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Upper bounds on the mass of the lightest neutralino

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We derive the general upper bounds on the mass of the lightest neutralino, as a function of the gluino mass, in different supersymmetry breaking models with minimal particle content and the standard model gauge group. This includes models with gravity mediated supersymmetry breaking, as well as models with anomaly mediated supersymmetry breaking. We include the next-to-leading order corrections in our evaluation of these bounds. We then expand the mass matrix in powers of M_Z/μ and find the upper bound on the mass of the lightest neutralino from this expansion. By scanning over all of the parameter space, we find that the bound we have obtained can be saturated. We compare the general upper bound on the lightest neutralino mass to the upper bound that is obtained when the radiative electroweak symmetry breaking scenario is assumed.

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I. INTRODUCTION

It is widely expected that at least some supersymmetric particles will be produced at the CERN Large Hadron Collider (LHC) that is starting operation in a few years time. However, most of these supersymmetric particles will not be detected as such, since they will decay into the particles of the standard model (SM), or to the lightest supersymmetric particle (LSP), which is stable as long as the R parity is conserved. Thus, the experimental study of supersymmetry involves the study of cascade decays of the supersymmetric particles to the LSP and the reconstruction of the subsequent decay chains. The LSP in a large class of supersymmetry breaking models is the lightest neutralino, which has thus been a subject of intense study for a long time [1–6]. A stable lightest neutralino is also an excellent candidate for dark matter [5]. As such it is important to have information on the mass of the lightest neutralino state.

In view of this, the properties of the lightest neutralino and also heavier neutralinos and charginos, which often appear in the cascade decays, are of considerable importance. In the minimal version[1] of the supersymmetric extension of the standard model at least two Higgs doublets H_1 and H_2 with hypercharge (Y) having values -1 and $+1$, respectively, are required. The fermionic partners of these Higgs doublets mix with the fermionic partners of the gauge bosons to produce four neutralino states $\tilde{\chi}_i^0, i=1,2,3,4$, and two chargino states $\tilde{\chi}_i^{\pm}, i=1,2$.

The neutralino mass matrix $\hat{\mathcal{M}}$ depends on the ratio of the vacuum expectation values (VEVs) of the two Higgs doublets denoted by $\tan\beta \equiv v_2/v_1$, where $v_1=\langle H_1^0\rangle$ and $v_2=\langle H_2^0\rangle$ are the vacuum expectation values of the two Higgs doublets with opposite hypercharge, the supersymmetry breaking $U(1)_Y$ and $SU(2)_L$ gaugino masses M_1 and M_2, and the supersymmetry conserving Higgs(ino) mixing parameter μ. The mass matrix is symmetric, but not necessarily real. The mass parameters can have arbitrary complex phases, as can also the Higgs boson VEVs. However, all of these are not actually independent—one can choose the two nontrivial phases to be in M_1 and μ. The electric dipole moments strictly constrain the phases in supersymmetric (SUSY) models. However, these bounds are for products of the phases. Thus, if there are cancellations between phases, a single phase can be larger than the limits for the product [7].

Recently, it has been demonstrated that at the linear collider one can determine the above parameters of the neutralino and chargino sectors from the masses of charginos and three lightest neutralinos, or alternatively from two lightest neutralinos and the cross section $e^+e^- \to \chi_1^0\chi_2^0$ [1,8]. The linear collider is likely to be available several years after the completion of the LHC, and thus all the information that is available now or can be obtained at the LHC will be very valuable.

In this paper we obtain the theoretical upper bound on the mass of the lightest neutralino state in the most commonly studied supersymmetry breaking models. These include the gravity mediated supersymmetry breaking model and the anomaly mediated supersymmetry breaking model, with the minimal particle content.[2] In a general model with an arbitrary particle content, an upper bound for the lightest neutralino mass was calculated in [2]. For specific supersymme-

[1]By minimal version we here mean the model with the minimal particle content and the standard model gauge group.

[2]In the gauge mediated supersymmetry breaking (GMSB) models the lightest neutralino is not the lightest supersymmetric particle. However, in many models it is the next-to-lightest particle. Here we will also comment on the upper bound in the GMSB models.

try breaking scenarios we can give a more accurate bound. In Sec. II we obtain the general upper bound on the mass of the lightest neutralino in the minimal version of the supersymmetric standard model. We then evaluate this upper bound for the two most popular supersymmetry breaking models, namely, the gravity mediated supersymmetry breaking (SUGRA) models and the anomaly mediated supersymmetry (AMSB) breaking models. We include the next-to-leading order corrections in the numerical evaluation of this upper bound.

In Sec. III, we then study the expansion of the neutralino mass matrix in powers of M_Z/μ, which at second order is accurate to 1% for large values of μ. The extremum (maximum) of the lightest neutralino mass gives an upper bound from this expansion. This value is lower than the upper bound obtained directly from the mass matrix. By a numerical scan over real and complex parameter values we confirm that this bound is accurately saturated in the supersymmetric models that we study in this paper. We compare the general upper bound with the largest value of the lightest neutralino mass that is obtained when radiative electroweak symmetry breaking is assumed (and the μ parameter determined from the radiative electroweak symmetry breaking). For this purpose we have used the numerical program SOFTSUSY [9]. In Sec. IV we present our conclusions.

II. THE GENERAL UPPER BOUND ON THE MASS OF THE LIGHTEST NEUTRALINO

We start by recalling the neutralino mass matrix in supersymmetric models in the basis

$$\psi_j^0=(-i\lambda',-i\lambda_3,\psi_{H_1}^1,\psi_{H_2}^2), \quad j=1,2,3,4, \tag{1}$$

which can be written as [10]

$$\hat{\mathcal{M}}=\begin{pmatrix} M_1 & 0 & -M_Z\cos\beta\,\sin\theta_W & M_Z\sin\beta\,\sin\theta_W \\ 0 & M_2 & M_Z\cos\beta\,\cos\theta_W & -M_Z\sin\beta\,\cos\theta_W \\ -M_Z\cos\beta\,\sin\theta_W & M_Z\cos\beta\,\cos\theta_W & 0 & -\mu \\ M_Z\sin\beta\,\sin\theta_W & -M_Z\sin\beta\,\cos\theta_W & -\mu & 0 \end{pmatrix}, \tag{2}$$

where λ' and λ_3 are the two-component gaugino states corresponding to the $U(1)_Y$ and the third component of the $SU(2)_L$ gauge groups, respectively, and $\psi_{H_1}^1,\psi_{H_2}^2$ are the two-component Higgsino states. Furthermore, g' and g are the gauge couplings associated with the $U(1)_Y$ and the $SU(2)_L$ gauge groups, respectively, with $\tan\theta_W=g'/g$, and $M_Z^2=(g^2+g'^2)(v_1^2+v_2^2)/2$. Assuming CP conservation, this mass matrix is real. We shall denote the eigenstates of the neutralino mass matrix by $\chi_1^0,\chi_2^0,\chi_3^0,\chi_4^0$ labeled in order of increasing mass. Since some of the neutralino masses resulting from diagonalization of the mass matrix can be negative, we shall consider the squared mass matrix $\hat{\mathcal{M}}^\dagger\hat{\mathcal{M}}$. An upper bound on the squared mass of the lightest neutralino χ_1^0 can be obtained by using the fact that the smallest eigenvalue of $\hat{\mathcal{M}}^\dagger\hat{\mathcal{M}}$ is smaller than the smallest eigenvalue of its upper left 2×2 submatrix

$$\begin{pmatrix} M_1^2+M_Z^2\sin^2\theta_W & -M_Z^2\sin\theta_W\cos\theta_W \\ -M_Z^2\sin\theta_W\cos\theta_W & M_2^2+M_Z^2\cos^2\theta_W \end{pmatrix}, \tag{3}$$

thereby resulting in the upper bound

$$M_{\chi_1^0}^2\leq\frac{1}{2}[M_1^2+M_2^2+M_Z^2 - \sqrt{(M_1^2-M_2^2)^2+M_Z^4-2(M_1^2-M_2^2)M_Z^2\cos 2\theta_W}]. \tag{4}$$

We emphasize that the upper bound (4) is independent of the supersymmetry conserving parameter μ and also independent of $\tan\beta$, but depends on the supersymmetry breaking gaugino mass parameters M_1 and M_2. Despite this dependence on the unknown supersymmetry breaking parameters, we will show that Eq. (4) leads to a useful bound on $M_{\chi_1^0}$.

A. Gravity mediated supersymmetry breaking

In the gravity mediated minimal supersymmetric standard model, the soft gaugino masses M_i satisfy the renormalization group equations (RGEs) ($|M_3|=m_{\tilde{g}}$, the gluino mass)

$$16\pi^2\frac{dM_i}{dt}=2b_iM_ig_i^2, \quad b_i=\left(\frac{33}{5},1,-3\right), \tag{5}$$

at the leading order. Here $g_1=\frac{5}{3}g'$, $g_2=g$, and g_3 is the $SU(3)_C$ gauge coupling. The RGEs (5) imply that the soft supersymmetry breaking gaugino masses scale like gauge couplings:

$$\frac{M_1(M_Z)}{\alpha_1(M_Z)}=\frac{M_2(M_Z)}{\alpha_2(M_Z)}=\frac{M_3(M_Z)}{\alpha_3(M_Z)}, \tag{6}$$

where $\alpha_i=g_i^2/4\pi, i=1,2,3$.

The relation (6) reduces the three gaugino mass parameters to one, which we take to be the gluino mass $m_{\tilde{g}}$. The other gaugino mass parameters are then determined through

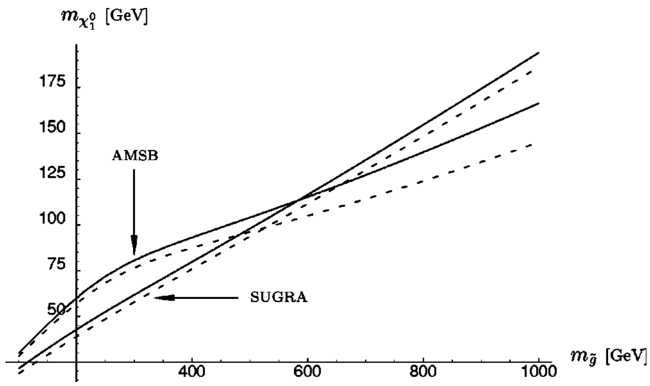

FIG. 1. The upper bound on the mass of the lightest neutralino in the SUGRA and AMSB models. The tree level results are given by dashed lines and the next-to-leading order results by the solid lines.

$$M_1(M_Z)=\frac{5\alpha}{3\alpha_3\cos^2\theta_W}m_{\tilde g}\simeq 0.14 m_{\tilde g}, \tag{7}$$

$$M_2(M_Z)=\frac{\alpha}{\alpha_3\sin^2\theta_W}m_{\tilde g}\simeq 0.28\ \ m_{\tilde g}, \tag{8}$$

where we have used the value of various couplings at the Z^0 mass

$$\alpha^{-1}(M_Z)=127.9,\quad \sin^2\theta_W=0.23,\quad \alpha_3(M_Z)=0.12. \tag{9}$$

Using Eqs. (7) and (8) in Eq. (4), we get the upper bound on the mass of the lightest neutralino.[3] For a gluino mass of 200 GeV, the upper bound (4) for the lightest neutralino mass is about 35 GeV. Similarly, for a gluino mass of 1 TeV, the upper bound (4) becomes 186 GeV.

We have plotted the upper bound (4) on the mass of the lightest neutralino in Fig. 1. The almost straight dashed line corresponds to the SUGRA model at the tree level. From Fig. 1, we observe that $m_{\chi_1^0}<186$ GeV for $m_{\tilde g}<1$ TeV.

We now include next-to-leading order (NLO) corrections coming from α_3 and from the top-quark Yukawa coupling α_t ($\equiv h_t^2/4\pi$) two-loop contributions to the beta functions and logarithmically enhanced weak threshold corrections. In this approximation, one finds [11]

$$M_1^{NLO}=M_1(Q)\left\{1+\frac{\alpha}{8\pi\cos^2\theta_W}\left[-21\ln\frac{Q^2}{M_1^2}+11\ln\frac{m_{\tilde q}^2}{M_1^2}+9\ln\frac{m_{\tilde \ell}^2}{M_1^2}\ln\frac{\mu^2}{M_1^2}+\frac{2\mu}{M_1}\sin 2\beta\frac{m_A^2}{\mu^2-m_A^2}\ln\frac{\mu^2}{m_A^2}\right]+\frac{2\alpha_3}{3\pi}-\frac{13\alpha_t}{66\pi}\right\}, \tag{10}$$

$$M_2^{NLO}=M_2(Q)\left\{1+\frac{\alpha}{8\pi\sin^2\theta_W}\left[-13\ln\frac{Q^2}{M_2^2}+9\ln\frac{m_{\tilde q}^2}{M_2^2}+3\ln\frac{m_{\tilde \ell}^2}{M_2^2}+\ln\frac{\mu^2}{M_2^2}+\frac{2\mu}{M_2}\sin 2\beta\frac{m_A^2}{\mu^2-m_A^2}\ln\frac{\mu^2}{m_A^2}\right]+\frac{6\alpha_3}{\pi}-\frac{3\alpha_t}{2\pi}\right\}, \tag{11}$$

$$M_3^{NLO}=M_3(Q)\left\{1+\frac{3\alpha_3}{4\pi}\left[\ln\frac{Q^2}{M_3^2}+F\left(\frac{m_{\tilde q}^2}{M_3^2}\right)-\frac{14}{9}\right]+\frac{\alpha_t}{3\pi}\right\} \tag{12}$$

$$F(x)=1+2x+2x(2-x)\ln x+2(1-x)^2\ln|1-x|. \tag{13}$$

Here $M_i(Q)$ are the leading order results given by Eq. (6). Notice that the next-to-leading order corrections are of the same form in all models. It is only the leading order $M_i(Q)$ that are different for different models.

In order to calculate the next-to-leading order upper bound on the lightest neutralino mass, we need to know M_1^{NLO} and M_2^{NLO}. As with the leading order result, we express M_1^{NLO} and M_2^{NLO} as a function of M_3^{NLO} (the NLO physical gluino mass), using Eqs. (10)–(13) and substitute it in Eq. (4). We plot the NLO corrected upper bound on the lightest neutralino mass as a function of the gluino mass as a solid curve in Fig. 1. As input we have used here $\tan\beta=10$, $m_t(\text{pole})=174$ GeV, $m_0=300$ GeV, $A_0=1$ TeV, $\mu=-460$ GeV, and $Q=890$ GeV. Since dependence on these model parameters appears only at the loop level, the upper bound is not very sensitive to these parameters. We note that the NLO corrections increase the upper bound from its tree level result by only a few GeV for a wide range of the gluino mass. Indeed, we find that at the NLO the upper bound on the mass of the lightest neutralino is $m_{\chi_1^0}<194$ GeV for $m_{\tilde g}<1$ TeV. In [12] full one-loop corrections to sparticle masses were calculated. The loop corrections to the lightest neutralino mass can be typically 10% for $m_{\chi_1^0}<40$ GeV. However, for $m_{\chi_1^0}=100$ GeV the full one-loop corrections to the lightest neutralino mass are $\lesssim 5\%$. We note that the experimental lower bound [13] on the lightest neutralino mass, valid for any $\tan\beta$ and m_0, is $m_{\chi_1^0}>37$ GeV.

B. The anomaly mediated supersymmetry breaking

The anomaly induced soft terms are always present in a broken supergravity theory, regardless of the specific form of the couplings between the hidden and observable sectors. They are linked to the existence of the superconformal anomaly. Indeed, they explicitly arise when one tries to eliminate from the relevant Lagrangian the supersymmetry breaking auxiliary background field by making a suitable Weyl rescaling of the superfields in the observable sector.

[3]In the GMSB models, one gets the same relations (6) at the messenger scale, since $M_a\propto\alpha_a$. Thus, the upper bound obtained in the SUGRA model can be applied in the GMSB model as well.

The soft terms in the anomaly mediated supersymmetry breaking models are especially interesting because they are invariant under the renormalization group transformations. The phenomenological appeal of the soft terms in AMSB resides precisely in this crucial property. In particular, it implies a large degree of predictivity, since all the soft terms can be computed from the known low-energy SM parameters and a single mass scale $m_{3/2}$. Also, it leads to robust predictions, since the RG invariance guarantees complete insensitivity of the soft terms to the ultraviolet physics. As demonstrated with specific examples in Ref. [14], heavy states do not affect the low-energy parameters, since their effects in the beta functions and threshold corrections exactly compensate each other. This means that the gaugino mass prediction

$$M_\lambda = \frac{\beta_g}{g} m_{3/2} \tag{14}$$

is valid irrespective of the grand unified theory gauge group in which the SM may or may not be embedded. A unique feature of the anomaly mediated supersymmetry is the gaugino mass hierarchy implied by Eq. (14). At the leading order, we thus have

$$M_1(Q) = \frac{11\alpha(Q)}{4\pi\cos^2\theta_W} m_{3/2} \simeq 8.9\times 10^{-3} m_{3/2}, \tag{15}$$

$$M_2(Q) = \frac{\alpha(Q)}{4\pi\sin^2\theta_W} m_{3/2} \simeq 2.7\times 10^{-3} m_{3/2}, \tag{16}$$

$$M_3(Q) = -\frac{3\alpha_3(Q)}{4\pi} m_{3/2} \simeq -2.8\times 10^{-2} m_{3/2} \tag{17}$$

at the scale M_Z. Using Eqs. (15)–(17) in Eq. (4), we obtain the leading order result for the upper bound on the lightest neutralino mass in the minimal AMSB model. We have plotted this upper limit as the upper dashed curve in the Fig. 1. It is interesting to note that there is a kink in this dashed curve around $m_{\tilde{g}} \simeq 210$ GeV. This is due to the competition between the diagonal terms in the 2×2 submatrix (3). The term containing M_1 is smaller, when the gluino mass is small, but with the increasing gluino mass the term with M_2 becomes smaller around 210 GeV. This is because the *W*-ino triplet mass parameter is always smaller than the *B*-ino mass parameter in the AMSB type model, in contrast to the SUGRA or GMSB type models where the *B*-ino mass parameter is smaller than M_2.

In the next-to-leading order corrections to the lightest neutralino mass in AMSB models, the complete sparticle spectrum becomes important. Unfortunately, it turns out that the pure scalar mass squared anomaly contribution for the sleptons is negative [15]. In order to avoid this problem we need to consider other positive soft contributions to the spectrum. This can arise in a number of ways, but most of the solutions will spoil the RG invariance of the soft terms and the consequent ultraviolet insensitivity. Nevertheless, there are various options to cure this problem without reintroducing the flavor problem [15–18].

The necessary cure for the slepton masses may also completely upset the mass relations for the other particles (as in the case of the model of Ref. [16]). However, here we will simply parametrize the new positive contributions to the squared sfermion masses with a common mass parameter m_0^2, assuming that the extra terms do not reintroduce the supersymmetric flavor problem. The low-energy soft supersymmetry breaking parameters for the scalars and the trilinear couplings are then obtained from

$$m_{\tilde{Q}}^2 = -\frac{1}{4}\left(\frac{\partial\gamma}{\partial g}\beta_g + \frac{\partial\gamma}{\partial y}\beta_y\right) m_{3/2}^2 + m_0^2, \tag{18}$$

$$A_y = -\frac{\beta_y}{y} m_{3/2}, \tag{19}$$

respectively. Using Eqs. (10)–(13), we obtain for the anomaly mediated supersymmetry breaking models the next-to-leading order results for the gaugino mass parameters as

$$M_1^{NLO} = 1.06\, M_1(Q), \tag{20}$$

$$M_2^{NLO} = 1.28\, M_2(Q), \tag{21}$$

$$M_3^{NLO} = 0.9\, M_3(Q), \tag{22}$$

where the $M_i(Q), i=1,2,3$ (the leading order result), is given in Eqs. (15)–(17). Here we have used as input $\tan\beta = 10$, $m_t(\text{pole}) = 174$ GeV, $m_{3/2} = 35$ TeV, $m_0 = 600$ GeV, $\mu = -600$ GeV, and $Q = 958$ GeV. The Higgsino corrections to M_1 and M_2 are proportional to $\mu/M_{1,2}$ and can become very important in models with large μ, as discussed in Ref. [14].

In Fig. 1 we have plotted the next-to-leading order upper bound on the mass of the lightest neutralino in anomaly mediated supersymmetry breaking models. The NLO result, obtained using Eqs. (20)–(22), is shown as a solid line. The NLO corrections are significant, of the order of 20%. The larger NLO correction in the AMSB model as compared to the SUGRA model is due to the fact that the α_3 corrections for the M_2 mass parameter are larger than for the M_1 parameter. For $m_{\tilde{g}} < 1$ TeV, the upper bound on the lightest gluino mass is 167 GeV, which is considerably less than in the SUGRA case.

III. LIGHTEST NEUTRALINO MASS BOUND FROM THE STRUCTURE OF THE MASS MATRIX

We can also obtain information on the neutralino masses by studying the expansion of the neutralino mass matrix in terms of the parameter M_Z/μ. This expansion can be obtained most conveniently by using the basis $(\tilde{\gamma}, \tilde{Z}^0, \tilde{H}_a^0, \tilde{H}_b^0)$. In this basis the mass matrix is given by

$$\hat{\mathcal{M}}=\begin{pmatrix} M_1c_W^2+M_2s_W^2 & (M_2-M_1)c_Ws_W & 0 & 0\\ (M_2-M_1)c_Ws_W & M_1s_W^2+M_2c_W^2 & M_Z & 0\\ 0 & M_Z & \mu s_{2\beta} & -\mu c_{2\beta}\\ 0 & 0 & -\mu c_{2\beta} & \mu s_{2\beta}\end{pmatrix}. \tag{23}$$

Here we have used the abbreviations $s_{2\beta}=\sin 2\beta$, $c_{2\beta}=\cos 2\beta$, $s_W^2=\sin^2\theta_W$ and $c_W^2=\cos^2\theta_W$. Let us start by supposing, as before, that all the mass parameters are real. The mass matrix is then real and symmetric.[4] The neutralino mass matrix $\hat{\mathcal{M}}$ can be cast into a form whereby the gaugino and Higgsino mass parameters are only at the diagonal positions by a similarity transformation with a matrix $\mathcal{A}$,

$$\mathcal{M}=\mathcal{A}^T\hat{\mathcal{M}}\mathcal{A}, \tag{24}$$

where

$$\mathcal{A}=\begin{pmatrix} c_W & s_W & 0 & 0\\ -s_W & c_W & 0 & 0\\ 0 & 0 & \cos(\pi/4-\beta) & \sin(\pi/4-\beta)\\ 0 & 0 & -\sin(\pi/4-\beta) & \cos(\pi/4-\beta)\end{pmatrix}. \tag{25}$$

The mass matrix can then be diagonalized by using perturbation theory. In the SUGRA model, for the mass of the lightest neutralino we get, up to terms of $\mathcal{O}(M_Z/\mu)^2$,

$$m_{\chi_1^0}=M_1-\frac{M_Z^2s_W^2}{\mu}\sin 2\beta-\left(M_Z^2s_W^2M_1+\frac{M_Z^4s_W^2c_W^2}{M_2-M_1}\sin^2 2\beta\right)\frac{1}{\mu^2}. \tag{26}$$

Similarly, for the second lightest neutralino χ_2^0 we obtain

$$m_{\chi_2^0}=M_2-\frac{M_Z^2c_W^2}{\mu}\sin 2\beta-\left(M_Z^2c_W^2M_2+\frac{M_Z^4s_W^2c_W^2}{M_1-M_2}\sin^2 2\beta\right)\frac{1}{\mu^2}. \tag{27}$$

If instead we were considering the AMSB model, Eq. (27) would represent the mass of the lightest neutralino χ_1^0, and Eq. (26) would give the formula for the mass of the second lightest neutralino. The dependence of the lightest neutralino mass on the specific SUSY breaking scenario is due to the fact that the ordering of the gaugino mass parameters is model dependent (for AMSB models $M_2<M_1$, whereas for SUGRA models $M_1<M_2$).

In Fig. 2 we plot the mass of the lightest neutralino obtained from the expansion of the mass matrix in (M_Z/μ) together with the exact results obtained from the numerical evaluation of the lightest neutralino mass from the mass matrix. The results for the other neutralinos are very similar in accuracy. The second order tree level expansion is generally better than 1% for $|\mu|>450$ GeV ($m_{\tilde{g}}<1600$ GeV and $\tan\beta=10$), with the exception of small gluino mass, when $m_{\chi_1^0}$ is very small, thus giving a larger relative error. For our purpose it is sufficient to calculate the expansion up to second order in (M_Z/μ).

Due to the simple functional form of Eqs. (26) and (27) the extremal values of the masses with respect to μ are easily calculated. These functions have only one extremum (maximum), which is given (within the limits of validity of the expansions) for the values of μ

$$\mu=-2\left(\frac{M}{\sin 2\beta}+\frac{M_Z^2\sin 2\beta}{\tilde{M}-M}\,\frac{s_W^2c_W^2}{t}\right), \tag{28}$$

and the maximum mass is then given by the upper bound

$$m_{\chi_1^0}\leq M+\frac{1}{4}\,\frac{M_Z^2t^2\sin^2 2\beta}{tM+M_Z^2s_W^2c_W^2\sin^2 2\beta/(\tilde{M}-M)}, \tag{29}$$

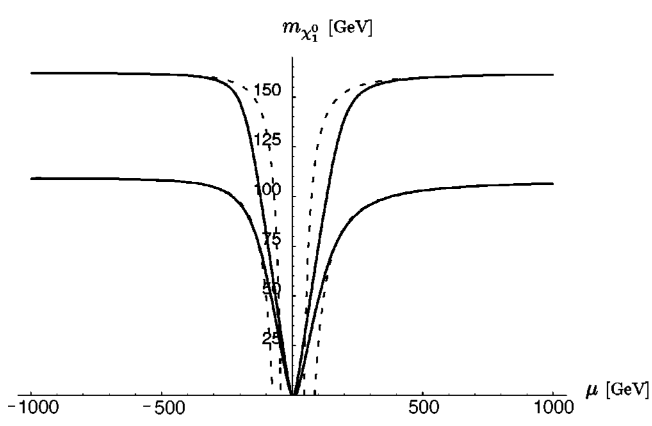

FIG. 2. Mass of the lightest neutralino χ_1^0 as a function of μ. Solid lines correspond to the numerical results from the mass matrix and dashed lines to the second order expansion from Eq. (26). The upper two lines represent masses in SUGRA models and the lower two in AMSB models. Here $\tan\beta=10$ and $m_{\tilde{g}}=900$ GeV.

[4]We note that in the specific models that we have been considering, SUGRA and AMSB, the phases of M_1 and M_2 are the same [see Eqs. (6) and (14)]. So if M_2 is real, then M_1 is also real. On the other hand, the μ parameter is in general complex. Complex parameters would imply a non-Hermitian mass matrix, giving generally complex eigenvalues. Such a situation can be handled by considering the eigenvalues of the matrix $\hat{\mathcal{M}}^\dagger\hat{\mathcal{M}}$.

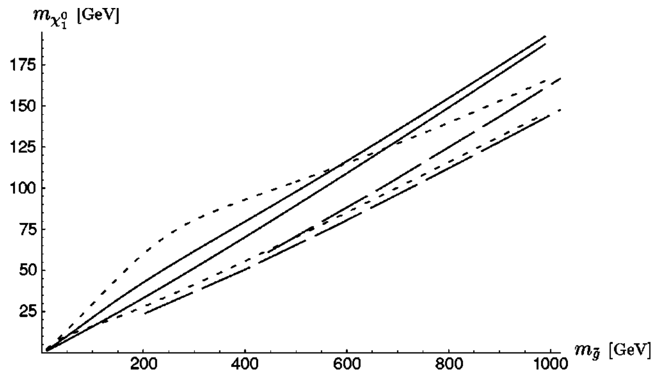

FIG. 3. The upper limit on $m_{\chi_1^0}$ as a function of $m_{\tilde{g}}$. Solid lines represent masses in SUGRA model and short dashed lines in AMSB model. The lower curve in each case corresponds to the upper limit obtained from the expansion in (M_Z/μ), and the upper curve corresponds to the general upper limit obtained from the mass matrix. The long dashed curves correspond to the case when the radiative electroweak symmetry breaking scenario is implemented.

where $M=\min(M_1,M_2),\tilde{M}=\max(M_1,M_2),t=s_W^2$ if $M=M_1$ (SUGRA), and $t=c_W^2$ if $M=M_2$ (AMSB). In Fig. 3 we plot the upper limit on the lightest neutralino mass obtained from Eq. (29) as a function of $m_{\tilde{g}}$, for both the SUGRA and AMSB models. We also plot the upper limit obtained from Eq. (4) in the same figure. The SUGRA results are represented as solid lines and the AMSB results as short dashed lines. The lower curve of each set corresponds to the upper bound obtained from the expansion in $(M_Z/\mu)^2$, and the upper curve corresponds to the upper bound obtained from Eq. (4). These results for SUGRA and AMSB in Fig. 3 are NLO results. We have plotted the results for the value of $\tan\beta=10$. In order to verify the accuracy of these results, we made an extensive scan over the parameter space, using both real and complex values of the μ parameter. The highest mass obtained from this corresponds extremely well to the upper limit obtained from the expansion in (M_Z/μ).

We have also made a scan over the parameter space using the SOFTSUSY program [9], in which the phenomenon of radiative electroweak symmetry breaking (REWSB) is implemented. Thus, the μ value in this program is given by the REWSB condition. The resulting spectrum includes one- and dominant two-loop corrections. The maximum mass obtained for the lightest neutralino is plotted in Fig. 3 as a function of $m_{\tilde{g}}$ with long-dashed lines. The upper long-dashed line corresponds to the SUGRA model and the lower one to the AMSB model. One can see that with radiative electroweak symmetry breaking, the $m_{\chi_1^0}$ in the AMSB model is close to the maximum mass obtained from the expansion in (M_Z/μ), while in the SUGRA model with REWSB the $m_{\chi_1^0}$ obtained is clearly lower than the maximum value from the expansion, indicating that μ_{REWSB} for the SUGRA model is not close to the value obtained from Eq. (28).

As in the case for χ_1^0 we can search for the upper bound on the mass of the second lightest neutralino χ_2^0. For light gluinos [lighter than $\mathcal{O}(60)$ GeV] the extremum in the mass for χ_2^0 is a minimum due to a sign change in the expansion, but for experimentally allowed masses the extremum for $m_{\chi_2^0}$ is a maximum. In Fig. 4 we have plotted the upper limits for both the lightest and second lightest neutralino obtained from the expansion in (M_Z/μ). The solid lines correspond to the SUGRA model, while the dashed lines correspond to the AMSB model. For $m_{\tilde{g}}<1$ TeV, the NLO upper bounds for the second lightest neutralino are 440 GeV for the SUGRA case and 419 GeV for the AMSB case.

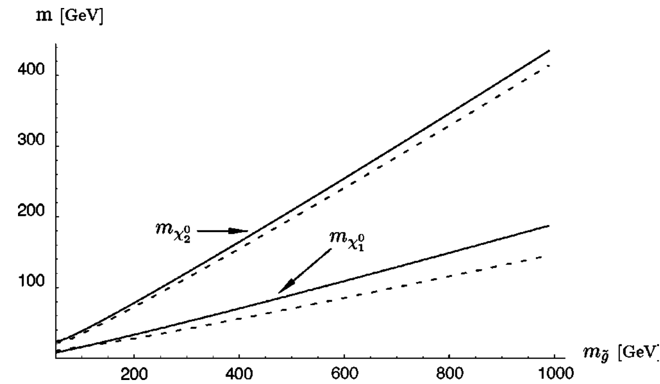

FIG. 4. The upper limits of $m_{\chi_1^0}$ and $m_{\chi_2^0}$ as a function of $m_{\tilde{g}}$. Solid lines represent masses in SUGRA and dashed in AMSB.

IV. CONCLUSIONS

In this paper we have studied the neutralino mass matrix for the minimal supersymmetric model with the aim of obtaining an upper bound on the mass of lightest neutralino. Knowledge of the mass of the lightest neutralino is of crucial importance for the supersymmetric phenomenology. We have shown that a general limit, valid for arbitrary values of parameters, can be obtained from the mass matrix. Even though such a bound depends on the supersymmetry breaking parameters M_1 and M_2, it nevertheless leads to a significant numerical bound on the lightest neutralino mass in the SUGRA and AMSB models. We have also obtained an upper bound on the lightest neutralino mass by expanding the neutralino mass matrix in terms of the parameter M_Z/μ. We see that the upper limit from this expansion is considerably lower for the AMSB model than for the SUGRA model for similar $m_{\tilde{g}}$. From this analysis we conclude that the upper bound on the mass of lightest neutralino is $m_{\chi_1^0}<200$ GeV for $m_{\tilde{g}}<1$ TeV.

In Fig. 3 we have three separate regions for the upper bound on the mass of the lightest neutralino: one which is valid in both SUGRA and AMSB cases, one which is valid in only one of the models, and a third one which is not available for any of the models that we have studied.

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[1] S.Y. Choi, J. Kalinowski, G. Moortgat-Pick, and P.M. Zerwas, Eur. Phys. J. C **22**, 563 (2001); **23**, 769 (2002); J. Kalinowski and G. Moortgat-Pick, hep-ph/0202083.
[2] P.N. Pandita, Phys. Rev. D **53**, 566 (1996); hep-ph/9701411.
[3] A. Bartl, H. Fraas, and W. Majerotto, Nucl. Phys. **B278**, 1 (1986); J.F. Gunion and H.E. Haber, Phys. Rev. D **37**, 2515 (1988); A. Bartl, H. Fraas, W. Majerotto, and N. Oshimo, *ibid.* **40**, 1594 (1989).
[4] S. Ambrosanio and B. Mele, Phys. Rev. D **55**, 1399 (1997).
[5] D. Majumdar, J. Phys. G **28**, 2747 (2002); Y. Gyun, T. Nihei, L. Roszkowski, and R. de Austri, J. High Energy Phys. **12**, 034 (2002).
[6] V. Berezinsky, A. Bottino, J.R. Ellis, N. Fornengo, G. Mignola, and S. Scopel, Astropart. Phys. **5**, 1 (1996).
[7] S. Pokorski, J. Rosiek, and C.A. Savoy, Nucl. Phys. **B570**, 81 (2000).
[8] S.Y. Choi, A. Djouadi, H.S. Song, and P.M. Zerwas, Eur. Phys. J. C **8**, 669 (1999); S.Y. Choi *et al.*, *ibid.* **14**, 535 (2000).
[9] B.C. Allanach, Comput. Phys. Commun. **143**, 305 (2002).
[10] H.P. Nilles, Phys. Rep. **110**, 1 (1984); Pran Nath, R. Arnowitt, and A.H. Chamseddine, in *Supersymmetry, Supergravity and Perturbative QCD*, edited by P. Roy and V. Singh (Springer, Heidelberg, 1984).
[11] T. Gherghetta, G.F. Giudice, and J.D. Wells, Nucl. Phys. **B559**, 27 (1999).
[12] D.M. Pierce, J.A. Bagger, K.T. Matchev, and R.J. Zhang, Nucl. Phys. **B491**, 3 (1997).
[13] Particle Data Group, K. Hagiwara *et al.*, Phys. Rev. D **66**, 010001 (2002).
[14] G.F. Giudice, M.A. Luty, H. Murayama, and R. Rattazzi, J. High Energy Phys. **12**, 027 (1998).
[15] L. Randall and R. Sundrum, Nucl. Phys. **B557**, 79 (1999).
[16] A. Pomarol and R. Rattazzi, J. High Energy Phys. **05**, 013 (1999).
[17] E. Katz, Y. Shadmi, and Y. Shirman, J. High Energy Phys. **08**, 015 (1999); R. Rattazzi, A. Strumia, and J.D. Wells, Nucl. Phys. **B576**, 3 (2000); Z. Chacko, M. Luty, E. Pontón, Y. Shadmi, and Y. Shirman, Phys. Rev. D **64**, 055009 (2001); Z. Chacko, M.A. Luty, I. Maksymsk, and E. Pontón, J. High Energy Phys. **04**, 001 (2000); I. Jack and D.R.T. Jones, Nucl. Phys. **B482**, 167 (2000); M. Carena, K. Huitu, and T. Kobayashi, *ibid.* **B592**, 164 (2001).
[18] For a recent study, see K. Huitu, J. Laamanen, and P.N. Pandita, Phys. Rev. D **65**, 115003 (2002); hep-ph/0303067.