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Quark-lepton compositeness: A three-body scenario

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Abstract. The Harari-Shupe model of quarks and leptons is viewed, not as a gauge theory, but as a quantum-mechanical three-body problem of the extreme relativistic type involving massless preons. Considerations based on S_3-symmetry in the available degrees of freedom (spin, isospin, space and hypercolour) are employed in conjunction with a spin-dependence ansatz on the three-preon forces ($\Sigma^a = \sigma^{(1)}_{\mu\nu}\sigma^{(2)}_{\nu\lambda}\sigma^{(3)}_{\lambda\mu}$) for an understanding of the three basic issues of (i) spin-1/2, (ii) generation structure and (iii) steeply rising mass patterns of quark-lepton families. The Σ^a-dynamics is compatible with the interpretation of colour as a manifestation of S_3-symmetry, as envisaged in the original Harari-Shupe proposal, while the interpretation of the generation structure devolves on the role of a certain quantum number N which takes on three different classes of values ($3n, 3n \pm 1; n = 0, 1, 2, \ldots$) according to the S_3-symmetry of the *spatial* wavefunction.

Keywords. Quark-lepton composites; Harari-Shupe model; lepton-quark substructure; preon interaction.

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1. Introduction

It is a privilege to be asked to present this paper in honour of Sudhir Pandya with whom I have had an association extending over four decades (since our student days at Delhi University). My view of Sudhir may be summarized in 3 words: "noble, relaxed, disarming". He patters, he banters and does the most serious things in apparently the lightest manner, but I cannot imagine any form of meanness, in the remotest sense of the term, emanating from him. His discriminating taste in physics is amply testified by the nature of his publications—a strong preference for symmetry principles, exemplified in the domain of nuclear properties. The famous Pandya theorem is but one example on his line of thinking in physics. I had once the good fortune to collaborate with him on the problem of energy levels in nuclei, using the so-called separable potentials. It was a delightful experience to see his mind work on certain techniques of transformation from individual nucleon coordinates to the *relative* coordinates of nucleon-pairs, in an almost playful spirit, which greatly simplified the matrix elements! Later these techniques were known as "Moshinsky brackets" and though Sudhir had discovered them earlier, I never noticed a trace of rancour in his utterances. It was just good sport for him.

I therefore found it quite challenging to be able to "choose" a topic for this occasion, which would suit his taste for symmetry principles without getting involved in mathematical complexities. I finally decided on the subject of composite leptons and quarks through a simple-minded quantum-mechanical treatment of a model 3-preon problem which I had formulated some years ago (Mitra 1983), in terms of a direct 3-preon interaction with a special spin-dependence (Σ^a), and had only recently revived some interest in it from the point of view of understanding "colour" as a form of S_3-symmetry, in keeping with the spirit of the original, Harari-Shupe (H-S) model (Harari 1979; Shupe 1979). The present paper is a short review based on the earlier attempt, together with some additional ideas (as yet unpublished) on the quantum-mechanical understanding of colour and generation structure under Σ^a-dynamics.

2. Lepton-quark substructures

In terms of the theoretical picture of today, quarks and leptons are regarded as the closest candidates for elementary particle status up to a distance 10^{-16} cm, representing the probing limits of modern experimental technology. However, the rapid proliferation of both these varieties (reaching the half century mark without supersymmetry (SUSY) and the century mark with SUSY) has understandably caused strong speculation about a "fourth-generation" of particle structure in accordance with the following scheme:

This is despite the influence of certain "all-time" theories which, by their very nature, must decide *once for all* in favour of "elementarity" of particles at a certain preassigned level and "refuse" to entertain any further possibilities of compositeness in their own structures. Such a decision must be made in advance, whether at the level of quark-leptons or even at the preon-level. However, once made, the frameworks of such theories are such as to render irrelevant the very questions of further substructures of these building blocks. This kind of philosophy is reminiscent of the Bootstrap theories of the early sixties (Carelton *et al* 1985) wherein all strongly interacting particles were thought to be bound states of all others (nuclear democracy), something akin to the Mach principle. In modern times a similar philosophy is in evidence in the popular versions of the superstring theories (Green *et al* 1984) where the contact with data has got relegated to the second place due to overwhelming concern for internal self-consistency of the theory.

This alternative point of view of successive substructures is not only in accordance with history (which has already seen 3 such generations) but is much less rigid in the conceptual formulation, leaving enough scope for further substructures, if need be, in response to subsequent observations. In principle, one could even conceive of an unending process of successive shell-structures (*a la* onion skins), were it not for the experimental limitations of the day which tend to "round off" such speculations beyond observational feasibility. At the present state of the experimental art, the window is just opening for a peep into a possible *fourth* stage of compositeness. However, the observational signatures of quark-lepton compositeness are much too few at this stage to warrant such conclusions based on experimental facts alone. Yet certain broad features of the observed quark-lepton families do suggest a composite

Table 1. Successive levels of compositeness.

	atoms	→ nuclei	→ hadrons	→ quark-leptons
Gauge th	QED	QHD	QCD	hyper-color?
Constituents	$(e^-, \mu^-, \ldots)$	$N, \pi, \ldots$	$q, \bar{q}, g$	preons?

picture. There are similarities in the characteristics of their successive "recurrences" as follows:

$$e\nu_e du; \quad \mu\nu_\mu sc; \quad \tau\nu_\tau bt(?) \tag{1}$$

reminiscent of the early multiplet classification of baryons and mesons which eventually led to the quark picture. These groupings, termed successive "generation", horizontal symmetry, etc. have led to various group assignments, e.g., SU(4) of Pati and Salam (1974), or SU(5) of Georgi and Glashow (1974). These have inevitably led to composite models carrying similar group signatures, all with the common feature of *proton instability* (Langacker 1981) against decay into lighter mesons and leptons which are no longer forbidden by selection rules for transitions between quark and lepton species, since both now have a *common* underlying substructure. On the other hand, the "refusal" of the proton so far to "decay" within observable time limits has put brakes on unrestricted model building.

3. The Harari-Shupe model

As an illustrative example of one of the "admissible" models of composite quarks and leptons, it is useful to consider a simple model which has shown the capacity to incorporate many of the observed quark-lepton patterns within its basic tenets in a very natural way. This is the two-component (t, V) preon model of Harari-Shupe, whose SU(2) structure (charge content 1/3, 0) helps it satisfy the so-called t'Hooft anomaly constraints (t'Hooft 1980) trivially (an important self-consistency requirement for a composite model), and gives a simple 3-preon characterization to each quark-lepton quartet as

$$e^+ = ttt; \quad \nu_e = VVV; \quad u = ttV; \quad \bar{d} = VVt, \tag{2}$$

and similarly for the higher sets in (1). As for the colour attribute, the original effort (Harari 1979; Shupe 1979) was to simulate this property combinatorially through constructions based on permutation (S_3) symmetry which prima facie yield colour-singlet (unique) assignments to e and ν_e, 3 to u and 3* to d. Taken literally this would imply that conservation of colour would have to be contingent on an exact satisfaction of certain formal S_3-symmetry requirements on the wavefunctions of 3 identical (t, V) preons in a broad quantum-mechanical sense. Such a requirement need not, in principle, be incompatible with the more accepted principle of colour-gauge invariance at the quark-gluon level, if it is remembered that the operation of the gauge principle at "composite" particle levels (quarks and gluons are now composites of preons) cannot be directly compared with the corresponding statement for "elementary" quark interactions with the gauge field of (elementary) gluons. In other words the operation of the usual QCD gauge principle has to be viewed in the larger perspective of composite dynamics not only for the fermions (quarks) but also for the fields (gluons are now tt and/or VV composites). The QED gauge invariance in this model would still be governed by an elementary photon field but with composite electrons. The question of compatibility of S_3-symmetry at the composite level with usual colour gauge invariance was not pursued further by the originators of the preon model reverting (Harari and Seiberg 1981) to the conventional gauge picture at the cost of a three-fold increase (colour triplets) in the number of preon constituents. However, this attractive possibility deserves a closer look.

For a dynamical understanding of the detailed structure of the successive families listed in (1), within the Harari-Shupe model, it is first necessary to identify their main observational features as follows (Mitra 1983; Mitra 1988).

(a) Spin-1/2 for all the members (without exception) with no trace of spin-3/2—a most unusual coincidence for a 3-body composite made up to only 1/2-spin constituents.
(b) Relative stability of successive generations against mutual transitions (e.g., $\mu \not\to e\gamma$) which presumably calls for a rather special kind of quantum number to characterize them.
(c) Steeply rising mass pattern of the successive generations. A possible mechanism to deal with these features is outlined in the next section.

4. A three-way preon interaction model (Mitra 1983, 1988)

By analogy with quark-gluon confinement, it is natural to suppose that preons are confined by "hyper-colour" interactions. Since little is known about them, only an effective description is possible through a suitable ansatz on the overall spin and momentum dependence of the preon interaction kernel, say, within a BSE type of framework. In other words, hypercolour as an attribute must be a "hidden" degree of freedom and manifests only through the effective preonic interactions for which a direct three-preon force was suggested as a possible alternative scenario to the beaten track of a 2-body type interaction. Within the 3-body kernel, a spin-dependence of the form (Mitra 1983)

$$\Sigma^a \equiv \sigma^{(1)}_{\mu\nu}\,\sigma^{(2)}_{\nu\lambda}\,\sigma^{(3)}_{\lambda\mu} \tag{3}$$

was suggested as a possible mechanism to keep out $S = 3/2$ states from the influence of this operator. This is most easily seen from its space-like form $\boldsymbol{\sigma}_1 . \boldsymbol{\sigma}_2 \times \boldsymbol{\sigma}_3$ which gives zero on a spin-3/2 function χ^s, thus making such states "force-free" and hence devoid of physical interest. On the other hand, Σ^a converts the two standard spin-1/2 states χ' and χ'' into each other, so that the complex combinations

$$\sqrt{2}\chi^{\pm} = \chi'' \pm i\chi' \tag{4}$$

of spin-1/2 states are eigenstates of Σ^a:

$$\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \times \boldsymbol{\sigma}_3 (\chi^{\pm}; \chi^s) = \pm 2\sqrt{3}(\chi^{\pm}; 0). \tag{5}$$

How can such an effective Σ^a-structure arise in a field-theoretic model? It was suggested (Mitra 1983) that an intrinsically antisymmetric hypergluon field $F_{\mu\nu} (= -F_{\nu\mu})$ can couple to a preon field ψ in the Pauli form $\bar{\psi}\sigma_{\mu\nu}F_{\mu\nu}\psi$. And if the $F_{\mu\nu}$ field has a self-coupling of the form $F_{\mu\nu}F_{\nu\lambda}F_{\lambda\mu}$, these two couplings together can generate a three-way, Mercedes-Benz type, diagram for a direct 3-preon interaction, as in figure 1a, b.

While (5) accounts quite qualitatively for feature (a) in the absence of $S = 3/2$ states, the Σ^a-dynamics also bears on the other two features (a) and (b), though in a less

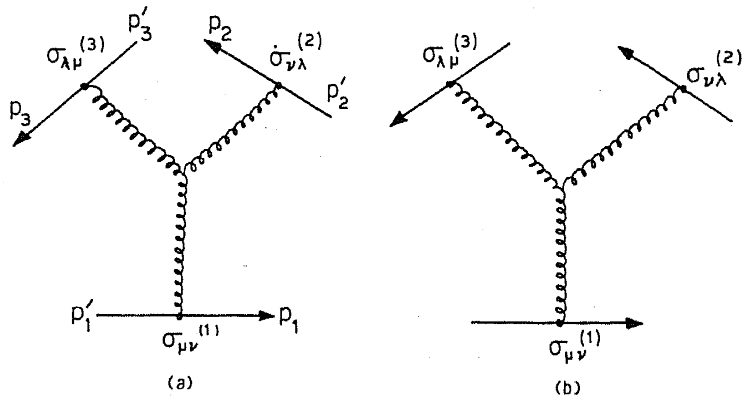

Figure 1. A 3-way hypergluonic mechanism for *ttt* and *Vtt* couplings in the Harari-Shupe model.

obvious way. Qualitatively, a harmonic springforce simultaneously in each of the hypergluon lines produce a strongly confining effect with a very steeply rising "potential". This provides the basis for a steeply rising mass spectrum for the successive generations as listed in item (c). Item (b) is more subtle and depends on a certain form of the 3-preon BSE. It may be shown that under certain general conditions this BSE for almost massless preons may be cast in the form of a six-dimensional Schrödinger-like equations in $(\boldsymbol{\xi}, \boldsymbol{\eta})$ where the roles of the coordinates and momenta are interchanged (Mitra 1983).

$$(\Delta_\xi + \Delta_\eta - a\rho^{1/3} + b)\theta(\boldsymbol{\xi}, \boldsymbol{\eta}) = 0, \tag{6}$$

where $\rho^2 = \xi^2 + \eta^2$ and θ is a spatial function of the appropriate S_3-symmetry associated with the spin-1/2 functions (χ', χ'') to produce the correct representation of S_3 in the combined space of spin and orbital degrees of freedom. It is easily seen that (6) corresponds to an R^6 confinement after due conversion to the co-ordinate representation. As to the item (b), the general structure of (6) offers the following insight into this question. The spatial function $\theta(\boldsymbol{\xi}, \boldsymbol{\eta})$ for orbitally unexcited states $(L = 0)$ can be expressed in terms of 3 independent scalars:

$$\xi^2 + \eta^2 = \rho^2, \quad \xi^2 - \eta^2 = -\gamma\cos\lambda, \quad 2\boldsymbol{\xi}\cdot\boldsymbol{\eta} = \gamma\sin\lambda. \tag{7}$$

Of these, the variables (ρ, γ) are carriers of full [3] symmetry, while λ which is akin to an 'angle' (*not* Euler angles) carries the information on the mixed [2, 1] symmetry content in the spatial wavefunction. This last is expressed in terms of the eigenvalues and eigenfunctions of the operator $-i(\partial/\partial\lambda)$:

$$-i\frac{\partial}{\partial\lambda}\theta_N = \frac{N}{2}\theta_N; \quad \theta_N \sim \gamma^{N/2}\exp\left(\pm\frac{i}{2}N\lambda\right), \tag{8}$$

where N can be one of the three possible forms

$$N = 3n, \quad 3n \pm 1; \quad (n = 0, 1, 2, \ldots). \tag{9}$$

Equation (8) also expresses the associated γ-dependence of θ_N (analogous to centrifugal barrier effects in L-excitation) which suggests a singular behaviour for $N = -1$, but which is certainly "allowed" for a wavefunction in six-dimensional space (see further below).

In this picture, therefore, the 3 classes of N are sought to be identified with the three horizontal generations whose lowest configurations are the 3 sets given in (1) and correspond to $n = 0$, i.e., $N = -1, 0, +1$ respectively. The higher values of n correspond to "vertical" excitations of these 3 horizontal generations. Other kinds of vertical excitations for each horizontal generation are: (i) radially excited states (ρ-dependent) and (ii) orbitally excited states of $L > 0$ (dependent on 3 Euler angles). Such a proliferation of vertical excitations for each horizontal generation is a necessary consequence of any composite model whose eventual success, if any, must depend on the details of dynamics, an essential condition being that the first (vertically) excited state of the first horizontal ($N = -1$) species, viz. e^+, should lie well above the ground state of the third horizontal species ($N = +1$), viz., τ^+. Between these two, the middle species has a ground state $N = 0$, corresponding to μ^+.

Note the qualitative difference in the highly isotropic structure of the μ^+ state ($N = 0$) from those of the e^+ and τ^+ states ($N = \mp 1$) which have mixed symmetry and are akin to each other. This could be a qualitative reason for the lack of radiative transition between $\mu^\pm$ and $e^\pm$. The corresponding selection rules need not be as rigid for radiative transition between $\tau^\pm$ and $e^\pm$, which may nevertheless be suppressed for other reasons (e.g., large energy denominators). For completeness, a more quantitative statement on the steeply rising mass spectrum (item c which, as already noted, is a general consequence of the three-way confining mechanism figure 1) is possible on the basis of the general equation (6) which effectively incorporates this 3-way mechanism. The mass spectrum prediction of (6) for the lepton series was found to be expressible in units of the electron mass as (Mitra 1983)

$$(M/m_e) = (\beta_r/\beta_0)^{21}, \tag{10}$$

where $\beta_r (r = 0, 1, 2, \ldots)$ is a function of N and the rth zero of the Airy function. The steeply rising nature of the kernel is reflected in the huge power dependence of M which, e.g., predicts (Mitra 1983)

$$M_\mu/m_e = 196{\cdot}7 \quad (\text{vs}\ 206{\cdot}7),$$
$$M_\tau/m_\mu = 14{\cdot}3 \quad (\text{vs}\ 16{\cdot}9). \tag{11}$$

Very similar results also hold for the other members of the quark-lepton families (Mitra 1983). The difference between the force mechanisms involved in the leptons vs quark structures is schematically indicated by the comparison of figure 1(a) (with 3 equal string lengths for leptons (e, ν_e) and figure 1(b) (with only 2 equal string lengths) for quarks (u, d).

The foregoing is only an illustration of the nature of the "issues" involved in attempting a dynamical understanding of the central features of the quark-lepton families within a composite description. The H-S model chosen in this context is merely one of many possible models of a certain general kind in which the constituents (preons) are recognized to have an existence independent of the attributes (colour, flavour, generation index) they are supposed to possess, in contrast to another class of models (Terazawa 1980) which the preons are considered to be almost a "paraphrase" for these very attributes. (The dynamical scenario, too, is very different for this class of models.)

5. Colour as S_3-symmetry?

Though the original form of the H-S model was soon abandoned in favour of colour as an intrinsic attribute it is nevertheless interesting to pursue the idea of colour as a manifestation of S_3-symmetry in keeping with the basic spirit of compositeness designed to *minimize* the number of basic building blocks (preons), postponing for the moment the more abstract problem of compatibility of this alternative notion with the gauge principle. To that end we shall continue in the spirit of the above formalism (Σ^a-dynamics) for the spin and momentum degrees of freedom, but make a more careful listing of the remaining degrees of freedom (including isospin) in order to obtain a fully symmetric wavefunction when its hypercolour part h^a is suppressed. Thus let ψ, χ, ξ stand for the momentum, spin and isospin (t, v) SU(2) degrees of freedom respectively with superscripts indicating the type [3], [2, 1] or [1, 1, 1], of the S_3-representation. Thus, in an obvious notation, we have

$$\sqrt{3}\psi^s = \psi_1 + \psi_2 + \psi_3, \quad \sqrt{2}\psi' = \psi_3 - \psi_2, \quad \sqrt{6}\psi'' = \psi_2 + \psi_3 - 2\psi_1,$$

and similarly for the χ- and ξ-functions. This gives 5 distinct S_3-symmetric wavefunction (suppressing hypercolour) as follows:

$$\psi_1 = \psi^s \chi^s \xi^s,$$

$$\psi_2 = \psi^s \frac{1}{\sqrt{2}}(\chi'\xi' + \chi''\xi''),$$

$$\psi_3 = \frac{1}{\sqrt{2}}(\psi'\xi' + \psi''\xi'')\chi^s,$$

$$\psi_4 = \frac{1}{\sqrt{2}}(\psi'\chi' + \psi''\chi'')\xi^s,$$

$$\psi_5 = \tfrac{1}{2}[\psi'(\chi'\xi'' + \chi''\xi') + \psi''(\chi'\xi' - \chi''\xi'')].$$

Now leptonic states ttt and VVV can be simply recovered from above through their proportionality to ξ^s, so that these are merely ψ_1 and ψ_4. Of these, the spin-3/2 state ψ_1 is ruled out by e.g., Σ^a-mechanism, equation (5), leaving only ψ_4 in the fray. Next, for the quarks, the 5 functions are again reduced through the Σ_a-mechanism to only *three* (ψ_2, ψ_4, ψ_5), just the number needed for the colour-designation. Thus the Σ^a-mechanism not only eliminates the spin-3/2 states, but leaves just enough states needed for the colour classification of quarks and leptons in accordance with S_3-symmetry. The rest of the discussion revolves round the dynamical question of ensuring an exact degeneracy of the states $\psi_{2,4,5}$ which are built up essentially of the space triplet (ψ_1, ψ_2, ψ_3). This is mainly a quantum-mechanical problem in which the Laplacian operator with the appropriate number of dimensions plays a central role, be it in coordinate space or with (conjugate) momentum space, largely dictated by the kinematics of the physical system on hand. In this respect, a non-relativistic system (in coordinate space) and an ultra-relativistic system (in momentum space) show considerable similarity, which is most easily recognized through an h.o. picture by interchanging the traditional roles of $\mathbf{x}$ and $\mathbf{p}$ (wherein the force-term looks like a "kinetic energy"). For our (limited) purpose of exhibiting energy degeneracy of the triplet χ_i, it is not necessary to distinguish between the two situations, since all that is needed is the effect of the relevant "Laplacian" $(-K)$ on the states ψ_i for different choices of their $\mathbf{x}$ (or $\mathbf{p}$) dependence. In this respect we can distinguish between *two* types of configurations which can both be covered by the "Mercedes-Benz" diagram, figure 1, through appropriate interpretations of its centre.

(a) With a "geometrical" centre there are effectively *two* internal variables $\boldsymbol{\xi}, \boldsymbol{\eta}$, since $\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 = 0$.
(b) With a very massive centre, $\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 \neq 0$ so that there are now *three* independent variables $\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3$.

For situation (a), the S-wave $(L=0)$ K.E. operator $-\hat{K}_s$ the form

$$-\hat{K}_s = \frac{1}{\xi^2}\frac{\partial}{\partial \xi}\left(\xi^2 \frac{\partial}{\partial \xi}\right) + \frac{1}{\eta^2}\frac{\partial}{\partial \eta}\left(\eta^2 \frac{\partial}{\partial \eta}\right) - \frac{\mathbf{L}_\xi^2}{\xi^2} - \frac{\mathbf{L}_\eta^2}{\eta^2}. \tag{12}$$

This case which was described in detail, using the complex variables z, z^*, (as summarized in §4, Mitra 1983) has a ground-state wavefunction z^{-1} (or z^{*-1}). This much singularity is "allowed" in a 6-D phase space, without violating the meaning of the "kinetic energy" (see below). To see the energy degeneracy of the 3 colour states (ψ_1, ψ_2, ψ_3) more transparently, the operator $-\hat{K}_s$ may be alternatively expressed as

$$-\hat{K}_s = \sum_{i=1}^{3} \frac{1}{p_i^2}\frac{\partial}{\partial p_i}\left(p_i^2 \frac{\partial}{\partial p_i}\right); \quad (\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 = 0), \tag{13}$$

whose eigenfunctions are found to be of the forms:

$$\psi_i = \frac{1}{p_i}\phi(\rho); \quad (\rho^2 = \xi^2 + \eta^2); \quad (i = 1, 2, 3) \tag{14}$$

showing the colour degeneracy explicitly (without changing the physical content of the earlier $1/z$ form of the solution).

Next, the configuration (b) is more easily discussed in terms of the form (13) but *without* the restriction $\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 = 0$, so that the effective phase space is 9-D. It is now possible to show through the transformations (reminiscent of "polar" coordinates)

$$p_1 = \rho \sin\theta \cos\phi, \quad p_2 = \rho \sin\theta \sin\phi, \quad p_3 = \rho \cos\theta$$

that the following types of singular functions ψ_1 are now "admissible" without violating the meaning of the kinetic energy:

$$\psi_1 = \left(\frac{1}{p_1}; \frac{1}{p_2 p_3}; \frac{1}{p_1 p_2 p_3}\right)\phi(\rho) \tag{15}$$

with cyclic permutations for ψ_2, ψ_3. Note that the stronger the singularity the lower is the *energy* (or mass) or the state conserved. However, there is a "rock-bottom" mathematical limit represented by the unique state $\sim (p_1 p_2 p_3)^{-1}$ below which the operator $\hat{K}_s$ loses its meaning.

As a short diversion, we now give the K.E. argument. A singularity $\psi \sim \rho^{-n}$ (n integral) in a D-dimensional space, preserves the meaning of K.E. provided

$D - 1 - 2n \geqslant 0$. This gives $n \leqslant 1$ for $D = 6$; $n \leqslant 3$ for $D = 9$, corresponding to the two situations (a) and (b), depicted by (14) and (15) respectively.

While, the colour triplet structure for quarks is thus realized, the generation hierarchy starts at the lowest (most singular) "allowed" state, and goes successively up the ladder. For configuration (a), the generation index is essentially $N = 3n, 3n + 1$, as already described (Mitra 1983). For configuration (b), on the other hand, the corresponding hierarchy is represented by the sequence (15) in the *reverse* order. Now for (a), it was possible to simulate the steeply rising mass pattern for successive generations, through a special ansatz (M-dependent) for the strength of the 3-way preon interaction, which leads to a structure of the form (10). In (b), the hierarchy of 3 successive singular structures themselves provides a viable mechanism for a steep escalation of the mass scales thus serving as a possible scenario for the generation structure with a lesser degree of reliance on a special ansatz on the strength of the 3-way interaction. An explicit construction is under way.

Some of the ideas presented in §5 arose in the course of a discussion with Prof. D B Lichtenberg and his colleagues at the Indiana University during the summer of 1986.

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