Pramāṇa – J. Phys., Vol. 26, No. 1, January 1986, pp. 21–41. © Printed in India.

Validity of the equivalent-photon approximation for the production of massive spin-1 particles

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MS received 11 July 1985

Abstract. We point out that the equivalent-photon approximation (EPA) for processes with massive spin-1 particles in the final state would have validity in a more restricted kinematic domain than for processes where it is commonly applied, viz., those with spin-1/2 or spin-0 particles in the final state. We obtain the criterion for the validity of EPA for the two-photon production of a pair of charged, massive, point-like spin-1 particles $V^{\pm}$, each of mass M and with a standard magnetic moment ($\kappa = 1$). In a process in which one of the photons is real and the other virtual with four-momentum q, the condition for the validity of EPA is $|q^2| \ll M^2$, in addition to the usual condition $|q^2| \ll W^2$, W being the V^+V^- invariant mass. In a process in which both photons are virtual (with four-momenta q and q'), our condition is $|q^2||q'^2|W^4 \ll 16M^8$, in addition to $|q^2| \ll M^2$, $|q'^2| \ll M^2$ and $|q^2| \ll W^2$, $|q'^2| \ll W^2$. Even when these extra conditions permitting the use of EPA are not fulfilled, convenient approximate expressions may still be obtained assuming merely $|q^2| \ll W^2$ and $|q'^2| \ll W^2$.

We also discuss how the extra conditions are altered when the vector bosons are incorporated in a spontaneously broken gauge theory. Examples of W boson production in Weinberg-Salam model are considered for which the condition $|q^2||q'^2|W^4 \ll 16M^8$ is shown to be removed.

Keywords. Equivalent-photon approximation; massive spin-1 particles; gauge theories; electron-positron annihilation; W^+W^- production.

PACS No. 13·10; 14·80; 12·10; 13·90

1. Introduction

The equivalent-photon approximation (EPA), or the Williams-Weizsäcker method (Bohr 1913, 1915; Fermi 1924; Williams 1933, 1934, 1935; Weizsäcker 1934) has for long been a useful theoretical tool (Kessler 1975; Budnev 1975) in studying reactions involving virtual photons, where the virtual photons can be assumed to be almost real. In such a case, the method enables one to relate the cross-section for a given photon-mediated process to that for a simpler process involving only real photons, and thus provides a convenient method of calculation. EPA has assumed even greater importance in recent times when e^+e^- colliding beams have been used to study "two-photon" reactions (Brodsky *et al* 1970; Terazawa 1973).

The EPA method is applied to processes of the type

$$AB \rightarrow A\gamma^* B \rightarrow AX, \qquad (1)$$

where A is a charged particle, B is any particle, and X is a multi-particle state. If the virtual photon γ^* can be treated as almost real, EPA consists in deducing the cross-section for the process $AB \to AX$ by calculating just the cross-section for $\gamma B \to X$ for a real photon γ having only transverse polarization, and with a spectrum which depends only on the nature of the particle A. A general derivation of EPA (Kessler 1975; Budnev 1975) depends on the fact that an amplitude containing the propagator of a virtual photon of four-momentum q is proportional to $1/q^2$, and hence the region $q^2 \approx 0$ dominates. Hence terms proportional to q^2 in the cross-section for $\gamma B \to X$ can be dropped. Moreover, the amplitude involving a longitudinally polarized virtual photon is proportional to $Q = \sqrt{-q^2}$ due to gauge invariance, and is expected to be suppressed for $Q^2 \approx 0$.

In a practical application of the EPA method, one must know how small Q^2 has to be for the approximation to be applicable, and a general criterion is usually expressed (see, for example, Kessler 1975 and Carimalo *et al* 1979) as $Q^2/W^2 \ll 1$, W being the invariant mass of the final state X produced by the photon subprocess. However, this criterion involves certain implicit assumptions about the general behaviour of the virtual-photon amplitude for $Q^2 \ll W^2$. For example, if another mass scale M is present in the problem, it is assumed that the longitudinal-photon cross-section, which gets a factor Q^2/W^2 from the photon polarization vectors, does not have, in addition, powers of W^2/M^2, which could be large for small M. In the latter case, the longitudinal terms could not, in general, be dropped. It is, moreover, assumed that the transverse cross-section does not contain terms with powers of Q^2/M^2 which, for $W \gg M$ cannot be neglected even if $Q \ll W$, unless $Q^2/M^2 \ll 1$[†].

In this paper, we show that the criterion $Q^2/W^2 \ll 1$ is not sufficient for the validity of EPA when massive spin-1 particles are produced in the process (1) considered above, i.e., if any particle in X is a massive vector particle. The reason is that amplitudes with external massive spin-1 particles have in them powers of $1/M$, M being the mass of the vector particle, and if M is smaller than W, some of the Q^2/W^2 terms in the longitudinal cross-section would get multiplied by possibly large factors of the type $(W^2/M^2)^n$, and EPA would not be valid. Moreover, powers of Q^2/M^2 would enter even the transverse cross-section and cannot be neglected.

Powers of $1/M$ might enter an amplitude involving massive vector particles in various ways: (i) In the presence of electromagnetic couplings with a "non-standard" magnetic moment ($\kappa \neq 1$) (Lee and Yang 1962) for the vector fields, the $(p_\mu p_\nu/M^2)(p^2 - M^2)^{-1}$ term in the massive vector propagator would contribute factors of $1/M^2$ (see, for example, the result for the cross-section for $\gamma q \to Wq$ in Mikaelian *et al* 1979). These terms drop out in three diagrams when $\kappa = 1$ (Vainshtein and Khriplovich 1971). (ii) Even in the case of $\kappa = 1$ (massive Yang-Mills fields, for example), or of the amplitude in question involving only couplings of the vector field to fermions, $1/M$ enters the amplitude for the zero-helicity mode of the spin-1 particle via the polarization vector.

It may happen that these factors of $(W^2/M^2)^n$ arising due to the presence of massive

[†]For a general formulation of the underlying assumptions for dropping the longitudinal contribution, see Carimalo *et al* (1979). If the criterion of "isotropy" of Carimalo *et al* can be shown to be satisfied, the longitudinal contributions can be dropped even in the presence of a small mass scale M. We shall see that this is not the case in specific examples. The Q^2/M^2 terms in the transverse contribution cannot be dropped for small M, in any case.

vector fields get altered due to more detailed dynamics, for example if form factors are present, or if the massive vector fields are gauge fields in a spontaneously broken gauge theory which has a smooth $M \to 0$ limit. Even in such cases, though a type of EPA can still be used, it will be quite different from the usual EPA. This could lead, for $W \gg M$, to results different from naive expectations based on EPA (for example, for $W \gg m_W, m_Z$ in electroweak gauge theories, or for $W \gg m_g$ in broken colour gauge theories, m_g being the gluon mass).

We have formalized the above arguments for the process (1) by considering a general analysis of the contributions of the ± 1 and 0 photon helicities, and then specializing to two processes, (i) when A is a fermion, B is a real photon, and X consists of a pair of charged vector particles V^+V^- i.e., the process

$$\text{(i)} \quad A\gamma \to A\gamma^*\gamma \to AV^+V^-,$$

and (ii) when A, B are fermions, and X consists of B and a pair of charged vector particles V^+V^- i.e., the process

$$\text{(ii)} \quad AB \to A\gamma^*B\gamma^* \to ABV^+V^-.$$

In the latter case, we would be discussing the validity of the so-called double equivalent-photon approximation (DEPA), where an effective real-photon spectrum is used for each of the two virtual photons.

For both processes, we assume only point-like electromagnetic couplings for the $V^\pm$, and take $\kappa = 1$, for simplicity. The case of $\kappa \neq 1$ would be more involved in terms of algebraic expressions, but could otherwise be discussed in the same way. Of course, we also have in mind possible applications to spontaneously broken gauge theories, where $\kappa = 1$. In fact, in order to study the effect of additional non-electromagnetic interaction, we consider as concrete examples the processes $e_R^-\gamma \to e_R^-W^+W^-$ and $e_R^-e_L^+ \to e_R^-e_L^+W^+W^-$ in the SU(2) × U(1) Weinberg-Salam model.

For simplicity in discussing our results, we assume $(M, Q) \ll (W, k_T)$, where k_T is the transverse momentum of either of the produced vector particles in the two-photon centre-of-mass (c.m.) frame.

Our results can be summarized as follows. EPA can be applied to process (i) only if $Q^2 \ll M^2$, in addition to the usual condition $Q^2 \ll W^2$. DEPA can be applied to process (ii) only if $Q^2Q'^2 \ll M^8/W^4$ (in addition to $Q^2 \ll M^2$, $Q'^2 \ll M^2$ and $Q^2 \ll W^2$, $Q'^2 \ll W^2$) which is a stringent condition for $W \gg M$. The restriction on Q^2 or Q'^2 may be imposed in an experiment by "tagging", or detecting the final A or B within a restricted scattering angle.

In these cases, if one does a full calculation without using EPA or DEPA, integrates over all azimuthal angles, and only drops terms of the type Q^2/W^2, Q'^2/W^2 or their higher powers, while retaining terms with merely Q^2/M^2, Q'^2/M^2 and their higher powers, a simplified expression for the cross-section can be obtained. This expression does not have the advantage of enabling one to calculate the cross-section for the full process from the cross-section for the real-photon subprocess. Nevertheless, it does lead to a more tractable phase-space integration, and can be used provided azimuthal correlations are not required.

The examples of $e_R^-\gamma \to e_R^-W^+W^-$ and $e_R^-e_L^+ \to e_R^-e_L^+W^+W^-$ in the Weinberg-Salam model illustrate how in a gauge theory the above results get modified. In particular, if the Higgs boson mass m_H is small compared to W in the latter reaction, the condition for the applicability of DEPA is merely $Q^2 \ll m_W^2$, $Q'^2 \ll m_W^2$, assuming that m_H is of the same order as m_W^2, and if an integration over the full range of Q^2 or Q'^2 is carried out, the well-known logarithm $\ln (E/m)$ in the equivalent photon spectrum (Williams *op. cit.*; Weizsäcker *op. cit.*),

$$F_\gamma(x, E) = \frac{\alpha}{\pi}\frac{1+(1-x)^2}{x}\ln\frac{E}{m}, \tag{2}$$

where x is the ratio of the photon energy to the electron energy E, and m is the electron mass, gets modified to $\ln (m_Z/m)$. This could invalidate calculations using EPA for $W \gg m_W$. Also, as the non-leading terms could be significant, a detailed calculation would be necessary.

It should be pointed out that all our discussion is with regard to the *a priori* validity of various approximations. Numerical comparison of the approximations with the exact results lies outside the scope of the present paper. A comparison with DEPA of our approximation scheme, which neglects $Q^2/W^2, Q'^2/W^2$, but not $Q^2/M^2, Q'^2/M^2$, for the process $e_R^- e_L^+ \to e_R^- e_L^+ W^+ W^-$ can be found elsewhere (Jayaraman *et al* 1985).

The organization of the paper is as follows. In §2 we present the analysis of the processes $AB \to A\gamma^* B \to AX$ and $AB \to A\gamma^* B\gamma^* \to ABX$ using helicity representations for the virtual-photon polarization vectors, and indicate how the usual EPA is obtained for $Q^2, Q'^2 \ll W^2$. In §3, we discuss the two processes (i) and (ii) for vector production mentioned above, and obtain the criteria for the validity of EPA in each case. In §4 we present results for $e_R^- \gamma \to e_R^- W^+ W^-$ and $e_R^- e_L^+ \to e_R^- e_L^+ W^+ W^-$ in the Weinberg-Salam model. In the last section (5), we comment on implications of our work for other interactions and processes. Appendices A, B and C list some complete expressions corresponding to the approximate forms used in the text.

2. Helicity formalism

In this section we relate the square of the matrix element for the process

$$AB \to A\gamma^* B \to AX, \tag{3}$$

involving one virtual photon to the helicity amplitudes for $A \to A\gamma^*$ and $\gamma^* B \to X$, and for the process

$$AB \to A\gamma^* B\gamma^* \to ABX, \tag{4}$$

involving two virtual photons to the helicity amplitudes for $A \to A\gamma^*$, $B \to B\gamma^*$ and $\gamma^*\gamma^* \to X$. We essentially use the formalism of Kessler (1970), specifically in the form employed for process (4) Carimalo *et al*, (*op. cit.*). It is essential to go through some details in order to make our notation clear and to facilitate the understanding of the procedure of the subsequent sections.

(i) $AB \to A\gamma^* B \to AX$ The matrix element for the process can be written as

$$M = a_\mu c_\nu (-g^{\mu\nu}/q^2), \tag{5}$$

where a_μ is the amplitude for $A \to A\gamma_\mu^*$ and c_ν is the amplitude for $\gamma_\nu^* B \to X$, q being the virtual photon four-momentum. In the $\gamma^* B$ c.m. frame, we define the direction of $\mathbf{q}$ to be the z axis, and the plane of the momenta of γ^* and X_1 to be the xz plane, where X_1 is a particle chosen from X. One can then write the following representations for photon polarization vectors with ± 1 and 0 helicities:

$$\varepsilon^{\mu}_{\pm 1} \equiv \pm\frac{1}{\sqrt{2}}(0,\ 1,\ \mp i,\ 0),$$

$$\varepsilon^{\mu}_{0} \equiv \frac{1}{iQ}(|\mathbf{q}|,\ 0,\ 0,\ q_0). \tag{6}$$

Here $Q = \sqrt{-q^2}$, and the notation for four-vectors is $V^{\mu} = (V^0, V^1, V^2, V^3)$. These ε^{μ}_{m} satisfy the closure relations

$$\sum_m (-1)^m \varepsilon^{\mu}_m \varepsilon^{\nu *}_m = g^{\mu\nu} - q^{\mu}q^{\nu}/q^2. \tag{7}$$

Using (7) to write $g^{\mu\nu}$ in (5) in terms of polarization vectors, and using gauge invariance ($a_{\mu}q^{\mu} = c_{\nu}q^{\nu} = 0$), we get

$$M = -(1/q^2)\sum_m (-1)^m a_{\mu}\varepsilon^{\mu}_m c_{\nu}\varepsilon^{\nu *}_m. \tag{8}$$

$(-1)^m c_{\nu}\varepsilon^{\nu *}_m$ is the helicity amplitude c_m for $\gamma^* B \to X$. If one were to define the helicity amplitude a_m for $A \to A\gamma^*$ with respect to the same z axis, but an xz plane chosen to be the plane of $\mathbf{q}$ and the momentum of A, this would be related to the helicity amplitude $a_{\mu}\varepsilon^{\mu}_m$ in the old frame by a rotation about the z axis through the azimuthal angle ϕ_1 of X_1 relative to the new xz plane ($A\gamma^*$ plane). Hence,

$$a_m = a_{\mu}\varepsilon^{\mu}_m \exp(-im\phi_1). \tag{9}$$

a_m, of course, does not depend on ϕ_1.

Equation (8) can now be written in terms of a_m and c_m as

$$M = -(1/q^2)\sum_m a_m c_m \exp(+im\phi_1), \tag{10}$$

where the complete ϕ_1 dependence of M appears explicitly. Consequently,

$$\sum |M|^2 = (1/q^4)\sum_{m,\bar{m}} A_{m\bar{m}} C_{m\bar{m}} \exp[i(m-\bar{m})\phi_1], \tag{11}$$

where

$$A_{m\bar{m}} = \Sigma a_m a_{\bar{m}}^*, \qquad C_{m\bar{m}} = \Sigma c_m c_{\bar{m}}^*, \tag{12}$$

and where Σ on the left hand side of (11) and in (12) stands for summation over spin states of the external particles. If desired, appropriate polarizations could be projected out, instead of summing over them.

We shall restrict ourselves, for simplicity, to the case when ϕ_1 is not measured, in which case we can average over ϕ_1 in (11) to get

$$\langle \sum |M|^2 \rangle = (1/q^4)\sum_{m,\bar{m}} A_{m\bar{m}} C_{m\bar{m}} \delta_{m\bar{m}}. \tag{13}$$

Using $A_{++} = A_{--}$ and $C_{++} = C_{--}$ following from parity invariance, we get

$$q^4 \langle \sum |M|^2 \rangle = 2A_{++}C_{++} + A_{00}C_{00}. \tag{14}$$

The usual next step in obtaining EPA is to note that because of gauge invariance, the zero-helicity amplitude c_0 is down for small Q by a factor $Q/q_0 = 2QW/[W^2 - Q^2 - M_B^2]$ (where M_B is the mass of B) relative to c_+, the factor Q/q_0 coming only from the differences in ε_0 and ε_+. Hence if $Q^2 \ll W^2$, c_0 is expected to be small compared to c_+, and can be neglected. We shall examine this argument critically a little later. Meanwhile, we go on with the procedure for obtaining EPA.

If c_0 terms can be dropped, only transverse photon helicities contribute, and consequently, neglecting Q^2/W^2 in C_{++}, we can relate the cross-section for process (3) to that for $\gamma B \to X$, where γ is a real photon with only transverse polarizations. Depending on the nature of the particle A, A_{++} will give an appropriate equivalent spectrum of photons. For example, for a charged lepton with charge e and with mass neglected,

$$A_{++}\ (\text{lepton}) = \frac{-e^2}{2}\left[Q^2 \frac{1+(1-x)^2}{x^2}\right], \tag{15}$$

where $x = (q_0 + |\mathbf{q}|)/(p_0 + p_z)$ in the γB c.m. frame, p being the incident lepton momentum. This leads to the well-known equivalent photon spectrum for leptons (Williams *op. cit.*, Weizsäcker *op. cit.*).

Coming to the assumption of c_0 being negligible compared to c_+ for $Q \ll W$, we note that if another mass scale (say M) is involved, we should be careful. For example, if c_ν contains terms like W/M which in the zero helicity amplitude would get multiplied by Q/W, we would have terms like Q/M, which cannot be neglected for small M. This happens to be the case with the production of massive spin-1 particles, for example, for which the amplitude for longitudinal polarization of these particles would have W/M type of terms. In such a case not only can C_{00} not be dropped in (14), but also all terms proportional to Q^2 in C_{++} cannot be dropped, since some of them could have M^2 in the denominator. Thus, one cannot connect the cross-section to that for a real photon, and EPA cannot be applied.

(ii) $AB \to A\gamma^* B\gamma^* \to ABX$ The procedure for writing the matrix element in terms of helicity amplitudes in this case closely follows the procedure described for process (3). We now write the matrix element for the process as

$$M = a^\mu(-g_{\mu\sigma}/q^2)c^{\sigma\tau}(-g_{\nu\tau}/q'^2)b^\nu, \tag{16}$$

where a_μ is the amplitude for $A \to A\gamma^*_\mu(q)$, b_ν is the amplitude for $B \to B\gamma^*_\nu(q')$, and $c^{\sigma\tau}$ is the amplitude for $\gamma^\sigma(q)^*\gamma^\tau(q')^* \to X$. We now go to the $\gamma^*\gamma^*$ c.m. frame, and choose $\mathbf{q}$ as the z axis. With $\gamma^* X_1$ plane chosen to be the xz plane, the polarization vectors ε and ε' for the two photons with momenta q and q' respectively are

$$\varepsilon^\mu_{\pm 1} = -\varepsilon'^\mu_{\pm 1} \equiv \pm\frac{1}{\sqrt{2}}(0,\ 1,\ \mp i,\ 0), \tag{17}$$

$$\varepsilon^\mu_0 \equiv \frac{1}{iQ}(|\mathbf{q}|,\ 0,\ 0,\ q_0),\ \varepsilon'^\mu_0 \equiv \frac{1}{iQ'}(|\mathbf{q}'|,\ 0,\ 0,\ -q'_0),$$

with $Q = \sqrt{-q^2}$, $Q' = \sqrt{-q'^2}$.

As before, using gauge invariance, the closure relation (7), and a similar one for ε', we can write M as

$$M = 1/(q^2 q'^2) \sum_{m,n} (a_\mu \varepsilon^\mu_m)\left[(-1)^{m+n}(\varepsilon^{\sigma *}_m c_{\sigma\tau} \varepsilon'^{\tau *}_n)\right](b_\nu \varepsilon'^\nu_n). \tag{18}$$

The helicity amplitude for $\gamma^*\gamma^* \to X$ is $c_{mn} = (-1)^m \varepsilon^{\sigma *}_m c_{\sigma\tau} \varepsilon'^{\tau *}_n (-1)^n$.

In analogy with the procedure of subsection (i), we write the helicity amplitudes for a_μ and b_ν in frames with the xz planes defined as the γA and γB momentum planes respectively, as

$$a_m = a_\mu \varepsilon^\mu \exp(-im\phi_1),\ b_n = b_\nu \varepsilon_n'^\nu \exp[in(\phi_1 - \phi)], \tag{19}$$

where ϕ_1 is as defined before, and ϕ is the azimuthal angle of the momentum of B in the original frames. Equation (18) can now be written as

$$M = 1/(q^2 q'^2) \sum_{m,n} a_m c_{mn} b_n \exp(in\phi) \exp[i(m-n)\phi_1]. \tag{20}$$

As before, all the ϕ and ϕ_1 dependence of M is explicit in (20).

We can now write for $\Sigma|M|^2$, where Σ stands for summation over spin of the external particles,

$$\sum |M|^2 = (1/q^4 q'^4) \sum_{m,\bar{m},n,\bar{n}} A_{m\bar{m}} C_{m\bar{m},n\bar{n}} B_{n\bar{n}} \exp\{i[(m-\bar{m})\phi + (n-\bar{n})(\phi - \phi_1)]\} \tag{21}$$

where

$$A_{m\bar{m}} = \Sigma a_m a^*_{\bar{m}},\quad B_{n\bar{n}} = \Sigma b_n b^*_{\bar{n}},$$
$$C_{m\bar{m},n\bar{n}} = \Sigma c_{mn} c^*_{\bar{m}\bar{n}}. \tag{22}$$

If azimuthal correlations are not required, we can average over ϕ and ϕ_1 to get (Carimalo *et al op. cit.*; Kessler 1970)

$$\begin{aligned}\langle \sum |M|^2 \rangle = (1/q^4 q'^4)[&A_{++}(C_{++,++} + C_{++,--} + C_{--,++} + C_{--,--})B_{++} \\ &+ A_{++}(C_{++,00} + C_{--,00})B_{00} + A_{00}(C_{00,++} + C_{00,--})B_{++} \\ &+ A_{00} C_{00,00} B_{00}],\end{aligned} \tag{23}$$

where we have used $A_{++} = A_{--}$, $B_{++} = B_{--}$ and $C^*_{m\bar{m},n\bar{n}} = C_{m\bar{m},n\bar{n}}$.

As before, to get DEPA, one usually drops all C's involving zero helicities for small Q/W, Q'/W using gauge invariance, and neglects terms of order Q/W, Q'/W or higher even in $C_{++,++} + C_{++,--} + C_{--,++} + C_{--,--}$. $\langle \Sigma |M|^2 \rangle$ would then be related to the squared matrix element for $\gamma\gamma \to X$ with real, transversely polarized photons. The equivalent photon spectrum for A, B could be obtained from A_{++} and B_{++}.

As we remarked in the case of process (3), if massive spin-1 particles are being produced, $C_{00,++}$, $C_{++,00}$ and $C_{00,00}$ could have terms proportional to $(Q^2/W^2)(W^2/M^2)^n$, and then these could not be dropped. We cannot drop terms proportional to Q/M in the transverse polarization contribution either, and DEPA fails.

3. Production of charged massive vectors

We discuss here the application of the formalism of § 2 to the processes

$$A\gamma \to A\gamma^*\gamma \to AV^+V^-$$

and (24)

$$AB \to A\gamma^* B\gamma^* \to ABV^+V^-, \tag{25}$$

where A, B are charged fermions, and V^+V^- are charged massive spin-1 particles with mass M and $\kappa = 1$. We assume that besides electrodynamics no other interactions are present. We also assume that V^+V^- state has a large invariant mass (as compared to Q^2

of the photons), hence, processes with intermediate states indicated in (24) and (25) are the dominant ones for V^+V^- production.

(i) $A\gamma \to A\gamma^*\gamma \to AV^+V^-$ As in § 2 (i), we can write the matrix element for this process as

$$M = -a_\mu c^\mu / q^2, \tag{26}$$

where now c_μ, the amplitude for $\gamma_\mu^* \gamma_\nu \to V_\alpha^+ V_\beta^-$ corresponding to the diagrams in figure 1, is given by the following expression, with the Lorentz indices for the vector field displayed explicitly:

$$c_{\mu\nu}^{\alpha\beta} = -e^2 \left[V_{\mu\alpha\sigma}(q,k)\, V_{\nu\beta\tau}(q',k') \left\{ \frac{-g^{\sigma\tau} + (q-k)^\sigma (q-k)^\tau / M^2}{(q-k)^2 - M^2} \right\} + V_{\mu\beta\sigma}(q,k')\, V_{\nu\alpha\tau}(q',k) \left\{ \frac{-g^{\sigma\tau} + (q-k')^\sigma (q-k')^\tau / M^2}{(q-k')^2 - M^2} \right\} - (2 g_{\alpha\beta} g_{\mu\nu} - g_{\alpha\nu} g_{\beta\mu} - g_{\alpha\mu} g_{\beta\nu}) \right]. \tag{27}$$

Here q, q' are the momenta of γ_μ^* and γ_ν respectively, k, k' are the momenta of V^+ and V^-.

$$V_{\mu\alpha\sigma}(q,k) = (2q-k)_\alpha g_{\mu\sigma} + (2k-q)_\mu g_{\alpha\sigma} - (q+k)_\sigma g_{\mu\alpha} \tag{28}$$

represents the $\gamma_\mu V_\alpha^+ V_\sigma^-$ vertex, and the V propagator for momentum p is written as $\{-g^{\sigma\tau} + p^\sigma p^\tau / M^2\} [p^2 - M^2]^{-1}$.

In order to obtain the counterpart of (14) for this process, we will directly calculate $C_{m\bar{m}}$ for unpolarized real photon, V^+ and V^-, rather than c_n for various helicities of these particles. Thus, the required $C_{m\bar{m}}$ is

$$C_{m\bar{m}} = (-1)^{m+\bar{m}} \varepsilon_m^{\mu *} \varepsilon_{\bar{m}}^{\mu'} C_{\mu\mu'}, \tag{29}$$

with

$$C_{\mu\mu'} = c_{\mu\nu}^{\alpha\beta} c_{\mu'\nu'}^{\alpha'\beta'} (-g^{\nu\nu'}) (-g_{\alpha\alpha'} + k_\alpha k_{\alpha'}/M^2)(-g_{\beta\beta'} + k'_\beta k'_{\beta'}/M^2), \tag{30}$$

and where the polarization sums for the real photon, V^+ and V^- have been put in. From Lorentz and gauge invariance considerations, $C_{\mu\mu'}$ can be written as

$$C_{\mu\mu'} = -W_1 g_{\mu\mu'} - W_2 \left(g_{\mu\mu'} + \frac{q^2}{(k\cdot q)^2} k_\mu k_{\mu'}\right) - W_3 \left(g_{\mu\mu'} + \frac{q^2}{(q\cdot k')^2} k'_\mu k'_{\mu'}\right) + W_4 \left(k_\mu k'_{\mu'} + k'_\mu k_{\mu'} - \frac{q\cdot k'}{q\cdot k} k_\mu k_{\mu'} - \frac{q\cdot k}{q\cdot k'} k'_\mu k'_{\mu'} \right), \tag{31}$$

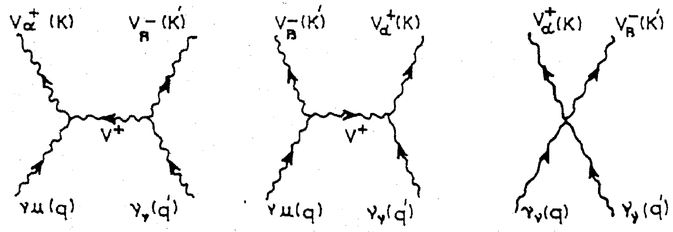

Figure 1. The Feynman diagrams for $\gamma_\mu(q) + \gamma_\nu(q') \to V_\alpha^+(k) + V_\beta^-(k')$ for virtual or real photons.

where W_1, W_2, W_3 and W_4 are invariant functions of q^2, $k\cdot q$, $k'\cdot q$ and $q\cdot q'$ (with $k\cdot q + k'\cdot q - q\cdot q' = q^2$). Terms proportional to $q_\mu, q_{\mu'}$ have been dropped in writing (31). In terms of W_i,

$$C_{++} = [W_1 + W_2 + W_3 + (q\cdot q')W_4]$$
$$C_{00} = -W_1. \tag{32}$$

The explicit forms of W_i, obtained after a lot of tedious algebra, are given in Appendix A. Here, for the sake of simplicity, we only give the W_i in the limit $(Q^2, M^2) \ll (k\cdot q, k'\cdot q, q\cdot q')$, (with Q^2/M^2 not neglected) which suffices for our present purposes.

$$W_1 \approx 2e^4(q^2/M^2)(y^2 + 1/y^2 + 2y + 2/y + 2)$$
$$W_2 \approx e^4\{[14 - 4(q^2/M^2) + \tfrac{1}{2}(q^2/M^2)^2] + 2(4 - q^2/M^2)[y^2 + y + 1/y]\}$$
$$W_3 \approx e^4\{[14 - 4(q^2/M^2) + \tfrac{1}{2}(q^2/M^2)^2 + 2(4 - q^2/M^2)[1/y^2 + y + 1/y]\} \tag{33}$$
$$W_4(q\cdot q') \approx 0,$$

where $y = (2q\cdot k - q^2)/(2q\cdot k' - q^2)$.

Notice that if we neglect q^2/M^2 and put q^2 exactly zero, $W_1 = 0$, and $C_{00} = 0$, as expected from gauge invariance. If, however, Q^2 and M^2 are comparable we have to retain W_1. Thus, even though Q^2/W^2 (with $W^2 = q^2 + 2q\cdot q'$) has been neglected, the zero-helicity photon contribution cannot be dropped unless $Q^2/M^2 \ll 1$†. Also, in C_{++}, we cannot retain just the real-photon terms by dropping Q^2, unless $Q^2/M^2 \ll 1$. Thus, we can apply EPA only if Q^2/W^2 as well as Q^2/M^2 can be neglected.

To complete the calculation we need A_{++} and A_{00}, which can be obtained using the helicity representations (6) and the expression

$$\sum a_\mu a^*_{\mu'} = e^2 \Sigma [\bar{u}(\bar{p})\gamma_\mu u(p)][\bar{u}(\bar{p})\gamma_{\mu'} u(p)]^*$$
$$= e^2[p_\mu \bar{p}_{\mu'} + \bar{p}_\mu p_{\mu'} - (Q^2/2)g_{\mu\mu'}], \tag{34}$$

where p, $\bar{p}$ are the momenta of incoming and outgoing fermion A, with $p - \bar{p} = q$. We obtain (Carimalo *et al op. cit.*)

$$A_{++} = e^2[\tfrac{1}{4}(Q^2 + 4m^2)(\sinh^2\alpha + 2) - 12m^2]$$
$$A_{00} = e^2[\tfrac{1}{2}(Q^2 + 4m^2)\sinh^2\alpha] \tag{35}$$

where††

$$\sinh^2\alpha = 4\left(\frac{1-x}{x^2}\right), \quad x = \frac{q_0 + q_z}{p_0 + p_z}. \tag{36}$$

In the limit of the fermion mass $m \to 0$, we get, on substituting from (35), (36) and (32) into (14), with W_i given by (3),

$$\langle \sum |M|^2 \rangle = \frac{e^6}{Q^2}\left[\frac{1 + (1-x)^2}{x^2}\left\{8\left(y + \frac{1}{y} + 1\right)^2 + 4 + 4(Q^2/M^2) + (Q^2/M^2)^2\right\} + 2\left(\frac{1-x}{x^2}\right)(Q^2/M^2)\left\{\left(y + \frac{1}{y} + 1\right)^2 - 1\right\}\right]. \tag{37}$$

†It is thus clear, a posteriori, that the criterion of 'isotropy' of Carimalo *et al* (1979) is not satisfied unless, possibly, if Q^2 (and Q'^2) satisfies the additional conditions we have discussed.

††Our definition of $\sinh^2\alpha$ differs from that of Carimalo *et al* (1979) but is identical to it for $m = 0$.

This is the complete expression which should be used to calculate the cross-section if Q^2/M^2 is not small, rather than the expression which is obtained using EPA, viz,

$$\langle \sum |M|^2 \rangle_{\text{EPA}} = \frac{e^6}{Q^2} \frac{1+(1-x)^2}{x^2} \left\{ 8\left(y + \frac{1}{y} + 1 \right)^2 + 4 \right\}. \tag{38}$$

We note that integration of (38) over Q^2 would give a logarithmic term, whereas (37) would give other terms as well.

(ii) $AB \to A\gamma^* B\gamma^* \to ABV^+V^-$ The matrix element for this process can be written in terms of the matrix elements a_μ for $A \to A\gamma^*_\mu$, b_ν for $B \to B\gamma^*_\nu$ and $c_{\mu\nu}$ for $\gamma^*_\mu\gamma^*_\nu \to V^+V^-$:

$$M = a^\mu b^\nu c_{\mu\nu}/(q^2 q'^2). \tag{39}$$

Here q, q' are the momenta of the virtual photons at the A and B vertices, respectively. $c_{\mu\nu}$ is obtained again from the diagrams in figure 1, and with Lorentz indices α and β for V^+ and V^-, the corresponding $c^{\alpha\beta}_{\mu\nu}$ is again given by (27).

Using the helicity representations (17), we calculate $C_{m\bar{m},n\bar{n}}$ of (22) directly, using polarization sums for V^+ and V^-:

$$C_{m\bar{m},n\bar{n}} = (-1)^{m+\bar{m}+n+\bar{n}} \varepsilon^{\mu *}_m \varepsilon^{\mu'}_{\bar{m}} \varepsilon'^{\nu *}_n \varepsilon'^{\nu'}_{\bar{n}} C_{\mu\mu',\nu\nu'}, \tag{40}$$

where

$$C_{\mu\mu',\nu\nu'} = c^{\alpha\beta}_{\mu\nu} c^{\alpha'\beta' *}_{\mu'\nu'} (-g_{\alpha\alpha'} + k_\alpha k_{\alpha'}/M^2)(-g_{\beta\beta'} + k'_\beta k'_{\beta'}/M^2). \tag{41}$$

Unlike in the case of $\gamma^*\gamma \to V^+V^-$, we have not attempted to write $C_{\mu\mu',\nu\nu'}$ in terms of the most general Lorentz and gauge invariant tensors. For our problem it is simpler to stick to the specific form for $C_{\mu\mu',\nu\nu'}$, which can be written as

$$C_{\mu\mu',\nu\nu'} = \sum_{i=1}^{21} T^i_{\mu\mu',\nu\nu'} W_i, \tag{42}$$

where $T^i_{\mu\mu',\nu\nu'}$ $(i = 1, 2, \ldots, 21)$ are tensors constructed out of k and k' (q_μ, $q_{\mu'}$, q'_ν, $q'_{\nu'}$ can be dropped due to current conservation at A and B vertices, and q_ν, $q_{\nu'}$, q'_μ, $q'_{\mu'}$ can be written in terms of the other momenta using energy-momentum conservation $q+q' = k+k'$, and the resulting q, q' can be dropped), and W_i $(i = 1, 2, \ldots, 21)$ are invariant functions of q^2, q'^2, $q\cdot q'$, $q\cdot k$, $q\cdot k'$ (with $q\cdot k + q\cdot k' - q\cdot q' = q^2$). W_i, whose calculation is simple in principle, but a time-consuming exercise in algebraic manipulation, and T^i, are listed in Appendix B.

We now present the expressions for the $C_{m\bar{m},n\bar{n}}$ occurring in (23), obtained by substituting (42) in (40):

$$\begin{aligned} C_{++,++} = C_{--,--} = {} & W_2 + W_3 - \tfrac{1}{2}k_T^2[(W_8 + W_9 - 2W_{10}) \\ & + (W_{11} + W_{12} - 2W_{13}) + 2(W_{14} + W_{15} - W_{16} - W_{17})] \\ & + \tfrac{1}{4}k_T^4[W_{18} + W_{19} + 2(W_{20} + W_{21})], \end{aligned} \tag{43}$$

$$\begin{aligned} C_{++,--} = C_{--,++} = {} & W_1 + W_2 - \tfrac{1}{2}k_T^2[2(W_4 + W_5 - W_6 - W_7) \\ & + (W_8 + W_9 - 2W_{10}) + (W_{11} + W_{12} - 2W_{13})] \\ & + \tfrac{1}{4}k_T^4[W_{18} + W_{19} + 2(W_{20} + W_{21})], \end{aligned} \tag{44}$$

$$\begin{aligned} C_{++,00} = C_{--,00} = {} & -W_2 + \tfrac{1}{2}k_T^2(W_{11} + W_{12} - 2W_{13}) + \frac{1}{q'^2}\{[(W_8 + W_9 + 2W_{10}) \\ & - \tfrac{1}{2}k_T^2(W_{18} + W_{19} - 2W_{20} - 2W_{21})]k_0^2|\mathbf{q}|^2 \end{aligned}$$

$$+[(W_8+W_9-2W_{10})-\tfrac{1}{2}k_T^2(W_{18}+W_{19}+2W_{20}+2W_{21})]k_z^2q_0'^2$$
$$+[(W_8-W_9)+\tfrac{1}{2}k_T^2(W_{18}-W_{19})]2k_0k_zq_0'\,|\mathbf{q}|\}, \qquad (45)$$

$$C_{00,++}=C_{00,--}=-W_2+\tfrac{1}{2}k_T^2(W_8+W_9-2W_{10})+\frac{1}{q^2}\{[(W_{11}+W_{12}+2W_{13})$$
$$-\tfrac{1}{2}k_T^2(W_{18}+W_{19}-2W_{20}-2W_{21})]k_0^2|\mathbf{q}|^2$$
$$+[(W_{11}+W_{12}-2W_{13})-\tfrac{1}{2}k_T^2(W_{18}+W_{19}+2W_{20}+2W_{21})k_z^2q_0^2$$
$$-[(W_{11}-W_{12})+\tfrac{1}{2}k_T^2(W_{18}-W_{19})]2k_0k_zq_0\,|\mathbf{q}|\}, \qquad (46)$$

$$C_{00,00}=\frac{(q\cdot q')^2}{q^2q'^2}(W_1+W_3)+W_2+\frac{2q\cdot q'}{q^2q'^2}[(W_4+W_{14}+W_5+W_{15})(k_0^2|\mathbf{q}|^2-k_z^2q_0q_0')$$
$$+(W_6+W_{16}+W_7+W_{17})(k_0^2|\mathbf{q}|^2+k_z^2q_0q_0')$$
$$+\{(W_4+W_{14}-W_5-W_{15})(q_0'-q_0)$$
$$-(W_6+W_{16}-W_7-W_{17})(q_0'+q_0)\}\,k_0k_z|\mathbf{q}|]$$
$$-\frac{1}{q^2q'^2}\{[(W_8+W_9+2W_{10})q^2+(W_{11}+W_{12}+2W_{13})q'^2k_0^2|\mathbf{q}|^2$$
$$+[(W_8+W_9-2W_{10})q^2q_0'^2+(W_{11}+W_{12}-2W_{13})q'^2q_0^2]k_z^2$$
$$+[(W_8-W_9)q^2q_0'-(W_{11}-W_{12})q'^2q_0]2|\mathbf{q}|k_0k_z\}$$
$$+\frac{1}{q^2q'^2}\{W_{18}(k_0|\mathbf{q}|-k_zq_0)^2(k_0|\mathbf{q}|-k_zq_0')^2$$
$$+W_{19}(k_0|\mathbf{q}|+k_zq_0)^2(k_0|\mathbf{q}|+k_zq_0')^2$$
$$+2(W_{20}+W_{21})(k_0^2|\mathbf{q}|^2-k_z^2q_0^2)(k_0^2|\mathbf{q}|^2-k_z^2q_0'^2)\}. \qquad (47)$$

In the above expressions, the components of k, q, q' can be written in a covariant form using

$$k_0=W/2,\; k_z=-[q\cdot(k-k')]/(2|\mathbf{q}|),\; |\mathbf{q}|=[(q\cdot q')^2-q^2q'^2]^{1/2}/W,$$
$$k_T^2=\frac{(2q\cdot k-q^2)(2q'\cdot k-q'^2)-q^2q'^2}{4|\mathbf{q}|^2}-M^2. \qquad (48)$$

Though these could be evaluated exactly using the expressions for W_i given in Appendix B, we have, for simplicity, used the approximation $(Q^2, Q'^2, M^2)\ll(q\cdot q', q\cdot k, q\cdot k')$, but we do not neglect Q^2/M^2, Q'^2/M^2. The result is:

$$C_{++,++}=C_{--,--}\approx e^4\left\{4\left(y^2+\frac{1}{y^2}+1\right)-4\frac{(q^2+q'^2)}{M^2}+\frac{(q^2+q'^2)^2+2q^2q'^2}{M^4}\right.$$
$$\left.-\frac{q^2q'^2(q^2+q'^2)}{M^6}+\frac{q^4q'^4}{4M^8}\right\}, \qquad (49)$$

$$C_{++,--}=C_{--,++}\approx e^4\left\{4\left(y^2+\frac{1}{y^2}\right)+16\left(y+\frac{1}{y}\right)+24+\frac{q^4q'^4}{4M^8}\right\}, \qquad (50)$$

$$C_{++,00}=C_{--,00}\approx -e^4\left(\frac{q'^2}{M^2}\right)\left\{\left[\left(y+\frac{1}{y}+1\right)^2-1\right]-\frac{q^2}{M^2}\left(y+\frac{1}{y}+2\right)+\frac{q^4}{2M^4}\right\}, \qquad (51)$$

$$C_{00,++} = C_{00,--} \approx -e^4\left(\frac{q^2}{M^2}\right)\left\{\left[\left(y+\frac{1}{y}+1\right)^2-1\right]-\frac{q'^2}{M^2}\left(y+\frac{1}{y}+2\right)+\frac{q'^4}{2M^4}\right\}, \tag{52}$$

$$C_{00,00} \approx e^4\left(\frac{q^2q'^2}{M^4}\right)\left\{\frac{(q\cdot q')^2}{4M^4}+\frac{(q\cdot q')}{2M^2}\left(y+\frac{1}{y}\right)+\frac{q^2q'^2}{8M^4}\left(y+\frac{1}{y}-6\right)\right.$$
$$\left.+\frac{3(q^2+q'^2)}{2M^2}+\frac{1}{4}\left(y^2+\frac{1}{y^2}-18\right)\right\}. \tag{53}$$

As before, $y = (2q\cdot k - q^2)/(2q\cdot k' - q^2)$. It can be seen from the above that the zero-helicity contribution can be dropped only if $Q^2/M^2 \ll 1$, $Q'^2/M^2 \ll 1$ and also $(q\cdot q')^2 Q^2 Q'^2/4M^8 \ll 1$.† The last inequality, which may be written as $Q^2Q'^2 \ll 16M^8/W^4$ is much more stringent than the other two, since $2M/W < 1$. Thus, DEPA cannot be used unless these conditions are satisfied. The range of Q^2 and Q'^2 for which DEPA is valid shrinks as W increases. In fact, for sufficiently large W, only the first term in $C_{00,00}$ need be kept and the other terms in $C_{00,00}$, as well as the other helicity combinations can be dropped.

Using A_{++}, A_{00} of (35), (36) and similar expressions for B_{++}, B_{00}, and substituting in (23) one can write the final expression for $\langle \Sigma |M|^2 \rangle$ in the approximation $(Q^2, Q'^2, M^2) \ll (q\cdot q', q\cdot k, q\cdot k')$. We do not give the expression here, since it is somewhat lengthy. It is clear that it would reduce to DEPA only if $Q^2/M^2 \ll 1$, $Q'^2/M^2 \ll 1$ and if $Q^2Q'^2 \ll 16M^8/W^4$.

Under what conditions can we use the single EPA for this process? That is, what is the condition on Q'^2 so that we can neglect longitudinal polarizations for the Q' photon, and also drop all terms proportional to Q'^2, and use (37) together with an appropriate B_{++}? For that, we should be able to drop all terms proportional to q'^2 in (49)–(53), and the condition for this is not just $Q'^2 \ll M^2$, as one would have naively thought, but also $Q^2Q'^2 \ll 16M^8/W^4$.

4. W-pair production in the Weinberg-Salam model

We have seen in §3 that in the case of vector-pair production EPA or DEPA cannot be used unless the Q^2 of the photon or photons is restricted. We have, of course, considered electrodynamics as the theory in that section. Strong interactions, or at high energy, weak boson effects could modify the picture. At high energies, a unified electroweak theory requires us to include the effect of gauge bosons other than the photon, or possible heavy leptons. In fact, the inverse powers of M which occur in the case of W-pair production, for example, would be removed in a gauge theory by the inclusion of other neutral gauge bosons, Higgs bosons, and other leptons. Taking the specific case of W^+W^- production in $e^+e^- \to e^+e^-W^+W^-$ in the Weinberg-Salam (WS) model (Salam 1968; Weinberg 1967) the leading diagrams contributing at high energies (and large W^+W^- invariant mass) would be the "two-photon" diagrams where a Z exchange is also included (figure 2a), as well as the diagrams like those in figure 2b

†It is thus clear, a posteriori, that the criterion of 'isotropy' of Carimalo *et al* (1979) is not satisfied unless, possibly, if Q^2 (and Q'^2) satisfies the additional conditions we have discussed.

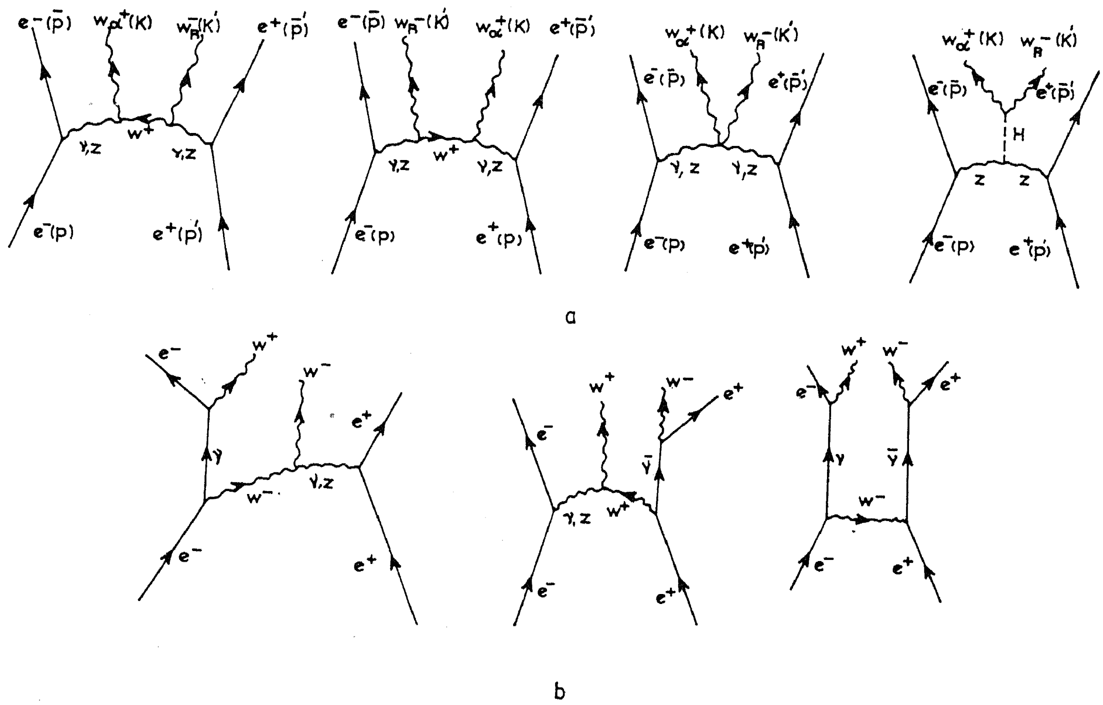

Figure 2. The Feynman diagrams for $e^-(p)+e^+(p')\to e^-(\bar{p})+e^+(\bar{p}')+W^+_\alpha(k)+W^-_\beta(l')$ in the WS model, relevant for large W^+W^- invariant mass.

involving neutrinos. When all these are taken into account, it is expected that there will be no inverse powers of M ("mass singularities"), as the theory is renormalizable.

We verify below that this is the case in a specific example where diagrams of figure 2b do not contribute. Our processes are $e^-_R\gamma\to e^-_RW^+W^-$ and $e^-_Re^+_L\to e^-_Re^+_LW^+W^-$, where R and L refer to right-handed and left-handed polarizations for the leptons. In the limit of zero electron mass (in which limit we calculate), $W^\pm$ do not couple to e^-_R and e^+_L, and we can drop diagrams of figure 2b. Calculating the unpolarized processes would be much more complicated, and though more relevant phenomenologically, it is not essential for our present purposes.

(i) $e^-_R\gamma\to e^-_RW^+W^-$ The diagrams relevant for large W^+W^- invariant mass are given in figure 3, involving photon and Z exchange. The "hard bremsstrahlung" diagrams are, therefore, not considered. The matrix element for $e^-(p)\gamma_\nu(q')\to e^-(\bar{p})W^+_\alpha(k)W^-_\beta(k')$ using the diagrams in figure 3 is

$$M=-e\bar{u}(\bar{p})\gamma^\mu(a+b\gamma_5)u(p)c^{\alpha\beta}_{\mu\nu}/q^2, \tag{54}$$

where a, b are functions of q^2, given by the WS theory to be

$$a=-1+\frac{4\sin^2\theta_W-1}{4\sin^2\theta_W}\frac{q^2}{q^2-m_Z^2},$$

$$b=\frac{1}{4\sin^2\theta_W}\frac{q^2}{q^2-m_Z^2}, \tag{55}$$

where m_z is the mass of the Z boson, θ_W is the weak mixing angle, and $c^{\alpha\beta}_{\mu\nu}$ is given by (27), with M replaced by m_W, the mass of W. Thus, the matrix element has been written in

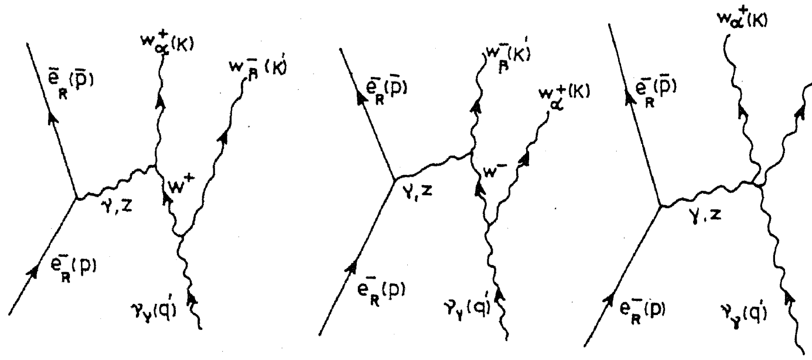

Figure 3. The Feynman diagrams for $e_R^-(p)+\gamma_\nu(q')\to e_R^-(\bar{p})+W_\alpha^+(k)+W_\beta^-(k')$ in the WS model, relevant for large W^+W^- invariant mass.

the form of an effective photon exchange; however, there is a q^2-dependent factor (55) arising from the γ-Z mixing. In obtaining (54), the $(q_\mu q_\nu/m_Z^2)(q^2-m_Z^2)^{-1}$ term in the Z propagator has been dropped, since for the electron mass $m\to 0$,

$$q_\mu\bar{u}\gamma^\mu u = q_\mu\bar{u}\gamma^\mu\gamma_5 u = 0. \tag{56}$$

If in (54) we further put in the condition that the e^- is right-handed, by introducing the projection operator $(1+\gamma_5)/2$ (in the limit $m\to 0$), and using $(a+b\gamma_5)(1+\gamma_5)/2=(a+b)(1+\gamma_5)/2$, we get

$$M = -e\frac{(a+b)}{2}\bar{u}(\bar{p})\gamma^\mu(1+\gamma_5)u(p)c^{\alpha\beta}_{\mu\nu}/q^2, \tag{57}$$

where $a+b$ from (55) is

$$a+b = m_Z^2/(q^2-m_Z^2). \tag{58}$$

We see now that we can use the results of § 3 for calculating $C_{m\bar{m}}$ from $c^{\alpha\beta}_{\mu\nu}$. $A_{m\bar{m}}$ will now be different in that it is only for right-handed helicities of the electron, and that it is multiplied by a factor $m_Z^4/(q^2-m_Z^2)^2$ coming from $(a+b)^2$. The modified $A_{m\bar{m}}$, called $A^{R'}_{m\bar{m}}$, are obtained from

$$A^{R'}_{m\bar{m}} = \frac{e^2}{2}\frac{m_Z^4}{(Q^2+m_Z^2)^2}\left[p_\mu\bar{p}_{\mu'}+\bar{p}_\mu p_{\mu'}-\frac{Q^2}{2}g_{\mu\mu'}+i\varepsilon_{\mu\mu'\alpha\beta}p^\alpha\bar{p}^\beta\right]\varepsilon^\mu_m\varepsilon^{\mu'*}_{\bar{m}}. \tag{59}$$

We find, neglecting m,

$$A^{R'}_{++} = \frac{e^2}{2}\frac{m_Z^4}{(Q^2+m_Z^2)^2}Q^2\frac{1}{x^2}, \tag{60}$$

$$A^{R'}_{--} = \frac{e^2}{2}\frac{m_Z^4}{(Q^2+m_Z^2)^2}Q^2\frac{(1-x)^2}{x^2}, \tag{61}$$

$$A^{R'}_{00} = e^2\frac{m_Z^4}{(Q^2+m_Z^2)^2}Q^2\left(\frac{1-x}{x^2}\right), \tag{62}$$

where x is as defined in (36). Equation (13), together with $C_{++}=C_{--}$ leads to

$$\langle\sum|M|^2\rangle = \frac{e^2}{2Q^2}\frac{m_Z^4}{(Q^2+m_Z^2)^2}\left[\frac{1+(1-x)^2}{x^2}C_{++}+\frac{2(1-x)}{x^2}C_{00}\right]. \tag{63}$$

Substituting for C_{++}, C_{00} from (32) and using W_i ($i = 1, 2, 3, 4$) from Appendix A (with $M = m_W$), we find that the expression obtained differs from that obtained in § 3 merely by a factor 1/2 and a factor $m_Z^4/(Q^2+m_Z^2)^2$, the former arising due to the fact that only the +1 helicity is used for e^-, and the latter due to $\gamma - Z$ mixing. As before, for $Q^2 \ll m_W^2$ (and m_Z^2), (63) reduces to the corresponding EPA equation. However, when the range of Q^2 includes values comparable to m_Z^2 or higher, the factor $m_Z^4/(Q^2+m_Z^2)^2$ leads to an important modification of the logarithm appearing in EPA. In EPA, the factor $1/Q^2$ on integration gives $\ln Q^2$, which for the full kinematic range gives $\ln(s/m^2)$. In (63), the leading term is now proportional to $(1/Q^2)(m_Z^4/(Q^2+m_Z^2)^2)$ which gives on integration

$$\int \frac{dQ^2}{Q^2}\frac{m_Z^4}{(Q^2+m_Z^2)^2} = \ln\left(\frac{Q^2}{Q^2+m_Z^2}\right) + \frac{m_Z^2}{Q^2+m_Z^2}, \qquad (64)$$

which is, for the full kinematic range, approximately $\ln(m_Z^2/m^2)$ for $s \gg m_Z^2$. Thus the $\ln(s/m^2)$ of EPA is replaced by $\ln(m_Z^2/m^2)$ for $s \gg m_Z^2$. However, there is another term of the form

$$\frac{1}{Q^2}\frac{Q^4}{(Q^2+m_Z^2)^2},$$

which on integration gives

$$\ln(Q^2+m_Z^2) + \frac{m_Z^2}{Q^2+m_Z^2}.$$

For $s \gg m_Z^2$, this becomes the leading term giving $\ln s/m_Z^2$. It is clear that EPA does not give a complete picture for large s, and one must be careful in extrapolating to higher energies.

(ii) $e_R^- e_L^+ \to e_R^- e_L^+ W^+ W^-$ For large W^+W^- invariant mass, the relevant diagrams are those in figure 2a. The diagrams in figure 2b do not apply in the limit $m \to 0$ for the chosen longitudinal polarizations of e^- and e^+. The matrix element for

$$e_R^-(p)e_L^+(p') \to e_R^-(\bar{p})e_L^+(\bar{p}')W_\alpha^+(k)W_\beta^-(k')$$

is given by

$$M = \frac{e^2}{4}\frac{(a+b)(a'+b')}{Q^2Q'^2}\bar{u}(\bar{p})\gamma_\mu(1+\gamma_5)u(p)\bar{v}(\bar{p}')\gamma_\nu(1+\gamma_5)v(p')(c_{\mu\nu}^{\alpha\beta}+\bar{c}_{\mu\nu}^{\alpha\beta}), \qquad (65)$$

where a, b are given in (55), a', b' are obtained from a, b by replacing q by q'. We have already incorporated the appropriate projection operators for e_R^- and e_L^+. $c_{\mu\nu}^{\alpha\beta}$ is given by (27) (with M replaced by m_W). $\bar{c}_{\mu\nu}^{\alpha\beta}$, which is given by

$$\bar{c}_{\mu\nu}^{\alpha\beta} = e^2\frac{Q^2Q'^2}{m_W^2}\frac{g^{\alpha\beta}g_{\mu\nu}}{(q+q')^2-m_H^2}, \qquad (66)$$

is obtained from the contribution of the s-channel Higgs diagram, by extracting the appropriate factors already included in $a+b$ and $a'+b'$. In (66), m_H is the mass of the Higgs boson in the Weinberg-Salam model, which we will assume to be comparable to, or smaller than m_W. In obtaining (66) we have also made use of the relation

$$m_W = m_Z\cos\theta_w. \qquad (67)$$

As in § 3, we write $C_{m\bar{m},n\bar{n}}$ in terms of $C_{\mu\mu',\nu\nu'}$ (equation (41)), the latter being now given by

$$C_{\mu\mu',\nu\nu'} = (c^{\alpha\beta}_{\mu\nu} + \bar{c}^{\alpha\beta}_{\mu\nu})(c^{\alpha'\beta'}_{\mu'\nu'} + \bar{c}^{\alpha'\beta'}_{\mu'\nu'})(-g_{\alpha\alpha'} + k_\alpha k_{\alpha'}/m_W^2) \times (-g_{\beta\beta'} + k'_\beta k'_{\beta'}/m_W^2). \quad (68)$$

As before, $C_{\mu\mu',\nu\nu'}$ can be written in terms of T^i and W^i as in (43), except that now W_i have an additional contribution due to the presence of the Higgs diagram, and the extra contributions to W_i are listed in Appendix C. Here, we shall merely present the results of a straightforward calculation of $C_{mm,nn}$ in the limit $(Q^2, Q'^2, m_W^2, m_H^2) \ll (q\cdot q', q\cdot k, q'\cdot k)$. We find that except for $C_{00,00}$, $C_{++,--}$ and $C_{--,++}$, the other $C_{mm,nn}$ are the same as given in (50)–(52), with M replaced by m_W. $C_{00,00}$ is now

$$C_{00,00} = e^4\left(\frac{q^2 q'^2}{4m_W^4}\right)\left\{\left(y^2 + \frac{1}{y^2}\right) + \left(y + \frac{1}{y}\right)\left(2 - \frac{m_H^2}{m_W^2}\right) + \left(5 - \frac{m_H^2}{m_W^2} + \frac{m_H^4}{4m_W^4}\right)\right\}. \quad (69)$$

$C_{++,--}$ and $C_{--,++}$ each get an extra term $-e^4 q^4 q'^4/(4m_W^8)$.

We see that the terms proportional to $(q\cdot q')^2/M^4$ and $q\cdot q'/M^2$ occurring in $C_{00,00}$ in the case of pure electromagnetism (equation (53)), have got cancelled by the Higgs contribution, and are replaced by better-behaved terms, as anticipated. Now we can use DEPA for Q^2/m_W^2, $Q'^2/m_W^2 \ll 1$ and the more stringent conditions obtained in § 3 are not needed. Moreover, the single EPA for either photon can be used provided the corresponding $Q^2 \ll m_W^2$, and m_H^2/m_W^2 is not large.

Using (60)–(62) for $A^{R'}_{mm}$, and similar equations for $B^{L'}_{nn}$ (obtained from $A^{R'}_{nn}$ by replacing Q by Q' and x by x'), we can write an expression for $\langle\Sigma|M|^2\rangle$, in the form

$$\langle\Sigma|M|^2\rangle = \frac{e^4}{4}\frac{m_Z^8}{(Q^2+m_Z^2)^2(Q'^2+m_Z^2)^2}\frac{1}{Q^2Q'^2}\left[\frac{(1-x)^2(1-x')^2}{x^2x'^2}C_{--,--} + \frac{1}{x^2x'^2}C_{++,++} + \frac{(1-x)^2}{x^2x'^2}C_{--,++} + \frac{(1-x')^2}{x^2x'^2}C_{++,--} + \frac{2(1-x)^2(1-x')}{x^2x'^2}C_{--,00} + \frac{2(1-x)(1-x')^2}{x^2x'^2}C_{00,--} + \frac{2(1-x')}{x^2x'^2}C_{++,00} + \frac{2(1-x)}{x^2x'^2}C_{00,++} + \frac{4(1-x)(1-x')}{x^2x'^2}C_{00,00}\right], \quad (70)$$

with $C_{mm,nn}$ listed earlier. In the same way as described in the previous subsection, the double logarithm obtained on integrating $dQ^2\,dQ'^2 m_Z^8[(Q^2+m_Z^2)(Q'^2+m_Z^2)]^{-2}$ will now be $[\ln(m_Z^2/m^2)]^2$, rather than $[\ln(s/m^2)]^2$ occurring in DEPA. Analogous to the single photon case, for large s the leading term is now $(\ln s/m_Z^2)^2$. Hence, as mentioned before, DEPA cannot be used for extrapolations to large s and W^2.

5. Conclusions and discussion

We have shown in the previous sections that the criterion for EPA and DEPA to be applicable in the case of massive vector production is not merely that Q^2/W^2, $Q'^2/W^2 \ll 1$, but in addition Q^2/M^2, Q'^2/M^2 have to be small compared to 1. In the case of DEPA, we found that for V^+V^- production by a two-photon process, a more stringent condition, viz. $Q^2Q'^2 \ll 16M^8/W^4$, has to be satisfied in addition. However, if the vector particles arise in a spontaneously broken gauge theory, the amplitudes would not be badly behaved for large W, and this latter condition may not be needed. We have shown this to be the case for W^+W^- production from longitudinally polarized e^+e^- beams via the two-photon type of process, in the Weinberg-Salam theory.

We have treated somewhat special interactions and processes in the main body of the paper, largely for the sake of simplicity. Nevertheless, we hope that the purpose of highlighting the problems in applying EPA or DEPA to processes involving massive spin-1 particles has been served. We discuss below possible implications for other interactions and processes.

If we did not assume $\kappa = 1$ for the anomalous magnetic moment of the vector particles in our general discussion, the amplitudes for V^+V^- production would have worse singularities for $M \to 0$, and we would possibly find that more stringent conditions are necessary for the applicability of EPA. In fact, even in the case of a process involving a single vector particle, with $\kappa \neq 1$ inverse powers of M occur (Mikaelian *et al* 1979). Thus, the cross-section for a process like $ep \to eWX$ calculated using EPA (Kamal *et al* 1981), if extrapolated to $W^2 \gg M^2$, for the full kinematic range of Q^2, could be in error. A numerical check by means of exact calculations would be useful.†

As pointed out earlier, because the leading logarithm $\ln(s/m^2)$ of EPA gets replaced for $s \gg m_W^2$ by $\ln(m_Z^2/m^2)$ in W production in the Weinberg-Salam model, EPA extrapolated to large $s(s \gg m_W^2)$ cannot be trusted. For example, the use of DEPA to estimate cross-sections for $e^+e^- \to e^+e^-W^+W^-$ in the TeV range, as in the work of Katuya (1983) may need critical examination (Jayaraman *et al* 1985).

The method described in § 3(i) for obtaining the cross-section for vector-pair production when Q^2/M^2 is not small, was earlier applied by us (Jayaraman *et al* 1982) in the calculation of $e^+e^- \to e^+e^- + 2$ jets in the integer-charge quark model based on a spontaneously broken colour gauge theory. In this theory, the gluons are charged and massive. Being charged, they contribute to the process in the lowest order (α^4) via the process $e^+e^- \to e^+e^-g^+g^-$. In order to compare the model with the PETRA single-tag experiments, this process was calculated using the single EPA, retaining all terms involving q^2/m_g^2 or powers of q^2/m_g^2, where q is the momentum of the tagged photon. The momentum of the untagged photon was put equal to zero, and the standard equivalent spectrum was used for that photon. In terms of our present analysis it can now be seen that this procedure was not complete (see the comment at the end of § 3), since the coloured Higgs bosons contributions were neglected, assuming the masses of the Higgs bosons to be large. However, we have shown that if the Higgs bosons are assumed light ($m_H \simeq m_g$), as noted in § 4, the potentially large terms of the type $Q^2Q'^2W^4/M^8$ occurring in $C_{00,00}$ (equation (53)) are cancelled by the Higgs contribution (Jayaraman 1983; Jayaraman *et al* 1983; Godbole *et al* 1984). The numerical results are altered somewhat, but the main conclusions are unaltered. We may note here that Cho *et al* (1983) have also calculated $e^+e^- \to e^+e^-g^+g^-$ in the integer-charged

† It is of interest to note that several calculations of W production, either exact or using EPA, were done in the 60's or even earlier. However, since all these were on hadron targets, form factors ensured that Q^2 was restricted to be effectively small. Nevertheless, the total cross-sections showed discrepancies between exact and EPA calculations (see Williamson and Salzman 1963; Berman and Tsai 1963). These discrepancies, as well as the sensitivity of the results to form factors, may well have arisen due to reasons discussed in this paper; only a detailed study can tell. For later calculations using EPA, see also Kim and Tsai (1972, 1973), Kessler (1975) and Renard (1979).

quark model but have ignored the careful consideration of the EPA which is necessary as pointed out in this paper.

A numerical check by means of exact calculation of the various conclusions of the present paper is essential and would be interesting.

The comments on the validity of EPA made in this paper are likely to be valid also for the production of particles with spin $\geqslant \frac{3}{2}$. This would be an interesting theoretical problem to investigate, though it may not be of great practical interest at the moment.

Acknowledgement

We thank Dr T Chandramohan for discussions and for drawing our attention to certain references.

Appendix A

The invariant functions W_i $(i = 1, 2, 3, 4)$ relevant to the reaction $\gamma^*(q)+\gamma(q') \to V^+(k) + V^-(k')$ discussed in §3 are listed here.

$$W_1 = e^4\Bigg\{4-\frac{4q^2}{M^2}+\frac{q^4}{M^4}-8q^2(q^2-4M^2)\left(\frac{1}{(2q'\cdot k)^2}+\frac{1}{(2q'\cdot k')^2}\right)$$
$$+8\left(\frac{2q'\cdot k'}{2q'\cdot k}+\frac{2q'\cdot k}{2q'\cdot k'}+1\right)^2+\frac{8q^2(q^2-2M^2)(q^2-4M^2)}{M^2(2q'\cdot k)(2q'\cdot k')}$$
$$+\frac{8q^2(q^2-4M^2)}{M^2}\left(\frac{1}{2q'\cdot k'}+\frac{1}{2q'\cdot k}\right)\Bigg\}-W_2-W_3. \tag{A.1}$$

$$W_2 = e^4\Bigg\{-(q\cdot k)^2\left[4\left(\frac{q^2}{M^2}-4-\frac{12M^2}{q^2}\right)\frac{1}{(2q'\cdot k')^2}\right.$$
$$\left.+8\left(\frac{q^2}{M^2}-4\right)\frac{1}{(2q'\cdot k)^2}\right]\Bigg\}-\frac{(q\cdot k)(q\cdot k')}{q^2}W_4. \tag{A.2}$$

$$W_3 = e^4\Bigg\{-(q\cdot k')^2\left[4\left(\frac{q^2}{M^2}-4-\frac{12M^2}{q^2}\right)\frac{1}{(2q'\cdot k)^2}\right.$$
$$\left.+8\left(\frac{q^2}{M^2}-4\right)\frac{1}{(2q'\cdot k')^2}\right]\Bigg\}-\frac{(q\cdot k)(q\cdot k')}{q^2}W_4. \tag{A.3}$$

$$W_4 = e^4\Bigg\{\frac{8q^2}{M^2}(q^2-4M^2)\left(\frac{1}{(2q'\cdot k)^2}+\frac{1}{(2q'\cdot k')^2}\right)$$
$$+\frac{(q^2-2M^2)}{(2q'\cdot k)(2q'\cdot k')}\left(-\frac{2q^4}{M^4}+\frac{8q^2}{M^2}-24\right)\Bigg\} \tag{A.4}$$

It may be mentioned that though W_2, W_3 appear to be singular at $q^2 = 0$ because of q^2 in the denominator in (A.2) and (A.3), they are not really so, as an explicit expression obtained on substituting for W_4 from (A.4) would show.

Appendix B

We list here the expressions for $T^i_{\mu\mu',\nu\nu'}$ and W_i ($i = 1, 2, \ldots, 21$) relevant to the reaction $\gamma^*(q)+\gamma^*(q') \to V^+(k)+V^-(k')$ discussed in §3. We use $T = (q-k)^2 - M^2$, $U = (q-k')^2 - M^2$, and $y = T/U$. The tensors $T^i_{\mu\mu',\nu\nu'}$ are:

$$T^1_{\mu\mu',\nu\nu'} = g_{\mu\nu}g_{\mu'\nu'},\ T^2_{\mu\mu',\nu\nu'} = g_{\mu\mu'}g_{\nu\nu'},\ T^3_{\mu\mu',\nu\nu'} = g_{\mu\nu'}g_{\mu'\nu},$$

$$T^4_{\mu\mu',\nu\nu'} = g_{\mu\nu}k_{\mu'}k_{\nu'}+g_{\mu'\nu'}k_{\mu}k_{\nu},\ T^5_{\mu\mu',\nu\nu'} = g_{\mu\nu}k'_{\mu'}k'_{\nu'}+g_{\mu'\nu'}k'_{\mu}k'_{\nu},$$

$$T^6_{\mu\mu',\nu\nu'} = g_{\mu\nu}k_{\mu'}k'_{\nu'}+g_{\mu'\nu'}k_{\mu}k'_{\nu},\ T^7_{\mu\mu',\nu\nu'} = g_{\mu\nu}k'_{\mu'}k_{\nu'}+g_{\mu'\nu'}k'_{\mu}k_{\nu},$$

$$T^8_{\mu\mu',\nu\nu'} = g_{\mu\mu'}k_{\nu}k_{\nu'},\ T^9_{\mu\mu',\nu\nu'} = g_{\mu\mu'}k'_{\nu}k'_{\nu'},\ T^{10}_{\mu\mu',\nu\nu'} = g_{\mu\mu'}(k_{\nu}k'_{\nu'}+k'_{\nu}k_{\nu'}),$$

$$T^{11}_{\mu\mu',\nu\nu'} = g_{\nu\nu'}k_{\mu}k_{\mu'},\ T^{12}_{\mu\mu',\nu\nu'} = g_{\nu\nu'}k'_{\mu}k'_{\mu'},\ T^{13}_{\mu\mu',\nu\nu'} = g_{\nu\nu'}(k_{\mu}k'_{\mu'}+k'_{\mu}k_{\mu'}),$$

$$T^{14}_{\mu\mu',\nu\nu'} = g_{\mu\nu'}k_{\mu'}k_{\nu}+g_{\mu'\nu}k_{\mu}k_{\nu'},\ T^{15}_{\mu\mu',\nu\nu'} = g_{\mu\nu'}k'_{\mu'}k'_{\nu}+g_{\mu'\nu}k'_{\mu}k'_{\nu'},$$

$$T^{16}_{\mu\mu',\nu\nu'} = g_{\mu\nu'}k_{\mu'}k'_{\nu}+g_{\mu'\nu}k_{\mu}k'_{\nu'},\ T^{17}_{\mu\mu',\nu\nu'} = g_{\mu\nu'}k'_{\mu'}k_{\nu}+g_{\mu'\nu}k'_{\mu}k_{\nu'},$$

$$T^{18}_{\mu\mu',\nu\nu'} = k_{\mu}k_{\mu'}k'_{\nu}k'_{\nu'},\ T^{19}_{\mu\mu',\nu\nu'} = k'_{\mu}k'_{\mu'}k_{\nu}k_{\nu'},$$

$$T^{20}_{\mu\mu',\nu\nu'} = k_{\mu}k'_{\mu'}k_{\nu}k'_{\nu'}+k_{\mu'}k'_{\mu}k_{\nu'}k'_{\nu},$$

$$T^{21}_{\mu\mu',\nu\nu'} = k_{\mu}k'_{\mu'}k'_{\nu}k_{\nu'}+k_{\mu'}k'_{\mu}k'_{\nu'}k_{\nu}.$$

The invariant functions W_i are:

$$W_1 = e^4\Bigg\{-4-4\frac{(q^2+q'^2)}{M^2}+\frac{(q^2+q'^2)^2}{M^4}-8\left(y+\frac{1}{y}\right)$$
$$-\left(\frac{1}{T}+\frac{1}{U}\right)\frac{2q^2q'^2}{M^2}\left(\frac{q^2+q'^2}{M^2}-8\right)+\frac{2q^4q'^4}{M^4}\frac{1}{TU}$$
$$+\left(\frac{1}{T^2}+\frac{1}{U^2}\right)q^2q'^2\left(16-\frac{4(q^2+q'^2)}{M^2}+\frac{q^2q'^2}{M^4}\right)\Bigg\},$$

$$W_2 = e^4\left\{4\left[\left(y+\frac{1}{y}+1\right)^2-1\right]-\frac{4q^2q'^2}{M^2}\left(y+\frac{1}{y}\right)\left(\frac{1}{T}+\frac{1}{U}\right)+\frac{q^4q'^4}{M^4}\left(\frac{1}{T^2}+\frac{1}{U^2}\right)\right\},$$

$$W_3 = e^4\left\{16+8\left(y+\frac{1}{y}\right)-\frac{8q^2q'^2}{M^2}\left(\frac{1}{T}+\frac{1}{U}\right)+\frac{2q^4q'^4}{M^4}\frac{1}{TU}\right\},$$

$$W_4 = e^4\Bigg\{-\frac{4q'^2}{M^2}\frac{(q'^2-4M^2)}{T^2}-\frac{4q^2}{M^2}\frac{(q^2-4M^2)}{U^2}-\frac{2}{M^4TU}[q^2q'^2(q^2+q'^2)$$
$$-2M^2(q^4+q'^4)+8M^4(q^2+q'^2)-12q^2q'^2M^2]\Bigg\},$$

$$W_6 = e^4\Bigg\{-\frac{1}{T}\left[8+\frac{8(q^2+q'^2)}{M^2}-\frac{2(q^2+q'^2)^2+2q^2q'^2}{M^4}+\frac{q^2q'^2(q^2+q'^2)}{M^6}\right]-\frac{16}{U}$$
$$-\frac{16U}{T^2}+\frac{1}{T^2}\left[16(q^2+q'^2)+\frac{8q^2q'^2-4q^4-4q'^4}{M^2}-\frac{2q^2q'^2(q^2+q'^2)}{M^4}+\frac{q^4q'^4}{M^6}\right]$$
$$+\frac{1}{TU}\left[-16(q^2+q'^2)+\frac{4(q^2+q'^2)^2+24q^2q'^2}{M^2}-\frac{4q^2q'^2(q^2+q'^2)}{M^4}+\frac{q^4q'^4}{M^6}\right]\Bigg\},$$

$$W_8 = e^4\left\{\frac{1}{T^2}\left[16q'^2 - \frac{4q'^4}{M^2}\right] + \frac{1}{U^2}\left[16q'^2 + 32q^2 - \frac{8q^4 + 4q'^4 + 16q^2q'^2}{M^2} + \frac{4q^2q'^2(q^2+q'^2)}{M^4} - \frac{q^4q'^4}{M^6}\right] + \frac{16q^2q'^2}{M^2TU}\right\},$$

$$W_{10} = e^4\left\{\left[16q'^2 - \frac{8q^2q'^2 + 4q'^4}{M^2} + \frac{2q^2q'^4}{M^4}\right]\left(\frac{1}{T^2} + \frac{1}{U^2}\right) - \frac{2}{TU}\left[16q^2 - \frac{8q^2q'^2 + 4q^4}{M^2} + \frac{2q^4q'^2}{M^4}\right]\right\},$$

$$W_{14} = e^4\left\{\frac{4q'^2}{T^2M^2}(q'^2 - 4M^2) + \frac{4q^2}{U^2M^2}(q^2 - 4M^2) + \frac{1}{TU}\left[16(q^2+q'^2) - \frac{4(q^2+q'^2)^2 + 24q^2q'^2}{M^2} + \frac{4q^2q'^2(q^2+q'^2)}{M^4} - \frac{q^4q'^4}{M^6}\right]\right\},$$

$$W_{16} = e^4\left\{\frac{32}{T} + \frac{16}{U} + \frac{16U}{T^2} + \frac{1}{T^2}\left[-16(q^2+q'^2) + \frac{4(q^2+q'^2)^2}{M^2} - \frac{2q^2q'^2(q+q')^2}{M^4}\right] + \frac{1}{TU}\left[16(q^2+q'^2) - \frac{4(q^2+q'^2)^2 + 16q^2q'^2}{M^2} + \frac{2q^2q'^2(q^2+q'^2)}{M^4}\right]\right\},$$

$$W_{18} = \frac{e^4}{T^2}F(q^2, q'^2), \quad W_{19} = \frac{e^4}{U^2}F(q^2, q'^2),$$

$$W_{20} = 32e^4\left(\frac{1}{T} + \frac{1}{U}\right)^2, \; W_{21} = e^4\left\{-32\left(\frac{1}{T^2} + \frac{1}{U^2}\right) + \frac{1}{TU}[F(q^2, q'^2) - 64]\right\},$$

where

$$F(q^2, q'^2) = 48 - \frac{16(q^2+q'^2)}{M^2} + \frac{4(q^2+q'^2)^2 + 8q^2q'^2}{M^4} - \frac{4q^2q'^2(q^2+q'^2)}{M^6} + \frac{q^4q'^4}{M^8}.$$

Of the remaining, W_5, W_7, W_9, W_{15} and W_{17} are obtained by interchange of T and U from W_4, W_6, W_8, W_{14} and W_{16}, respectively. W_{11}, W_{12} and W_{13} are obtained from W_8, W_9 and W_{10} respectively, by interchange of q and q', which includes interchange of T and U, since $T = q^2 - 2q\cdot k = q'^2 - 2q'\cdot k'$ and $U = q^2 - 2q\cdot k' = q'^2 - 2q'\cdot k$.

Appendix C

After the inclusion of the Higgs diagram shown in figure 2a the invariant functions W_i ($i = 1, 2, \ldots, 21$) occurring in the cross-section for $\gamma^*\gamma^* \to W^+W^-$ (§4(ii)) are altered as follows (with $T = (q-k)^2 - m_W^2$, $U = (q-k')^2 - m_W^2$ and $y = T/U$):

$$W_1 \to W_1 + \frac{q^4q'^4}{4m_W^8}\frac{W^4 - 4W^2m_W^2 + 12m_W^4}{(W^2 - m_H^2)^2} + \frac{2q^2q'^2}{m_W^4}\frac{1}{W^2 - m_H^2}\left[-W^2\frac{(q^2 + q'^2 - 2m_W^2)}{2m_W^2} - 2m_W^2\right.$$

$$+q^2+q'^2-\frac{q^2q'^2}{m_W^2}-\left(y+\frac{1}{y}\right)\left(4m_W^2-q^2-q'^2+\frac{q^2q'^2}{2m_W^2}\right)$$

$$+\frac{q^2q'^2}{2m_W^2}(q^2+q'^2-4m_W^2)\left(\frac{1}{T}+\frac{1}{U}\right)\Bigg],$$

$$W_4\to W_4+\frac{q^2q'^2}{m_W^4}\frac{1}{W^2-m_H^2}\left(\frac{1}{T}+\frac{1}{U}\right)\left(2q^2+2q'^2-8m_W^2-\frac{q^2q'^2}{m_W^2}\right),$$

$$W_5\to W_5+\frac{q^2q'^2}{m_W^4}\frac{1}{W^2-m_H^2}\left(\frac{1}{T}+\frac{1}{U}\right)\left(2q^2+2q'^2-8m_W^2-\frac{q^2q'^2}{m_W^2}\right),$$

$$W_6\to W_6+\frac{q^2q'^2}{m_W^4}\frac{1}{W^2-m_H^2}\Bigg\{\left(-2+\frac{q^2+q'^2}{m_W^2}-\frac{q^2q'^2}{2m_W^4}\right)\left(\frac{U}{T}+1\right)-\frac{1}{T}\frac{(q^2+q'^2)^2}{m_W^2}$$

$$-2(q^2+q'^2)-4m_W^2+\frac{q^2q'^2}{m_W^2}-\frac{q^2q'^2(q^2+q'^2)}{2m_W^4}+\frac{2}{U}[q^2+q'^2-4m_W^2]\Bigg\}.$$

W_7 is obtained from the altered W_6 by interchange of T and U. All the remaining W_i are unaltered.

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