Universal conductance fluctuations in three dimensional metallic single crystals of Si

Arindam Ghosh
Department of Physics, Indian Institute of Science, Bangalore 560 012, India

A. K. Raychaudhuri
National Physical Laboratory, K.S. Krishnan Road, New Delhi 110 012, India

(February 1, 2008)

In this paper we report the measurement of conductance fluctuations in single crystals of Si made metallic by heavy doping ($n \approx 2-2.5n_c$, n_c being critical composition at Metal-Insulator transition). Since all dimensions (L) of the samples are much larger than the electron phase coherent length L_ϕ ($L/L_\phi \sim 10^3$), our system is truly three dimensional. Temperature and magnetic field dependence of noise strongly indicate the universal conductance fluctuations (UCF) as predominant source of the observed magnitude of noise. Conductance fluctuations within a single phase coherent region of L_ϕ^3 was found to be saturated at $\langle(\delta G_\phi)^2\rangle \approx (e^2/h)^2$. An accurate knowledge of the level of disorder, enables us to calculate the change in conductance δG_1 due to movement of a single scatterer as $\delta G_1 \sim e^2/h$, which is ~ 2 orders of magnitude higher than its theoretically expected value in 3D systems.

72.70.+m, 72.80.Cw, 72.80.Ng

In disordered systems with strong impurity scattering at low temperatures, the noise can arise from the mechanism of universal conductance fluctuations (UCF) [1]. The origin of the UCF lies in the quantum interference of multiply backscattered electrons from the scattering centers and is intimately related to the mechanism of weak localization [8]. Theory of UCF [1], showed that at $T = 0$ the electrical conductance (G) of a metallic system, is an extremely sensitive function of its impurity configuration and alteration of position of even a single impurity over a sufficient length scale may induce a conductance change $\delta G_1 \sim e^2/h$. At finite temperatures, one considers diffusive electronic motion in regions bounded by electron phase coherence length L_ϕ, within which the interference effects are relevant. Total conductance change due to motion of a number of scatterers inside L_ϕ^d is additive (as long as $\delta G_1 \ll e^2/h$) and when the number of such scatterers is sufficiently large, UCF noise inside one phase coherent region is saturated with the total noise power $\langle(\delta G_\phi)^2\rangle \approx (e^2/h)^2$. At higher temperatures or in cleaner samples with longer elastic mean free path of the electrons, the noise due to UCF gives way to that due to mechanisms like local interference (LI) [9].

Extensive experimental studies on UCF has been carried out in 1D and 2D disordered systems like, metallic films of Bi [2], Ag [3], C-Cu composites [4], Li wires [5] GaAs/AlGaAs heterostructures [6] or silicon inversion layers [7] and the existence of UCF has been convincingly established in these systems. However, the absolute magnitude of noise has always been a very roughly estimated parameter since the nature of the defect and the the level of disorder causing the UCF were unknown in most of the cases. In this paper we report conductance noise measurement in a completely different class of "metal", namely heavily doped single crystalline Si, where disorder is primarily introduced as substitutional impurities of P (and B) to the Si matrix. These are metallic systems with a low carrier density (n) ($\approx 10^{-3}$ times the carrier density of a metal) and low conductivity (σ) ($\approx 10^{-2}$ times the conductivity of bad metals) and are close to the critical composition for Metal-Insulator (MI) transition.

There are the following compelling reasons to carry out the experiment on single crystals of Si made metallic by heavy doping: (1) It is a very well defined system where the number as well as the nature of defects are known and the samples used are extremely well characterized unlike most of the previous studies. This will allow us to quantitatively compare the experiment to the magnitude of the noise predicted by the theory. (2) The defects which give rise to the noise are in the bulk of the solid and being single crystal, issues like defects at the surface or grain boundary (which often is the case in polycrystalline metallic thin films) are absent here, (3) This has been the most extensively studied system in investigations of weak localization and M-I transition and is often taken as model solid in which new concepts in theories have been tested. Investigation of electronic phase induced fluctuation phenomena and UCF has thus been a long outstanding need of the field. In particular, it will be search for UCF in a bulk 3D system in contrast to past studies which were in 2D or 1D.

Polished, $\langle 111\rangle$ - Czochralski grown, P and B doped wafers of Si with thickness ≈ 300 μm were sized down to a length of 2 mm, width 0.10-0.15 mm and were thinned down by chemical etching to a thickness of ≈ 30 μm. These wafers were used extensively in conductivity studies earlier [10]. We have chosen two systems with similar concentration of donors (P), but one was compensated with B (Si:P,B), whereas the other was left uncompensated (Si:P)(see table I). They are in the weak localization regime with $5.5 < k_F l < 2.3$, where l is the elas-

tic mean free path. We have studied several samples of the same system for our experiment. Noise, electrical conductivity and magnetoresistance (MR) were all measured in the same sample to avoid any ambiguity. For noise measurement (done with a temperature stability $|\Delta T/T| < 0.01\%$) we used a five probe ac technique (carrier frequency 377 Hz), aided by digital signal processing methods [11]. Sample volume for noise detection (Ω) $\approx 1.5 - 2.0 \times 10^{-12}$ m^3. The peak current density was kept at $\leq 10^6$ A/m^2 (power dissipation $< 50\mu$Watt) to avoid heating. Electrical contacts were made by a specially fabricated thermal wire bonder using gold wires of diameter $\approx 25\mu$m with average separation of electrodes $\approx 200 - 250\mu$m. The contacts, with a temperature independent resistance $\ll 1$ ohm, were ohmic. The Hall coefficient was found to be essentially temperature independent down to 2 K with the variation in the whole range being $\leq 20\%$. This ensured that issues like carrier freeze out are of no consequence here.

Both the samples are metallic with $\sigma_{4.2K}/\sigma_{300K} > 1$ (see inset of figure 1). At $T < 4.2$ K the correction to conductivity, $\Delta\sigma(T)$ $(=\sigma(T) - \sigma(T=0))$, is different in the two samples. The Si:P sample showed a dominant correction which is from the electron-electron interaction [8,10] and $\Delta\sigma(T) \approx mT^{1/2}$ with $m < 0$. The compensated sample Si:P,B, with higher disorder showed $\Delta\sigma(T) > 0$ as expected in a sample with correction to conductivity coming from the weak localization (WL) [8,10].

The phase coherence length L_ϕ was determined from MR ($\Delta\rho/\rho$) measurements. The data are shown in fig.1. For both the samples MR is negative at low H due to the WL contribution and at higher H the interaction contribution dominates. Observed $\Delta\rho/\rho$ were fitted to an expression of MR consisting of contributions from weak-localization (consisting of three field scales due to the inelastic scattering H_ϕ, spin-orbit scattering H_{so} and the spin-flip scattering H_s) and interaction. Given the space considerations we are not giving the detailed fit procedure which are available elsewhere [8]. The lines through the data are the calculated MR. In both the systems we observed $H_\phi \gg H_{so}, H_s$. The L_ϕ as obtained from the MR data using the relation $H_\phi = \hbar/4eL_\phi^2$ are shown as function of T in fig.2. $L_\phi \propto T^{-p/2}$ where $p \approx 1.0$. Even at the lowest temperature, $L_x/L_\phi \approx 10^3$, where L_x is the smallest dimension (thickness) of the sample. This clearly shows that our system is truly 3D.

For all the samples studied, the spectral power was $\propto V_{bias}^2$ within experimental accuracy and found to scale with sample volume as $\propto \Omega^{-\nu}$ [12]. Typically, $\nu \approx 1.1$-1.3. This scaling was found to be independent of the surface to volume ratio and valid till low temperatures. This implies that predominant contribution to the noise arises from the bulk. This clean inverse dependence is a very important observation because almost all the past noise studies done on doped Si ($n \leq 10^{21}$ m^{-3}) the source of noise was the surface or interface [14].

In fig.3 we show the typical data of relative conductance fluctuations $N\langle(\delta G)^2\rangle/G^2$ as function of T. The variance shows a minima at around 150 K - 175 K. In the temperature range $T > 175$ K, $\langle(\delta G)^2\rangle$ rises very rapidly following a near exponential dependence on T. For $T < 100$ K, $\langle(\delta G)^2\rangle$ rises again as T is reduced as $\langle(\delta G)^2\rangle \propto T^{-q}$, where $q = 0.53 \pm 0.03$ for Si:P and 0.57 ± 0.03 for Si:P,B. Very often the noise is expressed through the normalized value γ defined as:

$$\gamma = f^\alpha \, S_v(f) \, (\Omega n)/V_{bias}^2 \qquad (1)$$

$\alpha \approx$ 1-1.2 for $T \geq 10$ K. At $T < 10$ K, the spectral dependence of the noise power differs significantly from $1/f^\alpha$ form as can be seen in the inset of fig.3. For comparison, in both systems at $f = 3$ Hz, $\gamma \approx 1.5$ at $T = 300$ K. It is ≈ 0.3 at the minima at $T = 150$ K - 175 K and at $T = 2$ K it has a value ≈ 1. This is about three orders of magnitude higher than that seen in conventional thin metallic films. In our experiment, the absolute value of γ is reproducible within $\sim 20\%$ from run to run and 50% from sample to sample of the same system, arising primarily from uncertainty in Ω. The temperature dependence of $\langle(\delta G)^2\rangle$ (and hence of γ) including the minima at $T \approx 150$ K, is identical to that seen in films of Bi [2], Ag [3] and C-Cu [4] and is qualitatively different from that seen in lightly doped Si films on sapphire which are deep within the insulating side [14]. It is amazing that the temperature dependence of γ in these three widely different materials are so identical although the absolute value of γ differ. It is even more spectacular considering the fact that the physical form of the samples (thin film vs bulk crystal) are completely different and so is the nature of defect that may cause the $1/f$ noise in these materials.

In investigating the origin of noise, we note that the effect of weak localization was evident from $\Delta\sigma(T)$ as well as MR for both the samples. UCF being a quantum interference related phenomenon, is expected to be the likely origin of noise in this case. Below 100 K the diffusive dynamics of electrons is ensured by $l < L_\phi$. The increase in noise magnitude below 100 K also indicates strongly towards a UCF dominated noise mechanism. However, the cleanest signature of UCF is the sensitivity of $\langle(\delta G)^2\rangle$ to an external magnetic field H. The zero field noise is expected to get reduced by a factor of 1/2 at some characteristic field scale $H_{c1} \approx \mathcal{A}(h/e)/L_\phi^2$, where $\mathcal{A}$ is a constant of the order of unity [1]. This is due to the breaking of time reversal symmetry as the magnetic field introduces an extra random phase to the electron's wave function. In systems with weak spin-orbit scattering, a further drop by another factor of 1/2 is expected due to spin-symmetry breaking from the Zeeman splitting of the conduction electrons at a characteristic field $H_{c2} \sim k_B T/g\mu_{imp}$, where g is the g-factor and μ_{imp} is

the magnetic moment at an impurity site [13]. In fig.4, we plot the observed variance $\langle(\delta G)^2\rangle$ as a function of H. The first reduction of 1/2 occurs very distinctly at field $H_{c1} \approx 2.5$ mT for both the samples. As the temperature is raised, H_{c1} becomes larger. At higher magnetic field, the fluctuations are reduced further by an additional factor of 1/2. We identify this as H_{c2}, which agrees well with the calculated value of ≈ 1.4 T. The distinct dependence of the noise on the magnetic field is a clear indication that a substantial portion of the low temperature noise indeed arises from the UCF mechanism.

From the observed magnitude of rms fluctuations over the experimental bandwidth, we can calculate the average variance, $\langle(\delta G_\phi)^2\rangle$, in a single phase coherent box of volume L_ϕ^3. At length scales larger than L_ϕ, noise from different phase coherent boxes get superposed classically and hence, the observed conductance variance $\langle(\delta G)^2\rangle$ is related to $\langle(\delta G_\phi)^2\rangle$ as [1],

$$\frac{\langle(\delta G)^2\rangle}{G^2} = \frac{L_\phi^3}{\Omega}\,\frac{\langle(\delta G_\phi)^2\rangle}{G_\phi^2} \tag{2}$$

where $G_\phi = \sigma L_\phi$. Thus, at $T \approx 2$ K, from the observed $\langle(\delta G)^2\rangle/G^2 = 1.3\times10^{-13}$ for Si:P and 3.1×10^{-13} for Si:P,B, we get, $\langle(\delta G_\phi)^2\rangle^{1/2} \approx 1.5\times(e^2/h)$ for Si:P and $1.2\times(e^2/h)$ for Si:P,B. This clearly shows that the noise magnitude within L_ϕ^3 in both the samples are saturated. This saturation of $\langle(\delta G_\phi)^2\rangle$ is also reflected in the temperature dependence of the noise. Using $G_\phi = \sigma L_\phi$, we observe $N\langle(\delta G)^2\rangle/G^2 \propto L_\phi \propto T^{-p/2}$, where $p \approx 1.0$. (N is the total number of carriers in volume Ω.) This matches very well with our experimental observation.

One of the most basic feature of the UCF theory is the sensitivity of the fluctuations towards the movement of a single scatterer which is referred to as $\langle(\delta G_1)^2\rangle$ [15]. We show the important result that within a phase coherent volume L_ϕ^3 even $\langle(\delta G_1)^2\rangle$ is saturated to the value $(e^2/h)^2$. From the UCF theory one obtains that [1,2] $\langle(\delta G_\phi)^2\rangle = n_s\langle(\delta G_1)^2\rangle \times L_\phi^3 \times (L_T/L_\phi)^2$ where n_s is the density of "active" scatterers and $L_T = \sqrt{\hbar D/k_B T}$. Using $\langle(\delta G_\phi)^2\rangle \approx (e^2/h)^2$ for both the samples, we get $n_s\langle(\delta G_1)^2\rangle \approx 2\times10^{23}\times(e^2/h)^2$ /m^3 for Si:P and $6\times10^{23}\times(e^2/h)^2$ /m^3 for Si:P,B. For both the samples total dopant concentration, $n_d \approx 10^{25}$ /m^3 (see table I). Thus for both the samples $n_s \approx (0.03-0.04)\times n_d$ is all that is needed to saturate the fluctuations $\langle(\delta G_1)^2\rangle \approx (e^2/h)^2$.

An independent estimate of n_s can be obtained from the analysis of the high temperature noise ($T > 150$ K). At this temperature range $l \le L_\phi$. Under this condition the likely mechanism of noise can be the LI mechanism [9]. From the observed $\langle(\delta G)^2\rangle$ it is possible to estimate the fraction of active sites n_{LI} taking part in the noise production by using the relation [9]

$$N\frac{\langle(\delta G_{LI})^2\rangle}{G^2} = \left[n_{Si}\lambda(T)\beta_c\delta_c\right]^2\frac{n_{LI}}{n_{Si}} \tag{3}$$

where $n_{Si} = 5\times10^{28}$ m^{-3}, is the atomic density of Si, $\beta_c \approx 0.25$ is the anisotropy parameter, $\delta_c \approx 4\pi/k_F^2$, the average defect cross-section and $\lambda(T)$ is the net mean free path. We find that within the observed bandwidth at 300 K, $n_{LI} \approx 5\times10^{-6}n_{Si} \approx 0.03\times n_d$. As T decreases n_{LI} also decreases and below 100 K n_{LI} is $\approx 0.01\times n_d$. Although n_s and n_{LI} are not the same, they will be of similar magnitude. Thus extending over ~ 10 decades of frequency, we obtain an independent estimate of $n_s \approx 0.03\times n_d$ which agrees well with that found in the previous paragraph.

The saturation of $\langle(\delta G_1)^2\rangle$ to $\approx (e^2/h)^2$ is a surprising result in a 3D system. According to UCF theory, in a sample of side L in $d \ge 2$ dimensions, only a fraction $(l/L)^{d-2}$ of the Feynman paths passes through a particular scattering site. Assuming that the transmission amplitude due to this fraction of paths is statistically independent from that due to rest of the paths, one would expect, $\langle(\delta G_1)^2\rangle \sim (e^2/h)^2(l/L)^{d-2}$. More detailed calculation shows in 3D in a sample of size L_ϕ,

$$\langle(\delta G_1)^2\rangle = C\,(e^2/h)^2\,\frac{\alpha(k_F\delta r)}{(k_F l_e)^2}\left(\frac{l}{L_\phi}\right) \tag{4}$$

where $C \approx 4\sqrt{3}\pi$ and $\alpha(x) = 1-\sin^2(x/2)/(x/2)^2 \le 1.0$. δr is the length scale over which the atomic displacement takes place. Using the known value of other parameters and assuming $\alpha \le 1$, we get $\langle(\delta G_1)^2\rangle \le 0.01\times(e^2/h)^2$ in Si:P and $\le 0.03\times(e^2/h)^2$ in Si:P,B. This value of $\langle(\delta G_1)^2\rangle$ is nearly two orders of magnitude less than the saturation value of $(e^2/h)^2$ observed experimentally.

Thus UCF theory underestimates $\langle(\delta G_1)^2\rangle$ for 3D systems. Our result indicates that the assumption of the theory of uncorrelated (statistically independent) Feynman paths for 3D system may not be the correct assumption. From the experimental observation of the saturation of noise, we have to reach the conclusion that when a scattering center in the system moves, transmission probability associated to even those paths which are not passing through the center gets affected by correlation effects. It would be interesting to know whether this correlation among the Feynman paths is due to electronic phase coherence intrinsic to any 3D system or a finite correlation length effect in systems near MI transition.

In conclusion, we have observed saturated universal conductance fluctuations in a 3D disordered system, namely single crystals of silicon made metallic by heavy doping. We find that the existing theory of UCF grossly underestimates the absolute magnitude of noise in such systems even though the qualitative features do show satisfactory agreement [16].

We would like to thank Prof. D. Holcomb of Cornell University for the Si samples and Prof. T.V. Ramakrishnan for a number of helpful discussions.

[1] P.A. Lee, A.D. Stone and H. Fukuyama, Phys. Rev. B **35**, 1039 (1987); N. Giordano, in *Mesoscopic Phenomena in Solids*, *ed.* B.L. Altshuler, P.A. Lee and R.A. Webb, Elsevier Sc. Publ. B.V. (1991)
[2] N.O. Birge, B. Golding and W.H. Haemmerle, Phys. Rev. B **42**, 2735 (1990)
[3] P. McConville and N.O. Birge, Phys. Rev. B **47**, 16667 (1993)
[4] G.A. Garfunkel, G.B. Alers, M.B. Weissman, J.M. Mochel and D.J. VanHarlingen, Phys. Rev. Lett. **60**, 2773 (1988)
[5] J.S. Moon, N.O. Birge and B. Golding, Phys. Rev. B **56**, 15124 (1997)
[6] P. Debray, J.-L.Pichard, J. Vicente and P.N. Tung, Phys. Rev. Lett. **63**, 2264 (1989)
[7] W.J. Skocpol, P.M. Mankeiwich, R.E. Howard, L.D. Jackel, D.M. Tennant and A.D. Stone, Phys. Rev. Lett. **56**, 2865 (1986)
[8] P.A. Lee and T.V. Ramakrishnan, Rev. Mod. Phys. **57**, 287 (1985)
[9] J. Pelz and J. Clarke, Phys. Rev. B **36** 4479 (1987)
[10] U. Thomanschefsky and D.F. Holcomb, Phys. Rev. B **45**, 13356 (1992)
[11] J. Scofield, Rev. Sci. Instrum. **58**, 985 (1987); A. Ghosh, PhD Thesis, Indian Institute of Science, Bangalore (1999)
[12] A. Ghosh and A.K. Raychaudhuri, J. Phys. C **11**, 457 (1999)
[13] A.D. Stone, Phys. Rev. B **39**, 10736 (1989)
[14] R.D. Black, M.B. Weissman and P.J. Restle, J. Appl. Phys. **53**, 6280 (1982)
[15] S. Feng, P.A. Lee and A.D. Stone, Phys. Rev. Lett. **56**, 1960 (1986)
[16] We also find that H_{c1} obtained in noise experiments is much less than the estimate $H_{c1} \approx (h/e)/L_\phi^2$. Further investigations on this issue will be published in a future publication.

TABLE I.

Sample	$\sigma(0)$ (S/m)	n_d (m^{-3})	K	l (nm)	Ω (m^3)	L_T (nm)
Si:P	3.1×10^4	1×10^{25}	0.0	8.2	1.9×10^{-12}	8.7
Si:P,B	1.2×10^4	1×10^{25}	0.4	3.4	1.5×10^{-12}	6.9

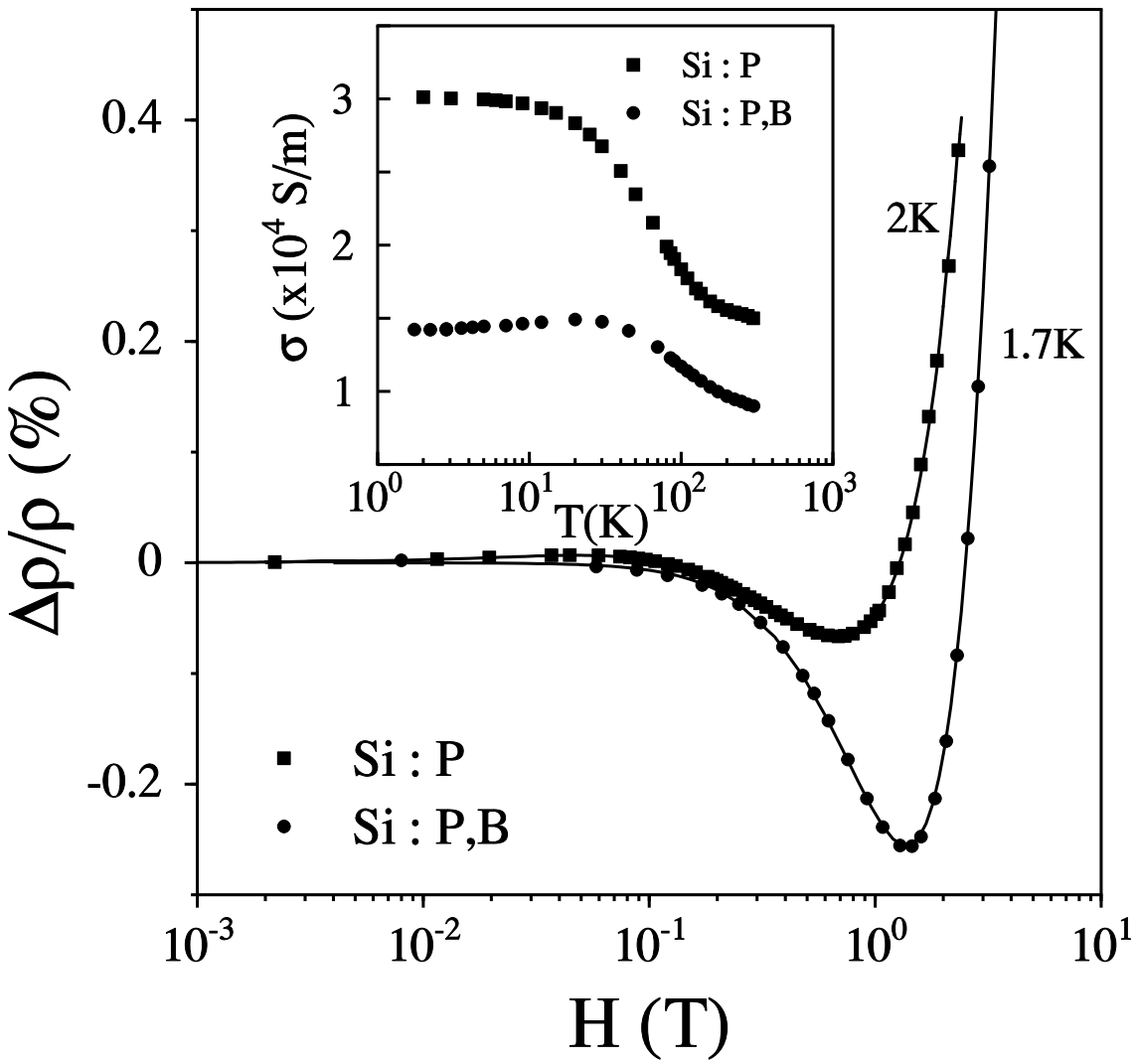
Figure caption:

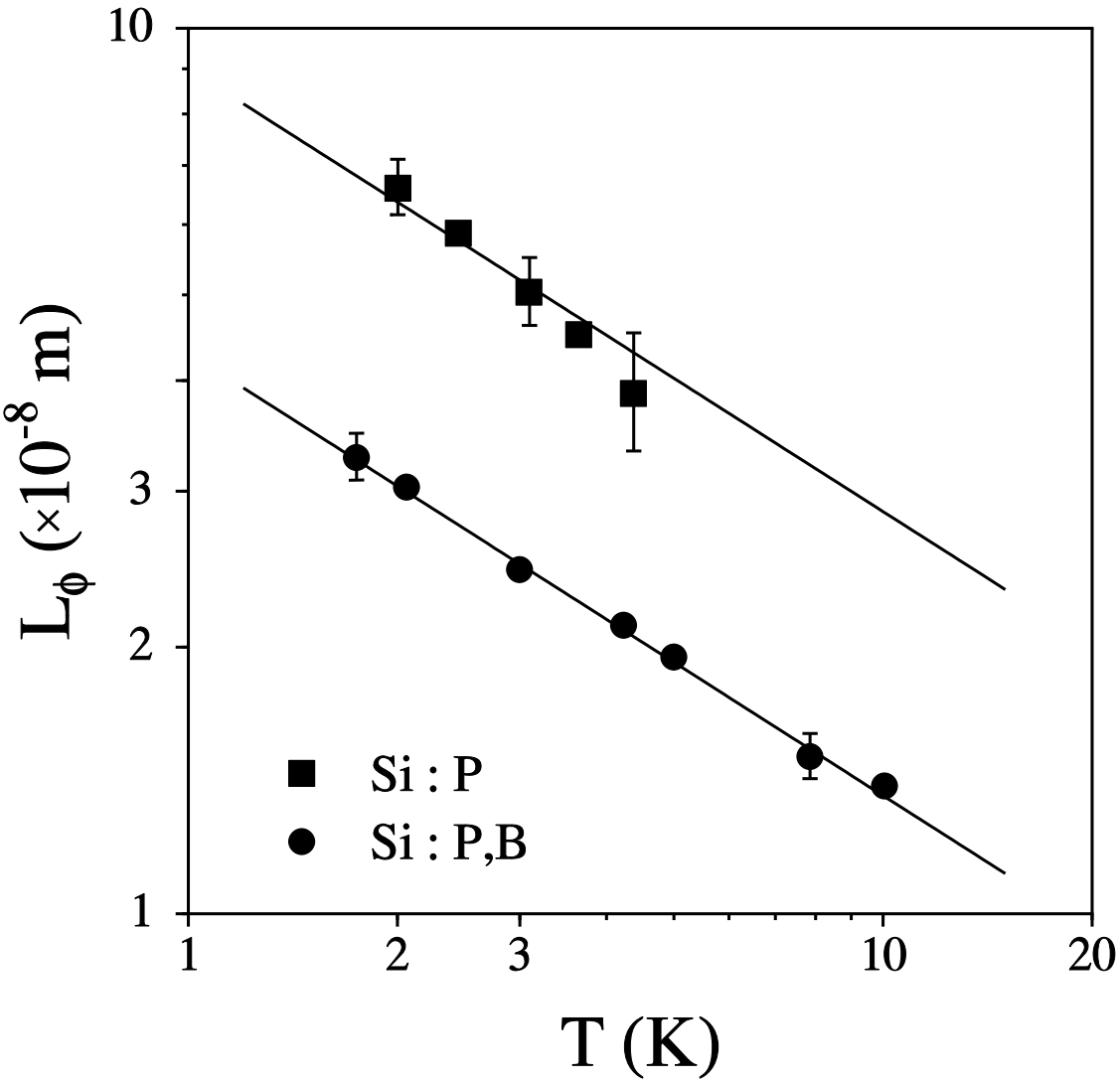
figure 1: Magnetoresistance (MR) as a function of magnetic field (H). The inset shows temperature variation of the zero field conductivity. The solid lines are fit to the MR data according to the weak localization and electron-electron interaction correction.

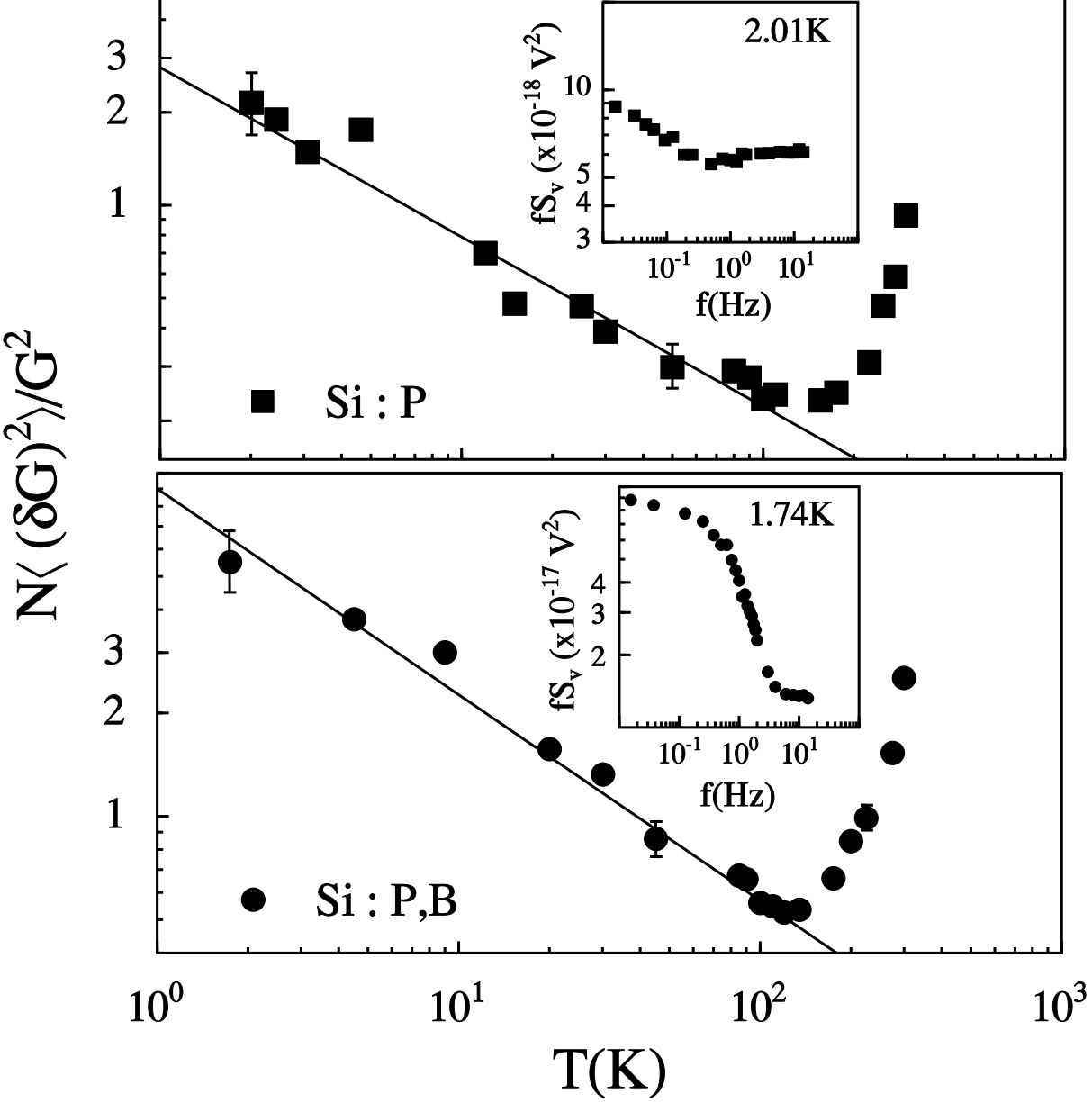
figure 2: The temperature dependence of the phase breaking length L_ϕ determined from magnetoresistance fits. The solid lines show, $L_\phi \propto T^{-1/2}$ in this temperature range.

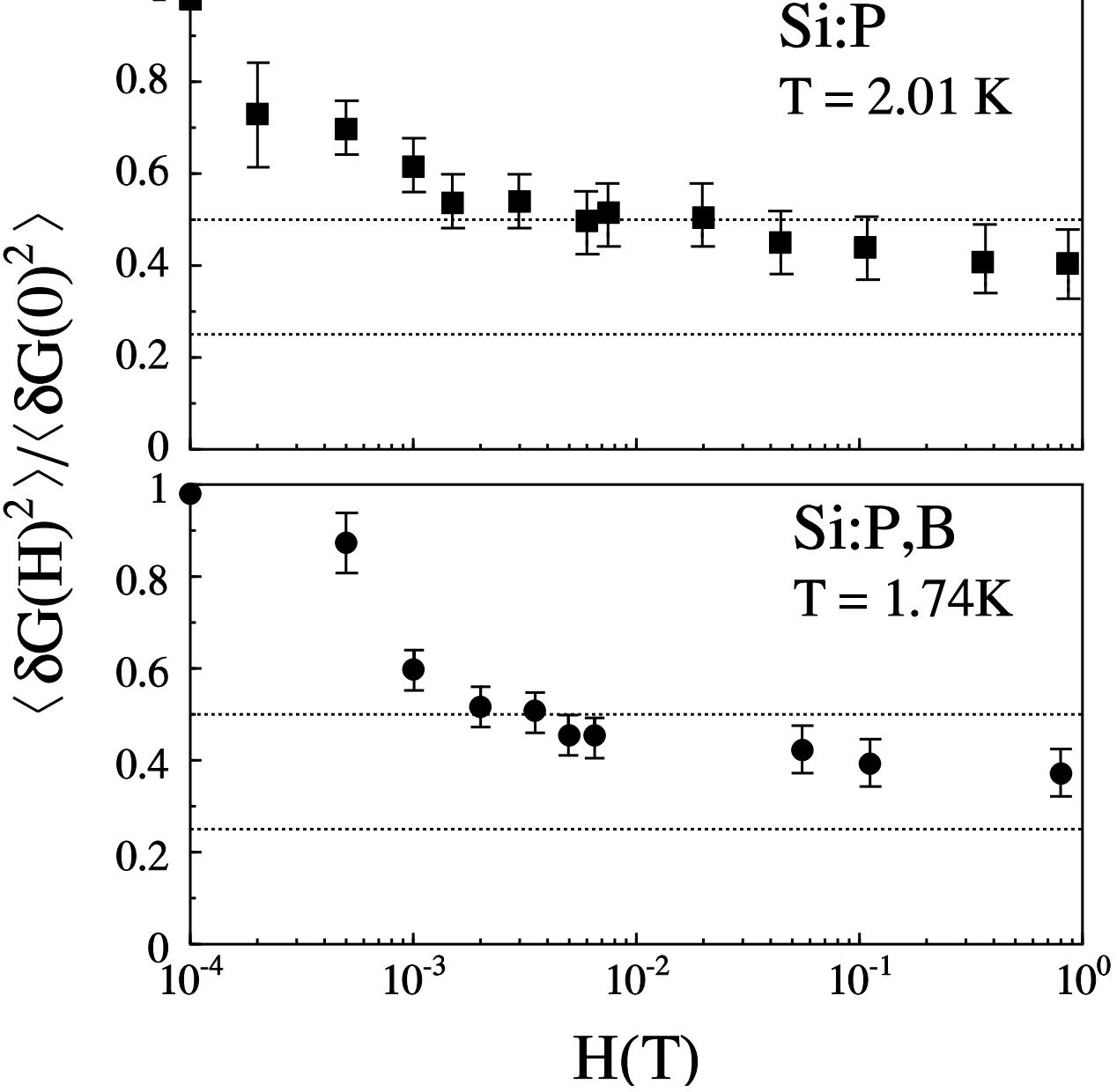
figure 3: Conductance fluctuations as a function of temperature. The inset shows the spectral dependence of noise power at the lowest T.

figure 4: The magnetic field dependence of the conductance fluctuations.


0.4
0.2
0
-0.2
Δρ/ρ (%)
10^-3
10^-2
10^-1
10^0
10^1
H (T)
Si : P
Si : P,B
2K
1.7K
σ (x10^4 S/m)
3
2
1
10^0
10^1
10^2
10^3
T(K)
Si : P
Si : P,B


10
3
2
1
L_ϕ ($\times 10^{-8}$ m)
Si : P
Si : P,B
1
2
3
10
20
T (K)


N⟨(δG)²⟩/G²
3
2
1
Si : P
2.01K
fS_v (x10^-18 V²)
10
5
4
3
10^-1 10^0 10^1
f(Hz)
Si : P,B
1.74K
fS_v (x10^-17 V²)
4
3
2
10^-1 10^0 10^1
f(Hz)
10^0
10^1
10^2
10^3
T(K)


Si:P
T = 2.01 K
Si:P,B
T = 1.74K
$\langle \delta G(H)^2 \rangle / \langle \delta G(0)^2 \rangle$
H(T)
0
0.2
0.4
0.6
0.8
1
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}