QSSC re-examined for the newly discovered SNe Ia

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Abstract

We examine the possible consistency of the quasi-steady state model with the newly discovered SNe Ia. The model assumes the existence of metallic dust ejected from the SNe explosions, which extinguishes light travelling over long distances. We find that the model shows a reasonable fit to the data, which improves if one takes account of the weak gravitational lensing effect of the SNe which have been observed on the brighter side.

Subject heading: cosmology: alternative theories, SNe Ia.
Key words: quasi-steady state cosmology, SNe Ia observations.

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1. INTRODUCTION

The high redshift supernovae (SNe) Ia explosions look fainter than they are expected in the Einstein-deSitter model, which used to be the favoured model before these observations were made a few years ago. This observed faintness is generally explained by invoking some hypothetical source with negative pressure often known as *'dark energy'*, the simplest and the most favoured candidate being a positive cosmological constant Λ. This happens because the metric distance of an object out to any redshift can be increased by incorporating a *'fluid'* with negative pressure in Einstein's equations. However, a constant Λ, is plagued with the so called cosmological constant problem: why don't we see the large vacuum energy density $\rho_{\rm v} \equiv \Lambda/8\pi G \approx 10^{61}$ GeV^4, required to drive inflation in the primordial epochs of the universe which is $\approx 10^{108}$ times larger than the value required by the SNe observations? It is more natural to believe that Λ dropped to zero, after the inflation was over, rather than try to explain its relic at such a small but highly fine-tuned value. Variable Λ or 'quintessence' models also fail to solve this problem without proper fine tuning, apart from the fact that they have a somewhat ad-hoc nature and certainly do not share the elegance of the overall structure of general relativity.

An alternative way to explain the faintness of the high redshift SNe Ia is to consider the absorption of light by metallic dust ejected from the SNe explosions−an issue which, in the standard approach, is generally ignored while discussing m-z relation for SNe Ia. However, there is at least one theory, the quasi-steady state cosmology (QSSC), which considers this effect as a main ingredient of the theory. The QSSC is a Machian theory proposed by Hoyle, Burbidge and Narlikar (1993, 2000) as an alternative to the Standard Big Bang Cosmology (SBBC). This cosmology does not have any cosmic epoch when the universe was hot and *is free from the initial singularity* of the SBBC. It has been shown earlier (Banerjee, et al 2000; Vishwakarma 2002; Narlikar, et al 2002) that by taking into account the absorption of SNe light by the intergalactic metallic dust, the QSSC explains successfully the SNe Ia data from Perlmutter et al (1999), together with SN 1997ff, the highest redshift SN observed so far (Riess, et al 2001). Extending that work further, we examine in this paper how well (or badly !) the QSSC fits the new data discovered with the Hubble Space Telescope (HST) which include 7 highest redshift SNe Ia known, all at $z > 1.25$ (Riess, et al 2004).

Being based on a Machian theory of gravity, the magnitudes and signs of the creation field energy and Λ are determined by the large scale structure of the universe (Hoyle, et al 1995). The role of Λ (which is negative in this theory) in the dynamics of the model is to energize the creation-field by controlling the expansion of the universe. The repulsive effects (akin to that from a positive Λ in the SBBC) are generated by the creation field which has a negative energy density. Therefore the model has cycles of expansion and contraction (regulated by the creation- and the negative Λ-fields respectively) of comparatively shorter period (around 50 Gyr) superposed on a long term (around 1000 Gyr) steady state-like expansion. Creation of matter is also periodic, being confined to pockets of strong gravitational fields around compact massive objects. These creation centers are 'turned on' close to the minimum scale size in a typical cycle and are gradually 'turned off' during expansion to maximum scale size.

The theory seems to meet all the available observational constraints at the present time. According to the QSSC, the cosmic microwave background (CMB) is the relic starlight left by the stars of the previous cycles which has been thermalized by the metallic whisker dust emitted by the supernovae. It is very interesting to note that *the energy available from this process is just right to give a radiation background of 2.7 K at the present epoch* (Hoyle, et al 1994). SBBC, on the other hand, does not predict the present temperature of the CMB. The theory also explains the observed anisotropy of CMB, including the peaks at $l \sim 200$ and $l \sim 600$ which are related, in this cosmology, to the clusters and groups of clusters (Narlikar, et al 2003). The theory does not face the cosmological constant problem mentioned earlier. In fact, the Λ in the QSSC does not represent the energy density of the quantum fields, as this model does not experience the energy scales of quantum gravity except within the local centres of creation.

It may also be noted that, unlike SBBC, the QSSC allows the dark matter to be baryonic. It may be recalled that the SBBC predicts the existence of non-baryonic, though as yet undetected, particles to solve the problems of structure formation and of the missing mass in bound gravitational systems such as galaxies and clusters of galaxies. Although there has been a steady evolution of views on whether dark matter is predominantly cold or hot, there is no satisfactory observational evidence for the postulated particles from laboratory physics. The predicted density distribution of dark halos which result from N-body simulations (Navarro, et al 1996), appears to be inconsistent with observations of spiral galaxies (de Blok, et al 2001) or with strong lensing in clusters of galaxies (Treu, et al 2003). It is therefore fair to say that this scheme has still to demonstrate its viability. However, in the framework of the QSSC, the dark matter need not be necessarily non-baryonic. It can be in the form of baryonic matter being the relic of very old stars of the previous cycles.

For the sake of completeness and for the ready reference, we describe briefly the mathematical formulation of QSSC in the Appendix. The details of this cosmology can be found in the papers of Hoyle, Burbidge and Narlikar mentioned above and in the paper by Sachs et al (1996).

2. EXTINCTION BY METALLIC DUST

Chitre and Narlikar (1976) were the first to discuss the role of intergalactic dust in the m-z relation, which was however largely ignored at that time. It is, however, generally accepted now that the metallic vapours are ejected from the SNe explosions which are subsequently pushed out of the galaxy through pressure of shock waves (Hoyle & Wickramasinghe 1988; Narlikar, et al 1997). Experiments have shown that metallic vapours on cooling, condense into elongated whiskers of $\approx 0.5-1$ mm length and $\approx 10^{-6}$ cm cross-sectional radius (Donn & Sears 1963; Nabarro & Jackson 1958). Indeed this type of dust extinguishes radiation travelling over long distances (Aguire 1999; Banerjee, et al 2000; Narlikar, et al 2002; Vishwakarma 2002; 2003). The density of the dust can be estimated along the lines of Hoyle, et al (2000). If the metallic whisker production is taken as 0.1 $M_\odot$ per SN and if the SN production rate is taken as 1 per 30 years per galaxy, the total production per galaxy (of spatial density ≈ 1 per 10^{75} cm^3) in 10^{10} years is $\approx 2/3 \times 10^{41}$ g. The expected whisker density, hence, becomes $2/3 \times 10^{41} \times 10^{-75} \approx 10^{-34}$ g cm^{-3}. We shall see later that this value is in striking agreement with the best-fitting value coming from the SNe Ia data.

Earlier work (Banerjee, et al 2000; Vishwakarma 2002; Narlikar, et al 2002) has shown that the extinction due to dust adds an extra magnitude $\Delta m(z)$ to the apparent magnitude $m(z)$ of a supernova of redshift z, where

$$\Delta m(z) = 1.0857 \times \ \kappa \ \rho_{\rm g0} \int_0^z (1+z')^2 \frac{{\rm d}z'}{H(z')}. \tag{1}$$

Here κ is the mass absorption coefficient which is effectively constant over a wide range of wavelengths and is of the order 10^5 cm^2 g^{-1} (Wickramasinghe

& Wallis 1996); and $\rho_{\rm go}$ is the whisker grain density at the present epoch: $\rho_{\rm g}\ S^3 = \rho_{\rm g0}\ S_0^3$. The net apparent magnitude is then given by

$$m^{\rm net}(z) = m(z) + \Delta m(z). \tag{2}$$

The usual apparent magnitude $m(z)$ arising from the cosmological evolution is given by

$$m(z) = 5\log[H_0\ d_{\rm L}(z)] + \mathcal{M}, \tag{3}$$

with the luminosity distance $d_{\rm L}$ given by

$$d_{\rm L}(z) = (1+z)\int_0^z \frac{{\rm d}z'}{H(z')}, \tag{4}$$

for the $k = 0$ case of the RW metric. The constant $\mathcal{M}$ appearing in equation (3) is given by $\mathcal{M} \equiv M - 5\ \log H_0 + \textit{constant}$, where M is the absolute magnitude of the SNe. The Hubble parameter $H(z)$ appearing in equations (1) and (4) is given by (A.16).

3. DATA FITTING

We consider the data recently published by Riess et al (2004) which, in addition to having previously observed SNe, also include 16 newly discovered SNe Ia by the HST, 6 of them being among the 7 highest redshift SNe Ia known, all at redshift > 1.25. We particularly focus on their 'gold sample' of 157 SNe Ia which is claimed to have a 'high confidence' quality of the spectroscopic and photometric record for individual supernovae. We note that the data points of this sample are given in terms of distance modulus $\mu_o = m^{\rm net} - M = 5\log d_{\rm L} + \textit{constant}$. However, the zero-point absolute magnitude or Hubble constant were set arbitrarily for this sample. Therefore, while fitting the data, we can compare the observed μ_o with our predicted $m^{\rm net}$ given by equation (2) and compute χ^2 from

$$\chi^2 = \sum_{i=1}^{157}\left[\frac{m^{\rm net}(z_i;\ \Omega_{\Lambda 0},\ \kappa\rho_{\rm g0}H_0^{-1},\ \mathcal{M}) - \mu_{o,i}}{\sigma_{\mu_{o,i}}}\right]^2, \tag{5}$$

the constant $\mathcal{M}$ thus playing the role of the normalization constant. The quantity $\sigma_{\mu_{o,i}}$ is the uncertainty in the distance modulus $\mu_{o,i}$ of the i-th SN. We consider the simplest QSSC model with $k = 0$ case of the RW metric (A.4). Thus there are only three independent free parameters to be estimated

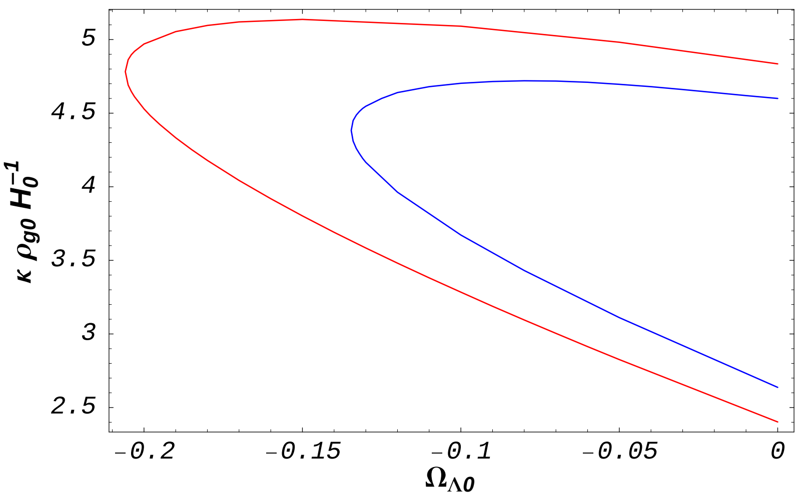

Figure 1: The allowed regions by the 'gold sample' of SNe Ia data (with 157 points) are shown at the 95% (inner contour) and 99% (outer contour) confidence levels, by marginalizing over $\mathcal{M}$. (The parameters κ, $\rho_{\rm g0}$ and H_0 are measured in units of 10^5 cm^2 g^{-1}, 10^{-34} g cm^{-3} and 100 km s^{-1} Mpc^{-1} respectively.) For an average values $\kappa = 5 \times 10^5$ cm^2 g^{-1} and $H_0 = 70$ km s^{-1} Mpc^{-1}, the estimated range of the whisker grain density from the data is 3.5×10^{-34} g cm^{-3} $\leq \rho_{\rm g0} \leq 8 \times 10^{-34}$ g cm^{-3} at 99% confidence level.

from the data: $\mathcal{M}$, $\kappa\rho_{\rm g0}H_0^{-1}$ and $\Omega_{\Lambda 0}$ (only the last one comes from the field equations, see Appendix). The parameter $\kappa\rho_{\rm g0}H_0^{-1}$, which is dimensionless, is of the order of unity if one considers κ of the order 10^5 cm^2 g^{-1}, $\rho_{\rm g0}$ of the order 10^{-34} g cm^{-3} and $H_0 \sim 70$ km s^{-1} Mpc^{-1}. However, we have kept it as a free parameter to be estimated from the data.

By varying the free parameters of the model, we find that χ^2 decreases as $\Omega_{\Lambda 0}$ increases. Thus for the theoretically allowed region $\Omega_{\Lambda 0} < 0$, the best-fitting χ^2 is > 201.41 [at 155 degrees of freedom (dof), i.e., $\chi^2/{\rm dof} = 1.3$]. For example, for the models

$\Omega_{\Lambda 0} = -0.1$: $\chi^2/{\rm dof} = 205.85/155 = 1.33$;
$\Omega_{\Lambda 0} = -0.2$: $\chi^2/{\rm dof} = 210.36/155 = 1.36$;

$\Omega_{\Lambda 0} = -0.3$: $\chi^2/\mathrm{dof} = 214.91/155 = 1.39$.

In order to compare these results with the SBBC, we note that for a constant Λ, the best-fitting flat model ($\Omega_{m0} + \Omega_{\Lambda 0} = 1$) and the global best-fitting model (without any such constraint) are obtained as

$\Omega_{\Lambda 0} = 1 - \Omega_{m0} = 0.69$, with $\chi^2/\mathrm{dof} = 177.07/155 = 1.14$;
$\Omega_{\Lambda 0} = 0.98$, $\Omega_{m0} = 0.46$, with $\chi^2/\mathrm{dof} = 175.04/154 = 1.14$.

The so called '*concordance*' model $\Omega_{m0} = 1 - \Omega_{\Lambda 0} = 0.27$ gives $\chi^2 = 178.17$, which is slightly higher than the best-fitting value of χ^2 for the flat model mentioned above. In fact, there is an almost flat valley around $\Omega_{m0} = 0.3$ on the Ω_{m0}-$\mathcal{M}$-χ^2 surface where χ^2 does not vary significantly and hover around 177-178. For example, the models $\Omega_{m0}(= 1 - \Omega_{\Lambda 0}) = 0.28$ and 0.3, respectively, give $\chi^2 = 177.67$ and 177.13.

Though the fit in the QSSC is certainly not as good as in the SBBC, it is by no means rejectable. Moreover the estimated value of $\kappa\rho_{g0}H_0^{-1}$ is indeed of the order of unity: the best-fitting values of $\kappa\rho_{g0}H_0^{-1}$ for the cases $\Omega_{\Lambda 0} =$ -0.1, -0.2 and -0.3 are respectively 4.19, 4.75 and 5.30, as expected from the theory. Here, the parameters κ, ρ_{g0} and H_0 are measured in units of 10^5 cm^2 g^{-1}, 10^{-34} g cm^{-3} and 100 km s^{-1} Mpc^{-1} respectively. In Figure 1, we have shown the allowed regions in the parameter space $\Omega_{\Lambda 0} - \kappa\rho_{g0}H_0^{-1}$ at 95% and 99% confidence levels, by marginalizing over the parameter $\mathcal{M}$. Figure 2 compares the best-fitting theoretical models with the actual data points.

4. EFFECTS OF WEAK LENSING

Weak gravitational lensing is an unavoidable systematic uncertainty in the use of SNe Ia as standard candles. As the universe is inhomogeneous in matter distribution, the SNe fluxes are magnified by foreground galaxy excess and demagnified by foreground galaxy deficit, compared to a smooth matter distribution. Recently Williams & Song (2004) have reported such a correlation between the magnitudes of 55 SNe from the sample of Tonry et al (2003) and foreground galaxy overdensity. They have found the difference between the most magnified and the most demagnified SNe as about 0.3-0.4 mag. Wang (2004) has claimed further evidence of gravitational magnification of three brightest SNe from the Riess et al (2004) sample:

SN1997as ($z = 0.508$, $\mu_o = 41.64$): magnified by 2.10 ± 0.68;
SN2000eg ($z = 0.540$, $\mu_o = 41.96$): magnified by 1.80 ± 0.70;
SN1997as ($z = 0.886$, $\mu_o = 42.91$): magnified by 2.42 ± 1.98.

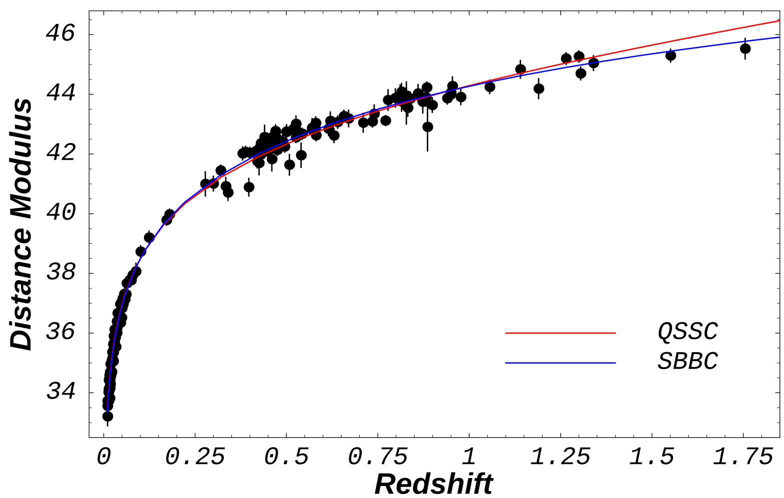

Figure 2: Best-fitting flat models in QSSC and SBBC are compared with the 'gold sample' of SNe Ia data from Riess et al (2004). The QSSC model corresponds to $\Omega_{\Lambda 0} = -0.2$.

These high magnification factors ~ 2 from weak lensing are though somewhat surprising, as also mentioned by Menard & Dalal (2004) who claim not to find any significant correlation between SN magnification and foreground galaxy overdensity. However, even if we assume a mild magnification of the above-mentioned SNe just by an average 0.5 mag, this improves the fit considerably:

$\Omega_{\Lambda 0} = -0.1$: $\chi^2_{\rm improved}/{\rm dof} = 198.55/155 = 1.28$;
$\Omega_{\Lambda 0} = -0.2$: $\chi^2_{\rm improved}/{\rm dof} = 203.17/155 = 1.31$;
$\Omega_{\Lambda 0} = -0.3$: $\chi^2_{\rm improved}/{\rm dof} = 207.82/155 = 1.34$.

Though the weak lensing effects are estimated to be small for SNe at $z < 1$, they are non-negligible for higher redshift SNe. As more SNe are discovered at higher redshifts, it becomes increasingly important to minimize the effect of weak lensing, say, by considering the flux-averaging (Wang, 2000).

5. CONCLUDING REMARKS

The redshift magnitude test has had a chequered history. During the 1960s and the 1970s it was used to draw very categorical conclusions. The deceleration parameter q_0 was then claimed to lie between 0 and 1 and thus it was claimed that the universe is decelerating. Gunn and Oke (1975), however, pointed out that there remained observational errors to be allowed for, that vitiated that conclusion. For example, aperture correction, luminosity evolution, etc. were to be allowed for. It was then realized that the test is not as conclusive in selecting a cosmological model as it earlier appeared to be.

Today's situation, we feel, is hardly different. There has been considerable progress in our understanding of the physics of supernovae, yet it is hard to imagine that the peak luminosity of Type Ia supernovae remains a standard candle over a redshift exceeding 1. Evolution has long been assumed in various other cosmological tests, like the counts of galaxies and radio sources, the variation of angular size with redshift, etc. The assumption of a non-evolving standard candle for supernovae therefore needs to be more critically examined than has been hitherto.

The possible role of gravitational lensing in amplifying the supernova luminosity at high redshifts has been discussed by several authors and we have applied those ideas here to illustrate the difference it can make to any conclusion drawn from the data. Additionally, the role of intergalactic dust still remains to be appreciated fully and we have demonstrated here the possible difference it can make to the viability of a model.

Contrary to the widespread belief that these caveats do not matter or have already been allowed for, we retain a healthy skepticism of this test as contributing to a 'precise' determination of cosmological parameters. For this reason we are satisfied with the level of 'goodness of fit' obtained here for the QSSC. The fit could no doubt be improved by tinkering with the parameters; but given the observational uncertainties, we do not feel it worthwhile to undertake that exercise.

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APPENDIX

The field equations of QSSC, which arise from a Machian theory of gravity, are more general than the Einstein field equations:

$$R^{ij} - \frac{1}{2}R\ g^{ij} - \Lambda\ g^{ij} = -8\pi G\left[T^{ij}_{\rm matter} + T^{ij}_{\rm creation}\right]. \qquad (A.1)$$

Here the speed of light is taken as unity. The first term on the right is the usual energy momentum tensor of matter

$$T^{ij}_{\rm matter} = (\rho + p)u^i\ u^j + p\ g^{ij}, \qquad (A.2)$$

whereas the second term denotes the contribution from a trace-free zero rest mass scalar field c of *negative* energy and stresses with gradient $c_i \equiv \partial c/\partial x^i$:

$$T^{ij}_{\rm creation} = -f\left(c^i c^j + \frac{1}{4}c^\ell c_\ell\ g^{ij}\right), \qquad (A.3)$$

with a positive coupling constant f. In the case of the homogeneous isotropic spacetime described by the RW metric

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + S^2(t)\left\{\frac{\mathrm{d}r^2}{1-kr^2} + r^2(\mathrm{d}\theta^2 + \sin^2\theta\ \mathrm{d}\phi^2)\right\}, \qquad (A.4)$$

the field equations (A.1–A.3) lead to the following two equations for the 'dust' universe ($p = 0$):

$$\frac{\dot{S}^2}{S^2} + \frac{k}{S^2} = \frac{\Lambda}{3} + \frac{8\pi G}{3}(\rho_{\rm m} + \rho_{\rm c}), \quad \rho_{\rm c} \equiv -\frac{3}{4}f\dot{c}^2; \qquad (A.5)$$

$$2\frac{\ddot{S}}{S} + \frac{\dot{S}^2}{S^2} + \frac{k}{S^2} = \Lambda - 8\pi G p_{\rm c}, \quad p_{\rm c} \equiv -\frac{1}{4}f\dot{c}^2. \qquad (A.6)$$

By assuming that the present epoch is represented by the non-creative mode of the model, i. e., $T^{ij}_{{\rm matter}\ ;j} = T^{ij}_{{\rm creation}\ ;j} = 0$, giving

$$\frac{8\pi G}{3}\rho_{\rm m} = \frac{A}{S^3}, \quad \frac{8\pi G}{3}\rho_{\rm c} = \frac{B}{S^4}, \quad A, B = \text{constants}, \qquad (A.7)$$

equation (A.5) can be solved to give the scale factor in the form

$$S = \bar{S}[1 + \eta\cos\psi(t)]. \qquad (A.8)$$

The parameter η lies in the range $0 < \eta < 1$ and the function ψ is given by

$$\dot{\psi}^2 = -\frac{\Lambda}{3}(1 + \eta \cos \psi)^{-2}\{6 + 4\eta \cos \psi + \eta^2(1 + \cos^2 \psi)\}. \qquad \text{(A.9)}$$

Obviously Λ is negative, as has been mentioned earlier. It is also obvious from equation (A.8) that S never becomes zero and oscillates between

$$\bar{S}(1 - \eta) \equiv S_{\text{min}} \leq S \leq S_{\text{max}} \equiv \bar{S}(1 + \eta). \qquad \text{(A.10)}$$

The constant $\bar{S}$, appearing in (A.8) and (A.10), is given by

$$A = 2k\bar{S} - \frac{4}{3}\Lambda \bar{S}^3(1 + \eta^2), \qquad \text{(A.11)}$$

$$B = k\bar{S}^2(1 - \eta^2) - \frac{1}{3}\Lambda \bar{S}^4(1 - \eta^2)(3 + \eta^2), \qquad \text{(A.12)}$$

which can be obtained from equations (A.5) and (A.6) using (A.7) and (A.8) therein. Equations (A.11) and (A.12) can be recast in the following forms in terms of the different energy components computed at the present epoch:

$$\Omega_{\text{m0}} = 2\Omega_{k0}(1 + \bar{z})^{-1} - 4\Omega_{\Lambda 0}(1 + \bar{z})^{-3}(1 + \eta^2), \qquad \text{(A.13)}$$

$$\Omega_{\text{c0}} = -\Omega_{k0}(1 + \bar{z})^{-2}(1 - \eta^2) - 4\Omega_{\Lambda 0}(1 + \bar{z})^{-4}(1 - \eta^2)(3 + \eta^2), \qquad \text{(A.14)}$$

where, as usual,

$$\Omega_{\text{m0}} \equiv \frac{8\pi G}{3H_0^2}\rho_{\text{m0}}, \quad \Omega_{k0} \equiv \frac{k}{H_0^2 S_0^2}, \quad \Omega_{\Lambda 0} \equiv \frac{\Lambda}{3H_0^2}, \quad \Omega_{\text{c}} \equiv \frac{8\pi G}{3H_0^2}\rho_{\text{c0}}. \qquad \text{(A.15)}$$

In terms of these dimensionless parameters, equations (A.5) and (A.6), by the use of (A.7), reduce to

$$H(z) = H_0[\Omega_{\Lambda 0} - \Omega_{k0}(1 + z)^2 + \Omega_{\text{m0}}(1 + z)^3 + \Omega_{\text{c0}}(1 + z)^4]^{1/2}, \qquad \text{(A.16)}$$

$$2q(z) = \left[\frac{H_0}{H(z)}\right]^2 [\Omega_{\text{m0}}(1 + z)^3 - 2\Omega_{\Lambda 0} + 2\Omega_{\text{c0}}(1 + z)^4]. \qquad \text{(A.17)}$$

It should be noted that not all the parameters, introduced above, are independent. For example, equation (A.16) suggests that

$$\Omega_{\Lambda 0} - \Omega_{k0} + \Omega_{\text{m0}} + \Omega_{\text{c0}} = 1. \qquad \text{(A.18)}$$

This equation also suggests that at the maximum redshift $z_{\rm max}$ (say, in the present cycle), one has the following identity:

$$\Omega_{\Lambda 0} - \Omega_{k0}(1+z_{\rm max})^2 + \Omega_{\rm m0}(1+z_{\rm max})^3 + \Omega_{c0}(1+z_{\rm max})^4 = 0. \qquad (A.19)$$

Also equation (A.10) suggests that $z_{\rm max}$, $\bar{z}$ and η are related by

$$1+\bar{z} \equiv \frac{S_0}{\bar{S}} = (1-\eta)(1+z_{\rm max}). \qquad (A.20)$$

Thus out the 7 parameters $\Omega_{\Lambda 0}$, $\Omega_{\rm m0}$, $\Omega_{\rm c0}$, Ω_{k0}, $\bar{z}$, η and $z_{\rm max}$, only 3 parameters, say, Ω_{k0}, $\Omega_{\Lambda 0}$ and $z_{\rm max}$ are independent. For the case $k=0$, which gives the simplest one of the QSSC models, only two independent parameters $\Omega_{\Lambda 0}$ and $z_{\rm max}$ are left out. We further consider $z_{\rm max}=5$, as has been done in the earlier papers on the QSSC, which leaves only one free parameter, say, $\Omega_{\Lambda 0}$ coming from the field equations.

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