Inertial Mass of a Vortex in Cuprate Superconductors

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Abstract

We present here a calculation of the inertial mass of a moving vortex in cuprate superconductors. This is a poorly known basic quantity of obvious interest in vortex dynamics. The motion of a vortex causes a dipolar density distortion and an associated electric field which is screened. The energy cost of the density distortion as well as the related screened electric field contribute to the vortex mass, which is small because of efficient screening. As a preliminary, we present a discussion and calculation of the vortex mass using a microscopically derivable phase-only action functional for the far region which shows that the contribution from the far region is negligible, and that most of it arises from the (small) core region of the vortex. A calculation based on a phenomenological Ginzburg-Landau functional is performed in the core region. Unfortunately such a calculation is unreliable, the reasons for it are discussed. A credible calculation of the vortex mass thus requires a fully microscopic, non-coarse grained theory. This is developed, and results are presented for a s-wave BCS like gap, with parameters appropriate to the cuprates. The mass, about 0.5 m_e per layer, for magnetic field along the c axis, arises from deformation of quasiparticle states bound in the core, and screening effects mentioned above. We discuss earlier results, possible extensions to d-wave symmetry, and observability of effects dependent on the inertial mass.

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1. **Introduction**

The discovery of high temperature superconductors has led to a renewed interest in the mixed phase. Several novel phenomena arising from their short coherence length, layered nature and large superconducting transition temperatures have been theoretically and experimentally studied(1). One area of interest is the existence of effects connected with vortex dynamics, e.g. quantum creep (2,3), anomalies in the Hall effect (4-6), and ac electromagnetic response (7,8). These phenomena are not fully understood, partly because of the lack of a well developed first principles theory of vortex dynamics, especially in the quantum, interacting vortex regime. A number of recent contributions address parts of the problem (9-12), especially the Magnus force driven dynamics in the presence of dissipation.

A necessary ingredient in all considerations of the motion of a vortex is its inertial mass. This quantity, generally believed to be small, is surprisingly ill known and its origin is not well understood. (See Ref. 1, for example). Not much attention has been paid to this question because, for some phenomena, the dynamics is governed by the large dissipation (13) or the strong Magnus force (9-12) and the inertial mass could be irrelevant. However, there is experimental evidence for a low dissipation regime in cuprate superconductors (5), and for a Magnus force smaller (14) than standard estimates (9-12). It is thus quite possible that the inertial mass could affect dynamical processes involving vortices. (These questions are taken up in Section IV). Also, in the absence of an understanding of what contributes to the vortex mass, and how much, it is difficult to meaningfully discuss the question of whether or how such a mass influences vortex dynamics. We therefore present here an extensive discussion and a calculation of the vortex mass, and go into the question of mass related phenomena in Section IV. This work was first reported in 1994(15).

We first estimate the mass using a phase-only functional (Section II) which is known to give a good description of the system far from the vortex core, where the amplitude of the order parameter is nearly constant and the only relevant degree of freedom is the phase. Recently, Duan and Leggett (17) and Duan (18) have given a careful discussion of this approach where the time dependent order parameter (phase) of a moving vortex causes the electronic density to fluctuate. This in turn gives rise to an electric field which is screened. The energy of the density distortion and electric field energy are the cause of the mass. The phase-only functional used has been derived microscopically (19) and the result obtained thus has a microscopic significance and gives an accurate estimate of the contribution to the vortex mass which accrues from transitions induced in the electronic scattering states of a vortex by the vortex motion. The contribution of this process from the far region turns out to be very small due to the efficient screening; thus most of the mass comes from the core of the vortex.

We then calculate the core contribution to the mass using a phenomenological Ginzburg-Landau functional as has been conventional since the early work of Suhl (16). This functional is not derivable microscopically and is used mainly as an interpolating formula which reduces to the correct phase-only functional in the far region. However, obviously the coarse grained GL approach is unrealistic for effects within the core which is of the same size (ξ) as the coarse graining scale of the theory. Further the screening length in GL theory is proportional to $|\psi|^{-1}$, where ψ is the superconducting order parameter, and thus diverges at the core. This is clearly an artefact of the GL approach, as the efficiency of Coulomb screening is related to the electronic compressibility, and is expected to be largely independent of the superconducting order parameter. The GL estimate of the vortex mass is clearly unreliable and is presented here mainly to contrast the correct microscopic calculation of the mass which forms the main body of this paper. The correct, microscopically obtained vortex mass, is very different from the GL estimate in its dependence on the basic parameters of a superconductor. Inspite of this, the numerical value of the mass obtained in the GL approach is (largely by accident) in the same range as that obtained from the microscopic theory for parameters appropriate to the cuprates.

We then present the first correct microscopic calculation of the vortex mass (Section III). We employ the self consistent pair field approximation which has been used extensively for static vortex structure, quasiparticle energy levels, etc. (20-23). We make a Galilean transformation to a frame of reference where the vortex is at rest. In this frame, the motion of the vortex acts like a perturbation of the form $\vec{u}.\vec{p}^{op}$ where $\vec{u}$ is the vortex velocity and $\vec{p}^{op}$ is the momentum operator for the electrons. The inertial mass is obtained by integrating out the electronic degrees of freedom to second order in $\vec{u}$. The coefficient of the $(u^2/2)$ term in the effective action is the effective mass of the vortex. The mass is found to originate from a polarization process involving the virtual excitation of the lowest energy quasiparticles in the bound electronic states (quasiparticle states) localized in the core of the vortex. The small core size ($\xi \sim 15\mathring{A}$) in the cuprate superconductors implies that the lowest unoccupied state is separated by a sizeable gap ($\sim 100K$) from the highest occupied state below the Fermi level (23), in strong contrast to conventional superconductors. The existence of this large gap in the core quasiparticle spectrum has been recently observed (43) by STM measurements in the vortex core region in $YBa_2Cu_3O_{7-\delta}$. This virtual transition process gives rise to a large vortex mass ($m^* \approx 25m_e$). However, strong dielectric screening drastically reduces the mass and leads to a value $m^* \simeq 0.5m_e$ per CuO_2 layer. We discuss the physical reason for a mass of this size in terms of the basic length scales and the screening process.

In the final section (Section IV) we discuss the calculation critically, compare with other results, consider the calculation of a vortex mass for a non s-wave superconductor and go into the question of when effects due to the small vortex mass might be observable.

II. **Ginzburg Landau calculation of the vortex effective mass**

The most natural way of discussing the motion of a singularity in the phase θ of the superconducting order parameter is the time dependent Ginzburg Landau theory (17 - 19) where the free energy (or action) is expressed as a functional of the phase of the superconducting order parameter. This functional provides a good description of the region far from the centre of the vortex where the amplitude of the superconducting gap is nearly constant. We describe the functional and briefly summarize known results for the mass contribution from the region outside the core (17,18).

Outside the core, a phase only Hamiltonian is sufficient, and the action functional S per length L (at $T = 0$) is given by

$$(S/L) = S_\theta + S_{em} \qquad (1a)$$

where

$$S_\theta = \int_{-\infty}^{+\infty} dt \int d\vec{r} \left[\frac{\alpha_1}{2} \left(\dot{\theta} - \frac{2eA_0}{\hbar} \right)^2 - \frac{\alpha_2}{2} \left((\nabla\theta - \frac{2e\vec{A}}{\hbar c})^2 + \frac{4m\dot{\theta}}{\hbar} \right) \right] \qquad (1b)$$

and

$$S_{em} = \int_{-\infty}^{+\infty} dt \int d\vec{r} \left[\frac{(\nabla A_0)^2 - (\nabla \times \vec{A})^2}{8\pi} \right] \tag{1c}$$

where $\vec{r}$ is the coordinate in the plane perpendicular to the magnetic field. In Eq. (1b), the first two terms are the energies associated with pair charge and pair current (or velocity) fluctuations respectively. The term linear in $\dot{\theta}$ is a total time derivative which has no physical consequences in the absence of vortices. Its physical origin can be understood (10) in terms of the Berry phase associated with the adiabatic motion of a vortex and it gives rise to the Magnus force. Notice the absence of any term linear in A_0 which is constrained to be zero because of charge neutrality in the electron-ion system.

The action functional of Eq. (1a) can be obtained microscopically by starting with electrons interacting with a pair potential (whose magnitude is nearly constant in the far region), going to a gauge where the order parameter is real and then integrating out the fermions. The details of the derivation have been outlined in (19). In Fig. (1), we show the Feynman diagrams which contribute to the action of Eq. (1a) at $T = 0$ in the clean limit. The coefficients α_1 and α_2 are the appropriate polarizabilities. For a weakly interacting, clean Fermi gas, they have the values

$$\alpha_1 = \left(\frac{\hbar}{2e}\right)^2 \frac{\lambda_{TF}^{*-2}}{4\pi} \tag{2a}$$

and

$$\alpha_2 = \left(\frac{\hbar c}{2e}\right)^2 \frac{\lambda_L^{-2}}{4\pi} \tag{2b}$$

where λ_{TF}^* (the Thomas Fermi screening length) is given by $\lambda_{TF}^{*-2} = (6\pi n e^2/\epsilon_F)$ and λ_L (the London penetration depth) is given by $\lambda_L^{-2} = (4\pi n e^2)/(m_e c^2)$. A_0 and $\vec{A}$ are the scalar and vector electromagnetic potentials respectively, the two terms in Eq. (1c) being just the field energies. In case there is an external magnetic field $\vec{H}_0$, the term $\left\{(\nabla \times \vec{A})^2/8\pi\right\}$ is modified to $\{(\nabla \times \vec{A} - \vec{H}_0)^2/8\pi\}$. The functional is clearly gauge invariant. In Eq. (1) the displacement current term $(\frac{1}{c}\frac{\partial \vec{A}}{\partial t})^2$ has been omitted because of the largeness of c.

The motion of the vortex in a charged superconductor gives rise to an electric field (or potential A_0). To find this A_0, we minimize S with respect to A_0 which yields

$$\frac{\nabla^2 A_0}{4\pi} = \frac{-2e}{\hbar}\alpha_1 \left(\dot{\theta} - \frac{2eA_0}{\hbar}\right) \tag{3}$$

Now for a vortex moving with a (small) uniform velocity $\vec{u}$, we make the assumption that

$$\theta(\vec{r}, t) = \theta_0(\vec{r} - \vec{u}t) \tag{4}$$

where $\theta_0(\vec{r})$ is the phase around a static vortex at the origin. Thus we have

$$\dot{\theta}(\vec{r}, t) = -\vec{u}.\vec{\nabla}\theta_0 \tag{5}$$

Using this in combination with the known θ_0, one finds the potential A_0 from Eq. (3) and the extra energy from Eq. (1). The result is

$$E_{KE} = \frac{\pi\alpha_1 u^2}{4} ln(1 + \frac{\xi^{-2} - R_c^{-2}}{\lambda_{TF}^{*-2} + R_c^{-2}}) \tag{6a}$$

where R_c is a long distance cutoff (whose magnitude will be taken to infinity in the end). Then it is easy to see that in the limit $e \to 0$ which implies $\lambda_{TF}^* \to \infty$, the expression for E_{KE} reduces to $\frac{\pi\alpha_1 u^2}{2} ln(\frac{R_c}{\xi})$ which diverges logarithmically as the long-distance cutoff R_c is taken to ∞. This is the expected result for a neutral superfluid (17, 18). For the present case of a charged superconductor we are always in the limit $\lambda_{TF}^* \ll \xi$ and the expression for the kinetic energy simplifies to

$$E_{KE} \simeq \frac{\pi\alpha_1 u^2}{4}\frac{\lambda_{TF}^{*2}}{\xi^2} = \frac{u^2}{16\xi^2}\left(\frac{\hbar}{2e}\right)^2 \tag{6b}$$

The expression in Eq. (6b) can be rewritten as

$$E_{KE} = E_{SV}\left(\frac{u^2}{v_F^2}\right)\left(\frac{\lambda_{TF}^{*2}}{\xi^2}\right)\left(\frac{3}{4ln(\lambda_L/\xi)}\right) \tag{6c}$$

where $E_{SV} = (\frac{\phi_0}{4\pi\lambda_L})^2 ln(\frac{\lambda_L}{\xi})$ is the London energy (per unit length) of a static vortex which results from the terms involving transverse fluctuations in the current and magnetic field in the phase functional of Eq. (1). The result is then easily understood on physical grounds; the velocity u is to be compared with the natural velocity scale v_F of the Fermi system. The second bracketed factor in Eq. (6c) is due to the reduction of charge fluctuations by screening. The former occur on a length scale ξ, while the screening length is $\lambda_{TF}^* \ll \xi$. On substituting appropriate numbers in Eq. (6b), i.e. $\xi \simeq 15\mathring{A}$ and $\lambda_{TF}^* \simeq 1\mathring{A}$ we find that the contribution of this source to $m_f^* \simeq 7 \times 10^{-4} m_e$ per layer, (m_e being the electron mass) assuming an interlayer spacing $d \simeq 10\mathring{A}$. This is an extremely small number; one reason for its smallness is the screening factor $(\lambda_{TF}^*/\xi)^2 \simeq (1/200)$. The result, (Eq. (6b)), has been obtained earlier by Duan (18), where details may be found.

The functional S_θ of Eq. (1) is not Galilean invariant and there are additional terms involving higher order derivatives of θ, whose inclusion is necessary to restore this symmetry. These arise from the fact that the Gaililean invariant combination involving the time dependent order parameter phase θ is the local electrochemical potential $\delta\mu(\vec{r},t) = -\frac{\hbar}{2}\dot{\theta} + eA_0 - \frac{(\frac{\hbar\nabla\theta}{2} - \frac{e\vec{A}}{c})^2}{2m}$ The correct functional was recently obtained by Aitchison and co-workers (24) and is of the form

$$S_\theta = \int_{-\infty}^{+\infty} dt \int d\vec{r} \left[\frac{\alpha_1}{2}[(\dot{\theta} - \frac{2eA_0}{\hbar}) + \frac{\hbar(\nabla\theta - \frac{2e\vec{A}}{\hbar c})^2}{4m}]^2 - \frac{\alpha_2}{2}[\frac{4m}{\hbar}\dot{\theta} + (\nabla\theta - \frac{2e\vec{A}}{\hbar c})^2]\right] \tag{7a}$$

The additional diagrams contributing here, are shown in Fig. (2). The harmonic electric field terms in Eqs. (7a) and (1c) can be integrated out to give the action functional

$$S_\theta = \int dt \int \frac{d\vec{q}}{(2\pi)^2}\frac{2\alpha_1}{\hbar^2} \mid \delta\mu(\vec{q})\mid^2 \frac{q^2}{q^2 + \lambda_{TF}^{-2}} - \frac{\alpha_2}{2}\int dt \int d\vec{r}[\frac{4m}{\hbar}\dot{\theta} + (\nabla\theta - \frac{2e\vec{A}}{\hbar c})^2] \tag{7b}$$

It is clear from Eq. (7b), that additional contributions accruing to the vortex mass come from the coupling of charge fluctuations to the supercurrent fluctuations leading to non-adiabatic corrections in the supercurrent distribution that are proportional to the vortex velocity. However, an explicit calculation shows that all such contributions to the vortex mass are screened at least doubly more efficiently and are therefore smaller than that estimated above (Eq. (6b)) by a factor of $(\lambda_{TF}/\xi)^2 \approx .005$. Since the far mass estimated before is already small, this additional contribution is smaller still, and therefore negligible. In fact in the perfect screening approximation, where the total electro-chemical potential $\delta\mu$ is set equal to zero locally, we get back the result of Eq. (6), the contribution of the additional terms being zero as expected. It is worth emphasizing that the results obtained from the phase-only functional are essentially microscopic (see the Feynman diagrams in Fig. 1 and Fig. 2) and give an accurate estimate of the contribution to the vortex mass from the electronic scattering states which are extended in nature and live mainly outside the core.

We now consider the contribution to the vortex mass arising from the core. The amplitude of the superconducting order parameter changes as a function of radial distance from the center of the vortex in the core region. We use a phenomenological Ginzburg - Landau action functional per unit length

$$S = \int dt \int d\vec{r} \left[\frac{3}{2mv_F^2} \left| \left(\frac{\hbar}{i}\frac{\partial}{\partial t} - 2eA_0 \right) \psi(\vec{r},t) \right|^2 - \frac{1}{2m} \left| \left(\frac{\hbar}{i}\nabla - \frac{2e}{c}\vec{A} \right) \psi \right|^2 - V(|\psi|^2) \right] + \int dt \int d\vec{r} \left[\frac{(\nabla A_0)^2 - (\nabla \times \vec{A})^2}{8\pi} \right] \tag{8}$$

to estimate the core contribution to the vortex mass.

This functional is not derivable microscopically and has been chosen primarily as an interpolating formula which reduces to the correct phase-only functional in the far region in the limit $\Psi \to \sqrt{n_s}e^{i\theta}$ and gives the correct (linear in r) dependence for the amplitude of the order parameter near the center of the vortex. The parameter m which appears above is therefore fixed by requiring this functional to reduce to the action functional of Eq. (1) in the "phase-only" approximation. For the potential $V(|\psi|^2)$ we assume the standard form $V(|\psi|^2) = \frac{\alpha}{2}|\psi|^2 + \frac{\beta}{4}|\psi|^4$.

Starting with the early work of Suhl (16) and subsequent work by others (see Ref. (1) and references therein) all estimates of the vortex mass have proceeded from this functional. The value of the mass obtained from this functional is not expected to be very accurate for reasons that are discussed below. However we present a calculation of the vortex mass from this functional mainly to contrast and highlight the microscopic calculation presented in Sec. III.

We again (see Eq. (4)) make the ansatz that for small velocity $\vec{u}$, the vortex moves rigidly i.e.

$$\psi(\vec{r},t) = \psi_0(\vec{r} - \vec{u}t) \tag{9}$$

where $\psi_0(\vec{r})$ is the order parameter configuration associated with a static vortex. We further assume a common and fairly accurate explicit form for $\psi_0(\vec{r})$, namely

$$\psi_0(\vec{r}) = \sqrt{n_s}\tanh(r/\xi)\exp(i\phi) \tag{10}$$

where n_s is the superfluid density far from the vortex, and (r,ϕ) are the radial and angular coordinates of the two dimensional vector $\vec{r}$. Again, minimizing Eq. (7) with respect to A_0, we find

$$\frac{\nabla^2 A_0}{4\pi} = \frac{3e\hbar}{imv_F^2}\left(\psi\frac{\partial\psi^*}{\partial t} - \psi^*\frac{\partial\psi}{\partial t} \right) + \frac{3}{mv_F^2}(2e)^2|\psi|^2 A_0 \tag{11}$$

. Using this condition in the action functional of Eq. (8) we find that the extra energy due to vortex motion is

$$\Delta E = \int d\vec{r} \left[\frac{3\hbar^2}{2mv_F^2} |\frac{\partial \psi}{\partial t}|^2 - \frac{3e\hbar A_0}{2mv_F^2 i} \left(\psi^* \frac{\partial \psi}{\partial t} - \psi \frac{\partial \psi^*}{\partial t} \right) \right] \quad (12).$$

The first term in Eq. (12) is due to the density fluctuations induced by vortex motion, and the second describes the reduction due to screening. The first term is readily computed using the ansatz Eq. (9) for the time dependence of $\psi(\vec{r},t)$ and the form Eq. (10) for the coordinate dependence of $\psi_0(\vec{r})$. We are interested here only in the core contribution to the mass, the contribution from the far region having been previously determined (Eq. (6)). The radial integration in Eq. (12) is therefore performed over the range $0 < r < \xi$ (core region). It gives a mass per unit length

$$m^*{}_{\text{unscreened}}^{\text{core}} \simeq m_u^{*\ c} = 0.61 \left(\frac{m_e}{a_0} \right) \left(\frac{a_0}{4\lambda_{TF}^*} \right)^2 \quad (13)$$

where $a_0 = \frac{\hbar^2}{m_e e^2}$ is the Bohr radius, and $(\lambda_{TF}^*)^{-2} = 4\pi n_s (2e)^2/(mv_F^2/3)$. For the cuprate superconductors, we find that the unscreened core vortex mass per layer is

$$m_u^{*\ c} \approx 0.19 m_e \quad (14)$$

An unexpected, and incorrect, feature of Eq. (13) is the lack of dependence of the vortex mass on the core size ξ. This is a consequence of the fact that the core mass is proportional to the gradient energy per unit length in the core, which is scale invariant and does not depend on the natural length scale ξ in two dimensions. This is inevitable in any local continuum free energy theory.

We now consider the second or screening term in Eq. (12). One clearly needs to know the electric potential A_0 induced by vortex motion. Setting

$$A_0(\vec{r}) = v(r) \vec{u} \cdot \hat{\phi} \quad (15)$$

we find that $v(r)$ satisfies the radial equation

$$\left[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - \frac{1}{r^2} - \frac{1}{(\lambda_{TF}^*)^2} \frac{|\psi(r)|^2}{n_s} \right] v(r)$$

$$= \frac{1}{4\pi} \frac{\hbar}{2e} \lambda_{TF}^{*-2} \left[\frac{\tanh^2(r/\xi)}{r} \right] \quad (16)$$

This is just the radial Poisson equation with a screening term (last term on the left hand side), and a source term (on the right hand side) appropriate for a two dimensional system. The 'effective screening length' $\lambda_{TF}^* \sqrt{n_S}/|\psi(r)|$ is r dependent, and increases as $|\psi(r)|$ decreases, i.e. with $r \to 0$. This is yet another unrealistic feature of the Ginzburg Landau theory, since one expects screening which is related to the electronic compressibility, to be relatively independent of superconducting order. Eq. (16) is solved numerically, with appropriate boundary conditions at $r = \infty$ and $r = 0$. The $v(r)$ and the $A_0(\vec{r})$ thus obtained (see Eq. (15)) are used to calculate the second term in the vortex core energy (Eq. (12)). Adding the contributions of both the terms, the final result for the mass per layer is

$$m^* \quad (\text{core}) = m_c^* = \left(\frac{a_0}{4\lambda_{TF}^*}\right)^2 \left(\frac{d}{a_0}\right) [0.61 - 0.04]\, m_e \tag{17}$$

where d is the interlayer separation. In Eq. (17), the first term in the square brackets is due to the density distortion in the core, and the second term is the negligibly small correction due to screening. Thus the total mass per layer, for a cuprate superconductor at $T = 0K$ is

$$m_c^* \simeq 0.19 m_e \qquad (18).$$

In the absence of a microscopic theory, the phenomenological Ginzburg Landau approach has been used, mainly as a dimensional aid, to estimate the vortex inertial mass. The values quoted (Ref. (1)) are in the range 0.2 to 2 m_e, and turn out to be (largely by accident) not far from our microscopic result (see below) for parameters appropriate to the cuprates. As a preliminary to the microscopic calculation which forms the main result of this paper, we have performed a detailed calculation using this functional to bring out its inadequacies. The above detailed analysis of the different contributions to the mass shows that within the phenomenological GL theory it is due to the essentially unscreened density distortion induced in the core. The unscreened mass estimated above too is incorrect as the gradient expansion implicit in a phenomenological theory like the GL functional of Eq. (8), breaks down at short length scales and strongly non-local (in space) effects lead to a much larger mass (see Sec. III. below) in the unscreened case than calculated here. This picture is clearly incorrect on another count as well. Electronic screening processes are not expected to be affected much by the onset of superfluid order and a strong reduction of the mass is expected because of Coulomb screening. This aspect too is explicitly seen in the microscopic theory. In short, the microscopic calculation shows that the phenomenological GL picture is wrong on all counts, as we shall see now.

III. Microscopic calculation of the vortex mass

We present now a microscopic calculation of the vortex inertial mass for a layered superconductor at $T = 0$. The dynamics of the system of paired electrons is described by the action

$$S = \int dt \int dz \int d\vec{r} \sum_l \left[L_l^f(\vec{r},t) + \tilde{L}_l^f(\vec{r},t) + L_l^{pair}(\vec{r},t)\right] \delta(z - ld) + S_{em} \tag{19a}$$

where

$$L_l^f(\vec{r},t) = \sum_\sigma \bar{\psi}_{l\sigma}(\vec{r},t) \left[i\hbar\frac{\partial}{\partial t} - \frac{(\frac{\hbar}{i}\nabla - \frac{e}{c}\vec{A})^2}{2m} + \epsilon_F\right] \psi_{l\sigma}(\vec{r},t) \tag{19b}$$

$$\tilde{L}_l^f(\vec{r},t) = -eA_0(\vec{r},ld) \sum_\sigma \bar{\psi}_{l\sigma}(\vec{r},t)\psi_{l\sigma}(\vec{r},t) \tag{19c}$$

$$L_l^{pair}(\vec{r},t) = -[\Delta_l(\vec{r},t)\bar{\psi}_{l\uparrow}(\vec{r},t)\bar{\psi}_{l\downarrow}(\vec{r},t) + h.c.] - \frac{|\Delta_l(\vec{r},t)|^2}{V} \tag{19d}$$

and

$$S_{em} = \int dt \int dz \int d\vec{r} \frac{(\nabla A_0)^2 - (\nabla \times \vec{A})^2}{8\pi} \qquad (19e)$$

The electrons at $(\vec{r}, t)$ on layer l, with spin σ are represented by the Grassmann field variables $\bar{\psi}_{l\sigma}(\vec{r}, t)$, $\psi_{l\sigma}(\vec{r}, t)$. The electronic kinetic energy is described by the term $L_l^f(\vec{r}, t)$, i.e. Eq. (19b) and the electronic coupling to the Coulomb potential by the term $\tilde{L}_l^f(\vec{r}, t)$ (Eq. (19c)). Eq. (19d) is the mean pair field decomposition of the Cooper pair attraction, appropriate for a BCS s-wave superconductor and V is the strength of the attractive contact interaction. The last term S_{em} (Eq. (19e)) is the electro-magnetic field contribution.

Several possible modifications and generalizations, such as order parameter symmetries other than s-wave and effects due to quasiparticles at $T \neq 0$ are briefly discussed in Section IV. We have neglected above the small interlayer (Josephson) coupling, so that the (pancake) vortices in different layers

are coupled only via electric and magnetic fields. This neglect has very little effect on the vortex mass, which is overwhelmingly due to processes occuring within a layer, and as a matter of fact, to processes within the small core.

Every layer has one pancake vortex. All the pancake vortices are assumed to lie along a straight line parallel to the magnetic field which is perpendicular to the layers and to move with a uniform velocity $\vec{u}$, so that the vortex (core) coordinate is $(\vec{R}_0, ld)$ where $\vec{R}_0 = \vec{u}t$ is a vector in a plane parallel to the layers and the z co-ordinate ld specifies the position of the layer. The pair potential $\Delta_l(\vec{r},\ t)$ in the presence of such a uniformly moving vortex is a function of $(\vec{r} - \vec{R}_0(t))$. It can thus be written in an adiabatic approximation as

$$\Delta_l(\vec{r}, t) = \Delta_0(|\vec{r} - \vec{R}_0(t)|)\; e^{i\theta(\vec{r} - \vec{R}_0(t))} \qquad (20).$$

Here $\Delta_0(r)$ is the magnitude of the pair potential and $\theta(\vec{r})$ the phase, for a static vortex situated at $\vec{r} = 0$. The pair potential does not depend on l. The action functional relevant for vortex dynamics is found by an expansion of the microscopic action in powers of the vortex velocity u after integrating out the electronic degrees of freedom. The vortex mass is then determined from the term quadratic in u (the dissipative and Magnus forces coming from the linear term). This calculation is most conveniently carried out in the rest frame of the vortex. In this frame, the pair potential seen by the electrons is $\Delta_0(r)\ \exp[i(\theta(\vec{r}) - \frac{2m\vec{u}\cdot\vec{r}}{\hbar} + \frac{mu^2 t}{\hbar})]$ i.e. the pair potential corresponding to a vortex at rest with extra phase factors coming from the macroscopic supercurrent and kinetic energy of the electrons which acquire an extra velocity $-\vec{u}$ in this frame. The Coulomb potential seen by the electrons in this frame becomes $A_0 - \frac{\vec{u}}{c} \cdot \vec{A}$ while the vector potential is the same as before. All physical quantities can be calculated in this frame and then transformed to the lab frame as necessary. Note that this does not assume Galilean invariance.

After a gauge transformation, the action for the system in this moving frame can be written as

$$S = S_0 + S_1 \qquad (21)$$

where the static vortex action S_0 is

$$S_0 = \int dt \int d\vec{r} \sum_l \left[L_l^f(\vec{r}, t) - \Delta_0(r)[e^{i\theta(\vec{r})}\bar{\psi}_{l\uparrow}(\vec{r}, t)\bar{\psi}_{l\downarrow}(\vec{r}, t) + h.c.] - \frac{\Delta_0^2(r)}{V} \right]$$

$$-\int dt \int dz \int d\vec{r}\frac{(\nabla\times\vec{A})^2}{8\pi} \qquad (22a)$$

and the perturbation due to vortex motion is contained in

$$S_1 = \int dt \int d\vec{r} \sum_l \left[\vec{u}\cdot\vec{p}_l(\vec{r},t) - eA_0(\vec{r},ld)\rho_l(\vec{r},t)\right] + \int dt \int dz \int d\vec{r}\frac{(\nabla A_0)^2}{8\pi} \qquad (22b)$$

In Eq. (22b), $\vec{p_l}(\vec{r},t)$ is the momentum density operator for the lth layer,

$$\vec{p_l}(\vec{r},t) = \sum_\sigma \bar{\psi}_{l\sigma}(\vec{r},t)\frac{\hbar}{i}\nabla\psi_{l\sigma}(\vec{r},t) \qquad (23a)$$

and $\rho_l(\vec{r},t)$ is the density operator

$$\rho_l(\vec{r},t) = \sum_\sigma \bar{\psi}_{l\sigma}(\vec{r},t)\psi_{l\sigma}(\vec{r},t) \qquad (23b)$$

The first term in Eq. (22b) is linear in $\vec{u}$, coupling to the electron momentum. The second term is the electric potential energy of the (nonuniform) electron density around the moving vortex. The associated electric potential has to be determined self consistently.

Since the London screening length (λ) is much larger than the core size (ξ), the magnetic field in the core is nearly the same as the external magnetic field, deviations from this being of order $(\xi/\lambda)^2$. However, the vector potential associated with this field is negligible in comparison with the gradient of the phase of the superconducting order parameter and has been ignored in the following calculations. The former $\approx H\,r$ whereas the latter $\approx \phi_0/r$. Thus for the core region ($r \leq \xi$), we find that for $H \ll \phi_0/\xi^2$ the vector potential may be ignored (20). Corrections due to the vector potential can be estimated to be of order H/H_{c2} where H is the external magnetic field and H_{c2} is the upper critical field and are therefore small in the dilute vortex limit.

Now a systematic expansion in powers of $\vec{u}$ becomes possible by integrating out the electrons giving rise to the Lagrangian that describes vortex dynamics. Dynamics of vortices (classical or quantum) can be studied either by working directly with the vortex action or alternately by introducing a canonically conjugate momentum which permits one to go over to the Hamiltonian formalism.

The Magnus and dissipative forces come from the term linear in $\vec{u}$ and can be obtained by taking the gradient of this term with respect to the vortex co-ordinate (37). The inertial term in the action is quadratic in the vortex velocity. We are thus interested in the change of action to second order in $\vec{u}$, the coefficient of $(u^2/2)$ in the change being the vortex effective mass m^*. Clearly, this is calculable by going to second order in u and A_0. For ease of presentation and also to emphasize the importance of Coulomb screening in the core, we do this in two stages. First, we find the unscreened vortex mass, i.e. in S_1 (Eq. (22 b)), we turn off the Coulomb interaction by putting $e = 0$, and calculate the second order shift. We then calculate the effect of Coulomb interactions, i.e. the effect of the electric potential due to the electron charge density change consequent on vortex motion. We would like to emphasize that the unscreened mass calculated here corresponds to the contribution of the *bound states* localised in the vortex core. The contribution of bound states is finite. For a (hypothetical) neutral superconductor, this term is to be added to the contribution from the region outside the core. This can be calculated from the (microscopically derived) phase-only functional (Eq. 1) by setting $e = 0$, and as mentioned earlier, is logarithmically divergent because longitudinal density fluctuations associated with vortex motion are not screened. Thus for the uncharged superconductor, the finite core contribution can be neglected in comparision with the log diverging far contribution.

For a charged superconductor, the core contribution has to be calculated with the inclusion of screening effects which, we show (Eq. (43) below) reduces the 'unscreened' or $e = 0$ mass by a factor of fifty or more. To this we have to add the m^* due to the far region, which because of Coulomb screening is finite, and is actually negligibly small (Eq. (6c)).

The second order effective Lagrangian due to just the $\vec{u}.\vec{p}$ term in S_1 is given using standard many body perturbation theory by $\frac{m_0^*}{2}u^2$ where

$$m_0^* = i \int d\vec{r} \int d\vec{r'} \int dt \left\langle T\left[p_l^x(\vec{r},t)p_l^x(\vec{r'},0)\right]\right\rangle = -\int d\vec{r} \int d\vec{r'} \chi_l^{xx}(\vec{r},\vec{r'}) \tag{24}$$

In Eq. (24), the vortex velocity has been assumed to be in the x-direction. Note that intralayer averages such as Eq. (24) do not depend on the layer index. The correlation function in Eq. (24) is calculated with respect to the unperturbed action S_0 (Eq. (22a)) which describes a single static vortex. This last problem of a static vortex has been studied extensively using the Bogoliubov-de Gennes self-consistent field theory (20-23). The electronic eigenstates, in the presence of a static vortex, are bound states which are localised and have appreciable amplitude only in the core of the vortex, or extended states which are scattered by the superfluid velocity and are primarily in the region outside the vortex core where the amplitude of the order parameter is nearly constant. The latter are well described by neglecting the variation in the amplitude of the order parameter and the scattering processes contributing to the mass involving these states are just those considered in the Feynman diagrams (Fig. 1) contributing to the phase-only functional in Eq. (1). Thus the mass contribution from the deformation of the scattering states is accurately estimated by the calculation proceeding from the phase-only functional of Eq. (1) and has been shown to be negligibly small due to efficient screening. We will therefore concentrate in the following only on the contribution of the localised states to the correlation function of Eq. (24) to find the core contribution to the vortex mass.

The eigenfunctions of the Bogoliubov-de Gennes equations for a single layer which are localised in the vortex core are the amplitudes $u_\mu(\vec{r})$ and $v_\mu(\vec{r})$, labelled by the (azimuthal) angular momentum quantum number μ because of the cylindrical symmetry of the single vortex problem. The Bogulibov amplitudes are related to the fermion field operators by the relations

$$\psi_\uparrow(\vec{r},t) = \sum_{\mathrm{all}_\mu} u_\mu(\vec{r})\gamma_\mu(t) \tag{25a}$$

and

$$\psi_\downarrow^\dagger(\vec{r},t) = \sum_{\mathrm{all}_\mu} v_\mu(\vec{r})\gamma_\mu(t) \tag{25b}$$

Here μ runs over all half-odd integers (positive as well as negative). The quasi-particle annihilation operators γ_μ correspond to the empty (particle) states for $\mu > 0$ and filled (hole) states for $\mu < 0$. In terms of the quasi-particle operators γ_μ, the Hamiltonian for the static vortex system described by the action S_0 can be written in a diagonal form as,

$$H_0 = \sum_{\{\mu\}} \epsilon_\mu \gamma_\mu^+ \gamma_\mu + \quad \text{constant} \tag{26a}$$

Our definition of the quasi-particle operators (Eq. (25)) differs from the standard definition (20) by a particle-hole transformation. In doing this, use has been made of the fact that if $(u_\mu(\vec{r}), v_\mu(\vec{r}))$ is an eigenstate with eigenvalue ϵ_μ, the state $(-v_\mu^*(\vec{r}), u_\mu^*(\vec{r}))$ is also an eigenstate with eigenvalue $-\epsilon_\mu$. Thus,

$$(u_\mu(\vec{r}), v_\mu(\vec{r})) = (-v_{-\mu}^*(\vec{r}), u_{-\mu}^*(\vec{r})) \tag{26b}$$

with

$$\epsilon_\mu = -\epsilon_{-\mu} \tag{26c}$$

for $\mu < 0$. The ground state of the system has all states with $\mu < 0$ occupied and all states with $\mu > 0$ empty.

The correlation function of Eq. (24) is easily evaluated in this representation. The vortex mass can then be understood as arising from a polarization process involving a virtual (quasi-particle) transition from the highest occupied to the lowest unoccupied state. This process can also be viewed as a deformation of the ground state by the perturbation which mixes in higher energy states. In terms of u and v, the momentum-momentum correlation function can be written as

$$i\int dt \left\langle T\left[p_l^x(\vec{r},t)p_l^x(\vec{r'},0)\right]\right\rangle$$

$$= 2\sum_{\mu>0}^{unocc.}\sum_{\mu'<0}^{occ.}(\epsilon_\mu - \epsilon_{\mu'})^{-1}\times$$

$$\left[u_\mu^*(\vec{r})\frac{\hbar}{i}\frac{\partial u_{\mu'}(\vec{r})}{\partial x}\left(u_{\mu'}^*(\vec{r'})\frac{\hbar}{i}\frac{\partial u_\mu(\vec{r'})}{\partial x'} - v_\mu(\vec{r'})\frac{\hbar}{i}\frac{\partial v_{\mu'}^*(\vec{r'})}{\partial x'}\right) + h.c.\right] \tag{27}$$

The various terms here correspond to the different quasi-particle processes that contribute to the polarization. There is a simple selection rule for nonvanishing matrix elements, namely $\mu = -\mu' = 1/2$. We discuss this now. The details are worked out in Appendix I. The operator p_x has an angular dependence of the form $\cos\phi$ (radial derivative term) and $\sin\phi$ (angular derivative term) where ϕ is the angle in the 2d plane of the vector $\vec{r}$ with respect to the x axis. The amplitudes $u_\mu(\vec{r}),\ v_\mu(\vec{r})$ can be written (20-23) as

$$\begin{pmatrix} u_\mu(\vec{r}) \\ v_\mu(\vec{r}) \end{pmatrix} = e^{-i\mu\phi}\begin{pmatrix} e^{i\phi/2} & f_\mu^-(r) \\ e^{-i\phi/2} & f_\mu^+(r) \end{pmatrix} \tag{28}$$

where $f_\mu^\pm(r)$ are functions only of the radial coordinate r. The ϕ dependence of $u_\mu(\vec{r}),\ v_\mu(\vec{r})$ above implies that on integration of the matrix elements in Eq. (27) over the angle ϕ, the only nonvanishing terms are $(\mu - \mu') = \pm 1$. We <u>also</u> need one of the states μ to be unoccupied and the other μ' to be occupied. The only possibility among bound states is $\mu = -\mu' = \frac{1}{2}$. To understand this, we exhibit in Fig. 3, the spectrum of eigenstates for parameters appropriate to the cuprate superconductor (23). The bound states within the gap have quantum numbers $\mu = \pm\frac{1}{2},\ \pm\frac{3}{2},$......... The occupied (unoccupied) states have negative (positive) μ. It is now clear that the only virtual transitions that satisfy the selection rule $\Delta\mu = \pm 1$, and are between occupied and unoccupied states, correspond to $\mu = -\mu' = \frac{1}{2}$, i.e. the highest occupied and lowest unoccupied bound states. With this simplification used in Eq. (27) and the latter substituted into the expression Eq. (24) for m_0^*, we have

$$m_0^* = \frac{4|g_x|^2}{\epsilon_{1/2}} \tag{29a}$$

where

$$g_x = \int d\vec{r} v_{1/2}(\vec{r}) \frac{\hbar}{i} \frac{\partial u_{1/2}(\vec{r})}{\partial x} \tag{29b}$$

We thus need to know the energy $\epsilon_{1/2}$ and the core bound state wavefunctions $f_{1/2}^{\pm}(r)$ in order to find m_0^*. These have been determined self-consistently and numerically by Zhu et. al. (22). We use here the variational forms

$$\Delta_0(r) = \Delta_0 \tanh(r/\xi) \tag{30a}$$

$$f_{1/2}^{-}(r) = A_{1/2} J_0(k_F r) e^{-r/2\xi} \tag{30b}$$

and

$$f_{1/2}^{+}(r) = A_{1/2} J_1(k_F r) e^{-r/2\xi} \tag{30c}$$

where

$$A_{1/2}^{-2} = \int d\vec{r} [J_0^2(k_F r) + J_1^2(k_F r)] e^{-r/\xi} \tag{30d}$$

is the normalisation factor. With $\xi = 15\mathring{A}$, $k_F^{-1} = 3.36\mathring{A}$ and $\Delta_0 = 60$ meV, the expectation value of the energy $< \epsilon_{1/2} >$ is 69 K, close to the self-consistent numerical value of 66 K obtained by Zhu et. al. (22). The wavefunctions are also very close. Using these, the expression m_0^* can be evaluated (see Appendix I for details), giving a value

$$m_0^* \simeq 25 m_e \tag{31}$$

The vortex mass obtained above can be estimated by the following simple physical arguments. The correlation function in Eq. (24) can be estimated as follows. Each of the momentum operators gives a factor of $\hbar k_F$ which is the typical electronic momentum. The energy denominator of the correlation function (see Eq. (27)) is twice the bound state energy $\epsilon_{1/2}$ which is the energy cost of the polarisation process involving creation of a 'particle-hole' pair. Finally there is a factor of two corresponding to electron spin. The bound state energy (20) $\epsilon_{1/2}$ is about $\frac{\Delta_0^2}{2\epsilon_F}$. The core contribution to the unscreened vortex mass (for parameters appropriate to the cuprates) is thus estimated to be

$$m_0^* \simeq \frac{\hbar^2 k_F^2}{\epsilon_{1/2}} \simeq 100 m_e \qquad (32).$$

This is larger than the value calculated above for the vortex mass (Eq. 31) by a factor of four. This discrepancy arises because the matrix element in the detailed calculation is smaller (by a factor of half approximately) than the dimensional estimate.

We now consider the effect of Coulomb interactions. The dipolar charge distribution induced by vortex motion is screened efficiently by the electrons. This greatly reduces the vortex kinetic energy. The reduction of the vortex kinetic energy is calculated using standard self-consistent linear response theory. The dipolar charge distribution produces an extra electric potential i.e. changes the electrochemical potential of the system. Any change in the electrochemical potential causes a change in density which therefore needs to be calculated self-consistently. In the following, we outline the calculation of the reduction of the vortex mass because of Coulomb screening. The details are provided in Appendix II. Varying the action in Eqs. (21) and (22) with respect to the electric potential A_0, we find at the extremum, the Poisson equation

$$\nabla^2 A_0(\vec{r}, z) = -4\pi e \sum_l \langle \rho_l(\vec{r}) \rangle \, \delta(z - ld) \tag{33.}$$

We find $< \rho_l(\vec{r}) >$, the electron density, to linear order in u and A_0 to be

$$\langle \rho_l(\vec{r}) \rangle = e \int d\vec{r'} \chi_l^{00}(\vec{r}, \vec{r'}) A_0(\vec{r'}, ld) - u \int d\vec{r'} \chi_l^{0x}(\vec{r}, \vec{r'}) \tag{34.}$$

In Eq. (34), the density-density and density-current response functions χ^{00} and χ^{0x} are given by

$$\chi_l^{00}(\vec{r}, \vec{r'}) = -i \int dt \theta(t) \left\langle \left[\rho_l(\vec{r}, t), \rho_l(\vec{r'}, 0) \right] \right\rangle \tag{35}$$

and

$$\chi_l^{0x}(\vec{r}, \vec{r'}) = -i \int dt \theta(t) \left\langle \left[\rho_l(\vec{r}, t), p_l^x(\vec{r'}, 0) \right] \right\rangle \tag{36}$$

The second term on the right hand side of Eq. (34) is the source term for $< \rho_l >$; it has to be determined by integrating the correlation function $\chi_l^{0x}(\vec{r}, \vec{r'})$ over the co-ordinate $\vec{r'}$. This is evaluated to be

$$\int \chi_l^{0x}(\vec{r}, \vec{r'}) d\vec{r'} = \eta(\vec{r}) \lambda \tag{37}$$

where

$$\eta(\vec{r}) = 2 \sqrt{\frac{2}{\epsilon_{1/2}}} f_{1/2}^-(r) f_{1/2}^+(r) \sin\phi \tag{38a}$$

and

$$\lambda = -\sqrt{\frac{2}{\epsilon_{1/2}}} \; \hbar \int d\vec{r} v_{1/2}^*(\vec{r}) \frac{\partial u_{1/2}^*(\vec{r})}{\partial x} \tag{38b}$$

It is thus clear from Eq. (37) substituted into Eq. (34) that the charge distribution generated by the vortex motion is proportional to $\sin\phi$ i.e. it is dipolar in nature. We thus seek a self-consistent solution to the Poisson equation of the form

$$A_0(\vec{r}, z) = V(r, z) \sin\phi \tag{39}$$

Substituting this form in Eqs. (33) and (34), and using Eq. (37) we solve for $A_0(\vec{r}, z)$ (see Appendix II for details), to get

$$A_0(\vec{q}, ld) = -\frac{4\pi e u \lambda \eta(\vec{q})}{1 + 2\pi e^2 M(0)} \frac{1}{2q} \left(\frac{\sinh qd}{\cosh qd - 1} \right) \tag{40}$$

where $A_0(\vec{q}, ld)$ and $\eta(\vec{q})$ are the two dimensional Fourier transforms of $A_0(\vec{r}, ld)$ and $\eta(\vec{r})$ respectively. In Eq. (40), $M(0)$ is given by

$$M(0) = \int \frac{d\vec{q}}{(2\pi)^2} \frac{|\eta(\vec{q})|^2}{2q} \left(\frac{\sinh qd}{\cosh qd - 1} \right) \tag{41}$$

and has the physical significance of an irreducible polarizability . Knowing the electric potential $A_0(\vec{q}, ld)$ (which is linear in u) we can integrate out the electrons and the harmonic electric potential fluctuations in the action (Eq. (22)) to second order in u. The result for the extra action per layer is

$$S_{KE} = \int dt (u\lambda)^2 \left[1 - \frac{2\pi e^2 M(0)}{1 + 2\pi e^2 M(0)} \right] \tag{42}$$

where the first term in the square bracket is the large unscreened contribution calculated earlier (Eq. (31)), and the cancelling second term gives the reduction because of screening. Combining both these terms, we find m^* to be

$$m^* = \frac{m_0^*}{(1 + 2\pi e^2 M(0))} \tag{43}$$

We see from Eq. (43) that there is a 'dielectric' screening of the vortex mass, i.e. the factor in the denominator is a finite large number. This is due to the discrete level spectrum in the core, in contrast to the continuum of states in a metal or the near continuum in conventional superconductors giving 'metallic' screening. The core dielectric constant (ϵ_{core}) can be evaluated using the variational wave functions of Eqs. (28) and (30). The screening reduces m_0^* by a factor of about 50. The large dielectric constant can be understood as being due to the high polarizability of the core quasiparticle system. Approximately, the dielectric constant (ϵ_{core}) is dimensionally given by the ratio of the core Coulomb energy and the excitation energy ($\epsilon_{+-} = 2\epsilon_{1/2}$) necessary to create the particle-hole excitation which contributes to the polarization process leading to screening (see Appendix II). Thus,

$$\epsilon_{\text{core}} \simeq (E_{\text{Coulomb}}/E_{\text{excitation}}) \simeq \{(e^2/\xi)/\epsilon_{+-}\} \simeq 75 \qquad (44).$$

The detailed calculation yields a dielectric constant of 53. Thus the screened or effective inertial mass of a vortex is rather small, being equal to

$$m^* = (m_0^*/53) \simeq 0.5 m_e \tag{45}$$

This is our main result, for the mass per layer, at $T = 0$, when the field is along the c-axis.

We note that the mass Eq. (45) can be roughly estimated qualitatively by using the physical estimates Eqs. (32) and (44), i.e.

$$m^* \simeq \left(\hbar^2 k_F^2 / \epsilon_{1/2} \right) / \left\{ \left(e^2/\xi \right) / 2\epsilon_{1/2} \right\} = (2 k_F^2 a_0 \xi) m_e \qquad (46).$$

This gives a value $m^* \simeq 1.3 m_e$ close to the the result Eq. (45) of the detailed calculation! It is clear from the formal expressions, Eq. (29) and Eq. (43) for the effective mass, as well as the approximate form Eq. (46) that m^* is effectively, the ratio of two polarizabilities, current-current and charge-charge. Vortex motion (in the rest frame of the vortex) gives rise to both supercurrent fluctuations and electric potential fluctuations. They are connected because of gauge invariance. Since both polarizabilities involve the same energy denominator, this drops out of m^* (Eq. (46)). It is also clear, from the occurrence of k_F, a_0 and ξ in this equation, that the vortex mass is related to the carrier density, core size as well as Coulomb interactions. The small vortex mass in the cuprates is a consequence of the small core size and the low carrier density in comparision with conventional superconductors.

In the next, concluding section, we discuss the approximations involved in the result obtained by us, its generalization, and the question of when vortex mass related effects may be observed.

IV. Discussion and Conclusion

A. Discussion of the Vortex Mass The microscopic calculation above uses an approximate variational order parameter Δ and corresponding Bogoliubov-deGennes amplitudes (u_μ, v_μ) (Eq. (30)). An obvious improvement would be to solve exactly for these quantities given only the parameters Δ_0, ξ and the BCS relation $\xi = (\hbar v_F/\pi\Delta_0)$. This has been done numerically (22). There is of course a fair amount of uncertainty in these parameters for cuprate superconductors, quite apart from the question of whether an s-wave, BCS like order parameter with a conventional kinetic energy functional is at all appropriate for cuprate superconductors. However, within the s-wave BCS model, the exact expression Eq. (29) and (43) for m^* can be evaluated, once u_μ, v_μ and their first spatial derivative are known for $\mu = 1/2$. We have ignored the contribution due to transitions from the bound states to the continuum, satisfying the selection rule $\Delta\mu = \pm 1$. The reasons are the smallness of the matrix elements, the largeness of the energy denominator and very good screening. A rough estimate shows that these change the mass estimated by about 10-20%. We have also ignored contributions to the mass from polarisation processes involving the collective excitations of the superconducting state. In the case of a neutral Fermi superfluid it has been found by Niu et. al. (40) that the inclusion of these excitations, which correspond to long wavelength density fluctuations, leads to a finite vortex mass in contrast to the logarithmically divergent result obtained in our approach (see Section I.). However for the case of a charged superconductor, which is the primary concern of this paper, the corresponding mode is pushed up to the plasma frequency which is much larger than the other energies in the problem and is therefore unlikely to contribute to the vortex mass in a significant way.

With increasing temperature, the gap $\Delta(T)$ and the inverse coherence length $\xi(T)^{-1}$ decrease. The structure of the vortex core also changes. In principle, one can repeat the $T = 0$ calculation with temperature dependent input parameters. At low temperatures ($k_B T \ll \epsilon_{1/2}$) this would roughly have the effect of *increasing* the effective inertial mass of the vortex as a function of temperature (Eq. 46). However, at higher temperatures (when $\epsilon_{1/2}$ is of order or less than $k_B T$) an additional contribution to the vortex mass will accrue from transitions involving thermally excited quasiparticles in the vortex core. A qualitatively new effect which arises in this regime is that due to quasiparticles scattering off the moving vortex, there is a damping of vortex motion. This dissipative term is generally included phenomenologically (it is linear in vortex velocity u and contributes an imaginary term to the action), though microscopic theories have been developed (25, 37). It is not clear whether the two effects viz. thermal renormalization of the effective mass, and dissipation, both due to thermal quasiparticles, are completely independent. Also, the regime where the bound core level spacing $\epsilon_{1/2}$ is of order or less than $k_B T$ is clearly very different from the low temperature regime $\epsilon_{1/2} >> k_B T$. We have not considered the former dissipation dominated 'high temperature' limit.

There is the related question of adiabaticity, which is contained in the assumption that the order parameter of the system with one moving vortex is the same as that of the static vortex, but with $\vec{r} \rightarrow (\vec{r} - \vec{u}t)$. It is expected that the vortex motion would distort the gap function from its form in the static case. The distortions induced have to be determined self-consistently at every order of $\vec{u}$. Simanek (26) has recently considered this question and has pointed out that the vortex velocity needs to be small enough such that $(\hbar u k_F) \leq |\epsilon_{1/2}| = (\Delta_0^2/\epsilon_F)$ or $u \leq (u_{BCS})(\Delta_0/\epsilon_F)$ where u_{BCS} is the BCS critical velocity i.e. $\hbar u_{BCS} k_F \simeq \Delta_0$. The violation of this criterion means that the vortex motion can significantly distort the gap function as well as the magnetic field and supercurrent distribution. The changes induced by the vortex motion have to be self-consistently determined and it's effects included in the calculation of the vortex mass. However, at low vortex velocities ($u \ll u_{BCS}$), these effects are small and have therefore been ignored.

We have made a Galilean transformation to the rest frame of the vortex to simplify the calculation of change in action to second order in $\vec{u}$. Inclusion of the effects of the periodic lattice and impurity scattering is also possible within our formalism. Their interaction potential with the electrons becomes explicitly time-dependent and leads to inelastic scattering (38) of electrons whose effects have to be included while evaluating the correlation functions of Eqs. (24), (35) and (36).

So far our discussion has been restricted to a single vortex. However, at larger magnetic fields, in the presence of a vortex lattice, additional contributions coming from the vortex lattice will have to be included. The periodicity of the pair potential broadens the localised quasiparticle levels into energy bands and new contributions to the vortex mass as well as the forces experienced by vortices are expected to arise from collective effects arising from the vortex lattice (15). A totally different approach becomes necessary in the presence of strong magnetic fields in the dense vortex limit near H_{c2}. The strong amplitude fluctuations which allow the dissociation of a Cooper pair make the dynamics of the order parameter diffusive in this regime. A calculation of the vortex mass will proceed from the Abrikosov solution (36) of the GL equations for a triangular vortex lattice. Both the vortex effective mass as well as the nature of the forces that drive the vortex dynamics in this regime are subjects that require further study.

The core contribution to the vortex mass was estimated earlier microscopically by by T. Hsu (42). He obtained an answer which is of the same order of magnitude as the core contribution calculated by us in the abscence of Coulomb screening for parameters appropriate to the cuprates. Hsu used the Bogulibov De-Gennes formalism to obtain the vortex acceleration in response to a transport supercurrent. The vortex mass is then deduced from this equation by a comparision with a hypothetical force equation obtained by setting the unknown vortex mass times the acceleration of the vortex equal to a Magnus force of the size suggested by Nozieres and Vinen using arguements of fluid hydrodynamics. While many aspects of the formalism are similar to ours we believe this method is unreliable for determining the vortex mass. The Coulomb screening effects which are found to be vital for giving rise to a small mass have been ignored. Even for the case of a neutral superfluid, the uncertainty in the size as well as sign of the Magnus force cast doubts about a procedure which relies on a knowledge of the Magnus force to deduce the core contribution to the vortex mass.

Recently, after completion of this work(15), a paper by Simanek (27) appeared where a calculation of the vortex mass using discrete core states is presented. This calculation is based on a TDGL functional, in terms of a single complex superconducting order parameter, which is derived in the presence of a moving vortex and leads to a mass which is dimensionally the same as our estimate for the unscreened mass. However, the all important screening effect which reduces m^* by a factor of 50-75, has not been considered at all in Simanek's calculation. Further extensions (41) along the lines of Ref. (27) have appeared during the reviewing process. However, once again the substantial reduction in the vortex mass because of Coulomb screening has been ignored.

The assumption of an s-wave like order parameter is not realistic. There is increasing direct experimental evidence for a strongly anisotropic order parameter, or an order parameter with vanishing amplitude at some points in k (or r space) (28,29). Consider for example a $d_{x^2-y^2}$ like order parameter. The pair amplitude is nonlocal i.e. $< \bar{\psi}_{\uparrow}(\vec{r'})\bar{\psi}_{\downarrow}(\vec{r}) >$ vanishes for $\vec{r} = \vec{r'}$, and has a dependence on the direction of $(\vec{r} - \vec{r'})$ with a square symmetry. It is thus clear that the Bogoliubov-deGennes equations are nonlocal mixing different angular momentum eigenstates. Recent work by Volovik (30) and Ren and co-workers (35) (see also Ref. 44 and references therein) has suggested that vortices in superconductors with $d_{x^2-y^2}$ symmetry have a non-zero s-wave component in the core of the vortex which vanishes at the vortex centre. Thus, the gapless bound state spectrum, which might have been expected for d-wave superconductors with lines of nodes in the gap function (in k-space) is absent. However, gapless excitations are available in the far region, where the s-wave component vanishes and are likely to give rise to strong dissipative effects so that the nature of vortex dynamics would be qualitatively very different. This is an area which needs much further work (see for example ref. 30 and 35).

B. Observability of effects due to inertial mass

It is clear that if there is no dissipation and no Magnus force, both of which produce a term in the action linear in velocity (10,11), the small inertial mass of a vortex would give rise to strong quantum effects. The vortices are bosonic particles whose degeneracy temperature is $(eH/m^*c)(\hbar/k_B)$. This is of order 20K for an external field of 10T, if the effective mass m^* is about $0.5m_{el}$. If this limit is realized, then several novel possibilities would arise, especially in strongly layered cuprate superconductors such as 2212. In these systems the vortex liquid phase extends to very low temperatures, especially in high magnetic fields (31). This liquid instead of becoming a solid, could on cooling become a quantum Bose liquid and then a genuine vortex superfluid (32). Such a vortex superfluid is a new ground state with unusual properties, most likely a new kind of insulator. The vortex superfluid could persist till $T = 0$, or freeze into a quantum solid, whose spectrum of collective excitations (phonons) would depend on the mass m^*. The dynamics of vortices in this regime would be that of interacting bosons in a random potential.

It is not clear however that the quantum Bose regime of the many vortex system is experimentally realizable. Firstly, at least for higher temperatures, there is strong dissipation which dominates the dynamics in both the quantum and classical regimes. Secondly there is a large Magnus force (9,10,11). If only the former were present, the mass could still be relevant for phenomena like quantum creep. If only the Magnus force were present, as is believed to be the case in the cuprates where the onset of a dissipationless regime has been reported (5), the system of vortices is like that of bosons in a strong 'magnetic' field, the Magnus force being the analogue of the Lorentz force. The Hamiltonian of the system of 2d vortices can be written as

$$H_{vort} = \sum_i (\vec{p}_i - \vec{a}_i)^2/2m^* + (1/2)\sum_{i,j} V(\vec{r}_i - \vec{r}_j)$$

where $(a_x, a_y) = (\pi\hbar n/2)(-x_i, y_i)$ and $V(\vec{r}_i - \vec{r}_j)$ is the interaction between vortices. The $\vec{a}$ term is due to the Magnus force with n being the electron density per layer and m^* is the vortex effective mass. The 'magnetic' field associated with this Magnus term is rather large, the cyclotron frequency being about 0.7 eV for $m^* \simeq 0.5m_e$. Thus the Landau level separation is large, and the vortex system is in the lowest Landau level with a low filling fraction of (n_v/n) where n_v is the vortex density and n the electron density. The magnetic length of the system, i.e. the cyclotron orbit size is rather small $\sim 7\AA$ so that the dynamics is that of the guiding centre; the inertial mass is irrelevant. Even in the strong Magnus force limit, a large vortex mass has been shown (10) to give rise to quantum effects in phenomena involving vortex tunneling. In particular the semi-classical action develops a linear in T dependence, which would reflect in the observed rate of flux creep at low temperatures. In the language of our paper, a large vortex mass would result in a reduction of the cyclotron frequency, mixing in higher Landau levels and thus enhancing quantum effects. However, the rather small value of the vortex mass obtained by us implies that this scenario is actually not realised. The main uncertainty in vortex dynamics is the actual size of the Magnus force. The contribution of the bound states i.e. of localised quasiparticles to the Magnus force and the effect of disorder on it are major unsettled issues; there are a number of suggestions (33,34) that these could reduce, cancel or reverse the Magnus force! The dynamics of an isolated vortex with inertial mass, in the presence of a large Magnus force and dissipation had been investigated recently (9-12). Another possibility is that additional Magnus like forces could arise from the pair potential in the dense vortex lattice limit (15) with an opposite sign. However, there is a lack of a clear microscopic theory. There is a growing body of experimental evidence based on quantum creep (3), Hall measurements (6) and a.c. electromagnetic response (14) that the Magnus force is actually much smaller than current theoretical estimates (9, 10, 11). In that case, there exists the intriguing possibilty of the formation of a correlated quantum Hall fluid of the bosonic vortices at low temperatures (39). With the mounting evidence in the cuprates for a superconducting order parameter which has $d_{x^2-y^2}$ symmetry and quasiparticles whose mean free paths could be very long for $T \ll T_c$, a realistic picture of this whole field awaits a microscopic calculation of the inertial mass, the Magnus force on a moving vortex and dissipation of its momentum for a d-wave superconductor at low temperatures.
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Appendix I

In this appendix we outline the evaluation of the correlation function (Eq. 24) which determines the unscreened core contribution to the vortex mass. Using the Bogulibov transformation (Eq. 25) the field operators in Eq. (24) can be rewritten in terms of the quasiparticle operators (γ_μ) to give

$$m_0^* = \int dt \int d\vec{r} \int d\vec{r'} D(\vec{r}, \vec{r'}; t) \qquad (AI.1)$$

where the correlation function $D(\vec{r}, \vec{r'}; t)$ is given by

$$D(\vec{r}, \vec{r'}; t) = \sum_{\mu,\nu,\lambda,\eta} \sum_{\sigma,\sigma'} f_\mu^{\sigma *}(\vec{r}) \frac{\hbar}{i} \frac{\partial f_\nu^\sigma(\vec{r})}{\partial x} f_\lambda^{\sigma' *}(\vec{r'}) \frac{\hbar}{i} \frac{\partial f_\eta^{\sigma'}(\vec{r'})}{\partial x'} P_{\mu,\nu}^{\lambda,\eta}(t) \qquad (AI.2a)$$

where

$$P_{\mu,\nu}^{\lambda,\eta}(t) = i\langle T[\gamma_\mu^\dagger(t)\gamma_\nu(t)\gamma_\lambda^\dagger(0)\gamma_\eta(0)]\rangle \qquad (AI.2b)$$

. Here $f_\mu^\uparrow(\vec{r}) = u_\mu(\vec{r})$ and $f_\mu^\downarrow(\vec{r}) = v_\mu^*(\vec{r})$ and the summation with respect to μ, ν, λ and η runs over both positive and negative values. The correlation function in Eq. (AI.2) is easily evaluated using the diagonalized Hamiltonian (Eq. (26)) to yield the expression given in Eq. (27).

We will now derive the selection rule mentioned in Section III. To find m_0^* from the correlation function of Eq. (27) we need to integrate with respect to the co-ordinates $\vec{r}$ and $\vec{r'}$. The $\vec{r}$ -integration requires the evaluation of the integral

$$I_1 = \int d\vec{r} u_\mu^*(\vec{r}) \frac{\hbar}{i} \frac{\partial u_{\mu'}(\vec{r})}{\partial x} \qquad (AI.3)$$

Substituting the explicit forms of $u_\mu(\vec{r})$ and $u_{\mu'}(\vec{r})$ (Eq. (28)) we find that

$$I_1 = \frac{\hbar}{i} \int dr r f_\mu^-(r) \frac{\partial f_{\mu'}^-(r)}{\partial r} \int d\phi \cos\phi e^{i(\mu-\mu')\phi}$$
$$+\hbar(\mu' - 1/2) \int dr\, f_\mu^-(r) f_{\mu'}^-(r) \int d\phi \sin\phi e^{i(\mu-\mu')\phi} \qquad (AI.4)$$

The angular integrals in Eq. (AI.4) are zero unless $\mu - \mu' = \pm 1$. This together with the constraint $\mu > 0$, $\mu' < 0$ implies that the only non-zero contribution to m_0^* comes from $\mu = -\mu' = 1/2$. Making use of this selection rule, we find, after a little algebra, that the expression for m_o^* reduces to Eq. (29). In arriving at this relation, we have used Eqs. (26b) and (26c). The only remaining task is to evaluate the matrix element g_x occuring in Eq. (29). Substituting the explicit functional forms (Eq. (30)) into Eq. (29b) we find that

$$g_x = \frac{\hbar}{2\xi} \frac{\int_0^\infty dx\, x e^{-x} J_1(k_F\xi x)[-J_0(k_F\xi x)/2 + k_F\xi J_0'(k_F\xi x)]}{\int_0^\infty dx\, x e^{-x}[J_0^2(k_F\xi x) + J_1^2(k_F\xi x)]} \qquad (AI.5)$$

Evaluating the dimensionless integrals on the R.H.S. of Eq. (AI.5) we find that for parameters appropriate to the cuprates ($k_F\xi \simeq 4.47$)

$$\mid g_x \mid \simeq 1.12 \frac{\hbar}{\xi} \qquad (AI.6)$$

The largest contribution to g_x in Eq. (AI.5) comes from the term involving $k_F\xi J_0'(k_F\xi x)$. However, unlike conventional superconductors, the relative smallness of the dimensionless parameter $k_F\xi$ implies that the other term cannot be ignored.

Appendix II

In the following, we present details of the calculation of the large reduction in the core contribution to the vortex mass due to Coulomb screening. To solve the Poisson equation (Eqs. (33) and (34)), it is necessary to determine the source term on the R.H.S. of Eq. (34) by integrating the correlation function $\chi_l^{0x}(\vec{r},\vec{r'})$ over the co-ordinate $\vec{r'}$. Making use of the Bogulibov transformation (Eq. (25)) and the diagonalised Hamiltonian (Eq. (26)) we find (after some algebra) that

$$\chi_l^{0x}(\vec{r},\vec{r'}) = 2\sum_{\mu>0}^{unocc.}\sum_{\nu<0}^{occ.}\frac{u_\mu^*(\vec{r})u_\nu(\vec{r})}{(\epsilon_\mu-\epsilon_\nu)}[v_\mu(\vec{r'})\frac{\hbar}{i}\frac{\partial v_\nu^*(\vec{r'})}{\partial x'} - u_\nu^*(\vec{r'})\frac{\hbar}{i}\frac{\partial u_\mu(\vec{r'})}{\partial x'}] + h.c. \qquad (AII.1)$$

The operator $\frac{\hbar}{i}\frac{\partial}{\partial x}$ behaves like $\cos\phi$ ($\sin\phi$) and as before, the integration over the co-ordinate $\vec{r'}$ (see Eq. (AI.4) and the discussion following it) gives the selection rule $\mu-\nu=\pm1$ which together with the constraint $\mu>0$ and $\nu<0$ implies $\mu=-\nu=1/2$. We therefore find

$$\int\chi_l^{0x}(\vec{r},\vec{r'})d\vec{r'} = \frac{-2u_{1/2}^*(\vec{r})v_{1/2}^*(\vec{r})}{\epsilon_{1/2}}\int d\vec{r'}v_{1/2}(\vec{r'})\frac{\hbar}{i}\frac{\partial u_{1/2}(\vec{r'})}{\partial x'} + h.c. \qquad (AII.2)$$

In writing Eq. (AII.2) we have used Eqs. (26b) and (26c). Now substituting the explicit variational forms for $u_{1/2}(\vec{r})$ and $v_{1/2}(\vec{r})$ (Eqs. (28) and (30)) we arrive at Eqs. (37) and (38). To proceed further, we have to solve the Poisson equation and get a self-consistent solution for the scalar potential $A_0(\vec{r},z)$ of the form assumed in Eq. (39). We will now consider the other term on the R.H.S. of Eq. (34). This term represents the screening charge induced by the Coulomb potential consequent to the electron density change induced by the vortex motion; it has to be determined by integrating the product of $\chi_l^{00}(\vec{r},\vec{r'})$and $A_0(\vec{r'},ld)$ with respect to the co-ordinate $\vec{r'}$. Since the latter has an angular dependence of the form $\sin\phi$ we once again find that the only process which contributes to the polarization involves a transition from the highest occupied state ($\nu=-1/2$) to the lowest unoccupied state ($\mu=1/2$). We thus find,

$$\int d\vec{r'}\chi_l^{00}(\vec{r},\vec{r'})A_0(\vec{r'},ld) = -\frac{\eta(\vec{r})}{2}\int d\vec{r'}A_0(\vec{r'},ld)\eta(\vec{r'}) \qquad (AII.3)$$

Combining Eqs. (37) and (AII.3) with the Poisson equation (Eqs. (33) and (34)) we get

$$\frac{\nabla^2 A_0(\vec{r},z)}{4\pi} = e\eta(\vec{r})\sum_l\delta(z-ld)[u\lambda+\frac{e}{2}\int d\vec{r'}\eta(\vec{r'})A_0(\vec{r'},ld)] \qquad (AII.4)$$

Transforming to Fourier space, this can be rewritten in terms of the corresponding Fourier components as

$$\frac{[-q^2-k^2]A_0(\vec{q},k)}{4\pi} = eu\lambda\eta(\vec{q})\sum_l\exp[-ikld] + \frac{e^2\eta(\vec{q})}{2d}\sum_m\int\frac{d\vec{q'}}{(2\pi)^2}\eta(-\vec{q'})A_0(\vec{q'},k-\frac{2\pi m}{d}) \qquad (AII.5)$$

This is an integral equation for the scalar potential A_0. To solve for A_0, we find it convenient to introduce the quantity

$$X(k) = \frac{1}{d}\sum_m\int\frac{d\vec{q}}{(2\pi)^2}\eta(-\vec{q})A_0(\vec{q},k-\frac{2\pi m}{d}) \qquad (AII.6)$$

Substituting Eq. (AII.6) in (AII.5) and making use of the property $X(k) = X(k - \frac{2\pi m}{d})$ for any integer m, we solve for $X(k)$ to get

$$X(k) = \frac{-4\pi eu\lambda M(k) \sum_l e^{-ikld}}{1 + 2\pi e^2 M(k)} \qquad (AII.7)$$

where

$$M(k) = \int \frac{d\vec{q}}{(2\pi)^2} \frac{\mid \eta(\vec{q}) \mid^2}{2q} \left(\frac{\sinh qd}{\cosh qd - \cos kd} \right) \qquad (AII.8)$$

Substituting Eqs. (AII.7) and (AII.8) in Eq. (AII.5) we can now solve for $A_0(\vec{q}, k)$ to get

$$A_0(\vec{q}, k) = \frac{-4\pi eu\lambda}{q^2 + k^2} \frac{\eta(\vec{q})}{1 + 2\pi e^2 M(k)} \sum_l \exp(-ikld) \qquad (AII.9)$$

Fourier transforming the above equation with respect to the wave vector k we get $A_0(\vec{q}, ld)$ (Eq. (40)).

We now consider the action Eq. (22). We first consider the electric field energy. Integrating by parts, and making use of the Poisson equation (Eq. (33)) we get

$$\int dz \int d\vec{r} \frac{(\nabla A_0(\vec{r}, z))^2}{8\pi} = \frac{-e}{2} \sum_l \int d\vec{r} \int d\vec{r'} A_0(\vec{r}, ld)[u\chi_l^{0x}(\vec{r}, \vec{r'}) - e\chi_l^{00}(\vec{r}, \vec{r'}) A_0(\vec{r'}, ld)] \qquad (AII.10)$$

On integrating out the electrons to second order in the vortex velocity u and the Coulomb potential A_0 we get the effective action

$$S' = \sum_l \int dt \int d\vec{r} d\vec{r'} [\frac{-u^2}{2} \chi_l^{xx}(\vec{r}, \vec{r'}) - \frac{e^2}{2} A_0(\vec{r}, ld) \chi_l^{00}(\vec{r}, \vec{r'}) A_0(\vec{r'}, ld) + ueA_0(\vec{r}, ld) \chi_l^{0x}(\vec{r}, \vec{r'})] \qquad (AII.11)$$

Combining Eqs. (AII.10) and (AII.11) we find that the effective action for the system, to second order in u, is given by

$$S_{KE} = \sum_l \int dt \int d\vec{r} d\vec{r'} [\frac{-u^2}{2} \chi_l^{xx}(\vec{r}, \vec{r'}) + \frac{ue}{2} A_0(\vec{r}, ld) \chi_l^{0x}(\vec{r}, \vec{r'})] \qquad (AII.12)$$

The first term in S_{KE} is the unscreened core contribution to the vortex mass evaluated earlier (Eq. (24)) while the other term represents the reduction because of Coulomb screening. We now substitute the explicit forms for χ_l^{xx} and χ_l^{0x}. Doing a calculation very similar to the one leading to Eqs. (37), (38) and (AII.2) we find

$$\int d\vec{r} d\vec{r'} \chi_l^{xx}(\vec{r}, \vec{r'}) = -2\lambda^2 \qquad (AII.13)$$

Substituting Eqs. (37) and (AII.13) into Eq. (AII.12) we get

$$S_{KE} = \sum_l \int dt [u^2\lambda^2 + \frac{eu\lambda}{2} \sum_l \int d\vec{r} \eta(\vec{r}) A_0(\vec{r}, ld)] \qquad (AII.14)$$

Fourier transforming the second term in Eq. (AII.14) and substituting the expression for A_0 (Eq. (40)) we finally arrive at Eq. (42).

The only remaining task is to evaluate the polarizability $M(0)$. Using Eq. (38a), we find that

$$\eta(\vec{q}) = 2\sqrt{\frac{2}{\epsilon_{1/2}}} \int d\vec{r} f^{-}_{1/2}(r) f^{+}_{1/2}(r) \sin\phi e^{-i\vec{q}\cdot\vec{r}} \qquad (AII.15)$$

On substituting the variational forms (Eq. 30) for $f^{-}_{1/2}(r)$ and $f^{+}_{1/2}(r)$ and integrating over the angular co-ordinate ϕ this reduces to

$$\eta(\vec{q}) = 2\sqrt{\frac{2}{\epsilon_{1/2}}} \frac{2\pi i}{A^2_{1/2}} \frac{q_y}{q} \int_0^\infty dr r J_0(k_F r) J_1(k_F r) J_1(qr) e^{-r/\xi} \qquad (AII.16)$$

Combining Eq. (AII.16) with the expression for $M(0)$ (Eq. (41)) we finally get

$$M(0) = \frac{4\pi}{A^4_{1/2}\epsilon_{1/2}} \int dq \frac{\sinh(qd)}{\cosh(qd) - 1} I^2(q) \qquad (AII.17)$$

where

$$I(q) = \int_0^\infty dr r J_0(k_F r) J_1(k_F r) J_1(qr) e^{-r/\xi} \qquad (AII.18)$$

Using Eqs. (30d), (43), (AII.17) and (AII.18) we are now in a position to calculate the 'core dielectric constant' ϵ_{core}. We find that $\epsilon_{core} = 1 + 2\pi e^2 M(0)$ is given by

$$\epsilon_{core} = 1 + \frac{e^2/\xi}{\epsilon_{+-}} \frac{L_1}{L_2^2} \qquad (AII.19)$$

where

$$L_1 = 4\int_0^\infty dx \frac{\sinh(xd/\xi)}{\cosh(xd/\xi) - 1} f^2(x) \qquad (AII.20)$$

$$L_2 = \int_0^\infty dx\, x e^{-x} (J_0^2(k_F\xi x) + J_1^2(k_F\xi x)) \qquad (AII.21)$$

and

$$f(x) = \int_0^\infty dy y J_0(k_F\xi y) J_1(k_F\xi y) J_1(xy) e^{-y} \qquad (AII.22)$$

On evaluating these expressions (Eqs. (AII.19), (AII.20) and (AII.21) numerically we find $\epsilon_{core} \simeq 53$.

Figure Captions

Fig. 1 Feynman diagrams that contribute to the phase-only action functional of Eq. 1) at $T = 0$ in the clean limit on integrating out the electrons. The thick lines correspond to the fermions, the wavy lines indicate the density fluctuations $\rho = (\hbar\dot{\theta} - 2eA_0)$ and the dashed line stands for the supercurrent fluctuations $\vec{j} = (\hbar\nabla\theta - 2e\vec{A}/c)$.

Fig. 2 Additional diagrams that contribute to the Galilean invariant action functional of Eq. 7) on integrating out the electrons. The thick lines correspond to the fermions, the wavy lines indicate the density fluctuations $\rho = (\hbar\dot{\theta} - 2eA_0)$ and the dashed line stands for the supercurrent fluctuations $\vec{j} = (\hbar\nabla\theta - 2e\vec{A}/c)$.

Fig. 3 Spectrum of bound states within the core (within the range $-\Delta_0$ to Δ_0) and of continuum states outside it, with the angular momentum quantum numbers. The levels are appropriate to the parameters mentioned in the text for a cuprate superconductor at $T = 0$. The transition allowed by selection rules is shown by an arrow.

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