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Gamma ray flashes by plasma effects in the middle atmosphere

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In this paper a novel mechanism is identified for the generation of gamma ray flashes observed on the Compton Gamma Ray Observatory satellite. During typical cloud to ground lightning flashes, the electromagnetic pulse can create a self-focused whistler wave channel or duct to guide $10-10^2/\mathrm{cm}^{-3}$ of $\sim$1 MeV electrons (formed by static stratified electric field in clouds at 20 km), to a height of about 30 km where these electrons can create the gamma ray flash by bremsstrahlung. This scenario combines the various observational features of lightning-generated electromagnetic pulses and low altitude energetic electrons to provide a viable nonlinear transport mechanism of energetic electrons to the desired altitude of 30 km for conversion into gamma ray flashes. © *2001 American Institute of Physics.* [DOI: 10.1063/1.1407821]

I. INTRODUCTION

The observation of atmospheric gamma ray flashes[1] associated with thunderstorms and lightning appears mysterious for several reasons. The Compton Gamma Ray Observatory (GRO) observed these gamma ray flashes. In all likelihood these flashes are due to bremsstrahlung of a population of about $10^{16}-10^{17}$, $\sim$1 MeV electrons in the height region above 30–35 km. Whereas it is easy to see how these electrons could be produced in the thunderstorm regions (heights $\leqslant$ 20 km) by runaway discharge phenomena,[2,3] it is not obvious how these electrons could come up to the required heights in spite of energy loss and diffusive spreading due to scattering by atmospheric neutrals. Similarly, the generation and sustenance of runaway electrons in the lower ionosphere[4] (heights $\sim$ 70 km) by the much weaker fields ($\leqslant$500 V/m) has been called into question because of the strong magnetization of electrons in this region.[5]

In this paper we describe a new plasma phenomenon, which could be important at middle atmospheric altitudes (heights between 20 and 50 km) and could possibly sustain runaway discharges in these regions. There are three key ingredients which form the basis of physical effects described in the following, viz. (a) a runaway population of electrons produced by static stratified electric fields creates a magnetized plasma species ($\nu_{en} < \omega_{ce}$) at altitudes as low as 20 km; (b) trapping of the runaway population of $1-10^2/\mathrm{cm}^{-3}$ at these heights is enough to promote the propagation of the electromagnetic pulse (EMP) associated with thunderstorms and lightning (from lower altitudes $\sim$5–10 km) as a whistler mode in this region; (c) whistler waves can exhibit an ionization driven, self-focusing instability which self-consistently maintains the runaway population and channels the whistler energy along field-aligned filaments all the way to the required heights $\geqslant$30–35 km. These key aspects are schematically shown in Fig. 1, where a lightning stroke initially generates a whistler EMP. This whistler pulse propagates upward and traps a runaway electron beam caused by the static field at around 20 km. Nonlinear mechanisms, which sustain the whistler discharge, alluded to previously, transport the energetic electrons to the desired height where this runaway population produces gamma ray flashes. Before we go on to describe the various physical processes involved in this complex scenario, we would like to emphasize that the effects described here present a new channel for coupling a fraction of the thunderstorm/lightning energy in the lower atmosphere (globally about 5×10^{11} W on average) into the middle atmosphere.

In Sec. II, we present the basics of runaway electrons in the atmosphere. Section III is devoted to the two possible mechanisms of self-focusing of whistler waves. In Sec. IV we apply the ideas of runaway electrons (Sec. II) and the whistler wave self-focusing to the issue of creating whistler-mediated runaway discharges in the atmosphere and the subsequent generation of gamma ray flashes. Finally a brief conclusion is given in Sec. V.

II. BASICS OF RUNAWAY ELECTRONS IN THE ATMOSPHERE

We first briefly describe the runaway beam generation due to static electric fields from a thundercloud. It relies on the avalanche of relativistic electrons triggered by cosmic ray secondaries. The electric field has to be higher than the critical field[6]

$$E_c = \frac{4\pi N_n Z e^3}{mc^2} \ln \Lambda \simeq 2.2 P(\mathrm{atm}) \frac{\mathrm{kV}}{\mathrm{cm}}, \tag{1a}$$

where P(atm) is the atmospheric pressure. The ionization length λ (related to the ionization collision frequency $\nu_i \simeq c/\lambda$) is given by[6]

$$\lambda = c\beta\tau_i, \tag{1b}$$

where τ_i is the characteristic ionization time, $\beta = v/c$ is the relative velocity of high energy electrons, which is related to the applied electric field by[2]

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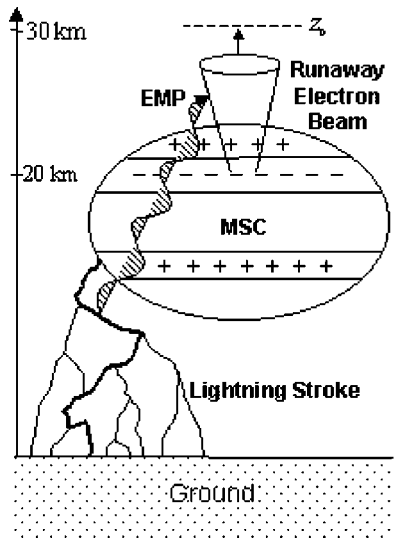

FIG. 1. Schematics of the proposed model for gamma ray flashes in the atmosphere.

$$\beta=[E/E_c(1+2\ln\beta/\ln\Lambda)]^{1/2}. \tag{1c}$$

For $E\simeq 3E_c$, we find from the implicit equation (1c) that $\beta=0.544$ for $\ln\Lambda=11$ as suggested in Ref. 6. Furthermore, the ionization time, or as it is often called, the avalanche time, has been recently studied in Refs. 7 and 8. Here we adopt the value $\tau_i=62$ ns computed for $E=3E_c$ for 5 km altitude.[8] We then obtain the scale length $\lambda(\mathrm{m})=5.5/P(\mathrm{atm})$ for an exponential atmosphere, while at height of 20 km this becomes

$$\lambda(\mathrm{m})=75\times\exp\{(z(\mathrm{km})-20)/6.4\}. \tag{2}$$

Starting from a single cosmic ray secondary, a runaway avalanche at 20 km height can generate a total of 10^{16}–10^{17} (1 MeV) electrons in a length of order $L\simeq\lambda\ln N_{\mathrm{tot}}\simeq 2.7$–$2.9$ km. This will be discussed at length in Sec. IV. The spreading of the beam of unmagnetized electrons in this region produces a cone with a maximum radius $r_{\max}\simeq 270$–300 m. For a total number of runaway electron $N_{\mathrm{tot}}=10^{16}$–10^{17},

$$n\simeq\frac{1}{2}\frac{N_{\mathrm{tot}}}{\pi r_{\max}^2\lambda}\simeq 10^3-10^4\ \ \mathrm{cm}^{-3}. \tag{3}$$

However, this number density drops off very rapidly as the electrons leave the region of the thundercloud due to beam stopping and beam spreading. The stopping length of 1.4 MeV electrons is given by $l(\mathrm{m})=5.6/P(\mathrm{atm})$ and so the electron number density propagating above the clouds changes with distance Δz as

$$n_0(\Delta z)=N_{\mathrm{tot}}\exp(-\Delta z/l)/l\pi r_b^2. \tag{4}$$

The beam radius r_b expands due to diffusive spreading and is given by[9]

$$r_b^2=r_{b0}^2+\theta^2\Delta z^2+0.025\left(\frac{(1+0.22\ln\overline{\gamma})}{\beta^4\overline{\gamma^2}}\right)(\Delta z)^3P(\mathrm{atm}),$$

where r_b is the initial radius, θ is the angular spread, $\beta=v/c$, and $\overline{\gamma}$ is the relativistic factor. Equation (4) predicts that the runaway density drops to $1-10^2$ per cm^{-3} in a distance of the order of a few hundred meters. Thus, unless additional physical effects sustain the runaways, they cannot reach heights of the order of 30–50 km to produce the gamma ray flashes, as observed by GRO.

III. WHISTLER SELF-FOCUSING MECHANISM

We have argued previously that those runaway electrons with density $\sim 1-10^2\ \mathrm{cm}^{-3}$ may survive up to a height of about 20 km. We also observe from Eq. (2) that at 20 km heights, $\nu_i\simeq c/\lambda$ becomes of order ω_{ce} (in the earth's field 0.3 G). Thus at higher altitudes the runaway behaves as a magnetized plasma species ($\nu_i<\omega_{ce}$). We now demonstrate that the above-mentioned number density is adequate to permit the propagation of electromagnetic pulses associated with thunderstorm activity ($f\leqslant 10^4$ Hz) as whistler modes. The dispersion relation for whistlers is given by[10]

$$\frac{c^2k^2}{\omega^2}=1+\frac{\omega_p^2}{\omega\omega_{ce}\cos\theta}, \tag{5}$$

where ω_p and ω_{ce} are the electron plasma and cyclotron frequencies, respectively. The plasma term on the right-hand side contributes significantly to the dispersion relation when

$$n_0>10^{-2}f\cos\theta. \tag{6}$$

Inequality (6) shows that an electron density of $10^2\ \mathrm{cm}^{-3}$ is enough to influence propagation of all frequencies up to 10^4 Hz. If we add a population of low energy electrons with number density n_c, they will form a highly collisional species and contribute a term $i\omega_{pc}^2/\omega\nu_{\mathrm{cold}}$ to the right-hand side of Eq. (5), ω_{pc} and ν_{cold} being the plasma and collision frequency of cold electrons, respectively. Thus cold electrons will lead to an intense absorption of the whistler wave with

$$\frac{\mathrm{Im}(k)}{\mathrm{Re}(k)}\approx\frac{\omega_{ce}\cos\theta}{\nu_{\mathrm{cold}}}\frac{n_c}{n_0}.$$

Here $\mathrm{Re}(k)$ and $\mathrm{Im}(k)$ are real and imaginary parts of the wave vector. This puts a limit on the number of cold electrons, which can be tolerated for the propagation of the whistler. In this paper we neglect the whistler absorption, which is justified by the following considerations. The ratio of density of the cold to runaway electrons can be estimated using the electron distribution function presented by Fig. 5 in Ref. 7. This distribution covers a wide energy spectrum of electrons from thermal electrons of a few eV up to runaways with energy in excess of 1 MeV. The ratio $n_c/n_0<100$ at $E/E_c=2$ and drops at higher electric field. Furthermore, the electron collision frequency $\nu_{\mathrm{cold}}(\mathrm{s}^{-1})\approx 5\times 10^{-8}N_n(\mathrm{cm}^{-3})$ according to Refs. 11 and 12, where N_n is the air density. Thus at the altitudes under 30 km $\nu_{\mathrm{cold}}\geqslant 2\times 10^{10}\ \mathrm{s}^{-1}$. Finally taking into account that the electron gyro frequency $\omega_{ce}\approx 10^7\ \mathrm{s}^{-1}$ we obtain that $\omega_{ce}n_c/\nu_{\mathrm{cold}}n_0\leqslant 0.05$ at $z\leqslant 30$ km.

A typical EMP associated with thunderstorms is several ms in length and will propagate as a wave packet of whistlers. The parallel phase and group velocities of the wave are given by

$$\frac{v_p}{c}=\frac{\omega}{k_{||}c}\simeq\frac{\omega_c}{\omega_p}\left(\frac{\omega}{\omega_c\cos\theta}\right)^{1/2},$$
$$\frac{v_g}{c}=\frac{v_p}{c}(1+\cos^2\theta). \quad (7)$$

It is important to have the parallel group and phase velocities of the whistler close to c (readily possible for oblique waves) so that the bunch of runaways stays with the wave packet and has resonant wave particle interaction with the parallel electric field of the oblique whistler mode. This will ensure that the runaway electrons are sustained at $\sim$1 MeV energy, in spite of the neutral collisions which try to slow them down. An alternative view is applicable when the resonant wave particle interaction is absent and the whistler waves interact collisionally with the background neutrals via a stochastic runaway breakdown mechanism. This mechanism can also sustain the runaway population and is discussed in detail in Refs. 12 and 13.

We now make an estimate of the amount of energy coupled into the whistler mode by a simple mode transformation process. Assuming that the energy is injected as a vacuum mode from lower heights (where $\nu>\omega_c$), we calculate the transmission coefficient into the region with $\nu<\omega_c$ where whistlers propagate as undamped modes. For simplicity, we consider propagation in the vertical direction (z), which is also assumed to be the direction of the inhomogeneity and that of the B field; furthermore we ignore the slow dependence of the plasma frequency on z and retain only the rapid z dependence of the collision frequency ν on z. The basic wave equation is

$$\frac{d^2\varepsilon}{dz^2}+\frac{\omega^2}{c^2}\left[1+\frac{\omega_p^2}{\omega(\omega_c-i\nu)}\right]\varepsilon=0, \quad (8)$$

where we take

$$\nu=\nu_0-\nu' z.$$

The solution to Eq. (8) can be expressed in terms of Whitaker functions as

$$\varepsilon=AW_{\mu,1/2}(\xi)+BW_{\mu,1/2}(-\xi), \quad (9)$$

where $\mu=\omega_p^2/2c\nu'$, $\xi=-2i(\omega/c)[z-(\nu_0+i\omega_c)/\nu']$ and A and B are arbitrary constants to be determined by boundary conditions. We use the upward propagating condition at $z\simeq\nu_0/\nu'$ where $\nu\ll\omega_c$ (viz. at high altitudes where undamped whistlers propagate) and use asymptotic forms at $|\xi|\gg|\mu|$ and $|\mu|\gg|\xi|$ to get an expression for the transmission coefficient

$$T\simeq\exp\left[-2^{3/2}\left(\frac{\omega_p^2\omega\omega_c}{c^2\nu'^2}\right)^{1/2}\right]. \quad (10)$$

Physically, the transmission coefficient is less than unity because of partial reflections from the gradients of the collision frequency ν and is essentially determined by the Wentzell–Kramers–Brillouin result: $\exp(-2\,\mathrm{Im}\int k_z dz)$ where $k_z^2\simeq(\omega^2/c^2)(\omega_p^2/\omega\omega_c)(1-i\nu' z/\omega_c)^{-1/2}$. We note that significant transmission into whistler waves will result when $(\omega\omega_c/\nu^2)(\omega_p L_v/c)^2$ is not too large compared to unity, where $L_v=\nu/\nu'$ is the beam scale length. This is readily satisfied for typical parameters like $\omega_c/\nu\simeq5$, $(\omega_p L_v/c)\simeq5$ and $\omega/\omega_c\simeq10^{-3}$.

We now go on to discuss the nonlinear aspects of whistler wave propagation which can lead to self-focusing and filamentation by runaway ionization and breakdown effects. Physically speaking the dielectric constant for whistler wave propagation $\varepsilon\simeq\omega_p^2/\omega\omega_c$ shows that the phase velocity $c/\sqrt{\varepsilon}$ goes up with decreasing density. Thus if a plane wave front of whistlers with varying intensity produces more ionization in the central strong field regions, it will produce a curvature in the wave front which focuses the whistler beam and gives positive feedback. This directly leads to a self-focusing instability, which favors the formation of channels and filaments with maintenance of high electron density in the central regions where the field intensity is maximum. Such effects have already been observed in laboratory plasmas with modest electron energies.[14]

Ignoring the unity on the right-hand side of the dispersion relation, Eq. (5), we may write the general wave equation for propagation of whistlers as

$$\left[\frac{\partial^2}{\partial t^2}+\omega_c^2\left(\frac{c^2}{\omega_p^2}\right)^2\frac{\partial^2}{\partial z^2}\left(\frac{\partial^2}{\partial z^2}+\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\right]b_1=0, \quad (11)$$

where b_1 is a component of the background earth's magnetic field of the wave and z is oriented along the direction of the magnetic field. We consider a steady-state problem in a frame moving with the parallel group velocity of the whistler wave. The basis whistler wave packet is assumed to propagate in the x–z plane and filaments in the y-direction. We may then write

$$b_1=b(y,z,t)\exp[-i(\omega t-k_{z0}z-k_{x0}x)+\text{c.c.}], \quad (12)$$

where the y,z,t dependence of b describes the modulation due to filamentation effects, $\partial/\partial z\ll k_{z0}$ so that $\partial^2/\partial z^2\simeq-k_{z0}^2+2ik_{z0}(\partial/\partial z)$. and similarly $\partial^2/\partial t^2\simeq-\omega^2-2i\omega(\partial/\partial t)$. We also write $\omega_p^2\simeq\omega_{pc}^2(1+\delta n/n_0)$ where δn refers to change in electron density induced by ionization effects due to the modulation of the whistler wave. Any changes in the density produced by the infinite plane wave before modulations are included in n_0. Using the zeroth-order dispersion relation to eliminate some terms and normalizing the y and z variables to new coordinates we get

$$i\frac{\partial b}{\partial Z}+\frac{\partial^2 b}{\partial Y^2}+\frac{\delta n}{n_0}b=0, \quad (13)$$

where $Y=\sqrt{2}y((k_{z0}^2+k_{x0}^2)^{1/2}$, $Z=k_{z0}(z-v_{gz}t)(k_{20}^2+k_{x0}^2)/(2k_{z0}^2+k_{x0}^2)$ and v_{gz} is the parallel group velocity given by $v_{gz}=k_{z0}\omega_c/\omega_p^2$.

Our next task is to express the relationship between the modulated density δn and the whistler wave amplitude b. We shall consider two extreme limits. In the first one, the whistler and the runaway population undergo resonant wave particle interactions such that the whistler keeps runaways accelerated and the runaways produce avalanche ionization. In the latter extreme, the whistler waves interact collisionally with the background neutrals via stochastic runaway breakdown mechanism. In either case we may write a model equation for the runaway density

$$\frac{\partial n_0}{\partial t}+c\frac{\partial n_0}{\partial z}=c\left(\frac{1}{\lambda}-\frac{1}{l}\right)n_0-an_0^2, \tag{14}$$

where the ionization rate is c/λ discussed after Eq. (1), the loss rate due to stopping is determined by l defined before Eq. (4), and we have introduced an additional density dependent loss rate a as a model for all other loss mechanisms. Equation (14) shows that in a frame moving with the runaways (which for $v_{gz}\sim c$ is the same as the frame of the whistler wave packet), the population reaches a steady state

$$n_0\simeq\left(\frac{1}{\lambda}-\frac{1}{l}\right)\frac{c}{a}. \tag{15}$$

Assuming, for concreteness, that $1/\lambda$ given by Eq. (1b), has a simple E_0^2 dependence, we may write

$$\frac{\delta n}{n_0}=\alpha[|b|^2-|b_0|^2], \tag{16}$$

where $\alpha\equiv(\omega^2/c^2k_0^2)(c/a)[d(1/\lambda)/dE_0^2]$ and b_0 is the amplitude of the unmodulated whistler wave. Later, for convenience, we shall absorb the coefficient α into the normalization of $|b|$ and $|b_0|$. We may now substitute Eq. (16) into (13) and finally get the model equation for nonlinear magnetic field perturbations:

$$i\frac{\partial b}{\partial Z}+\frac{\partial^2 b}{\partial Y^2}+[|b|^2-|b_0|^2]b=0. \tag{17}$$

Equation (17) is the nonlinear Schrödinger equation, which is known to display self-focusing instabilities, and trapped filament solutions. To study the filamentation instability, we write $b=b_0+u+iv$, separate the real and imaginary parts, and solve the resulting coupled set of equations in (u,v) by taking perturbation of the form $\exp\sim(iqY+\Gamma Z)$. The final dispersion relation takes the form

$$\Gamma^2=q^2(2b_0^2-q^2). \tag{18}$$

Equation (18) shows that all perturbations with $q<\sqrt{2}b_0$ are unstable. The growth parameter Γ maximizes at $q_m=b_0$ and has the maximum value $\Gamma_{\max}\simeq b_0^2$. These results show that an infinite plane whistler breaks up into slabs of thickness of order q_m^{-1} (or b_0^{-1}) in the y direction. The final nonlinear state of this instability may be obtained from the nonlinear equation

$$\frac{\partial^2 b}{\partial Y^2}-\gamma b+|b|^2 b=0, \tag{19}$$

where we assume $b\sim\exp(i\gamma z)$ corresponding to a small wave number shift due to nonlinear effects. Looking for nonlinear solutions which vanish at $\pm\infty$ we get the envelope solution

$$b=\gamma\ \mathrm{sech}^2(\sqrt{\gamma}Y/2)\exp(i\phi+iZ)+\text{c.c.}, \tag{20}$$

where ϕ is a constant phase factor.

Going back to unnormalized variables

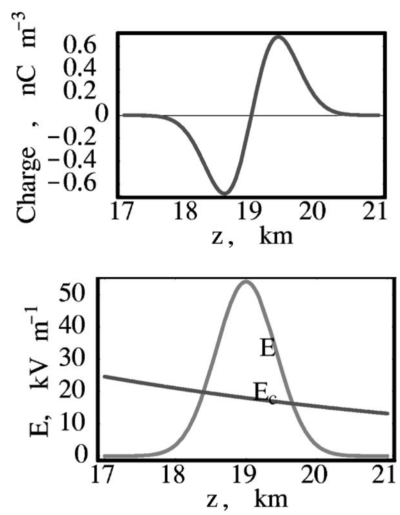

FIG. 2. Model of the charge density inside thundercloud (top panel), and the electric field distribution caused by this charge (bottom panel).

$$b=\frac{1}{\sqrt{\alpha}}\gamma\ \mathrm{sech}^2\left[\frac{\sqrt{\gamma}}{2}\sqrt{2}k_0 y\right]\cos[\omega t-k_{x0}x-k_{z0}z$$
$$-k_{z0}\gamma(z-v_{gz}t)/(1+\cos^2\theta)]. \tag{21}$$

The nonlinear slab solution shows that the plane whistler will break up because of ionization effects into filaments of transverse scale size $(\sqrt{\gamma/2}k_0)^{-1}$ where $\gamma=b\sqrt{\alpha}=b[(c/a)d(1/\lambda)/dE_0^2(\omega_0/ck_0)]$ is the normalized whistler wave amplitude. The maximum growth parameter for conversion into these filaments is $\Gamma\sim(b\sqrt{\alpha})^2$ giving a growth length in unnormalized variables $\sim[b\sqrt{\alpha}k_{z0}/(1+\cos^2\theta)]^{-1}$. These estimates indicate that when $\gamma\sim 1$, we get growth lengths of order λ_{z0} and perpendicular filament scale sizes of order λ_0.

IV. WHISTLER MEDIATED RUNAWAY ELECTRON DISCHARGE IN THE ARMOSPHERE

We now put the various pieces discussed previously on runaway electrons and self-focused whistlers together and apply it to the problem of gamma ray flashes. In the discussion of the runaway electron in Sec. II we had stated that a number density of 10^{16}–10^{17} electrons could be created at 20 km. This is addressed here. We recall that in our model we considered a runaway breakdown, which occurs at the top of a mesoscale convective system (MCS), which develops horizontal charge stratification. Due to the charge separation a strong vertical electric field E of a few kV/m is formed.[15] As soon as the amplitude $E(z)$ exceeds the critical field $E_c(z)$ the runaway breakdown starts, triggered by a flux of cosmic ray secondary electrons. Only if the electric field E is negative does the beam of runaway electrons move up until $E(z)>E_c(z)$, where, at the altitude of around 20 km, the electrons become magnetized.

We first discuss the issue of the strength of the electric field due to the charge separation. For this we consider two layers with charges of opposite sign, as shown in Fig. 2(a). Both charged layers have a Gaussian distribution $\rho=\pm\rho_{\text{peak}}\exp\{-(z-z_{1,2})^2/2\Delta^2\}$, where ρ_{peak} is the peak charge density; the layers are centered at the heights z_1 and z_2, and have the same half-width Δz. The static electric field caused by the charge separation can be found from the Poisson equation

$$\rho(z)=\varepsilon_0 \frac{\partial E}{\partial z}, \tag{22}$$

where ε_0 is the permittivity of free space. The electric field caused by the charge separation is shown in Fig. 2(b) along with the critical field E_c for runaway breakdown. It is apparent from Fig. 2(b) that a moderate charge density $\rho_{\text{peak}}=0.4-0.6\ \text{nC/m}^3$ can produce a significant flux of runaway electrons. We remind the reader that a peak charge density of a few nC/m^3 is typically observed in a thundercloud at 4–5 km altitude,[16] while at the top of the MCS cloud the charge density is much less than its peak value. Furthermore as shown in the discussion following Eq. (4) the electric field, which is three times the critical field, generates a runaway electron density of $10^1-10^2\ \text{cm}^{-3}$.

To reiterate the scenario, the magnetized electron population then channels the EMP into whistler waves propagating along the field lines; the coupling efficiency is significant when $(\omega\omega_c/\nu^2)(\omega_p L_v/c)^2 \leqslant 1$, a condition which is readily satisfied [see Eq. (10)]. It is this self-focused filament of whistler modes which keeps the runaways accelerated against the atmospheric slowing down process. For $E/E_c < 1$, we can readily have $\gamma \sim 1$ and obtain a growth length of filamentation $\sim k_0^{-1}$ ~few km [see the discussion after Eq. (20)]. The characteristic transverse size of the filament is $\sim(\sqrt{\gamma/2k_0}) \sim 1$ km, and the filament's cross section $S \sim 1\ \text{km}^2$.

Note that the duration and energy of the EMP controls the duration of the upward moving runaway beam along with its total energy. This in turn determines the number of runaway electrons delivered to the altitudes in excess of 30 km, where γ-rays due to bremsstrahlung can escape into space. A pulse width of a few ms is typical for EMP from lightning, which is consistent with the pulse width of γ-ray flashes observed by GRO.[1]

Taking the pulse width of the EMP from lightning stroke as $\Delta t \simeq 1-2$ ms, one can estimate the total number of ~ 1 MeV runaway electrons having velocity $v \sim c$, and moving through the filament, with a cross section S,

$$N_{\text{tot}}=n_0\, c\, S\, \Delta t. \tag{23}$$

In fact, for the runaway density $n_0 \sim 10-10^2\ \text{cm}^{-3}$, and for $S \sim 1\ \text{km}^2$ as discussed previously, the total number of runaway electrons could reach $N_{\text{tot}} \simeq 10^{16}-10^{17}$. The latter amount is consistent with the GRO observations.[1,17]

Another consistency check for the validity of the model is estimating the amount of energy from the EMP required to sustain the runaway beam against atmospheric slowing down. We note that the stopping length $l \simeq (5.6/P(\text{atm}))m \approx l_{20}\exp(z(\text{km})/6.4)$ [discussion before Eq. (4)] where $l_{20} \approx 300$ m is the stopping length at the heights of 20 km. The amount of energy required for sustaining the beam against stopping between the heights of 20 and 30 km is

$$\approx W_R\left(\frac{6400}{l_{20}}\right)\left[\exp\left(\frac{30-20}{6.4}\right)-1\right]$$
$$\approx 20\ \text{kJ},$$

where W_R is the energy in 10^{16}, 1 MeV electrons $\simeq 1.6$ kJ. This number has to be significantly smaller than the energy of the EMP. We estimate this energy in two different ways.

In order to estimate the total energy W_{EMP} carried by the EMP from the lightning, we assume that a charge of 50 C is released in a positive cloud-to-ground discharge (+CG). This number comes from observations.[18] If we recall that the potential difference between the cloud base and ground is of the order of 10^8 V,[19] and that about 0.01% of the total lightning discharge is converted into the EMP energy; the total energy of the pulse can be estimated to be 500 kJ.

An alternate estimate of W_{EMP} by using the same observations[18] relies on interpolation of the EMP fields observed at 400–500 km from the source, to a smaller distance of 20 km from the source. The source is a +CG discharge located at around 5 km altitude. For a typical +CG discharge of 50 C, which lasts 1 ms, we obtain that at 20 km from the source $E_0 \sim 600$ V/m. This corresponds to the power density $P=\varepsilon_0 c E_0^2/2 \simeq 530\ \text{W/m}^2$. For a filament cross section of $S=1\ \text{km}^2$ the energy is

$$W_{\text{EMP}}=P\, S\, \Delta t \simeq 530\ \text{kJ}, \tag{24}$$

which is consistent with the earlier estimate.

These estimates imply that about 4% of the energy in the EMP is used in sustaining the runaway beam of 10^{16} electrons. This runaway beam should readily reach the height of 30 km where it creates the gamma ray flash as the observable bremsstrahlung process. Thus based on the various considerations and consistency checks, the proposed scenario is a viable one.

V. CONCLUSION

In this paper we have discussed a novel scenario for the creation of gamma ray flashes. During a typical lightning discharge, a small fraction of the energy of the 500 kJ of EMP generated can sustain a population of 10^{16} energetic electrons (~1 MeV), which can be transported in self-focused whistler wave ducts to a height of about 30 km. At this height these energetic electrons can give rise to the 1 ms gamma ray flash by the process of bremsstrahlung and those flashes can escape from the atmosphere into space. It is only through this whistler-medicated, self-focusing instability that the energetic electrons can be delivered to the desired heights. The characteristic time scale of the gamma ray flash is also of the order of the time scale for the lightning flash. The observations of whistlers during lightning flashes is well documented.[19] Thus the proposed mechanism brings together a series of naturally occurring events to provide a viable transport mechanism for the energetic electrons to the 30–35 km height for conversion into gamma rays.

[1] G. J. Fishman, P. N. Bhat, R. Mallozzi *et al.*, Science **264**, 1313 (1994).
[2] A. V. Gurevich, G. M. Milikh, and R. Roussel-Dupre, Phys. Lett. A **165**, 463 (1992).
[3] R. Roussel-Dupre, A. V. Gurevich, T. Tunnel, and G. M. Milikh, Phys. Rev. E **49**, 2257 (1994).
[4] T. F. Bell, V. P. Pasko, and U. S. Inan, Geophys. Res. Lett. **22**, 2127 (1995).
[5] A. V. Gurevich, J. A. Valdivia, G. M. Milikh, and K. Papadopoulos, Radio Sci. **31**, 1541 (1996).
[6] V. Gurevich, G. M. Milikh, and R. Roussel-Dupre, Phys. Lett. A **187**, 197 (1994).
[7] G. Lehtinen, T. F. Bell, and U. S. Inan, J. Geophys. Res., [Atmos.] **104A**, 24699 (1999).
[8] E. M. D. Symbalisty, R. A. Roussel-Dupre, and V. A. Yukhimuk, IEEE Trans. Plasma Sci. **26**, 1575 (1998).
[9] S. P. Slinker, R. F. Hubbard, and M. Lampe, J. Appl. Phys. **62**, 1171 (1987).
[10] A. G. Engelhardt, A. V. Phelps, and C. G. Risk, Phys. Rev. **135**, A1566 (1964).
[11] R. D. Hake, Jr. and A. V. Phelps, Phys. Rev. **158**, 70 (1967).
[12] C. R. Menyuk, A. T. Drobot, K. Papadopoulos, and H. Karimabadi, Phys. Rev. Lett. **58**, 2071 (1987).
[13] G. Milikh and J. A. Valdivia, Geophys. Res. Lett. **26**, 525 (1999).
[14] R. L. Stenzel, J. M. Urrutia, and C. L. Rousculp, J. Phys. IV **5**, 61 (1995).
[15] T. C. Marshall, M. Stolzenburg, and W. D. Rust, J. Geophys. Res., [Atmos.] **101**, 6979 (1996).
[16] T. Marshall and W. D. Rust, J. Geophys. Res., [Atmos.] **96**, 297 (1991).
[17] R. J. Nemiroff, J. T. Bonnel, and J. P. Norris, J. Geophys. Res., [Atmos.] **102**, 9659 (1997).
[18] C. Gomes and B. Cooray, J. Atmos. Sol.-Terr. Phys. **60**, 693 (1998).
[19] M. A. Uman, *The Lightning Discharge* (Academic, Orlando, 1987).