A NOTE ON THE EVEN, ODD AND HALF-OVERTONES IN PIEZO-ELECTRIC CRYSTAL PLATES

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1. Introduction

It is well known that the fundamental and its odd harmonics are the most easily excited frequencies when piezo-electric plates are set into oscillation. Bergmann[1] has recorded that the even harmonics may also be excited by using very strong electric fields when there is want of symmetry in the plate or in the electric field. Working with quartz, he has actually measured a number of these in some cases.[2] Much work does not, however, appear to have been done in the direction of investigating the conditions favourable to the appearance of even harmonics. Even more interesting is the recent observation of Parthasarathy[3] and collaborators[4] that the crystal plates sometimes exhibit a half fundamental and its odd overtones. The authors of this note have, for some time past, been engaged in the study of elastic constants of a number of crystals by piezo-electric methods and had accordingly occasion to study the behaviour of these plates using quartz, tourmaline and sphalerite under various conditions. Some interesting results obtained are reported here.

In this investigation, the resonance frequencies are always detected by the appearance of the Debye-Sears patterns and the measurements which are made with the help of a standard wavemeter are checked against the fringe widths of the diffraction patterns. The crystal plate rests on a horizontal annular brass ring with the lower surface touching the liquid in a glass cell and the upper electrode is a brass plate smaller in diameter than the inner diameter of the lower ring, so that there is always an asymmetric field if the crystal is unsilvered.

2. Even and Odd Harmonics

It is well known that even overtones cannot be excited in the ideal case of an infinite piezo-electric plate with the electric field constant over the entire area of the plate. In practice, on the other hand, they have been produced under certain conditions of excitation in the longitudinal vibrations of X-cut quartz plates as has been done by Bergmann,[2] Parthasarathy and others.[4] The usual practice in observing the Debye-Sears[5] and

Lucas-Biquard[6] effect in liquids employing a X-cut quartz plate is to silver the two faces of the crystal uniformly with the result that only the odd overtones of the longitudinal vibration come out and with great intensity. We have seen that such a procedure in the case of Z-cut tourmaline plates also gives only the odd overtones of the longitudinal vibration. If, now, the silver is removed from small areas on both sides of the crystal by nitric acid or by rubbing lightly on an abrasive surface such that silvered portions on one side, lie roughly opposite to the unsilvered portions on the other, the even overtones come out in considerable strength, while the intensity of the odd overtones falls slightly. In a 2 mm. tourmaline plate thus treated, we have found the even overtones upto the 6th as fairly strong and could detect the 8th overtone also. A 1 mm. plate with a fundamental at nearly 3·75 Mc. giving only the first and the third overtones strongly when fully silvered, gave two orders in the 2nd overtone after the above treatment. The most interesting feature of this plate is that it is excited by this method, not only in the longitudinal mode but also in a shear mode, as detected again by the appearance of a number of diffraction orders at the overtone values of a different fundamental frequency 2·29 Mc. The second overtone of this shear mode is also present in considerable intensity. The elastic constant C_{44} obtained from this mode as $6 \cdot 56 \times 10^{11}$ dynes per sq. cm. assuming it to be a thickness shear mode, checks very well with the constants C_{44} (slightly coupled) obtained by us in a separate experiment from both X-cut and Y-cut plates of the same specimen as 6·45 and 6·51 respectively. Even when unsilvered, this 1 mm. Z-cut plate produced diffraction in both the modes and at even overtones also, but with a general loss in intensity of all the overtones of both modes. The shear mode in both the cases is always lower in intensity than the longitudinal mode.

Similarly a 2 mm. X-cut quartz plate with a fundamental at 1·44 Mc. gave the longitudinal even overtones upto the 6th, only when it was badly silvered in the above manner. Again when it was badly silvered or not silvered, a shear mode with a fundamental of 1·29 Mc. in all its overtones including even, upto the 7th overtone appeared. The constant obtained from this mode is very nearly the same as that calculated from the elastic constants of quartz determined by Atanasoff and Hart[7] and corrected by Lawson.[8] *It will be noted here that the 5th overtone of the shear mode coincides with 9/2 times that of the longitudinal mode and could easily be mistaken for the latter one.* Another 0·198 cm. quartz plate, not strictly X-cut but slightly inclined to the X-axis, has been studied *when fully silvered* and found to oscillate only in the odd harmonics of two modes, one longitudinal and the other a shear mode similar to that of the above quartz plate, with fundamentals very near those obtained for the X-cut plate. *When it is badly or not silvered, the even overtones also of both the modes come out. In all these cases an even overtone of any mode is always of lower intensity than the odd overtones between which it lies. Any overtone of the shear mode is always of a lower intensity than the corresponding overtone of the longitudinal mode.* Thus in each of the modes, the intensities of the odd overtones form a series in which the higher ones decrease progressively and those of the even overtones form another similar series of lower intensity.

Octahedral and (221) plates of a sphalerite crystal, from Galena, Kansas, the chemical analysis of which is given elsewhere,[9] gave the even overtones in considerable strength, although they were *very well silvered.* It may here be noted that these plates get heated up soon when they are in resonant vibration and this phenomenon in ZnS is most probably due to the presence of impurities in the crystal which cause consequential inhomogeneities.

3. Half-Overtones

Recently Parthasarathy, Pande and Pancholy[4] made some observations on the piezo-electric vibrations of quartz plates. They have not explicitly stated what the state of silver is on their crystal. They found certain frequencies other than the odd and even overtones of the fundamental and explained them as due to odd overtones of half the fundamental. This explanation appears to be very strained and does not fit with the intensities of these frequencies as may be seen from the following remarks. Firstly, the fundamental frequencey for a X-cut crystal of thickness 0·552 cm. comes to about 520 Kc., either from Hund's expression or from Voigt's or Atanasoff and Harts' value of C_{11} while Parthasarathy and collaborators observed it to be 580 Kc. This considerable discrepancy can only be attributed to an error in orientation of the crystal. Consequently, if their method of excitation is such that the even overtones of the longitudinal mode are present, in view of the above stated observations of ours, it is easily seen that a shear mode is also likely to have been excited not only in its odd but also in its even overtones. Assuming the fundamental of the shear mode to be 515 Kc., the reported 5/2, 7/2, 9/2, 11/2 and 13/2 harmonics of Parthasarathy and collaborators are surprisingly close to the 3rd, 4th, 5th, 6th and 7th overtones respectively of the shear mode. Even more convincing is the run of intensities of these frequencies. According to Parthasarathy and collaborators, the so-called 5/2, 9/2 and 13/2 which are the 3rd, 5th and 7th overtones in our explanation are the more prominent ones and this is just as we should expect. The even overtones, as has been stated above, are always of a lower intensity than the odd overtones in both the modes, and the shear mode is again of a lower intensity than the longitudinal mode, and the intensities recorded by Parthasarathy and collaborators find an immediate and natural explanation on this basis if the idea of a shear fundamental at 515 Kc. is accepted. The frequency of their 3/2 harmonic is not exactly equal to the 2nd overtone of the shear mode though its intensity fits in with such an explanation. This is presumably on account of the coupling and size effect present in such thick crystals near the fundamental frequency.

We have made a special effort to directly observe the half-fundamental. In no case have we been able to observe it. In the 1 mm. tourmaline and a 1 mm. X-cut quartz, when they are well silvered, we found that the resonance curve at the fundamental extended from beyond the frequency of the 2nd overtone to a frequency lower than half the fundamental of the longitudinal mode. At about half the fundamental, on account of the harmonic content of the oscillator circuit, the crystal was found to oscillate at its fundamental but this phenomenon is quite distinct and cannot be interpreted as the crystal showing up a half-fundamental. When the crystal is sufficiently badly silvered or not silvered such that the resonance curve is much sharper, the above phenomenon disappears and no diffraction is observed at half the fundamental.

The above interpretation of the vibrations of the piezo-electric plates naturally leads to the question as to the manner in which the shear mode is communicated to the liquid for producing a diffraction pattern. The physical explanation of this seems to lie in the fact that except along unique directions, the waves propagated in anisotropic media are not strictly longitudinal and transverse but only quasi-longitudinal and quasi-transverse while the three displacement vectors are always mutually perpendicular.

On the other hand, if the interpretation of the results leading to the recognition of the crystal plate oscillating in its half-fundamental is accepted, we are faced with serious difficulties of a theoretical nature. Mention may be made of one such consequence relating to free and unclamped boundary of a crystal remaining a node when the whole crystal is thrown into oscillation.

4. Summary

Conditions under which even and odd overtones of piezo-electric crystal plates may be excited have been investigated using tourmaline, quartz and sphalerite plates. Whenever there is a non-uniformity of the electric field or of the crystal plate it has been found that the even overtones are strongly excited. These features and also errors in orientation result in the appearance of characteristic shear modes along with the appropriate

even and odd harmonics. In all these cases, an even overtone is always of lower intensity than the odd overtones between which it lies. Any overtone of the shear mode is always of a lower intensity than the corresponding overtone of the longitudinal mode. All attempts to observe a half-fundamental have proved unsuccessful. Certain frequencies recorded by Parthasarathy and collaborators and attributed to odd overtones of a half-fundamental are explained as the usual (even and odd) overtones of a shear mode.

References

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