DAILY RELATIVE SUNSPOT NUMBER AND 10·7 CM. SOLAR FLUX

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Abstract

Auto and cross-spectra of relative sunspot number and slowly varying component of solar radiation at 10·7 cm. have been computed for a 52-month period beginning September 1, 1958. Significant features of the spectra are relatively high variance at periods corresponding to one, three and four solar rotations. Statistically significant spectral peaks have been observed at three other frequencies and are ascribed to amplitude modulation of the 27-day component. The cross-spectral analysis indicates that during the period under investigation the solar 10·7 cm. flux leads the sunspot number for periods in excess of about 27·7 days; for shorter periods the flux lags behind the sunspot number. The coherence between the two time series, after an initial decrease from unity at zero frequency, assumes a maximum value of 0·985 at 27·7 days. The phase and coherence indicate that long-lived radio emission regions and spots appeared to co-rotate during 1958-62 with a period of 27·7 days.

I. Introduction

The frequency-domain analysis of time series is a relatively new technique in the study of periodicities. The methods for carrying out these analyses have been outlined by Blackman and Tukey[1] and have been applied in the past several years to a variety of astrogeophysical time series. The spectra of the indices of solar activity, computed by Ward and Shapiro,[2] show a large variance near zero frequency and pronounced peaks at frequencies of 1/27 cycles/day and at its harmonics.

The spectra of the indices of solar activity, however, differ from one solar cycle to another. They also vary during different periods of the same solar cycle. In the present communication auto and cross-spectra of relative sunspot number and 10·7 cm. flux, two of the most valuable primary indices of activity, have been computed from daily values of these indices for a 52-month period beginning with September 1, 1958. It is well known that these two indices are well correlated when smoothed values, such as monthly means, are used to computed the correlation coefficients. However, the correlation is found to vary between wide ranges when the coefficients are computed for each month from daily values. The period of 52 months considered here was marked by a high degree of day-to-day similarity of the two indices, the correlation coefficients for 48 of the 52 months being 0·70 or higher. This period, a part of the declining phase of solar cycle 19 was, therefore, considered very suitable for an investigation of periodicities in the time series of daily values of the two indices. The phase and coherence have also been obtained by cross-spectral analysis to examine these near a frequency of 1/27 cycles-day, corresponding to period of one solar rotation.

II. ANALYTICAL PROCEDURE

The computations follow the method of Tukey as outlined by Munk *et al.*[8] The time series X_i and Y_i composed of 1583 consecutive daily values of relative sunspot number and flux respectively were centralised about the means of the respective series. Auto- and cross-correlations were computed according to

$$A(k) = \frac{1}{N-k}\sum_{i=k+1}^{N} X_{i-k}\, X_i - \frac{1}{(N-k)^2}\sum_{i=k+1}^{N} X_{i-k}\sum_{i=k+1}^{N} X_i$$

$$B(k) = \frac{1}{N-k}\sum_{i=k+1}^{N} Y_{i-k}\, Y_i - \frac{1}{(N-k)^2}\sum_{i=k+1}^{N} Y_{i-k}\sum_{i=k+1}^{N} Y_i$$

$$C(k) = \frac{1}{N-k}\sum_{i=k+1}^{N} X_{i-k}\, Y_i - \frac{1}{(N-k)^2}\sum_{i=k+1}^{N} X_{i-k}\sum_{i=k+1}^{N} Y_i$$

$$D(k) = \frac{1}{N-k}\sum_{i=k+1}^{N} Y_{i-k}\, X_i - \frac{1}{(N-k)^2}\sum_{i=k+1}^{N} Y_{i-k}\sum_{i=k+1}^{N} X_i$$

where $A(k)$ and $B(k)$ are auto-correlations of X_i and Y_i respectively and $C(k)$ and $D(k)$ are the cross-correlations, k is the lag and successively assumes values of $0, 1, 2, \cdots m$ with a maximum lag $m = 300$. N represents the number of data points in the series and is 1583.

The even and odd parts of the cross-correlation, E (k) and F (k), were computed from

$$\mathrm{E}(k) = \frac{\mathrm{D}(k) + \mathrm{C}(k)}{2}$$

$$\mathrm{F}(k) = \frac{\mathrm{D}(k) - \mathrm{C}(k)}{2}$$

The cosine transforms of auto- and cross-correlations were computed next to obtain auto and cross-spectra; the raw spectral estimates were smoothed with a Hanning window.

$$\mathrm{X}(h) = \frac{\delta_h}{m}\left[\sum_{k=1}^{m-1} 2\epsilon(k)\cos\frac{hk\pi}{m}\,\mathrm{A}(k) + \mathrm{A}(0)\right]$$

$$\mathrm{Y}(h) = \frac{\delta_h}{m}\left[\sum_{k=1}^{m-1} 2\epsilon(k)\cos\frac{hk\pi}{m}\,\mathrm{B}(k) + \mathrm{B}(0)\right]$$

$$\mathrm{Z}(h) = \frac{\delta_h}{m}\left[\sum_{k=1}^{m-1} 2\epsilon(k)\cos\frac{hk\pi}{m}\,\mathrm{E}(k) + \mathrm{E}(0)\right]$$

$$\mathrm{W}(h) = \frac{\delta_h}{m}\left[\sum_{k=1}^{m-1} 2\epsilon(k)\sin\frac{hk\pi}{m}\,\mathrm{F}(k) + \mathrm{F}(0)\right].$$

where X (h) and Y (h) are the auto-spectra of X_i and Y_i respectively, Z (h) and W (h) are the co- and quadrature-spectra, h is the frequency,

$$2\epsilon(k) = 1 + \cos\frac{\pi k}{m}$$

and

$$\delta_h = \tfrac{1}{2} \quad \text{for} \quad h = 0 \quad \text{or} \quad m$$
$$\delta_h = 1 \quad \text{otherwise.}$$

Finally, the coherence R (h) and phase Φ (h) were computed from

$$\mathrm{R}(h) = \frac{\mathrm{W}^2(h) + \mathrm{Z}^2(h)}{\mathrm{X}(h)\,\mathrm{Y}(h)}$$

$$\Phi(h) = \mathrm{ARCTAN}\,\frac{\mathrm{W}(h)}{Z(h)}.$$

The computations were carried out on the CDC 3600 160 A computer at the Tata Institute of Fundamental Research.

III. AUTO-SPECTRA OF RELATIVE SUNSPOT NUMBER AND 10·7 CM. FLUX

In any whole-disc index of solar activity a 13½-day smoothing is inherent because long-lived centres of activity remain on the solar disc facing the earth for 13½ days. The higher frequencies in the spectra are therefore considerably damped and the smoothing results in a sharp decrease in power of frequencies corresponding to periods shorter than 13½ days. This is obvious in Fig. 1 where the first 80 of the 301 spectral estimates of sunspot number (solid line) and 10·7 cm. flux (broken line) are shown.

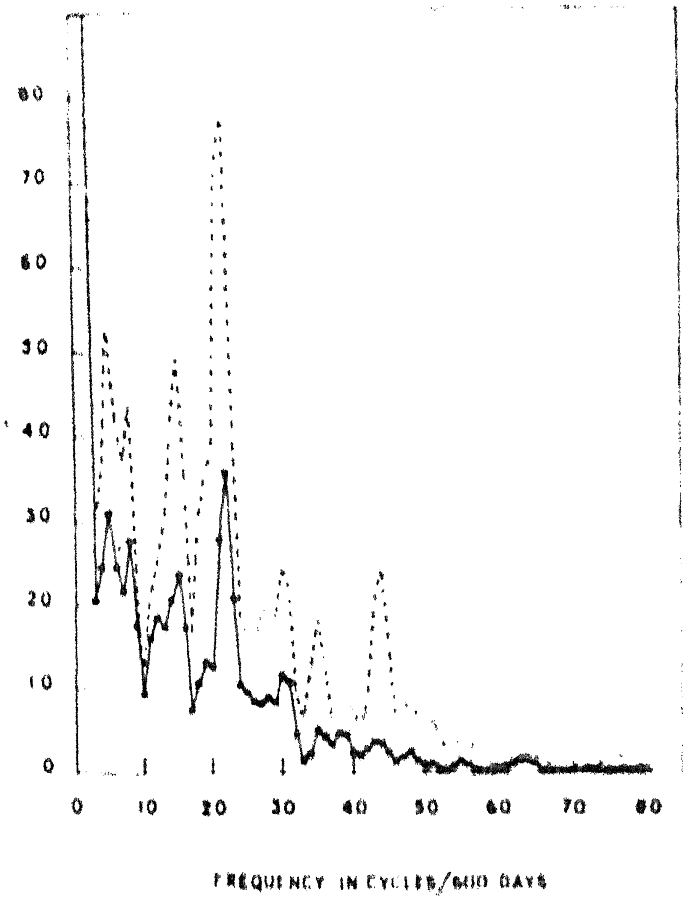

FIG. 1. Auto-spectra of relative sunspot number, $X(h)$ (broken line) and of 10·7 cm. flux, $Y(h)$ (solid line).

The spectra are essentially 'red', the noise continuum decreasing rapidly with increasing frequency. In addition to a large variance near $h = 0$ several prominent peaks appear in the spectra. In order to test whether these are significant or occur due to sampling fluctuations, statistical tests were applied. Tukey[4] has shown that the spectral estimates are distributed according to Chi-square/f, where f is the number of degrees of freedom defined by $(2N - m/2)/m$. For 1583 data points and 300 estimates, $f \approx 10$. The 5% limit of Chi-square for 10 degrees of freedom is about 18·3 and Chi-square/f = 1·83. The probability is, therefore, 5% that a sample may show an estimate of 1·83 times the power at neighbouring frequencies or larger. As a result of the test the significant peaks are listed in Table I.

TABLE I

No.	Frequency h (cycles/600 days)	Corresponding period (days)
1	5	108
2	8	81
3	15	40
4	21–22	27
5	30	20
6	35	17
7	44	13·6

While the resolution at lower frequencies is not adequate for precise determination of periods of the first few peaks, the peaks at 108 and 81 days, corresponding to four and three solar rotations respectively appear to arise from long-lived spots and radio emission regions. The largest peak listed at 4 in Table I is spread over frequencies of 21 to 22 corresponding to a period of 27 days and arises from spots and radio regions which last at least one solar rotation. The peak listed at No. 6 corresponds to a harmonic of the frequency 1/27 cycles/day and has been noticed by Ward and Shapiro.[9] An expected peak at 54 days corresponding to 2 solar rotations is surprisingly absent in the spectrum of sunspot number and is not significant in the spectrum of 10·7 radiation. Instead, three subsidiary peaks corresponding to periods of about 40, 20 and 17 days appear in both the spectra. A discussion of these maxima follows in Section V.

IV. COHERENCE AND PHASE OBSERVATIONS

Coherence is a measure of the degree of relationship between two time series as a function of frequency, disregarding phase difference. It can vary from zero to unity. With a high degree of correlation between the two indices during the period considered a large coherence should be observed. Meaningful values of coherence are however limited to periods of 13½ days and larger due to inherent smoothing of the whole-disc indices. The coherence spectra show that between $h=0$ and $h=44$, the coherence fluctuated between unity and 0·82. In the immediate vicinity of period of 27 days, the coherence and phase are shown in Fig. 2. It is noticed that the flux at 10·7 cm. leads the sunspot number between frequencies (in C/600 days) of 19 and 21, corresponding to periods of 31·6 and 28·6 days. Between frequencies of 22 and 24 corresponding to 27·3 and 25 days, the flux lags behind the sunspot number. The two indices are in phase at a frequency estimated at 21·68 cycles per 600 days which corresponds to a solar rotation period of 27·7 days. The coherence also assumes a maximum value (at a non-zero frequency) of 0·985. These features suggest that the long-lived radio emission regions and sunspots appear to co-rotate with a period very nearly equal to 27·7 days.

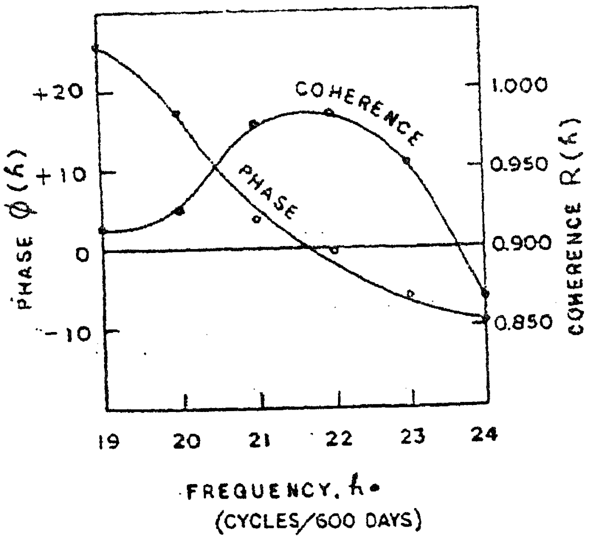

FIG. 2. Coherence and phase between frequencies 19 and 24 cycles/600 days.

V. RESULTS

The spectral features indicate that the 52-month period during the declining phase of solar cycle 19 was marked by the appearance on the solar disc of long-lived active regions. While the largest number of active regions persisted for 27 days, the variance spectra indicate that some regions persisted over periods corresponding to three and four solar rotations. A surprising feature of the variance spectra is the absence of a peak corresponding to 54 days, a period of two solar rotations.

Three subsidiary peaks corresponding to approximate periods of 40, 20 and 17 days also appear in the spectra of both the indices and cannot be accounted for by solar rotation. In a recent communication Coleman and Smith[5] have interpreted two subsidiary peaks near 27 days in a high resolution spectra of geomagnetic activity indices C_i and K_p as due to an amplitude modulation of the 27-day component by a modulating function with a period of one year. Similar considerations applied to the present analysis indicate that the modulation of the 27-day component by functions with periods of 54, 81 and 108 days could give rise to several side bands. The frequencies and periods of these side bands together with the periods from the observed spectra of sunspot number and 10·7 cm. radiation are given in Table II.

TABLE II

Side band	Frequency c.p.d.	Period days	Period (days) from computed spectra
$f_{27}+f_{108}$	0·04630	21·6	20
$f_{27}-f_{108}$	0·02778	36·0	..
$f_{27}+f_{54}$	0·05556	18·0	17
$f_{27}-f_{54}$	0·01852	54·0	54*
$f_{27}+f_{81}$	0·04939	20·2	20
$f_{27}-f_{81}$	0·02469	40·0	40

* A small peak in auto-spectra of 10·7 cm. radiation.

Except for the lower side band $f_{27}-f_{108}$ a close agreement between the periods due to side bands and periods of the three spectral peaks suggests that amplitude modulation of the 27-day component by a signal of period 81 and 108 days caused by longer lived solar activity accounts for the three spectral peaks at 40, 20 and 17 days. A 93-day periodicity in sunspot number for 57–58 and in cm. wave flux has recently been observed by Altschuler and Sastry.[6] If an amplitude modulation of 27-day component by a modulating function with a period of 93 days is considered the side band frequencies are 0·04779 and 0·02629, the corresponding periods being 20·9 days and

38·04 days. This suggests that two of the three subsidiary peaks could also be accounted for by an amplitude modulation by a sinusoidal function of a period of about 93 days.

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