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On the Conservation of Electric Charge Around a Monopole of Finite Size

YOICHI KAZAMA†
Department of Physics, Kyoto University, Kyoto 606, Japan

and

ASHOKE SEN
Fermi National Accelerator Laboratory
Batavia, Illinois 60510

† Address after Sept. 16, 1983: National Laboratory for High Energy Physics (KEK), Tsukuba, Ibaraki, 305 Japan.

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ABSTRACT

In monopole-fermion dynamics, the boundary condition, which is responsible for baryon number non-conservation, also violates electric and color hyper-charge conservation. We show, by detailed calculations, that actually the latter conservation laws are dynamically restored. It is shown that for a finite size monopole, there is a small but finite amplitude for the monopole ground state to make a virtual transition into a state containing a dyon and some fermions carrying equal and opposite charge as that of the dyon. But the amplitude for this state to make a virtual transition to a state carrying a net total charge is identically zero. The monopole ground state, as a result, is an eigenstate of electric charge even in the presence of massless fermions. We also calculate the four body charge and chirality conserving but baryon number violating condensates, which exist independently of the existence of anomaly and hence persist even in the presence of more generations of massless fermions.

I. INTRODUCTION

It was proposed by Rubakov [1], and subsequently by Callan [2], that monopoles of the 't Hooft-Polyakov type [3] in grand unified theories may catalyze baryon number violation at the strong interaction rate. Since then a number of investigations have been made to try to clarify the origin of this fascinating phenomenon and to calculate various fermion condensates around the monopole [4-11]. These studies have so far brought forth the following understanding: The baryon number violation is essentially caused by the peculiar non-abelian dynamics inside the monopole core in the J=0 partial wave sector, which is expressed through the effective boundary condition on the fermion fields at the monopole core. If we denote the unbroken generator of the SU(2) subgroup in which the monopole is embedded, by T_3, the boundary condition says that a left (right) handed fermion carrying negative (positive) T_3 charge, entering the monopole core, must be accompanied by a left (right) handed fermion carrying positive (negative) T_3 charge, coming out of the core. For embeddings for which the members of the SU(2) doublet carry different baryon numbers, it may effect baryon number non-conservation. That this mechanism operates without hindrance is essentially due to the nature of the J=0 partial waves, which are present because of the extra angular momentum of the monopole-charged particle system

[12]. Outside the monopole core, the interactions between the radial magnetic field and the Dirac spin and the extra angular momentum precisely cancel and the radial motions are effectively free. Because of these circumstances, the monopole ground state is not an eigenstate of the baryon number operator. It is rather a superposition of states with different baryon numbers and the monopole may absorb or emit fermions, carrying net baryonic charge, at no cost of energy. It has also been clarified [2,5,6,9] that the Adler-Bell-Jackiw anomaly plays a secondary role; it is needed only for those processes which violate chirality.

Now, the boundary condition described above seems to imply more than the baryon number violation. In fact, it implies that whatever quantum numbers are different for the upper and lower members of the SU(2) doublet should be non-conserved. These include electric and color charges in addition to the baryonic charge. Are these charges indeed not conserved? In the limit of a point-like monopole, studies of various fermionic condensates have revealed that they are actually conserved; charge non-conserving condensates all vanish due to a factor of the form $\exp(-\text{const.} \ln \infty)$ arising from the infinite coulomb energy [5,6,11]. The charge non-conserving boundary condition is effectively replaced by a charge conserving one.

This however immediately raises the question as to what happens if the finite size of the monopole is taken into account. One would expect that the infinity in the above exponent will then become finite. Does that then mean that the electric charge (and the color) symmetry is violated in the presence of the monopole of finite size? This is the question to which we shall address ourselves in this paper. We will show, by explicit calculations, that these charges are exactly conserved and explain how this comes about.

The above conclusion is by no means a trivial one. In fact, in the limit $e \to 0$, eg finite, (e=electric charge of the fermion, g=magnetic charge of the monopole) with finite monopole radius, a simple calculation shows that the electric charge conservation is violated in a monopole-fermion system, in exactly the same way as the baryon number conservation. It requires certain careful manipulations and definitions to restore the electric charge conservation in a monopole-fermion system, with any finite e and finite monopole radius. First of all, it is quite important to note that in the presence of a monopole of radius r_0, we may define two types of gauge invariant fermion creation operators. If ψ and A_μ refer to the fermion field and the gauge field respectively, the operator $\exp\left(-ie\int_{r_0}^{r} A_r dr\right) \psi^\dagger(r,t)$ creates a fermion with its string of gauge field lying between r_0 and r. Hence we may interpret this operator as the creation operator for a fermion and an equal and opposite charge at the monopole core. This creates a radial electric field lying between r_0 and r and hence the state has a large energy (of order e^2/r_0) associated with this electric field. (At this point we should mention that the bosonized fermion operators, defined in Ref.2, are precisely this type of fermion creation operators.) The second type of gauge invariant fermion creation operator is of the form $\exp(ie\int_r^\infty A_r dr)\psi^\dagger(r,t)$. This creates a fermion with its string of gauge field lying between r and ∞. This state has energy of order e^2/r, which remains finite even in the limit of a point monopole. The first type of fermion creation operators create charge neutral states, even if the fermion field carries a net charge, whereas the second type of fermion creation operators create charged states.

We shall demonstrate, by explicit calculation, that for a finite size monopole, the Green's function involving the first type of fermion creation operators may have finite value, even if the fermion fields in the Green's function carry a net total charge. This reflects the fact that the monopole may make virtual transitions to a state containing a dyon and fermions carrying equal and opposite charge (where by dyon we mean a state with a net charge within the monopole radius r_0, whatever be the way we choose to define r_0). This is analogous to the way in which the QED vacuum makes a virtual transition to a state containing e^+e^- pair, and does not imply charge non-conservation. The amplitude for such virtual transitions, however, falls off as the monopole radius goes to zero, because of the large energy associated with such intermediate states.

The Green's functions involving the second type of fermion creation operators, on the other hand, vanish identically, unless the total charge, carried by the fermionic fields in the Green's function, is zero. This shows that the electric charge is indeed conserved in a monopole fermion interaction.

We shall organize the rest of the paper as follows. In Sec. II, we briefly review the SU(2) model to be studied and fix our notations. Sec. III deals with the bosonization of the model and explains how we can introduce the two types of gauge invariant fermion operators in such a language. In Sec. IV, which constitutes the main part of the paper, we examine various two body fermionic Green's functions in detail for a monopole of finite size. Gauge invariant fermion operators of the first and the second type are clearly distinguished, and we focus on the Green's functions for which the total charge carried by the fermion field is non-zero. The result for a four body charge neutral condensate is also described briefly. Discussions of the various results, obtained in the paper, will be found in Sec. V. Three appendices are provided: In Appendix A we give the technical details of the evaluation of a complicated, yet important integral, encountered in the text. Appendix B describes the quantization of the system in a finite box of radius R, which is needed to regularize some divergences, and the computation of some integrals that appear in the Green's functions involving the second type of gauge invariant fermion operators. Appendix C describes the calculation of a four body charge neutral chirality conserving condensate.

II. THE MODEL

We consider an SU(2) gauge theory with a Dirac doublet of massless fermions and an adjoint Higgs. For convenience, we restrict ourselves to one Dirac doublet. The case of more than one Dirac doublet may be treated in a similar way. We assume that the SU(2) gauge symmetry is spontaneously broken down to U(1) by the vacuum expectation value of the adjoint Higgs. We shall refer to the charge associated with the unbroken U(1) generator as the electric charge. This model has magnetic monopoles of the 't Hooft Polyakov type. We wish to study the interaction of this monopole with the fermions.

It has been shown that if we restrict ourselves to the J=0 partial wave sector, the monopole-fermion system may be described by an effective two dimensional theory. In the γ^5 diagonal representation, the four component Dirac field may be written as $\begin{pmatrix} \psi_R \\ \psi_L \end{pmatrix}$, where ψ_R and ψ_L are two component spinors. In the J=0 partial wave sector, these spinors may be written as[5]:

$$\psi_{R\alpha i}(\vec{r},t) = (\sqrt{4\pi}\, r)^{-1} \left(R_+(r,t)\, \eta_{+\alpha}\, \eta_{-i} + R_-(r,t)\, \eta_{-\alpha}\, \eta_{+i} \right)$$

$$\psi_{L\alpha i}(\vec{r},t) = (\sqrt{4\pi}\, r)^{-1} \left(L_+(r,t)\, \eta_{+\alpha}\, \eta_{-i} + L_-(r,t)\, \eta_{-\alpha}\, \eta_{+i} \right) \tag{2.1}$$

i,α are respectively the isospin and the Lorentz spin indices. η_+ and η_- are respectively the eigenstates of $\vec{\sigma}\cdot\hat{r}$ with eigenvalues ± 1. The tensor basis in which we have expanded the field $\psi_{\alpha i}$ are slightly different from the basis chosen in Refs. [1,2], but the reader may easily verify that the tensors $\eta_{+\alpha}\,\eta_{-i}$ and $\eta_{-\alpha}\,\eta_{+i}$ are linear combinations of the tensors used in Refs. [1,2].

We now define the two component fields:

$$R = \begin{pmatrix} R_+ \\ -iR_- \end{pmatrix} \quad , \quad L = \begin{pmatrix} L_- \\ -iL_+ \end{pmatrix} \tag{2.2}$$

Next we introduce the collective co-ordinate $\lambda(r,t)$ in the same way as in Refs. [1,2]. In the $A_0^a=0$ gauge (we work in this gauge throughout our calculation), a monopole solution, together with dyon like collective co-ordinate fluctuations $\lambda(r,t)$ is written as,

$$A_i^a = \partial_r \lambda\, \hat{r}_a \hat{r}_i + \frac{1}{r}\epsilon_{aij}\hat{r}_j + \frac{K(r)}{r}\left\{\left(\delta_{ai} - \hat{r}_a\hat{r}_i\right)\sin\lambda - \epsilon_{aij}\hat{r}_j\cos\lambda\right\} \tag{2.3}$$

where the radial function $K(r)$, describing the deviation from the Abelian monopole, becomes 1 at the origin and vanishes exponentially as $r\to\infty$.

The interaction of the J=0 partial wave fermions with the monopole field is described by an effective two dimensional action (which may be obtained from the four dimensional action using Eqs. (2.1) and (2.3)),

$$I_{eff} = \int_{-\infty}^{\infty} dt \int_{0}^{r} dr \left\{ \frac{2\pi r^2}{e^2} (\dot{\lambda}')^2 + \frac{4\pi}{e^2} K^2 \dot{\lambda}^2 + \bar{R}\, i \not\partial R + \bar{L}\, i \not\partial L \right.$$
$$+ \frac{\lambda'}{2} (\bar{L} \gamma^1 \gamma^5 L - \bar{R} \gamma^1 \gamma^5 R) + \frac{K}{r} (\bar{R}\, i \gamma_5 R \cos\lambda - \bar{R} R \sin\lambda$$
$$\left. + \bar{L}\, i \gamma_5 L \cos\lambda + \bar{L} L \sin\lambda) \right\} \tag{2.4}$$

where the prime and the dot denotes ∂_r and ∂_t respectively. If we now define the gauge invariant fields R_N and L_N as,

$$R_N(r,t) = \exp\left(i \lambda(r,t) \gamma^5 / 2 \right) R(r,t)$$
$$L_N(r,t) = \exp\left(- i \lambda(r,t) \gamma^5 / 2 \right) L(r,t) \tag{2.5}$$

then I_{eff} takes the form,

$$I_{eff} = \int_{-\infty}^{\infty} dt \int_{0}^{\infty} dr \left\{ \frac{2\pi r^2}{e^2} (\lambda')^2 + \frac{4\pi}{e^2} K^2 \dot{\lambda}^2 + \bar{R}_N\, i \not\partial R_N + \bar{L}_N\, i \not\partial L_N \right.$$
$$\left. - \frac{\dot{\lambda}}{2} (\bar{L}_N \gamma^1 L_N - \bar{R}_N \gamma^1 R_N) + \frac{K}{r} (\bar{R}_N\, i \gamma^5 R_N + \bar{L}_N\, i \gamma^5 L_N) \right\} \tag{2.6}$$

Since $K(r) \to 1$ as $r \to 0$, the last two terms in the right hand side of (2.6) show that the fields R_N and L_N effectively become extremely massive near the origin. This makes the components of R_N and L_N vanish in a certain way as $r \to 0$ and leads to the boundary condition [1,2,5],[F1]

$$(R_{N+} + R_{N-})\big|_{r=r_0} = 0 \qquad (L_{N+} + L_{N-})\big|_{r=r_0} = 0 \tag{2.7}$$

ignoring terms of order r_0, and the fluctuation in the collective co-ordinate λ. Although the boundary conditions, in general, will be modified in the presence of λ field fluctuations, we shall be using Eq. (2.7) throughout our calculation. We shall justify the use of these boundary conditions in Sec. V. Here r_0 is any distance beyond which the classical monopole field, is indistinguishable from the Abelian monopole field, i.e. beyond which $K(r)$ is practically zero. We call r_0 the monopole radius. Once $R_{N\pm}$ and $L_{N\pm}$ are required to satisfy the boundary conditions (2.7), we may forget about the dynamics inside the monopole core, and describe the system by an effective action,

$$I_{eff} = \int_{-\infty}^{\infty} dt \int_{r_0}^{\infty} dr \left\{ \frac{2\pi r^2}{e^2} (\dot{\lambda}')^2 + \bar{R}_N i \not\partial R_N + \bar{L}_N i \not\partial L_N - \frac{\dot{\lambda}}{2} \left(\bar{L}_N \gamma^1 L_N - \bar{R}_N \gamma^1 R_N \right) \right\} \tag{2.8}$$

The fields R_N and L_N have the following interpretation. If $A_\mu(r,t)$ is the four dimensional gauge field associated with the unbroken generator of the gauge group, then, in the $A_0^a=0$ gauge, A_r is given by λ'. Thus the gauge invariant fields R_N and L_N refer to $\exp(i\gamma^5\int_0^r A_r dr)\, R(r,t)$ and $\exp(-i\gamma^5\int_0^r A_r dr)\, L(r,t)$ respectively. It is easy to see that $\gamma^5(-\gamma^5)$ measures the charge of the $R(L)$ fields. Hence the exponential factors in R_N and L_N describe strings of gauge field between the monopole core and r. In other words, the fermion creation operator $R_N^\dagger(r,t)\,(L_N^\dagger(r,t))$ creates a fermion at the point r and an equal and opposite charge at the monopole core and a string of gauge field extending from the monopole core to the fermion.

In our analysis in the next two sections, we shall use another type of gauge invariant fermion creation operator, which creates a fermion at a point r, and equal and opposite charge at ∞, and a string of gauge field lying between the point r and infinity. These operators are given by $\exp(-i\gamma^5\int_r^\infty A_r dr)R(r,t)$ and $\exp(i\gamma^5\int_r^\infty A_r dr)L(r,t)$, or, in terms of the λ field, $\exp(i\gamma^5(\lambda(r,t)-\lambda(\infty,t)))R(r,t)$ and $\exp(-i\gamma^5(\lambda(r,t)-\lambda(\infty,t)))L(r,t)$ respectively. We denote these fields by $\tilde{R}_N(r,t)$ and $\tilde{L}_N(r,t)$. In calculating the Green's functions involving these operators, we run into divergences in the spatial integrals from infinity, and we must regularize these divergences in a consistent way. This may be done by quantizing the system in a box of radius R, and taking the $R\to\infty$ limit at the end of the calculation, as has been described in Appendix B.

III. BOSONIZATION

Following Callan [2], we can map the two dimensional fermionic system, containing the fields R_N and L_N, into a two dimensional bosonic system, containing the fields ϕ_R and ϕ_L, by the following correspondence:

$$R_{N+} = \sqrt{\frac{\mu c}{2\pi}}\; N_\mu \; \exp\left[i\sqrt{\pi}\left(\varphi_R(r,t) - \int_{r_0}^{r} \dot{\varphi}_R(s,t)\, ds\right)\right]$$

$$-i R_{N-} = i\sqrt{\frac{\mu c}{2\pi}}\; N_\mu \; \exp\left[i\sqrt{\pi}\left(\varphi_R(r,t) + \int_{r_0}^{r} \dot{\varphi}_R(s,t)\, ds\right)\right]$$

$$L_{N-} = \sqrt{\frac{\mu c}{2\pi}}\ N_\mu\ \exp\left[i\sqrt{\pi}\left(\varphi_L(x,t) - \int_{x_0}^{x} \dot\varphi_L(s,t)\, ds\right)\right]$$

$$-i L_{N+} = i\sqrt{\frac{\mu c}{2\pi}}\ N_\mu\ \exp\left[i\sqrt{\pi}\left(\varphi_L(x,t) + \int_{x_0}^{x} \dot\varphi_L(s,t)\, ds\right)\right] \tag{3-1}$$

where c is a constant and N_μ denotes normal ordering with respect to an infinitesimal mass μ. The fields ϕ_R and ϕ_L satisfy the boundary conditions,

$$\varphi_R' = \varphi_L' = 0 \quad \text{at} \quad x = x_0 \tag{3.2}$$

We exhibit some details of the bosonization here since they will be useful later. The normal ordering operator N_μ is defined in the following way. We first define the operators a_f and $a_f^\dagger$ at any fixed time t, through the equations:

$$\varphi(x,t) = \int_0^\infty \frac{dk}{\sqrt{2\pi}} \frac{1}{\sqrt{2\sqrt{k^2+\mu^2}}} \left[a_f(k,\mu,t)\, \exp\left(-i\sqrt{k^2+\mu^2}\, t\right) + a_f^\dagger(k,\mu,t)\, \exp\left(i\sqrt{k^2+\mu^2}\, t\right)\right] 2\cos k(x-x_0) \tag{3-3}$$

and,

$$\dot\varphi(x,t) = \int_0^\infty \frac{dk}{\sqrt{2\pi}} \frac{i\,(k^2+\mu^2)^{1/4}}{\sqrt{2}} \left[-a_f(k,\mu,t)\, e^{-i\sqrt{k^2+\mu^2}\, t} + a_f^\dagger(k,\mu,t)\, e^{i\sqrt{k^2+\mu^2}\, t}\right]\ 2\cos k(x-x_0) \tag{3-4}$$

$$\equiv \tilde{\Delta}_0 (r,t,r',t')$$

$$= -\frac{1}{4\pi} [A_+ (r,t,r',t') + A_- (r,t,r',t') + B_+ (r,t,r',t')$$

$$+ B_- (r,t,r',t') + 2 \ln \mu^2 c^2]$$

$$(3.6)$$

where,

$$A_\pm (r,t,r',t') = \ln \{(r-r') \pm (t-t') \mp i\epsilon\}$$

$$B_\pm (r,t,r',t') = \ln \{(r+r'-2r_0) \pm (t-t') \mp i\epsilon\} \tag{3.7}$$

$$\lim_{\mu \to 0} a_f (k,\mu,t) \; |0\rangle_f = 0 \tag{3.8}$$

where $|0\rangle_f$ is the vacuum of the free massless field theory.

$$-4\pi \int_{r_0}^{r} ds \;\; \partial_t \tilde{\Delta}_0 (s,t,r',t') = A_+ - A_- + B_+ - B_- + 2\pi i \tag{3.9}$$

$$-4\pi \int_{r_0}^{r'} ds' \;\; \partial_{t'} \tilde{\Delta}_0 (r,t,s',t') = A_+ - A_- - B_+ + B_- \tag{3.10}$$

$$-4\pi \int_{r_0}^{r} ds \int_{r_0}^{r'} ds' \; \partial_t \; \partial_{t'} \; \tilde{\Delta}_0(s,t,s',t')$$

$$= A_+ + A_- - B_+ - B_- \tag{3.11}$$

Using Eqs. (3.1) and (3.5)-(3.8) we get,

$$\langle 0 | R_{N\eta}(r,t) \; R^{+}_{N\eta'}(r',t') | 0 \rangle$$

$$= \exp\Big[\pi \Big\{ \tilde{\Delta}_0(r,t,r',t') - \eta \int_{r_0}^{r} ds \frac{\partial}{\partial t} \tilde{\Delta}_0(s,t,r',t')$$

$$- \eta' \int_{r_0}^{r'} ds' \frac{\partial}{\partial t'} \tilde{\Delta}_0(r,t,s',t') + \eta\eta' \int_{r_0}^{r} ds \int_{r_0}^{r'} ds' \frac{\partial}{\partial t} \frac{\partial}{\partial t'} \tilde{\Delta}_0(s,t,s',t') \Big\} \Big] \tag{3.12}$$

Using Eqs. (3.6) and (3.9)-(3.11) one can show that the right hand side of (3.12) reduces to the correct free fermion propagator.

We shall now write down the fully interacting theory. To do this, we need to express the currents $\bar{L}_N \gamma^1 L_N$ and $\bar{R}_N \gamma^1 R_N$ in terms of the bosonized fields ϕ_R and ϕ_L. Using the point splitting method of Ref. [13], we find[F2],

$$\begin{aligned} \bar{L}_N \gamma^1 L_N &= -\varphi_L' \\ \bar{R}_N \gamma^1 R_N &= -\varphi_R' \end{aligned} \tag{3.13}$$

Substituting this in the effective action and integrating it once by parts, we get,

$$I_{eff} = \int_{-\infty}^{\infty} dt \int_{r_0}^{\infty} dr \left\{ \frac{2\pi r^2}{e^2} (\dot{\lambda}')^2 + \frac{1}{2} \partial_\mu \varphi_R \partial^\mu \varphi_R + \frac{1}{2} \partial^\mu \varphi_L \partial_\mu \varphi_L \right.$$

$$\left. - \dot{\lambda}' (\varphi_L - \varphi_R)/2\sqrt{\pi} \right\} + \text{SURFACE TERMS} \qquad (3.14)$$

where,

SURFACE TERMS

$$= \frac{1}{2\sqrt{\pi}} \int_{-\infty}^{\infty} dt \left[\dot{\lambda}(\infty,t) (\varphi_L(\infty,t) - \varphi_R(\infty,t)) - \dot{\lambda}(r_0,t) (\varphi_L(r_0,t) - \varphi_R(r_0,t)) \right] \qquad (3.15)$$

We shall come back to a discussion of the surface terms at the end of this section. For the time being let us ignore them. $\dot{\lambda}'$ may be interpreted as the radial electric field E_r. We may eliminate it by using the equations of motion:

$$E_r \equiv \dot{\lambda}' = \frac{e^2}{8\pi r^2 \sqrt{\pi}} (\varphi_L - \varphi_R) \qquad (3.16)$$

and obtain the effective Hamiltonian of the system:

$$H = \int_{r_0}^{\infty} dr \left[\frac{1}{2}(\dot{\varphi}_R)^2 + \frac{1}{2} (\varphi_R')^2 + \frac{1}{2} (\dot{\varphi}_L)^2 + \frac{1}{2} (\varphi_L')^2 \right.$$

$$\left. + \frac{e^2}{32\pi^2 r^2} (\varphi_L - \varphi_R)^2 \right] \qquad (3.17)$$

We now note that the fermion fields $R_{N\eta}$ and $L_{N\eta}$, defined in Eq. (3.1), create a non-zero value of $\phi_R - \phi_L$ between the points r_0 and r, while acting on the state $\phi_R - \phi_L = 0$. This can be seen by considering the commutator,

$$\left[-\frac{1}{\sqrt{\pi}}\left(\varphi_R(r,t)-\varphi_L(r,t)\right),\ R_{N\pm}(r',t)\right]$$
$$= \mp\,\Theta(r-r_0)\;\Theta(r'-r)\;R_{N\pm}(r',t) \tag{3.18}$$

and similarly for $L_{N\pm}$. This corresponds to creating a non-zero radial electric field, falling off as $1/r^2$, between the points r_0 and r. Hence the operators $L_{N\pm}$, $R_{N\pm}$ create an anti-fermion at the point r at time t and an equal and opposite charge at the monopole core at the same time. Thus these operators indeed correspond to the first type of gauge invariant fermion creation operators given by Eq. (2.5).

Next we try to see whether it is possible to represent the gauge invariant operators $\tilde{R}_N$ and $\tilde{L}_N$, introduced at the end of Sec. II, in the bosonized theory. These operators must satisfy the following properties: 1) For e=0, the λ field is frozen and hence the Green's functions involving the fields $\tilde{R}_N$ and $\tilde{L}_N$ must correctly reproduce the free fermion propagator. 2) The operators carrying the fields $\tilde{R}_N$ and $\tilde{L}_N$ must create a non-zero electric field between the points r and infinity. The most obvious guess for such operators is,

$$\tilde{R}_{N+} = \sqrt{\frac{\mu c}{2\pi}}\;N_\mu\;\exp\left[i\sqrt{\pi}\left(\varphi_R(r,t)+\int_r^\infty \dot{\varphi}_R(s,t)\,ds\right)\right]$$

$$-i\tilde{R}_{N-} = i\sqrt{\frac{\mu c}{2\pi}}\;N_\mu\;\exp\left[i\sqrt{\pi}\left(\varphi_R(r,t)-\int_r^\infty \dot{\varphi}_R(s,t)\,ds\right.\right.$$

$$\tilde{L}_{N-} = \sqrt{\frac{\mu c}{2\pi}}\; N_\mu \; \exp\left[i\sqrt{\pi}\left(\varphi_L(r,t) + \int_r^\infty \dot{\varphi}_L(s,t)\, ds\right)\right]$$

$$-i\tilde{L}_{N+} = i\sqrt{\frac{\mu c}{2\pi}}\; N_\mu \; \exp\left[i\sqrt{\pi}\left(\varphi_L(r,t) - \int_r^\infty \dot{\varphi}_L(s,t)\, ds\right)\right]$$

(3.19)

By considering the commutator

$$\left[-\frac{1}{\sqrt{\pi}}\left(\varphi_R(r,t) - \varphi_L(r,t)\right),\; \tilde{R}_{N\pm}(r',t)\right] = \mp\, \Theta(r-r')$$

(3.20)

and a similar commutator involving $\tilde{L}_{N\pm}$, we see that the operators $\tilde{R}_{N\pm}$, $\tilde{L}_{N\pm}$ indeed create a non-zero electric field between the point r' and infinity.

We must now proceed to show that in the e→0 limit, the fields defined in Eq. (3.19), reproduce the correct free field propagator. The Green's function $\langle 0|\tilde{R}_{N\eta}(r,t)\tilde{R}^{\dagger}_{N\eta}(r',t')|0\rangle$ is given by an expression similar to the right hand side of Eq. (3.12), with the integrals from r_0 to r replaced by integrals from r to infinity. Some of these integrals are ambiguous due to the lack of proper regularization at spatial infinity. If we regulate these integrals by quantizing the system in a box of radius R, then, as has been shown in Appendix B, the resulting expression correctly reproduces the free fermion propagator. Thus we see that the operators defined in Eq. (3.19), indeed correspond to the second type of fermion creation operators, mentioned at the end of Sec. II.

Finally, let us discuss the surface terms (3.15). To get an estimate of these terms, we consider a particular mode of excitation of the $\Phi(\equiv\phi_L-\phi_R)$ field of energy E (say). It will be shown in the next section that for $Er_0 \ll 1$, the value of the Φ field at r_0 in this particular mode goes as $(Er_0)^{1+0(e^2)}$. We get an estimate for $\lambda(r_0,t)$, by using the equation,

$$\dot{\lambda}' \propto \Phi/r^2 \sim (E r_0)^{1+o(e^2)} (r_0)^{-2} \quad \text{at } r=r_0 \tag{3.21}$$

and,

$$\lambda|_{r=0} = 0 \tag{3.22}$$

in order to ensure that the collective co-ordinate excitation is non-singular at the origin [1,2]. Eq. (3.21) may also be assumed to be an estimate of $\dot{\lambda}'$ for $r<r_0$. Then,

$$\dot{\lambda}(r=r_0) \sim (r_0)^{-1} (E r_0)^{1+o(e^2)} \tag{3.23}$$

Hence

$$\dot{\lambda}(r=r_0)\, \Phi(r=r_0) \sim (E r_0)^{1+o(e^2)} E \tag{3.24}$$

which is small compared to E, so long as $Er_0 \ll 1$. In our calculation in appendices A and B, we shall see that it is the excitation modes with $E \ll r_0^{-1}$ which give the major contribution to all the Green's functions. For these modes the boundary term $\lambda(r_0)\Phi(r_0)$ may be neglected.

Next, we must estimate the boundary term at infinity. After we regularize the theory by quantizing it in a box of radius R, the boundary term at infinity becomes,

$$\dot{\lambda}(R)\ \Phi(R) \tag{3.25}$$

$\dot{\lambda}$ (R) is determined from the equation,

$$\dot{\lambda}(R) = \dot{\lambda}(r_0) + (e^2/8\pi^{3/2})\int_{r_0}^{R}(\Phi/r^2)\,dr \tag{3.26}$$

As can be seen from Eq. (3.23), $\dot{\lambda}(r_0)$ is of order E. The second term on the right hand side of (3.26) is also finite. There is no divergence from the region of integration at small r, since $\Phi\sim(Er)^{1+0(e^2)}$ in the region of small r. There is no divergence from the region of large r either, since Φ goes to a cosine function in this region. Hence $\dot{\lambda}(R)$ is a finite number, independent of R in the limit of large R. $\Phi(R)$, on the other hand alternates between the values +1 and -1 as we move from one energy level to the next one. This may be seen from Eqs. (B.18) and (B.19). In the $R\rightarrow\infty$ limit, the spacing between these levels goes to zero, and $\Phi(R)$, expressed as a function of E, becomes a rapidly oscillating function. Hence, although $\Phi(R)\dot{\lambda}(R)$ is finite for a particular mode, its effect vanishes in any calculation, which involves sum over different modes.

IV. CALCULATION OF VARIOUS GREEN'S FUNCTIONS

We shall now calculate various Green's functions in the interacting theory in various limits. We define the fields ϕ and Φ as,

$$\Phi = (\varphi_R - \varphi_L)/\sqrt{2} \tag{4.1}$$

$$\varphi = (\varphi_R + \varphi_L)/\sqrt{2} \tag{4.2}$$

The Hamiltonian may then be written as,

$$H = \int_{r_0}^{r} dr \left[\tfrac{1}{2}\dot{\Phi}^2 + \tfrac{1}{2}\dot{\varphi}^2 + \tfrac{1}{2}\Phi'^2 + \tfrac{1}{2}\varphi'^2 + \frac{e^2}{16\pi^2 r^2}\Phi^2 \right] \tag{4.3}$$

Hence the equations for ϕ and Φ completely decouple. The ϕ field is a free field satisfying the boundary condition $\phi'=0$ at r_0. The Φ field satisfies the equation:

$$\left(\partial_t^2 - \partial_r^2 + e^2/8\pi^2 r^2\right)\Phi = 0 \tag{4.4}$$

and the boundary condition,

$$\Phi'\big|_{r=r_0} = 0 \tag{4.5}$$

with the solution,

$$\Phi = \int \frac{dE}{\sqrt{2\pi}\sqrt{2E}} \left(e^{-iEt}a(E) + e^{iEt}a^\dagger(E)\right)\sqrt{Er}\, f_\nu(E,r,r_0)$$

$$\equiv \Phi^{(-)} + \Phi^{(+)} \tag{4.6}$$

$f_\nu(E,r,r_0)$ is the solution of the equation,

$$\frac{d^2 f_\nu(r)}{dr^2} + \frac{1}{r}\frac{d f_\nu(r)}{dr} + \left(E^2 - \frac{\nu^2}{r^2}\right) f_\nu(r) = 0 \qquad (4.7)$$

with the boundary condition:

$$\frac{d}{dr}\left(\sqrt{Er}\; f_\nu(E,r,r_0)\right) = 0 \quad \text{at } r = r_0 \qquad (4.8)$$

and the normalization:

$$\int_0^\infty dE\; E\sqrt{rr'}\; f_\nu(E,r,r_0)\, f_\nu(E,r',r_0) = 2\pi\,\delta(r-r') \qquad (4.9)$$

$$\int_{r_0}^\infty dr\; \sqrt{EE'}\; r\; f_\nu(E,r,r_0)\, f_\nu(E',r,r_0) = 2\pi\,\delta(E-E') \qquad (4.10)$$

Eq. (4.7) is the Bessel's equation of order ν. Here,

$$\nu = \left(\frac{1}{4} + \frac{e^2}{16\pi^2}\right)^{1/2} \qquad (4.11)$$

The true vacuum of the system satisfies the equation:

$$a(E)\,|0\rangle = 0 \quad \forall E \qquad (4.12)$$

Normal ordering operation upon this vacuum, which we shall denote by $N_{1/r}$, is defined by the negative-positive frequency decomposition in Eq. (4.6). The general solution of Eq. (4.7) may be written as,

$$f_\nu(E, r, r_0) = a_\nu J_\nu(Er) + b_\nu J_{-\nu}(Er) \tag{4.13}$$

The boundary conditions (4.8) then gives,

$$\frac{a_\nu}{b_\nu} = -\frac{d}{d\rho}\left(\sqrt{\rho}\, J_{-\nu}(\rho)\right)\Big|_{\rho=Er_0} \Big/ \frac{d}{d\rho}\left(\sqrt{\rho}\, J_\nu(\rho)\right)\Big|_{\rho=Er_0} \tag{4.14}$$

We define,

$$\tilde{\Delta}(r,t,r',t') = [\,\Phi^{(-)}(r,t),\ \Phi^{(+)}(r',t')]$$

$$= \int_0^\infty \frac{dE}{4\pi E}\, e^{-iE(t-t')}\, E\sqrt{rr'}\, f_\nu(E,r,r_0)\, f_\nu(E,r',r_0) \tag{4.15}$$

$\tilde{\Delta}$ may be calculated in various limits by knowing f_ν. The various Green's functions, involving the fermion fields defined in Eqs. (3.1) and (3.19) may then be calculated by using the following results [14],

$$N_\mu(e^A) = N_{1/r}(e^A)\ \exp\left(\tfrac{1}{2}[A^{(-)}, A^{(+)}]_{1/r}\right) / \exp\left(\tfrac{1}{2}[A_f^{(-)}, A_f^{(+)}]_\mu\right) \tag{4.16}$$

where $[A_f^{(-)},\ A_f^{(+)}]_\mu$ means that while evaluating the commutator, we must assume that the field Φ satisfies the equations of motion of a free field with infinitesimal mass μ, while in evaluating $[A^{(-)}, A^{(+)}]_{1/r}$ we must assume that the field Φ satisfies the equations of motion (4.4). These commutators may be expressed in terms of $\tilde{\Delta}_0$ and $\tilde{\Delta}$, defined in Eqs. (3.6) and (4.15) respectively. Eq. (4.16), together with the equation analogous to (3.5),

$$N_{1/r}\,(e^{A})\,N_{1/r}\,(e^{B}) = e^{[A^{(-)},B^{(+)}]}\,N_{1/r}\left(e^{A+B}\right) \qquad (4.17)$$

helps us in evaluating all the Green's functions.

(A) TWO BODY FERMIONIC GREEN'S FUNCTIONS INVOLVING THE OPERATORS DEFINED IN EQ. (3.1):

We want to calculate,

$$\langle 0|\,R^{\dagger}_{N\eta}(x,t)\;R_{N\eta'}(x',t')\,|0\rangle$$
$$= \eta\eta'\,\frac{\mu c}{2\pi}\,\langle 0|\,N_{\mu}\,\exp\left\{-i\sqrt{\pi}\left(\varphi_R(x,t)-\eta\int_{x_0}^{x}\dot{\varphi}_R(s,t)\,ds\right)\right\}$$

$$N_{\mu}\,\exp\left\{i\sqrt{\pi}\left(\varphi_R(x,t)-\eta'\int_{x_0}^{x}\dot{\varphi}_R(s,t)\,ds\right)\right\}|0\rangle \qquad (4.18)$$

where $\eta,\eta'=\pm 1$. We do this in the following way: 1) We first express ϕ_R in terms of the fields ϕ and Φ. 2) For the Φ field, we convert the normal ordering with respect to μ to normal ordering with respect to $1/r$ using Eq. (4.16). 3) We then combine the product of normal ordered operators in terms of normal ordered product of operators using Eqs. (3.5) and (4.17). 4) Finally, we calculate the vacuum expectation value of normal ordered operators using the fact that $\Phi^{(-)}$ and $\phi_f^{(-)}$ annihilates the vacuum. The result is,

$$\frac{\eta\eta'\mu c}{2\pi}\;\exp(E_1)\;\exp(E_2)\;\exp(E_3) \qquad (4.19)$$

where,

$$E_1 = \frac{\pi}{2}\left\{ \tilde{\Delta}_0(r,t,r',t') - \eta\int_{r_0}^{r} ds\ \partial_t \tilde{\Delta}_0(s,t,r',t') - \eta'\int_{r_0}^{r'} ds'\ \partial_{t'} \tilde{\Delta}_0(r,t,s',t') \right.$$

$$\left. + \eta\eta' \int_{r_0}^{r} ds \int_{r_0}^{r'} ds'\ \partial_t\ \partial_{t'}\ \tilde{\Delta}_0(s,t,s',t') \right\} \tag{4.20}$$

$$E_2 = \frac{\pi}{2}\left\{ \tilde{\Delta}(r,t,r',t') - \eta\int_{r_0}^{r} \partial_t \tilde{\Delta}(s,t,r',t') - \eta'\int_{r_0}^{r'} ds'\ \partial_{t'} \tilde{\Delta}(r,t,s',t') \right.$$

$$\left. + \eta\eta' \int_{r_0}^{r} ds \int_{r_0}^{r'} ds'\ \partial_t\ \partial_{t'}\ \tilde{\Delta}(s,t,s',t') \right\} \tag{4.21}$$

and,

$$E_3 = -\frac{\pi}{4}\left\{ \tilde{\Delta}(r,t,r,t) - \tilde{\Delta}_0(r,t,r',t')\right\} - \eta\int_{r_0}^{r} ds\ \partial_t\left(\tilde{\Delta}(s,t,r,\tau)\right.$$

$$\left. - \tilde{\Delta}_0(s,t,r,\tau)\right)\Big|_{\tau=t} - \eta\int_{r_0}^{r} ds'\ \partial_\tau\left(\tilde{\Delta}(r,t,s',\tau) - \tilde{\Delta}_0(r,t,s',\tau)\right)\Big|_{\tau=t}$$

$$+ \int_{r_0}^{r} ds \int_{r_0}^{r} ds'\ \partial_t\ \partial_\tau\ \left(\tilde{\Delta}(s,t,s',\tau) - \tilde{\Delta}_0(s,t,s',\tau)\right)\Big|_{\tau=t}$$

$$+ \tilde{\Delta}(r',t',r',t') - \tilde{\Delta}_0(r',t',r',t') - \eta'\int_{r_0}^{r'} ds\ \partial_{t'}\left(\tilde{\Delta}(s,t',r',\tau)\right.$$

$$\left. - \tilde{\Delta}_0(s,t',r',\tau)\right)\Big|_{\tau=t'} - \eta'\int_{r_0}^{r'} ds'\ \partial_\tau\left(\tilde{\Delta}(r',t',s',\tau) - \tilde{\Delta}_0(r',t',s',\tau)\right)\Big|_{\tau=t'}$$

$$+ \int_{r_0}^{r'} ds \int_{r_0}^{r'} ds'\ \partial_t\ \partial_\tau\left(\tilde{\Delta}(s,t',s',\tau) - \tilde{\Delta}_0(s,t',s',\tau)\right)\Big|_{\tau=t'} \Big\}$$

$$(4.22)$$

In Eq. (4.19), the term exp (E_1) comes from the contribution of the ϕ field, the term exp (E_2) exp (E_3) comes from the contribution of the Φ field. We shall now consider various limits. First, note that if we take the $e^2 \to 0$ limit at fixed r_0, $\tilde{\Delta}$ becomes identical to $\tilde{\Delta}_0$. As a result, E_3 is zero, and exp (E_1) exp (E_2) $\eta\eta'\mu c/2\pi$ reproduces the free field propagator (3.12).

Next, we shall take the limit $r_0 \to 0$ first, keeping e^2 finite, and then take the $e^2 \to 0$ limit. We must first evaluate the propagator $\tilde{\Delta}(r,t,r',t')$ in this limit, using Eq. (4.15). We use the property,

$$\frac{d}{d\rho}\left(\sqrt{\rho}\, J_\nu(\rho)\right) \sim \rho^{\nu-1/2} \qquad \frac{d}{d\rho}\left(\sqrt{\rho}\, J_{-\nu}(\rho)\right) \sim \rho^{-\nu-1/2} \qquad (4.23)$$

near $\rho=0$. Eq. (4.14) then tells us that in the $r_0 \to 0$ limit,

$$a_\nu = \text{constant} \qquad b_\nu = 0 \tag{4.24}$$

We now take the limit $e \to 0$ $(\nu \to 1/2)$ and use the property,

$$J_{1/2}(\rho) = \sqrt{2/\pi\rho} \;\; \sin\rho \tag{4.25}$$

to get,

$$f_\nu(E, r, 0) \simeq 2\sin Er \tag{4.26}$$

Hence, from (4.15),

$$\tilde{\Delta}(r, t, r', t') = \int_0^\infty \frac{dE}{4\pi E} \; e^{-iE(t-t')} \; 4\sin Er \sin Er'$$

$$= \left(-\frac{1}{4\pi}\right) \left(A_+ + A_- - B_+ - B_-\right) \tag{4.27}$$

where $A_\pm$ and $B_\pm$ are defined in Eq. (3.7).

We also have the following relations:

$$-4\pi \int_0^r ds \;\; \partial_t \; \tilde{\Delta}(s, t, r', t')$$

$$= A_+ - A_- - B_+ + B_- + 2\left[\ln(r' + t - t' - i\epsilon) - \ln(r' - (t - t') + i\epsilon)\right] \tag{4.28}$$

$$-4\pi \int_0^{r'} ds' \, \partial_{t'} \tilde{\Delta}(r,t,s',t')$$

$$= A_+ - A_- + B_+ - B_- - 2\left[\ln(r+t-t'-i\epsilon) - \ln(r-(t-t')+i\epsilon)\right]$$

$$(4.29)$$

$$-4\pi \int_0^{r} ds \int_0^{r'} ds' \, \partial_t \, \partial_{t'} \tilde{\Delta}(s,t,s',t')$$

$$= A_+ + A_- + B_+ + B_- - 2\left[\ln(r'+t-t'-i\epsilon) + \ln(r'-(t-t')+i\epsilon)\right]$$

$$-2\left[\ln(r+t-t'-i\epsilon) + \ln(r-(t-t')+i\epsilon)\right]$$

$$+2\left[\ln(t-t'-i\epsilon) + \ln(-(t-t')+i\epsilon)\right]$$

$$(4.30)$$

First note that the μ dependence of E_1, E_2 and E_3, coming from the $\ln\mu^2c^2$ term in $\tilde{\Delta}_0$, cancels with the explicit multiplicative factor of μc in Eq. (4.19), and hence the final result is μ independent.

We now look for divergent terms in the exponential of (4.19). To do this, we first express E_1, E_2 and E_3 in terms of the functions $A_\pm$ and $B_\pm$, using Eqs. (3.7), (3.9)-(3.11), (4.27)-(4.30). In E_3, we get divergent terms of the form $A_\pm(r,t,r,t)$, but they all cancel between the $\tilde{\Delta}$ and $\tilde{\Delta}_0$ terms. The only divergence comes from the terms

$$\int_0^r ds \int_0^r ds' \, \partial_t \, \partial_\tau \{ \tilde{\Delta}(s,t,s',\tau) - \tilde{\Delta}_0(s,t,s',\tau) \} |_{\tau=t} \tag{4.31}$$

and,

$$\int_0^{r'} ds \int_0^{r'} ds' \, \partial_{t'} \, \partial_\tau \{ \tilde{\Delta}(s,t',s',\tau) - \tilde{\Delta}_0(s,t',s',\tau) \} |_{\tau=t'} \tag{4.32}$$

since these expressions contain terms of the form $\ln(t-\tau)$ and $\ln(t'-\tau)$, which blow up at $\tau=t$ and at $\tau=t'$ respectively (See Eq. (4.30)). As a result, in E_3, we get a term of the form $-\ln\infty$ and the Green's function (4.18) vanishes identically. The origin of this divergence will become clear when we discuss the effect of finite monopole radius. But it is worth mentioning that in the $r_0 \to 0$ limit the function f_ν satisfies the equation $f_\nu(\rho)=0$ as well as $d/d\rho(\sqrt{\rho} f_\nu(\rho))=0$ at $\rho=0$. $f_\nu(\rho)=0$ corresponds to the condition $\Phi(r=0)=0$. This is a charge conserving boundary condition, since $\dot{\Phi}(r=0)$ measures the net flow of the U(1) charge into the monopole core. Thus, although we started with a charge non-conserving boundary condition, the dynamics turns the boundary condition into a charge conserving one, because of the infinite energy of the dyon in the point monopole limit.

We shall now turn to the evaluation of the Green's function keeping e^2 and r_0 finite, and then consider various limits. We need to do this only for the integrals (4.31) and (4.32), with the lower limits of integration replaced by r_0, since the other terms are finite in the $r_0 \to 0$ limit and hence the effect of having a finite r_0 and e^2 will be a small correction, as long as e^2 and r_0 are small, and e^2r/r_0 is large. We start with Eqs. (3.6) and (4.15) and obtain,

$$\int_{r_0}^{r} ds \int_{r_0}^{r} ds' \; \partial_t \, \partial_\tau \left[\tilde{\Delta}(s,t,s',\tau) - \tilde{\Delta}_0(s,t,s',\tau) \right]_{\tau=t}$$

$$= \int_0^\infty \frac{dE}{4\pi E} \Big[\int_{r_0}^{r} \sqrt{Es}\, f_\nu(E,s,r_0)\, E\, ds \int_{r_0}^{r} \sqrt{Es'}\, f_\nu(E,s',r_0)\, E\, ds'$$

$$- 4 \int_{r_0}^{r} \cos(E(s-r_0))\, E\, ds \int_{r_0}^{r} \cos(E(s'-r_0))\, E\, ds' \qquad (4.33)$$

The calculation of the triple integral is rather complicated but we can reliably extract the terms divergent in the limit $r_0 \to 0$. [See Appendix A for the details of the calculation]. We consider the following limits:

1) e^2 small, r_0 small, e^2r/r_0 large: In this limit, the term divergent in the limit $e^2r/r_0 \to \infty$ is given by,

$$(1 + 2\sigma \ln 2 + O(\sigma^2)) \frac{1}{\pi} \ln \frac{\sigma r}{r_0} + \text{finite} \qquad (4.34)$$

where σ is defined as,

$$\sigma = \nu - 1/2 = e^2/16\pi^2 + O(e^4) \qquad (4.35)$$

which shows that the Green's function (4.18) falls off as $(r_0/\sigma r)^{1/2+\sigma\ln 2+O(\sigma^2)}$ as $\sigma r/r_0 \to \infty$.

2) $\sigma=1$ (special case), r/r_0 large: In this case the function

f_ν may be expressed in terms of the trigonometric functions and hence the integral (4.33) may be evaluated exactly. The result is,

$$\frac{\pi}{4} \ln \frac{r}{r_0} \tag{4.36}$$

which corresponds to a suppression factor

$$(r_0/r)^{\pi^2/4} \tag{4.37}$$

We now point out the following important features of our result:

1) Note that for a finite r_0 and e^2, the Green's function (4.18) is always finite, even when $\eta \neq \eta'$, i.e. the fermion fields in the Green's function carry a net total charge. This, however, does not mean that charge conservation is violated, since the fermion fields $R_{N\pm}$ and $L_{N\pm}$ are always accompanied by an equal and opposite charge at the monopole core, and hence the net charge carried by all the operators in the Green's function (4.18) is zero.

2) The Green's function (4.18) is suppressed by a power of $(r_0/e^2 r)$ in the limit $r_0 \to 0$. This reflects the fact that the excitation energy of a dyon state is large $(\sim e^2/r_0)$ and hence such excitations become more and more difficult in the $r_0 \to 0$ limit. Note that the suppression factor is present even for $\eta = \eta'$, i.e. even when the net charge carried by the fermion fields in the Green's function is zero. This may be

explained as follows. For $t \neq t'$ the fermion fields $R_{N\eta}(r',t')$ and $R^{\dagger}_{N\eta}(r,t)$ create equal and opposite charges at the monopole core, but at different times and hence we do not avoid creating a high energy state even for $\eta=\eta'$. We expect the suppression factor to disappear for $t=t'$. ($|t-t'| \ll r_0$). This is indeed the case, because at $t=t'$, there is a divergent integral in E_2,

$$\frac{\pi}{2}\,\eta\eta' \int_{r_0}^{r} ds \int_{r_0}^{r'} ds'\ \partial_t\, \partial_{t'}\, \tilde{\Delta}(s,t,s',t')\big|_{t=t'} \tag{4.38}$$

which exactly cancels the divergent term in E_3 for $\eta\eta'=+1$, i.e. if $\eta=\eta'$, and we get a finite answer. In this case we do not create any net charge at the monopole core at any time. We create equal and opposite charges at the points r and r' at time t. In fact definitions (3.1) and (3.19) give the same answer for this particular Green's function since the electric field extends from the point r to r' at time t in both the cases. More generally, for the calculation of any charge neutral <u>condensate</u>, we get the same answer, irrespective of whether we use definitions (3.1) or (3.19) for the gauge invariant fermion fields.

Next, we turn to the Green's functions of the form:

$$\langle 0|\, R^{\dagger}_{N\eta}(r,t)\ L_{N\eta'}(r',t')\,|0\rangle$$

$$= -\eta\eta'\,\frac{hc}{2\pi}\ \langle 0|\, N_\mu \exp\{-i\sqrt{\pi}\,(\varphi_R(r,t)-\eta\int_{r_0}^{r}\dot{\varphi}_R(s,t)\,ds\}$$

$$N_\mu \exp\{\,i\sqrt{\pi}\,(\varphi_L(r,t)-\eta'\int_{r_0}^{r}\dot{\varphi}_L(s,t)\,ds\}\,|0\rangle \tag{4.39}$$

We may analyze this in the same way as (4.18), and obtain an expression similar to (4.19), with the difference that η' is replaced by $-\eta'$ in E_2, E_3 and the multiplicative factor $\eta\eta'\mu c/2\pi$, but not in E_1. We obtain the same suppression factors as in the previous case, except that in this case we get a finite answer in the $r_0 \to 0$ limit if $t=t'$ and $\eta=-\eta'$. (R_η and L_η carry opposite charge, that is why we get a finite answer for $\eta=-\eta'$ in this case, as opposed to for $\eta=\eta'$ in the previous case).

(B) TWO BODY FERMIONIC GREEN'S FUNCTIONS INVOLVING THE OPERATORS DEFINED IN EQ. (3.19):

We shall now turn to the Green's functions involving the fermionic operators defined in Eq. (3.19). The Green's function

$$\langle 0 | \tilde{R}^{+}_{N\eta}(r,t)\ \tilde{R}_{N\eta'}(r',t') | 0 \rangle \tag{4.40}$$

is given by an expression similar to (4.19), with all the integrals $\int_{r_0}^{r,r'}$ in E_1, E_2 and E_3 replaced by the integrals $-\int_{r,r'}^{\infty}$. The result is,

$$\frac{\eta\eta'\mu c}{2\pi} \exp(\tilde{E}_1)\exp(\tilde{E}_2)\exp(\tilde{E}_3) \tag{4.41}$$

where,

$$\tilde{E}_1 = \frac{\pi}{2}\{\tilde{\Delta}_o(r,t,r',t') + \eta\int_r^\infty ds\ \partial_t\ \tilde{\Delta}_o(s,t,r',t')$$

$$+\eta'\int_{r'}^\infty ds'\ \partial_{t'}\tilde{\Delta}_o(r,t,s',t') + \eta\eta'\int_r^\infty ds\int_{r'}^\infty ds'\ \partial_t\ \partial_{t'}\ \tilde{\Delta}_o(s,t,s',t') \tag{4.42}$$

$$\tilde{E}_2 = \frac{\pi}{2}\{\tilde{\Delta}(r,t,r',t') + \eta\int_r^\infty ds\ \partial_t\tilde{\Delta}(s,t,r',t')$$

$$+\eta'\int_{r'}^\infty ds'\ \partial_{t'}\ \tilde{\Delta}(r,t,s',t') + \eta\eta'\int_r^\infty ds\int_{r'}^\infty ds'\ \partial_t\ \partial_{t'}\tilde{\Delta}(s,t,s',t') \tag{4.43}$$

and,

$$\tilde{E}_3 = -\frac{\pi}{4}\{\tilde{\Delta}(r,t,r,t) - \tilde{\Delta}_o(r,t,r,t) + \eta\int_r^\infty ds\ \partial_t\ (\tilde{\Delta}(s,t,r,\tau)$$

$$-\tilde{\Delta}_o(s,t,r,\tau))|_{\tau=t} + \eta\int_r^\infty ds'\ \partial_t\ (\tilde{\Delta}(r,t,s',\tau) - \tilde{\Delta}_o(r,t,s',\tau))|_{\tau=t}$$

$$+\int_r^\infty ds\int_{r'}^\infty ds'\ \partial_t\ \partial_\tau\ (\tilde{\Delta}(s,t,s',\tau) - \tilde{\Delta}_o(s,t,s',\tau))|_{\tau=t}$$

$$+\tilde{\Delta}(r',t',r',t') - \tilde{\Delta}_o(r',t',r',t') + \eta'\int_{r'}^\infty ds\ \partial_{t'}\{\tilde{\Delta}(s,t',r',\tau)$$

$$-\tilde{\Delta}_o(s,t',r',\tau))|_{\tau=t'} + \eta'\int_{r'}^\infty ds'\ \partial_\tau\ (\tilde{\Delta}(r',t',s',\tau) - \tilde{\Delta}_o(r',t',s',\tau))|_{\tau=t'}$$

$$+\int_r^\infty ds\int_r^\infty ds'\ \partial_t\ \partial_\tau\ (\tilde{\Delta}(s,t',s',\tau) - \tilde{\Delta}_o(s,t',s',\tau))|_{\tau=t'}]$$

$$\tag{4.44}$$

The integrals of the form,

$$\int_{r}^{\infty} ds \int_{r'}^{\infty} ds' \; \partial_t \; \partial_{t'} \; (\tilde{\Delta}(s,t,s',t') - \tilde{\Delta}_0(s,t,s',t'))|_{r=r',\, t=t'} \tag{4.48}$$

and,

$$\int_{r}^{\infty} ds \int_{r'}^{\infty} ds' \; \partial_t \; \partial_{t'} \; \tilde{\Delta}(s,t,s',t') \tag{4.49}$$

diverge from the region of large s,s'. We regulate these divergences by quantizing the system in a finite box of radius R (see Appendix B). Then, in the first integral the divergence appears as $(1/\pi)\ln(R\xi)$ where $\xi=\min(1/r,e^2/r_0)$. In the second integral the divergence appears as $(1/\pi)\ln\ (R\chi)$ where $\chi=\min(r^{-1},(r')^{-1},(t-t')^{-1},e^2/r_0)$. If we add the divergent terms from $\tilde{E}_2$ and $\tilde{E}_3$, we get,

$$\frac{1}{\pi} \; \frac{\pi}{2} \; (\eta\eta'-1) \;\; \ln R + \text{finite} \tag{4.50}$$

Thus if $\eta=-\eta'$, the term in the exponential diverges to $-\infty$ in the limit $R\to\infty$, and the Green's function vanishes. On the other hand, if $\eta=\eta'$, i.e. the total charge carried by the fermion fields in the Green's function is zero, the divergent terms in the exponent cancel and we get a finite answer.

The following points are worth mentioning here:

1) The divergences of the integrals (4.48) and (4.49) which make the charge non-conserving Green's functions vanish, is independent of the $r_0\to 0$ limit, it persists for finite r_0. In fact, the region of integration responsible for the divergence is $R^{-1}<<E<<(r')^{-1},r^{-1},(t-t')^{-1},e^2/r_0$, i.e. the region of small E or large s,s' (Appendix B). This is to be contrasted with the region of integration $(r')^{-1},r^{-1},(t-t')^{-1}<<E<<e^2/r_0$, which was responsible for large contribution to the integral (4.33) (See Appendix A).

2) In the cases of physical interest, where $r^{-1},(r')^{-1},(t-t')^{-1}<<e^2/r_0$, the ξ and χ in $\ln(R\xi)$ and $\ln(R\chi)$ are $(r')^{-1},r^{-1}$ or $(t-t')^{-1}$. However if we take the limit $e^2\to 0$, keeping all other quantities fixed, then both ξ and χ are equal to e^2/r_0 and the divergent term is given by $\ln(Re^2/r_0)$. If we now take $e^2\to 0$ limit keeping R fixed, Re^2/r_0 is no longer a large number and there is no divergence in the integrals (4.48) and (4.49). This shows

us how to recover the free field limit for $e^2=0$. This is a consistency check on our calculations, since we know that in the free field limit there exist charge non-conserving Green's functions which are finite. For any finite non-zero e^2, the charge symmetry is restored.

It can easily be checked that the vacuum satisfies the cluster property, i.e. the Green's function

$$\langle 0 | \mathcal{O}^\dagger(r, t+T, r', t'+T) \; \mathcal{O}(r, t, r', t') | 0 \rangle \tag{4.51}$$

where $\mathcal{O}$ is the operator $\tilde{R}^{+}_{N\eta}(r,t)\tilde{R}_{N\eta'}(r',t')$ with $\eta'=-\eta$, vanishes in the $T\to\infty$ limit. The operator $\mathcal{O}^\dagger\mathcal{O}$ as a whole is charge neutral, hence it does not have any lnR term in the exponential. But some of the divergent terms in the exponential have the form $\ln(R/T)$, and the logs of T are left in the exponential after the large logs of R cancel. These logs of T are responsible for the vanishing of the matrix element (4.51) in the $T\to\infty$ limit.

Green's functions of the form $\tilde{R}^{\dagger}_{N\eta}\tilde{L}_{N\eta'}$ may be calculated in a similar way. We find a non-vanishing answer only for $\eta=-\eta'$.

(C) OTHER GREEN'S FUNCTIONS:

Other two body Green's functions of the form $<0|\tilde{R}_{N\eta}\tilde{R}_{N\eta'}|0>$, $<0|\tilde{L}_{N\eta}\tilde{L}_{N\eta'}|0>$, $<0|\tilde{R}_{N\eta}\tilde{L}_{N\eta'}|0>$, and similar Green's functions involving the fields $R_{N\eta}$ and $L_{N\eta'}$ should vanish identically, since they carry a non-zero fermion number, which is an exactly conserved quantum number of the theory. This corresponds to the symmetry $\phi\to\phi+a$ in the bosonized action. In our calculation, these Green's functions have a net multiplicative factor of μ and hence vanish as $\mu\to 0$. [Although in the definition (3.1) μ is any arbitrary mass, we can use the relation ${}_{free}<0|N_\mu(e^A)|0>_{free}=1$ only in the $\mu\to 0$ limit. Since in our calculation we use this result quite often, we must take the $\mu\to 0$ limit in our final result].

Calculation of Green's functions involving more than two fermionic fields either in the present model, or in the model with more than one Dirac doublet of fermions, show the same general features. The Green's function may vanish for two reasons. If it violates a charge which is violated as a consequence of a continuous global symmetry of the theory, then we get a net extra factor of μ in the final answer, which vanishes in the $\mu\to 0$ limit. On the other hand, if the Green's function violates the conservation of the charge associated with the unbroken U(1) generator of the gauge group, then we get a divergent term in the exponential, which makes the integral vanish. Thus, the charge conservation in a monopole-fermion system is a consequence of the Coulomb interaction term. In fact, if e is identically zero, then we may get a finite answer for a charge violating Green's function around the monopole. This is a reflection of the fact that for e=0, the dyon state is degenerate with the monopole ground state, and as a result, the true monopole ground state may be a superposition of states carrying different electric charges.

A sample calculation of a four fermion condensate in a theory with two Dirac doublets of fermions (which is the relevant case for SU(5) GUT) is given in Appendix C. The condensate is chirality conserving but baryon number violating. This calculation illustrates the fact that it is the non-trivial boundary condition at the monopole core, rather than the Adler-Bell-Jackiw anomaly, which is responsible for baryon number violation in the monopole-fermion interaction. Since the existence of this condensate does not depend on the existence of anomaly, it exists even in the presence of arbitrary number of higher generations of massless fermions.

V. DISCUSSION

In this paper we have calculated various fermionic Green's functions for a monopole-fermion system. Although the boundary conditions on the fermion fields are charge exchange ones, so that for a finite size monopole, one may expect the Green's functions to be charge non-conserving in general, we have shown that all the Green's functions obey the charge conservation law.

The crucial observation made in this paper is that in order to define a gauge invariant fermion creation operator, we must also specify the string of electric flux emanating from the fermion. In the present model, we have two choices, we may either take the string to extend from the fermion to the monopole core, or we can take it to extend from the fermion to infinity. In the first case, we create a gauge invariant operator by creating an equal and opposite charge at the monopole core, together with the fermion, hence we essentially create a neutral dyon-fermion system. In the second case we create a gauge invariant operator by creating an equal and opposite charge at infinity. The Green's function involving these two types of operators have drastically different behavior. The fermion creation operators of the first type create a dyon state of energy $\sim e^2/r_0$, where r_0 is the radius of the monopole. As a result, we shall expect the Green's functions involving these operators to be suppressed in the limit $r_0 \to 0$. Our calculation shows that this is indeed the case, these Green's functions carry factors of order $\exp(-c \ln(e^2 r/r_0))$ where c is a constant, irrespective of whether the total charge carried by the fermion fields is zero or not. On the other hand, for finite r_0, these Green's functions are finite, even if the fermion fields involved in these Green's function carry a net total charge. This, however, does not imply non-conservation of charge, since each of these fermion creation operators is charge neutral. Also, if the Green's function involves product of two fermion fields of opposite charge at the same time, then it is finite even in the $r_0 \to 0$ limit, since the fermion creation operators do not create any charge at the monopole core at any time.

The finiteness of the Green's functions at finite r_0 only tells us that the ground state of the monopole-fermion system has a finite probability of making a transition into a virtual dyon-fermion state, in the same way that the vacuum of QED has a finite probability of making a transition into a virtual e^+e^- pair. There is one subtle point which is worth mentioning here. We know that the propagator of a free field of mass m falls off as $\exp(-m|x|)$ as $x \to \infty$, where x is the space-time separation between the two points. This reflects the fact that when we create a particle-antiparticle pair, separated by a distance x, we create a state of energy 2m, and the state must exist at least for a time x/2, before the particle-antiparticle pair may annihilate. Thus the action of the solution, which interpolates between the vacuum and a particle-antiparticle pair separated by a distance x, is of order mx. This produces the suppression factor of $\exp(-mx)$. We have the same situation here, but instead of producing a particle-antiparticle pair, we create a fermion at the point r, and an equal and opposite charge at the monopole core.

This state has an energy of order e^2/r_0, and naively we would expect an exponential suppression of the form $\exp(-ce^2r/r_0)$. However, in the region of physial interest, for which $e^2r/r_0 >> 1$, we only get a suppression factor of the form $\exp(-c\ \ln(e^2r/r_0))$. This is a result of summation of an infinite number of terms in the perturbation series in e^2. Physically this reflects the fact that even when we create a fermion at a point r, and an equal and opposite charge at the monopole core, the system need not be in a state of energy e^2/r_0 for a time of order r. The monopole core may release its charge by emitting charged fermions, which then annihilates the oppositely charged fermion at the point r. As a result, the suppression factor is much softer.

The Green's functions involving the second class of fermion creation operators are finite even in the limit of zero monopole radius, provided the total charge carried by all the fermion operators in the Green's function is zero. This is due to the fact that these fermion creation operators do not create states of large energy. On the other hand, if in the Green's function the total charge carried by the fermion operators is not zero, it vanishes identically, even for finite monopole radius. This shows that the ground state of the fermion-monopole system does not have a finite amplitude for transition into a state containing a neutral core and a set of fermions carrying a net total charge. This establishes the charge conservation rule for a monopole-fermion system.

To calculate the scattering of low energy fermions from the monopole, both in the initial and the final state the monopole core must be kept charge neutral. Hence the fermion creation and annihilation operators that one must use in the calculation are the ones whose electric fields extend to infinity. As a result, the total charge carried by the incoming fermions must be equal to the total charge carried by the outgoing fermions.

Finally, we make a comment on the boundary condition on the field Φ at the monopole core. The condition $\Phi'=0$ may be written in terms of the four dimensional field ψ as,

$$\left(\bar{\psi}_R\ \vec{\gamma}\cdot\hat{r}\ \psi_R - \bar{\psi}_L\ \vec{\gamma}\cdot\hat{r}\ \psi_L\right)\Big|_{r=r_0} = 0 \tag{5.1}$$

which says that the total chiral current flowing into the monopole core at any instant of time is zero. This boundary condition was derived in the absence of the electric field, in which case the chiral current does not have any anomaly and is exactly conserved. However, in the presence of the electric field, the chiral current is no longer conserved because of the anomaly. The non-conservation of the chiral charge outside the monopole core is taken care of by the $e^2\Phi^2/r^2$ in the Hamiltonian [9], but the contribution to the chiral charge non-conservation from inside the monopole core must be taken care of by changing the boundary condition at $r=r_0$. Also, when we take into account the effect of the electric field, the monopole offers a resistance to the flow of a net charge into its core, because of the large energy associated with the electric field. Again, the effect of the electric field energy outside the core is taken care of by the $e^2\Phi^2/r^2$ term in the Hamiltonian, but its effect inside the core must be taken care of by modifying the boundary conditions. Since for $e^2=0$, $\Phi'=0$ is the correct boundary condition, whereas for large e^2/r_0, we expect a charge conserving boundary condition $\Phi=0$, a generous estimate for the modified effective boundary condition is,

$$\Phi' + (\beta e^2/r_0)\,\Phi = 0 \tag{5.2}$$

where β is a constant of order unity. This boundary condition amounts to adding a term $(\beta e^2/Er_0)\cos Er_0$ in the numerator and $(\beta e^2/Er_0)\sin Er_0$ in the denominator of Eq. (4.33). This does not change the results (4.34) or (4.35) so long as β is a finite constant and hence all our results are valid even with the boundary condition (5.2).

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APPENDIX A: EVALUATION OF SOME INTEGRALS

In this appendix, we shall exhibit the details of the calculation which leads to the Eqs. (4.34-4.37) of the text.

The integral we wish to evaluate is of the form,

$$\int_0^\infty \frac{dE}{4\pi E}\left[\left\{\int_{r_0}^{r}\sqrt{Es}\; f_\nu(E,s,r_0)\,E\,ds\right\}\left\{\int_{r_0}^{r'}\sqrt{Es'}\; f_\nu(E,s',r_0)\,E\,ds'\right\}\right.$$

$$\left.-4\sin E(r-r_0)\sin E(r'-r_0)\right] \qquad (A.1)$$

as given in Eq. (4.33) of the text. Here,

$$f_\nu(E,s,r_0) = a_\nu J_\nu(x) + b_\nu J_{-\nu}(x) \qquad (A.2)$$

where x and x_0 denote respectively Er and Er_0. a_ν and b_ν are determined from the boundary condition (4.8) and the normalization condition (4.9). (4.8) gives,

$$\frac{a_\nu}{b_\nu} = -\frac{d}{dx_0}\left(\sqrt{x_0}\,J_{-\nu}(x_0)\right) \Big/ \frac{d}{dx_0}\left(\sqrt{x_0}\,J_\nu(x_0)\right) \qquad (A.3)$$

The normalization condition (4.9) is easy to incorporate at the region of large r,r', where,

$$\sqrt{\frac{\pi x}{2}}\;J_\nu(x) = \cos\left(x-\left(\frac{\nu}{2}+\frac{1}{4}\right)\pi\right) \qquad (A.4)$$

giving,

$$\left(a_\nu^2 + b_\nu^2 - 2a_\nu b_\nu \sin\pi\sigma\right) = 2\pi \qquad (A.5)$$

a_ν and b_ν may be obtained by solving Eqs. (A.3) and (A.5). But just from Eq. (A.5) we can see that,

$$|a_\nu|,\ |b_\nu| \leq \left(1 - \sin^2 \pi\sigma\right)^{-1/2} \sqrt{2\pi} \tag{A.6}$$

since it describes an ellipse.

The integral (A.1), with the co-efficients a_ν, b_ν given above, is too complicated to evaluate exactly. But as we shall show, we may reliably compute the leading contribution for small r_0. The following cases have been examined:

Case (a): $\sigma \ll 1$, $\sigma r/r_0 \gg 1$

Case (b): σ takes special value=1, $r/r_0 \gg 1$

Hereafter we shall concentrate on the case (a), which is of most interest. Calculations for the case (b) can be done in a similar way. We shall briefly comment on it later.

First we need to study the behaviour of the co-efficients $a_\nu(x_0)$ and $b_\nu(x_0)$ for various x_0:

(i) $x_0 \ll 1$: We may use the expression for the Bessel functions near the origin. Then we find, from (A.3),

$$\frac{a_\nu}{b_\nu} = \frac{2^{2\nu}}{2 - \sigma/\nu} \; \frac{\Gamma(\nu)}{\Gamma(1-\nu)} \; \sigma \, x_0^{-1-2\sigma} \tag{A.7}$$

(ii) $x_0 \gg 1$: In this region we may use the asymptotic form (A.4) for the Bessel functions, giving us:

$$a_\nu / b_\nu = \sin(x_0 + \sigma\pi/2) / \cos(x_0 - \sigma\pi/2) \qquad (A.8)$$

(iii) General x_0: For any general x_0, we may make small σ approximation to evaluate the right hand side of (A.3). In this approximation we expand the Bessel's function $J_{\pm\nu}$ about the point $\nu=1/2$ in a power series in $\sigma(\equiv\nu-1/2)$ and keep only terms up to the first power of σ. We use the equations:

$$\sqrt{\frac{\pi\rho}{2}}\, J_{\sigma+1/2}(\rho) \simeq \sin\rho + \sigma\{Ci(2\rho)\sin\rho - Si(2\rho)\cos\rho\}$$

$$\sqrt{\frac{\pi\rho}{2}}\, J_{-\sigma-1/2}(\rho) \simeq \cos\rho - \sigma\{Ci(2\rho)\cos\rho + Si(2\rho)\sin\rho\} \qquad (A.9)$$

to obtain,

$$\frac{a_\nu}{b_\nu} = \frac{\sin x_0 + \sigma(\cos x_0/x_0 + Si(2x_0)\cos x_0 - Ci(2x_0)\sin x_0) + O(\sigma^2)}{\cos x_0 + \sigma(-\sin x_0/x_0 + Ci(2x_0)\cos x_0 + Si(2x_0)\sin x_0) + O(\sigma^2)} \qquad (A.10)$$

Here Ci and Si are the cosine and the sine integral functions, defined by,

$$Ci(x) = \gamma_E + \ln x + \int_0^x \frac{\cos t - 1}{t}\, dt$$

$$Si(x) = \int_0^x \frac{\sin t}{t}\, dt \qquad \gamma_E = \text{Euler's constant} \qquad (A.11)$$

Using the small and large x behavior of the Ci and the Si functions,

$$Ci(x) \sim \ln x + \gamma_E \quad \text{as } x \to 0$$

$$\sim C/x^\rho \quad \text{as } x \to \infty$$

$$Si(x) \sim x \quad \text{as } x \to 0$$
$$\sim \frac{\pi}{2} + \tilde{c}/x^{\rho} \quad \text{as } x \to \infty \tag{A.12}$$

where ρ is a positive constant and $c, \tilde{c}$ are constants, we can verify that we get back (A.7) and (A.8) from (A.10) in the appropriate limits.

We now proceed to evaluate the integral (A.1). To do this, we divide the whole integration region over E into different regions and evaluate the contribution from each region separately.

(i) $0 \leq E \leq \alpha/r$, where α is a small but fixed number: In this region, Er and Er_0 are both small. We may evaluate a_ν and b_ν from Eqs. (A.5) and (A.7), which gives $a_\nu = \sqrt{2\pi}$, $b_\nu = 0$. $J_\nu(Es)$ may be obtained by using the small Es approximation, which gives,

$$\sqrt{\frac{\pi\rho}{2}}\ J_\nu(\rho) \simeq \sqrt{\frac{\pi}{2}} \left(\frac{1}{2}\right)^{\nu} \rho^{\nu+1/2} / \Gamma(\nu+1) \tag{A.13}$$

leading to,

$$I_1 \equiv \frac{1}{4\pi}\int_0^{\alpha r^{-1}} \frac{dE}{E}\left[\left\{\int_{r_0}^{r}\sqrt{Es}\, f_\nu(E,s,r_0)\, E\, ds\right\}\left\{\int_{r_0}^{r'}\sqrt{Es'}\, f_\nu(E,s',r_0)\, E\, ds'\right\}\right.$$

$$\left. - 4\sin E(r-r_0)\,\sin E(r'-r_0)\right]$$

$$\simeq \left(\frac{1}{2}\right)^{2\nu+2}\frac{1}{(\Gamma(\nu+1))^2}\,\frac{\alpha^{2\nu+3}}{(\nu+3/2)^3}\left\{1-\left(\frac{r_0}{r}\right)^{2\nu+3}\right\}$$

$$-\frac{1}{\pi}\left\{\frac{1}{r-r'}\sin\left(\alpha(r-r')/r\right)-\frac{1}{r+r'-2r_0}\sin\left(\alpha(r+r'-2r_0)/r\right)\right\} \qquad (A.14)$$

which is finite in the limit $r_0 \to 0$.

(ii) $\alpha/r \le E \le \alpha\sigma/r_0$: In this region Er_0/σ is small, Er is of order unity or larger. Thus we can still use Eq. (A.7) for evaluating a_ν and b_ν, which gives $a_\nu=\sqrt{2\pi}$, $b_\nu=0$. In evaluating the integral over $\sqrt{ES}\,J_\nu(ES)$, we use the small σ expansion (A.9) for J_ν, which gives,

$$\int_{x_0}^{x} dz\sqrt{\frac{\pi z}{2}}\, J_\nu(z)$$

$$= (\cos x_0-\cos x)+\sigma\left\{Ci(x)-\cos x\, Ci(2x)-\sin x\,(Si(2x)-\pi/2)\right.$$

$$\left. -\frac{\pi}{2}\sin x - Ci(x_0)+\cos x_0\, Ci(2x_0)+\sin x_0\, Si(2x_0)\right\}+O(\sigma^2)$$

$$(A.15)$$

Thus,

$$I_2 \equiv \int_{\alpha/r}^{\alpha\sigma/r_0}\frac{dE}{4\pi E}\left[\left\{\int_{r_0}^{r}\sqrt{Es}\, f_\nu(E,s,r_0)\, E\, ds\right\}\left\{\int_{r_0}^{r'}\sqrt{Es'}\, f_\nu(E,s',r_0)\right.\right.$$

$$\left.\left. E\, ds'\right\} - 4\sin E(r-r_0)\,\sin E(r'-r_0)\right]$$

$$= \frac{1}{\pi}\int_{\alpha/r}^{\sigma\alpha/r_0}\frac{dE}{E}\ \{(\cos x_0 - \cos x)(\cos x_0 - \cos x') + \sigma[(\cos x - \cos x_0)$$

$$\{Ci(x') - Ci(x_0) - \cos x'\, Ci(2x') - \sin x'\, Si(2x') + \cos x_0\, Ci(2x_0)$$

$$+ \sin x_0\ Si(2x_0)\} + (x \leftrightarrow x')] + O(\sigma^2) - \sin(x - x_0)\sin(x' - x_0)\} \tag{A.16}$$

We can further simplify this integral by using small x_0 formulae given in (A.12). Then we obtain,

$$I_2 \simeq \frac{1}{\pi}\int_{\alpha/r}^{\sigma\alpha/r_0}\frac{dE}{E}\ \{(1-\cos x)(1-\cos x') - \sin x \sin x'\}$$

$$+ (\sigma/\pi)\int_{\alpha/r}^{\sigma\alpha/r_0}\frac{dE}{E}\ [\{(1-\cos x')\ (Ci(x) - \cos x\, Ci(2x) - \sin x\,(Si(x) - \tfrac{\pi}{2})$$

$$-(\pi/2)\sin x + \ln 2)\} + \{x \leftrightarrow x'\} + O(\sigma^2)] \tag{A.17}$$

In the second integral in (A.17), the terms with Ci(x), Ci(2x) and (Si(x)$-\pi/2$) give only a finite contribution, since they provide sufficient damping. The rest of the integral can be done with the formulae:

$$\int_a^b (\cos Er/E)\, dE = Ci(br) - Ci(ar)$$

$$\int_a^b (\sin Er/E)\, dE = Si(br) - Si(ar) \tag{A.18}$$

and then using the asymptotic behavior of the Ci and Si functions given in Eq. (A.12). Then we get:

$$I_2 = \frac{1}{\pi}\ (1 + 2\sigma \ln 2)\ \ln\frac{\sigma r}{r_0} + \text{finite} \tag{A.19}$$

(iii) $\alpha\sigma/r_0 \le E \le \beta\sigma/r_0$, β is a large but fixed number: In this region $Er_0 \sim \sigma, Er >> 1$. Thus a_ν and b_ν are both of order unity. We may use Eq. (A.10) to find a_ν and b_ν and Eq. (A.15) to evaluate the integral $\int_{r_0}^{r} \sqrt{Es} J_\nu(Es) E ds$. Using the fact that Er_0 is small, and Er is large, and using the asymptotic behavior of the Ci and the Si functions from Eq. (A.12), we get,

$$\int_{x_0}^{x} dz \sqrt{\frac{\pi z}{2}}\; J_\nu(z) \simeq (1-\cos x) + \sigma\left(\ln z - \frac{\pi}{2} \sin x\right)$$

$$\int_{x_0}^{x} dz \sqrt{\frac{\pi z}{2}}\; J_{-\nu}(z) \simeq \sin x - \frac{\sigma\pi}{2}(1-\cos x) \tag{A.20}$$

Then, since a_ν and b_ν are bounded from above by $\sqrt{2\pi}$,

$$I_3 \equiv \frac{1}{4\pi} \int_{\alpha\sigma/r_0}^{\beta\sigma/r_0} \frac{dE}{E} \left[\left\{ \int_{r_0}^{r} \sqrt{Es}\; f_\nu(E,s,r_0)\; E ds \right\} \right.$$

$$\left. \left\{ \int_{r_0}^{r'} \sqrt{Es'}\; f_\nu(E,s',r_0)\; E ds' \right\} - 4 \sin E(r-r_0) \sin E(r'-r_0) \right] \tag{A.21}$$

is bounded by,

$$I_3 \leq M \int_{\alpha\sigma/r_0}^{\beta\sigma/r_0} dE/E = M \ln \frac{\beta}{\alpha} \tag{A.22}$$

where M is a finite positive number.

(iv) $\beta\sigma/r_0 << E << \alpha/r_0$: In this range $\sigma << x_0 << 1, x >> 1$. The ratio a_ν/b_ν may be determined from Eq. (A.10). Using the limiting behavior of the Ci and the Si functions, we get,

$$\frac{a_\nu}{b_\nu} = \tan x_0 + \sigma/x_0 + O(\sigma \ln x_0) + O(\sigma^2) \tag{A.23}$$

where $0(\sigma . \ln x_0)$ term contains terms proportional to $\sigma . \ln x_0$, and less singular terms in the small x_0 limit. Using the normalization condition (A.5), we get,

$$a_\nu = \sqrt{2\pi}\,(\sin x_0 + \sigma/x_0) + O(\sigma \ln x_0) + O(\sigma^2) \tag{A.24}$$

$$b_\nu = \sqrt{2\pi}\,\cos x_0 + O(\sigma) \tag{A.25}$$

where in the second term of the above equation the $O(\sigma)$ term does not have a σ/x_0 term. We shall now show that the σ/x_0 term in a_ν gives a finite contribution to,

$$I_4 \equiv \int_{\beta\sigma/r_0}^{\alpha/r_0} \frac{dE}{4\pi E} \left\{ \left(\int_{r_0}^{r} f_\nu(E,s,r_0)\sqrt{Es}\, E\, ds \right) \left(\int_{r_0}^{r'} f_\nu(E,s',r_0) \sqrt{Es'}\, E\, ds' \right) - 4 \sin E(r-r_0) \sin E(r'-r_0) \right\} \tag{A.26}$$

To see this, first note that $\int_{r_0}^{r} \sqrt{ES}J_{\pm\nu}(ES)E ds$ is bounded from above by a finite number, so that I_4 is bounded by,

$$\int_{\alpha\sigma/r_0}^{\beta\sigma/r_0} \frac{dE}{E} \left(C_1 a_\nu^2 + C_2 b_\nu^2 + C_3 |a_\nu b_\nu| \right) \tag{A.27}$$

where the C_i's are finite positive constants. Thus the contribution to I_4 from the σ/x_0 term in a_ν is bounded by,

$$C\sigma \int_{\beta\sigma/r_0}^{\alpha/r_0} \frac{1}{E r_0} \frac{dE}{E} = C\left(\frac{1}{\beta} - \frac{\sigma}{\alpha} \right) \tag{A.28}$$

which is finite since β^{-1} and σ/α are both small numbers. Hence we may keep only the zeroth order terms in the expansion of a_ν and b_ν as a power series in σ. Using Eq. (A.20), we get,

$$I_4 = \frac{1}{\pi}\int_{\beta\sigma/r_0}^{\alpha/r_0} \frac{dE}{E} \Big[\{ \sin x_0 (1-\cos x) + \sigma \sin x_0 (\ln 2 - \frac{\pi}{2}\sin x)$$

$$+ \cos x_0 \sin x - \frac{\sigma\pi}{2}\cos x_0 (1-\cos x) \} \{ x \leftrightarrow x' \}$$

$$- \sin(x-x_0)\,\sin(x'-x_0) + \text{finite} \qquad (A.29)$$

Again the $O(\sigma)$ term is bounded by,

$$C\sigma \int_{\beta\sigma/r_0}^{\alpha/r_0} dE/E \sim C\sigma \ln(\alpha/\beta\sigma) \qquad (A.30)$$

and can be ignored. The result is,

$$I_4 = \frac{1}{\pi}\int_{\beta\sigma/r_0}^{\alpha/r_0} \{ \sin x_0 \sin(x'-x_0) + \sin x_0 \sin(x-x_0)$$

$$+ \sin^2 x_0 \} \frac{dE}{E} + \text{finite} \qquad (A.31)$$

These integrals may be expressed in terms of the sine and the cosine integral functions and may be shown to be finite in the $r_0 \to 0$ or $\sigma \to 0$ limit.

(v) $\alpha/r_0 \le E \le \beta/r_0$: In this interval $x_0 \sim 1, x \gg 1$. The analysis of the integral in this interval is similar to that for I_3. We can bound the modulus of the integrand by a finite number, and show that the integral gives at most a finite contribution of order (β/α).

(vi) $\beta/r_0 \le E \le \infty$: In this region $x_0 \gg 1, x \gg 1$. Thus, we may use Eq. (A.8) for the ratio a_ν/b_ν and also the asymptotic form (A.4) for $J_{\pm\nu}(ES)$, getting,

$$\sqrt{Er}\; f_\nu(E,r,r_0) = \cos E(r-r_0) \qquad (A.32)$$

As a result the integrand in (A.1) vanishes identically in this region and hence the contribution to the integral also vanishes.

Thus we see that in the $r_0 \to 0$ limit, the expression (A.1) diverges as,

$$\frac{1}{\pi}\left(1 + 2\sigma \ln 2 + O(\sigma^2)\right) \ln \frac{\sigma r}{r_0} + \text{finite} \qquad (A.33)$$

the finite part does not contain any $\ln\sigma$ term, so that the σ dependence for the leading term is indeed as exhibited.

We now briefly comment on the case (b). In this case we can utilize the fact that the Bessel functions of half integer order can be written in terms of sines and cosines, in particular,

$$\sqrt{\frac{\pi\rho}{2}}\; J_{3/2}(\rho) = \frac{\sin\rho}{\rho} - \cos\rho \qquad (A.34)$$

$$\sqrt{\frac{\pi\rho}{2}}\; J_{-3/2}(\rho) = -\frac{\cos\rho}{\rho} - \sin\rho \qquad (A.35)$$

Using these equations we can find out the leading divergent part in the integral (A.1). This is given by,

$$\frac{\pi^2}{4} \ln\left(\frac{r}{r_0}\right) \qquad (A.36)$$

APPENDIX B: QUANTIZATION IN A FINITE BOX

In this appendix, we discuss the quantization of a boson field Φ in a radial box of inner radius r_0 and outer radius R, satisfying the boundary condition $\Phi'|_{r=r_0}=\Phi'|_{r=R}=0$. First, let us assume that the field Φ satisfies the free field equations of motion. Then we may expand it as,

$$\Phi(r,t) = \frac{\pi}{R}\sum_{n=1}^{\infty}\frac{1}{\sqrt{2\pi}}\frac{1}{\sqrt{2E_n^{(0)}}}\left(a(k_n)\,e^{-iE_n^{(0)}t} + a^{\dagger}(k_n)\,e^{iE_n^{(0)}t}\right)$$

$$2\cos k_n(r-r_0) \equiv \Phi_f^{(-)} + \Phi_f^{(+)} \tag{B.1}$$

with,

$$k_n = n\pi/(R-r_0) \tag{B.2}$$

$$E_n^{(0)} = \sqrt{k_n^2+\mu^2} \tag{B.3}$$

μ being an infinitesimal mass of the Φ field. Φ, as given in (B.1), satisfies the equation,

$$\int_{r_0}^{R}\Phi_f^{(\pm)}(r,t)\,dr = 0 \tag{B.4}$$

since $\sin k_n(R-r_0)=0$. A similar equation is satisfied by any time derivative of $\Phi_f^{(\pm)}$. This implies that in the free field limit, the gauge invariant fermion operators of type II, defined in Eq. (3.19), are identical to the gauge invariant fermion operators of type I, defined in Eq. (3.1). This can be easily seen if we ignore the normal ordering in Eqs. (3.1) and (3.19), but this is also true for the normal ordered operators, as can be shown by using Eq. (3.5).

Another way of showing this is to note that the Green's function,

$$\tilde{\Delta}_0(r,t,r',t') = [\,\Phi^{(-)}(r,t),\ \Phi^{(+)}(r',t')\,]$$

$$= \frac{\pi}{R}\sum_{n=1}^{\infty}\frac{1}{4\pi E_n^{(0)}}\,4\cos k_n(r-r_0)\,\cos k_n(r'-r_0)\,e^{-iE_n^{(0)}(t-t')} \qquad (B.5)$$

satisfies,

$$\int_{r_0}^{R} ds\ \tilde{\Delta}_0(s,t,r,t') = \int_{r_0}^{R} ds'\ \tilde{\Delta}_0(r,t,s',t') = 0 \qquad (B.6)$$

for all values of r, t and t'. Hence any time derivative of $\int\tilde{\Delta}_0 ds$ is also zero. In particular,

$$\int_{r_0}^{R} ds\ \partial_t\,\tilde{\Delta}_0(s,t,r',t') = \int_{r_0}^{R} ds'\ \partial_{t'}\,\tilde{\Delta}_0(r,t,s',t') = 0 \qquad (B.7)$$

and,

$$\int_{r_0}^{R} ds\int_{r_0}^{R} ds'\ \partial_t\,\partial_{t'}\,\tilde{\Delta}_0(s,t,s',t') = 0 \qquad (B.8)$$

Thus in the free field case the fermionic Green's function involving the fields $\tilde{R}_{N\pm}$, $\tilde{L}_{N\pm}$ are identical to those involving the fields $R_{N\pm}$, $L_{N\pm}$, since the former may be obtained by replacing the integrals $\int_{r_0}^{r}$ by $-\int_{r}^{R}$ in expression (3.12). Since we know that the Green's function involving the fields $R_{N\pm}$, $L_{N\pm}$ correctly reproduces the free fermion propagator, so does the Green's functions involving the fields $\tilde{R}_{N\pm}$, $\tilde{L}_{N\pm}$.

Let us now turn to the case of interacting fields, where, by interacting fields, we mean that the field Φ satisfies the equation:

$$\ddot{\Phi} - \Phi'' + (e^2/8\pi^2 r^2)\,\Phi = 0 \tag{B.9}$$

The field Φ may then be expanded as,

$$\Phi(r,t) = \frac{\pi}{R}\sum_{n=1}^{\infty} \frac{1}{\sqrt{2\pi}}\,\frac{1}{\sqrt{2E_n}}\left(a(E_n)\,e^{-iE_n t} + a^{\dagger}(E_n)\,e^{iE_n t}\right)$$

$$\sqrt{E_n r}\;\; f_\nu(E_n, r, r_0, R) \tag{B.10}$$

where,

$$f_\nu(E_n, r, r_0, R) = a_\nu\, J_\nu(E_n r) + b_\nu\, J_{-\nu}(E_n r) \tag{B.11}$$

$$\nu = \left(\frac{1}{4} + \frac{e^2}{16\pi^2}\right)^{1/2} \tag{B.12}$$

a_ν, b_ν are determined from the boundary conditions and the normalization condition:

$$\frac{d}{dr}\left[\sqrt{E_n r}\;\left(a_\nu\, J_\nu(E_n r) + b_\nu\, J_{-\nu}(E_n r)\right)\right]_{r=r_0} = 0 \tag{B.13}$$

$$\frac{d}{dr}\left[\sqrt{E_n r}\;\left(a_\nu\, J_\nu(E_n r) + b_\nu\, J_{-\nu}(E_n r)\right)\right]_{r=R} = 0 \tag{B.14}$$

and

$$\int_{r_0}^{R} \left(a_\nu J_\nu (E_n r) + b_\nu J_{-\nu} (E_n r) \right)^2 E_n r \, dr = R/\pi \qquad (B.15)$$

The ratio a_ν/b_ν may be determined in various limits in the same way as in Appendix A. Hence we can directly use the results of Eqs. (A.7), (A.8) and (A.10) in our calculation. We shall evaluate the integral,

$$\begin{aligned} I &= \int_{r}^{R} ds \int_{r'}^{R} ds' \; \partial_t \, \partial_{t'} \{ \tilde{\Delta}(s,t,s',t') - \tilde{\Delta}_0(s,t,s',t') \} \\ &= \frac{\pi}{R} \sum_{n=1}^{\infty} \frac{1}{4\pi E_n} \{ \int_{r}^{R} ds \; E_n \sqrt{E_n s} \; f_\nu (E_n, s, r_0, R) \} \\ &\quad \{ \int_{r'}^{R'} ds' \, E_n \sqrt{E_n s'} \; f_\nu (E_n, s', r_0, R) \} \, e^{-iE_n (t-t')} \\ &- \frac{\pi}{R} \sum_{n=1}^{\infty} \frac{1}{4\pi E_n^{(0)}} \left(2 \sin E_n^{(0)} (r - r_0) \right) \left(2 \sin E_n^{(0)} (r' - r_0) \right) e^{-iE_n^{(0)}(t-t')} \end{aligned} \qquad (B.16)$$

The right hand side of Eq. (B.16) may be analyzed in the same way as the integral (A.1). In this case, however, we shall simplify our calculation by keeping only the leading divergent terms in the exponent, i.e. by ignoring all divergent terms of order e^2 or higher. Then, after we evaluate a_ν and b_ν by using the appropriate equations (Eq. (A.7), (A.8) or (A.10)), we may replace

$J_{\pm\nu}(Es)$ by $J_{\pm 1/2}(Es)$ in evaluating the integral $\int\sqrt{E}s f_\nu(Es)Eds$, since these integrals are smooth functions of $\sigma=\nu-1/2$ at $\sigma=0$. Thus we may write,

$$f_\nu(E_n, s, r_0, R) \simeq (a_\nu \sin E_n s + b_\nu \cos E_n s)\sqrt{\frac{2}{\pi}} \qquad (B.17)$$

Combining the results (A.7), (A.8), (A.10), (A.5) and (B.17), we get,

$$\begin{aligned}\sqrt{E_n s}\; f_\nu(E_n, s, r_0, R) &\simeq 2\cos E_n(s-r_0) \quad \text{for } E_n r_0 >> \sigma \\ &\simeq 2\sin E_n s \quad \text{for } E_n r_0 << \sigma << 1\end{aligned} \qquad (B.18)$$

E_n is determined from the boundary condition (B.14). Thus,

$$\begin{aligned}E_n &= (n-\tfrac{1}{2})\,\pi/R \quad \text{for } E_n r_0 << \sigma << 1 \\ &= n\pi/R \quad \text{for } E_n r_0 >> \sigma\end{aligned} \qquad (B.19)$$

Hence we are led to,

$$\begin{aligned}\int_r^R \sqrt{E_n s}\; f_\nu(E_n, s, r_0, R)\, E_n\, ds &= \frac{R}{(n-\frac{1}{2})\pi}\, 2\cos E_n r \quad \text{for } E_n r_0 << \sigma << 1 \\ &= -\frac{R}{n\pi}\, 2\sin E_n(r-r_0) \quad \text{for } E_n r_0 >> \sigma\end{aligned}$$

$$(B.20)$$

If we now examine the right hand side of Eq. (B.16), we see that for $E_n r_0 >> \sigma, E_n^{(0)} r_0 >> \sigma$, the contribution to the sum, coming from the $\tilde{\Delta}$ and the $\tilde{\Delta}_0$ terms cancel. Contribution from the region $E_n r_0 \sim \sigma$ may be bounded from above by a finite constant, exactly as we did in Appendix A. Thus, we are left with the contribution,

$$\frac{1}{R} \sum_{n=1}^{(\alpha\sigma/r_0)R/\pi} \Big\{ \frac{R}{(n-\frac{1}{2})\pi} \cos \frac{(n-\frac{1}{2})\pi r}{R} \cos \frac{(n-\frac{1}{2})\pi r'}{R} e^{-i\frac{(n-\frac{1}{2})\pi(t-t')}{R}}$$

$$- \frac{R}{n\pi} \sin \frac{n\pi(r-r_0)}{R} \sin \frac{n\pi(r'-r_0)}{R} e^{-in\pi(t-t')/R} \qquad (B.21)$$

where α is a small but finite number.

We are interested only in the divergent part of the integral. Since each individual term is finite, the only divergence in the $R\to\infty$ limit may come from a sum over infinite number of terms. Hence we may focus our attention on the region of large n. For large n and small r/R, r'/R, we may replace (n-1/2) by n everywhere in the first term inside { }. Also, in the region of summation $n\pi r_0/R$ is a small number and hence we may drop these terms from the second term inside { }. The result is, then,

$$\frac{1}{\pi} \sum_{n=1}^{(\alpha\sigma/r_0)R/\pi} \frac{1}{n} e^{-in\pi(t-t')/R} \cos \frac{n\pi(r+r')}{R} \tag{B.22}$$

If we define,

$$\Theta = \max\left(|t-t'|,\ r+r',\ r_0/\sigma \right) \tag{B.23}$$

then (B.22) may be decomposed as,

$$\frac{1}{\pi} \sum_{n=1}^{\alpha R/\Theta} \frac{1}{n} e^{-in\pi(t-t')/R} \cos \frac{n\pi(r+r')}{R}$$

$$+ \frac{1}{\pi} \sum_{n=\alpha R/\Theta+1}^{(\alpha\sigma/r_0)R/\pi} \frac{1}{n} e^{-in\pi(t-t')/R} \cos \frac{n\pi(r+r')}{R} \tag{B.24}$$

The second term in the above expression is finite, since for $n \gtrsim R/\theta$, at least one of the terms $e^{-in\pi(t-t')/R}$ or $\cos n\pi(r+r')/R$ begins to oscillate, thus guaranteeing the convergence of the sum. In the first term, on the other hand, we may replace both the terms $e^{-in\pi(t-t')/R}$ and $\cos n\pi(r+r')/R$ by unity and the sum diverges in the $R\to\infty$ limit, giving us,

$$\frac{1}{\pi} \ln(R/\Theta) + \text{finite} \tag{B.25}$$

We should remind the reader that in our calculation we have kept only the leading divergent terms in the exponential, as a result we might have lost the divergent terms of the form $e^2 \ln R$. Such terms may be calculated by going through a detailed analysis of the integral, as in Appendix A.

The integral,

$$\frac{1}{4\pi}\int_{R}^{R} ds\ \ \partial_t\ \tilde{\Delta}(s,t,s',t') \qquad (B.26)$$

may be analyzed in a similar way and may be shown to be finite. Thus the divergent terms in the exponent of (4.41) come from the divergences in $\tilde{E}_2$ and $\tilde{E}_3$. The divergence in $\tilde{E}_2$ is due to the integral (4.49) of the text, and that of $\tilde{E}_3$ is from the integral (4.48) and a similar integral with (r,t) replaced by (r',t'). As a result, the net leading divergent term in the exponent of (4.41) is given by,

$$\frac{\pi}{2}\ \frac{1}{4\pi}\left(\eta\eta'-1\right)\ \ln R \qquad (B.27)$$

APPENDIX C: FOUR BODY CHIRALITY CONSERVING CONDENSATE

In this appendix, we shall describe a sample calculation of four fermion condensates in SU(5) grand unified theory with one generation of quarks and leptons. We shall focus on chirality preserving ones, since this discussion lucidly illustrates that the Adler-Bell-Jackiw anomaly is not the primary cause of the baryon number violation.

As has been repeatedly discussed, the four SU(2) Weyl doublets which are relevant in the above setting are

$$\psi^{(1)}_{R,L} = \begin{pmatrix} d_3 \\ e^+ \end{pmatrix}_{R,L} \qquad \psi^{(2)}_{R,L} = \begin{pmatrix} u_2^c \\ u_1 \end{pmatrix}_{R,L} \tag{C.1}$$

For each of these doublets, we need a corresponding boson operator. The following linear combinations of them completely decouple dynamically:

$$\begin{aligned}
\Phi &= \tfrac{1}{2} (\varphi^{(1)}_R - \varphi^{(1)}_L + \varphi^{(2)}_R - \varphi^{(2)}_L) \\
\varphi_1 &= \tfrac{1}{2} (\varphi^{(1)}_R - \varphi^{(1)}_L - \varphi^{(2)}_R + \varphi^{(2)}_L) \\
\varphi_2 &= \tfrac{1}{2} (\varphi^{(1)}_R + \varphi^{(1)}_L + \varphi^{(2)}_R + \varphi^{(2)}_L) \\
\varphi_3 &= \tfrac{1}{2} (\varphi^{(1)}_R + \varphi^{(1)}_L - \varphi^{(2)}_R - \varphi^{(2)}_L)
\end{aligned} \tag{C.2}$$

White ϕ_i $(i=1,2,3)$ remain free fields with infinitesimal mass μ, Φ is the combination which acquires r dependent mass due to the Coulomb interaction.

Let us consider the following operator which preserves both the charge and the chirality (but violates baryon number):

$$\begin{aligned}
\hat{\Theta}(a, b, a', b') &= \bar{\psi}^{(1)} \gamma^\mu (a + b\gamma^5) \tau_+ \psi^{(1)} \\
&\times \bar{\psi}^{(2)} \gamma_\mu (a' + b'\gamma^5) \tau_- \psi^{(2)}
\end{aligned} \tag{C.3}$$

where a,b,a',b' are constants and $\tau_\pm$ are the charge raising and lowering Pauli matrices. Expressed in terms of components, this operator reduces to,

$$\bar{d}_3 \gamma^\mu (a+b\gamma^5)\, e^+\, \bar{u}_1 \gamma^\mu (a'+b'\gamma^5)\, u_2^c \tag{C.4}$$

which is a baryon number violating, but charge and chirality conserving condensate. (A similar operator without $\tau_\pm$ suffers from short distance singularities and will not lead to an unambiguous finite answer). Expanding the fermi fields as in Eq. (2.1), we get,

$$\langle 0|\hat{\mathcal{O}}(a,b,a',b')|0\rangle = -\left(\frac{1}{4\pi\ell^2}\right)^2 \langle 0|\left[(a+b)\, R_-^{(1)\dagger} R_+^{(1)} - (a-b)\, L_-^{(1)\dagger} L_+^{(1)}\right]\left[(a'+b')\, R_+^{(2)\dagger} R_-^{(2)} - (a'-b')\, L_+^{(2)\dagger} L_-^{(2)}\right]|0\rangle$$

$$\eta_-^\dagger \sigma_i \eta_+\, \eta_+^\dagger \sigma_i \eta_- \tag{C.5}$$

where we have used orthogonality property $\eta_\pm^\dagger \eta_\mp = 0$. The Fierz identity for the Pauli matrices gives

$$\eta_-^\dagger \sigma_i \eta_+\, \eta_+^\dagger \sigma_i \eta_- = 2 \tag{C.6}$$

Now we evaluate the four-body condensates appearing in the above expression. Consider for example $\langle 0|R_-^{(1)\dagger} R_+^{(1)} R_+^{(2)\dagger} R_-^{(2)}|0\rangle$. Substituting the bosonized formula (3.1) (since we are evaluating charge neutral condensate, definitions (3.1) and (3.19) give the same answer), and using the rules for combining normal ordered exponentials, we get, after simple calculations,

$$\langle 0| R_-^{(1)\dagger} R_+^{(1)} R_+^{(2)\dagger} R_-^{(2)} |0\rangle = \left(\frac{\mu c}{2\pi}\right)^2 \left(\frac{1}{2\pi \mu c}\right)^2 \langle 0| N_\mu \, e^{-2i\sqrt{\pi} \int_{\pi_c}^{\pi} (\dot{\varphi}_1 + \dot{\varphi}_3) ds} |0\rangle \tag{C.7}$$

The fact that this expression does not involve $\int \dot{\Phi} ds$ or Φ in the exponent is a manifestation of the fact that we are dealing with an operator which does not carry either charge or chirality. As stated before, ϕ_1 and ϕ_3 remain free fields, so the expression $\langle 0|N_\mu(\ )|0\rangle$ in Eq. (C.7) is simply unity. Other condensates can be evaluated in a similar manner. Putting them all together, we obtain,

$$\langle 0| \bar{\psi}^{(1)} \gamma^\mu (a + b\gamma^5) \tau_+ \psi^{(1)} \; \bar{\psi}^{(2)} \gamma_\mu (a' + b'\gamma^5) \tau_- \psi^{(2)} |0\rangle$$

$$= -\frac{8}{(4\pi)^4} \frac{1}{\pi^6} a a' \tag{C.8}$$

Notice that the axial vector parts do not contribute. This is a consequence of a discrete symmetry: It can be easily checked that the Lagrangian of the system and the boundary conditions are invariant under the transformation

$$\psi^{(s)}_{R\alpha i} \to -\epsilon_{\alpha\beta}\, \epsilon_{ij}\, \psi^{(s)\dagger}_{L\beta j}$$

$$\psi^{(s)}_{L\alpha i} \to \epsilon_{\alpha\beta}\, \epsilon_{ij}\, \psi^{(s)}_{R\beta j} \tag{C.9}$$

for both s=1 and s=2 separately. Under this symmetry operation, the axial vector currents change sign while the vector currents do not.

In this example, it is clearly the boundary condition which is responsible for the baryon number violation, while the anomaly is completely irrelevant. As a result, such four body condensates exist even in the presence of higher generation massless fermions, as opposed to the chirality violating condensates, in which the total number of fields in the condensate must be equal to the number of Weyl multiplets.

FOOTNOTES

F1 These boundary conditions are valid only for those modes of excitation for which the energy of excitation (E) is small compared to the inverse radius of the monopole (r_0^{-1}). Since in our calculation most of the important contribution comes from such modes, this is a consistent approximation. For a detailed study of how the boundary conditions change when $Er_0 \gtrsim 1$, see Ref. 5.

F2 At this point, the necessity of working with gauge invariant fermion operators becomes clear. If we work with the bare fermion fields R and L, the point splitting method of Ref. 14 would have given us the current $\bar{\psi}(x+\varepsilon)\gamma^{\mu}\psi(x)$, and not the gauge invariant current $\bar{\psi}(x+\varepsilon)\gamma^{\mu}\exp(-ie\int_x^{x+\epsilon} A\cdot dx)\psi(x)$. This is automatically taken care of by working with the gauge invariant fermion fields R_N, L_N, or $\tilde{R}_N$, $\tilde{L}_N$.

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