CP-violating $Z\gamma\gamma$ and top-quark electric dipole couplings in $\gamma\gamma \to t\bar{t}$

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Abstract

An effective anomalous CP-violating $Z\gamma\gamma$ coupling can give rise to observable CP-odd effects in $\gamma\gamma \to t\bar{t}$. We study certain asymmetries in the decay lepton distributions in $\gamma\gamma \to t\bar{t}$ arising from top decay in the presence of a CP-violating $Z\gamma\gamma$ coupling as well as a top-quark electric dipole coupling. We find that a photon linear collider with geometric luminosity of 20 fb^{-1} can put limits of the order of 0.1 on the imaginary part of the CP-violating anomalous $Z\gamma\gamma$ coupling using these asymmetries.

While gauge-boson couplings to fermions have been measured with great accuracy and agreement of these measurements with predictions from the Standard Model (SM) is overwhelmingly precise, the area of pure gauge bososn couplings is not explored with that precision. Deviation of the gauge boson couplings from the SM values could be used to infer the presence of new physics. Such couplings arising from new physics could even be CP-violating. There have been detailed discussions on the anomalous triple gauge boson couplings WWV and $Z\gamma V$, where $V = \gamma, Z$, in the literature [1, 2, 3]. Experimental bounds on these couplings obtained at LEP [4, 5, 6, 7] and at the Tevatron [8, 9] are fairly weak, and are found to be consistent with SM. While future experiments would improve these limits, effects of these couplings are expected to be more visible at higher energies, for example, at the proposed linear e^+e^- colliders. CP-violating triple gauge boson couplings get contributions only beyond one loop in SM. This makes them good candidates to study new physics effects.

Theoretical studies have largely concentrated on γW^+W^- and ZW^+W^- couplings, and less attention has been paid to neutral gauge-boson self-couplings. In particular, CP-violating $Z\gamma\gamma$ and $ZZ\gamma$ couplings have been the subject of few discussions. These couplings are absent at the tree level in SM, and any observation of these at a substantial level would signal new physics beyond SM. Our work concerns the measurement of the CP-violating $Z\gamma\gamma$ coupling.

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Most experimental studies of anomalous gauge boson couplings at e^+e^- or hadron colliders would have to deal with the problem of separating not only several different types of form factors for the same effective vertex, but also separating couplings involving $Z\gamma\gamma$ couplings from $ZZ\gamma$ couplings. In this respect, projected $\gamma\gamma$ colliders would have an advantage that with the initial state fixed as $\gamma\gamma$, only the $Z\gamma\gamma$ vertex would contribute, ignoring triple-photon couplings. For this reason we would like to advocate here the use of a $\gamma\gamma$ collider for a study of $Z\gamma\gamma$ couplings.

With these points in mind we discuss in this letter the effect of CP-violating $Z\gamma\gamma$ coupling in the process $\gamma\gamma \to t\bar{t}$. The top quark is expected to decay before it hadronizes [10], and therefore one has the hope of using decay correlations to deduce polarization information of the production process. In this process, however, one has to contend with a possible extra source of CP violation, viz., the CP-violating electric dipole coupling of the top quark. We have also discussed here this possibility, and ways of obtaining separate limits on the CP-violating $Z\gamma\gamma$ and top dipole couplings.

Photon linear colliders have been widely discussed in the literature. In such a collider an intense low-energy laser beam would be scattered in the backward direction by a high-energy electron beam, transferring most of the electron energy to the photon in the process. The photon beam thus produced is made to collide with another photon beam produced in a similar way. The main features of such a photon linear collider are described in Ref. [11]. The luminosity and the polarizations of the photon beams would depend on the initial electron and laser beam helicities as well as their energies. When the electron and the laser beam helicities are of the opposite sign, the photon spectrum peaks at higher energies. Also, in this case, the higher energy photons will have the same helicity as that of the initial electrons [11]. But in general the scattered photon will be in a mixed polarization state. As we shall see, polarization plays an important role in improving the sensitivity of the experiments we suggest. Here we concentrate on longitudinally polarized electron beams and circularly polarized laser beams.

For the purpose of studying the anomalous CP-violating $Z\gamma\gamma$ coupling, we consider the effective Lagrangian

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{SM} + \mathcal{L}_{AC}, \tag{1}$$

where $\mathcal{L}_{SM}$ is the usual SM Lagrangian, and

$$\mathcal{L}_{AC} = \frac{e}{16M_Z^2 \cos\theta_w \sin\theta_w} \left[\lambda_1\, F_{\mu\nu}\, F^{\nu\lambda}\, (\partial_\lambda Z^\mu + \partial^\mu Z_\lambda) + \lambda_2\, F_{\mu\nu}\, F^{\mu\nu}\, \partial^\lambda Z_\lambda \right], \tag{2}$$

with

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \tag{3}$$

A_μ and Z_μ being the photon and Z-boson fields, and λ_1 and λ_2 are dimensionless couplings. $\mathcal{L}_{AC}$ is the most general CP-violating Lagrangian consistent with Lorentz and electromagnetic gauge invariance, if the photons are on-shell, or coupled to conserved currents. In our case, the photons are on-shell. The second term in $\mathcal{L}_{AC}$ is absent if the Z is on-shell or is coupled to a conserved current. The CP-violating Lagrangian with only the first term was used in [3] for calculating CP-violating forward-backward asymmetry in $e^+e^- \to \gamma Z$. It would be possible to include terms with more derivatives on the fields, but these can be taken care of by assuming that $\lambda_{1,2}$ are not just constants, but in momentum space, they are form factors depending on invariants constructed out of momenta.

We now derive the $Z\gamma\gamma$ vertex arising from the above effective Lagrangian $\mathcal{L}_{AC}$. It turns out that the contributions of the two terms in $\mathcal{L}_{AC}$ are proportional to each other, and the final vertex $ie\Gamma^{AC}_{\alpha\beta\mu}$ can be written as

$$
\begin{aligned}
\Gamma^{AC}_{\alpha\beta\mu}(k_1, k_2, q) \;=\;& \frac{i\lambda}{16\, M_Z^2\, \cos\theta_w \sin\theta_w} \\
& \times \big[\, g_{\alpha\nu}\, (k_2.q\, k_{1\beta} - k_1.k_2\, q_\beta) \\
& + g_{\beta\nu}\, (k_1.q\, k_{2\alpha} - k_1.k_2\, q_\alpha) \\
& - g_{\alpha\beta}\, (k_2.q\, k_{1\nu} + k_1.q\, k_{2\nu}) \\
& + k_{2\alpha} q_\beta k_{1\nu} + k_{2\nu} q_\alpha k_{1\beta}\,\big]
\end{aligned}
\tag{4}
$$

where k_1, k_2 and q are the four momenta of the photons and the Z boson, and α, β and μ are the corresponding Lorentz indices. Now λ is a linear combination (albeit momentum dependent) of λ_1 and λ_2. We will henceforth discuss constraints only on this combined form factor λ.

The process $\gamma\gamma \to t\bar{t}$ gets contribution, apart from the standard model t exchange diagrams, also from the anomalous $Z\gamma\gamma$ vertex, with a virtual Z exchanged in the s channel. (We neglect a possible $\gamma\gamma\gamma$ vertex). Using the method discussed by Vega and Wudka [12] we compute the helicity amplitudes for the process $\gamma\gamma \to t\bar{t}$. It turns out that the CP-violating $Z\gamma\gamma$ coupling contributes only when both of the photon beams have the same helicity, as well as the top quark and the top antiquark have the same helicity. The amplitude in this case is given below, including the effect of the top-quark electric dipole form factor (EDFF). The EDFF occurs in the Lagrangian term,

$$
\mathcal{L}_{edff} \;=\; ie\, d_t\, \overline{\psi}_t\, \sigma^{\mu\nu}\, \gamma_5\, \psi_t\, F_{\mu\nu},
\tag{5}
$$

and its effects were discussed in the earlier work [13, 14].

The helicity amplitudes for the process $\gamma\gamma \to t\bar{t}$ with these two CP-violating couplings, for the case when the two photons have equal helicities, as do the t and $\bar{t}$, are given by

$$
\begin{aligned}
M(\lambda_\gamma,\; \lambda_\gamma, \lambda_t,\; \lambda_t) \;=\;& -\frac{4m_t\, e^2\, Q_t^2}{\sqrt{s}(1 - \beta_t^2 \cos^2\theta_t)} \left\{ (\lambda_\gamma + \lambda_t \beta_t) \right. \\
& \left. -i\, d_t\, 2m_t \left[2 + \frac{s}{4m_t^2}\beta_t(\beta_t - \lambda_t\lambda_\gamma)\sin^2\theta_t \right] \right\} \\
& + ie^2\, \lambda\, \frac{m_t}{8\sqrt{s}\, x_w(1 - x_w)} \left(\frac{s}{4m_Z^2} \right)^2,
\end{aligned}
\tag{6}
$$

where λ_γ, $\lambda'_\gamma, \lambda_t$, $\lambda_{\bar{t}}$ in $M(\lambda_\gamma,\; \lambda'_\gamma, \lambda_t,\; \lambda_{\bar{t}})$ are the helicities of the two photon beams and the top quark and the top antiquark. $\sqrt{s}$ is the c. m. energy, m_t and m_Z are the top quark and Z boson masses, Q_t is the electric charge of the top quark, x_w is given in terms of the weak mixing angle θ_w by $x_w = \sin^2\theta_w$, and β_t and θ_t are the velocity and the scattering angle of the top quark in the c.m. frame. We have dropped terms quadratic in d_t. Rest of the amplitudes do not depend on the $Z\gamma\gamma$ coupling and are given in Ref. [13]. It is interesting to note that in the last term in the amplitude, not only is the factor of $(s - m_Z)^{-2}$ from the Z propagator cancelled by a factor coming from the anomalous vertex, but there is an additional $s^{3/2}$ dependence which increases with energy.

We construct CP-violating asymmetries which can be used to study the effect of the new coupling in experiments. In principle, there would be definite predictions for top

and antitop polarizations in the presence of CP-violating terms. These could be used to isolate CP violation. In particular, the anomalous coupling contributes only to the amplitude with equal t and $\bar{t}$ helicities. The interference of this amplitude with the standard model amplitude can give rise to definite prediction for the polarizations. However, it is the final top decay products which have to be used to analyze the polarization. From a practical point of view, it is better to work with asymmetries in terms of the decay leptons.

The asymmetries discussed below were studied earlier [13] in the context of CP violation effects induced by a possible top-quark electric dipole form factor (EDFF). These asymmetries are (i) the asymmetry in the number of leptons and the antileptons produced as decay products of the top quark and the top antiquark (the charge asymmetry) and (ii) the sum of the forward-backward asymmetries of the leptons and the antileptons. Being independent of the top quark momentum these asymmetries are experimentally favourable.

The two asymmetries are written in terms of the differential cross section as follows.

$$A_{ch}(\theta_0) = \frac{\int_{\theta_0}^{\pi-\theta_0} d\theta_l \left(\dfrac{d\sigma^+}{d\theta_l} - \dfrac{d\sigma^-}{d\theta_l} \right)}{\int_{\theta_0}^{\pi-\theta_0} d\theta_l \left(\dfrac{d\sigma^+}{d\theta_l} + \dfrac{d\sigma^-}{d\theta_l} \right)} \tag{7}$$

and

$$A_{fb}(\theta_0) = \frac{\int_{\theta_0}^{\frac{\pi}{2}} d\theta_l \left(\dfrac{d\sigma^+}{d\theta_l} + \dfrac{d\sigma^-}{d\theta_l} \right) - \int_{\frac{\pi}{2}}^{\pi-\theta_0} d\theta_l \left(\dfrac{d\sigma^+}{d\theta_l} + \dfrac{d\sigma^-}{d\theta_l} \right)}{\int_{\theta_0}^{\pi-\theta_0} d\theta_l \left(\dfrac{d\sigma^+}{d\theta_l} + \dfrac{d\sigma^-}{d\theta_l} \right)}. \tag{8}$$

In the above equations, $\frac{d\sigma^+}{d\theta_l}$ and $\frac{d\sigma^-}{d\theta_l}$ refer respectively to the l^+ and l^- distributions in the c.m. frame of the $\gamma\gamma$ pair, θ_l is the polar angle of l^+ or l^- and θ_0 is the cut-off in θ_l in the forward and backward directions. Not only is a cut-off in θ_l necessary from the experimental detection point of view, it also helps to tune the sensitivity of the experiments, as discussed below.

The asymmetries discussed above being CPT-odd should be proportional to the absorptive part of the amplitude.[3]

Photon-photon collisions would be achieved at an e^+e^- linear collider. So the actual collision rate for e^+e^- into a given final state is a convolution of the two-photon collision rate into that final state with the spectra of photons from laser backscattering. Denoting the effective two-photon luminosity by $L_{\gamma\gamma}$, we use the expression for the differential luminosity $dL_{\gamma\gamma}$ derived in [11]. We refer the reader to this work for details. Since it is easier to calculate the differential cross section in the center of mass (c.m.) frame of the $\gamma\gamma$ pair, we modify the above expression changing the limits to take care of the boost needed to go to the lab frame from the $\gamma\gamma$ c.m. frame. We can then write

$$\begin{aligned} A_{ch} &= \frac{1}{2N} \left\{ \int \frac{dL_{\gamma\gamma}}{d\omega_1 \, d\omega_2} \, d\omega_1 \, d\omega_2 \int_{-1}^{1} d\cos\theta_t \right. \\ &\quad \times \left. \int_{g(\theta_0)}^{f(\theta_0)} d\cos\theta_l \left[\frac{d\sigma^+}{d\cos\theta_t \, d\cos\theta_l} - \frac{d\sigma^-}{d\cos\theta_t \, d\cos\theta_l} \right] \right\}, \end{aligned} \tag{9}$$

[3]Here T refers to the naive time-reversal operator, which reverses spins and momenta of the particles involved, while not interchanging the initial and final states.

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and

$$A_{fb} = \frac{1}{2\,N} \int \frac{dL_{\gamma\gamma}}{d\omega_1\,d\omega_2}\,d\omega_1\,d\omega_2 \int_{-1}^{1} d\cos\theta_t$$
$$\times \left\{ \int_{-\beta_\gamma}^{f(\theta_0)} d\cos\theta_l \left[\frac{d\sigma^+}{d\cos\theta_t\,d\cos\theta_l} + \frac{d\sigma^-}{d\cos\theta_t\,d\cos\theta_l} \right] \right.$$
$$\left. - \int_{g(\theta_0)}^{-\beta_\gamma} d\cos\theta_l \left[\frac{d\sigma^+}{d\cos\theta_t\,d\cos\theta_l} + \frac{d\sigma^-}{d\cos\theta_t\,d\cos\theta_l} \right] \right\}. \qquad (10)$$

Here

$$f(\theta_0) = \frac{\cos\theta_0^{cm} - \beta_\gamma}{1 - \beta_\gamma \cos\theta_0^{cm}}$$

and

$$g(\theta_0) = \frac{\cos(\pi - \theta_0^{cm}) - \beta_\gamma}{1 - \beta_\gamma \cos(\pi - \theta_0^{cm})}$$
$$= \frac{-\cos\theta_0^{cm} - \beta_\gamma}{1 + \beta_\gamma \cos\theta_0^{cm}},$$

ω_1 and ω_2 are the two photon beam energies in the lab frame, θ_t is the scattering angle of the top quark in the c.m. frame of the $t\bar{t}$ system, θ_0^{cm} is the cut-off in this c.m. frame, and $\beta_\gamma = (\omega_1 - \omega_2)/(\omega_1 + \omega_2)$. N is the total number of events produced.

It is most advantageous to make use of the the semileptonic decay of $t\bar{t}$ pair, wherein either of t or $\bar{t}$ decays leptonically, while the other decays hadronically. While hadronic decays have large branching ratios, purely hadronic events are difficult to detect because of the large background. On the other hand, branching ratio for decay into leptons is small, even though the signal is clear. With a semileptonic final state, the overall branching ratio is still not too low, and since our asymmetries involve measurement only on a single lepton, the signal is easily measureable. In our calculations, all integrations except the ω_1, ω_2 and θ_t are done analytically. These three integrations are done numerically.

Sensitivity of the measurement of these asymmetries at specific colliders can be calculated considering the statistical fluctuations. For the asymmetry to be observable at the 90% confidence level (C.L.), the number of asymmetric events should be larger than $1.64\,\sqrt{N}$, where N is the total number of semi-leptonic events produced. This means that the asymmetry has to have a minimum value of

$$A_{min} = \frac{1.64}{2\sqrt{N}}. \qquad (11)$$

We have kept only the linear terms in the anomalous coupling, assuming that the anomalous coupling is small, and that higher-order terms are negligible. Thus the asymmetry can be written as

$$A = C_{ac}\,\mathrm{Im}\lambda, \qquad (12)$$

where the coefficient C_{ac} is independent of the coupling parameter, λ.

Hence the asymmetries will be proportional to the imaginary part of the couplings. Thus eqn. 12 would give a limiting value for the coupling

$$\mathrm{Im}\lambda\,|_{max} = \frac{1.64}{2\sqrt{N}}\,\frac{1}{C_{ac}} \qquad (13)$$

in the case that the symmetry is not observed.

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In an earlier work [13] we had considered these same asymmetries arising due to the top quark electric dipole form factor (EDFF) in the process $\gamma\gamma \to t\bar{t}$. If we consider the simultaneous presence of both the EDFF and the $Z\gamma\gamma$ coupling, the asymmetry would be a function of these two parameters. In such a case we could get limits on both these parameters simultaneously by plotting a contour in the parameter plane. This is done as follows.

In the presence of both these sources of CP violation, the asymmetry could be written as

$$A = C_{ac} \operatorname{Im}\lambda + C_{edff} \operatorname{Im}\tilde{d}_t. \tag{14}$$

We have redefined the EDFF (see eqn. 5) in terms of a dimensionless parameter: $d_t = \tilde{d}_t/2\,m_t$. C_{ac} and C_{edff} are coefficients independent of the parameters, λ and $\tilde{d}_t$.

To determine the sensitivity of the measurement, we now need the expression corresponding to two degrees of freedom. For the asymmetry to be observable, in this case, the number of asymmetric events should be greater than $2.15\sqrt{N}$. This gives a linear relation between the 90% C.L. limiting values of the two parameters:

$$C_{ac} \operatorname{Im}\lambda^{max} + C_{edff} \operatorname{Im}\tilde{d}_t^{max} = \pm\frac{2.15}{2\sqrt{N}}. \tag{15}$$

The $\pm$ comes in because the asymmetry could be of either sign. Contours plotted in the $\operatorname{Im}\lambda - \operatorname{Im}\tilde{d}_t$ plane would give a band of allowed values of $\operatorname{Im}\lambda$ and $\operatorname{Im}\tilde{d}_t$ for a single asymmetry. Using more than one asymmetry will give an allowed area, which is the area of intersection of the bands obtained for the individual asymmetries. Alternatively, a single asymmetry can be used with two different polarization combinations for the initial beams, and a similar allowed region of intersection can be determined.

For our numerical calculations, we have assumed that the electron beams have axial symmetry and a Gaussian distribution. We also assume that the conversion distance, i.e., the distance between the conversion points of the lasers and the interaction point of the colliding photons, is negligible. With these assumptions, as discussed in [11], the expressions for the cross sections for the case with longitudinally polarized electrons and circularly polarized laser photons simplify considerably.

We have assumed, for most of our calculations, a cut-off of at least $30°$ in the forward and backward directions of the lepton momentum. This should be sufficient for the practical purposes of suppressing the background of forward (and backward) moving particles due to standard-model processes. However, if a minimum energy or transverse momentum cut-off for the detection of leptons is required, our results would still be valid with such a cut-off, since most events with a lepton energy less than about 45 GeV are found to be suppressed.

We discuss our numerical results in the following.

We first switch off the dipole term and consider the effect of the triple gauge boson ($Z\gamma\gamma$) coupling. We study the asymmetries varying the helicities of the initial beams, the cut-off angle and the beam energy. The forward-backward asymmetry is seen to be more sensitive in all cases.

Studying the asymmetries for different combinations of the initial electron and laser beam helicities, it is seen, as expected, that the forward-backward asymmetry is absent when both of the electron beams have the same helicity as well as both of the laser beams have the same polarization. This is because, in this case, the two photon beams are identical and there is no distinction between the forward and the backward directions.

Table 1 displays the asymmetries and the limits which can be obtained from them for different helicity combinations for an initial electron beam energy of 250 GeV, a laser

λ_e^1	λ_e^2	λ_l^1	λ_l^2	N	A_{ch}	A_{fb}	$|A_{ch}|$	$|A_{fb}|$
					Asymmetries		Limits on Im λ from	
-0.5	-0.5	-1	-1	55	-0.0031	0.000	35.098	
-0.5	-0.5	-1	1	215	-0.0049	0.412	11.361	0.136
-0.5	-0.5	1	1	631	-0.0090	0.000	3.637	
-0.5	0.5	-1	-1	62	-0.0035	-0.403	29.502	2.569
-0.5	0.5	-1	1	23	-0.0037	0.256	50.354	0.661
-0.5	0.5	1	-1	163	-0.0004	-0.101	144.456	0.635
Unpolarized				179	-0.0056	0	11.004	

Table 1: Asymmetries and limits on the coupling obtained from them at different helicity combinations. Asymmetries are for Im $\lambda = 1$. Numbers are obtained assuming an integrated luminosity of 20 fb^{-1}. Initial electron beam energy is taken to be 250 GeV and a cut-off angle of $30°$ is assumed.

beam energy of 1.24 eV, an integrated geometrical luminosity of 20 fb^{-1} for the e^+e^-, and a cut-off angle $\theta_0 = 30°$. We find that the best limits would be obtained from the charge asymmetry when both $\lambda_e^1 = \lambda_e^2$ and $\lambda_l^1 = \lambda_l^2$ but the product, $2\lambda_e^i\lambda_l^i = -1$, where λ_e^i and λ_l^i are the electron and laser helicities. We get a limit of 3.6 on Im λ in this case. On the other hand the forward-backward asymmetry gives best limits when $\lambda_e^1 = \lambda_e^2$ and $\lambda_l^1 = -\lambda_l^2$. The limit obtained in this case is 0.14. We have considered this helicity combination while studying the variation of asymmetry with other parameters like the cut-off angle or energy.

It is clear from Table 1 that the limits that would be obtained in the absence of polarization are poor, and the importance of using polarized beams cannot be overemphasized.

We next consider the variation of asymmetries with the cut off angle. The result is shown in Table 2. As expected, there is no charge asymmetry in the absence of a cut-off. This is because when there is no cut-off, the asymmetry is just the difference in the number t and $\bar{t}$, which is zero from charge conservation. We see from the table that the limit from charge asymmetry is best for a cut-off around $60°$, while the limit from the forward-backward asymmetry gets better for smaller cut-off angles.

Asymmetries are best for higher $x = 4E_b\omega_0/m_e^2$ (ω_0 is the laser beam energy and E_b, the electron beam energy) values. However, there is a limit to which the x value can be increased. For $x > 4.83$ e^+e^- production due to the collision of high energy photon beam with laser beam is considerable [11]. This introduces additional e^+e^- beam backgrounds as well as degrading the photon spectrum. We use a value of 4.75 for x. With higher E_b the sensitivity increases considerably upto a point, and then increases more slowly. The improvement is by an order of magnitude in going from $E_b = 250$ GeV to $E_b = 500$ GeV. Table 3 displays the values obtained with varying electron beam energy.

Considering the case where both the EDFF and the triple gauge boson coupling λ are present, we get simultaneous limits on these parameters by plotting contours in the Imλ − Im$\tilde{d}_t$ plane. We get a band of allowed region in the parameter space for each

		Asymmetries		Limits on Im λ from	
θ_0 (deg.)	N	A_{ch}	A_{fb}	$|A_{ch}|$	$|A_{fb}|$
0	249	0.0000	0.476		0.109
10	245	−0.0006	0.469	87.339	0.112
20	233	−0.0023	0.447	23.192	0.120
30	215	−0.0049	0.412	11.361	0.136
40	189	−0.0081	0.364	7.340	0.164
50	159	−0.0115	0.305	5.668	0.213
60	123	−0.0146	0.237	5.049	0.311
70	84	−0.0172	0.162	5.202	0.551
80	42	−0.0188	0.082	6.660	1.525

Table 2: Variation of asymmetries and limits on the couplings obtained from them with cut-off angle. Asymmetries are for Im $\lambda = 1$. Numbers are obtained assuming an integrated luminosity of 20 fb^{-1}. Initial electron beam energy is taken to be 250 GeV and the helicity combination considered is $\lambda_e^1 = -0.5$, $\lambda_e^2 = -0.5$, $\lambda_l^1 = -1$ and $\lambda_l^2 = 1$.

		Asymmetries		Limits on Im λ from	
E_b (GeV)	N	A_{ch}	A_{fb}	$|A_{ch}|$	$|A_{fb}|$
250	215	−0.005	0.412	11.361	0.136
500	1229	−0.348	3.914	0.067	0.006
750	1032	−1.087	6.695	0.024	0.004
1000	850	−1.879	8.142	0.015	0.004

Table 3: Asymmetries and the limits at different electron beam energies. Cut off is taken to be 30° and a helicity combination of $\lambda_e^1 = -0.5$, $\lambda_e^2 = -0.5$, $\lambda_l^1 = -1$ and $\lambda_l^2 = 1$ is considered. Asymmetries are for Im $\lambda = 1$. An integrated luminosity of 20 fb^{-1} is assumed.
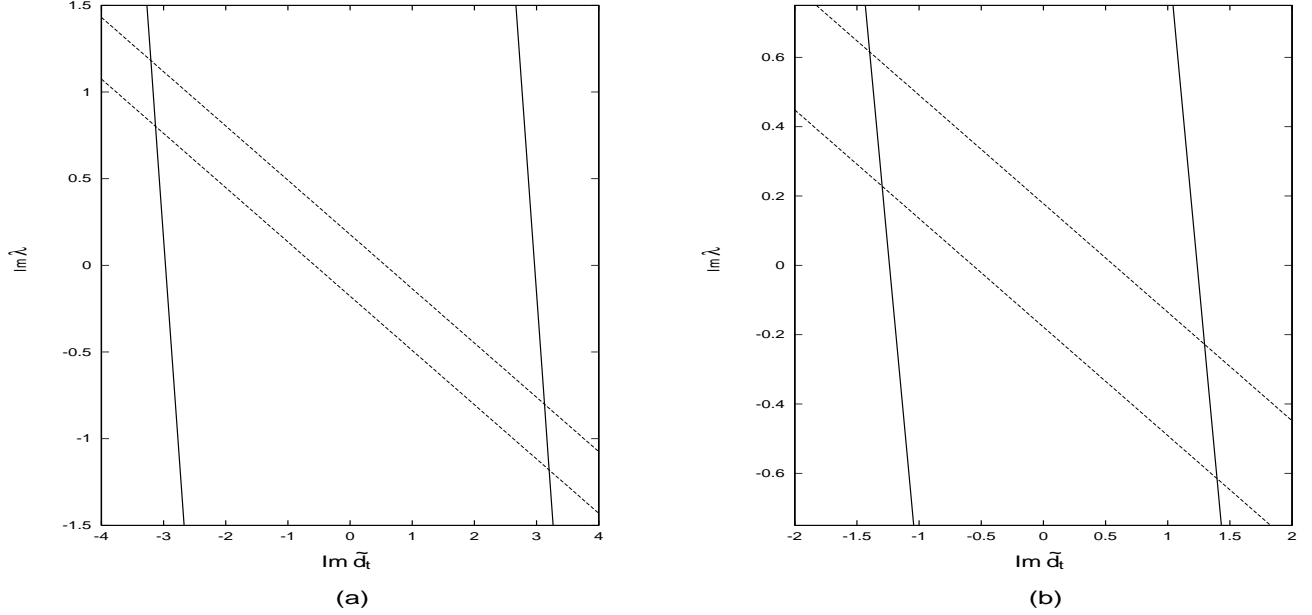
Figure 1: (a) Contours in the $\text{Im}\lambda - \text{Im}\tilde{d}_t$ plane obtained from the charge asymmetry (solid lines) and forward-backward asymmetry (dashed lines) with helicity combination $\lambda_e^1 = \lambda_e^2 = -0.5$, $\lambda_l^1 = -\lambda_l^2 = -1$. (b) Contours in the $\text{Im}\lambda - \text{Im}\tilde{d}_t$ plane obtained from the charge asymmetry with the helicity combination $\lambda_e^1 = \lambda_e^2 = -0.5$, $\lambda_l^1 = \lambda_l^2 = 1$ (solid lines) and forward-backward asymmetry with the helicity combination $\lambda_e^1 = \lambda_e^2 = -0.5$, $\lambda_l^1 = -\lambda_l^2 = -1$ (dashed lines). A cut-off $\theta_0 = 30°$, initial electron beam energy $E_b = 250$ GeV, and an integrated geometric luminosity of the 20 fb^{-1} are assumed.

asymmetry by using eqn.15. By considering two asymmetries we get two bands whose area of intersection would give the allowed values of the parameters. With the charge asymmetry and the forward-backward asymmetry we find that the best simultaneous limits are obtained for the case $\lambda_e^1 = \lambda_e^2$ and $\lambda_l^1 = -\lambda_l^2$. As shown in Fig 1(a) we get a limiting value of 1.2 for $\text{Im}\lambda$ and 0.79×10^{-16} e cm for the EDFF, $\text{Im}d_t$. When the charge asymmetry alone or forward-backward asymmetry alone was considered for different helicity combinations, the limits worsened. But by combining the case of charge asymmetry for $\lambda_e^1 = \lambda_e^2 = -0.5$, $\lambda_l^1 = \lambda_l^2 = 1$ and forward-backward asymmetry for $\lambda_e^1 = \lambda_e^2 = -0.5$, $\lambda_l^1 = -\lambda_l^2 = -1$ we get better limits, viz., 0.6 on $\text{Im}\lambda$, and 0.68×10^{-16} e cm on $\text{Im}d_t$ (Fig 1(b)).

Some remarks about the magnitudes of the limits on λ are in order. $|\lambda|$ would be bounded by unitarity. However, the corresponding limits are quite weak upto a fairly high energy scale [3].

The electron electric dipole moment (EDM) has been measured with very high precision and the experimental value has been presented in [15] The experimental limit is $|d_e| < 6.2 \times 10^{-27} ecm$ at 95% C.L.. The presence of a CP-violating $Z\gamma\gamma$ coupling can in principle be severely constrained from this limit on the EDM of the electron [3]. The effective interaction considered here can induce, at one loop, an EDM for the electron. This calculation depends on a momentum cut-off. It was shown in [3] that with an assumed cut-off of the order of 1 TeV, the experimental limit on the EDM of the electron gives a limit of about 10^{-3} on $|\lambda|$. The limit would be even more stringent, about 10^{-4}, if the cut-off is assumed to be of the order of a grand unification scale, viz., 10^{16} GeV. Thus, for values of λ which are relevant for the experiments we discuss in this paper,

the induced electron dipole moment would be in conflict with experiment. However, the procedure for obtaining a dipole moment from λ at one loop in a non-renormalizable effective theory is not rigorous. Such a calculation does not take into account the dependence of λ on q^2 values of γ and Z. Moreover, there is the possibility of cancellations between the γZZ and $Z\gamma\gamma$ contributions to the electron edm, which is assumed to be absent. For these reasons, it would therefore be desirable to obtain a direct experimental limit on λ rather than an indirect one.

We should compare the limits we discuss here with those that could be achieved in the process $e^+e^- \rightarrow \gamma Z$ discussed in [3]. The limits mentioned there are an order of magnitude better for comparable linear collider parameters. However, whereas we have taken into account top decay, [3] did not take into account details of Z decay, and it remains to be seen how much the results in [3] would be effected by Z detection efficiencies.

We have neglected CP violation in the decay of the top quark. A complete study should take this into account. It is quite conceivable that linear e^+e^- colliders would achieve better luminosities than anticipated here. In that case, there would be a corresponding improvements in the limits we derive.

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