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TWISTING CLASSICAL SOLUTIONS IN HETEROTIC STRING THEORY

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ABSTRACT

We show that, given a classical solution of the heterotic string theory which is independent of d of the space time directions, and for which the gauge field configuration lies in a subgroup that commutes with p of the $U(1)$ generators of the gauge group, there is an $O(d) \otimes O(d+p)$ transformation, which, acting on the solution, generates new classical solutions of the theory. With the help of these transformations we construct black 6-brane solutions in ten dimensional heterotic string theory carrying independent magnetic, electric and antisymmetric tensor gauge field charge, by starting from a black 6-brane solution that carries magnetic charge but no electric or antisymmetric tensor gauge field charge. The electric and the magnetic charges point in different directions in the gauge group.

Prefitem

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1. INTRODUCTION

It has been shown previously [1 − 5] that in any string theory, if we look for solutions that are independent of d of the space-time coordinates $\hat{Y}^m$, then the space of such solutions has an $O(d) \otimes O(d)$ or $O(d-1,1) \otimes O(d-1,1)$ symmetry, depending on whether the coordinates $\hat{Y}^m$ have Euclidean or Minkowski signature. (For definiteness, we shall assume from now on that the coordinates $\hat{Y}^m$ have Minkowski signature). Similar symmetries had been seen earlier in the context of supergravity theories [6], and invariance of classical equations of motion of the two dimensional σ model under such transformations of the background was shown in refs.[7][8] following an earlier work of Gaillard and Zumino [9]. Under the action of this $O(d-1,1) \otimes O(d-1,1)$ transformation a given solution is in general mapped to an inequivalent solution. In this paper we shall show that in heterotic string theory, if we look for solutions that are independent of d of the space-time coordinates, and for which the background gauge field lies in a subgroup that commutes with p of the $U(1)$ generators of the gauge group, then the space of such solutions has an $O(d-1,1) \otimes O(d+p-1,1)$ symmetry.† We shall also use this $O(d-1,1) \otimes O(d+p-1,1)$ transformation to generate new classical solutions of heterotic string theory starting from the known ones.

The plan of the paper is as follows. In sect. 2 we present general arguments showing the existence of $O(d-1,1) \otimes O(d+p-1,1)$ symmetry in the heterotic string theory for restricted class of backgrounds of the type mentioned above. In sect. 3 we study the manifestation of this symmetry in the low energy effective field theory. Section 4 contains application of this $O(d-1,1) \otimes O(d+p-1,1)$ transformation on the known solutions of the heterotic string theory, namely, the black p-brane solution in ten dimensions carrying magnetic charge [10]. We show that by applying the $O(d-1,1) \otimes O(d+p-1,1)$ transformation on this solution we can generate a black p-brane solution carrying electric, magnetic and antisymmetric

† Invariance of the classical equations of motion in the two dimensional σ-model under such transformations of the background was discussed in ref.[7].

tensor gauge field charge. We summarise our results in sect. 5.

2. $O(d-1,1) \otimes O(d+p-1,1)$ SYMMETRY IN HETEROTIC STRING THEORY

The origin of the $O(d-1,1) \otimes O(d-1,1)$ symmetry discussed in refs.[1]- [5] can be traced to the fact [3] that if we restrict to backgrounds that are independent of d coordinates $\hat{Y}^m$, then the interaction involving such backgrounds is governed by correlation functions of vertex operators carrying zero $\hat{Y}^m$ momenta in the two dimensional field theory of d scalar fields $\hat{Y}^m$. Such correlation functions factorise into products of correlation functions in the holomorphic sector and the antiholomorphic sector, each of which is separately invariant under the Lorentz transformations involving the coordinates $\hat{Y}^m$. In other words, these correlation functions have an $O(d-1,1) \otimes O(d-1,1)$ symmetry. As a result, the action involving $\hat{Y}^m$ independent background also has an $O(d-1,1) \otimes O(d-1,1)$ symmetry. The manifestation of this symmetry in the context of low energy effective field theory was found in refs.[1][2] (see also refs.[6][7][8]) and was applied in refs.[4][5][11] to generate new classical solutions in string theory from the known ones.

In heterotic string theory, besides the usual space-time coordinates we also have 16 internal coordinates which have only right moving (anti-holomorphic) component but no left moving (holomorphic) component. If we consider backgrounds which are independent of d of the space-time coordinates, and carry zero momentum (and winding number) in p of the 16 internal coordinates, then the previous argument can be generalized easily to conclude that the space of such solutions should have an $O(d-1,1) \otimes O(d+p-1,1)$ symmetry. For the sake of clarity, we shall now present this argument in some detail.

The argument is best presented in the language of string field theory, so let us first assume that there is an underlying string field theory that governs the dynamics of heterotic string theory, and that the vertices in this string field theory are given in terms of correlation functions in the conformal field theory describing the first quantised heterotic string theory. We consider heterotic string theory formulated in 10 dimensional flat space time, although the argument can easily be generalised to the case where some of the dimensions are replaced by an arbitrary (1,0) super-conformal field theory of the correct central charge. Let $\{\Phi_a\}$ denote the set of basis states in the conformal field theory (including the ghost part). The string field is given by $|\Psi\rangle = \sum_a \psi_a|\Phi_a\rangle$, with ψ_a's as the dynamical variables of the string field theory. Then the general N point interaction vertex is given by $\psi_{a_1}\ldots\psi_{a_N} f_N(a_1,\ldots a_N)$, where $f_N(a_1,\ldots a_N)$ denotes a quantity constructed out of an N point correlation function of some conformal transform of the fields $\Phi_{a_1},\ldots\Phi_{a_N}$.‡ For describing the string field configurations which are independent of d of the space-time coordinates (say $\hat{Y}^m$) and carry zero momentum in p of the internal directions associated with the coordinates $\check{Y}^R$ (say), only those components ψ_a will be non-zero, for which the corresponding basis states $|\Phi_a\rangle$ have zero momentum in these d space-time directions, and also zero momentum in these p internal directions. A basis of such states in the conformal field theory can be chosen in the form $|\chi_l\rangle\otimes|\bar{\chi}_{\bar{l}}\rangle\otimes|\Phi'_{a'}\rangle$, where $|\chi_l\rangle$ and $\bar{\chi}_{\bar{l}}\rangle$ are the basis states in the holomorphic and the antiholomorphic sectors respectively of the conformal field theory described by the coordinates $\hat{Y}^m$ and $\check{Y}^R$, and $|\Phi'_{a'}\rangle$ denote the basis of states in the conformal field theory describing the rest of the system. The correlation functions involving these basis states on the sphere factorise into correlation functions involving the states $|\chi_l\rangle$, correlation functions involving the states $|\bar{\chi}_{\bar{l}}\rangle$ and correlation functions involving the states $|\Phi'_{a'}\rangle$. Thus f_N also has the factorized form:

$$f_N(l_1,\bar{l}_1,a'_1,\ldots l_N,\bar{l}_N,a'_N) = f^{(1)}_N(l_1,\ldots l_N) f^{(2)}_N(\bar{l}_1,\ldots\bar{l}_N) f^{(3)}_N(a'_1,\ldots a'_N) \quad (2.1)$$

We now note that $f^{(1)}$ and $f^{(2)}$ are separately invariant under $O(d-1,1)$ and $O(d-1+p,1)$ transformations respectively, which simply correspond to Lorentz

‡ Such a representation for the string field theory action is known explicitly for the bosonic string theory [12 − 14] , but unfortunately not for the heterotic or the super-string theory.

transformations acting on the coordinates $\hat{Y}^m$ and $\check{Y}^R$. (Although the general correlation functions in the conformal field theory describing the internal coordinates $\check{Y}^R$ has no rotational symmetry due to the fact that the torus is not invariant under an arbitrary rotation, in the zero momemtum sector the correlation functions are completely ignorant of the compactification of the coordinates $\check{Y}^R$, and as a result the correlation functions are not only symmetric under a rotation among the coordinates $\check{Y}^R$, but also the ones which mix $\check{Y}^R$ and $\hat{Y}^m$.) This, in turn, implies that restricted to such backgrounds, the string field theory action will have an $O(d-1,1) \otimes O(d+p-1,1)$ symmetry, with the $O(d-1,1)$ transformations acting on the index l of $\psi_{l,\bar{l},a'}$, and the $O(d-1+p,1)$ transformations acting on the index $\bar{l}$ of $\psi_{l,\bar{l},a'}$.

In the absence of a field theory describing the heterotic string theory, the above arguments can be used to establish an $O(d-1,1) \otimes O(d+p-1,1)$ symmetry of the S-matrix elements when the external states are restricted to carry zero momentum in d of the space-time directions and zero charge under p of the U(1) generators of the gauge group. Since the effective action of the theory is constructed from the S-matrix, the $O(d-1,1) \otimes O(d+p-1,1)$ symmetry must manifest itself as a symmetry of the effective action also. Note that this argument holds to all orders in the α' expansion, since nowhere we had to assume that the momenta carried by the external states in directions other than these d directions are small.

Let us now see how this transformation acts on some specific components of the string field. Let $h_{\mu\nu}$, $b_{\mu\nu}$ and $a_{\mu R}$ denote the components of the stri ng field which couple to the graviton, the antisymmetric tensor and the gauge field vertex operators respectively. We shall choose normalizations such that $h_{mn}+b_{mn}$ couples to the vertex operator $c\bar{c}\partial\hat{Y}^m\bar{\partial}\hat{Y}^n e^{i\tilde{k}.\tilde{Y}}$ and a_{mR} couples to the vertex operator $c\bar{c}\partial\hat{Y}^m\bar{\partial}\check{Y}^R e^{i\tilde{k}.\tilde{Y}}$, where $\tilde{Y}^\alpha$ denote the set of coordinates other than $\hat{Y}^m$. Let S and R be the $O(d-1,1)$ and $O(d-1+p,1)$ matrices associated with the Lorentz transformations involving the unbarred and the barred indices respectively. Then the $O(d-1,1)\otimes O(d+p-1,1)$ transformation acts on these fields as:

$$(\, (h'+b') \quad a' \,) = S \, (\, (h+b) \quad a \,) \, R^T \tag{2.2}$$

where $(\, (h+b) \quad a \,)$ is regarded as a $d\times(d+p)$ matrix. Similarly, if $(h_{m\alpha}+b_{m\alpha})$ couples to the vertex operator $c\bar{c}\partial\hat{Y}^m\bar{\partial}\tilde{Y}^\alpha e^{i\tilde{k}.\tilde{Y}}$, $(h_{\alpha m}+b_{\alpha m})$ couples to the vertex operator $c\bar{c}\partial\tilde{Y}^\alpha\bar{\partial}\hat{Y}^m e^{i\tilde{k}.\tilde{Y}}$, and $a_{\alpha R}$ couples to the vertex operator $c\bar{c}\partial\tilde{Y}^\alpha\bar{\partial}\check{Y}^R e^{i\tilde{k}.\tilde{Y}}$, then these fields transform as,

$$\begin{aligned} h'_{m\alpha}+b'_{m\alpha} =& S_{mn}(h_{n\alpha}+b_{n\alpha}) \\ h'_{\alpha m}+b'_{\alpha m} =& (h_{\alpha n}+b_{\alpha n})R_{nm}+a_{\alpha R}R_{Rm} \\ a'_{\alpha R} =& (h_{\alpha n}+b_{\alpha n})R_{nR}+a_{\alpha S}R_{SR} \end{aligned} \tag{2.3}$$

Similar transformation laws can be derived for other fields as well, but we shall not list them here.

Note that the $O(d-1,1)\otimes O(d+p-1,1)$ 'symmetry' described above holds for any background that is independent of d of the space-time coordinates, and is neutral under p of the U(1) subgroups of the gauge group. This includes background massive fields as well.

3. SYMMETRY OF THE LOW ENERGY EFFECTIVE ACTION

Although the general argument guarantees the existence of an $O(d-1,1)\otimes O(d+p-1,1)$ symmetry, realisation of this symmetry in terms of fields that appear in the low energy effective action is somewhat non-trivial, since the explicit relationship between the string field components $h_{\mu\nu}$, $b_{\mu\nu}$ and the fields that appear in the low energy effective action is not known. What we would like to do now is to see how these transformations can be realised in the context of low energy effective field theory. To do this we start with the low energy effective action of heterotic string theory and rewrite it in such a way that its symmetry becomes manifest. The action is given by,

$$S = -\int d^D x \sqrt{\det G} e^{-\Phi} (\Lambda - R^{(D)}(G) + \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} - G^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi + \frac{1}{8} \sum_a F^a_{\mu\nu} F^{a\mu\nu})) \tag{3.1}$$

where $G_{\mu\nu}$, $B_{\mu\nu}$, A^a_μ and Φ denote the graviton, the antisymmetric tensor field, the gauge field, and the dilaton, respectively, $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^{abc} A^a_\mu A^b_\nu$, $H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} +$ cyclic permutations $-(\Omega^{(3)}_A)_{\mu\nu\rho}$, $R^{(D)}$ denotes the D dimensional Ricci scalar, and Λ is the cosmological constant equal to $(D-10)/2$ for heterotic string. $\Omega^{(3)}_A$ is the gauge Chern-Simons term given by $(\Omega^{(3)}_A)_{\mu\nu\rho} = (1/4)(A^a_\mu F^a_{\nu\rho} +$ cyclic permutations $-f^{abc} A^a_\mu A^b_\nu A^c_\rho)$. The full effective action also involves Lorentz Chern Simons term, but these are higher derivative terms and can be ignored to this order. Let us now split the coordinates X^μ into two sets $\hat{Y}^m$ and $\tilde{Y}^\alpha$ ($1 \le m \le d$, $1 \le \alpha \le D-d$) and consider backgrounds independent of $\hat{Y}^m$. Let us further concentrate on backgrounds where the gauge field background lies in a subgroup that commutes with p of the right moving $U(1)$ generators $\bar{\partial} X^I$ associated with the internal coordinates X^I of the heterotic string theory. Let us denote the corresponding internal coordinates by $\check{Y}^R$ ($1 \le R \le p$). Let $\tilde{a}$ denote the gauge indices corresponding to the gauge generators that commute with the $U(1)^p$ subgroup, and lie outside this subgroup. Thus the allowed non-vanishing components of the gauge fields are $A^{\tilde{a}}_\mu$ and A^R_μ.

To begin with, we shall further restrict to background field configurations for which $G_{m\alpha} = B_{m\alpha} = A^R_\alpha = A^{\tilde{a}}_m = 0$; i.e. to backgrounds of the form $G = \begin{pmatrix} \hat{G}_{mn} & 0 \\ 0 & \tilde{G}_{\alpha\beta} \end{pmatrix}$, $B = \begin{pmatrix} \hat{B}_{mn} & 0 \\ 0 & \tilde{B}_{\alpha\beta} \end{pmatrix}$, $A^a_\mu = \{\tilde{A}^{\tilde{a}}_\alpha, \hat{A}^R_m\}$.[⋆] Afterwards we shall see how to write down the transformation laws in the general case when such restrictions are not there. In this case, after an integration by parts, the action

⋆ For such backgrounds, the equations of motion obtained by varying the action with respect to the field components that we have set to zero are satisfied identically. Hence any invariance of the action for such restricted set of backgrounds will also imply invariance of the complete set of equations of motion.

(3.1) can be shown to take the form:

$$S = -\int d^d\hat{Y} \int d^{D-d}\tilde{Y}\sqrt{\det\tilde{G}}e^{-\chi}\Big[\Lambda - \tilde{G}^{\alpha\beta}\tilde{\partial}_\alpha\chi\tilde{\partial}_\beta\chi - \frac{1}{32}\tilde{G}^{\alpha\beta}Tr(\tilde{\partial}_\alpha ML\tilde{\partial}_\beta ML)$$
$$- \tilde{R}^{(D-d)}(\tilde{G}) + \frac{1}{12}\tilde{H}_{\alpha\beta\gamma}\tilde{H}^{\alpha\beta\gamma} + \frac{1}{8}\sum_{\tilde{a}}\tilde{F}^{\tilde{a}}_{\alpha\beta}\tilde{F}^{\tilde{a}\alpha\beta}\Big] \tag{3.2}$$

where,

$$L = \begin{pmatrix} \eta_d & 0 \\ 0 & -\eta_{d+p} \end{pmatrix} \tag{3.3}$$

$$\chi = \Phi - \ln\sqrt{\det\hat{G}} \tag{3.4}$$

and,

$$M = \begin{pmatrix} (K^T-\eta_d)\hat{G}^{-1}(K-\eta_d) & (K^T-\eta_d)\hat{G}^{-1}(K+\eta_d) & -(K^T-\eta_d)\hat{G}^{-1}\hat{A} \\ (K^T+\eta_d)\hat{G}^{-1}(K-\eta_d) & (K^T+\eta_d)\hat{G}^{-1}(K+\eta_d) & -(K^T+\eta_d)\hat{G}^{-1}\hat{A} \\ -\hat{A}^T\hat{G}^{-1}(K-\eta_d) & -\hat{A}^T\hat{G}^{-1}(K+\eta_d) & \hat{A}^T\hat{G}^{-1}\hat{A} \end{pmatrix} \tag{3.5}$$

Here η_m denotes the m dimensional Minkowski metric $diag(-1, 1, \ldots 1)$, $\hat{A}$ is the matrix $\hat{A}_{mR} \equiv \hat{A}^R_m$, and,

$$K = -\hat{B} - \hat{G} - (1/4)\hat{A}\hat{A}^T \tag{3.6}$$

The action (3.2) is manifestly invariant under,

$$M \to \Omega M \Omega^T \tag{3.7}$$

$$\chi \to \chi, \quad \tilde{G}_{\alpha\beta} \to \tilde{G}_{\alpha\beta}, \quad \tilde{B}_{\alpha\beta} \to \tilde{B}_{\alpha\beta}, \quad \tilde{A}^{\tilde{a}}_\alpha \to \tilde{A}^{\tilde{a}}_\alpha \tag{3.8}$$

where,

$$\Omega = \begin{pmatrix} S & \\ & R \end{pmatrix} \tag{3.9}$$

S and R being $O(d-1, 1)$ and $O(d+p-1, 1)$ matrices respectively. At the linearised level, $\hat{G}_{mn} = \eta_{mn} + h_{mn}$, $\hat{B}_{mn} = b_{mn}$ and $\hat{A}^R_m = a_{mR}$, and the transformations given above agree with the transformations of h, b and a given in eq.(2.2). The transformation law of Φ can also be shown to agree with the linearised transformations [3]. Also note that the action is in fact invariant under any $O(d, d+p)$ transformation generated by the matrices Ω satisfying $\Omega L \Omega^T = L$, but the transformations outside the $O(d-1,1) \otimes O(d+p-1,1)$ subgroup can be shown to be pure gauge deformations [3].

One way to derive the transformation laws given in eq.(3.7) under the $O(d-1,1) \otimes O(d+p-1,1)$ transformation is as follows.[⋆] Let us imagine for the time being that all the dimensions associated with the coordinates $\hat{Y}^m$ have been compactified (the effective action does not depend on whether these dimensions are compact or not). In that case the low energy effective field theory involving the moduli of this compact space (together with the moduli associated with the internal coordinates) is governed by the Zamolodchikov metric for these moduli, which, in turn, is invariant under the $O(d, d+p)$ group introduced in refs.[16]. Thus the action of the $O(d-1,1) \otimes O(d+p-1,1)$ group on the various fields may be obtained from the action of this $O(d, d+p)$ group on the moduli space. This action, in turn, may be read out directly from the analysis of ref.[17] and is given by,

$$M \to \Omega M \Omega^T \tag{3.10}$$

where M is the same matrix as given in eq.(3.5) and Ω is an $O(d, d+p)$ matrix which preserves the matrix $\mathrm{diag}(\eta_d, -\eta_{d+p})$.

Let us now consider the case where $G_{\alpha m}$, $B_{\alpha m}$ and $A_{\alpha R}$ are non zero. For simplicity, we shall assume here that the background gauge field is Abelian, and belongs to the $U(1)^{16}$ subgroup of the gauge group. Since this subgroup commutes with all the 16 U(1) generators, we can take $p = 16$. In this case, we shall define

⋆ This argument was pointed out by C. Vafa [15].

a $(2D+16)\times(2D+16)$ matrix $\mathcal{M}$ as:

$$\mathcal{M}=\begin{pmatrix}(\mathcal{K}^T-\eta_D)G^{-1}(\mathcal{K}-\eta_D) & (\mathcal{K}^T-\eta_D)G^{-1}(\mathcal{K}+\eta_D) & -(\mathcal{K}^T-\eta_D)G^{-1}A\\ (\mathcal{K}^T+\eta_D)G^{-1}(\mathcal{K}-\eta_D) & (\mathcal{K}^T+\eta_D)G^{-1}(\mathcal{K}+\eta_D) & -(\mathcal{K}^T+\eta_D)G^{-1}A\\ -A^TG^{-1}(\mathcal{K}-\eta_D) & -A^TG^{-1}(\mathcal{K}+\eta_D) & A^TG^{-1}A\end{pmatrix} \tag{3.11}$$

where,

$$\mathcal{K}=-B-G-(1/4)AA^T \tag{3.12}$$

The gauge index of A now runs over all the 16 coordinates. In this case, the full action can be expressed as,

$$\begin{aligned}-\int d^d\hat{Y}\int d^{D-d}\tilde{Y}e^{-\psi}\Big[\Lambda-G^{\alpha\beta}\tilde{\partial}_\alpha\psi\tilde{\partial}_\beta\psi-\frac{1}{32}G^{\alpha\beta}Tr(\tilde{\partial}_\alpha\mathcal{M}\mathcal{L}\tilde{\partial}_\beta\mathcal{M}\mathcal{L})\\ +\tilde{\partial}_\alpha\psi\tilde{\partial}_\beta G^{\alpha\beta}-\frac{1}{2}P^{\beta\alpha}_{(\alpha\beta)}\Big]\end{aligned} \tag{3.13}$$

where,

$$\mathcal{L}=\begin{pmatrix}\eta_D & 0\\ 0 & -\eta_{D+16}\end{pmatrix} \tag{3.14}$$

and,

$$\psi=\Phi-\ln\sqrt{\det G} \tag{3.15}$$

$P^{\beta\alpha}_{(\alpha\beta)}$ is defined as follows. We first define the matrices:

$$V=\begin{pmatrix}\eta_D/\sqrt{2} & -\eta_D/\sqrt{2} & 0\\ 1/\sqrt{2} & 1/\sqrt{2} & 0\\ 0 & 0 & 1\end{pmatrix} \tag{3.16}$$

$$\check{\mathcal{M}}=\frac{1}{2}V\mathcal{M}V^T=\begin{pmatrix}G^{-1} & -G^{-1}\mathcal{K} & G^{-1}A/\sqrt{2}\\ -\mathcal{K}^TG^{-1} & \mathcal{K}^TG^{-1}\mathcal{K} & -\mathcal{K}^TG^{-1}A/\sqrt{2}\\ A^TG^{-1}/\sqrt{2} & -A^TG^{-1}\mathcal{K}/\sqrt{2} & A^TG^{-1}A/2\end{pmatrix} \tag{3.17}$$

$$\check{\mathcal{L}} = V\mathcal{L}V^T = \begin{pmatrix} 0 & 1_D & 0 \\ 1_D & 0 & 0 \\ 0 & 0 & -1_{16} \end{pmatrix} \tag{3.18}$$

We now define the matrices $P_{(\alpha\beta)}, \ldots, Z_{(\alpha\beta)}$ through the relations:

$$(\partial_\alpha \check{\mathcal{M}} \check{\mathcal{L}} (\check{\mathcal{M}} - \check{\mathcal{L}}) \check{\mathcal{L}} \partial_\beta \check{\mathcal{M}}) = \begin{pmatrix} P_{(\alpha\beta)} & Q_{(\alpha\beta)} & R_{(\alpha\beta)} \\ S_{(\alpha\beta)} & T_{(\alpha\beta)} & W_{(\alpha\beta)} \\ X_{(\alpha\beta)} & Y_{(\alpha\beta)} & Z_{(\alpha\beta)} \end{pmatrix} \tag{3.19}$$

In the above, $P_{(\alpha\beta)}$ is a $D \times D$ matrix. We now define $P^{\mu\nu}_{(\alpha\beta)}$ to be the $\mu\nu$ component of this matrix.

The action given in eq.(3.13) can be shown to be invariant under a transformation of the form:

$$\mathcal{M} \to \Omega \mathcal{M} \Omega^T, \qquad \psi \to \psi \tag{3.20}$$

with,

$$\Omega = \begin{pmatrix} 1_{D-d} & & & \\ & S & & \\ & & 1_{D-d} & \\ & & & R \end{pmatrix} \tag{3.21}$$

where S and R are the $O(d-1,1)$ and $O(d+p-1,1)$ matrices discussed previously (with $p = 16$). Note that when $G_{m\alpha}$, $B_{m\alpha}$ and $A_{\alpha R}$ are zero, these transformation laws are identical to those given in eq.(3.10). Also, these transformations reduce to the ones given in eqs.(2.2) and (2.3) when $G_{\mu\nu} - \eta_{\mu\nu}$, $B_{\mu\nu}$ and $A_{\mu R}$ are small and hence can be identified with $h_{\mu\nu}$, $b_{\mu\nu}$ and $a_{\mu R}$ respectively. The invariance of the action given in eq.(3.13) under the symmetry transformation given in eq.(3.21) follows from the fact that $G^{\alpha\beta}$ and $P^{\beta\alpha}_{(\alpha\beta)}$ remain invariant under these transformations.

Before we conclude this section, let us remark that although the general arguments of sect. 2 guarantee the existence of an $O(d-1,1) \otimes O(d+p-1,1)$

'symmetry' of the string theory for appropriate backgrounds to all orders in α', it does not guarantee that the transformation laws, when expressed in terms of the fields $G_{\mu\nu}$, $B_{\mu\nu}$ and Φ will remain unchanged when we include corrections that are higher order in α'. This is due to the fact that the functional relationship between the string fields and the fields that appear in the effective field theory may undergo modification when we include the effect of higher derivative terms. Evidence of such modification in the transformation laws has already been seen [18] [19] [3].

4. APPLICATION OF THE $O(d-1,1) \otimes O(d+p-1,1)$ TRANSFORMATION

We shall now apply the above transformations to known solutions of heterotic string theory to generate new solutions. In particular we shall take our starting solution to be the black six brane solution of ref.[10] carrying a magnetic charge. (For related work see refs.[$20-35$] .) The solution is given by the following form of the metric and other fields:

$$ds^2 = -\frac{(1-r_+/r)}{(1-r_-/r)}dt^2 + \frac{dr^2}{(1-r_+/r)(1-r_-/r)} + r^2 d\Omega_2^2 + \sum_{i=1}^{6} dX^i dX^i \tag{4.1}$$

$$\Phi = -\ln(1-r_-/r) + \Phi_0 \tag{4.2}$$

$$F^1 = 2\sqrt{2} Q_M \epsilon_2 \tag{4.3}$$

where, $d\Omega_2$ is the line element on a two sphere, and ϵ_2 is the volume form on the same two sphere. Φ_0, r_+ and r_- are the three independent parameters labelling the solution ($r_+ > r_-$), and Q_M is the (quantised) magnetic charge carried by the black hole, given by,

$$Q_M = \sqrt{r_+ r_-/2} \tag{4.4}$$

For definiteness, we have taken the magnetic field to lie in the $U(1)$ subgroup generated by the first internal coordinate. We shall now perform the $O(d-1,1) \otimes$

$O(d+p-1,1)$ transformation on this solution to generate new solutions. To this end, note that the solution is independent of the coordinate t and also the six coordinates X^i, thus here $d=7$. Furthermore, the presence of the magnetic field requires A^1 to have a non-vanishing component tangent to the 2-sphere, thus if we want to satisfy the condition $A_{\alpha R}=0$, we must exclude the direction 1 from the set of directions R. Although we have shown that this is not necessary, we shall first consider this case. Thus here $p=15$. Although we can involve all the 7 space-time coordinates, and all the 15 internal coordinates in the transformation, a general transformation of this kind will generate solutions which will be related by rotation in the external and/or internal space.⋆ Thus the set of inequivalent field configurations are generated by taking the appropriate 'Lorentz transformations'among the coordinate t, one of the space coordinates (say X^1) and one of the internal coordinates (say $\check{Y}^2$). The symmetry group in this case is $O(1,1)\otimes O(2,1)$. The diagonal $O(1,1)$ subgroup corresponds to Lorentz transformation of the solution in the $t-X^1$ space, we may fix a Lorentz frame by choosing the matrix S to be the identity matrix. Thus we are left with the $O(2,1)$ matrix R parametrized by the three Euler angles. A further reduction of the parameters may be made by noting that the original solution is left invariant if we choose R to be a rotation in the $X^1-\check{Y}^2$ plane. Thus the general solution is obtained by taking R to be a boost in the $t-\check{Y}^2$ direction followed by a boost in the $t-X^1$ direction:

$$R=\begin{pmatrix}\cosh\alpha_2 & \sinh\alpha_2 & 0\\ \sinh\alpha_2 & \cosh\alpha_2 & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}\cosh\alpha_1 & 0 & \sinh\alpha_1\\ 0 & 1 & 0\\ \sinh\alpha_1 & 0 & \cosh\alpha_1\end{pmatrix}\tag{4.5}$$

We can now calculate the transformed solution in a straightforward way using

⋆ For $E_8\times E_8$ heterotic string theory rotation among the 15 internal coordinates can generate inequivalent field configurations since $O(16)$ is not a subgroup of the gauge group. But this only changes the direction of the gauge field in the final solution without modifying the essential properties of the solution.

eqs.(3.7), (3.8). The transformed solution is given by,

$$
\begin{aligned}
ds^2 &= -\frac{1}{4(r-r_0)^2}(4(r-r_+)(r-r_-)-(r_+-r_-)^2\beta^2)dt^2+\beta\frac{r_+-r_-}{(r-r_0)}dX^1dt \\
&\quad +\sum_{i=1}^{6}dX^idX^i+\frac{dr^2}{(1-r_+/r)(1-r_-/r)}+r^2d\Omega_2^2 \\
B_{t1} &= \beta\frac{r_+-r_-}{2(r-r_0)} \\
A_t^2 &= \gamma\frac{r_+-r_-}{(r-r_0)} \\
A_1^2 &= 0 \\
F^1 &= 2\sqrt{2}Q_M\epsilon_2 \\
\Phi &= -\ln(1-r_0/r)+\Phi_0
\end{aligned}
\tag{4.6}
$$

where,

$$
\begin{aligned}
\gamma &= \sinh\alpha_1 \\
\beta &= \cosh\alpha_1\sinh\alpha_2 \\
r_0 &= \frac{1}{2}((r_++r_-)-(r_+-r_-)\sqrt{1+\beta^2+\gamma^2})
\end{aligned}
\tag{4.7}
$$

This solution is characterized by an electric field as well as an antisymmetric tensor field strength, given by,

$$
\begin{aligned}
F_{rt}^2 &= \partial_r A_t^2 = -\gamma\frac{(r_+-r_-)}{(r-r_0)^2} \\
H_{rt1} &= \partial_r B_{t1} = -\beta\frac{(r_+-r_-)}{2(r-r_0)^2}
\end{aligned}
\tag{4.8}
$$

Hence, besides carrying the magnetic charge, the new solution carries both, electric and antisymmetric tensor gauge field charge, proportional to γ and β respectively.

Let us now discuss singularities of the solution (4.6). It can be easily seen that the matrix

$$
\begin{pmatrix} G_{tt} & G_{t1} \\ G_{t1} & G_{11} \end{pmatrix}
$$

has zero eigenvalues at $r=r_+$ and $r=r_-$. The component G_{rr} has poles at precisely these values of r, as can be seen from eq.(4.1). These singularities represent coordinate singularities, and can be removed by appropriate coordinate choice. To see this, let us first define new coordinates t', X' and ρ through the relations:

$$\begin{aligned} t =& t' \cosh\theta - X' \sinh\theta \\ X^1 =& X' \cosh\theta - t' \sinh\theta \\ r =& \rho + r_+ \end{aligned} \tag{4.9}$$

where,

$$\tanh\theta = \frac{\beta}{2}\frac{r_+ - r_-}{r_+ - r_0} \tag{4.10}$$

In this coordinate system, the metric near $r = r_+$ takes the form:

$$\begin{aligned} ds^2 =& -\frac{2\cosh\theta\sinh\theta}{\beta(r_+ - r_0)}\rho dt^2(1+\mathcal{O}(\rho)) \\ &+ \frac{2(r_+ - r_0)}{\beta(r_+ - r_-)}\Big[1 - \frac{\beta^2(r_+ - r_-)^2}{4(r_+ - r_0)^2}\Big]^2 \cosh\theta\sinh\theta(dX')^2(1+\mathcal{O}(\rho)) \\ &- \frac{2\rho}{r_+ - r_0}\Big[1 - \frac{1}{4}\frac{\beta^2(r_+ - r_-)^2}{(r_+ - r_0)^2} - \frac{r_+ - r_-}{r_+ - r_0}\Big] \cosh\theta\sinh\theta dX'dt'(1+\mathcal{O}(\rho)) \\ &+ \frac{(r_+)^2}{(r_+ - r_-)\rho}(d\rho)^2(1+\mathcal{O}(\rho)) + (r_+ + \rho)^2(d\Omega_2)^2 + \sum_{i=2}^{6} dX^i dX^i \end{aligned} \tag{4.11}$$

From this we see that the metric has a singularity at $\rho = 0$. This singularity is removed by defining new coordinates u, v as,

$$u = \sqrt{\rho}e^{at'}, \quad v = \sqrt{\rho}e^{-at'} \tag{4.12}$$

where,

$$a = \sqrt{\frac{\cosh\theta\sinh\theta(r_+ - r_-)}{2(r_+)^2(r_+ - r_0)\beta}} \tag{4.13}$$

In this coordinate system the metric takes the form:

$$
\begin{aligned}
ds^2 =&\frac{4(r_+)^2}{(r_+-r_-)}dudv + \frac{2(r_+-r_0)}{\beta(r_+-r_-)}\Big[1-\frac{\beta^2(r_+-r_-)^2}{4(r_+-r_0)^2}\Big]^2\cosh\theta\sinh\theta(dX')^2\\
&+(r_+)^2(d\Omega_2)^2+\sum_{i=2}^{6}dX^idX^i+\mathcal{O}(u,v)
\end{aligned}
\tag{4.14}
$$

From this we see that the metric is non-singular in this coordinate system at $u=0$ or $v=0$. It can also be seen easily that both the electric and the antisymmetric tensor fields are non-singular at $r=r_+$ in the new coordinate system. Similar change of coordinates can also be carried out near $r=r_-$ to show that this also represents a coordinate singularity.⋆ On the other hand, the point $r=r_0$ as well as $r=0$ represents genuine singularities of the solution. ($\Phi\to\pm\infty$ near these points.) Since for real β and γ, $r_0\le r_\pm$, we see that the solution represents a genuine singularity surrounded by two horizons. Solvable conformal field theories corresponding to black string solutions with two horizons have been found previously by Horne and Horowitz [35].

Note that if we take $\beta=0$, then the solution represents the direct product of a four dimensional black hole carrying magnetic and electric charge, and a six dimensional flat space described by the coordinates X^i ($1\le i\le 6$). If we compactify the coordinates X^i (say on a Calabi-Yau manifold, or a six dimensional torus), the result would be a four dimensional black hole carrying electric and magnetic charge. (The full solution, on the other hand, may be regarded as a black string in 5 dimensions by compactifying the coordinates $X^2,\ldots X^6$.) The two charges, however, lie in different U(1) subgroups of the gauge group. These

⋆ In this case we again look for a coordinate transformation of the form $t=t''\cosh\phi-X''\sinh\phi$, $X^1=X''\cosh\phi-t''\sinh\phi$, $r=r_-+\rho'$ as in eq.(4.9), so as to bring the metric in the standard form near the singular surface. It can be easily seen that if $|\beta|<\gamma^2/2$, then it is possible to find a ϕ ($\tanh\phi=\beta(r_+-r_-)/2(r_--r_0)$) for which $G_{t''t''}$ and $G_{X''t''}$ are of order ρ', and $G_{X''X''}$ is of order 1 as $r\to r_-$. On the other hand, if $|\beta|>\gamma^2/2$, then it is possible to find a ϕ ($\coth\phi=\beta(r_+-r_-)/2(r_--r_0)$) for which $G_{X''X''}$ and $G_{X''t''}$ are of order ρ' and $G_{t''t''}$ is of order unity as $r\to r_-$. Thus the global structure resembles that of a Reissner-Nordstrom black hole in the first case, and that of the black string solution of ref.[35][4] in the second case.

solutions are different from the ones discussed in ref.[33] in that in their solution the electric and the magnetic charge lie in the same U(1) subgroups of the gauge group. On the other hand, these solutions can be identified to the black hole solutions of Gibbons and Maeda [28] carrying electric and magnetic charge, if we interprete the electric and magnetic charge in their solution to belong to different U(1) subgroups of the gauge group. (Note that this is the only way to interprete the solutions of Gibbons and Maeda in the context of string theory, since if the electric and the magnetic fields belong to the same U(1) subgroup, we need to take into account the effect of the gauge Chern Simons term coupling to the antisymmetric tensor gauge field strength, which was not included in the analysis of ref.[28].) If we further set the magnetic charge Q_M to zero, the solution reduces to the charged black hole solution of ref.[22]. (Note that the metric $\hat{ds}^2$ considered in refs.[28][22] is related to the metric ds^2 given in eq.(4.6) through the relation $\hat{ds}^2 = e^{-\Phi} ds^2$ [10].)

Since we have derived the transformation laws of various fields under $O(d-1,1) \otimes O(d+p-1,1)$ transformation even when $G_{m\alpha}$, $B_{m\alpha}$ and $A_{\alpha R}$ are non zero, we could, in principle, perform an $O(d-1,1) \otimes O(d+p-1,1)$ rotation that includes the 1 direction of the gauge field. Note, however, that in this case, the initial gauge potential needs to be defined in separate coordinate patches; and are related by a gauge transformation on the overlap. This, in general, implies that the transformed fields also need to be defined in separate coordinate patches, and are related by gauge and general coordinate transformation on the overlap. To see this let us consider the transformation of the fields in the asymptotic region $r \to \infty$, so that $G_{\mu\nu} - \eta_{\mu\nu}$, $B_{\mu\nu}$ and $A_{\mu R}$ are small. If we choose $S = 1$, and R to be a $O(1,1)$ transformation that mixes the t coordinate with the 1 direction in the internal space, the transformed fields take the form:

$$\begin{aligned} G'_{\alpha t} &= \frac{1}{2} \sinh\theta A^1_\alpha \\ B'_{\alpha t} &= \frac{1}{2} \sinh\theta A^1_\alpha \\ A^{1\prime}_\alpha &= A^1_\alpha \cosh\theta \end{aligned} \tag{4.15}$$

where α denotes any of the three directions x, y or z on which the original solution depends. Let A^1_α and $\bar{A}^1_\alpha$ be the components of the original gauge field in the two different coordinate patches, then $A^1_\alpha - \bar{A}^1_\alpha = \partial_\alpha \Lambda$, where Λ is a function which is not single valued under a 2π rotation about the z axis (although $e^{ie\Lambda}$ is). From eq.(4.15) we see that $G'_{\alpha t} - \bar{G}'_{\alpha t}$ is now given by $(1/2)\sinh\theta\partial_\alpha\Lambda$, where G' and $\bar{G}'$ denote the transformed metric in the two coordinate patches. This shows that G' and $\bar{G}'$ are related by a coordinate transformation of the form $t \to t + \Lambda\sinh\theta$. However, since Λ is not a single valued function of the coordinates, this coordinate transformation is not globally well defined.

We could also have started with the metric which represents black holes carrying quantised antisymmetric tensor gauge field charge, instead of magnetic charge. This solution is given by [10]:

$$\begin{aligned}
ds^2 =& -\frac{1 - r_+^2/r^2}{1 - r_-^2/r^2} dt^2 + \frac{dr^2}{(1 - r_+^2/r^2)(1 - r_-^2/r^2)} + r^2 d\Omega_3^2 + \sum_{i=1}^{5} dX^i dX^i \\
\Phi =& -\ln(1 - r_-^2/r^2) + \Phi_0 \\
\tilde{H}_{\alpha\beta\gamma} =& Q(\epsilon_3)_{\alpha\beta\gamma}
\end{aligned} \tag{4.16}$$

where $d\Omega_3$ is the line element on a three sphere, ϵ_3 is the volume form on the same three sphere, and $Q = r_+ r_-$. In this solution, the expressions for G_{tt} and Φ are similar to those given in eqs.(4.1) and (4.2), except that the ratios $r/r_\pm$ are replaced by $(r/r_\pm)^2$. As a result, the final transformed solution will have the same form as given in eqs.(4.6) and (4.7) with r, r_0 and $r_\pm$ replaced by r^2, $(r_0)^2$ and $(r_\pm)^2$ everywhere in the expression for $\hat{G}$, $\hat{B}$ and $\hat{A}$. Thus the final solution will take the form:

$$
\begin{aligned}
ds^2 =& -\frac{1}{4(r^2-r_0^2)^2}(4(r^2-r_+^2)(r^2-r_-^2)-(r_+^2-r_-^2)\beta^2)dt^2+\beta\frac{r_+^2-r_-^2}{r^2-r_0^2}dX^1dt\\
&+\sum_{i=1}^{5}dX^idX^i+\frac{dr^2}{(1-r_+^2/r^2)(1-r_-^2/r^2)}+r^2d\Omega_3^2\\
\hat{B}_{t1} =&\beta\frac{r_+^2-r_-^2}{2(r^2-r_0^2)}\\
A_t =&\gamma\frac{r_+^2-r_-^2}{r^2-r_0^2}\\
A_1 =&0\\
\Phi =& -\ln(1-r_0^2/r^2)\\
\tilde{H}_{\alpha\beta\gamma} =&Q(\epsilon_3)_{\alpha\beta\gamma}
\end{aligned}
\tag{4.17}
$$

where,

$$
r_0^2=\frac{1}{2}((r_+^2+r_-^2)-(r_+^2-r_-^2)\sqrt{1+\beta^2+\gamma^2}) \tag{4.18}
$$

Note that in this case the antisymmetric tensor gauge field has a 'magnetic' type component denoted by $\tilde{H}_{\alpha\beta\gamma}$ and also an electric type component denoted by $\hat{H}_{rt1}\equiv\partial_r\hat{B}_{t1}$. Again, by taking the directions $X^2,\ldots X^5$ to be compact, this solution may be regarded as a black string solution in six dimensions.

In some cases, one can get solvable conformal field theories describing black hole solutions $[36-41]$ [19]. One expects that by twisting these solutions one will get solutions that again correspond to solvable conformal field theories. In fact the black p-brane solution obtained by twisting the solution [3] given in ref [36] are also described by solvable conformal field theories [34] [35].

5. CONCLUSION

In this paper we have shown that given a classical solution of the heterotic string theory which is independent of d of the space-time coordinates, and for which the background gauge field lies in a subgroup that commutes with p of the $U(1)$ generators of the gauge group, we can generate other classical solutions by applying an $O(d-1,1)\otimes O(d+p-1,1)$ transformation on the original solution. By using these transformations on the known black 6-brane solution of the heterotic string theory carrying magnetic charge, we have generated new solutions carrying magnetic, electric and antisymmtric tensor gauge field charge. These solutions are labelled by four continuous and one discrete parameters, characterizing the mass, the electric charge, the antisymmetric tensor gauge field charge, the asymptotic value of the dilaton field, and the magnetic charge of the 6-brane respectively. By compactifying 5 of the directions this solution may be regarded as a black string solution in five dimensions. Using this method we have also constructed black string solutions in six dimensions carrying electric charge, and both, electric and magnetic type antisymmetric tensor gauge field charge.

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