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## Weak dipole moment of $\tau$ in $e^+e^-$ collisions with longitudinally polarized electrons

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### Abstract

It is pointed out that certain CP-odd momentum correlations in the production and subsequent decay of tau pairs in  $e^+e^-$  collisions get enhanced when the  $e^-$  is longitudinally polarized. Analytic expressions for these correlations are obtained for the single-pion decay mode of  $\tau$  when  $\tau^+\tau^-$  have a “weak” dipole form factor (WDF) coupling to  $Z$ . For  $e^+e^-$  collisions at the  $Z$  peak, a sensitivity of about  $1.5 \times 10^{-17} e \text{ cm}$  for the  $\tau$  WDF can be reached using a *single*  $\tau^+\tau^-$  decay channel, with  $10^6$   $Z$ 's likely to be available at the SLC at Stanford with  $e^-$  polarization of 62%-75%.

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The standard model (SM), which has been well tested in recent experiments, can adequately accommodate CP violation in the K-meson system, the only place where it has been experimentally observed. Nevertheless, there remains the intriguing possibility

that the underlying theory is an extension of SM which would give observable CP violation in other hadronic, or even leptonic systems. For example, the observation of electric dipole moments would signal physics beyond SM, since these are predicted to be unobservably small in SM. It is therefore necessary to keep an eye open for such signals in the planning and analysis of experiments.

The study of CP-odd correlations has been proposed in the past as a test CP [1]. The observation of such correlations, for example in  $e^+e^-$  or  $p\bar{p}$  experiments, would signal violation of CP. In particular, CP-odd observables arising due to a possible  $\tau$ -lepton electric or “weak” dipole moment have been analyzed in great detail [2,3]. Such dipole moments can arise in extensions of SM with CP violation coming from sectors other than the standard  $3 \times 3$  Cabibbo-Kobayashi-Maskawa matrix.

Recent experiments at the LEP collider in CERN have also put an experimental upper limit [4] on the weak dipole form factor (Wdff) of  $\tau$  at the  $Z$  resonance by looking for the tensor correlation  $T_{ij} = (\mathbf{q}_+ - \mathbf{q}_-)_i (\mathbf{q}_+ \times \mathbf{q}_-)_j$ , where  $\mathbf{q}_+$  ( $\mathbf{q}_-$ ) represents the momentum of a charged particle arising from the decay of  $\tau^+$  ( $\tau^-$ ) produced in  $Z$  decay.

Recently, the Stanford Linear Collider (SLC) has achieved a longitudinal polarization ( $P_e$ ) of 62% for the electron beam, which is likely to be increased to about 75% [5]. Longitudinal polarization of  $e^-$  of 22% has already been used for the determination of  $\sin^2 \theta_W$  using left-right asymmetry [6]. Experiments with the present polarization of 62% can lead to better accuracy than obtained so far at LEP [7].

We study here the effect of longitudinal  $e^-$  polarization on CP-odd momentum correlations in  $e^+e^- \rightarrow \tau^+\tau^-$  with the subsequent decay of  $\tau^+$  and  $\tau^-$ . Since we have mainly the ongoing experiments at SLC in mind, we concentrate on the  $e^+e^-$  centre-of-mass (c.m.) energy tuned to the  $Z$  resonance. The CP-odd momentum correlations [8] are associated with the c.m. momenta  $\mathbf{p}$  of  $e^-$ ,  $\mathbf{q}_+$  of  $\pi^+$  and  $\mathbf{q}_-$  of  $\pi^-$ , where the  $\pi^+$  and  $\pi^-$  arise in the decays  $\tau^+ \rightarrow \pi^+\bar{\nu}_\tau$  and  $\tau^- \rightarrow \pi^-\nu_\tau$ . We restrict ourselves to the single-pion decay mode of the tau, for which case we can obtain analytic expressions for the correlations.

It must be noted that correlations which are CP violating in the absence of initial beam polarization are not strictly CP odd for arbitrary  $e^+$  and  $e^-$  polarizations, since the initial state is then not necessarily CP even. We argue, however, that this is true to a high degree of accuracy in the case at hand.

Our main result is that certain CP-odd correlations, which are relatively small in the absence of  $e^-$  polarization, since they come with a factor  $r = 2g_{Ve}g_{Ae}/(g_{Ve}^2 + g_{Ae}^2)$  ( $\approx 0.16$ ), get enhanced in the presence of polarization, now being proportional to  $(r - P_e)/(1 - rP_e)$  ( $\approx 0.71$  for  $P_e = -.62$ ) [9]. Here  $g_{Ve}, g_{Ae}$  are the vector and axial vector couplings of  $e^-$  to  $Z$ . The correlations which have this property are those

which have an odd number of factors of the  $e^+$  c.m. momentum  $\mathbf{p}$ , since this would need P and C violation at the electron vertex. Furthermore, we suggest a procedure for obtaining these correlations from the difference in the event distributions for a certain polarization  $P_e$  and the sign-flipped polarization  $-P_e$ . With this procedure, the correlations are further enhanced, leading to increased sensitivity.

More specifically, we have considered the observables  $O_1 \equiv \hat{\mathbf{p}} \cdot (\mathbf{q}_+ \times \mathbf{q}_-)$  and  $O_2 \equiv \hat{\mathbf{p}} \cdot (\mathbf{q}_+ + \mathbf{q}_-)$  (the caret denoting a unit vector) and obtained analytic expressions for their mean values and standard deviations in the presence of longitudinal  $e^-$  polarization  $P_e$ . By the procedure outlined above, the magnitude of the WDFD at the  $Z$ ,  $\tilde{d}_\tau(m_Z)$  which can be measured at  $1\sigma$  level is about  $1\text{-}2 \times 10^{-16} e \text{ cm}$  ( $3\text{-}5 \times 10^{-17} e \text{ cm}$ ) for a sample of 50,000 ( $10^6$ )  $Z$ 's, using a single  $\tau$  decay channel. Moreover,  $O_2$ , being CPT-odd, measures  $\text{Im } \tilde{d}_\tau$ , whereas  $O_1$  measures  $\text{Re } \tilde{d}_\tau$ . Inclusion of other exclusive  $\tau$  decay modes (not studied here) would improve the sensitivity further.

The process we consider is

$$e^-(p_-) + e^+(p_+) \rightarrow \tau^-(k_-) + \tau^+(k_+), \quad (1)$$

with the subsequent decays

$$\tau^-(k_-) \rightarrow \pi^-(q_-) + \nu_\tau, \quad \tau^+(k_+) \rightarrow \pi^+(q_+) + \bar{\nu}_\tau. \quad (2)$$

Under CP, the various three-momenta transform as

$$\mathbf{p}_- \leftrightarrow -\mathbf{p}_+, \quad \mathbf{k}_- \leftrightarrow -\mathbf{k}_+, \quad \mathbf{q}_- \leftrightarrow -\mathbf{q}_+. \quad (3)$$

In the  $e^+e^-$  c.m. frame, the only CP-odd vectors which can be constructed out of the directly observable three-momenta  $\mathbf{p} \equiv -\mathbf{p}_- = \mathbf{p}_+$ ,  $\mathbf{q}_+$  and  $\mathbf{q}_-$ , are  $\mathbf{q}_+ + \mathbf{q}_-$  and  $\mathbf{q}_+ \times \mathbf{q}_-$ , while  $\mathbf{p}$  and  $\mathbf{q}_+ - \mathbf{q}_-$  are the CP-even ones. It is therefore simple to list possible scalars which are odd under CP:  $\mathbf{p} \cdot (\mathbf{q}_+ + \mathbf{q}_-)$ ,  $\mathbf{p} \cdot (\mathbf{q}_+ \times \mathbf{q}_-)$  and  $(\mathbf{q}_+ + \mathbf{q}_-) \cdot (\mathbf{q}_+ - \mathbf{q}_-)$ . The CP-even scalars are  $(\mathbf{q}_+ + \mathbf{q}_-)^2$ ,  $(\mathbf{q}_+ - \mathbf{q}_-)^2$  and  $\mathbf{p} \cdot (\mathbf{q}_+ - \mathbf{q}_-)$ . One can, of course, take products among these six to construct more CP-odd quantities.

We choose for our analysis the two CP-odd observables  $O_1 \equiv \hat{\mathbf{p}} \cdot (\mathbf{q}_+ \times \mathbf{q}_-)$  and  $O_2 \equiv \hat{\mathbf{p}} \cdot (\mathbf{q}_+ + \mathbf{q}_-)$ , which have an odd number of factors of  $\hat{\mathbf{p}}$ . As mentioned before, they are expected to get enhanced in the presence of  $e^-$  polarization. Though these observables are CP odd, their observation with polarized  $e^+$  and  $e^-$  beams is not necessarily an indication of CP violation, unless the  $e^+$  and  $e^-$  longitudinal polarizations are equal and opposite, so that the initial state is described by a CP-even density matrix. However, in the limit of  $m_e = 0$ , the couplings of like-helicity  $e^+e^-$  pairs to spin-1 states like  $\gamma$  and  $Z$  drop out, effectively giving rise to a CP-even initial state to a very good accuracy for arbitrary  $e^+$  and  $e^-$  polarizations [10].

Of  $O_1$  and  $O_2$ ,  $O_1$  is even under the combined CPT transformation, and  $O_2$  is CPT-odd. A CPT-odd observable can only have a non-zero value in the presence of an absorptive part of the amplitude. It is therefore expected that  $\langle O_2 \rangle$  will be

proportional to the imaginary part of the weak dipole form factor  $\text{Im } \tilde{d}_\tau$ , since final-state interaction, which could give rise to an absorptive part, is negligible in the weak  $\tau$  decays. Since  $\langle O_1 \rangle$  and mean values of other CPT-even quantities will be proportional to  $\text{Re } \tilde{d}_\tau$ , phase information on  $\tilde{d}_\tau$  can only be obtained if  $\langle O_2 \rangle$  (or some other CPT-odd quantity) is also measured.

We assume SM couplings for all particles except  $\tau$ , for which an additional WDFF interaction is assumed, viz.,

$$\mathcal{L}_{WDFF} = -\frac{i}{2} \tilde{d}_\tau \bar{\sigma}^{\mu\nu} \gamma_5 \tau (\partial_\mu Z_\nu - \partial_\nu Z_\mu), \quad (4)$$

where  $\tilde{d}_\tau \equiv \tilde{d}_\tau(s=m_Z^2)$ . Using (4), we now proceed to calculate  $\langle O_1 \rangle$  and  $\langle O_2 \rangle$  in the presence of longitudinal polarization  $P_e$  for  $e^-$ .

We can anticipate the effect of  $P_e$  in general for the process (1). We can write the matrix element squared for the process in the leading order in perturbation theory, neglecting the electron mass, as

$$|M|^2 = \sum_{i,j} L_{\mu\nu}^{ij}(e) L^{ij\mu\nu*}(\tau) \frac{1}{s-M_i^2} \frac{1}{s-M_j^2}, \quad (5)$$

where the summation is over the gauge bosons ( $\gamma, Z, \dots$ ) exchanged in the  $s$  channel, and  $L_{\mu\nu}^{ij}(e, \tau)$  represent the tensors arising at the  $e$  and  $\tau$  vertices:

$$L_{\mu\nu}^{ij} = V_\mu^i V_\nu^{j*}. \quad (6)$$

For the electron vertex, with only the SM vector and axial vector couplings,

$$V_\mu^i(e) = g_i \bar{v}(p_+, s_+) \gamma_\mu (g_{V_e}^i - \gamma_5 g_{A_e}^i) u(p_-, s_-), \quad (7)$$

$g_i$  being the appropriate coupling constant ( $g_\gamma = e, g_Z = g/(2 \cos \theta_W)$ ), and  $g_{V_e}^i$  and  $g_{A_e}^i$  are given by

$$g_{V_e}^\gamma = -1, \quad g_{A_e}^\gamma = 0; \quad (8)$$

$$g_{V_e}^Z = -\frac{1}{2} + 2 \sin^2 \theta_W, \quad g_{A_e}^Z = -\frac{1}{2}. \quad (9)$$

It is easy to check, by putting in helicity projection operators, that

$$\begin{aligned} L_{\mu\nu}^{ij}(e) &= g_i g_j \\ &\times \left\{ \left[ (1 - P_e P_{\bar{e}}) (g_{V_e}^i g_{V_e}^j + g_{A_e}^i g_{A_e}^j) - (P_e - P_{\bar{e}}) (g_{V_e}^i g_{A_e}^j + g_{A_e}^i g_{V_e}^j) \right] \text{Tr}(\not{p}_- \gamma_\mu \not{p}_+ \gamma_\nu) \right. \\ &\left. + \left[ (P_e - P_{\bar{e}}) (g_{V_e}^i g_{V_e}^j + g_{A_e}^i g_{A_e}^j) - (1 - P_e P_{\bar{e}}) (g_{V_e}^i g_{A_e}^j + g_{A_e}^i g_{V_e}^j) \right] \text{Tr}(\gamma_5 \not{p}_- \gamma_\mu \not{p}_+ \gamma_\nu) \right\} \end{aligned} \quad (10)$$

in the limit of vanishing electron mass, where  $P_e$  ( $P_{\bar{e}}$ ) is the degree of the  $e^-$  ( $e^+$ ) longitudinal polarization. Note that the combinations  $P_e - P_{\bar{e}}$  and  $1 - P_e P_{\bar{e}}$  occurring

in (10) are indeed CP-even, showing that the initial state is effectively CP even for arbitrary  $P_e, P_{\bar{e}}$ .

Eq.(10) gives a simple way of incorporating the effect of the longitudinal polarization. In particular, at the  $Z$  peak, where photon effects can be neglected, to go from the unpolarized to the polarized one, one has to make the replacement (henceforth, we drop the superscript  $Z$ , as we shall only deal with  $Z$  couplings):

$$\begin{aligned} g_{V_e}^2 + g_{A_e}^2 &\rightarrow g_{V_e}^2 + g_{A_e}^2 - P_e 2g_{V_e}g_{A_e}, \\ 2g_{V_e}g_{A_e} &\rightarrow 2g_{V_e}g_{A_e} - P_e (g_{V_e}^2 + g_{A_e}^2). \end{aligned} \quad (11)$$

We have set  $P_{\bar{e}} = 0$ , as is the case at SLC, for example. It is then clear from (9), using  $\sin^2 \theta_W \approx 0.23$ , that quantities which are suppressed in the absence of polarization because of the small numerical value of  $2g_{V_e}g_{A_e}$  will get considerably enhanced in the presence of polarization.

To calculate correlations of  $O_1$  and  $O_2$ , we need the differential cross section for (1) followed by (2) at the  $Z$  peak, arising from SM  $Z$  couplings of  $e$  and  $\tau$ , together with a weak-dipole coupling of  $\tau$  arising from eq.(4). (We neglect electromagnetic effects completely). The calculation may be conveniently done, following ref.[3] (see also ref.[11]), in steps, by first determining the production matrix  $\chi$  for  $\tau^+\tau^-$  in spin space, and then taking its trace with the decay matrices  $\mathcal{D}^\pm$  for  $\tau^\pm$  decays into single pions:

$$\frac{1}{\sigma} \frac{d\sigma}{d\Omega_k d\Omega_-^* d\Omega_+^* dE_-^* dE_+^*} = \frac{k}{8\pi m_Z^2 \Gamma(Z \rightarrow \tau^+\tau^-)} \frac{1}{(4\pi)^3} \chi^{\beta\beta', \alpha\alpha'} \mathcal{D}_{\alpha'\alpha}^- \mathcal{D}_{\beta'\beta}^+, \quad (12)$$

where  $d\Omega_k$  is the solid angle element for  $\mathbf{k}_+$  in the overall c.m. frame,  $k = |\mathbf{k}_+|$ , and  $d\Omega_\pm^*$  are the solid angle elements for  $\mathbf{q}_\pm^*$ , the  $\pi^\pm$  momenta in the  $\tau^\pm$  rest frame. The  $\mathcal{D}$  matrices are given by

$$\mathcal{D}^\pm = \delta(E_\pm^* - E_0) \left[ 1 \mp \sigma_\pm \cdot \hat{\mathbf{q}}_\pm^* \right], \quad (13)$$

where  $\sigma_\pm$  are the Pauli matrices corresponding to the  $\tau^\pm$  spin,  $E_\pm^*$  are the  $\pi^\pm$  energies in the  $\tau^\pm$  rest frame, and

$$E_0 = \frac{1}{2} m_\tau (1 + p); \quad p = m_\pi^2 / m_\tau^2. \quad (14)$$

The expressions for  $\chi$  arising from SM as well as WDFP coupling of  $\tau$  are rather long, and we refer the reader to ref.[3] for these expressions in the absence of polarization. It is straightforward to incorporate polarization using (11).

The expressions for the correlations  $\langle O_1 \rangle$  and  $\langle O_2 \rangle$  obtained are, neglecting  $\tilde{d}_\tau^2$ ,

$$\langle \hat{\mathbf{p}} \cdot (\mathbf{q}_+ \times \mathbf{q}_-) \rangle = -\frac{m_Z^2}{18e} m_\tau \text{Re} \tilde{d}_\tau c_W s_W (1-x^2) \left( \frac{r - P_e}{1 - rP_e} \right) \frac{g_{A\tau}(1-p)^2 - 3g_{V\tau}(1-p^2)}{g_{V\tau}^2(1 + \frac{1}{2}x^2) + g_{A\tau}^2(1-x^2)}, \quad (15)$$

and

$$\langle \hat{\mathbf{p}} \cdot (\mathbf{q}_+ + \mathbf{q}_-) \rangle = \frac{2m_Z}{3e} m_\tau \text{Im} \tilde{d}_\tau c_W s_W \left( \frac{r - P_e}{1 - rP_e} \right) \frac{g_{A\tau}(1-x^2)^2(1-p)}{g_{V\tau}^2(1+\frac{1}{2}x^2) + g_{A\tau}^2(1-x^2)}, \quad (16)$$

where  $c_W = \cos \theta_W$ ,  $s_W = \sin \theta_W$ ,  $x = 2m_\tau/m_Z$ , and  $r = 2g_{V_e}g_{A_e}/(g_{V_e}^2 + g_{A_e}^2)$ .

We have also obtained analytic expressions for the variance  $\langle O^2 \rangle - \langle O \rangle^2 \approx \langle O^2 \rangle$  in each case, arising from the CP-invariant SM part of the interaction, but since they are lengthy, they will be presented elsewhere [12].

Our numerical results are presented in the tables. In Tables 1 and 2 we have presented, as in ref.[3], the values of  $c_{\pi\pi}$  for  $O_1$  and  $O_2$  respectively, defined as the correlation for a value of  $\text{Re} \tilde{d}_\tau$  or  $\text{Im} \tilde{d}_\tau$  (as the case may be) equal to  $e/m_Z$ , for various values of  $P_e$ . (By our convention, a positive value of  $P_e$  means right-handed polarization, and vice-versa). We have also presented the value of  $\sqrt{\langle O^2 \rangle}$  and  $\delta \tilde{d}_\tau$ , which represents the 1 s.d. upper limit unit on  $\tilde{d}_\tau$  which can be placed with a certain sample of events, for 50,000  $Z$ 's currently seen at SLC with 62% polarization, and for  $10^6$   $Z$ 's, eventually hoped to be achieved. This 1 s.d. limit is the value of  $\tilde{d}_\tau$  which gives a mean value of  $O_i$  equal to the s.d.  $\sqrt{\langle O_i^2 \rangle}/N_{\pi\pi}$  in each case.

$P_e$	$c_{\pi\pi}$ (GeV <sup>2</sup> )	$\sqrt{\langle O_1^2 \rangle}$ (GeV <sup>2</sup> )	$\delta \text{Re} \tilde{d}_\tau$ (e cm) for	
			$5 \times 10^4$ $Z$ 's	$10^6$ $Z$ 's
0	0.898	12.861	$6.6 \times 10^{-16}$	$1.5 \times 10^{-16}$
+0.62	-2.890	12.861	$2.0 \times 10^{-16}$	$4.6 \times 10^{-17}$
-0.62	4.007	12.861	$1.5 \times 10^{-16}$	$3.3 \times 10^{-17}$
+0.75	-3.792	12.861	$1.6 \times 10^{-16}$	$3.5 \times 10^{-17}$
-0.75	4.589	12.861	$1.3 \times 10^{-16}$	$2.9 \times 10^{-17}$

Table 1: Results for the observable  $O_1 \equiv \hat{\mathbf{p}} \cdot (\mathbf{q}_+ \times \mathbf{q}_-)$

$P_e$	$c_{\pi\pi}$ (GeV)	$\sqrt{\langle O_1^2 \rangle}$ (GeV)	$\delta \text{Im} \tilde{d}_\tau$ (e cm) for	
			$5 \times 10^4$ $Z$ 's	$10^6$ $Z$ 's
0	-0.157	9.572	$2.8 \times 10^{-15}$	$6.2 \times 10^{-16}$
+0.62	0.505	9.572	$8.7 \times 10^{-16}$	$1.9 \times 10^{-16}$
-0.62	-0.700	9.572	$6.3 \times 10^{-16}$	$1.4 \times 10^{-16}$
+0.75	0.662	9.572	$6.6 \times 10^{-16}$	$1.5 \times 10^{-16}$
-0.75	-0.802	9.572	$5.5 \times 10^{-16}$	$1.2 \times 10^{-16}$

Table 2: Results for the observable  $O_2 \equiv \hat{\mathbf{p}} \cdot (\mathbf{q}_+ + \mathbf{q}_-)$

As can be seen from Tables 1 and 2,  $\langle O_1 \rangle$  and  $\langle O_2 \rangle$  can probe respectively  $\text{Re} \tilde{d}_\tau$  and  $\text{Im} \tilde{d}_\tau$  down to about  $2 \times 10^{-16} e \text{ cm}$  and  $6 \times 10^{-16} e \text{ cm}$  with the current data from SLC. These limits can be improved by a factor of 5 in future runs of SLC.

These limits can be considerably improved by looking at correlations of the same observables, but in a sample obtained by counting the difference between the number

of events for a certain polarization, and for the corresponding sign-flipped polarization. If the differential cross section for the process for an  $e^-$  polarization  $P_e$  is given by

$$\frac{d\sigma}{d^3q_-d^3q_+}(P_e) = A(\mathbf{p}, \mathbf{q}_-, \mathbf{q}_+) + P_e \cdot B(\mathbf{p}, \mathbf{q}_-, \mathbf{q}_+), \quad (17)$$

then one calculates average values over the distribution given by the difference

$$\frac{d\sigma(P_e)}{d^3q_-d^3q_+} - \frac{d\sigma(-P_e)}{d^3q_-d^3q_+} = 2P_e B(\mathbf{p}, \mathbf{q}_-, \mathbf{q}_+). \quad (18)$$

The correlations are considerably larger, whereas the variances are unchanged. The event sample is however smaller (by a factor of about  $.16P_e$ , which is the left-right asymmetry times degree of polarization), leading to a larger statistical error. The sensitivity is nevertheless improved, as can be seen from Table 3. There we present, for the two variables  $O_1$  and  $O_2$ ,  $c_{\pi\pi}$  (defined as before) for the polarization asymmetrized sample described above, and the corresponding quantities  $\sqrt{\langle O_i^2 \rangle}$  and the 1 s.d. limits on  $\text{Re } \tilde{d}_\tau$  and  $\text{Im } \tilde{d}_\tau$ , respectively, both called  $\delta \tilde{d}_\tau$  in this table for convenience. We give the limits for two luminosities, but only for one polarization,  $P_e = .62$ . Only the last column changes with  $P_e$ , and the improvement in going to  $P_e = .75$  is marginal.

	$c_{\pi\pi}$	$\sqrt{\langle O_i^2 \rangle}$	$\delta \tilde{d}_\tau (e \text{ cm})$ for	
			$5 \times 10^4 Z$ 's	$10^6 Z$ 's
$O_1$	35.545 GeV <sup>2</sup>	12.861 GeV <sup>2</sup>	$5.3 \times 10^{-17}$	$1.2 \times 10^{-17}$
$O_2$	-6.208 GeV	9.572 GeV	$2.2 \times 10^{-16}$	$5.0 \times 10^{-17}$

Table 3: Results for the polarization asymmetrized sample of events, for  $P_e = 0.62$ .

As seen from Table 3, the current data can yield upper limits of about  $5\text{-}22 \times 10^{-17} e$  cm for  $\text{Re } \tilde{d}_\tau$  or  $\text{Im } \tilde{d}_\tau$ . An integrated luminosity corresponding to  $10^6 Z$ 's at SLC can improve the limits to about  $1\text{-}5 \times 10^{-17} e$  cm. These should be compared with the 95% C.L. limit of  $\text{Re } \tilde{d}_\tau < 7.0 \times 10^{-17} e$  cm from OPAL and  $\text{Re } \tilde{d}_\tau < 3.7 \times 10^{-17} e$  cm from ALEPH obtained by looking for tensor correlations from a sample of about 650,000  $Z$ 's at LEP, and including several decay channels of  $\tau$  [4]. The real advantage of polarization can be seen in the sensitivity for the measurement of  $\text{Im } \tilde{d}_\tau$ . The limit obtainable at LEP under ideal experimental conditions from the  $2\pi$  channel using the tensor correlation  $\langle \hat{\mathbf{p}} \cdot (\hat{\mathbf{q}}_+ + \hat{\mathbf{q}}_-) \hat{\mathbf{p}} \cdot (\hat{\mathbf{q}}_+ - \hat{\mathbf{q}}_-) \rangle$  is  $10^{-16} e$  cm with  $10^7 Z$ 's [3], whereas the limit we obtain here in the presence of polarization is as low as  $5 \times 10^{-17} e$  cm with only  $10^6 Z$ 's.

To conclude, our results show that the CP-odd correlations  $\langle O_1 \rangle$  and  $\langle O_2 \rangle$  are considerably enhanced due to longitudinal  $e^-$  polarization. By considering values of these correlations on changing the sign of the polarization, a much greater sensitivity is obtained. Considering both these correlations together can give phase information on the WDFP, which cannot be obtained using any single correlation.

We must emphasize that we have considered here only one  $\tau$  decay channel. Since the number of events in any single channel is small, inclusion of additional decay modes is necessary and can improve the sensitivity. The sensitivity of other channels will be discussed in a future work [12]. The limits obtainable may be comparable to those from tensor correlations [3,4] with unpolarized beams, as used at LEP. Moreover, studying  $O_2$  can constrain  $\text{Im} \tilde{d}_\tau$ , the tensor correlation could only constrain  $\text{Re} \tilde{d}_\tau$ . It should also be noted that the tensor correlations considered in refs. [3,4] are not changed in the presence of polarization.

We have neglected errors in measurement of  $P_e$ , whose effect on the results would be small compared to that of statistical errors.

It is possible that longitudinal polarization may be available at tau-charm factories or other future linear colliders planned to operate at higher energies. The advantages of polarization presented here can be studied by extending the analysis to the relevant energies, including the effect of photon exchange and the  $\tau$  electric dipole coupling. The study could also be extended to dipole moments of the top-quark or  $W^\pm$  which could be pair produced at the high energy accelerators.

It would also be interesting to consider event asymmetries corresponding to  $O_1$  and  $O_2$  (rather than their mean values), and also other CP-odd correlations with an odd number of  $\hat{\mathbf{p}}$ 's, like  $\langle (\mathbf{q}_+^2 - \mathbf{q}_-^2) \hat{\mathbf{p}} \cdot (\mathbf{q}_+ - \mathbf{q}_-) \rangle$ , which may have a different sensitivity.

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## References and Footnotes

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