $\alpha_2$  (radians) parameter space for the four scenarios. The regions above the contours are excluded by data for that particular scenario.

### B. $\theta_{13}$ to discriminate among scenarios

The four scenarios that we consider here give different predictions for the values of  $\theta_{13}^\Lambda$ :  $8.9^\circ$  (QLC1),  $1.6^\circ$  (QLC2),  $0^\circ$  (TBM1) and  $3.1^\circ$  (TBM2). In the absence of RG running, therefore, a discrimination between some of these scenarios should be possible in the near future. For example, if QLC1 is realised in nature, the value of  $\theta_{13}$  would be accessible to the current generation of experiments. However, the value of  $\theta_{13}$  changes with RG evolution and can either increase or decrease depending on the values of Majorana phases, as can be seen from (48). It is conceivable that the allowed ranges of  $\theta_{13}$  values for all the scenarios will then overlap and the power of discrimination will be lost. It is thus worthwhile to check whether one retains this discrimination capability in spite of the RG running. What helps in this is the fact that at higher values of  $m_0^\Lambda \tan \beta$ , where one expects large RG effects, not all  $\alpha_{2,3}^\Lambda$  values are allowed: the observed values of  $\theta_{12}$  (as shown in Sec. V A) as well as of  $\theta_{23}$  restrict the values of the Majorana phases, which in turn restrict the allowed values of  $\theta_{13}$ .

In Fig. 4, we show the permitted values of  $\theta_{13}$  in the four scenarios, subject to the constraints of the  $3\sigma$  allowed current ranges of the mixing angles. With the current constraints, it should be possible to distinguish between QLC1 and the other scenarios in the next round

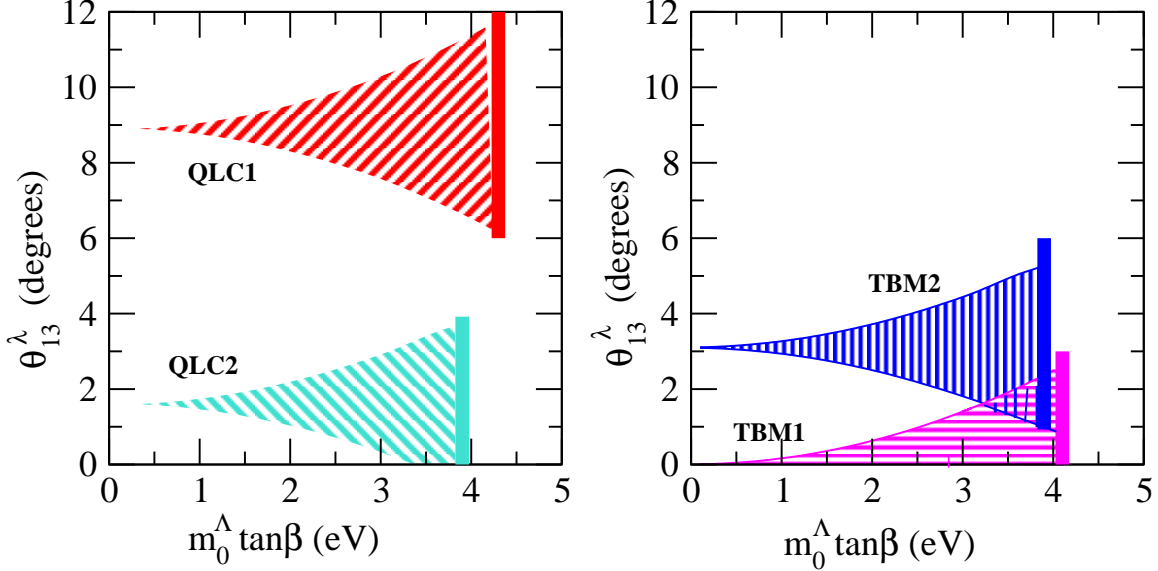


FIG. 4: Values of  $\theta_{13}$  in the QLC scenarios (left panel) and the TBM scenarios (right panel) as a function of  $m_0 \tan \beta$ , allowed with the current constraints on the mixing angles and mass squared differences. The shaded region represents the allowed values of  $\theta_{13}$  at the laboratory scale in each scenario. The thick bands represent upper limits on  $m_0^\Lambda \tan \beta$  from current measurements.

of experiments [39] probing  $\theta_{13}$ . For example, if  $\theta_{13}$  is shown to be less than  $6^\circ$ , QLC1 will be excluded.

The scenario TBM2 may be distinguishable from the remaining two cases if  $m_0^\Lambda \tan \beta \lesssim 2$  eV (i.e.  $m_0 \tan \beta \lesssim 1.4$  eV). So this needs, in addition to the information on  $\theta_{13}$ , stronger constraints on the values of  $m_0$  or  $\tan \beta$ . A more accurate measurement [40] of  $\theta_{12}$  will also indirectly limit the extent of RG evolution and may enable one to distinguish TBM2 from the other two scenarios, viz. QLC2 and TBM1. The last two scenarios have very similar predictions on  $\theta_{13}$ . This is on account of the close values of  $\theta_{13}^\Lambda$  in the two scenarios, and the nearly identical evolution  $\Delta\theta_{13} \equiv \theta_{13} - \theta_{13}^\Lambda$  in them, whatever the values of  $\alpha_2^\Lambda, \alpha_3^\Lambda, m_0^\Lambda$  and  $\tan \beta$ . From (48), the value of  $\Delta\theta_{13}$  differs in all the scenarios only through a factor of  $\sin 2\theta_{12}^\Lambda \sin 2\theta_{23}^\Lambda$  [see eq. (48)], which is equal to unity within 5% for all the four scenarios. Therefore, distinguishing between these two scenarios on the basis of a measurement of  $\theta_{13}$  alone will be difficult unless the values of the other parameters –  $m_0^\Lambda, \tan \beta, \alpha_2^\Lambda, \alpha_3^\Lambda$  – are also known to a good accuracy.

Though the constraints on  $m_0^\Lambda \tan \beta$  and  $\alpha_2^\Lambda$  in sec. V A are logarithmically sensitive to the choice of the high scale  $\Lambda$ , the limits on  $\theta_{13}$  in a given scenario are almost independent of

$\Lambda$ . This is because the maximum allowed value of  $|\Delta\theta_{13}|$  is dictated mainly by the maximum allowed value of  $\Delta\theta_{12}$  through  $|\Delta\theta_{13}|/\Delta\theta_{12} \approx |k_{13}|/k_{12}$ , whereas the maximum allowed value of  $\Delta\theta_{12}$  in turn is an experimentally determined quantity, quite independent of  $\Lambda$ .

### C. Effect of RG evolution on $\theta_{23}$

The predictions of all the scenarios for  $\theta_{23}$  are almost identical. The values of  $\theta_{23}^\Lambda$  are very close:  $42.1^\circ$  (QLC1),  $43.4^\circ$  (QLC2),  $45^\circ$  (TBM1) and  $42.7^\circ$  (TBM2). Moreover, the deviations  $\Delta\theta_{23} \equiv \theta_{23} - \theta_{23}^\Lambda$  are almost independent of the scenario, but depend on the value of  $\alpha_3^\Lambda$ . This may be shown as follows: at large values of  $m_0^\Lambda$  that are needed to have significant RG running, at the high scale  $|\rho_A^\Lambda| \ll 1$ , so that  $\Gamma^\Lambda \approx 1/\rho_A^\Lambda$ , and the value of  $\alpha_2^\Lambda$  is restricted to be very close to  $\pi$ . As a result, (48) gives

$$\Delta\theta_{23} \approx \frac{\Delta_\tau}{\rho_A^\Lambda} \sin 2\theta_{23}^\Lambda (1 - \cos 2\theta_{12}^\Lambda \cos \alpha_3^\Lambda) \quad (56)$$

which is almost independent of the scenario on account of similar values of  $\theta_{23}^\Lambda$  and  $\theta_{12}^\Lambda$ , but may vary by  $\approx \cos 2\theta_{12}^\Lambda \approx 30\%$  depending on the value of  $\alpha_3^\Lambda$ . As a consequence, a measurement of  $\theta_{23}$  cannot discriminate between the four scenarios unless it is accurate to the level of a degree. However, if a scenario has already been identified, the measured value of  $\theta_{23}$  will restrict the allowed values of the Majorana phase  $\alpha_3^\Lambda$ .

## VI. DISCUSSION AND SUMMARY

The current data on neutrino masses and mixings angles are consistent with symmetry-based schemes like quark-lepton complementarity (QLC) or tribimaximal mixing (TBM). These scenarios predict specific values of the neutrino mixing angles, which need to be compared with their forthcoming more accurately measured values in order to confirm or exclude a particular postulated symmetry pattern. However, the symmetry relations need to be imposed at a high scale, e.g. the seesaw scale  $\sim 10^{12}$  GeV, where the neutrino masses originate. Radiative corrections to the neutrino masses in general do not respect the symmetries involved in QLC or TBM. As a result, predictions of the neutrino parameters measured at laboratory energies become different from those given by these symmetries at the high scale. It is therefore necessary to obtain low scale predictions of these scenarios.

We have calculated radiative corrections to the neutrino mixing angles  $\theta_{ij}$  in the context of the minimal supersymmetric standard model (MSSM). We have taken the low scale to be the supersymmetry breaking scale  $\sim 10^3$  GeV and have neglected threshold effects, if any, during this renormalisation group (RG) evolution. We have presented a technique to calculate the deviations  $\Delta\theta_{ij}$  of the mixing angles from their high scale values. This technique yields analytically transparent results where the errors, caused by the approximations made, are small and under control. The analytic treatment clarifies the dependence of the RG running of neutrino mixing angles on currently unknown parameters like the mass scale  $m_0$  of neutrinos, the value of  $\tan\beta$  in MSSM and the values of the Majorana phases. We have also solved the RG equations numerically to confirm that the results are indeed closely approximated by our analytical expressions.

We have pointed out certain important patterns in the RG evolution of the mixing angles that are valid in any scenario. The RG running of  $\theta_{12}$  always increases its value from the high to the low scale. Therefore, if a scenario predicts the value of  $\theta_{12}^\Lambda$  at the high scale  $\Lambda$ , the low scale measurement must be  $\theta_{12} > \theta_{12}^\Lambda$  in order for the scenario to stay valid. Similarly, the value of  $\theta_{23}$  increases (decreases), while running from a high to a low scale, for a normal (inverted) neutrino mass ordering. The value of  $\theta_{13}$  is controlled not only by mass ordering, but also by the values of the Majorana phases, depending on which it may increase or decrease with RG running.

We have considered two versions of the QLC principle (QLC1 and QLC2) and two versions of the TBM scheme (TBM1 and TBM2), whose predictions at the high scale are consistent with the measured neutrino mixing angles at laboratory energies. The PMNS mixing matrices at the high scale predicted within these scenarios are  $V_{\text{CKM}}^\dagger U_{\nu,\text{bm}}$  (QLC1),  $U_{\nu,\text{bm}} V_{\text{CKM}}^\dagger$  (QLC2),  $U_{\nu,\text{tbm}}$  (TBM1) and  $V_{\ell L}^\dagger U_{\nu,\text{tbm}}$  (TBM2) respectively, where  $V_{\text{CKM}}$  is the CKM matrix,  $U_{\nu,\text{bm}}$  the bimaximal mixing matrix,  $U_{\nu,\text{tbm}}$  the tribimaximal [12] mixing matrix, and  $V_{\ell L}$  a charged lepton mixing matrix inspired by the Georgi-Jarlskog relation [36] at the GUT scale.

We summarise our findings in three items:

(i) The RG running of the mixing angles should not be too large lest the low energy values differ too much from their high energy predictions. Since  $\theta_{12}$  is the most accurately measured angle currently, that puts strong constraints on the allowed values of  $m_0^\Lambda \tan\beta$  as well as on the Majorana phase  $\alpha_2^\Lambda$ . It is observed that, for  $m_0^\Lambda \tan\beta \gtrsim 2$  eV (i.e.  $m_0 \tan\beta \gtrsim 1.4$



eV), the allowed range of  $\alpha_2^\Lambda$  is severely restricted. In all the scenarios,  $m_0^\Lambda \tan \beta$  has to be  $\lesssim 4.4$  eV (i.e.  $m_0 \tan \beta$  has to be  $\lesssim 3.1$  eV) for consistency with data. Moreover, the larger the value of  $m_0 \tan \beta$ , the closer to  $\pi$  has to be the value of  $\alpha_2^\Lambda$ , and hence that of  $\alpha_2$  at the laboratory scale, cf. eq. (55). Thus we find a preference for the approximate equality  $m_1 \simeq -m_2$  (especially for large  $\tan \beta$ ), as also suggested by considerations of leptogenesis [21]. A reduction of errors on the  $\theta_{12}$  measurement would decrease the allowed area in the  $m_0^\Lambda \tan \beta - \alpha_2^\Lambda$  plane. Moreover, since  $\theta_{12}$  always increases from higher to lower scales, if its value is measured to be smaller than what is predicted at the high scale in a scenario, that particular scenario would get excluded. *All the scenarios considered in this paper would be excluded if  $\theta_{12}$  were measured to be less than  $32^\circ$ .*

(ii) The measurement of  $\theta_{13}$  is most likely to serve as a discriminator among the four scenarios considered here. The predicted values of  $\theta_{13}$  in these scenarios at the high scale are  $8.9^\circ$  (QLC1),  $1.6^\circ$  (QLC2),  $0^\circ$  (TBM1) and  $3.1^\circ$  (TBM2). RG running can modify the value of  $\theta_{13}$  in either direction; however, the restrictions on  $m_0^\Lambda \tan \beta$  and  $\alpha_2^\Lambda$  from the  $\theta_{12}$  measurements limit the extent of this modification. We find, for example, that the value of  $\theta_{13}$  in QLC1 cannot be less than  $6^\circ$  ( $3\sigma$ ), whereas in none of the other cases can  $\theta_{13}$  be as large as  $6^\circ$  within  $3\sigma$ . Neutrino experiments during the next decade should be able to measure the value of  $\theta_{13}$  if it is greater than  $\approx 5^\circ$  or to put an upper bound of  $\approx 5^\circ$  on it. In either case, the scenario QLC1 will be distinguishable from the others. Both QLC2 and TBM1 predict almost identical  $\theta_{13}$  ranges:  $\theta_{13} < 3^\circ$  ( $3\sigma$ ). The allowed  $3\sigma$  range of  $\theta_{13}$  for TBM2 overlaps with the QLC2/TBM1 range for  $m_0^\Lambda \tan \beta \gtrsim 2$  eV (i.e.  $m_0 \tan \beta \gtrsim 1.4$  eV). Limiting  $m_0 \tan \beta$  to lesser values would also help in discriminating between TBM2 on one hand, and QLC1/TBM2 on the other.

(iii) It is not possible for a  $\theta_{23}$  measurement to discriminate among the four scenarios unless it is accurate to the level of a degree. However the value of  $\theta_{23}$  within any scenario is strongly dependent on the Majorana phase  $\alpha_3^\Lambda$ . Therefore, if a scenario has already been identified, the measured value of  $\theta_{23}$  will restrict the allowed values of  $\alpha_3^\Lambda$ .

In conclusion, we have shown how the high scale predictions on neutrino mixing angles get modified with RG running in MSSM for four symmetry-inspired scenarios that are consistent with the current neutrino data. With a combination of analytical insights and numerical calculations, we show that this limits the allowed ranges of parameters like  $m_0^\Lambda$ ,  $\tan \beta$  and the Majorana phases. We also indicate the extent to which future measurements

can discriminate among various scenarios and how the values of the parameters may be further restricted.

### Acknowledgements

A.D. and S.G. would like to thank W. Rodejohann for useful discussions. P.R. acknowledges the hospitality of the University of Hawaii at Manoa where part of this work was carried out. The work of A.D. is partly supported through the Partner Group program between the Max Planck Institute for Physics and Tata Institute of Fundamental Research. Part of the computational work for this study were carried out at cluster computing facility in the Harish-Chandra Research Institute (<http://www.cluster.mri.ernet.in>).

- 
- [1] R. N. Mohapatra *et al.*, arXiv:hep-ph/0510213; R. N. Mohapatra and A. Yu. Smirnov, *Ann. Rev. Nucl. Part. Sci.* **56**, 569 (2006).
  - [2] J. Schechter and J. W. F. Valle, *Phys. Rev. D* **22**, 2227 (1980).
  - [3] S. Antusch and S. F. King, *Nucl. Phys. B* **705**, 239 (2005); their Refs. [13-22]; G. C. Branco, M. N. Rebelo and J. I. Silva-Marcos, *Phys. Rev. Lett.* **82**, 683 (1999); E. J. Chun and K. Turzyski, arXiv:hep-ph/0703070.
  - [4] V. Barger, S. L. Glashow, P. Langacker and D. Marfatia, *Phys. Lett. B* **540**, 247 (2002); S. Pascoli, S. T. Petcov and W. Rodejohann, *Phys. Lett. B* **549**, 177 (2002); K. Matsuda, T. Fukuyama and H. Nishiura, *Mod. Phys. Lett. A* **18**, 1803 (2003); S. Pascoli, S. T. Petcov and T. Schwetz, *Nucl. Phys. B* **734**, 24 (2006).
  - [5] S. Blanchet and P. Di Bari, *JCAP* **0703**, 018 (2007).
  - [6] A. Dighe, S. Goswami and P. Roy, *Phys. Rev. D* **73**, 071301(R) (2006).
  - [7] M. Jezabek and Y. Sumino, *Phys. Lett. B* **457**, 139 (1999); C. Giunti and M. Tanimoto, *Phys. Rev. D* **66**, 053013 (2002); *Phys. Rev. D* **66**, 113006 (2002); P. H. Frampton, S. T. Petcov and W. Rodejohann, *Nucl. Phys. B* **687**, 31 (2004).
  - [8] M. Raidal, *Phys. Rev. Lett.* **93**, 161801 (2004).
  - [9] H. Minakata and A. Yu. Smirnov, *Phys. Rev. D* **70**, 073009 (2004); A. Yu. Smirnov, *J. Phys. Conf. Ser.* **39**, 232 (2006).

- [10] P. H. Frampton and R. N. Mohapatra, JHEP **0501**, 025 (2005); S. Antusch, S. F. King and R. N. Mohapatra, Phys. Lett. B **618**, 150 (2005); S. K. Kang, C. S. Kim and J. Lee, Phys. Lett. B **619**, 129 (2005); K. Cheung, S. K. Kang, C. S. Kim and J. Lee, Phys. Rev. D **72**, 036003 (2005); Z.-Z. Xing, Phys. Lett. B **618**, 141 (2005); A. Datta, L. Everett and P. Ramond, Phys. Lett. B **620**, 42 (2005); L. L. Everett, Phys. Rev. D **73**, 013011 (2006); A. Ghosal and D. Majumdar, Mod. Phys. Lett. A **21**, 1067 (2006); A. S. Joshipura and A. Yu. Smirnov, Nucl. Phys. B **750**, 28 (2006).
- [11] J. Ferrandis and S. Pakvasa, Phys. Rev. D **71**, 033004 (2005).
- [12] P. F. Harrison, D. H. Perkins and W. G. Scott, Phys. Lett. B **458**, 79 (1999); P. F. Harrison, D. H. Perkins and W. G. Scott, Phys. Lett. B **530**, 167 (2002).
- [13] F. Vissani, arXiv:hep-ph/9708483; V. D. Barger, S. Pakvasa, T. J. Weiler and K. Whisnant, Phys. Lett. B **437**, 107 (1998); A. J. Baltz, A. S. Goldhaber and M. Goldhaber, Phys. Rev. Lett. **81**, 5730 (1998); G. Altarelli and F. Feruglio, Phys. Lett. B **439**, 112 (1998); M. Jezabek and Y. Sumino, Phys. Lett. B **440**, 327 (1998); D. V. Ahluwalia, Mod. Phys. Lett. A **13**, 2249 (1998).
- [14] C. S. Lam, Phys. Lett. B **507**, 214 (2001); P. F. Harrison and W. G. Scott, Phys. Lett. B **547**, 219 (2002).
- [15] S. Choubey and W. Rodejohann, Eur. Phys. J. C **40**, 259 (2005); T. Ota and W. Rodejohann, Phys. Lett. B **639**, 322 (2006).
- [16] R. N. Mohapatra and S. Nussinov, Phys. Lett. B **441**, 299 (1998).
- [17] E. Ma and G. Rajasekaran, Phys. Rev. D **64**, 113012 (2001); E. Ma, Mod. Phys. Lett. A **20**, 2601 (2005); K. S. Babu, E. Ma and J. W. F. Valle, Phys. Lett. B **552**, 207 (2003); G. Altarelli and F. Feruglio, Nucl. Phys. B **741**, 215 (2006); R. R. Volkas, arXiv:hep-ph/0612296.
- [18] P. F. Harrison and W. G. Scott, Phys. Lett. B **557**, 76 (2003); W. Grimus and L. Lavoura, JHEP **0508**, 013 (2005); N. Haba, A. Watanabe and K. Yoshioka, Phys. Rev. Lett. **97**, 041601 (2006); R. N. Mohapatra, S. Nasri and H. B. Yu, Phys. Lett. B **639**, 318 (2006).
- [19] P. Minkowski, Phys. Lett. B **67**, 421 (1977); T. Yanagida, *Proceedings of the Workshop on the Unified Theory and the Baryon Number in the Universe* (O. Sawada and A. Sugamoto, eds.), KEK, Tsukuba, Japan, 1979, pp 95; M. Gell-Mann, P. Ramond, and R. Slansky, *Supergravity* (P. van Nieuwenhuizen and D. Z. Freedman, eds.), North Holland, Amsterdam, 1979, pp 315; S. L. Glashow, *Proceedings of the 1979 Cargèse Summer Institute on Quarks and Leptons*

- (M. Lévy, J.-L. Basdevant, D. Speiser, J. Weyers, R. Gastmans, and M. Jacob, eds.), Plenum Press, New York, 1980, pp. 687; R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980).
- [20] R. N. Mohapatra, N. Setzer and S. Spinner, Phys. Rev. D **73**, 075001 (2006).
- [21] W. Buchmuller, P. Di Bari and M. Plumacher, Nucl. Phys. B **665**, 445 (2003).
- [22] K. S. Babu, C. N. Leung and J. T. Pantaleone, Phys. Lett. B **319**, 191 (1993); M. Tanimoto, Phys. Lett. B **360**, 41 (1995); J. A. Casas, J. R. Espinosa, A. Ibarra and I. Navarro, Nucl. Phys. B **569**, 82 (2000); K. R. S. Balaji, A. S. Dighe, R. N. Mohapatra and M. K. Parida, Phys. Rev. Lett. **84**, 5034 (2000); Phys. Lett. B **481**, 33 (2000); N. Haba, Y. Matsui and N. Okamura, Eur. Phys. J. C **17**, 513 (2000); S. Antusch, M. Drees, J. Kersten, M. Lindner and M. Ratz, Phys. Lett. B **519**, 238 (2001); Phys. Lett. B **525**, 130 (2002); T. Fukuyama and N. Okada, JHEP **0211**, 011 (2002); G. Bhattacharyya, A. Raychaudhuri and A. Sil, Phys. Rev. D **67**, 073004 (2003); A. S. Joshipura, S. D. Rindani and N. N. Singh, Nucl. Phys. B **660**, 362 (2003); R. N. Mohapatra, M. K. Parida and G. Rajasekaran, Phys. Rev. D **69**, 053007 (2004).
- [23] J. A. Casas, J. R. Espinosa, A. Ibarra and I. Navarro, Nucl. Phys. B **573**, 652 (2000).
- [24] S. Antusch, J. Kersten, M. Lindner and M. Ratz, Nucl. Phys. B **674**, 401 (2003).
- [25] M. Drees, R. Godbole and P. Roy, “Theory and phenomenology of sparticles,” *Hackensack, USA: World Scientific (2004)*.
- [26] M. A. Schmidt and A. Yu. Smirnov, Phys. Rev. D **74**, 113003 (2006).
- [27] S. Luo and Z. Z. Xing, Phys. Lett. B **632**, 341 (2006).
- [28] Particle Data group, W.-M. Yao *et al.*, Journal of Physics G **33**, 1 (2006).
- [29] A. Strumia and F. Vissani, Nucl. Phys. B **726**, 294 (2005); B. Aharmim *et al.* [SNO Collaboration], Phys. Rev. C **72**, 055502 (2005); S. Goswami, Int. J. Mod. Phys. A **21**, 1901 (2006); G. L. Fogli, E. Lisi, A. Marrone and A. Palazzo, Prog. Part. Nucl. Phys. **57**, 742 (2006); T. Schwetz, Phys. Scripta **T127**, 1 (2006).
- [30] J. W. Mei and Z. Z. Xing, Phys. Rev. D **70**, 053002 (2004); R. N. Mohapatra, M. K. Parida and G. Rajasekaran, Phys. Rev. D **71**, 057301 (2005); J. R. Ellis, A. Hektor, M. Kadastik, K. Kannike and M. Raidal, Phys. Lett. B **631**, 32 (2005); S. K. Kang, C. S. Kim and J. Lee, Phys. Lett. B **619**, 129 (2005); S. T. Petcov, T. Shindou and Y. Takanishi, Nucl. Phys. B **738**, 219 (2006).

- [31] H. Arason, D. J. Castano, E. J. Piard and P. Ramond, Phys. Rev. D **47**, 232 (1993).
- [32] P. H. Chankowski and S. Pokorski, Int. J. Mod. Phys. A **17**, 575 (2002).
- [33] J. R. Ellis and S. Lola, Phys. Lett. B **458**, 310 (1999); P. H. Chankowski, W. Krolkowski and S. Pokorski, Phys. Lett. B **473**, 109 (2000).
- [34] S. F. King, JHEP **0508**, 105 (2005).
- [35] F. Plentinger and W. Rodejohann, Phys. Lett. B **625**, 264 (2005).
- [36] H. Georgi and C. Jarlskog, Phys. Lett. B **86**, 297 (1979).
- [37] R. Barbieri, J. R. Ellis and M. K. Gaillard, Phys. Lett. B **90**, 249 (1980); E. K. Akhmedov, Z. G. Berezhiani and G. Senjanovic, Phys. Rev. Lett. **69**, 3013 (1992); E. K. Akhmedov, Z. G. Berezhiani, G. Senjanovic and Z. J. Tao, Phys. Rev. D **47**, 3245 (1993); A. S. Joshipura, Phys. Rev. D **60**, 053002 (1999); A. de Gouvea and J. W. F. Valle, Phys. Lett. B **501**, 115 (2001); F. Vissani, M. Narayan and V. Berezinsky, Phys. Lett. B **571**, 209 (2003); B. S. Koranga, M. Narayan and S. U. Sankar, hep-ph/0611186.
- [38] A. Dighe, S. Goswami and W. Rodejohann, Phys. Rev. D **75**, 073023 (2007).
- [39] F. Ardellier *et al.* [Double Chooz Collaboration], arXiv:hep-ex/0606025; X. Guo *et al.* [Daya Bay Collaboration], arXiv:hep-ex/0701029.
- [40] A. Bandyopadhyay, S. Choubey and S. Goswami, Phys. Rev. D **67**, 113011 (2003); H. Minakata, H. Nunokawa, W. J. C. Teves and R. Zukanovich Funchal, Phys. Rev. D **71**, 013005 (2005); A. Bandyopadhyay, S. Choubey, S. Goswami and S. T. Petcov, Phys. Rev. D **72**, 033013 (2005).