Parity in left-right symmetric models

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Abstract

We considered parity breaking in some left-right symmetric models. We studied spontaneous and explicit parity violation in two cases with doublet and triplet Higgs scalars. The minimization condition in these two cases differ significantly. A comparative study of these models is presented emphasizing their phenomenological consequences.

1 Introduction

The $V - A$ structure [1] of the weak interaction and parity violation [2] suggested that the low energy gauge group should treat the left-handed particles preferentially. This leads to the standard model [3] gauge group $\mathcal{G}_{std} \equiv SU(3)_c \times SU(2)_L \times U(1)_Y$, in which all right-handed particles transform trivially under $SU(2)_L$. Although this explains the parity violation at low energy, this does not explain the origin of the parity violation. With an attempt to explain parity violation starting from a parity conserved theory, the standard model group was extended to a left-right symmetric group $\mathcal{G}_{LR} \equiv SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, where the group treats all fermions in a similar way [4]. The left-handed fermions transform trivially under $SU(2)_R$, while all right-handed fermions transform trivially under $SU(2)_L$. One may then define a left-right parity, which interchanges these two groups $SU(2)_L \Leftrightarrow SU(2)_R$. Since one interchanges all left-handed fermions with the right-handed fermions under parity, this becomes the parity of the Lorentz group, associated with space reflection.

It is then possible to break parity spontaneously or explicitly in left-right symmetric models. When the right-handed symmetry group $SU(2)_R$ is broken, parity breaks down. One may also break parity even before breaking the group $SU(2)_R$ by giving a vacuum expectation value to a Higgs scalar, which changes sign when the two $SU(2)$ groups are interchanged. These spontaneous breaking of parity is quite natural in grand unified theories. However, in some superstring inspired models it is also possible that the parity breaks down during compactification and one ends up with explicitly broken parity in left-right symmetric model.

Here we shall study the consequences of breaking parity in some left-right symmetric models with triplet and doublet Higgs scalars. Spontaneous parity breaking in models with triplet Higgs was studied extensively [5]. Here we discuss the models with doublet Higgs and explicit parity breaking. We study some aspects of these models after explaining the question of parity in some more details. The minimization conditions differs in these two cases with triplet and doublet Higgs scalars and imply different low energy phenomenology.

2 Parity of Lorentz group and left-right symmetric models

To define the parity, consider the local Lorentz group $SO(4)$ which is isomorphic to $SU(2) \times SU(2)$. There are inequivalent embeddings of the two $SU(2)$ in the $SO(4)$, in one of these embeddings the two $SU(2)$ groups act on the left-handed and right-handed fermions. Since the two $SU(2)$ groups are indistinguishable, there is a Z_2 symmetry, which interchanges the two $SU(2)$ groups. This is the usual discrete parity symmetry or the space-reflection symmetry $\mathcal{P}$.

A scalar [pseudo-scalar] field $\phi(x)$ is even [odd] under parity and transform under $\mathcal{P}$ as

$$\phi(x) \xrightarrow{\mathcal{P}} \pm\phi(-x).$$

Similarly, a vector [pseudo-vector] field is even [odd] under $\mathcal{P}$. However, for fermions we have to consider their chiral decomposition. A left-handed fermion $\psi_L = \frac{1}{2}(1-\gamma_5)\psi$ transforms under the Lorentz group $SU(2)_L \times SU(2)$ as [2,0], while a right-handed fermion $\psi_R = \frac{1}{2}(1+\gamma_5)\psi$ transforms as [0,2]. Under $\mathcal{P}$ parity they transform as

$$\psi_L \xrightarrow{\mathcal{P}} \psi_R \quad \text{and} \quad \psi_R \xrightarrow{\mathcal{P}} \psi_L. \tag{1}$$

Parity does not act on any internal group space. In the standard model, the left-handed fields are doublets under $SU(2)_L$, while all right-handed fields are singlets. So, $\mathcal{P}$ parity does not commute with the group $SU(2)_L$ and $\mathcal{P}$ is maximally broken.

In the left-right extension of the standard model, the gauge group is $\mathcal{G}_{LR} \equiv SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. Under $\mathcal{G}_{LR}$ the left-handed and the right-handed quarks and leptons transform as

$$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \equiv [3,2,1,1/3] \qquad q_R \begin{pmatrix} u_R \\ d_R \end{pmatrix} \equiv [3,1,2,1/3]$$
$$\ell_L \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \equiv [1,2,1,-1] \qquad \ell_R \begin{pmatrix} \nu_R \\ e_R \end{pmatrix} \equiv [1,1,2,-1].$$

The electric charge Q and the hypercharge Y are defined by

$$Q = T_{3L} + T_{3R} + \frac{B-L}{2} = T_{3L} + Y. \tag{2}$$

The $SU(2)_L$ and the $SU(2)_R$ groups are now related by a discrete Z_2 symmetry, called the $\mathcal{D}$ parity. Under $\mathcal{D}$ the gauge groups transform as

$$SU(2)_L \overset{\mathcal{D}}{\leftrightarrow} SU(2)_R$$

and the fermions transform as

$$\psi_L \overset{\mathcal{D}}{\leftrightarrow} \psi_R.$$

Thus the fermions cannot distinguish between the $\mathcal{D}$ and the $\mathcal{P}$ parity.

Let us now consider the gauge bosons. The $SU(2)_L$ gauge bosons W_L and the $SU(2)_R$ gauge bosons W_R are not parity eigenstates. The linear combinations

$$V = W_L + W_R \qquad \text{and} \qquad A = W_L - W_R$$

have definite parities:

$$V \overset{\mathcal{P}}{\leftrightarrow} V \qquad \text{and} \qquad A \overset{\mathcal{P}}{\leftrightarrow} -A.$$

This will imply that under parity,

$$W_L \leftrightarrow W_R$$

so that we can identify the left-right parity with the Lorentz parity. We shall now consider a couple of left-right symmetric models with triplet and doublet Higgs scalars and study the consequences of spontaneous and explicit parity breaking.

3 Left-right symmetry with triplet Higgs

In the left-right symmetric models, a bi-doublet Higgs scalar

$$\Phi \equiv [1, 2, 2, 0]$$

gives masses to all fermions and breaks the electroweak symmetry. This acquires a vacuum expectation value of 246 GeV. Then the neutrinos will also become as heavy as the other fermions, unless the Yukawa coupling for the neutrinos are too small which is unnatural. This problem is solved in these models, when one includes a new triplet Higgs scalar to break the left-right symmetry at a large scale M_R. In the left-right symmetric models the scale M_R is same as the scale of lepton number violation. So, one right-handed triplet Higgs with $B-L=2$ can break the $SU(2)_R \times U(1)_{B-L}$ symmetry and give Majorana masses to the right-handed neutrinos, which in turn gives a tiny see-saw mass to the left-handed neutrinos [6].

In the simplest left-right symmetric model, one assumes parity to be conserved before the left-right symmetry ($\mathcal{G}_{LR}$) breaking. The group $\mathcal{G}_{LR}$ is broken when the right-handed triplet Higgs scalar ($\Delta_R \equiv [1, 1, 3, -2]$) acquires a vacuum expectation value (*vev*) $\langle \Delta_R \rangle = u_R$. The *vev* of this right-handed triplet Higgs scalar breaks $B-L$ by 2 units and hence it can give a Majorana mass to the right-handed neutrinos. Parity then dictates that there should also be a left-handed triplet Higgs scalar $\Delta_L \equiv [1, 3, 1, -2]$. However, the *vev* of $\langle \Delta_L \rangle = u_L$ should be very small (less than a GeV) for the theory to be consistent with Z decay width. Since u_L can also give neutrino masses, it should be less than a few eV.

In general, the masses (μ_L and μ_R) of Δ_L and Δ_R could be different, breaking parity explicitly. However, when parity is conserved these masses are same ($\mu_L = \mu_R$). In models of spontaneous parity breaking, one introduces an additional singlet Higgs scalar ($\sigma \equiv [1, 1, 1, 0]$, with $\langle \sigma \rangle = \eta$), which is odd under $\mathcal{D}$ parity:

$$\sigma \xrightarrow{\mathcal{D}} -\sigma.$$

For a general analysis of the scalar potential, we shall include the scalar σ and also allow $\mu_L \neq \mu_R$. The parity conserving scenario, spontaneous parity breaking scenario and the explicit parity breaking scenario will then come out as special cases of this analysis.

The scalar potential will contain several terms including all these fields: the bi-doublet Φ, the right-handed triplet Δ_R, the left-handed triplet Δ_L and the odd $\mathcal{D}$ parity singlet σ. To study the minima of this potential we replace these fields by their *vev* without loss of generality. We write the potential in several components:

$$V(\kappa, u_L, u_R, \eta) = V_\kappa + V_L + V_R + V_\eta + V_{\kappa LR} + V_{\eta LR} + V_{\kappa\eta} + V_{LR} \qquad (3)$$

where $\langle\Phi\rangle = \begin{pmatrix} 0 & \kappa \\ \kappa' & 0 \end{pmatrix}$ with $\kappa \gg \kappa'$. The first few terms contain the usual quadratic and quartic couplings

$$V_a = -\frac{1}{2}\mu_a^2\phi_a^2 + \frac{1}{4}\lambda_a\phi_a^4 \qquad (4)$$

where $a = \kappa, L, R, \eta$ and $\phi_a = \kappa, u_L, u_R, \eta$, respectively. The cross terms are:

$$\begin{aligned} V_{\kappa LR} &= \frac{1}{2}\left(g_L u_L^2 + g_R u_R^2 + 2g_{LR}u_L u_R\right)\kappa^2 \\ V_{\eta LR} &= \frac{1}{2}M\eta(u_L^2 - u_R^2) + \frac{1}{2}\lambda_2\eta^2(u_L^2 + u_R^2) \\ V_{\kappa\eta} &= \bar{M}\eta\kappa^2 + \frac{1}{2}\lambda_1\eta^2\kappa^2 \\ V_{LR} &= \frac{1}{2}g u_L^2 u_R^2 \end{aligned} \qquad (5)$$

The term with coefficient g_{LR} plays a very significant role, as we shall discuss later.

$SU(2)_R$ and parity

We shall first consider the most popular model in which parity is broken when the group $SU(2)_R$ is broken. In this case, $\eta = 0$ signifies there is no spontaneous symmetry breaking. Conservation of the $\mathcal{D}$ parity will constrain some of the parameters:

$$\mu_L = \mu_R = \mu_\Delta, \qquad g_L = g_R = g' \qquad \lambda_L = \lambda_R = \lambda_\Delta. \tag{6}$$

Minimization of the potential then gives a condition:

$$u_L \frac{\partial V}{\partial u_R} - u_R \frac{\partial V}{\partial u_L} = 0 = (u_L^2 - u_R^2)[g_{LR}\kappa^2 + (g - \lambda_\Delta) u_L u_R]. \tag{7}$$

One of the solution $u_L = u_R$ preserves parity and left-right symmetry is not broken, which is not acceptable phenomenologically. The other solution breaks parity and determines the *vev* of the left-handed triplet Higgs scalar Δ_L to be,

$$u_L = \frac{g_{LR}\kappa^2}{(\lambda_\Delta - g) u_R}. \tag{8}$$

The smallness of the *vev* of the left-handed triplet Higgs scalar is thus guaranteed in the limit of large u_R *i.e.*, a large lepton number violating scale. The term with coefficient g_{LR} is very crucial for this condition. After Δ_R acquires a *vev*, this term allows an interaction of the left-handed triplet Higgs with two bi-doublets. This term and the Yukawa coupling of the triplet Higgs with the leptons together then breaks lepton number. Thus the see-saw suppressed *vev* of the triplet Higgs and the lepton number violation is intimately related [7].

Although the *vev* of the left-handed triplet Higgs scalar is tiny, the masses of both the triplet Higgs scalars are very large. For a neutrino mass of less than eV (as required by present experiments), the scale of lepton number violation is required to be greater than 10^7 GeV. Thus these Higgs scalars will not be accessible to the next generation accelerators.

Spontaneous parity violation

For spontaneous parity violation [5], we start with a parity conserving model, so that the constraints on the couplings, given by equation 6 is satisfied. We then consider $\eta \neq 0$. The minimization of the potential then gives the condition:

$$u_L \frac{\partial V}{\partial u_R} - u_R \frac{\partial V}{\partial u_L} = 0 = 2M\eta u_L u_R + (u_L^2 - u_R^2)[g_{LR}\kappa^2 + (g - \lambda_\Delta) u_L u_R]. \quad (9)$$

In this case, $g_L = g_R$ is no longer a solution of the minimization condition, except for $\eta = 0$.

All the solutions of this minimization condition breaks parity. For simplification we consider the solution of our interest, *i.e.*, $u_L \ll u_R$. This leads to a *vev* of the left-handed triplet Higgs scalar

$$u_L = \frac{g_{LR} u_R \kappa^2}{(\lambda_\Delta - g) u_R^2 - 2M\eta}. \quad (10)$$

The scale of parity violation η now determines the *vev* of the triplet Higgs. Thus, the left-right symmetry breaking scale can be much lower. The mass of the triplet Higgs can also be small, which can then be detected in the next generation accelerators.

Explicit parity violation

Since parity is violated explicitly, we do not require the scalar σ. However, if we include this field in our analysis, this will not change anything. For comparison with the spontaneous parity breaking scenario, we shall retain both these contributions. The minimization condition now reads

$$\begin{aligned} u_L \frac{\partial V}{\partial u_R} - u_R \frac{\partial V}{\partial u_L} = 0 &= u_L u_R[(\mu_L^2 - \mu_R^2) + 2M\eta + (g_L - g_R)\kappa^2 \\ &+ (\lambda_L u_L^2 - \lambda_R u_R^2)] + (u_L^2 - u_R^2)[g_{LR}\kappa^2 + g u_L u_R]. \end{aligned} \quad (11)$$

The explicit breaking contribution $\mu_L^2 - \mu_R^2$ can now substitute for the spontaneous symmetry breaking term $2M\eta$. In this scenario also, there is no solution with $g_L = g_R$. The parity violating solution correspond to

$$u_L = \frac{g_{LR} u_R \kappa^2}{(\mu_L^2 - \mu_R^2) + 2M\eta - \lambda_R u_R^2 + g u_R^2}. \tag{12}$$

In this expression we neglected κ^2 and u_L^2 terms compared to u_R^2 and η^2, assuming there is no fine tuning.

There is no distinction between the spontaneously broken parity and explicit breaking. In grand unified theories spontaneously broken $\mathcal{D}$ parity is quite natural, while in superstring inspired models the $\mathcal{D}$ parity can be broken by Wilson loops at the time of compactification. In these theories explicit breaking of parity appear to be more natural. A light triplet Higgs with rich phenomenology is allowed even with explicit parity violation.

4 Left-right symmetry with doublet Higgs

The left-right symmetry may also be broken by a doublet Higgs scalar [8]. For charged fermion masses one may retain a bi-doublet scalar Φ, which also breaks the electroweak symmetry. Although it is possible to construct models without any bi-doublet, here we restrict ourselves to models including the bi-doublet. The $SU(2)_R \times U(1)_{B-L}$ group is broken by a right-handed Higgs doublet $\chi_R \equiv [1, 1, 2, +1]$, which acquires a *vev* $\langle \chi_R \rangle = v_R$. Since the *vev* of this doublet breaks lepton number by 1 unit, it cannot contribute to the neutrino masses. However, including a singlet heavy fermion this problem can be solved, which we shall discuss later.

The left-right parity then ensures that there is another left-handed doublet Higgs scalar field $\chi_L \equiv [1, 2, 1, +1]$, which acquires a small *vev* $\langle \chi_L \rangle = v_L$. Unlike the left-handed triplet Higgs *vev* u_L, there is no restriction on the *vev* of the doublet Higgs v_L except that $v_L < \kappa$, since it breaks the group $SU(2)_L$.

The scalars $\chi_{L,R}$ break lepton number by one unit, so they cannot give masses to the neutrinos. Let us now consider the effective dimension-5 terms, which can contribute to the neutrino masses,

$$\mathcal{L}_\nu = \frac{h_L}{M_L}\ell_L \ell_L \chi_L \chi_L + \frac{h_R}{M_L}\ell_R \ell_R \chi_R \chi_R. \tag{13}$$

M_L is the lepton number violating scale. Since $B-L$ is broken at a scale v_R, we expect $M_L \sim v_R$. There are no terms involving the bi-doublet Φ, since it does not carry any $B-L$ quantum number.

For a realization of this effective operator, we introduce a heavy singlet fermion $S \equiv [1,1,1,0]$. This field S will interact with the neutrinos and the Higgs scalars and will affect the neutrino masses. We write down the terms which contribute to the neutrino masses

$$\mathcal{L}_S = f_{iL}\bar{\ell}_{iL} S \chi_L + f_{aR}\bar{\ell}_{aR} S \chi_R + M_S S S + f_{ia}\bar{\ell}_L \ell_R \Phi, \tag{14}$$

where $i = 1,2,3$ represents the three generations of left-handed leptons and $a = 1,2,3$ represents the three generations of right-handed leptons. When the scalars acquire *vev*s, they contribute to the neutrino mass matrix. We write down the neutrino mass matrix in the basis $\begin{pmatrix} \nu_{iL} & \nu_{aR} & S \end{pmatrix}$

$$M_\nu = \begin{pmatrix} 0 & f_{ia}\kappa & f_{iL}v_L \\ f_{ia}\kappa & 0 & f_{aR}v_R \\ f_{iL}v_L & f_{aR}v_R & M_S \end{pmatrix}. \tag{15}$$

The singlet S is the heaviest fermion with mass M_S. The right-handed neutrinos get an effective Majorana mass of v_R^2/M_S. The left-handed neutrinos get two contributions to their mass, a see-saw contribution from the left-handed doublets and a double-see-saw contribution from the right-handed doublets. They are,

$$m_{\nu ij} = -\frac{f_{iL}f_{jL}v_L^2}{M_S} + \frac{f_{ia}f_{jb}}{f_{aR}f_{bR}}\frac{M_S\kappa^2}{v_R^2}. \tag{16}$$

In the triplet Higgs scenario u_L is too small and hence the first term could be negligible. But as we shall see later, v_L could be comparable to κ and hence could dominate over the second term.

To break the parity we introduce a field $\sigma \equiv [1,1,1,0]$ (with $\langle\sigma\rangle = \eta$), which is odd under $\mathcal{D}$. The scalar potential contains the usual quadratic and quartic terms and in addition there are some cross terms given by,

$$\begin{aligned} V_\chi &= \frac{1}{2}\left(g_L v_L^2 + g_R v_R^2\right)\kappa^2 + \mu v_L v_R \kappa + \frac{1}{2} g v_L^2 v_R^2 \\ &\quad + \frac{1}{2}\eta[\bar{M}\kappa^2 + M(v_L^2 - v_R^2)] + \frac{1}{2}\eta^2[\lambda_1\kappa^2 + \lambda_2(v_L^2 + v_R^2)] \end{aligned} \tag{17}$$

This potential is somewhat similar to the triplet Higgs scenario, but still there are some important differences.

$SU(2)_R$ and parity

Conservation of parity would require $\eta = 0$ and

$$\mu_L = \mu_R = \mu_\chi, \qquad g_L = g_R = g' \qquad \lambda_L = \lambda_R = \lambda_\chi. \tag{18}$$

With these conditions we minimize the potential to obtain a condition relating the different *vev*s given by

$$v_L \frac{\partial V}{\partial v_R} - v_R \frac{\partial V}{\partial v_L} = 0 = (v_L^2 - v_R^2)[\mu\kappa + (g - \lambda_\chi) v_L v_R]. \tag{19}$$

One of the solutions $v_L = v_R$ conserves left-right symmetry even after symmetry breaking, which is of no interest. The other solution determines the *vev* of the left-handed triplet Higgs

$$v_L = \frac{\mu\kappa}{(\lambda_\Delta - g) v_R}. \tag{20}$$

A natural assumption for the mass scales is $\mu \sim v_R$ and hence $v_L \lesssim \kappa$, since κ breaks the electroweak symmetry and give masses to the fermions. The masses of both the doublets $\chi_{L,R}$ are large $\mu_\chi \sim v_R$. Thus this scenario does not lead to any interesting phenomenology.

Spontaneous parity violation

Here we assume the $\mathcal{D}$ parity conserving constraints on the coupling constants as mentioned above, but $\eta \sim v_R$. This would then give us the minimization condition

$$v_L \frac{\partial V}{\partial v_R} - v_R \frac{\partial V}{\partial v_L} = 0 = 2M\eta v_L v_R + (v_L^2 - v_R^2)[\mu\kappa + (g - \lambda_\chi) v_L v_R]. \quad (21)$$

Similar to the triplet Higgs models, $g_L = g_R$ is no longer a solution of the minimization condition. Assuming, $v_L \ll v_R$ we obtain the *vev* of the doublet Higgs to be

$$v_L = \frac{\mu v_R \kappa}{(\lambda_\chi - g) v_R^2 - 2M\eta}. \quad (22)$$

The *vev* of the doublet Higgs can again be of the order of electroweak symmetry breaking scale. In this case the mass of the left-handed doublet Higgs scalar can also be as small as the electroweak symmetry breaking scale. This will make this scenario phenomenologically interesting.

Explicit parity violation

We take both $\eta \neq 0$ and $\mu_L \neq \mu_R$. For simplicity, we shall impose $\mathcal{D}$ parity on other couplings, although relaxing this condition will not change any of the conclusions. The minimization condition now reads

$$v_L \frac{\partial V}{\partial v_R} - v_R \frac{\partial V}{\partial v_L} = 0 = v_L v_R [(\mu_L^2 - \mu_R^2) + 2M\eta] + (v_L^2 - v_R^2)[\mu\kappa + (\lambda_\chi - g) v_L v_R]. \quad (23)$$

It is apparent that the explicit parity breaking contribution $\mu_L^2 - \mu_R^2$ is similar to the spontaneous parity breaking contribution $2M\eta$. The *vev* of the left-handed doublet is now given by

$$v_L = \frac{-\mu v_R \kappa}{(\mu_L^2 - \mu_R^2) + 2M\eta + (g - \lambda_\chi) v_R^2}. \quad (24)$$

This result is similar to the spontaneous parity breaking scenario, except that now we can decouple the left-handed and right-handed doublets by considering $\mu = 0$. The $SU(2)_R$ symmetry breaking can take place with a Higgs doublet χ_R at a very high scale. Since parity is explicitly broken, the left-handed doublet χ_L can now be light and can have a *vev* $v_L \sim \mu_L$. No fine tuning is required to maintain this solution. The light Higgs doublet will then contribute to the low energy phenomenology.

5 Summary

We discussed the question of parity violation in left-right symmetric model. The left-right parity, acting in the left-right gauge group space, can be identified with the usual Lorentz parity or the space reflection symmetry. Since the left-right parity is identified as the space reflection parity, one may construct left-right symmetric model in which parity is conserved initially and broken along with $SU(2)_R$ symmetry breaking. Parity could also be spontaneously broken at a different scale compared to the left-right symmetry breaking. We also studied the explicit parity breaking in a couple of left-right symmetric models. Depending on whether the Higgs field required for the $SU(2)_R$ breaking is a triplet or a doublet, the minimization of the potential and phenomenological consequences are different.

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