PRAMANA
— journal of
physics

Vol. 52, No. 5
May 1999
pp. L529–L536

Solar neutrino oscillation diagnostics at SuperKamiokande and Sudbury Neutrino Observatory

DEBASISH MAJUMDAR and AMITAVA RAYCHAUDHURI
Department of Physics, University of Calcutta, 92 Acharya Prafulla Chandra Road, Calcutta 700 009, India
Email: debashish@tnpdec.saha.ernet.in; amitava@cubmb.ernet.in

MS received 4 December 1998; revised 16 March 1999

Abstract. Results for solar neutrino detection from the SuperKamiokande collaboration have been presented recently while those from the Sudbury Neutrino Observatory are expected in the near future. These experiments are sensitive to the ^{8}B neutrinos from the sun, the shape of whose spectrum is well-known but the normalization is less certain. We propose several variables, insensitive to the absolute flux of the incident beam, which probe the shape of the observed spectrum and can sensitively signal neutrino oscillations. They provide methods to extract the neutrino mixing angle and mass splitting from the data and also to distinguish oscillation to sequential neutrinos from those to a sterile neutrino.

Keywords. Neutrino oscillation; solar neutrino; SuperKamiokande; Sudbury Neutrino Observatory.

PACS Nos 26.65; 14.60

The recent evidence of neutrino oscillations in the atmospheric neutrino data presented by SuperKamiokande (SK) [1] has moved neutrino physics to the centrestage of research activity. A non-zero neutrino mass will have impact on many areas of particle physics, astrophysics, and cosmology [2] and new results are eagerly awaited. It is widely expected that important information will emerge from the data on solar neutrinos. All earlier experiments have consistently signalled a depletion of the solar neutrino flux [3] and high statistics results from SK (some already published [4]) and the other experiment of comparable size, the Sudbury Neutrino Observatory (SNO) [5], will further sharpen the situation.

There are several issues pertaining to the solar neutrino problem which still remain unsettled. The observed flux depletion could be a consequence of vacuum neutrino oscillations or resonant flavour conversion [6]. It is not possible to rule out any of these alternatives on the basis of the available data. Further, the electron neutrino may be mixed with either a sequential or a sterile neutrino. Three neutrinos are expected in association with the three known charged leptons. The inclusion of a fourth neutrino – sterile, in view of the LEP and SLC results – is suggested from the several evidences indicative of neutrino oscillations, namely, the solar neutrino puzzle, the atmospheric neutrino anomaly and the results of the LSND experiment, all of which cannot be accommodated together in a three neutrino framework [7]. Finally, it is expected that the mass splitting and mixing angle will be tightly constrained from the new data.

The solar neutrinos are produced in standard reactions (the p-p chain, CNO cycle, etc.) responsible for the generation of heat and light. Though the spectrum of neutrinos from each of the processes is well known, their absolute normalisations vary from one solar model to another [8,9]. The two latest detectors, SNO and SK, are sensitive to neutrinos from only the boron reaction in the p-p chain whose normalisation, for example, is known to vary like T_c^{18}, where T_c is the temperature at the solar core.

In this paper we examine the vacuum oscillation scenario. We propose several variables relevant for SK and SNO which are insensitive to the absolute normalisation of the ^{8}B neutrino flux and may be used (a) to distinguish mixing of the electron neutrino with a sequential neutrino from that to a sterile neutrino and (b) to determine the neutrino mass splitting and mixing angle. Other variables, insensitive to the absolute normalisation of the incident flux, have been explored earlier in [10,11] where the focus has been on the energy spectrum of the scattered electron neutrino at SNO, the MSW mechanism etc.

The SuperKamiokande detector uses 32 ktons of light water in which electrons scattered by ν_e – through both charged current (CC) and neutral current (NC) interactions – are identified via their Čerenkov radiation. If a sequential neutrino is produced by oscillation, it will contribute to the signal only through the NC interactions (roughly one sixth of the ν_e case) while a sterile neutrino will be entirely missed by the detector. The SNO detector has 1 kton of D_2O and neutrinos are primarily detected through the charged and neutral current disintegration of the deuteron: $\nu+d \to e^-+p+p$, $\nu+d \to \nu+p+n$, respectively. While the e^- in the CC reaction is identified through its Čerenkov radiation and can be used to determine the shape of the incident neutrino spectrum, the NC measurement, signalled by the detection of the neutron, is calorimetric. If oscillations to sequential neutrinos occur then they will not contribute to the CC signal while the NC channel will be unaffected. On the other hand if the ν_e oscillates to a sterile neutrino, which has no interactions whatsoever, then both the CC and NC signals will suffer depletions.

The first class of variables to probe the shape of the observed neutrino spectra that we propose are M_n, the normalized n-th moments of the solar neutrino distributions seen at SK and SNO. Specifically,

$$M_n = \frac{\int N_i(E) E^n \mathrm{d}E}{\int N_i(E) \mathrm{d}E} \tag{1}$$

where i stands for SK or SNO. It is seen from the definition that the uncertainty in the overall normalization of the incident neutrino flux cancels out from M_n.

To see how these variables are affected by neutrino oscillations, first consider oscillation of the electron neutrino to a sequential neutrino, say ν_μ. Since the oscillation probability is a function of the energy, the shape of the spectrum will be affected. As noted earlier, at SK the muon neutrino will only undergo NC reactions. Thus, for oscillation to a sequential neutrino, we have

$$N_{\rm SK}(E) = \epsilon_{\rm SK} f(E) \left\{ P_{\nu_e \to \nu_e}(E, \Delta, \vartheta) \sigma^e_{\rm SK}(E) + P_{\nu_e \to \nu_\mu}(E, \Delta, \vartheta) \sigma^\mu_{\rm SK}(E) \right\} N^0_{\rm SK}. \tag{2}$$

Here, $f(E)$ stands for the incident boron-neutrino fluence, $\epsilon_{\rm SK}$ for the detection efficiency which, for the sake of simplicity, is assumed to be energy independent, and $N^0_{\rm SK}$ for the number of electrons in the SK detector off which the neutrinos may scatter. $\sigma^e_{\rm SK}(E)$ is the

ν_e scattering cross-section with both NC and CC contributions whereas $\sigma^{\mu}_{\rm SK}(E)$ is the ν_μ cross-section obtained from the NC interaction alone.

Only the CC contributions are relevant at SNO for the determination of the spectrum and we get:

$$N^{\rm c.c}_{\rm SNO}(E) = \epsilon^{\rm c.c.}_{\rm SNO} f(E) P_{\nu_e \to \nu_e}(E, \Delta, \vartheta) \sigma^{\rm c.c.}_{\rm SNO}(E) N^0_{\rm SNO}. \tag{3}$$

$N^0_{\rm SNO}$ is the number of deuteron nuclei in the SNO detector and $\epsilon^{\rm c.c}_{\rm SNO}$ represents the CC detection efficiency assumed to be independent of the energy.

If the ν_e oscillates to a sterile neutrino, which is decoupled from the weak interactions, it will escape the SK and SNO detectors completely. Thus, for sterile neutrinos, in place of eq. (2) we have

$$N_{\rm SK}(E) = \epsilon_{\rm SK} f(E) \left\{ P_{\nu_e \to \nu_e}(E, \Delta, \vartheta) \sigma^e_{\rm SK}(E) \right\} N^0_{\rm SK} \tag{4}$$

while eq. (3) is unchanged.

In the two-flavour case, the probability of an electron neutrino of energy E_ν to oscillate to another neutrino (sequential or sterile), ν_x, after the traversal of a distance L is:

$$P_{\nu_e \to \nu_x} = \sin^2(2\vartheta) \sin^2\left(\frac{\pi L}{\lambda}\right) \tag{5}$$

where ϑ is the mixing angle, and the oscillation length, λ, is given in terms of the mass-squared difference Δ by:

$$\lambda = 2.47 \left(\frac{E_\nu}{\rm MeV}\right) \left(\frac{\rm eV^2}{\Delta}\right) \text{ metre.} \tag{6}$$

From probability conservation: $P_{\nu_e \to \nu_e} = 1 - P_{\nu_e \to \nu_x}$.

In figure 1 we present the results for M_1, M_2, and M_3 as a function of the mass splitting Δ for oscillation to sequential as well as sterile neutrinos. Results for the mixing angle $\vartheta = 45°$ and 15° are shown. As expected, for the smaller mixing angle the effects of neutrino oscillation are not very prominent. On the other hand, for $\vartheta = 45°$, the impact of neutrino oscillation is quite significant, especially for the smaller values of Δ, and it holds promise for distinguishing between the sequential and sterile neutrino alternatives.

In order to evaluate the usefulness of these variables in conjunction with the actual data, it needs to be noted first that for both the SNO CC and SK signals, what is experimentally measured via the Čerenkov technique is the energy of the outgoing electron. In the case of SNO, the large mass of the deuteron forces the electron to move in the direction of the incident neutrino. Further, since the recoiling hadrons are heavy, the electron's energy equals the incident neutrino energy less the threshold energy for the CC reaction, 1.44 MeV. For SK there is a unique correlation between the electron's energy and scattering angle with the neutrino energy. Thus the neutrino spectrum can be readily reconstructed from the measured electron energy for both experiments using the well-known cross-sections for the appropriate scattering process. The huge sizes of both detectors ensure that the error in the final results will be dominated by systematic uncertainties and careful estimates put these down to a few per cent [10].

If the errors on the extracted neutrino spectrum are at the expected few per cent level, it is easy to convince oneself from figure 1 that M_1, M_2, and M_3 will be useful diagnostic

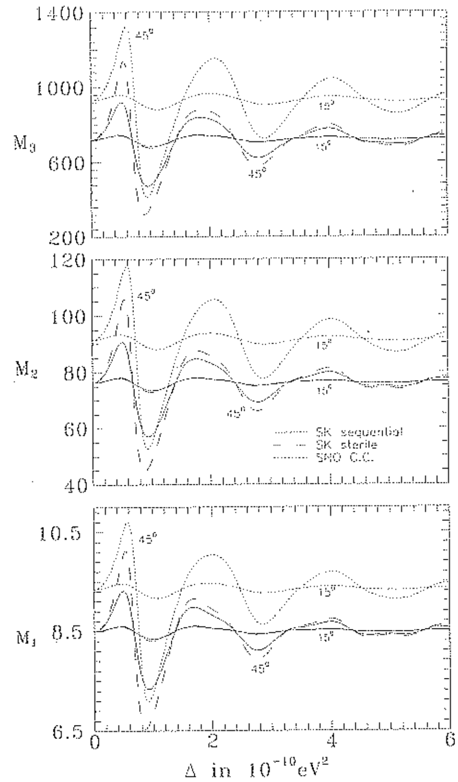

Figure 1. The variables (a) M_1, (b) M_2, and (c) M_3 as a function of the mass splitting Δ for the SK and SNO detectors. Results are presented for two values (45° and 15°) of the mixing angle ϑ. Note that the SNO (charged current) signal does not distinguish between the sequential and sterile neutrino scenarios.

tools. This gives us confidence that, if the mixing angle ϑ is not small (as indicated by the data from the other earlier experiments), the experimental results will enable a distinction between the sequential and the sterile neutrino alternatives and help focus on the mixing angle ϑ and mass splitting Δ involved.

We have also considered the ratios of the moments $r_i = (M_i)_{\rm SK}/(M_i)_{\rm SNO}$ as variables for the search for neutrino oscillations. We do not discuss these in this preliminary communication and results will be reported elsewhere [12].

The SNO experiment will enable separate measurements of the neutrino flux through charge current and neutral current reactions. As noted earlier, if ν_μ or ν_τ are produced through oscillation of solar neutrinos then they will register via neutral current interactions with full strength but their energy will not permit charged current interactions. The ratio, $R_{\rm SNO}$, of the calorimetrically measured signal in the NC channel, $\int N_{\rm SNO}^{\rm n.c.}$, to the total (energy integrated) signal in the CC channel, $\int N_{\rm SNO}^{\rm c.c.}$, is therefore a good probe for oscillations. Thus

$$R_{\rm SNO} = \frac{\int N^{\rm n.c.}_{\rm SNO}}{\int N^{\rm c.c.}_{\rm SNO}}, \tag{7}$$

where

$$\int N^{\rm n.c.}_{\rm SNO} = \int \epsilon^{\rm n.c.}_{\rm SNO} f(E) \sigma^{\rm n.c.}_{\rm SNO}(E) N^0_{\rm SNO} {\rm d}E, \tag{8}$$

where $\epsilon^{\rm n.c.}_{\rm SNO}$ is the efficiency of detection for the NC channel and

$$\int N^{\rm c.c.}_{\rm SNO} = \int \epsilon^{\rm c.c.}_{\rm SNO} f(E) P_{\nu_e \to \nu_e}(E, \Delta, \vartheta) \sigma^{\rm c.c.}_{\rm SNO}(E) N^0_{\rm SNO} {\rm d}E. \tag{9}$$

Clearly, $R_{\rm SNO}$ is independent of the absolute normalization of the incident neutrino flux $f(E)$ and only depends on its shape.

If oscillations to sterile neutrinos take place then (8) is replaced by:

$$\int N^{\rm n.c.}_{\rm SNO} = \int \epsilon^{\rm n.c.}_{\rm SNO} f(E) P_{\nu_e \to \nu_e}(E, \Delta, \vartheta) \sigma^{\rm n.c.}_{\rm SNO}(E) N^0_{\rm SNO} {\rm d}E \tag{10}$$

while (9) is unchanged.

The results for $R_{\rm SNO}$ are presented in table 1. For simplicity, we have assumed $\epsilon^{\rm n.c.}_{\rm SNO}$ to be independent of the energy and further equal to the efficiency for the CC reaction $\epsilon^{\rm c.c.}_{\rm SNO}$. If instead, $\epsilon^{\rm n.c.}_{\rm SNO}/\epsilon^{\rm c.c.}_{\rm SNO} = r_\epsilon$ and it can be taken to be independent of the energy to a good approximation, then our results for $R_{\rm SNO}$ will be multiplied by this factor.

If no oscillations take place then we find $R_{\rm SNO} = 0.382$. Oscillation to sequential neutrinos decreases the denominator of eq. (7) while the numerator is unaffected. Thus $R_{\rm SNO}$ increases if such oscillations take place. From table 1 it is seen that, especially for larger

Table 1. $R_{\rm SNO}$ for different values of the mixing angle, ϑ, and the mass splitting, Δ. Results are presented for both mixing with sequential and sterile neutrinos.

Δ in $10^{-10}{\rm eV}^2$	$R_{\rm SNO}$					
	$\vartheta = 15°$		$\vartheta = 30°$		$\vartheta = 45°$	
	Sequential	Sterile	Sequential	Sterile	Sequential	Sterile
0.0	0.382	0.382	0.382	0.382	0.382	0.382
0.3	0.422	0.384	0.532	0.389	0.613	0.392
0.6	0.480	0.383	0.991	0.387	2.117	0.396
0.9	0.467	0.378	0.848	0.362	1.428	0.337
1.2	0.438	0.380	0.623	0.375	0.788	0.370
1.5	0.422	0.383	0.537	0.387	0.620	0.390
1.8	0.417	0.383	0.512	0.386	0.577	0.388
2.1	0.431	0.383	0.582	0.387	0.706	0.390
2.4	0.444	0.382	0.660	0.383	0.873	0.384
2.7	0.444	0.380	0.658	0.375	0.867	0.370
3.0	0.444	0.381	0.659	0.379	0.869	0.377
3.5	0.431	0.382	0.582	0.381	0.705	0.380
4.0	0.434	0.383	0.597	0.386	0.735	0.388
4.5	0.435	0.382	0.606	0.381	0.753	0.380
5.0	0.440	0.382	0.634	0.381	0.813	0.381
5.5	0.437	0.382	0.614	0.381	0.770	0.381
6.0	0.434	0.382	0.597	0.382	0.735	0.382

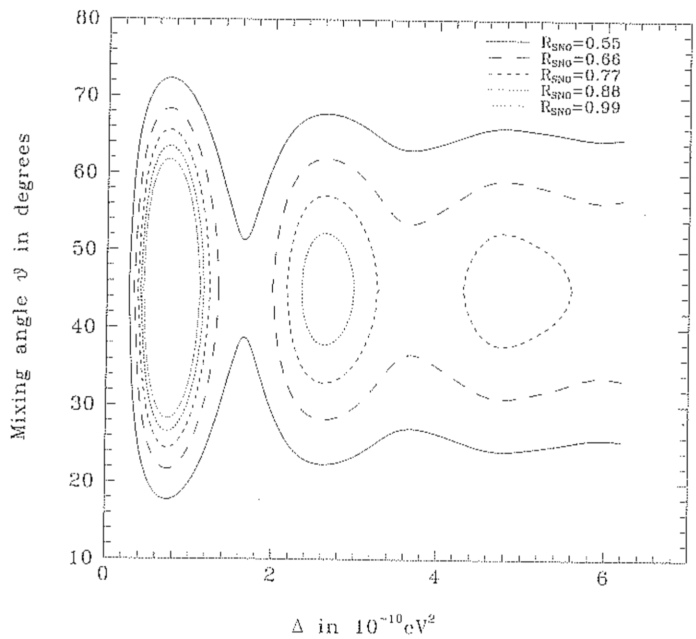

Figure 2. Contours of constant $R_{\rm SNO}$ – the ratio of the NC signal to the energy integrated CC signal at SNO – in the Δ-ϑ plane for oscillation to sequential neutrinos. No neutrino oscillation corresponds to $R_{\rm SNO} = 0.382$.

mixing angles $\vartheta = 30°$ or $45°$, $R_{\rm SNO}$ is significantly different from the no-oscillation limit for the sequential neutrino case. For the sterile neutrino alternative, the change in $R_{\rm SNO}$ is very marginal and it is unlikely that it will be observable. Thus $R_{\rm SNO}$ offers a clear method for the distinction between the sequential and sterile neutrino alternatives, independent of the uncertainty in the overall normalization of the incident neutrino flux.

In figure 2 we present contours of constant values of $R_{\rm SNO}$ in the Δ-ϑ plane for oscillation to sequential neutrinos. The symmetry of the contours about $\vartheta = 45°$ is expected. At $\Delta = 0$ or $\vartheta = 0°$ or $90°$ the limit of no oscillations will be obtained. Values of $R_{\rm SNO}$ as high as 0.99 can only be achieved for smaller values of Δ.

In this work, we have considered the oscillation of ν_e to either (a) a sequential neutrino or (b) sterile neutrino. We have restricted ourselves to vacuum neutrino oscillations. We plan to examine the alternative of matter enhanced MSW resonant flavour conversion later. We have not extended the analysis to a three (or four) neutrino mixing scheme. This would have introduced too many parameters. We have also ignored a small contribution from hep-neutrinos. These variables can also be utilized to study oscillation of supernova neutrinos. Some results are presented in [13].

The behaviour of the variables M_1, M_2, and M_3 and that of $R_{\rm SNO}$ leads us to believe that, as data from SK and SNO accumulate, used in conjunction, they may be fruitful not only to look for oscillations of solar neutrinos but also to zero in on the mass splitting and mixing angles for solar neutrino oscillations.

Note added: After this paper was submitted for publication the SuperKamiokande collaboration released first results, based on the data collected over 504 days, of the measured

Table 2. The values of Δ, ϑ, and $(\chi^2)_{\min}$ obtained by fitting the first six moments of the observed electron spectrum with the theoretical expectation including vacuum oscillations to either a sequential or a sterile neutrino. There are local minima which give almost equally good fits. Two such cases are presented.

Best fits	Sequential			Sterile		
	χ^2	Δ (10^{-10}eV^2)	$\sin^2 2\vartheta$	χ^2	Δ (10^{-10}eV^2)	$\sin^2 2\vartheta$
Case (a)	5.17	4.80	1.0	4.96	4.83	1.0
Case (b)	5.46	0.71	1.0	4.72	0.84	1.0

electron spectrum originating from solar neutrinos [14]. In order to test this data for neutrino oscillations using the methods proposed in this work, one has to consider the normalized moments of the electron spectrum rather than that of the neutrino. The modification is a straightforward procedure using the $\nu - e$ scattering cross-sections, detector resolutions, etc. available in the literature [8]. The moments of the observed electron spectrum can be compared with the theoretically calculated results with vacuum neutrino oscillations to get the best fit values of Δ and ϑ. A detailed analysis of the sequential and sterile alternatives, including a comparison of vacuum oscillations with MSW resonant flavour conversion, is in progress. We summarise the key features here. The results for Δ and ϑ obtained by fitting the first six moments of the electron spectrum are presented in table 2. There are several local minima which give almost equally good fits to the data for both the sequential and sterile options. For both the cases presented in table 2 the preferred mixing is maximal but the best values of Δ are somewhat different: 4.80 $\times 10^{-10}$ eV^2 and 0.71 $\times 10^{-10}$ eV^2. These results can be compared with those obtained in [15] for vacuum oscillations to sequential neutrinos by fitting the electron spectrum. Of the two values of Δ presented in table 2, the larger one is just above the best fit point of [15] while the other falls in their allowed region. The sterile neutrino alternative gives a slightly smaller $(\chi^2)_{\min}$ but the difference is not large enough yet to be deemed significant. The results are remarkably stable. For example, if the first five or four moments of the electron spectrum are fitted, the best fit values of Δ are changed by less than one per cent while ϑ remains the same. The χ^2 corresponding to the fit of case (a) of table 2 for oscillation to sequential (sterile) neutrinos is 3.92 (3.76) and 2.73 (2.62) for fits to the first five or four moments of the spectrum respectively.

Acknowledgements

This work is partially supported by the Eastern Centre for Research in Astrophysics. AR also acknowledges a research grant from the Council of Scientific and Industrial Research.

References

[1] Super-Kamiokande Collaboration, Y Fukuda *et al*, *Phys. Rev. Lett.* **81**, 1562 (1998)
T Kajita, Talk at '*Neutrino '98*', Takayama, Japan (1998)

[2] Rabindra N Mohapatra and Palash B Pal, *Massive neutrinos in physics and astrophysics*, 2nd Ed., World Scientific, Singapore (1998)

[3] B T Cleveland *et al*, *Nucl. Phys.* **B38**, 47 (1995)
Kamiokande Collaboration, Y Fukuda *et al*, *Phys. Rev. Lett.* **77**, 1683 (1996)
GALLEX Collaboration, W Hampel *et al*, *Phys. Lett.* **B388**, 384 (1996)
SAGE Collaboration, J N Abdurashitov *et al*, *Phys. Rev. Lett.* **77**, 4708 (1996)
J N Bahcall and M H Pinsonneault, *Rev. Mod. Phys.* **67**, 781 (1995)
[4] Super-Kamiokande Collaboration, Y Fukuda *et al*, *Phys. Rev. Lett.* **81**, 1158 (1998)
[5] Sudbury Neutrino Observatory Proposal, Report No. SNO-87-12 (1987)
[6] L Wolfenstein *Phys. Rev.* **D34**, 969 (1986)
S P Mikheyev and A Yu Smirnov, *Sov. J. Nucl Phys.* **42(6)**, 913 (1985); *Nuovo Cimento* **9c**, 17 (1986)
[7] S Goswami, *Phys. Rev.* **D55**, 2931 (1997)
S M Bilenky, C Giunti and W Grimus, *Eur. Phys. J.* **C1**, 247 (1998); hep-ph/9711311
[8] J N Bahcall, *Neutrino Astrophysics*, Cambridge Univ. Press (1989)
[9] S Turck-Chieze and I Lopes, *Astrophys. J.* **408**, 347 (1993)
S Turck-Chieze, S Cahen, M Casse and C Doom, *Astrophys. J.* **335**, 415 (1988)
[10] J N Bahcall and E Lisi, *Phys. Rev.* **D54**, 5417 (1996)
J N Bahcall, P I Krastev and E Lisi, *Phys. Rev.* **C55**, 494 (1997)
[11] G Fiorentini, M Lissia, G Mezzorani, M Moretti and D Vignaud, *Phys. Rev.* **D49**, 6298 (1994)
W Kwong and S P Rosen, *Phys. Rev.* **D51**, 6159 (1995)
[12] Debasish Majumdar and Amitava Raychaudhuri, hep-ph/9901401
[13] Debasish Majumdar, Kamales Kar, Alak Ray, Amitava Raychaudhuri and Firoza K Sutaria, astro-ph/9807100, *Phys. Rev.* **D.** (to appear)
[14] Super-Kamiokande Collaboration, Y Fukuda *et al*, hep-ex/9812011
[15] J N Bahcall, P I Krastev and A Yu Smirnov, *Phys. Rev.* **D58**, 096016 (1998)